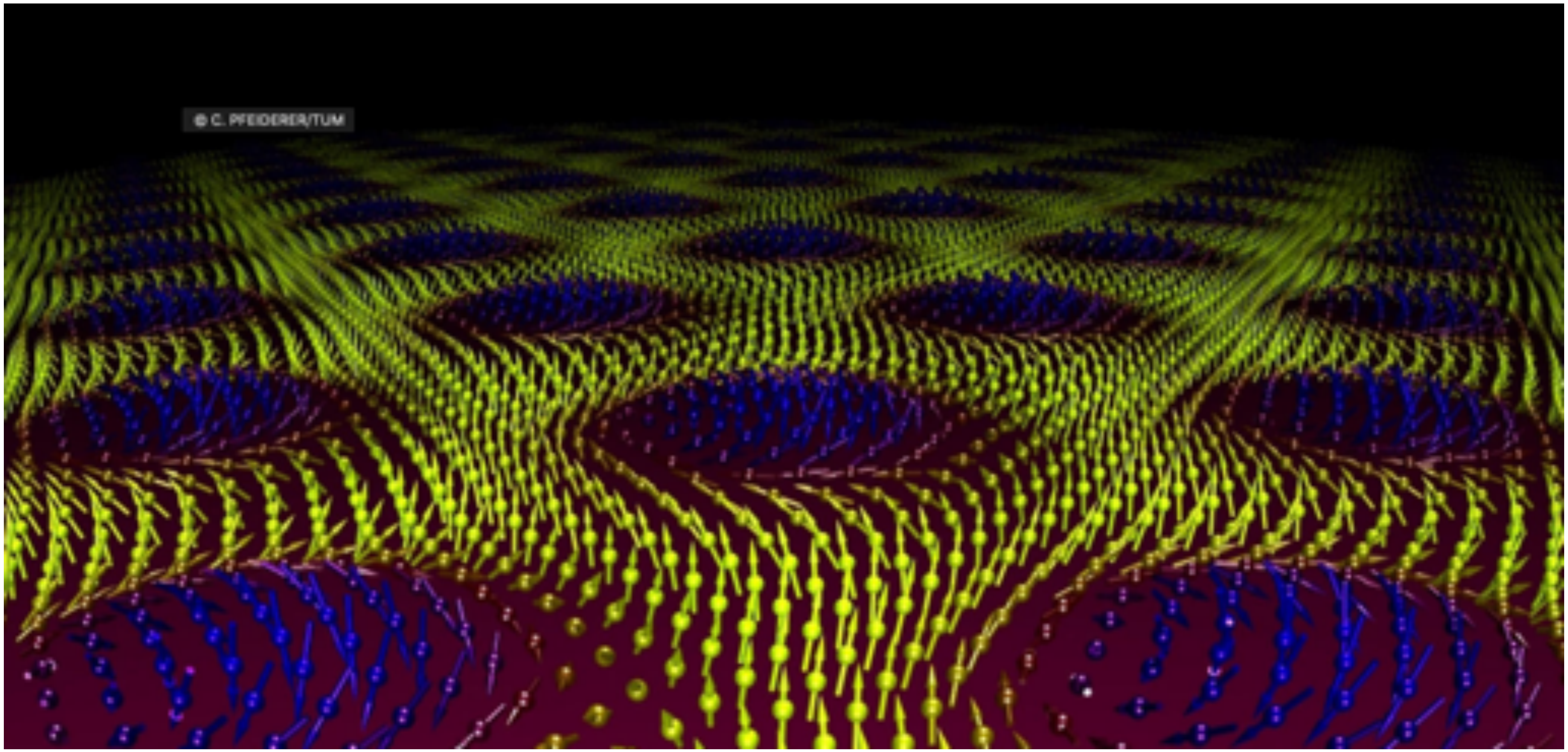
Holography of the spin current

Spin and hydrodynamics workshop,
8.10.2020

Umut Gürsoy
ITP, Utrecht University

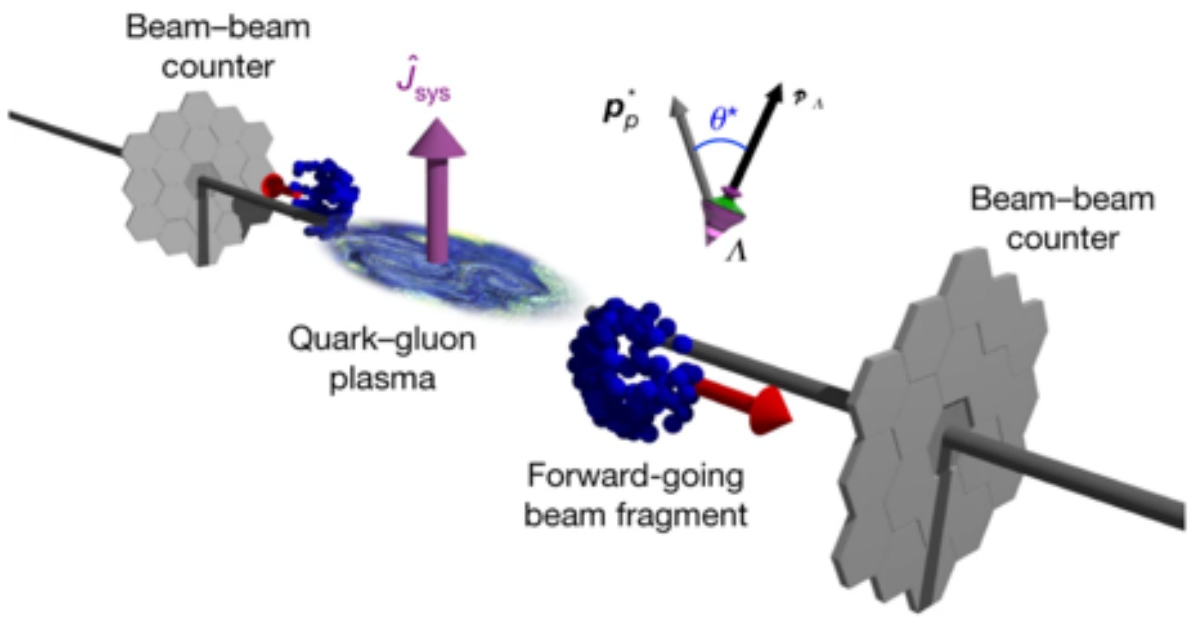
Based on: D. Gallegos, UG, arXiv:2004.05148

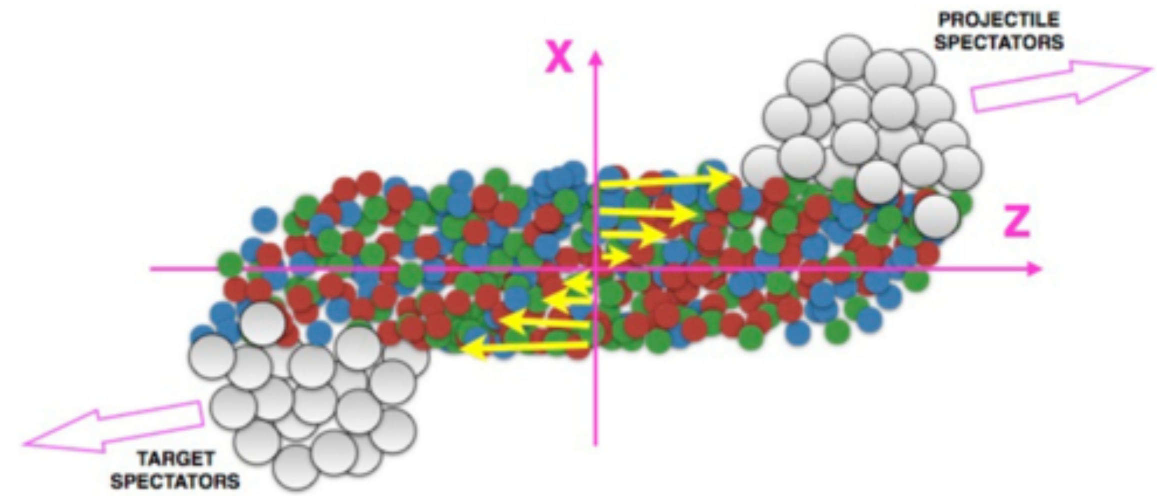
Spin transport

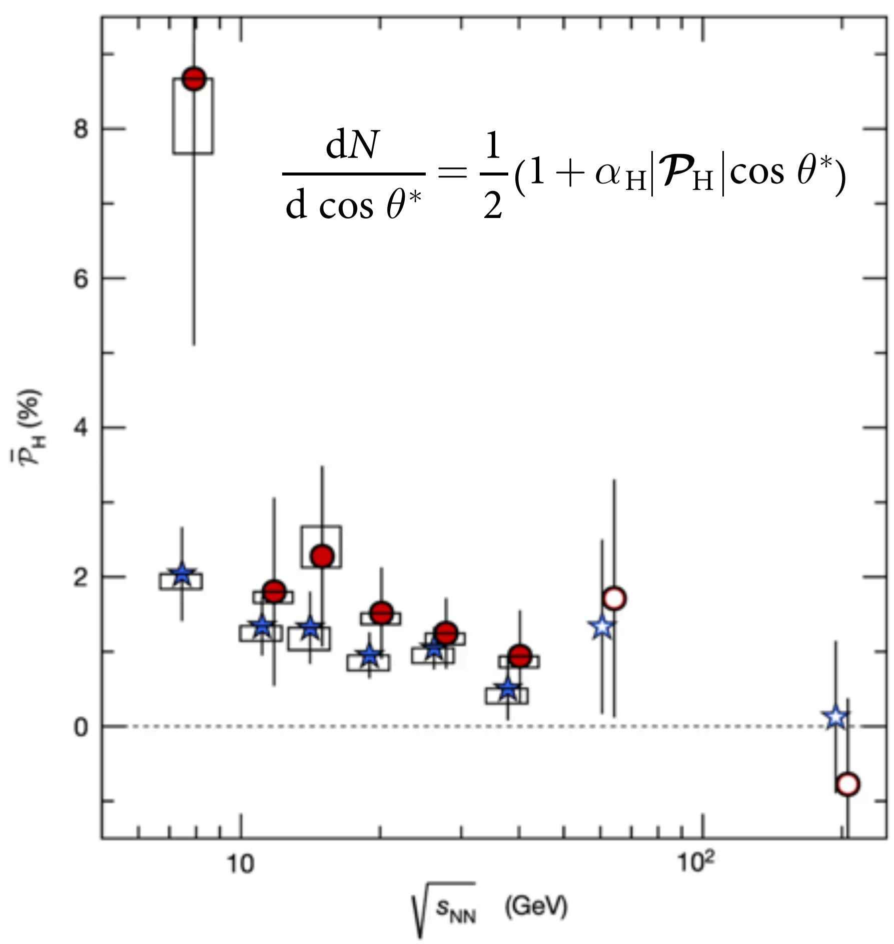


- Spintronics, liquid spintronics
- Graphene, Dirac/Weyl semimetals
- Quark gluon plasma
- …

Quark gluon plasma





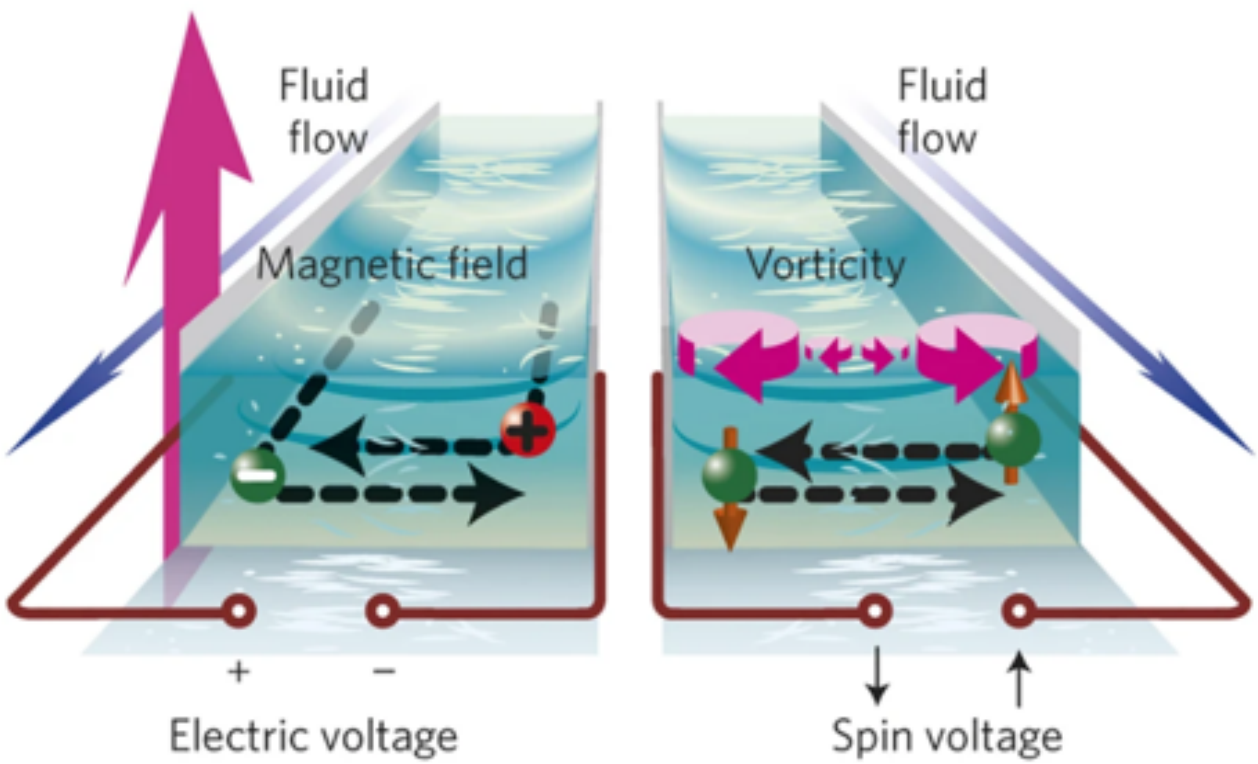


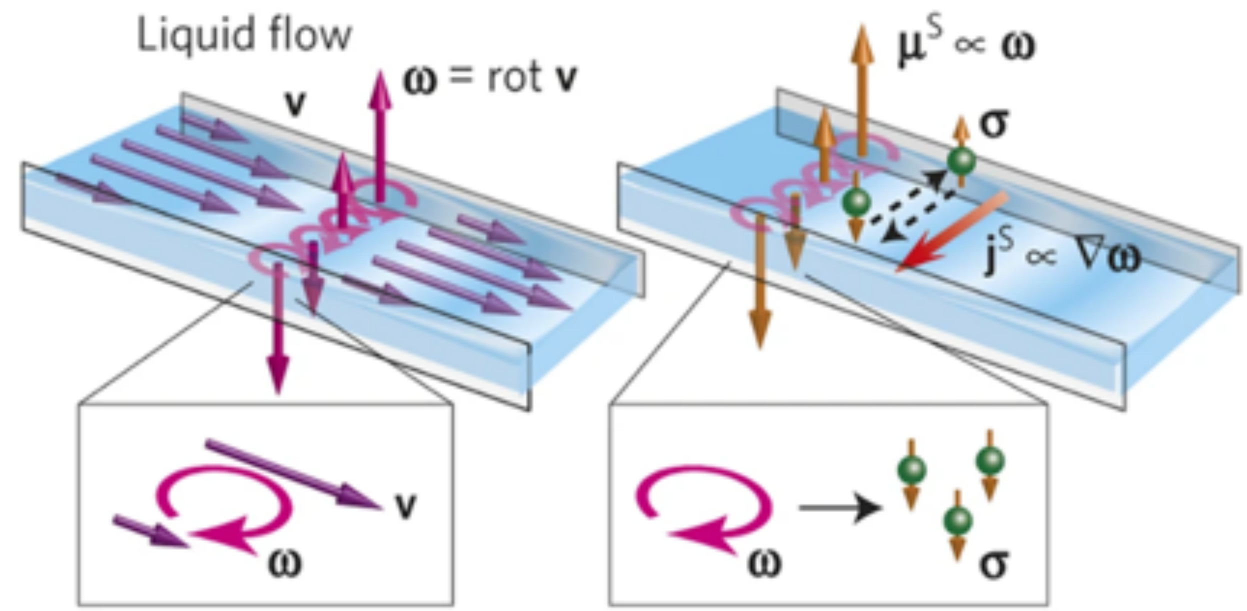
Global hyperon polarization at RHIC
by spin-orbit coupling $\vec{S} \cdot \vec{J}$

QGP: most vortical fluid: $\omega \sim 10^{22}\ s^{-1}$

Z.T. Liang, X.N. Wang ’05
Becattini, Karpenko, Lisa, Upsal, Voloshin ’17
STAR collaboration ‘19

Liquid spintronics





R. Takahashi et al. ’16

Generation of spin density by vorticity in liquid metals (Hg, GaInSn)

$\mu_s \propto \omega, J_s \propto \nabla\omega$

Hydrodynamics with spin current

- All possible sources of spin density/current
- Relation of spin flow to energy/charge/chirality flow
- Classification of spin transport coefficients

Hydrodynamics with spin current

- All possible sources of spin density/current
- Relation of spin flow to energy/charge/chirality flow
- Classification of spin transport coefficients

$\Longrightarrow$ construction of hydrodynamics with spin

$$T_{\mu\nu} \qquad\qquad S^{\lambda}_{\mu\nu}$$

Spin in effective action

Consider quantum field in a nontrivial Lorentz representation

$$e^{iW[e,\omega]} = \int D\Psi e^{iI[e,\omega,\Psi]}$$

Variations define the energy-momentum and spin current

$$T^{\mu\nu} = \frac{\delta W}{\delta e^a_\mu} e^\nu_a, \qquad S^\lambda_{ab} = \frac{\delta W}{\delta \omega^{ab}_\lambda}$$

Spin in effective action

Consider quantum field in a nontrivial Lorentz representation

$$e^{iW[e,\omega]} = \int D\Psi e^{iI[e,\omega,\Psi]}$$

Variations define the energy-momentum and spin current

$$T^{\mu\nu} = \frac{\delta W}{\delta e^a_\mu} e^\nu_a, \qquad S^\lambda_{ab} = \frac{\delta W}{\delta \omega^{ab}_\lambda}$$

Dependent in the absence of torsion:

$$T^a = de^a + \omega^a_b \wedge e^b$$

$\Longrightarrow$ spin current unambiguous when sourced by torsion

Spin in effective action

Consider quantum field in a nontrivial Lorentz representation

$$e^{iW[e,\omega]} = \int D\Psi e^{iI[e,\omega,\Psi]}$$

Variations define the energy-momentum and spin current

$$T^{\mu\nu} = \frac{\delta W}{\delta e^a_\mu} e^\nu_a, \qquad S^\lambda_{ab} = \frac{\delta W}{\delta \omega^{ab}_\lambda}$$

Dependent in the absence of torsion:

$$T^a = de^a + \omega^a_b \wedge e^b$$

$\Longrightarrow$ spin current unambiguous when sourced by torsion

Easier to formulate in terms of contorsion:

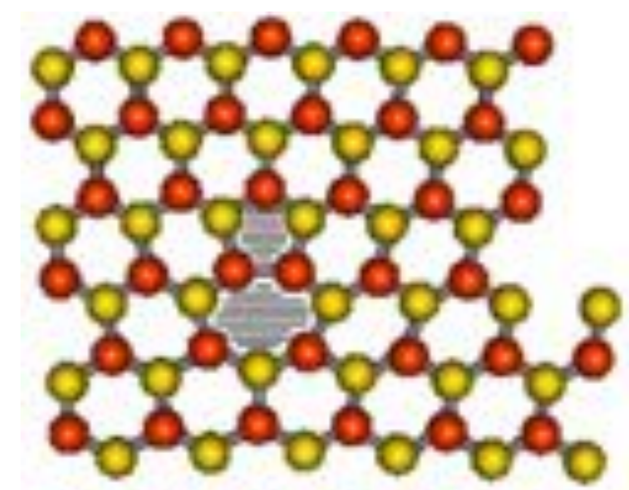
$$\omega^{ab}_\mu = \mathring{\omega}^{ab}_\mu + K^{ab}_\mu, \qquad \mathring{\omega} \sim \partial e$$

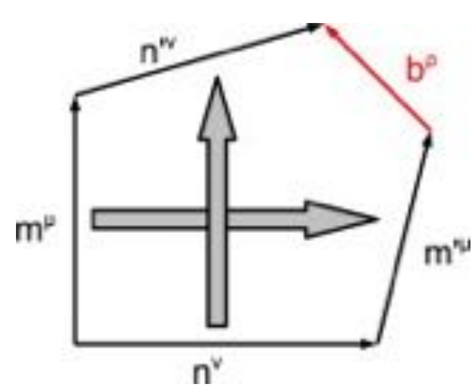
Lowest order source of spin flow in flat space e = 1

Torsion: lowest order source of spin flow

Kondo ’52; Katanaev, Volovich ’92; Furtado, Moraes ’99; Mesaros, Sadri, Zaanen ’09;
de Juan, Cortijo, Vozmediano ‘10

Example:
dislocations in graphene





$$m'^{\lambda} - n'^{\lambda} = \left[\Gamma^{\lambda}_{\mu\nu} - \Gamma^{\lambda}_{\nu\mu}\right] m^{\mu} n^{\nu} = T^{\lambda}_{\nu\mu} m^{\mu} n^{\nu}$$

Hydrodynamic equations

Require invariance of W[e,ω] under

1. Diffeomorphisms

$$\delta_\xi e^a = \mathcal{L}_\xi e^a \,, \qquad \delta_\xi \omega^{ab} = \mathcal{L}_\xi \omega^{ab}$$

2. Local Lorentz transformations

$$\delta_\lambda e^a = -\lambda^a{}_b e^b \,, \qquad \delta_\lambda \omega^{ab} = D\lambda^{ab}$$

Hydrodynamic equations

Require invariance of W[e,ω] under

1. Diffeomorphisms

$$\delta_\xi e^a = \mathcal{L}_\xi e^a \,, \qquad \delta_\xi \omega^{ab} = \mathcal{L}_\xi \omega^{ab}$$

2. Local Lorentz transformations

$$\delta_\lambda e^a = -\lambda^a{}_b e^b \,, \qquad \delta_\lambda \omega^{ab} = D\lambda^{ab}$$

Relativistic hydrodynamics with spin current

$$\mathring{\nabla}_\mu T^{\mu\nu} = R^{\rho\sigma\nu\lambda} S_{\lambda\rho\sigma} - T_{[\rho\sigma]} K^{\rho\sigma\nu} \,,$$ analogous to EM work j · E

$$\mathring{\nabla}_\lambda S^\lambda{}_{\mu\nu} = T_{[\mu\nu]} - 2S^\lambda{}_{\rho[\mu} K^\rho{}_{\nu]\lambda}$$ antisymm. EMT generates S

Classification of torsion sources

Lorentz invariant decomposition: $\omega^{ab} = \omega^{ab}_{\text{vector}} + \omega^{ab}_{\text{axial}} + \omega^{ab}_{\text{tensor}}$

$$(\omega^{ab}_{\text{vector}})_\mu = \delta^a_\mu \tilde{V}^b - \delta^b_\mu \tilde{V}^a \qquad (\omega^{ab}_{\text{axial}})_\mu = \epsilon^{abc}{}_d \, \tilde{A}_c \, \delta^d_\mu$$

In hydro with flow velocity u^μ further decompose: Gallegos, UG ,'20

$$\omega^{ab}_\beta = e^a_\mu e^b_\nu \Big[-\epsilon^{\mu\nu}{}_{\rho\sigma} u^\rho \left(\mu_A \delta^\sigma_\beta + \mathcal{C}^\sigma_\beta - \mathcal{A}^\sigma_1 u_\beta \right) + 2 \left(\mu_V u^{[\mu} - \mathcal{V}^{[\mu}_2 \right) \Delta^{\nu]}_\beta$$
$$+ 2u^{[\mu} \epsilon^{\nu]}{}_{\rho\sigma\beta} u^\rho \mathcal{A}^\sigma_2 - 2u^{[\mu} \left(\mathcal{V}^{\nu]}_1 u_\beta + \mathcal{H}^{\nu]}_\beta \right) \Big]$$

1 scalar, 1 pseudo-scalar, 2 transverse vectors, 2 axial transverse vectors, 2 traceless-transverse symmetric tensors

Constitutive relations

Energy-momentum tensor:

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu - p\Delta^{\mu\nu} + \underbrace{\bar{q}^\mu u^\nu + u^\mu q^\nu}_{\text{heat currents}} + \underbrace{\pi^{\mu\nu}}_{\substack{\text{symmetric} \\ \ni \text{ shear}}} + \underbrace{\tau^{\mu\nu}}_{\substack{\text{intrinsic} \\ \text{torque}}}$$

Spin current:

$$\begin{aligned} S^{\lambda\mu\nu} = {} & u^\lambda \left(u^\nu n_V^\mu - u^\mu n_V^\nu + \epsilon^{\mu\nu}{}_{\rho\sigma} u^\rho n_A^\sigma \right) + \rho_A \epsilon^{\lambda\mu\nu\rho} u_\rho + \rho_V \left(\Delta^{\lambda\nu} u^\mu - \Delta^{\lambda\mu} u^\nu \right) \\ & + N^{\lambda\mu} u^\nu - N^{\lambda\nu} u^\mu - \epsilon^{\mu\nu\rho\sigma} u_\rho \bar{N}_\sigma^\lambda + \Delta^{\lambda\nu} \bar{n}_V^\mu - \Delta^{\lambda\mu} \bar{n}_V^\nu \\ & + \epsilon^{\lambda\mu}{}_{\rho\sigma} u^\nu u^\rho \bar{n}_A^\sigma - \epsilon^{\lambda\nu}{}_{\rho\sigma} u^\mu u^\rho \bar{n}_A^\sigma \end{aligned}$$

Gallegos, UG ,'20

Constitutive relations

Energy-momentum tensor:

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu - p\Delta^{\mu\nu} + \underbrace{\bar{q}^\mu u^\nu + u^\mu q^\nu}_{\text{heat currents}} + \underbrace{\pi^{\mu\nu}}_{\text{symmetric} \ni \text{shear}} + \underbrace{\tau^{\mu\nu}}_{\text{intrinsic torque}}$$

Spin current:

$$\begin{aligned} S^{\lambda\mu\nu} = {} & u^\lambda \left(u^\nu n_V^\mu - u^\mu n_V^\nu + \epsilon^{\mu\nu}{}_{\rho\sigma} u^\rho n_A^\sigma \right) + \rho_A \epsilon^{\lambda\mu\nu\rho} u_\rho + \rho_V \left(\Delta^{\lambda\nu} u^\mu - \Delta^{\lambda\mu} u^\nu \right) \\ & + N^{\lambda\mu} u^\nu - N^{\lambda\nu} u^\mu - \epsilon^{\mu\nu\rho\sigma} u_\rho \bar{N}_\sigma^\lambda + \Delta^{\lambda\nu} \bar{n}_V^\mu - \Delta^{\lambda\mu} \bar{n}_V^\nu \\ & + \epsilon^{\lambda\mu}{}_{\rho\sigma} u^\nu u^\rho \bar{n}_A^\sigma - \epsilon^{\lambda\nu}{}_{\rho\sigma} u^\mu u^\rho \bar{n}_A^\sigma \quad = u^\lambda \rho^{\mu\nu} + S_\perp^{\lambda\mu\nu} \end{aligned}$$

Gallegos, UG ,'20

Constitutive relations

Energy-momentum tensor:

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu - p\Delta^{\mu\nu} + \underbrace{\bar{q}^\mu u^\nu + u^\mu q^\nu}_{\text{heat currents}} + \underbrace{\pi^{\mu\nu}}_{\text{symmetric} \ni \text{shear}} + \underbrace{\tau^{\mu\nu}}_{\text{intrinsic torque}}$$

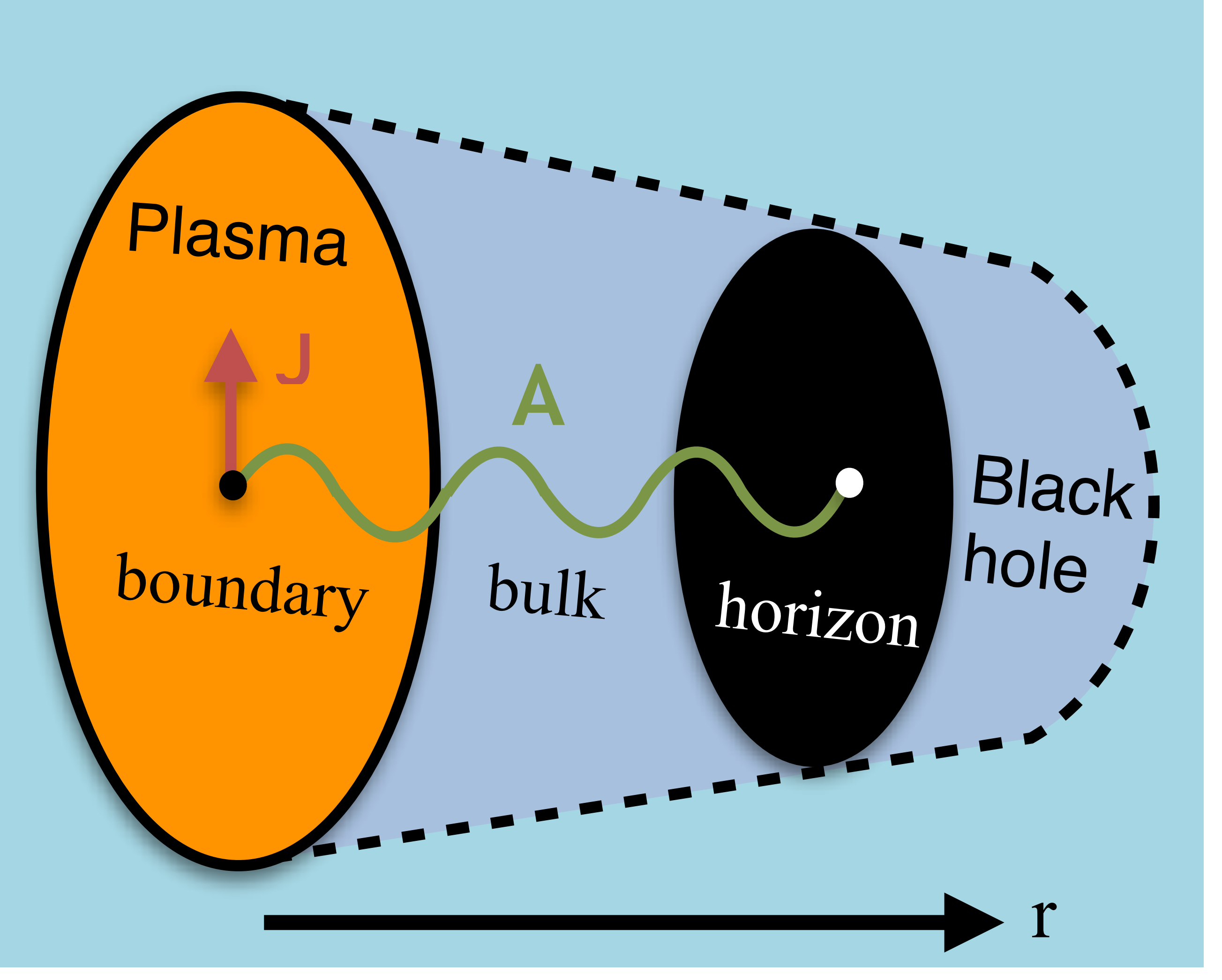
Spin current:

$$\begin{aligned} S^{\lambda\mu\nu} = &\, u^\lambda \left(u^\nu n_V^\mu - u^\mu n_V^\nu + \epsilon^{\mu\nu}{}_{\rho\sigma} u^\rho n_A^\sigma \right) + \rho_A \epsilon^{\lambda\mu\nu\rho} u_\rho + \rho_V \left(\Delta^{\lambda\nu} u^\mu - \Delta^{\lambda\mu} u^\nu \right) \\ &+ N^{\lambda\mu} u^\nu - N^{\lambda\nu} u^\mu - \epsilon^{\mu\nu\rho\sigma} u_\rho \bar{N}_\sigma^\lambda + \Delta^{\lambda\nu} \bar{n}_V^\mu - \Delta^{\lambda\mu} \bar{n}_V^\nu \\ &+ \epsilon^{\lambda\mu}{}_{\rho\sigma} u^\nu u^\rho \bar{n}_A^\sigma - \epsilon^{\lambda\nu}{}_{\rho\sigma} u^\mu u^\rho \bar{n}_A^\sigma \quad = u^\lambda \rho^{\mu\nu} + S_\perp^{\lambda\mu\nu} \end{aligned}$$

Spin Source	μ_V	μ_A	$\mathcal{V}_1^\mu$	$\mathcal{A}_1^\mu$	$\mathcal{V}_2^\mu$	$\mathcal{A}_2^\mu$	$\mathcal{H}^{\mu\nu}$	$\mathcal{C}^{\mu\nu}$
Spin current	ρ_V	ρ_A	n_V^μ	n_A^μ	$\bar{n}_V^\mu$	$\bar{n}_A^\mu$	$N^{\mu\nu}$	$\bar{N}^{\mu\nu}$
Degrees of Freedom	1	1	3	3	3	3	5	5

Gallegos, UG ,'20

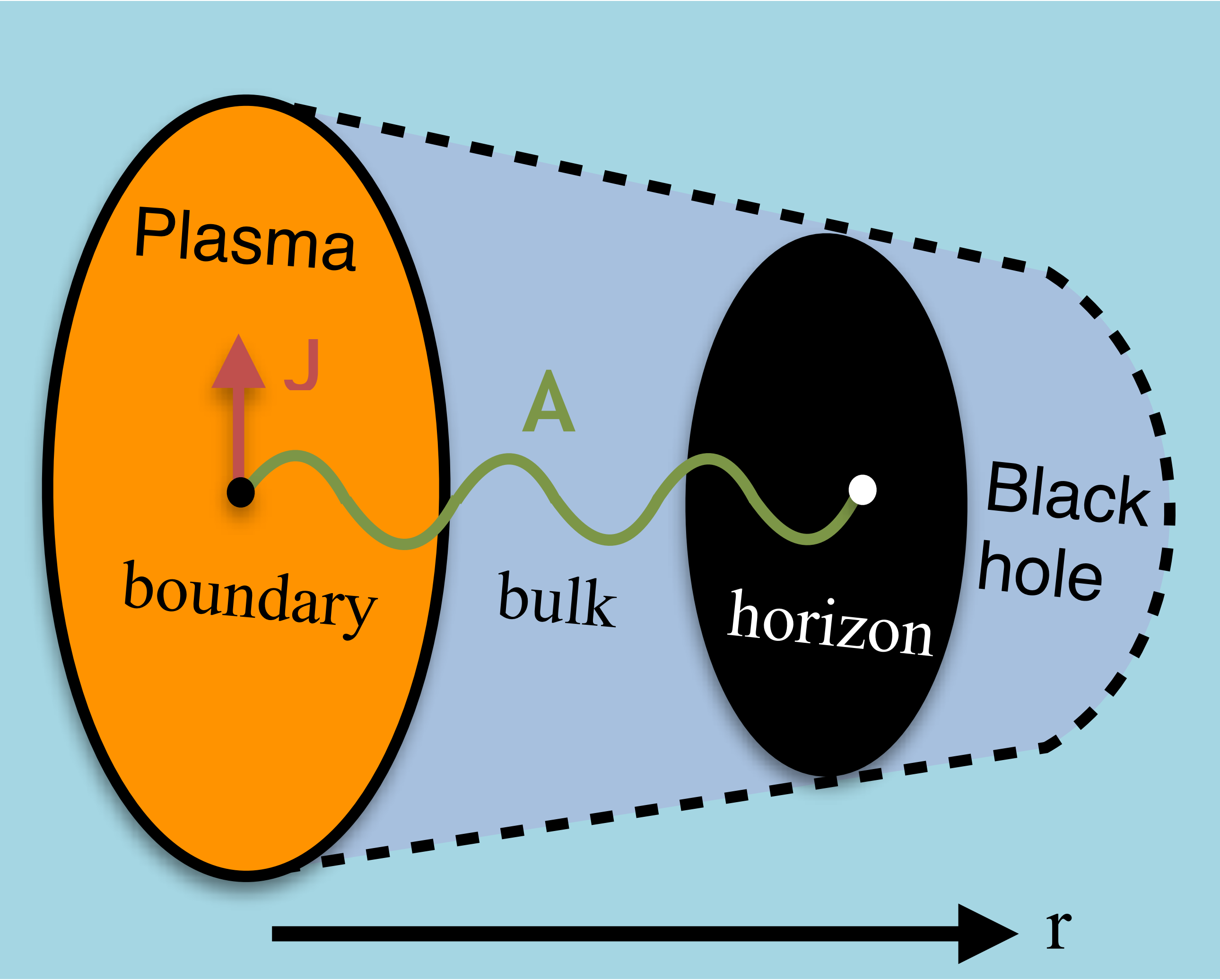
Holography



Strongly coupled large N gauge theory ⟺ semi-classical asymptotically AdS gravity

’t Hooft `93; Susskind `94; Maldacena `97; Witten `98; Gubser, Klebanov, Polyakov `98

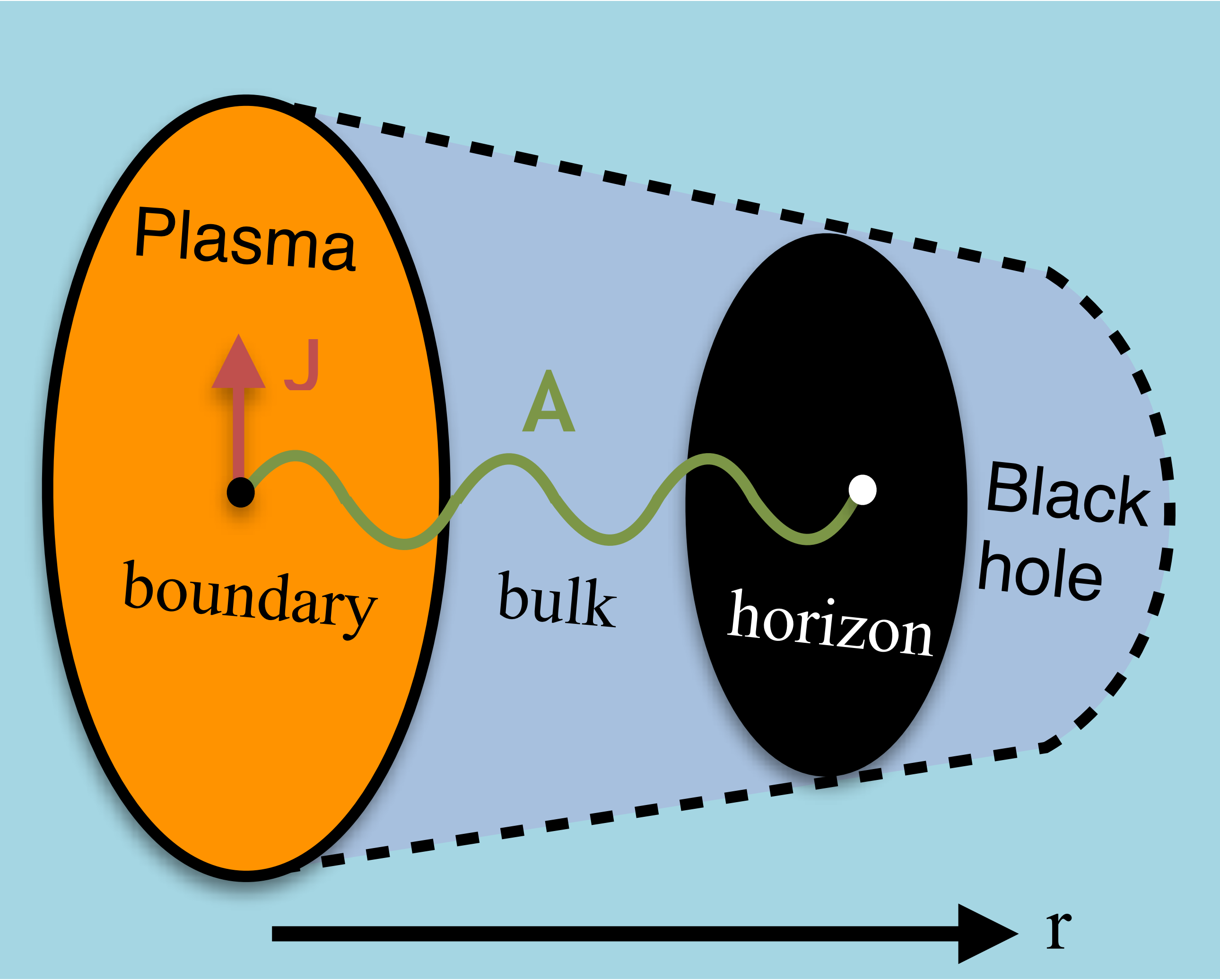
Holography



- Current J is mapped onto fluctuations on the horizon
- Generic properties of near horizon geometry

  $\Rightarrow$ universal transport e.g. $\eta/s = 1/4\pi$ Policastro, Son, Starinets `01
  Kovtun, Son, Starinets `04

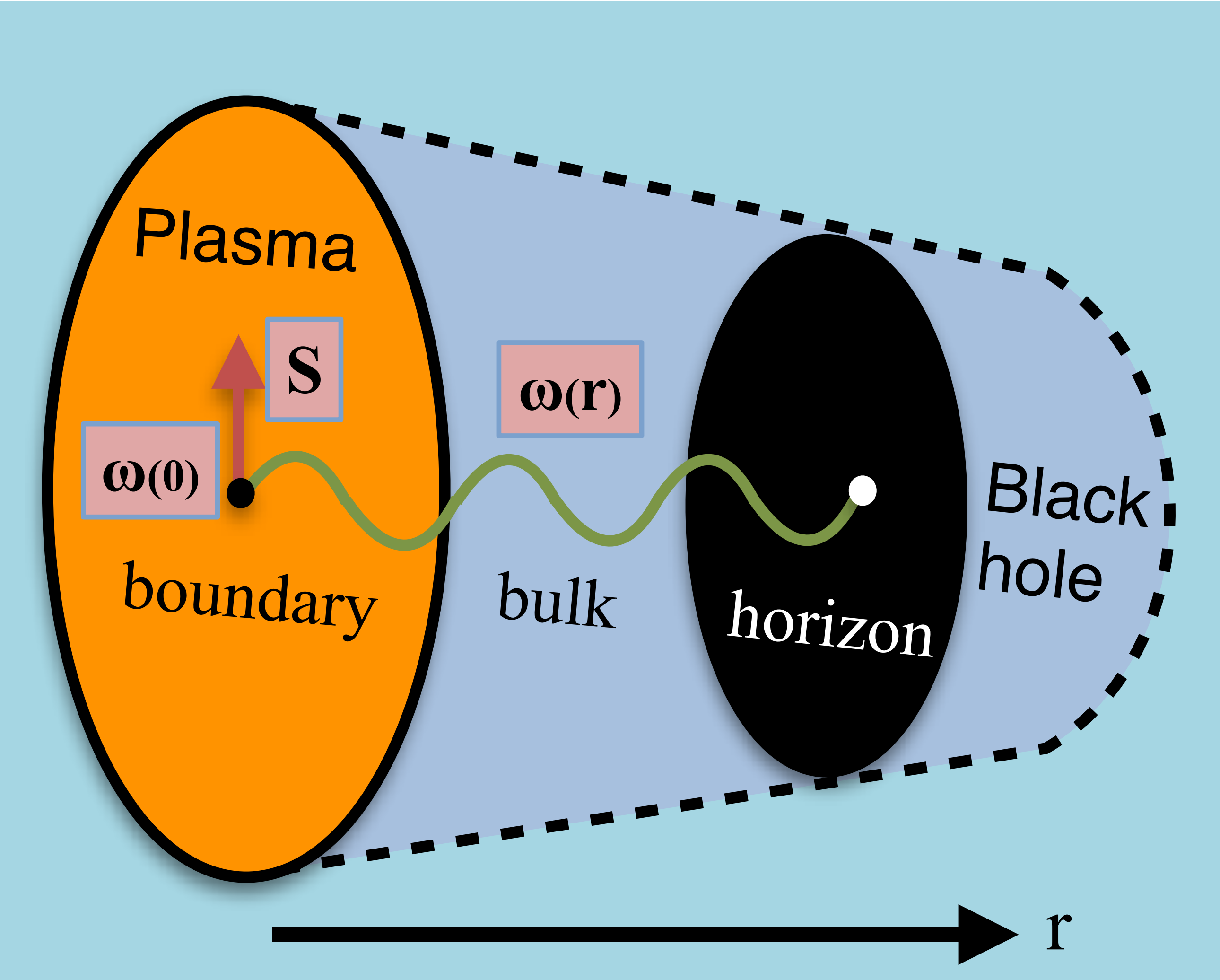
Holography



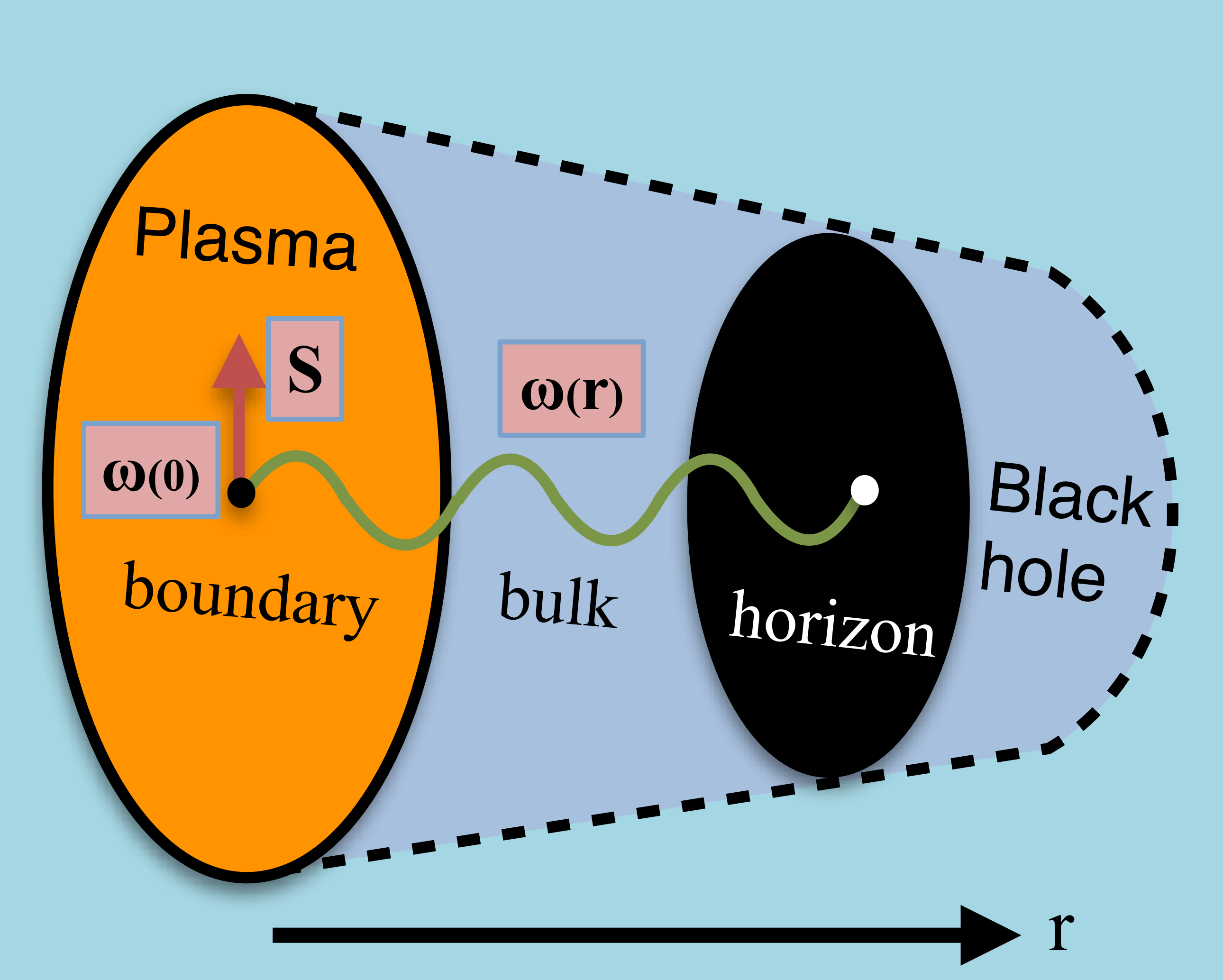
- Current J is mapped onto fluctuations on the horizon
- Generic properties of near horizon geometry
  $\Rightarrow$ universal transport e.g. $\eta/s = 1/4\pi$

Similar universality in spin transport?

Holography of spin flow



Holography of spin flow



- Need a first order formulation with independent e^a and ω^{ab}
- Practical to have analytic black-hole solutions

Lovelock-Chern-Simons AdS-gravity in 5D

Chamseddine ’89; Banados, Garay, Henneaux ’96; Zanelli 05;
Banados, Miskovic, Theisen ’06; Cvetkovic, Miskovic, Simic ‘17

$$S = \int \mathrm{Tr}\left\{\mathcal{F}\wedge\mathcal{F}\wedge\mathcal{A} - \frac{1}{2}\mathcal{F}\wedge\mathcal{A}\wedge\mathcal{A}\wedge\mathcal{A} + \frac{1}{10}\mathcal{A}\wedge\mathcal{A}\wedge\mathcal{A}\wedge\mathcal{A}\wedge\mathcal{A}\right\}$$

based on the G = SO(4,2) group algebra

Lovelock-Chern-Simons AdS-gravity in 5D

Chamseddine '89; Banados, Garay, Henneaux '96; Zanelli 05;
Banados, Miskovic, Theisen '06; Cvetkovic, Miskovic, Simic '17

$$S = \int \mathrm{Tr}\left\{\mathcal{F}\wedge\mathcal{F}\wedge\mathcal{A} - \frac{1}{2}\mathcal{F}\wedge\mathcal{A}\wedge\mathcal{A}\wedge\mathcal{A} + \frac{1}{10}\mathcal{A}\wedge\mathcal{A}\wedge\mathcal{A}\wedge\mathcal{A}\wedge\mathcal{A}\right\}$$

based on the G = SO(4,2) group algebra

Relation to Einstein-Cartan in 5D: $\mathcal{J}_{A6} = P_A, \mathcal{J}_{AB} = J_{AB}$

$$\mathcal{A} \equiv \hat{e}^A P_A + \frac{1}{2}\hat{\omega}^{AB} J_{AB}\,,$$
$$\mathcal{F} = \hat{T}^A P_A + \frac{1}{2}\left(R^{AB} + \hat{e}^A\hat{e}^B\right) J_{AB}$$

First order gravity in 5D:

$$S_{LCS} = \kappa \int_{M_5} \epsilon_{ABCDE}\left[\underbrace{\hat{R}^{AB}\hat{R}^{CD}\hat{e}^E}_{\text{Gauss-Bonnet with } \lambda_{GB}=1/4} + \frac{2}{3}\underbrace{\hat{R}^{AB}\hat{e}^C\hat{e}^D\hat{e}^E}_{\text{Ricci}} + \frac{1}{5}\underbrace{\hat{e}^A\hat{e}^B\hat{e}^C\hat{e}^D\hat{e}^E}_{\text{Cosmological Constant}}\right]$$

Solution I: single axial

Thermodynamics:

$$\varepsilon = 96\pi^4\kappa T^4\left(1+\frac{\mathcal{A}_2^2}{6\pi^2T^2}\right)\,,$$

$$p = -32\pi^4\kappa T^4\left(1+\frac{\mathcal{A}_2^2}{6\pi^2T^2}\right)\,,$$

$$\pi^{\mu\nu} = 16\pi^2\kappa T^2\left(\Delta^{\mu}_{(\rho}\Delta^{\nu}_{\sigma)}-\frac{1}{3}\Delta^{\mu\nu}\Delta_{\rho\sigma}\right)\mathcal{A}_2^{\rho}\mathcal{A}_2^{\sigma}$$

Spin current:

$$S^{\lambda\mu\nu} = -32\pi^2\kappa T^2\left(u^{\lambda}\epsilon^{\mu\nu\alpha\beta}+\epsilon^{\lambda\alpha\beta[\mu}u^{\nu]}\right)u_{\alpha}\mathcal{A}_{2\beta}$$

Solution I: single axial

Thermodynamics:

$$\varepsilon = 96\pi^4 \kappa T^4 \left(1 + \frac{\mathcal{A}_2^2}{6\pi^2 T^2}\right) ,$$

$$p = -32\pi^4 \kappa T^4 \left(1 + \frac{\mathcal{A}_2^2}{6\pi^2 T^2}\right) ,$$

$$\pi^{\mu\nu} = 16\pi^2 \kappa T^2 \left(\Delta^{\mu}_{(\rho}\Delta^{\nu}_{\sigma)} - \frac{1}{3}\Delta^{\mu\nu}\Delta_{\rho\sigma}\right) \mathcal{A}_2^{\rho}\mathcal{A}_2^{\sigma}$$

Spin current:

$$S^{\lambda\mu\nu} = -32\pi^2 \kappa T^2 \left(u^{\lambda}\epsilon^{\mu\nu\alpha\beta} + \epsilon^{\lambda\alpha\beta[\mu}u^{\nu]}\right) u_{\alpha}\mathcal{A}_{2\beta}$$

- Conformal fluid with axial vector spin source
- Non-trivial spin current *even for symmetric stress tensor!*

Solution II: Vector-axial-tensor

Thermodynamics:

$$p = -\epsilon/3 = -32\pi^4\kappa T^4 \left(1 + \frac{\mathcal{C}^{\alpha\beta}\mathcal{C}_{\alpha\beta} - 2\mathcal{V}_2^2 - 6\mu_A^2}{12\pi^2 T^2}\right)$$

$$q^\mu = -16\pi^2\kappa T^2 \left(\mathcal{C}^\mu_\alpha \mathcal{A}_1^\alpha - 2\mu_A \mathcal{A}_1^\mu + \epsilon^{\mu\alpha\rho\sigma} u_\alpha \mathcal{A}_{1\rho} \mathcal{V}_{2\sigma}\right) , \bar{q}^\mu = 0$$

$$\pi^{\mu\nu} = -16\pi^2\kappa T^2 \left(C^{\mu\alpha} C^\nu_\alpha + \mathcal{V}_2^\mu \mathcal{V}_2^\nu - \frac{1}{3}\left(\mathcal{C}^\alpha_\beta \mathcal{C}^\beta_\alpha + \mathcal{V}_2^2\right)\Delta^{\mu\nu} - \mu_A \mathcal{C}^{\mu\nu}\right)$$

$$\tau^{\mu\nu} = 16\pi^2\kappa T^2 u_\rho \mathcal{V}_2^\beta \epsilon^{\mu\nu\rho\sigma} \left(\mathcal{C}^\sigma_\beta + \mu_A \delta^\sigma_\beta\right)$$

Spin current:

$$S^{\lambda\mu\nu} = 32\pi^2\kappa T^2 \left[\mu_A \epsilon^{\lambda\mu\nu\alpha} u_\alpha - u_\alpha \epsilon^{\lambda\alpha\beta[\mu} \left(\mathcal{C}^{\nu]}_\beta + u^{\nu]} \mathcal{A}_{1\beta}\right) + \Delta^{\lambda[\mu} \mathcal{V}_2^{\nu]} + 2u^\lambda u^{[\mu} \mathcal{V}_2^{\nu]}\right]$$

Solution II: Vector-axial-tensor

Thermodynamics:

$$p = -\epsilon/3 = -32\pi^4\kappa T^4\left(1 + \frac{\mathcal{C}^{\alpha\beta}\mathcal{C}_{\alpha\beta} - 2\mathcal{V}_2^2 - 6\mu_A^2}{12\pi^2 T^2}\right)$$

$$q^\mu = -16\pi^2\kappa T^2\left(\mathcal{C}^\mu_\alpha\mathcal{A}_1^\alpha - 2\mu_A\mathcal{A}_1^\mu + \epsilon^{\mu\alpha\rho\sigma}u_\alpha\mathcal{A}_{1\rho}\mathcal{V}_{2\sigma}\right)\ , \bar{q}^\mu = 0$$

$$\pi^{\mu\nu} = -16\pi^2\kappa T^2\left(C^{\mu\alpha}C^\nu_\alpha + \mathcal{V}_2^\mu\mathcal{V}_2^\nu - \frac{1}{3}\left(\mathcal{C}^\alpha_\beta\mathcal{C}^\beta_\alpha + \mathcal{V}_2^2\right)\Delta^{\mu\nu} - \mu_A\mathcal{C}^{\mu\nu}\right)$$

$$\tau^{\mu\nu} = 16\pi^2\kappa T^2 u_\rho\mathcal{V}_2^\beta\epsilon^{\mu\nu\rho\sigma}\left(\mathcal{C}^\sigma_\beta + \mu_A\delta^\sigma_\beta\right)$$

Spin current:

$$S^{\lambda\mu\nu} = 32\pi^2\kappa T^2\left[\mu_A\epsilon^{\lambda\mu\nu\alpha}u_\alpha - u_\alpha\epsilon^{\lambda\alpha\beta[\mu}\left(\mathcal{C}^{\nu]}_\beta + u^{\nu]}\mathcal{A}_{1\beta}\right) + \Delta^{\lambda[\mu}\mathcal{V}_2^{\nu]} + 2u^\lambda u^{[\mu}\mathcal{V}_2^{\nu]}\right]$$

- Conformal fluid with spin sources
- Non-vanishing intrinsic torque and energy currents

Solutions at first order in ∂

Fluid-gravity correspondence: O(∂) transport to corrections to background

Bhattacharyya, Hubeny, Minwalla, Rangamani ‘08

Boosted black hole with space-time dependent boost parameter:

$$ds^2 = \frac{dr^2}{r^2 g^2} + r^2 \left[-f^2 u_\mu u_\nu + h^2 \Delta_{\mu\nu} + j \left(2hc_I \, u_\mu v_\nu^I - 2fb_I \, u_\mu v_\nu^I + h \, d_I t_{\mu\nu}^I \right) \right] dx^\mu dx^\nu$$

$\{c_I v_\mu^I, b_I v_\mu^I, d_I t_{\mu\nu}^I\}$ are linearly independent combinations of u^μ, T, ω_μ^{ab}

For simplicity we only consider $\mu_A \neq 0$ and $\mathcal{V}_2^\mu \neq 0$

Transport for $\mu_A \neq 0$

Energy transport:

$$\bar{q}^\mu = -\frac{64\pi^4\kappa T^4}{\mu_A}\omega^\mu\,,$$

$$q^\mu = -\frac{64\pi^4\kappa T^4}{\mu_A}\omega^\mu\left(1-\frac{\mu_A^2}{2\pi^2T^2}\right)\,,$$

$$\pi^{\mu\nu} = 0\,,$$

$$\tau^{\mu\nu} = -16\pi^2\kappa T^2\epsilon^{\mu\nu\alpha\beta}u_\alpha\partial_\beta\mu_A$$

Spin transport:

$$J_A^\mu = 32\pi^2\kappa T^2\left(\mu_A u^\mu + \omega^\mu\right)\,.$$

Transport for $\mu_A \neq 0$

Energy transport:

$$\bar{q}^\mu = -\frac{64\pi^4\kappa T^4}{\mu_A}\omega^\mu \,,$$

$$q^\mu = -\frac{64\pi^4\kappa T^4}{\mu_A}\omega^\mu\left(1-\frac{\mu_A^2}{2\pi^2T^2}\right)\,,$$

$$\pi^{\mu\nu} = 0\,,$$

$$\tau^{\mu\nu} = -16\pi^2\kappa T^2\epsilon^{\mu\nu\alpha\beta}u_\alpha\partial_\beta\mu_A$$

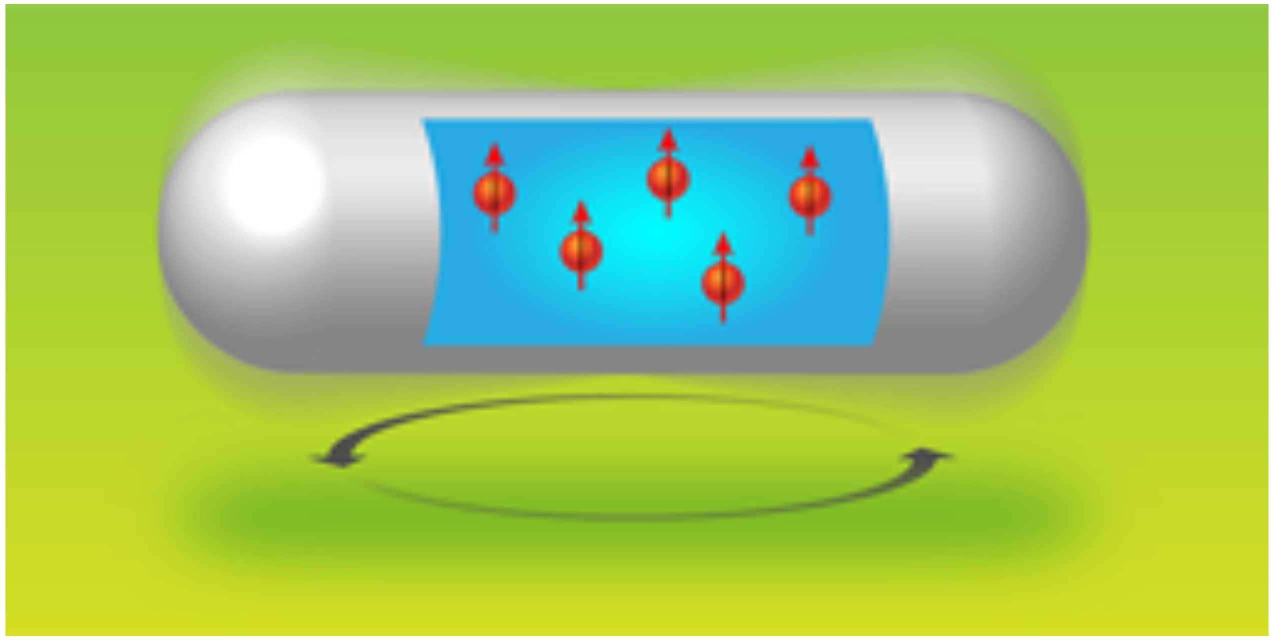
Spin transport:

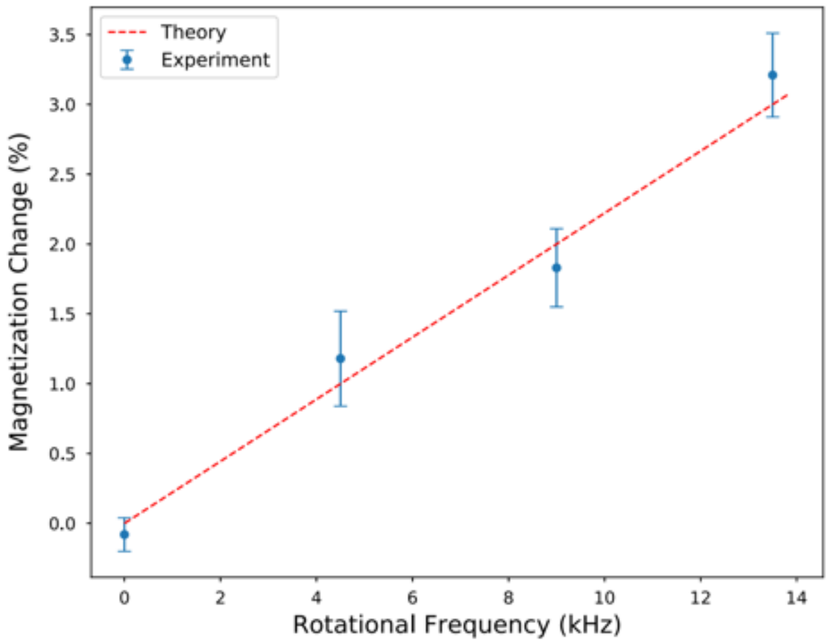
$$J_A^\mu = 32\pi^2\kappa T^2\left(\mu_A u^\mu + \omega^\mu\right)\,.$$

- Chiral vortical separation effect: *generation of chiral imbalance by vorticity*
- As well as "*chiral torsional effect*" Imaki and Yamamoto, '19
- Here also an intrinsic torque

The Barnett effect

Magnetization of an uncharged body by rotation S. Barnett, 1915





$$J_A^\mu = 32\pi^2 \kappa T^2 \left(\mu_A u^\mu + \omega^\mu\right) .$$

Nuclear Barnett effect

Arabgol and Sleator '19

Transport for $\mathcal{V}_2^\mu \neq 0$

New possibilities for spin transport:

$$J_V^\mu = \frac{32}{3}\pi^2 T^2 (\partial \cdot u) u^\mu \,,$$

$$\bar{J}_V^\mu = 32\pi^2 \kappa T^2 \mathcal{V}_2^\mu \,,$$

$$\bar{J}_A^\mu = 16\pi^2 \kappa T^2 \left(\eta^{\mu\nu} - \frac{\mathcal{V}_2^\mu \mathcal{V}_2^\nu}{\mathcal{V}_2^2} \right) \omega_\nu$$

- Spin current by sound waves ?!
- "Vortical-Hall spin current"

Summary

- Torsion is the lowest order source for the spin current
- Derived the hydro equations and constructed constitutive relations
- Lovelock-Chern-Simons provides a fruitful toy theory
- Holography helps identify novel spin transport and computes associated conductivities

Outlook

- A systematic study of relativistic spin transport:

  Entropy current, Onsager relations, etc.

  Montenegro, Tinti, Torrieri `17; Hattori et al. `19; Fukushima, Pu `20

- Analysis at weak coupling $\Longrightarrow$ universal transport coefficients?

- Derivation in more general holographic theories: e.g. supergravity

- Observable effects of the novel spin transport?

Back-up slides

Outline

- Definition of spin current
- Hydrodynamics with spin current and torsion
- Holographic description: first order gravity
- Lovelock-Chern-Simons theory
- Charges in LCS
- Analytic blackhole solutions
- Resulting energy and spin flow
- Discussion

Ambiguity in the spin current

Total angular momentum $$J^{\lambda\mu\nu} = x^{\mu}T^{\lambda\nu} - x^{\nu}T^{\lambda\mu} + S^{\lambda\mu\nu}$$

Conservation laws $$\partial_{\mu}T^{\mu\nu} = 0\,, \qquad \partial_{\lambda}J^{\lambda\mu\nu} = 0$$

Preserved by $$T'^{\mu\nu} = T^{\mu\nu} + \frac{1}{2}\partial_{\lambda}\left(\Phi^{\lambda\mu\nu} - \Phi^{\mu\lambda\nu} - \Phi^{\nu\lambda\mu}\right)\,,$$
$$S'^{\lambda\mu\nu} = S^{\lambda\mu\nu} - \Phi^{\lambda\mu\nu}$$

Belinfante-Rosenfeld $$\Phi^{\lambda\mu\nu} = S^{\lambda\mu\nu}$$

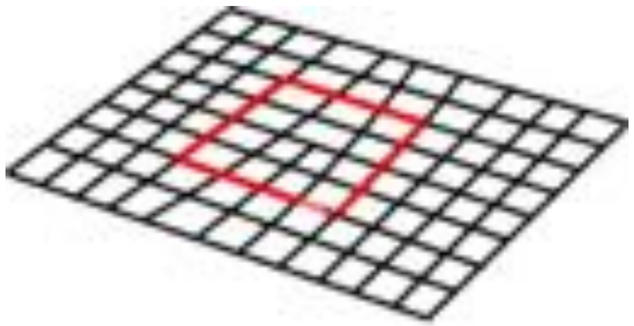
Removes spin current completely, renders energy-momentum tensor symmetric, compatible with GR

Classification of torsion sources

Assume flat 3+1D metric with nontrivial torsion $T^a_{\mu\nu} = (\omega^a_b)_\mu \wedge \delta^b_\nu$

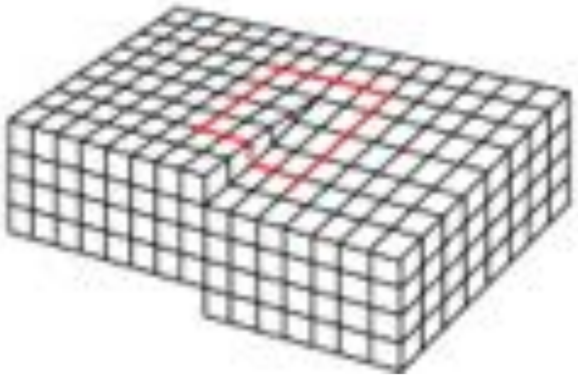
In 3D: ω_i^{ab} → loop plane → Burger's vector $\Longrightarrow$ 9 degrees of freedom

$i \parallel (ab)$



edge dislocation (6 d.o.f)

$i \perp (ab)$



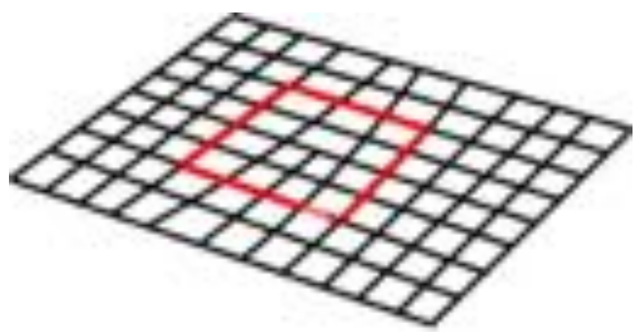
screw dislocation (3 d.o.f)

Classification of torsion sources

Assume flat 3+1D metric with nontrivial torsion $T^a_{\mu\nu} = (\omega^a_b)_\mu \wedge \delta^b_\nu$

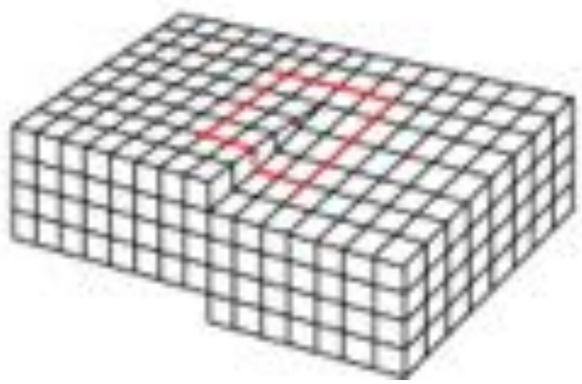
In 3D: ω_i^{ab} → loop plane, → Burger's vector $\Longrightarrow$ 9 degrees of freedom

$i \parallel (ab)$



edge dislocation (6 d.o.f)

$i \perp (ab)$



screw dislocation (3 d.o.f)

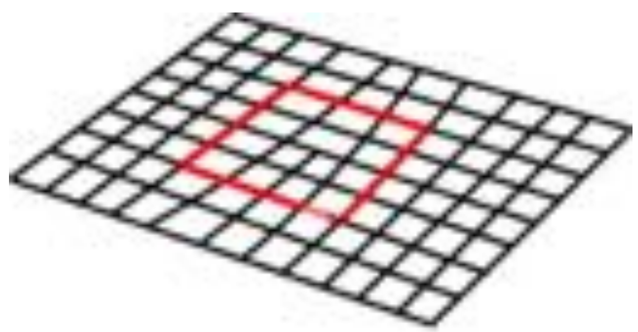
In 4D: ω_μ^{ab} $\Longrightarrow$ Wilson-edge (6 d.o.f.) + Wilson-screw (6 d.o.f) + Polyakov-edge (6 d.o.f) + Polyakov-screw (6 d.o.f)

Classification of torsion sources

Assume flat 3+1D metric with nontrivial torsion $T^a_{\mu\nu} = (\omega^a_b)_\mu \wedge \delta^b_\nu$

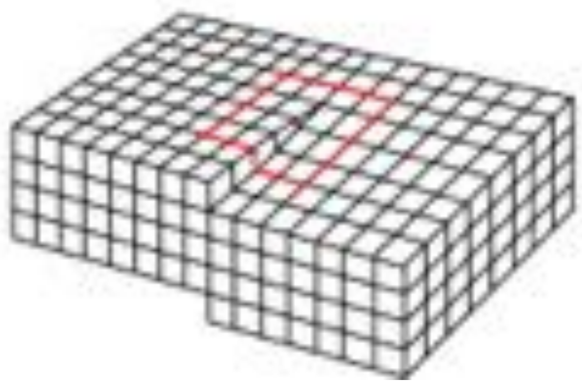
In 3D: ω_i^{ab} → loop plane, → Burger's vector $\Longrightarrow$ 9 degrees of freedom

$i \parallel (ab)$



edge dislocation (6 d.o.f)

$i \perp (ab)$



screw dislocation (3 d.o.f)

In 4D: ω_μ^{ab} $\Longrightarrow$ Wilson-edge (6 d.o.f.) + Wilson-screw (6 d.o.f) + Polyakov-edge (6 d.o.f) + Polyakov-screw (6 d.o.f)

Not a Lorentz invariant decomposition

Charges and thermodynamics

Holographic renormalization in first order gravity: Banados, Miskovic, Theisen '06

$$\beta\mathcal{F} = \lim_{\epsilon\to 0}\left[S_{\text{on-shell}}(\epsilon) - V(\epsilon)\right]$$

counterterm action

Entropy as a charge from the Lee-Wald formula at horizon: Gallegos, UG ,'20

$$\mathcal{S} = 4\pi\int_{M_3^h}\epsilon_{ABCDF}\left(\hat{R}^{CD}\hat{e}^F + \frac{1}{3}\hat{e}^C\hat{e}^D\hat{e}^F\right)n^{AB}\,, \qquad n^{AB}\equiv D^{[A}\xi^{B]}$$

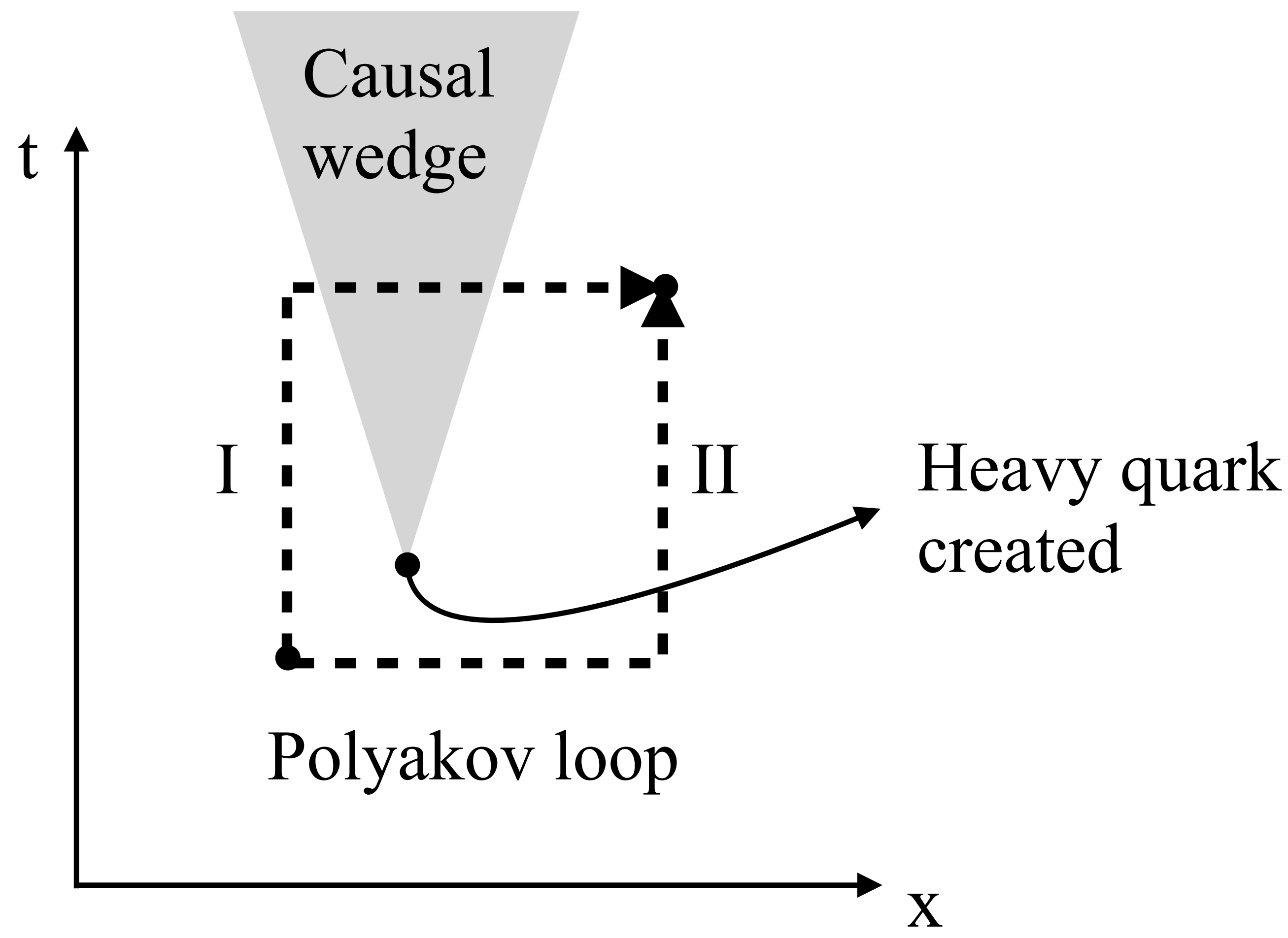
Total mass of the black hole:

$$M = M \equiv M_0 - \mu_I Q^I = \int_{M_3} d^3x\left[n_\mu\xi_\nu T^{\mu\nu} + \frac{1}{2}n_\lambda S^\lambda{}_{\mu\nu}\omega^{\mu\nu}{}_\alpha\xi^\alpha\right]$$

Satisfy the Smarr relation and the first-law:

$$\mathcal{F} = M_0 - T\mathcal{S} - \mu_I Q^I\,, \qquad d\mathcal{F} = -\mathcal{S}dT - Q^I d\mu_I\,,$$

Rocks in river



Route I ≠ route II not commute in a quantum liquid
with spontaneous creation of particle anti-particle pairs

Outlook

- Can we understand hyperon polarization in QGP by torsion?
- What are torsion defects in QGP?

Outlook

- Can we understand hyperon polarization in QGP by torsion?
- What are torsion defects in QGP?

Wild speculation: heavy quarks in a river of soft quarks

Counter action

$$V = 4\kappa\epsilon_{abcd}\int_{M_4}\left[(R^{ab}+e^a k^b)k^c e^d + \frac{\tilde{e}^a\tilde{e}^b\tilde{e}^c\tilde{e}^d}{6\epsilon^2} - \frac{\tilde{e}^a\tilde{e}^b\left(R^{cd}+\frac{4}{3}\tilde{e}^c\tilde{k}^d\right)}{2\epsilon}\right]$$

Finding solutions

Underlying gauge structure allows to find non-trivial analytic solutions:

- Introduce collective SO(4,2) indices: $\bar{A} = \{AB, A6\} \quad \text{with} \quad A = \{0,1,2,3,5\}$
- Equations of motion:

$$g_{\bar{A}\bar{B}\bar{C}}\epsilon^{\mu\nu\alpha\beta}\mathcal{F}^{\bar{B}}{}_{\mu\nu}\mathcal{F}^{\bar{C}}{}_{\alpha\beta} = 0\,, \quad \Longrightarrow \text{Ward identities i.e. hydro equations}$$

$$g_{\bar{A}\bar{B}\bar{C}}\epsilon^{\mu\nu\alpha\beta}\mathcal{F}^{\bar{B}}{}_{\mu\nu}\mathcal{F}^{\bar{C}}{}_{\alpha r} = 0 \quad \Longrightarrow \text{Can be solved analytically}$$

Finding solutions

Underlying gauge structure allows to find non-trivial analytic solutions:

- Introduce collective SO(4,2) indices: $\bar{A} = \{AB, A6\}$ with $A = \{0, 1, 2, 3, 5\}$

- Equations of motion:

$$g_{\bar{A}\bar{B}\bar{C}} \epsilon^{\mu\nu\alpha\beta} \mathcal{F}^{\bar{B}}{}_{\mu\nu} \mathcal{F}^{\bar{C}}{}_{\alpha\beta} = 0 \,, \quad \Longrightarrow \text{Ward identities i.e. hydro equations}$$

$$g_{\bar{A}\bar{B}\bar{C}} \epsilon^{\mu\nu\alpha\beta} \mathcal{F}^{\bar{B}}{}_{\mu\nu} \mathcal{F}^{\bar{C}}{}_{\alpha r} = 0 \quad \Longrightarrow \text{Can be solved analytically}$$

- Regularity of the Ricci scalar at the horizon $\Longrightarrow$ 2 sets of blackholes

1. Axial solution with $\mathcal{A}_2^\mu$
2. Scalar-vector-tensor solution with $\{\mathcal{C}^{\mu\nu}, \mathcal{V}_2^\mu, \mu_A, \mathcal{A}_1^\mu\}$

Gallegos, UG ,'20

Solution I

$$ds^2 = -\frac{dr^2}{r^2 g^2} + r^2 \left(-f^2 u_\mu u_\nu + h^2 \Delta_{\mu\nu}\right) dx^\mu dx^\nu \,,$$

$$f^2 = 1 - \frac{r_h^2}{r^2}\,,$$

$$g^2 = \left(1 - \frac{r_h^2}{r^2}\right)\left(1 - \frac{r_h^2 - 4\pi^2 T^2}{r^2}\right)$$

$$h^2 = 1 - \frac{r_h^2 - 4\pi^2 T^2}{r^2}\,,$$

$$T^a = r e^a_\mu u^\alpha \mathcal{A}_2^\beta \left(f \gamma^{\lambda\mu} \epsilon_{\alpha\beta\lambda\sigma} u_\rho - h u^\mu \epsilon_{\alpha\beta\rho\sigma}\right) dx^\rho \wedge dx^\sigma \,,$$

$$T^5 = 0\,,$$

Solution II

$$ds^2 = -\frac{dr^2}{r^2 g^2} + r^2 \left(-f^2 u_\mu u_\nu + h^2 \Delta_{\mu\nu}\right) dx^\mu dx^\nu$$

$$T^a = r h e^a_\mu \left[\epsilon^\mu{}_{\nu\rho\alpha} u^\nu \left(\mathcal{C}^\alpha_\sigma + \mathcal{A}^\alpha_1 u_\sigma - \mu_A \delta^\alpha_\sigma\right) - \mathcal{V}_{2\rho} \Delta^\mu_\sigma\right] dx^\rho \wedge dx^\sigma \,, \qquad T^5 = 0$$

$$f^2 = 1 - \frac{r_h^2}{r^2} \,, g^2 = \left(1 - \frac{r_h^2}{r^2}\right)\left(1 - \frac{r_h^2 - 4\pi^2 T^2}{r^2}\right) \,, h^2 = 1 - \frac{r_h^2 - 4\pi^2 T^2}{r^2}$$