Longitudinal exclusive heavy vector meson production at NLO

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Based on [2104.02349]

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30th of June, 2021

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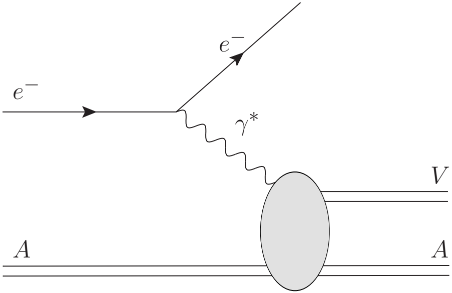
Exclusive vector meson production in deep inelastic scattering

- $\gamma^*_\lambda + A \to V_{\lambda'} + A$
- Heavy vector mesons: $V = \mathrm{J}/\psi, \Upsilon \dots$
- Polarization mixing highly suppressed
  $\Rightarrow \lambda = \lambda' = L, T$
  - See Farid's talk on Tuesday for the case $\lambda \neq \lambda'$
- In this talk: focus on J/ψ production in the longitudinal polarization case
- The momentum transfer Δ can be measured
  - Fourier transform: $\Delta \Leftrightarrow$ b
    $\Rightarrow$ Probes the spatial structure of the nucleus

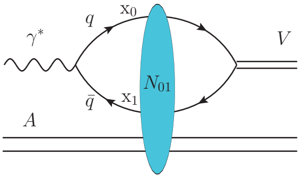


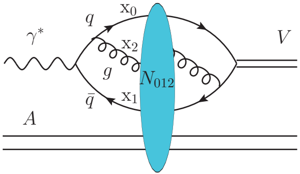
The dipole picture at NLO

$$\frac{\mathrm{d}}{\mathrm{d}t}\sigma^{\gamma^*+A\to V+A}\Big|_{t=0} = \frac{1}{16\pi}|\mathcal{A}|^2$$

$$-i\mathcal{A} = 2\int \mathrm{d}^2\mathbf{x}_0\,\mathrm{d}^2\mathbf{x}_1 \int \frac{\mathrm{d}z_0\,\mathrm{d}z_1}{(4\pi)}\delta(z_0+z_1-1)\Psi_{\gamma^*}^{q\bar{q}} N_{01} \Psi_V^{q\bar{q}*} + 2\int \mathrm{d}^2\mathbf{x}_0\,\mathrm{d}^2\mathbf{x}_1\,\mathrm{d}^2\mathbf{x}_2 \int \frac{\mathrm{d}z_0\,\mathrm{d}z_1\,\mathrm{d}z_2}{(4\pi)^2}\delta(z_0+z_1+z_2-1)\Psi_{\gamma^*}^{q\bar{q}g} N_{012} \Psi_V^{q\bar{q}g*}$$

- Mixed position-momentum fraction space $(\mathbf{x}_i, z_i)$
- Virtual photon light-front wave functions $\Psi_{\gamma^*}^{q\bar{q}}, \Psi_{\gamma^*}^{q\bar{q}g}$ from perturbative QCD [Beuf, Lappi, Paatelainen, 2103.14549]
- Energy dependence of the dipole-target amplitude N described by perturbative evolution equations (BK)
- Meson light-front wave functions $\Psi_V^{q\bar{q}}, \Psi_V^{q\bar{q}g}$ nonperturbative





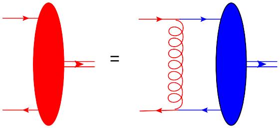
Nonrelativistic expansion

- Heavy vector meson Fock states: $|V\rangle = \Psi_V^{q\bar{q}} |q\bar{q}\rangle + \Psi_V^{q\bar{q}g} |q\bar{q}g\rangle +$ higher orders
- NRQCD: Parametrically $v \sim \alpha_s(vM_V) > \alpha_s(M_V)$

  $\Rightarrow$ Expansion in v and α_s: $1 > \alpha_s > v^2 > \dots$
- Nonrelativistic expansion [Escobedo, Lappi, 1911.01136]:

$$\Psi_V^n = \sum_{m,k} C_{n\leftarrow m}^k \int_0^1 \frac{\mathrm{d}z'}{4\pi} \left(\frac{1}{m_q}\nabla\right)^k \phi^m(\mathrm{r} = 0, z')$$

  - ϕ^m = leading-order wave function for Fock state m
  - α_s corrections included in $C_{n\leftarrow m}^k$
  - Relativistic corrections go as v^k in the index k



A loop correction to LFWF [1911.01136]

- This talk: order $\alpha_s v^0$ corrections to J/ψ production
- Relativistic corrections at $v^2\alpha_s^0$ calculated in [Lappi, Mäntysaari, JP, 2006.02830]

Nonrelativistic expansion

- Heavy vector meson Fock states: $|V\rangle = \Psi_V^{q\bar{q}} |q\bar{q}\rangle + \Psi_V^{q\bar{q}g} |q\bar{q}g\rangle +$ higher orders
- For NLO corrections in the nonrelativistic limit we need the following:

$$\Psi_V^{q\bar{q}} = C^0_{q\bar{q}\leftarrow q\bar{q}} \int_0^1 \frac{\mathrm{d}z'}{4\pi} \phi^{q\bar{q}}(\mathrm{r}=0, z') \qquad \Psi_V^{q\bar{q}g} = C^0_{q\bar{q}g\leftarrow q\bar{q}} \int_0^1 \frac{\mathrm{d}z'}{4\pi} \phi^{q\bar{q}}(\mathrm{r}=0, z')$$

- $C^0_{q\bar{q}\leftarrow q\bar{q}}$, $C^0_{q\bar{q}g\leftarrow q\bar{q}}$ calculated at NLO in [Escobedo, Lappi, 1911.01136]

  $q\bar{q}$ (virtual correction): $q\bar{q}g$ (real correction):

- Same Feynman diagrams in the NLO calculation for γ^* wave function

  [Beuf, Lappi, Paatelainen, 2103.14549]

Calculation of the production amplitude

- We have calculated the production at NLO with longitudinally polarized photon
  - Transverse calculations on the way
- UV divergences between the $q\bar{q}$ and $q\bar{q}g$ parts of the calculation cancel
- IR divergences cancel when one takes into account:
  - Renormalization of the leading-order wave function $\phi^{q\bar{q}}$
    - Can be written in terms of the leptonic decay width
  - The energy dependence of the dipole amplitude which can be described in terms of the Balitsky-Kovchegov (BK) equation:

$$\frac{\partial}{\partial \ln(1/x)} N_{01} = \frac{N_c \alpha_s}{2\pi^2} \int \mathrm{d}^2 \mathbf{x}_2 \, \frac{\mathbf{x}_{01}^2}{\mathbf{x}_{20}^2 \mathbf{x}_{21}^2} \left[N_{02} + N_{12} - N_{01} - N_{02} N_{12} \right]$$

$\Rightarrow$ The total production amplitude is finite and can be numerically evaluated

Final expression

$$-i\mathcal{A}^L = -Q\sqrt{\Gamma(V\to e^-e^+)\frac{3M_V}{16\pi^2\alpha_{\mathrm{em}}}}\int \mathrm{d}^2\mathbf{x}_{01}\int \mathrm{d}^2\mathbf{b}\left\{\mathcal{K}^{\mathrm{LO}}_{q\bar q}(Y_0) + \frac{\alpha_s C_F}{2\pi}\mathcal{K}^{\mathrm{NLO}}_{q\bar q}(Y_{\mathrm{dip}}) + \frac{\alpha_s C_F}{2\pi}\int \mathrm{d}^2\mathbf{x}_{20}\int_{z_{\min}}^{1/2}\mathrm{d}z_2\,\mathcal{K}_{q\bar q g}(Y_{\mathrm{qqg}})\right\}$$

where $\mathcal{K}^{\mathrm{LO}}_{q\bar q}(Y_0) = K_0(\zeta)N_{01}(Y_0)$, $\zeta = |\mathbf{x}_{01}|\sqrt{\frac{1}{4}Q^2+m_q^2}$,

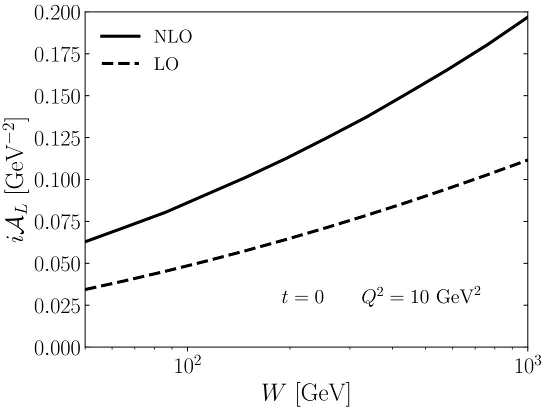
$$\mathcal{K}^{\mathrm{NLO}}_{q\bar q}(Y_{\mathrm{dip}}) = \left[\mathcal{K} + \tilde{\mathcal{I}}_\nu\left(z=\frac{1}{2},\mathbf{x}_{01}\right) + K_0(\zeta)\left(6-\frac{\pi^2}{3}+\Omega_{\mathcal{V}}\left(\gamma; z=\frac{1}{2}\right) + L\left(\gamma; z=\frac{1}{2}\right) - 3\log\left(\frac{|\mathbf{x}_{10}|m}{2}\right) - 3\gamma_E\right)\right] N_{01}(Y_{\mathrm{dip}})$$

and

$$\mathcal{K}_{q\bar q g}(Y_{\mathrm{q\bar qg}}) = -32\pi m_q\left\{\frac{i\mathbf{x}_{20}^i}{|\mathbf{x}_{20}|}K_1(2m_q z_2|\mathbf{x}_{20}|)\left[\left((1-z_2)^2+z_2^2\right)\mathcal{I}^i_{(f)} + (2z_2^2-1)(1-2z_2)\mathcal{I}^i_{(g)}\right] N_{012}(Y_{\mathrm{q\bar qg}})\right.$$
$$\left. + 4m_q z_2^3 K_1(2m_q z_2|\mathbf{x}_{20}|)\left[\mathcal{I}_{(f)} - \frac{1-2z_2}{1+2z_2}\mathcal{I}_{(g)}\right] N_{012}(Y_{\mathrm{q\bar qg}}) + \frac{1}{8\pi^2}\left((1-z_2)^2+z_2^2\right)\frac{1}{m_q z_2|\mathbf{x}_{20}|^2}K_0(\zeta)e^{-\mathbf{x}_{20}^2/(\mathbf{x}_{10}^2 e^{\gamma_E})}N_{01}(Y_{\mathrm{q\bar qg}})\right\}.$$

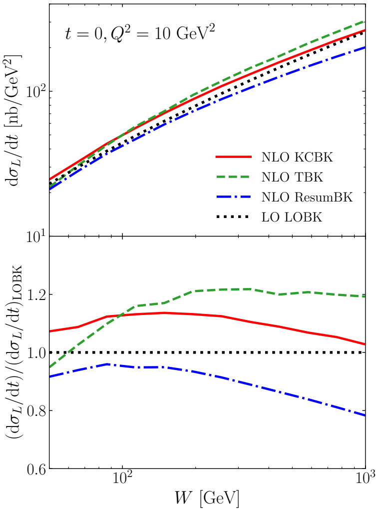
Production amplitude as a function of the center-of-mass energy W

- We evaluated the amplitude numerically
- NLO corrections are large
  - $\sim 75\%$ of the LO result
- Significant increase of the amplitude
- However, the production amplitude depends on the dipole amplitude used
  - Here the same dipole amplitude was used for both cases
  - But this is not actually consistent as we'll see...



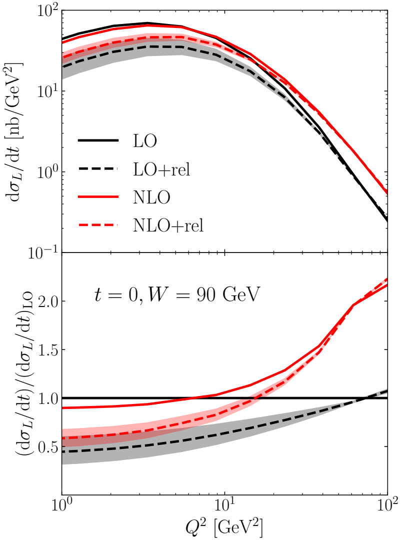
Cross section with different dipole amplitudes

- Compare dipole amplitudes from different fits
- Initial condition fitted to HERA structure function data
  - *LOBK* = LO fit [Lappi, Mäntysaari, 1309.6963]
  - *KCBK*, *TBK*, *ResumBK* = NLO fits
    [Beuf et al., 2007.01645]
- The difference between the LO and NLO results is smaller than what the amplitude plot indicates
  - LO fit compensates for NLO effects
    $\Rightarrow$ Important to use the fit with the same order!
- Some variation between the different NLO fits
  - Complementary information from VM production



Cross section – NLO vs relativistic corrections

- We include the first relativistic corrections of order v^2 for comparison [Lappi, Mäntysaari, JP, 2006.02830]
- Relativistic corrections smaller at NLO than at LO
  - Explained by the large NLO corrections
- At small Q^2, relativistic corrections more important
- Large Q^2:
  - Relativistic effects become insignificant
  - Difference between LO and NLO results explained by a different rapidity evolution in the dipole amplitude



Summary

- We calculated the NLO corrections to the longitudinal heavy vector meson production
- NLO corrections found to be significant
  - However, LO dipole amplitude fit can capture most of the NLO effects
- Some deviations between different NLO fits

  ⇒ Complementary probe to structure functions
- Future: transverse production
  - Calculations very similar to the longitudinal case
  - Will allow comparison of the NLO results to the data
- Important developments: precise measurements expected at ultra-peripheral collisions at the LHC and the future Electron-Ion Collider

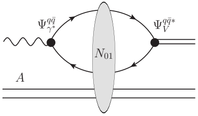
Backup

$q\bar{q}$ and $q\bar{q}g$ parts of the calculation

LO result: $-\frac{ee_f Q}{2\pi}\sqrt{\frac{N_c}{2}}2\int_0^1 \frac{\mathrm{d}z'}{4\pi}\phi^{q\bar{q}}(\mathrm{r}=0,z')\int \mathrm{d}^2\mathrm{x}_{01}\, N_{01} K_0(\zeta),\ \zeta = |\mathrm{x}_{01}|\sqrt{\frac{1}{4}Q^2+m_q^2}$

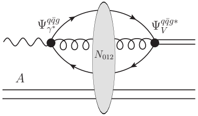
NLO, $q\bar{q}$ (dipole) part:

$$-\frac{ee_f Q}{2\pi}\sqrt{\frac{N_c}{2}}\frac{\alpha_s C_F}{2\pi}2\int_0^1 \frac{\mathrm{d}z'}{4\pi}\phi^{q\bar{q}}(\mathrm{r}=0,z')\int \mathrm{d}^{D-2}\mathrm{x}_{01}\, N_{01}\Bigg\{-\frac{2}{D-4}(4\log(2\alpha)+3)K_{D/2-2}(\zeta)+\mathcal{K}_0+\tilde{\mathcal{I}}_\nu\left(z_0=\frac{1}{2},\mathrm{x}_{01}\right)$$
$$+K_0(\zeta)\left[\frac{1}{\alpha}+3-\frac{\pi^2}{3}+L\left(\gamma;z_0=\frac{1}{2}\right)+\Omega_\mathcal{V}\left(\gamma;z_0=\frac{1}{2}\right)+4\gamma_E\log(2\alpha)+4\log(2\alpha)\log\left(\frac{2\pi^2|\mathrm{x}_{01}|^3\mu^2}{\sqrt{\frac{1}{4}Q^2+m^2}}\right)+3\log\left(\frac{4\pi^2|\mathrm{x}_{01}|^2\mu^2}{m\sqrt{\frac{1}{4}Q^2+m_q^2}}\right)\right]\Bigg\}$$

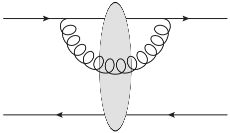


NLO, $q\bar{q}g$ part:

$$-\frac{ee_f Q}{2\pi}\sqrt{\frac{N_c}{2}}\frac{\alpha_s C_F}{2\pi}2\int_0^1 \frac{\mathrm{d}z'}{4\pi}\phi^{q\bar{q}}(\mathrm{r}=0,z')\int \mathrm{d}^{D-2}\mathrm{x}_{01}\,\mathrm{d}^{D-2}\mathrm{x}_{20}\int_\alpha^{1/2}\mathrm{d}z_2\,(-32\pi m_q)N_{012}\left(\frac{m_q z_2}{\pi|\mathrm{x}_{20}|\mu}\right)^{D/2-2}$$
$$\times\left\{i\frac{\mathrm{x}_{20}^i}{|\mathrm{x}_{20}|}K_{D/2-1}(2m_q z_2|\mathrm{x}_{02}|)\left[(1-z_2)^2+(D-3)z_2^2\right]\mathcal{I}^i_{(f)}+i\frac{\mathrm{x}_{20}^i}{|\mathrm{x}_{20}|}K_1(2m_q z_2|\mathrm{x}_{02}|)\left[(2z_2^2-1)(1-2z_2)\right]\mathcal{I}^i_{(g)}+4m_q z_2^3 K_0(2m_q z_2|\mathrm{x}_{02}|)\left[\mathcal{I}_{(f)}-\frac{1-2z_2}{1+2z_2}\mathcal{I}_{(g)}\right]\right\}$$



UV subtraction



- Divergences at the limit $D \to 4$ from virtual gluon loops
- Also present in the $q\bar{q}g$ part
- Solution: subtract the divergence from $q\bar{q}g$ and add it to the dipole part

$$N_{012} \mathbf{x}_{20}^i \mathcal{I}_{(f)}^i K_{D/2-1}(2mz_2|\mathbf{x}_{20}|) = \underbrace{\left\{ N_{012} \mathbf{x}_{20}^i \mathcal{I}_{(f)}^i K_{D/2-1}(2mz_2|\mathbf{x}_{20}|) - N_{01} \left[\mathbf{x}_{20}^i \mathcal{I}_{(f)}^i K_{D/2-1} \right]_{\text{UV}} \right\}}_{\text{UV finite}} + \underbrace{N_{01} \left[\mathbf{x}_{20}^i \mathcal{I}_{(f)}^i K_{D/2-1} \right]_{\text{UV}}}_{\text{UV divergent, combine with } q\bar{q} \text{ part}}$$

where

$$\left[\mathbf{x}_{20}^i \mathcal{I}_{(f)}^i K_{D/2-1} \right]_{\text{UV}} = \Gamma(D/2-1) \frac{i\mu^{2-D/2}}{4\pi^{D/2}} |\mathbf{x}_{20}|^{4-D} \left(\frac{\sqrt{\frac{1}{4}Q^2 + m^2}}{2\pi|\mathbf{x}_{10}|} \right)^{D/2-2} K_{D/2-2}\left(|\mathbf{x}_{10}| \sqrt{\frac{1}{4}Q^2 + m^2} \right) e^{-\mathbf{x}_{20}^2/(\mathbf{x}_{10}^2 e^{\gamma_E})} \cdot \frac{\Gamma(D/2-1)}{2} (mz_2|\mathbf{x}_{20}|)^{-D/2+1}$$

- Subtraction scheme from [Hänninen, Lappi and Paatelainen, 1711.08207] and [Beuf, Lappi and Paatelainen, 2103.14549]

$\Rightarrow$ UV divergences cancel in the dipole term!

Renormalization of the leading-order wave function

- We need to take into account the renormalization of the leading-order wave function $\phi^{q\bar{q}}$
- Easiest done using the NLO expression for the leptonic width [Escobedo, Lappi, 1911.01136]:

$$\Gamma(V \to e^- e^+) = \frac{2N_c e_f^2 e^4}{3\pi M_V} \sum_{h_0' h_1'} \left| \int \frac{\mathrm{d}z'}{4\pi} \phi^{q\bar{q}}_{h_0' h_1'} \right|^2 \left[1 + \frac{2\alpha_s C_F}{\pi} \left(\frac{1}{2\alpha} - 2 \right) \right].$$

- Solve $\int \mathrm{d}z' \, \phi^{q\bar{q}}$ from this and plug it into the equation for the production amplitude

  $\Rightarrow$ Cancels the divergence $1/\alpha$ from the dipole part

- Also allows us to replace the nonperturbative leading-order wave function with the leptonic width for which we can use the experimental value

Balitsky-Kovchegov equation

- The $q\bar{q}g$ part is singular at $\alpha \to 0$
- This is related to the rapidity evolution of the dipole amplitude, described by the Balitsky-Kovchegov (BK) equation:

$$\frac{\partial}{\partial Y} N_{01} = \frac{N_c \alpha_s}{2\pi^2} \int \mathrm{d}^2 \mathbf{x}_2 \, \frac{\mathbf{x}_{01}^2}{\mathbf{x}_{20}^2 \mathbf{x}_{21}^2} \left[N_{02} + N_{12} - N_{01} - N_{02} N_{12} \right]$$

- In fact, we can write:

$$\begin{aligned} \frac{\alpha_s}{2\pi} \int \mathrm{d}^2 \mathbf{x}_2 \int_{\alpha}^{1/2} \mathrm{d} z_2 \, \mathcal{K}_{q\bar{q}g} &= K_0(\zeta) \int \mathrm{d}^2 \mathbf{x}_2 \int_{\alpha}^{1/2} \mathrm{d} z_2 \, \frac{N_c \alpha_s}{2\pi^2 z_2} \frac{\mathbf{x}_{01}^2}{\mathbf{x}_{20}^2 \mathbf{x}_{21}^2} \left[N_{02} + N_{12} - N_{01} - N_{02} N_{12} \right] + \text{nonsingular part} \\ &= K_0(\zeta) \left[N_{01}\Big(Y(z_2 = 1/2) \Big) - N_{01}\Big(Y(z_2 = \alpha) \Big) \right] + \text{nonsingular part} \end{aligned}$$

- Combining this with the LO result, we get $Y(z_2 = \alpha) \to Y(z_2 = 1/2)$ for the evolution rapidity in the LO term

Relativistic corrections to the wave function

We use the wave function from [Lappi, Mäntysaari, JP, 2006.02830] which includes the relativistic corrections at the order v^2:

$$\Psi^{\lambda=0}_{+-}(\mathrm{r},z) = \Psi^{\lambda=0}_{-+}(\mathrm{r},z) = \frac{\pi\sqrt{2}}{\sqrt{m_c}}\left[A\delta(z-1/2) + \frac{B}{m_c^2}\left(\left(\frac{5}{2}+\mathrm{r}^2 m_c^2\right)\delta(z-1/2) - \frac{1}{4}\partial_z^2\delta(z-1/2)\right)\right]$$

$$\Psi^{\lambda=1}_{++}(\mathrm{r},z) = \Psi^{\lambda=-1}_{--}(\mathrm{r},z) = \frac{2\pi}{\sqrt{m_c}}\left[A\delta(z-1/2) + \frac{B}{m_c^2}\left(\left(\frac{7}{2}+\mathrm{r}^2 m_c^2\right)\delta(z-1/2) - \frac{1}{4}\partial_z^2\delta(z-1/2)\right)\right]$$

$$\Psi^{\lambda=1}_{+-}(\mathrm{r},z) = -\Psi^{\lambda=1}_{-+}(\mathrm{r},z) = \left(\Psi^{\lambda=-1}_{-+}(\mathrm{r},z)\right)^* = \left(-\Psi^{\lambda=-1}_{+-}(\mathrm{r},z)\right)^* = -\frac{2\pi i}{m_c^{3/2}}B\delta(z-1/2)(r_1+ir_2)$$

$$\Psi^{\lambda=1}_{--}(\mathrm{r},z) = \Psi^{\lambda=-1}_{++}(\mathrm{r},z) = \Psi^{\lambda=0}_{++}(\mathrm{r},z) = \Psi^{\lambda=0}_{--}(\mathrm{r},z) = 0$$

$$A = \phi(0) = 0.213\ \mathrm{GeV}^{3/2}, \quad B = \frac{1}{6}\nabla^2\phi(0) = -0.0157\ \mathrm{GeV}^{7/2}$$