Spin polarization in hydrodynamic calculations

Iurii KARPENKO

Faculty of Nuclear Sciences and Physical Engineering,
Czech Technical University in Prague

EUROPEAN UNION
European Structural and Investment Funds
Operational Programme Research,
Development and Education

MINISTRY OF EDUCATION,
YOUTH AND SPORTS

Theoretical ideas

1) Early ideas:

Initial angular momentum $\Rightarrow$ polarized quarks $\Rightarrow$ polarized hadrons

Z. T. Liang and X. N. Wang, Phys. Rev. Lett. 94, 102301 (2005)
Z. T. Liang and X. N. Wang, Phys. Lett. B 629, 20 (2005)

2) Later developments:

Initial angular momentum $\Rightarrow$ vortical fluid $\Rightarrow$ polarized hadrons

F. Becattini, V. Chandra, L. Del Zanna and E. Grossi, Annals Phys. 338, 32 (2013)

Theory side: polarization of fermions from the fluid

F. Becattini, V. Chandra, L. Del Zanna, E. Grossi, Ann. Phys. 338 (2013) 32

Also: Ren-hong Fang, Long-gang Pang, Qun Wang, Xin-nian Wang, Phys. Rev. C 94 (2016), 024904

Mechanism: spin-vorticity coupling at local thermodynamic equilibrium.

- Cooper-Frye prescription: $p^0 \frac{d^3N}{d^3p} = \int d\Sigma_\lambda p^\lambda \frac{1}{\exp\left(\frac{p\cdot u-\mu}{T}\right)\pm 1}$
- For the spin $1/2$ particles at the particlization surface: $\langle S^\mu(x,p)\rangle = \frac{1}{8m}(1-f(x,p))\varepsilon^{\mu\nu\rho\sigma}p_\sigma\partial_\nu\beta_\rho$, where $\beta_\mu = \frac{u_\mu}{T}$ is the inverse four-temperature field.

$$S^\mu(p) = \frac{\int d\Sigma_\lambda p^\lambda f(x,p)\langle S^\mu(x,p)\rangle}{\int d\Sigma_\lambda p^\lambda f(x,p)}$$

Polarization depends on the the thermal vorticity $\varpi_{\mu\nu} = -\frac{1}{2}(\partial_\mu\beta_\nu - \partial_\nu\beta_\mu)$.

- polarization is close or equal for particles and antiparticles
- caused not only by velocity, but also temperature gradients

Most of the calculations on the market are built as follows:

1) hydrodynamic calculations (**this talk**)

Hydrodynamic evolution → Freeze-out → Formula from above

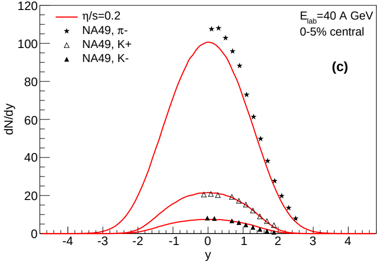
2) Microscopic(transport,cascade) model calculations (**Q. Wang's talk next Monday**)

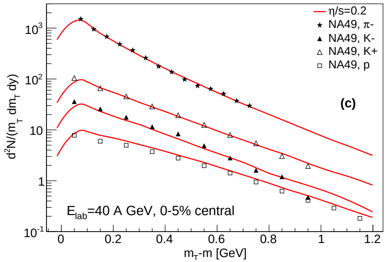
Parton/hadron cascade → Coarse-graining → Formula from above

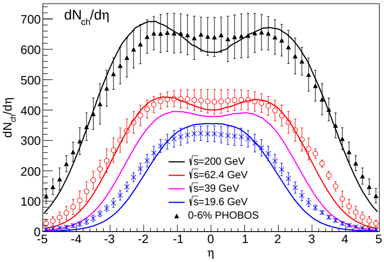
Hydrodynamic calculations

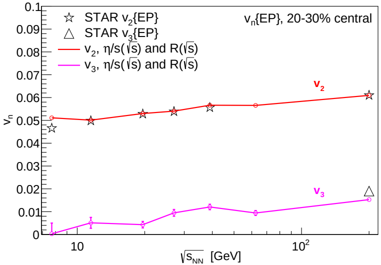
- **PICR**: Y.L. Xie, D.J. Wang, L.P. Csernai
  $\sqrt{s_{\rm NN}} = 7.7 \ldots 200$ GeV
- **ECHO-QGP**: F. Becattini, G. Inghirami, ...
  $\sqrt{s_{\rm NN}} = 200$ GeV
- **UrQMD+vHLLE, Glauber+vHLLE**: Iu. Karpenko, F. Becattini
  $\sqrt{s_{\rm NN}} = 7.7 \ldots 2760$ GeV
- **AMPT+CLVisc**: Long-Gang Pang, Hannah Elfner(Petersen), Qun Wang
  $\sqrt{s_{\rm NN}} = 62.4 \ldots 2760$ GeV
- **3FD** (3-fluid dynamics): Yu. Ivanov
  $\sqrt{s_{\rm NN}} = 4 \ldots 40$ GeV

All of the models had been tuned for the basic hadronic observables









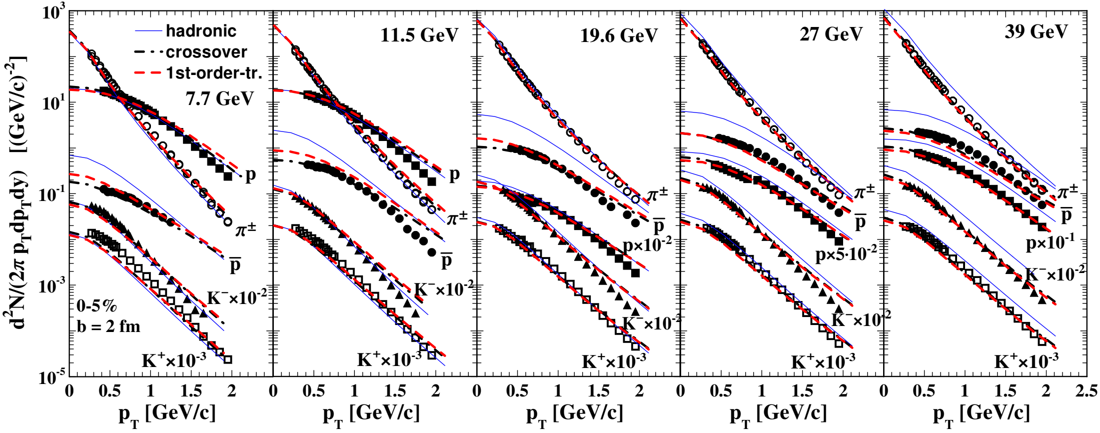
UrQMD+vHLLE

IK, Huovinen, Petersen, Bleicher, Phys.Rev. C91 (2015) no.6, 064901

There is no special tuning for the spin polarization observable!

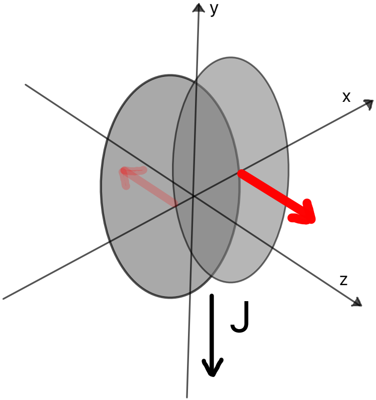
All of the models had been tuned for the basic hadronic observables

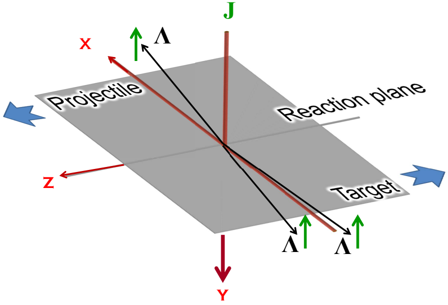
3-Fluid Hydro, Yu. B. Ivanov, A. A. Soldatov, Phys. Rev. C 97, 024908 (2018)



I. Global spin polarization

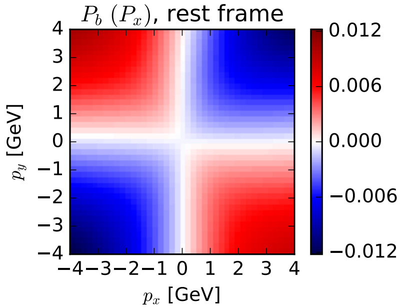
An expectation: all Λ are on average polarized along the angular momentum of the system.

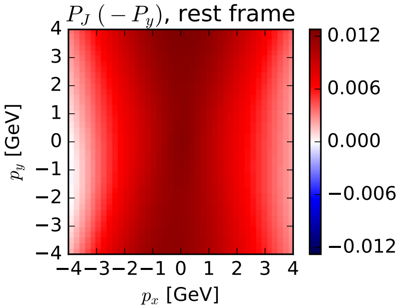


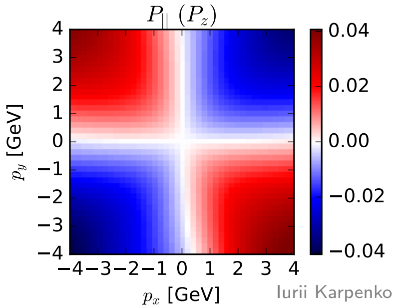


Different components of Λ polarization in a hydrodynamic model

UrQMD+vHLLE calculation for $\sqrt{s_{\mathrm{NN}}} = 19.6$ GeV, 40-50% Au-Au collisions





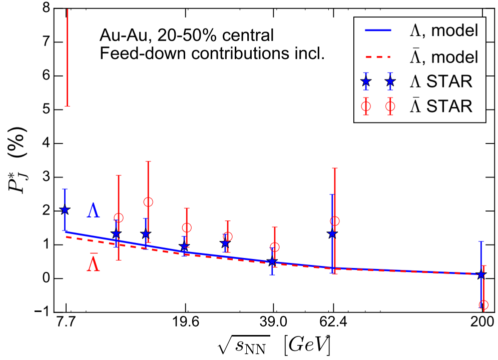


Result from a hydro model:

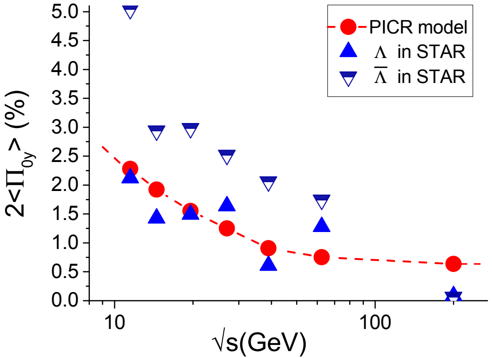
- only Λ produced at particlization
- $P_{||}$ is the largest component at large p_x and p_y (more about that later)
- P_b and $P_{||}$ average out to zero

Global Λ and $\bar{\Lambda}$ polarization: beam energy dependence in UrQMD+vHLLE and PICR

IK, F. Becattini, Eur. Phys. J. C 77, 213 (2017)



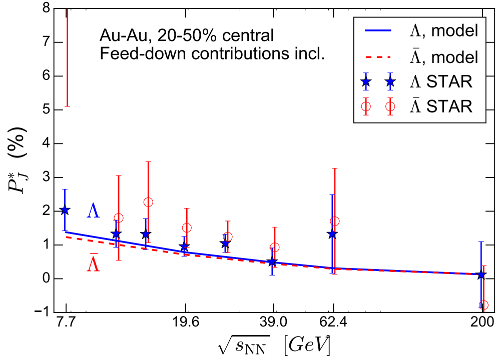
Y.L. Xie, D.J. Wang, L.P. Csernai PRC 95, 031901



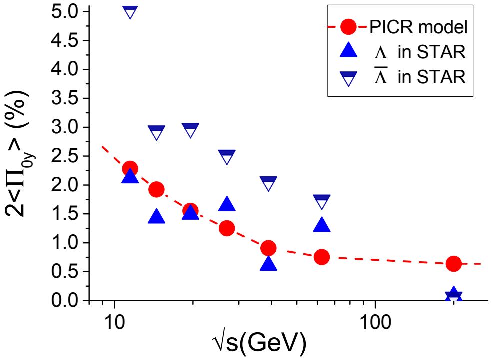
- Different centrality definitions: EbyE hydro with $b = 6.7 \ldots 10.4$ fm in UrQMD+vHLLE, $b = 0.7 \cdot 2R$ (R is nucleus radius), corresponding to centrality $\approx 49\%$ in PICR.
- Λ similar in both calculations **and** within experimental error bars.

Global Λ and $\bar{\Lambda}$ polarization: beam energy dependence in UrQMD+vHLLE and PICR

IK, F. Becattini, Eur. Phys. J. C 77, 213 (2017)



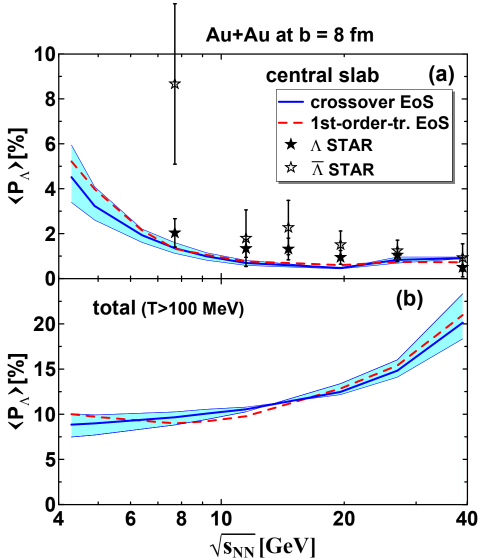
Y.L. Xie, D.J. Wang, L.P. Csernai PRC 95, 031901



- Much smaller and opposite sign $\bar{\Lambda}$-Λ splitting. Only μ_B effect in the model, and it is small.
- Possible explanations of the splitting: magnetic field (at particlization), different freeze-outs for Λ and $\bar{\Lambda}$ (arXiv:1910.06292 by Vitiuk, Bravina and Zabrodin; UrQMD calculation), core-corona picture (arXiv:2003.13757 by Alejandro Ayala et al.).

Global Λ and $\bar{\Lambda}$ polarization: beam energy dependence in 3FD

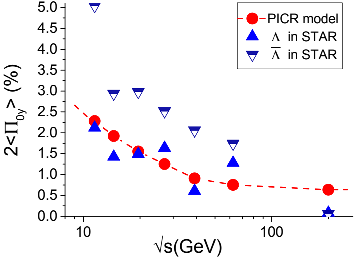
Yu.B. Ivanov, V.D. Toneev, A.A. Soldatov, Phys. Rev. C 100, 014908 (2019)



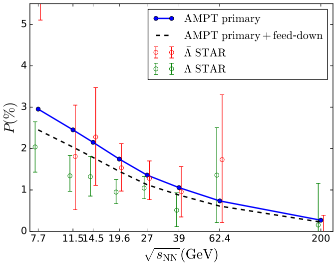
- Calculation at fixed $b = 8$ fm (smaller than the PICR calculation).
- Again, the polarization of Λ is consistent with STAR within error bars.
- The polarization is growing away from mid-rapidity.

A more broad picture: same $P(\sqrt{s_{NN}})$ trend in hydro and **non-hydro** models

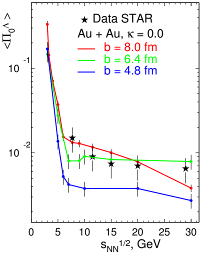
- Y.L. Xie, D.J. Wang, L.P. Csernai PRC 95, 031901



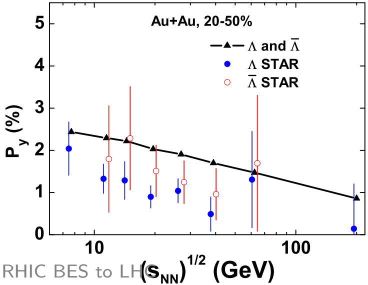
- Hui Li et al, PRC 96, 054908



- Baznat, Gudima, Sorin, Teryaev, PRC 97, 041902

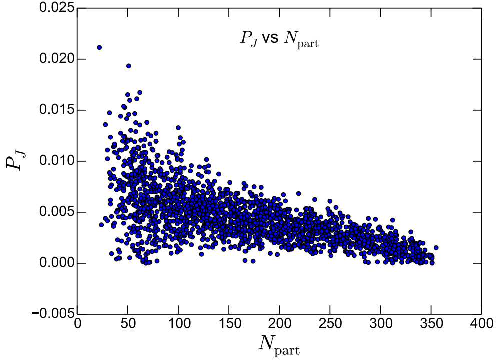


- Yifeng Sun, Che Ming Ko, PRC 96, 024906

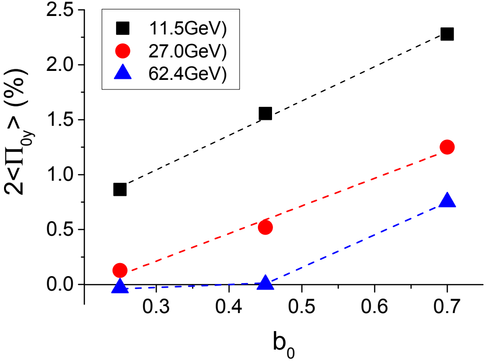


Centrality dependence

IK, F. Becattini, Eur. Phys. J. C 77, 213 (2017)
0-50% central Au-Au at $\sqrt{s_{\rm NN}} = 39$ GeV



Y.L. Xie, D.J. Wang, L.P. Csernai PRC 95, 031901



Ingredients of the two hydro models

UrQMD+vHLLE:

- Initial state: UrQMD
- 3D viscous hydrodynamics
- Cooper-Frye at fixed energy density $\varepsilon = 0.5$ GeV/fm^3.

  Polarization is evaluated at the same hypersurface of fixed energy density!

PICR:

- Initial state: Yang-Mills (?)
- 3D ideal hydrodynamics
- Cooper-Frye "at fixed time $t = 6.25 = (2.5 + 4.75)$ fm/c"
  Minkowski/Milne?
  6.25 or 7.25 fm/c?
  same at all collision energies?

3FD:

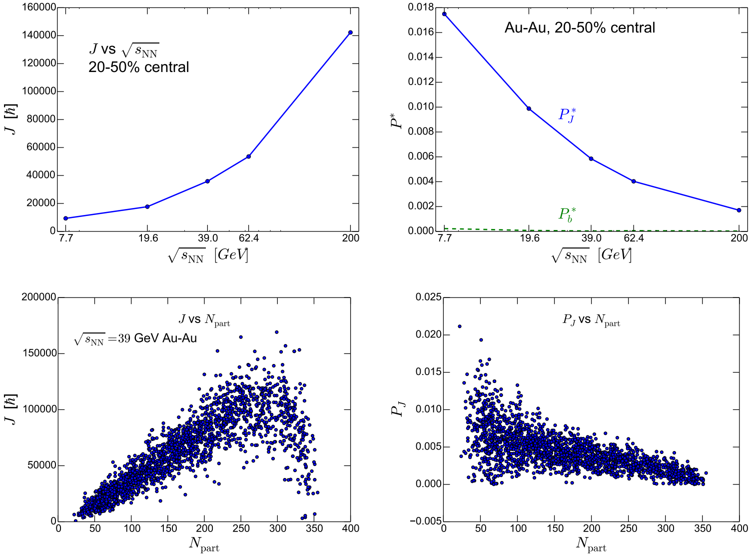
- Initial conditions: two blobs of fluid colliding
- Third fluid is being created dynamically via friction terms (ideal fluid)
- Cooper-Frye at fixed Minkowski time when $\langle \varepsilon \rangle = 0.4$ GeV/fm^3.
  (the freeze-out time changes with $\sqrt{s_{\rm NN}}$)

All the initial states have the global algular momentum

! 3FD uses an approximation for the total vorticity as: $\widetilde{\varpi}_{\mu\nu}(\mathbf{x},t) = \dfrac{\varpi^{\rm B}_{\mu\nu}(\mathbf{x},t)\varepsilon_{\rm B}(\mathbf{x},t) + \varpi^{\rm f}_{\mu\nu}(\mathbf{x},t)\varepsilon_{\rm f}(\mathbf{x},t)}{\varepsilon(\mathbf{x},t)}$

What does polarization correlate with?

IK, F. Becattini, Eur. Phys. J. C 77, 213 (2017)

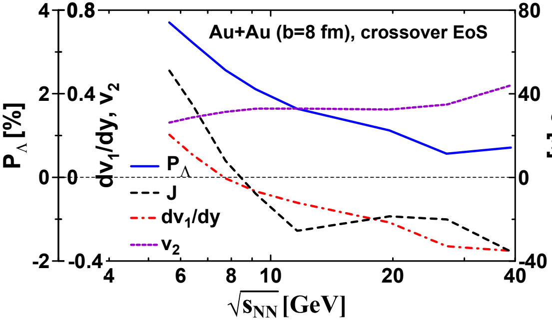


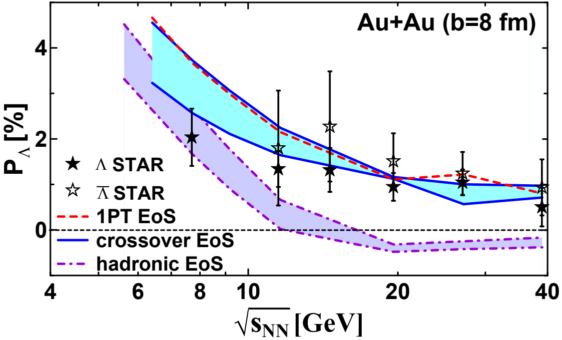
⇒ Angular momentum grows with energy, but polarization doesn't

⇒ Angular momentum peaks at large N_{part}, but polarization doesn't

What does polarization correlate with? (2)

Ivanov, Soldatov, Phys. Rev. C 102, 024916 (2020)



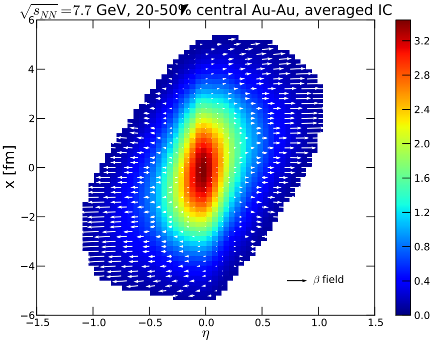


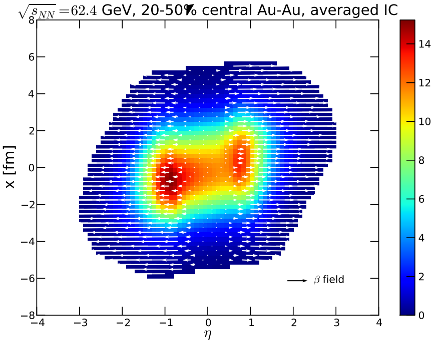
At mid-rapidity:

- The global polarization correlates **neither** with angular momentum **nor** with directed/elliptic flow.
- There is a correlation between the angular momentum and the directed flow!
- Hadronic EoS scenario produces global polarization in the opposite direction!
(although, that scenario does not work well for the other observables)

Beam energy dependence of P_J: explanation from UrQMD+vHLLE

1) Different initial vorticity distribution

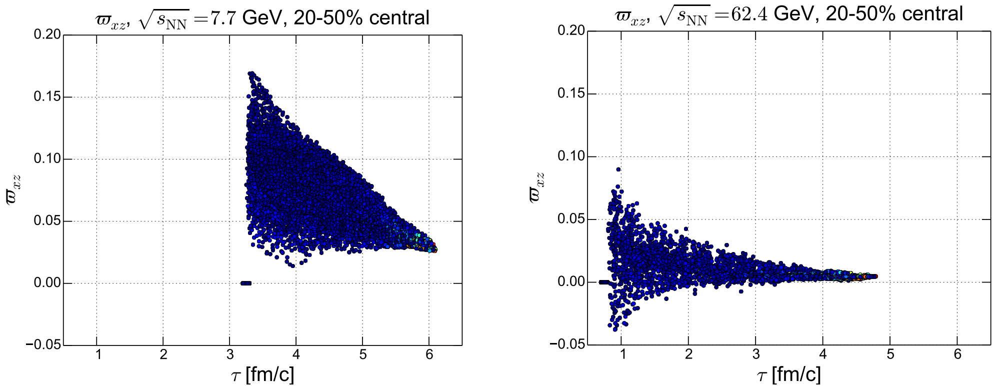




Colour = initial energy density in GeV/fm^3, arrows = initial inverse temperature field

Beam energy dependence of P_J: explanation from UrQMD+vHLLE (2)

2) Longer hydrodynamic evolution at higher $\sqrt{s_{\rm NN}}$ further dilutes the vorticity



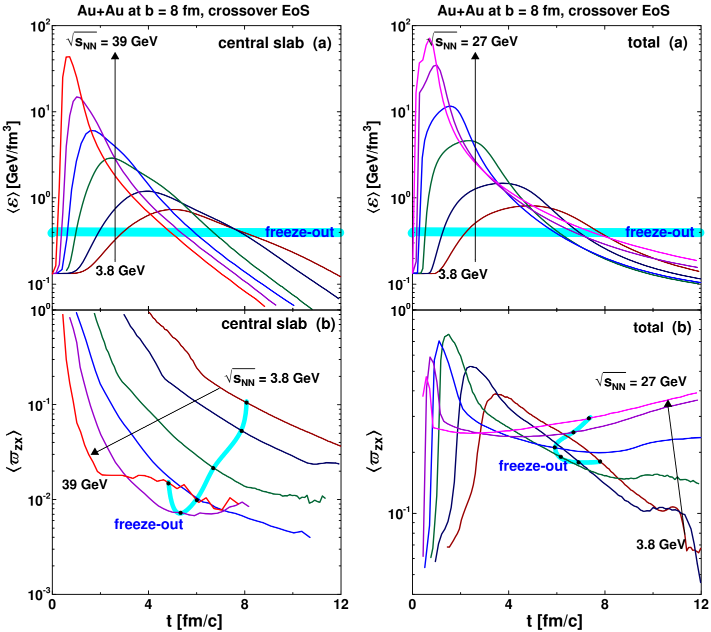
Figs: Distribution of xz component of thermal vorticity (responsible for P_J at $p_x = p_y = 0$) over particlization hypersurface.

- these two effects result in lower polarization at higher collision energies

Beam energy dependence of P_J: explanation from PICR

- "From a thermodynamical perspective, the polarization decreases with energy, and this can be attributed to the higher temperature in higher energy of collisions."
- "AMPT model has shown that the averaged classical vorticity decreases with the collision energy $\rightarrow$ decline of global polarization."

Beam energy dependence of P_J: explanation from 3FD



At mid-rapidity (central slab):

- $\langle\varpi_{zx}\rangle$ generally decreases with beam energy, because the vortical field is pushed out to the fragmentation of regions.
- At a given beam energy, the mean value of ϖ_{zx} decreases with time.
- Freeze-out time **increases** with the beam energy, in the region $\sqrt{s_{NN}} = 3.8 \ldots 27$ GeV.

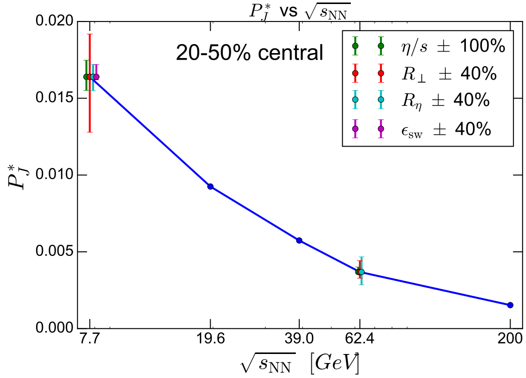
→ the first effect dominates the beam energy dependence of polarization.

For the full system:

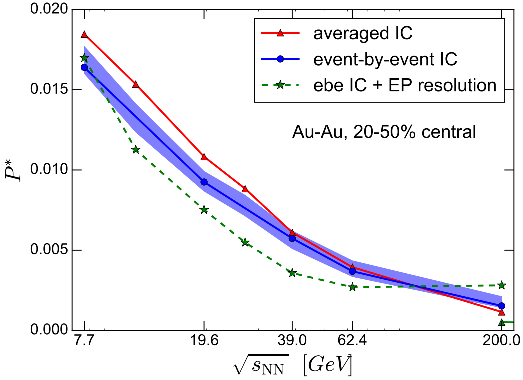
- $\langle\varpi_{zx}\rangle$ starts to increase with the beam energy at $\sqrt{s_{NN}} > 7.7$ GeV.

Robustness of the global polarization signal in UrQMD+vHLLE

variation of model parameters



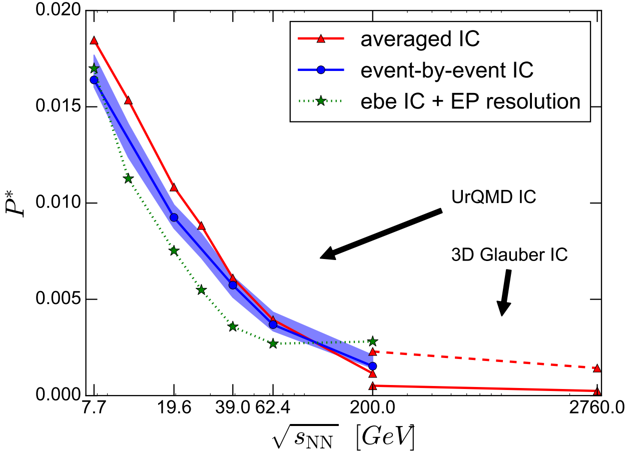
event-by-event vs. averaged



- Collision energy dependence is robust with respect to variation of the parameters of the model.
- There is no big difference between event-by-event and single shot hydrodynamic description.

II. Local spin polarization

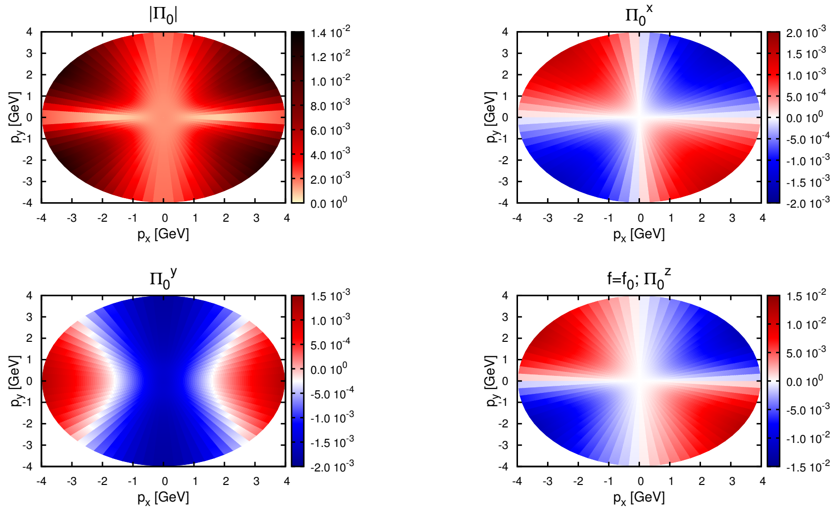
All the calculations show that, at mid-rapidity, global spin polarization decreases with beam energy.



Early calculations

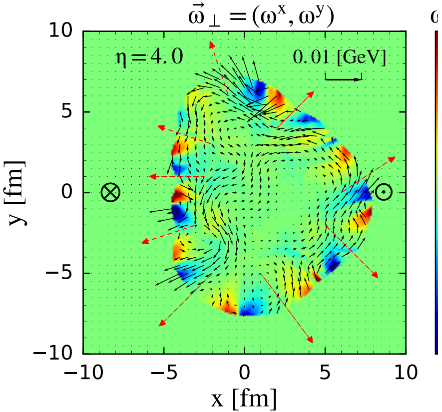
F. Becattini, G. Inghirami et al., EPJ C 75, 406 (2015): The quadrupole structure is already there!

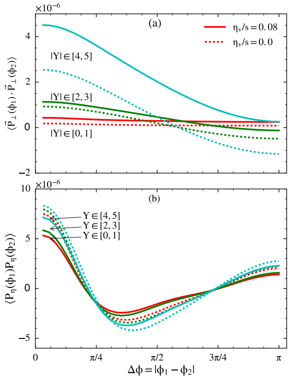
Tilted Monte Carlo Glauber IS + 3D viscous hydro (ECHO-QGP)



Early calculations (2)

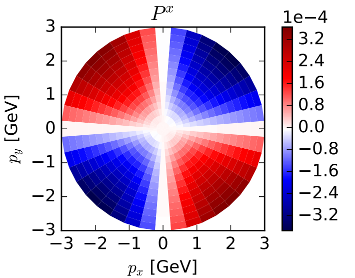
Long-Gang Pang, Hannah Petersen, Qun Wang, Xin-Nian Wang, Phys. Rev. Lett. 117, 192301 (2016)

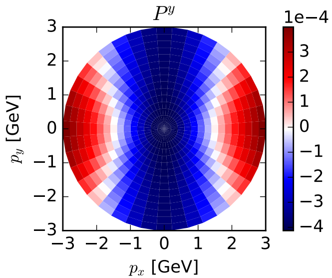


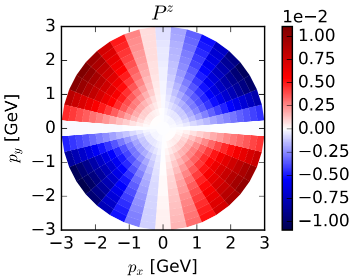


At high energies, the dominant component is P^z

Tilted Monte Carlo Glauber IS + 3D viscous hydro (vHLLE)







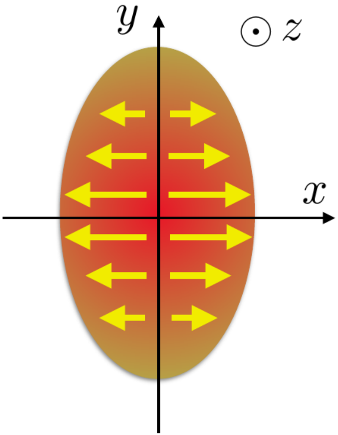
20-50% central Pb-Pb,
$\sqrt{s_{\rm NN}} = 2.76$ GeV

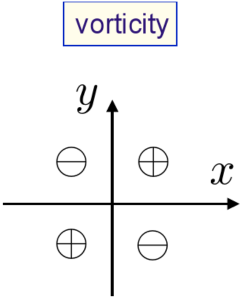
P^z is:

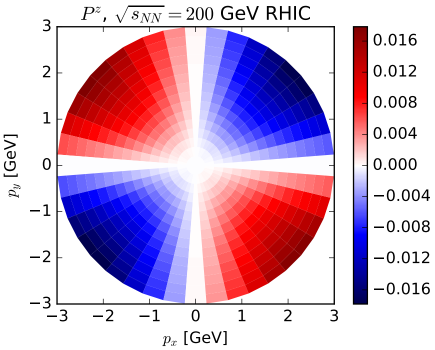
- nonzero in 2D boost-invariant hydrodynamics
- related to transverse expansion (more on that later)

An oversimplified explanation of the quadrupole structure

(c) Sergei Voloshin, SQM2017



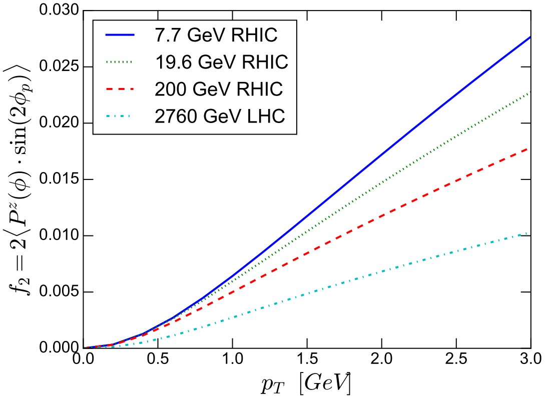




*It does not work quantitatively, since there are time derivatives and temperature gradients involved.

Fourier expansion for P^z

$$P^z(\mathbf{p_T}, y=0) = \sum_{k=1}^{\infty} f_{2k}(p_T)\sin 2k(\phi_p - \Psi)$$



- requires identification of event plane Ψ
- Blast-Wave model:

$$\boxed{f_2(p_T) = 2\frac{dT}{d\tau}\frac{1}{mT}v_2(p_T)}$$

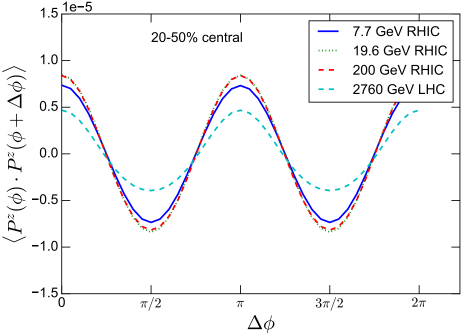
P^z emerges because of anisotropic transverse expansion, same way as v_2.

Corresponding Λ spin correlations

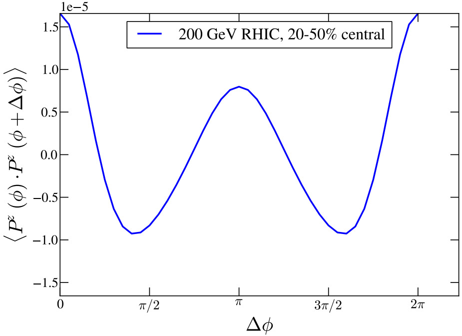
The quadrupole structure leads to correlations of P^z of Λ pairs

$$P^z = P_0^z \sin 2(\phi - \Psi) \quad \Rightarrow \quad \langle P^z(\phi) P^z(\phi + \Delta\phi) \rangle = \frac{1}{2} (P_0^z)^2 \cos 2\Delta\phi$$

average hydro



event-by-event hydro



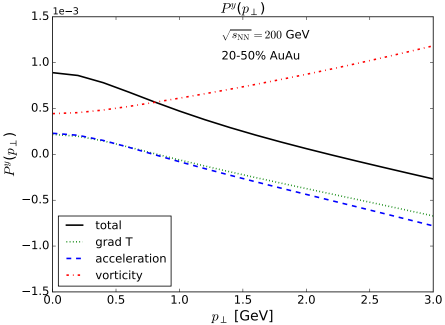
Similar results: Long-Gang Pang et al, Phys.Rev.Lett. 117 (2016) no.19, 192301
Λ spin correlations due to vorticity induced by initial state fluctuations

What causes the transverse and longitudinal components of polarization?

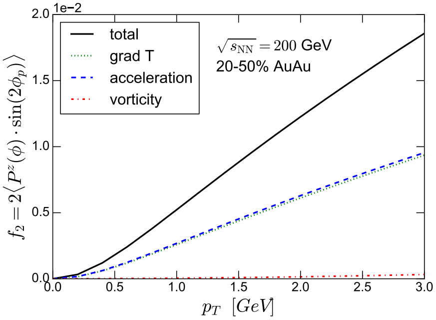
IK, F. Becattini, Nucl. Phys. A 982, 519-522 (2019); QM2019 proceeding

$$S^\mu \propto \varepsilon^{\mu\rho\sigma\tau}\varpi_{\rho\sigma}p_\tau = \varepsilon^{\mu\rho\sigma\tau}(\partial_\rho\beta_\sigma)p_\tau = \underbrace{\varepsilon^{\mu\rho\sigma\tau}p_\tau\partial_\rho\left(\frac{1}{T}\right)u_\sigma}_{\text{grad}T} + \underbrace{\frac{1}{T}2\left[\omega^\mu(u\cdot p) - u^\mu(\omega\cdot p)\right]}_{\text{“NR vorticity”}} + \underbrace{\varepsilon^{\mu\rho\sigma\tau}p_\tau A_\sigma u_\rho}_{\text{acceleration}}$$

Global transverse P_J:



Longitudinal quadrupole f_2:

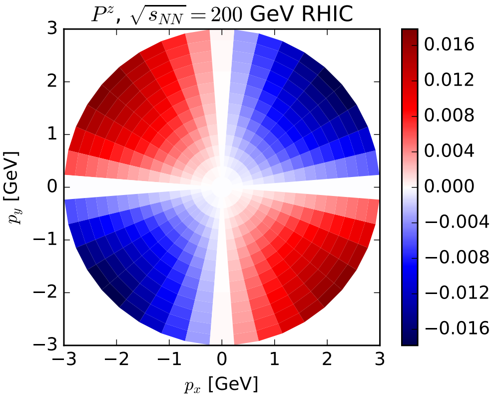


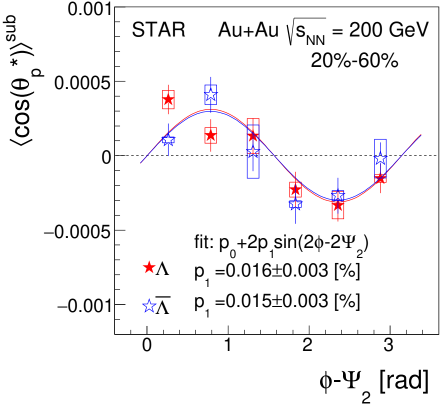
- P^J at low $p_\perp$ is dominated by vorticity
- P^z is dominated by acceleration and gradients of temperature

P^z: hydro versus $\sqrt{s_{\rm NN}} = 200$ GeV STAR data

vHLLE+Glissando IS: Becattini, IK, Phys. Rev. Lett. 120, 012302 (2018)

STAR data: Phys. Rev. Lett. 123, 132301 (2019)



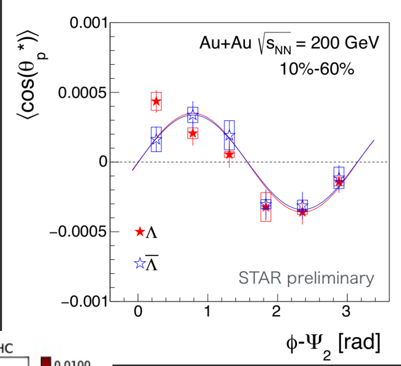


Similar $\sin(2\phi)$ structure is observed, but with an opposite sign!

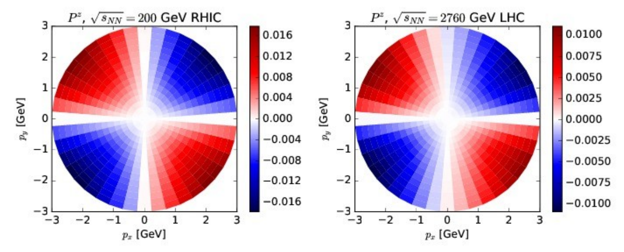
A *sign problem* for the longitudinal component

Quadrupolar structure of longitudinal polarization in the transverse momentum plane, as predicted.
Spectacular confirmation of hydro predictions...
yet with a flipped sign!

- Hydro initial conditions? (polarization is a sensitive probe of the initial flow)
- Incomplete local thermodynamic equilibrium for the spin degrees of freedom (spin kinetic theory)?
- Effect of spin dissipative transport coefficients?
- Effect of initial state fluctuations?
- Effect of decays?
- Error in the calculation



Z. Ye, T. Niida, Quark Matter 2018

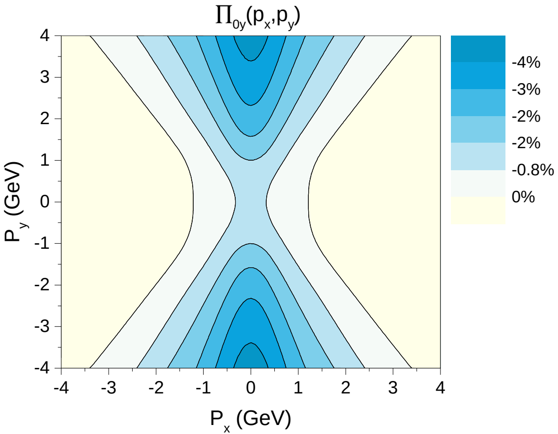


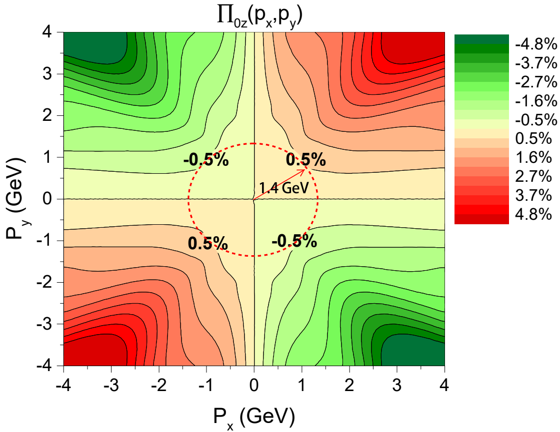
F. B., I. Karpenko, Phys. Rev. Lett. 120 (2018) 012302
S. Voloshin, in SQM 2017

Same pattern found in AMPT+thermal vorticity calculation X. L. Xia, H. Li, Z. B. Tang and Q. Wang, 1803.00867

Local spin polarization from PICR

Yilong Xie, Dujuan Wang, Laszlo Csernai, Eur. Phys. J. C 80, 39 (2020)





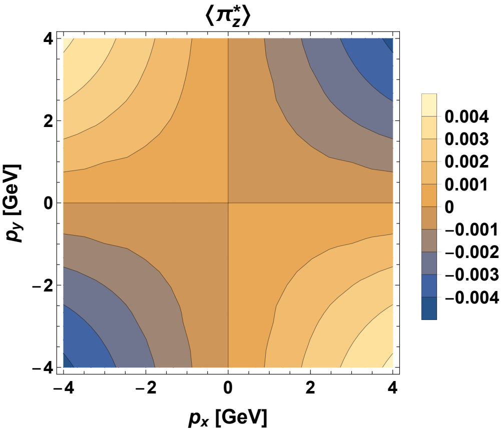
Transverse polarization: polarization decreases from in-plane to out-of-plane,agreeing with the experimental data, whose magnitude, how-ever, is about ten times smaller.

Longitudinal polarization: same sign as in experiment?

Longitudinal polarization in Blast-Wave (thermal) model

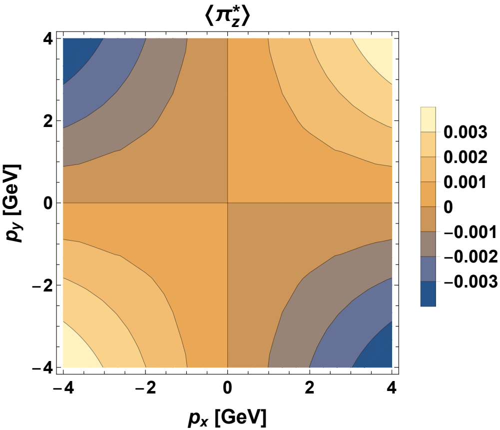
W. Florkowski, A. Kumar, R. Ryblewski, A. Mazeliauskas, Phys. Rev. C 100, 054907 (2019)

Polarization ∝ standard thermal vorticity
(opposite sign to experiment)



Standard thermal vorticity:
$\varpi^{\mu\nu} = -\frac{1}{2}\left(\partial^\mu \beta^\nu - \partial^\nu \beta^\mu\right)$

Polarization ∝ projected thermal vorticity
(same sign as in experiment)

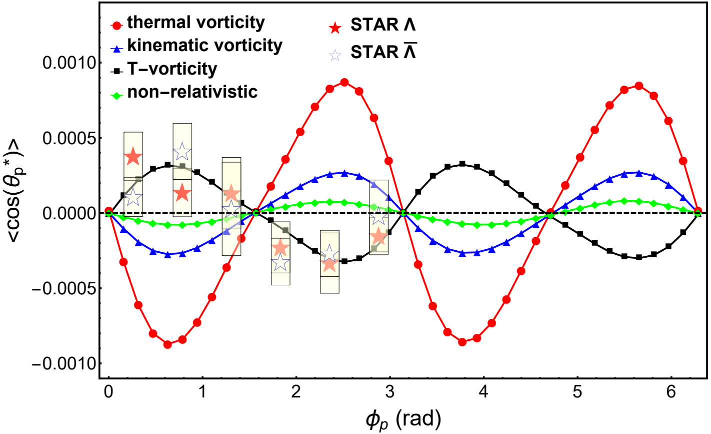


Projected thermal vorticity: $\varpi^{\mu\nu}_{\text{proj}} = \varpi_{\alpha\beta}\Delta^{\mu}_{\alpha}\Delta^{\nu}_{\beta}$

“Different definitions of vorticity vs experiment”

Hong-Zhong Wu, Long-Gang Pang, Xu-Guang Huang, Qun Wang, Phys. Rev. Research 1, 033058 (2019) + QM2019 proc

AMPT IS (includes angular momentum) + 3D viscous hydro (CLVisc)



$$\omega^{(\mathrm{th})}_{\mu\nu} = -\frac{1}{2}\left(\partial_\mu \beta_\nu - \partial_\nu \beta_\mu\right)$$

$$\omega^{(K)}_{\mu\nu} = -\frac{1}{2}\left(\partial_\mu u_\nu - \partial_\nu u_\mu\right)$$

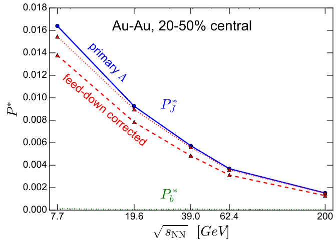
$$\omega^{(T)}_{\mu\nu} = -\frac{1}{2}\left[\partial_\mu (T u_\nu) - \partial_\nu (T u_\mu)\right]$$

$$\omega^{(\mathrm{NR})}_{\mu\nu} = \varepsilon_{\nu\mu\rho\eta} u^\rho \omega^\eta$$

Summary

GLOBAL polarization

- Hydrodynamic models with the initial state incorporating the angular momentum reproduce the increase of the global mean polarization of Λ towards lowest RHIC BES energies.
- The calculated *mean* Λ polarization is (almost) within the experimental error bars.
- Λ-$\bar{\Lambda}$ splitting: different explanations are possible.



LOCAL polarization

- At top RHIC/LHC energies, the largest component of polarization is P^z (along the beam axis), reaching 1% for $p_T = 3$ GeV Λ at midrapidity.
- $P^z(p_T)$ is a more generic effect, emerging in boost-invariant hydrodynamics due to anisotropy of transverse expansion (v_2). It probes velocity/temperature gradients at particlization surface.
- $P_J \Leftrightarrow$ vorticity(ϖ_{xz}), $P^z \Leftrightarrow$ transverse acceleration / grad T.
- There are puzzles: the sign of longitudinal polarization and the angular dependence of the transverse polarization.