

Color charge correlations in the proton and initial condition for the BK evolution

Heikki Mäntysaari

Mostly based on A. Dumitru, H. Mäntysaari, R. Paatelainen,
arXiv:2103.11682 [hep-ph] and 2105.08503 [hep-ph]

University of Jyväskylä, Department of Physics
Finland

Saturation and Diffraction at the LHC and the EIC / 2021

Motivation

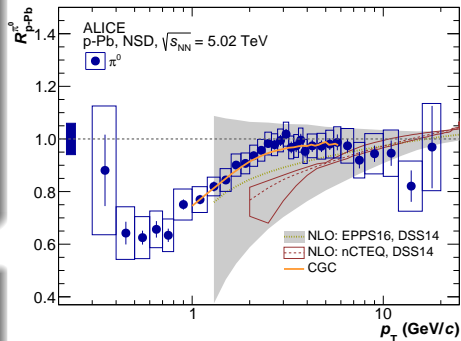
Recipe for successful small- x phenomenology

- Parametrize initial condition for the small- x (BK/JIMWLK) evolution
- Fit parameters describing non-perturbative structure at $x \sim 0.01$ to DIS data
- Compute e.g. particle production in pA, vector meson production at HERA, ...

A necessary ingredient

Dipole-proton scattering amplitude N at moderate x

- Evolution towards small- x perturbatively calculable
- This work: N from LCPT calculations consistently with high- x data $\Rightarrow x, \vec{r}, \vec{b}$ dependent IC for BK



ALICE, 1801.07051

Initial condition for the small-x evolution

Common approach

MV model (derived assuming a large nucleus)

$$\langle \rho^a(\vec{q}_1) \rho^b(\vec{q}_2) \rangle \sim g^2 \mu \delta^{(2)}(\vec{q}_1 + \vec{q}_2)$$

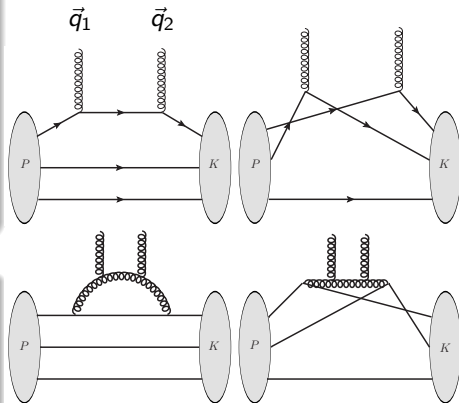
In coordinate space: correlator is local,
 \vec{b} dependence via $\mu \rightarrow \mu(\vec{b})$

This work: LCPT calculation

Dumitru, Skokov, Stebel, 2001.04516 and Dumitru, Miller, Venugopalan, 1808.02501

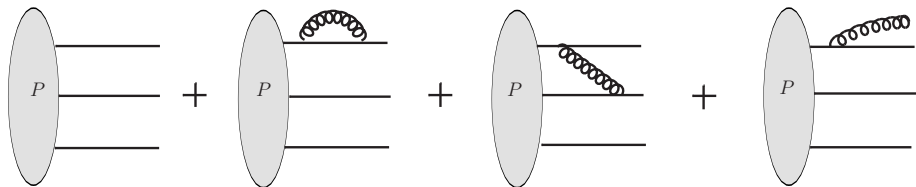
- Compute $\langle \rho^a(\vec{q}_1) \rho^b(\vec{q}_2) \rangle$ directly by attaching two gluons to the proton
- Extension to NLO: include gluon emission

Dumitru, Paatelainen, 2010.11245



A. Dumitru, H.M, R. Paatelainen, 2103.11682

Proton state at moderate x



Leading order:

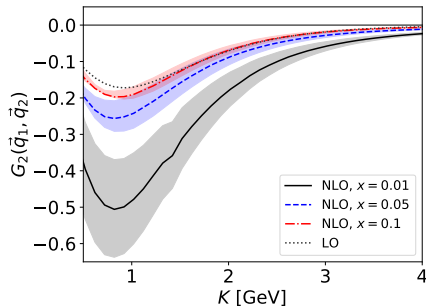
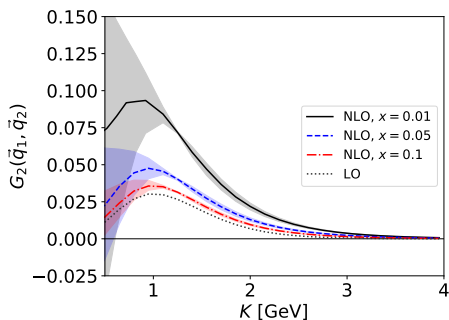
$$\begin{aligned}
 |P\rangle = & \frac{1}{\sqrt{6}} \int \frac{dx_1 dx_2 dx_3}{\sqrt{x_1 x_2 x_3}} \delta(1 - x_1 - x_2 - x_3) \int \frac{d^2 k_1 d^2 k_2 d^2 k_3}{(16\pi^3)^3} 16\pi^3 \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) \\
 & \times \psi(k_1, k_2, k_3) \sum_{i_1, i_2, i_3} \epsilon_{i_1 i_2 i_3} |p_1, i_1; p_2, i_2; p_3, i_3\rangle
 \end{aligned}$$

- Universal valence quark wave function $\psi(k_1, k_2, k_3)$ [Brodsky, Schlumpf, hep-ph/9402214](#)
 Constrained by high- x data (e.g. radius, anomalous magnetic moment)
- Perturbative gluon emission included in [Dumitru, Paatelainen, 2010.11245](#)

Color charge correlator at NLO

The NLO calculation of [2010.11245](#) numerically implemented in [2103.11682](#)

Momentum space: define $\langle \rho^a(\vec{q}_1) \rho^b(\vec{q}_2) \rangle = \delta^{ab} g^2 G_2(\vec{q}_1, \vec{q}_2)$

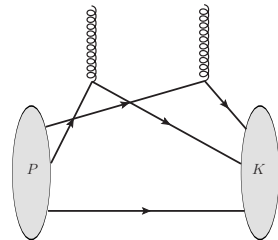
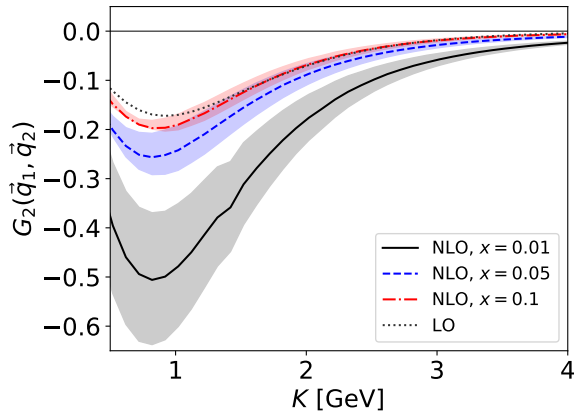


$\vec{q}_1 = \vec{q}_2 = (K/2, 0)$ (*parallel*)

$\vec{q}_1 = (K/\sqrt{2}, 0)$, $\vec{q}_2 = (0, K/\sqrt{2})$ (*perpend*)

- NLO (gluon emission) is small correction at $x = 0.1$ (gluon IR cut), important at $x = 0.01$
- Very different correlator depending on momentum configuration
- Bands: varying collinear regulator

2-body contribution

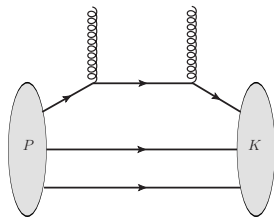
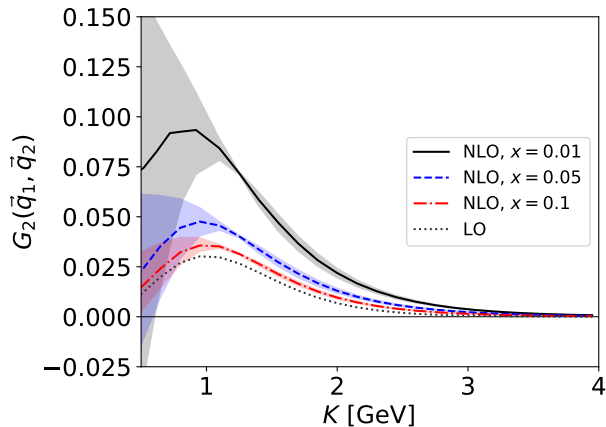


A. Dumitru, H.M, R. Paatelainen, 2103.11682

$$\vec{q}_1 = \vec{q}_2 = (K/2, 0) \text{ (parallel)}$$

- If all momentum is given to a one quark, $\langle P | - | K \rangle$ overlap is highly suppressed
- (Negative) “cat’s ears” diagram dominates at LO, this conclusion still holds at NLO

1-body contribution



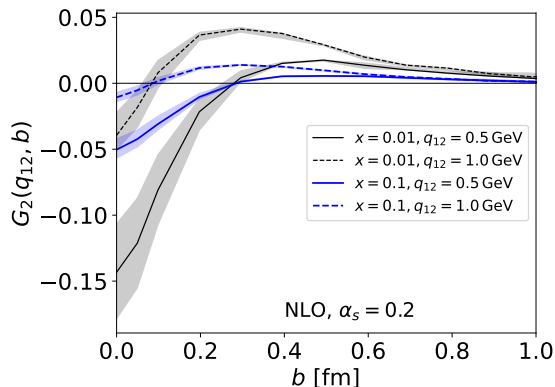
A. Dumitru, H.M, R. Paatelainen, 2103.11682

$$\vec{q}_1 = (K/\sqrt{2}, 0), \vec{q}_2 = (0, K/\sqrt{2}) \text{ (perpend)}$$

- Now much smaller penalty when attaching to only one quark
- (Positive) “handbag” diagram dominates at LO, this conclusion still holds at NLO

Study impact parameter dependence, define $\vec{q}_{12} = \vec{q}_1 - \vec{q}_2$ and

$$G_2(\vec{q}_{12}, \vec{b}) = \int \frac{d^2\vec{K}}{(2\pi)^2} e^{-i\vec{b}\cdot\vec{K}} G_2\left(\frac{\vec{q}_{12} - \vec{K}}{2}, -\frac{\vec{q}_{12} + \vec{K}}{2}\right).$$

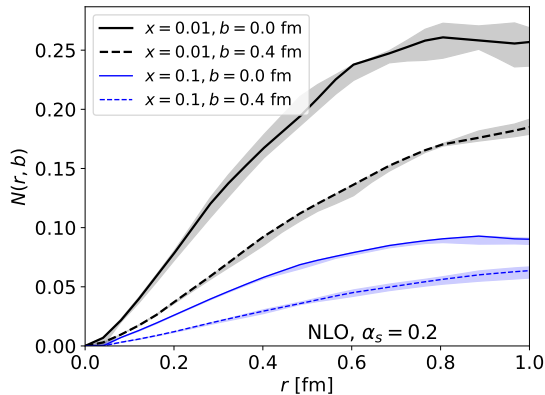


- Repulsive correlations at small b
- Attractive correlations at larger b
- Compare to b -dep MV models where
 - $\sim T_\rho(\vec{b})$
 - \sim our result at large b
- Increasing $|\vec{q}_{12}|$: “handbag” diag more important, less repulsive correlations

To the coordinate space: dipole scattering amplitude

When $N \ll 1$ can derive: [Dumitru et al, 1808.02501](#)

$$N(\vec{r}, \vec{b}) = -g^4 C_F \int \frac{d^2 \vec{K} d^2 \vec{q}}{(2\pi)^4} \frac{\cos(\vec{b} \cdot \vec{K})}{(\vec{q} - \frac{1}{2} \vec{K})^2 (\vec{q} + \frac{1}{2} \vec{K})^2} \times \left(\cos(\vec{r} \cdot \vec{q}) - \cos\left(\frac{\vec{r} \cdot \vec{K}}{2}\right) \right) G_2 \left(\vec{q} - \frac{1}{2} \vec{K}, -\vec{q} - \frac{1}{2} \vec{K} \right).$$

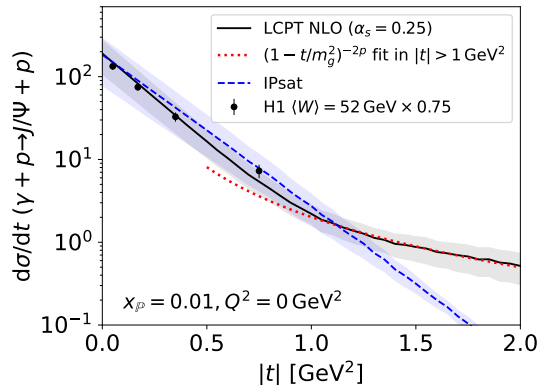


[A. Dumitru, H.M, R. Paatelainen, 2103.11682](#)

Direct calculation: find dipole-target scattering amplitude (with b dependence!) = IC for BK
Reminders: all input from high- x proton valence WF. x is the gluon long. momentum cutoff.

Proton shape

- In exclusive scattering $\sqrt{-t}$ is Fourier conjugate to b
- Coherent J/ψ spectra \leftrightarrow density profile
- Compare to IPsat where $Q_s^2 \sim e^{-b^2/(2B)}$
- LCPT at small t : close to Gaussian
- Large $t \leftrightarrow$ small b : powerlaw, large differences in the region where $\langle \rho\rho \rangle \sim T_\rho$ does not hold
- Scaled H1 data at lower $x_{\mathbb{P}} \approx 0.0035$ is shown for comparison



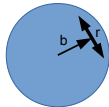
A. Dumitru, H.M, R. Paatelainen, 2105.08503 [hep-ph]

Bands: wave function uncertainty J. Penttala et al, 2006.02830

H1 data: 1304.5162

More differential imaging: angular dependence

Does the dipole prefer to scatter parallel or perpendicular to the impact parameter?



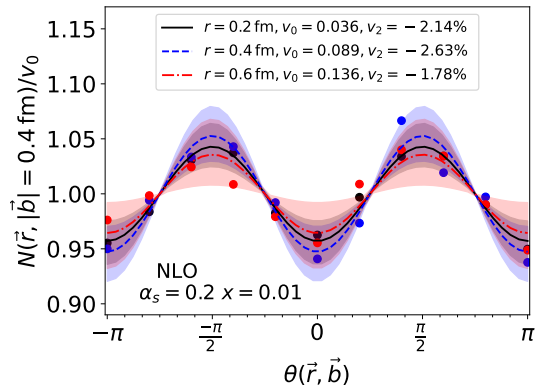
Parametrize:

$$N(\vec{r}, \vec{b}) = N_0(|\vec{r}|, |\vec{b}|)[1 + 2v_2 \cos(2\theta(r, b))]$$

Typically in CGC models:

$$\langle \rho\rho \rangle \sim T_p(b) \Rightarrow v_2 > 0 \quad \text{Iancu, Rezaeian, 1702.03943}$$

- We find $v_2 < 0$ from LCPT calculation
- Weak dependence on $|\vec{r}|$



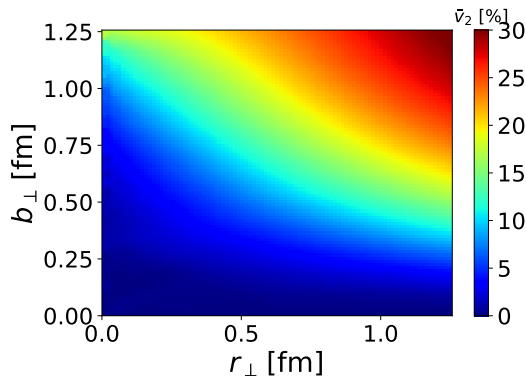
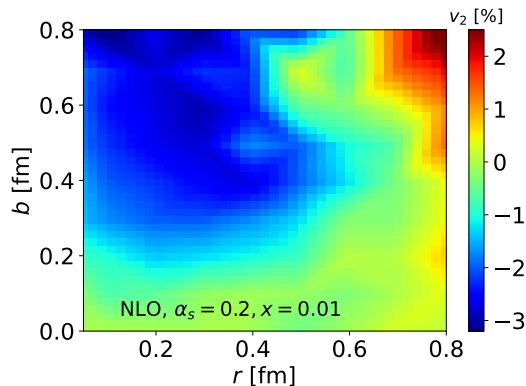
A. Dumitru, H.M. R. Paatelainen, 2103.11682

Dipole- v_2 : LCPT vs MV

Parametrize: $N(\vec{r}, \vec{b}) = N_0(|\vec{r}|, |\vec{b}|)[1 + 2v_2 \cos(\theta(r, b))]$.

LCPT calculation

MV model with $\langle \rho\rho \rangle \sim T_\rho(\vec{b})$



A. Dumitru, H.M, R. Paatelainen, 2103.11682

H.M, K. Roy, F. Salazar, B. Schenke, 2011.02464

Different sign and r, b systematics between LCPT and MV model calculations!

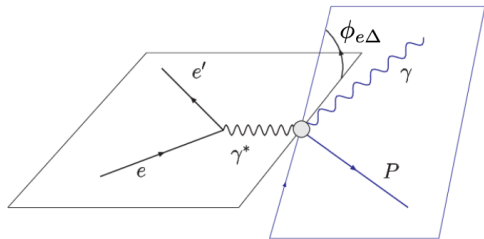
Accessing angular dependence of the dipole amplitude

See Farid's talk: study

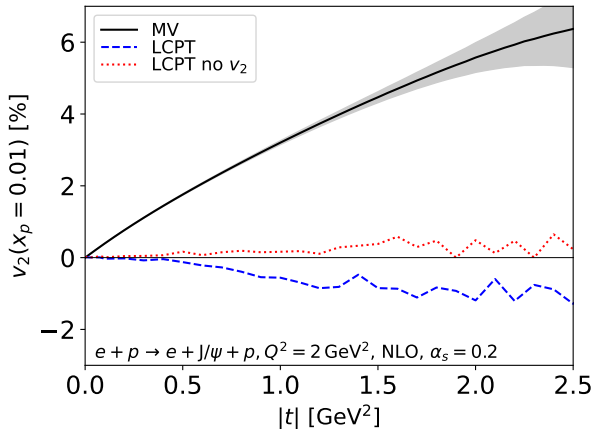
$$e + p \rightarrow J/\psi + e + p$$

Angular dependence in dipole contributes especially to v_2 in

$$d\sigma \sim v_0 [1 + 2v_1 \cos \phi_{e\Delta} + 2v_2 \cos 2\phi_{e\Delta}]$$



CLAS collaboration



Dumitru, H.M., Paatelainen, Roy, Salazar, Schenke, 2105.10144

- Negative v_2 in the LCPT dipole renders the v_2 in J/ψ production negative.
- Note: also kinematical contribution that gives $v_2 > 0$ if dipole does not depend on angles

- Starting point: proton valence quark Fock state
+ perturbative calculation of one gluon emission [A. Dumitru, R. Paatelainen, 2010.11245](#)
- Evaluated color charge correlator $\langle \rho\rho \rangle$ in coordinate and momentum spaces
- Observe significant deviations from “b-dep MV” results (where $\langle \rho\rho \rangle \sim T_p$) especially close to the center of the proton
- The obtained dipole $N(\vec{r}, \vec{b}, x)$ can be used as an initial condition for BK evolution
- Angular dependence of N is very different than in MV model, here find $v_2 < 0$
(=dipole prefers to scatter perpendicular to the impact parameter)