A probabilistic QCD picture of diffractive dissociation and predictions for future EICs

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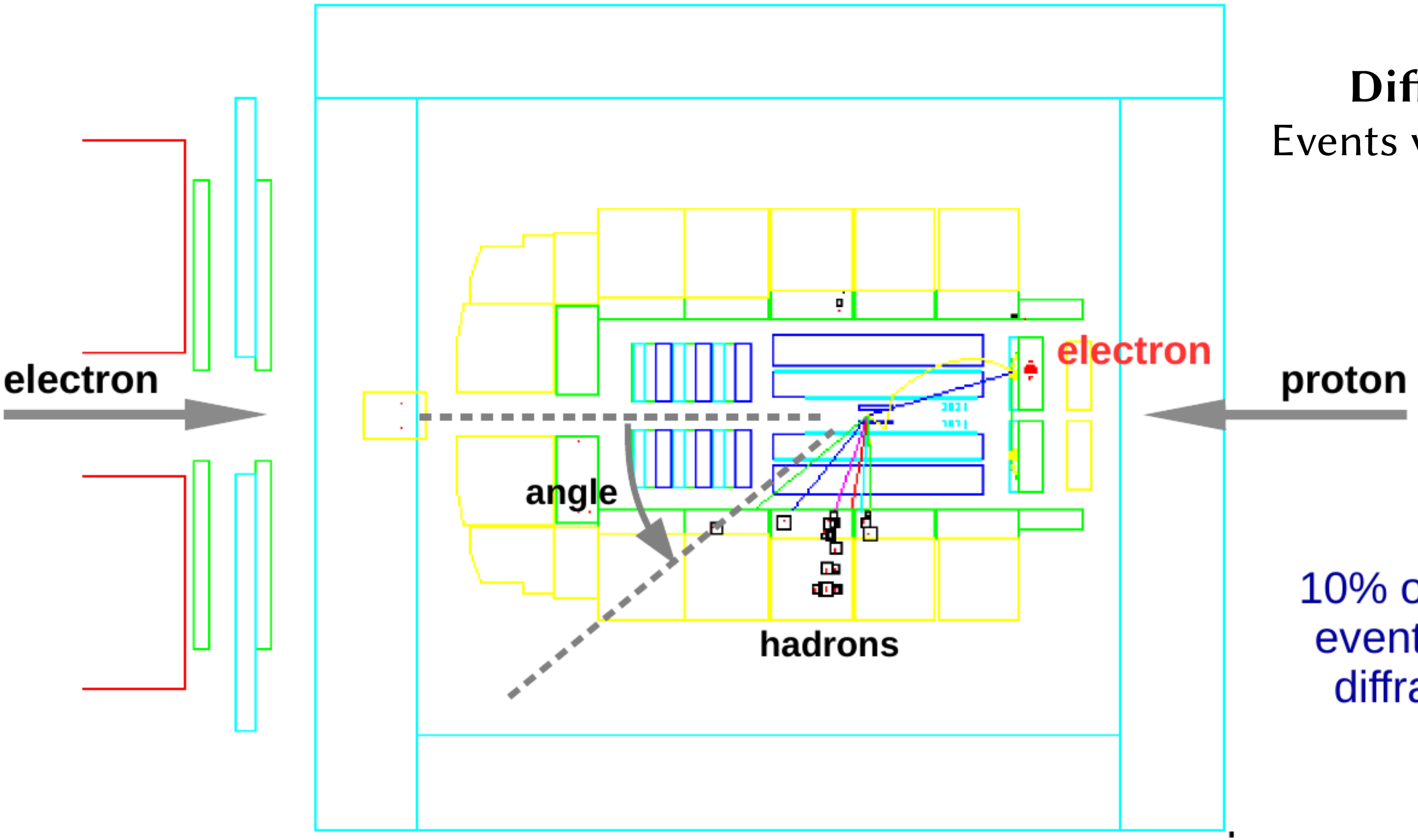
Based on:
ADL, Mueller & Munier, PRD 103 (2021) 054031
ADL, Mueller & Munier, arXiv:2103.10088 (submitted)
ADL, arXiv:2103.07724 (submitted)

Diffractive dissociation in deep-inelastic scattering

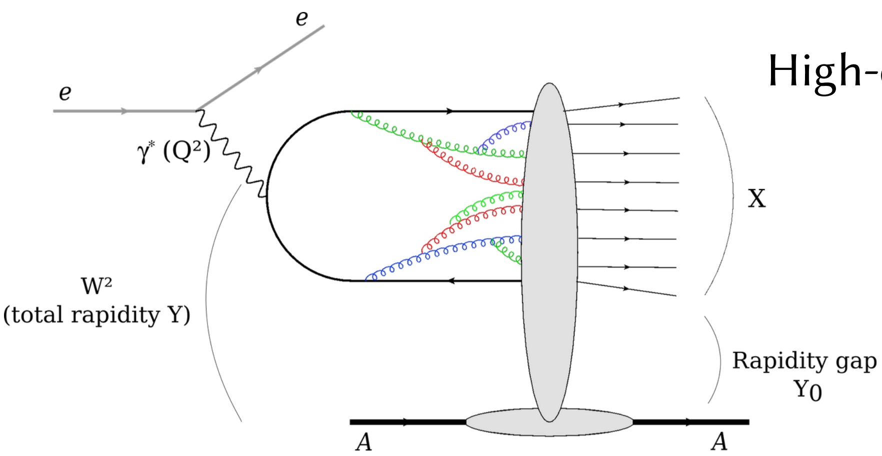


Diffraction at HERA:
Events with large angular gaps observed!

10% of HERA events were diffractive!

Similar observations at future EICs are expected !!!

Diffractive dissociation in deep-inelastic scattering



High-energy deep-inelastic $\gamma^* A$ scattering with dipole factorization

$$\langle \text{photon cross sections} \rangle = \int d^2 r \int_0^1 dz \, |\psi^{\gamma^* \to q\bar{q}}|^2 \, \langle \text{dipole cross sections} \rangle$$

Virtual photon diffraction ~ $q\bar{q}$ dipole diffraction

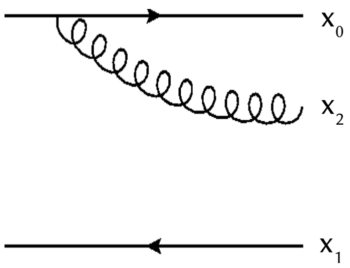
This talk:

- Diffraction in dipole-nucleus scattering
- Diffraction in virtual photon-nucleus scattering

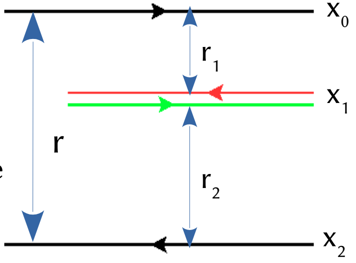
Distribution of the rapidity gap: $\left| \frac{1}{\sigma_{tot}} \frac{d\sigma_{diff}}{dY_0} \right|$?

Diffractive dissociation of small dipole

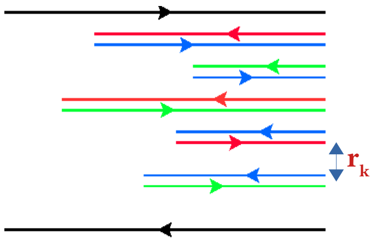
Diffraction in color dipole model



Large N_c
$\approx$
Transverse position space



High-energy boost



Dipole branching proba.

$$dP(r \rightarrow r_1, r_2) = \frac{\bar{\alpha}_s}{2\pi} \frac{r^2}{r_1^2 r_2^2} d^2 r_1 dY$$

Mueller (1993)

High-energy evolution ~ Color dipole branching process

Highly-evolved dipole ~ a set of dipoles with various transverse sizes

$\longrightarrow$ *random dipole density* $n(r_k)$

Diffraction in color dipole model: evolution equations

Forward elastic S-matrix element for dipole-nucleus scattering:

$$S(r, Y=0) = e^{-\frac{r^2 Q_A^2}{4}}$$

McLerran-Venugopalan (1993),
Golec-Biernat & Wusthoff (1998)

(Q_A: nuclear saturation scale)

$$\partial_Y S(r, Y) = \overline{\alpha}_s \int \frac{d^2 r_1}{2\pi} \frac{r^2}{r_1^2 r_2^2} [S(r_1, Y) S(r_2, Y) - S(r, Y)]$$

Balitsky (1996) & Kovchegov (1999)

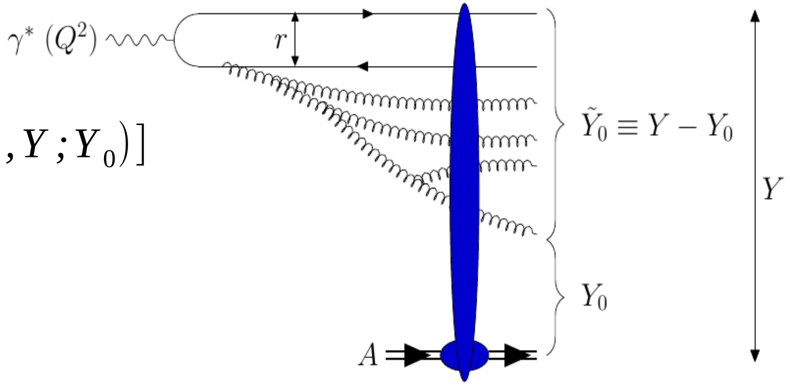
Define S_D at $Y=Y_0$ as: $S_D(r, Y=Y_0; Y_0) = [S(r, Y_0)]^2$

Diffraction with rapidity gap Y_0

For $Y>Y_0$:

$$\partial_Y S_D(r, Y; Y_0) = \overline{\alpha}_s \int \frac{d^2 r_1}{2\pi} \frac{r^2}{r_1^2 r_2^2} [S(r_1, Y; Y_0) S(r_2, Y; Y_0) - S(r, Y; Y_0)]$$

$$\Rightarrow \frac{d\sigma_{tot}}{dY_0} = -\frac{\partial}{\partial Y_0} S_D(r, Y; Y_0)$$



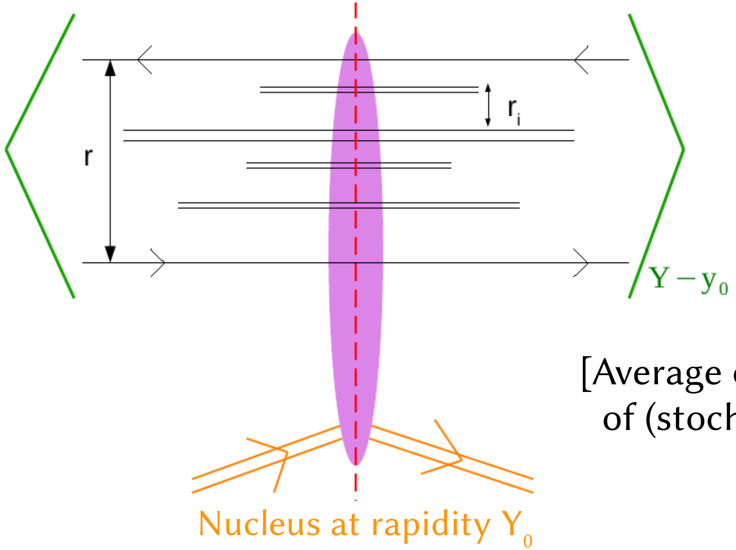
Challenging to solve analytically!

Kovchegov & Levin (2000)

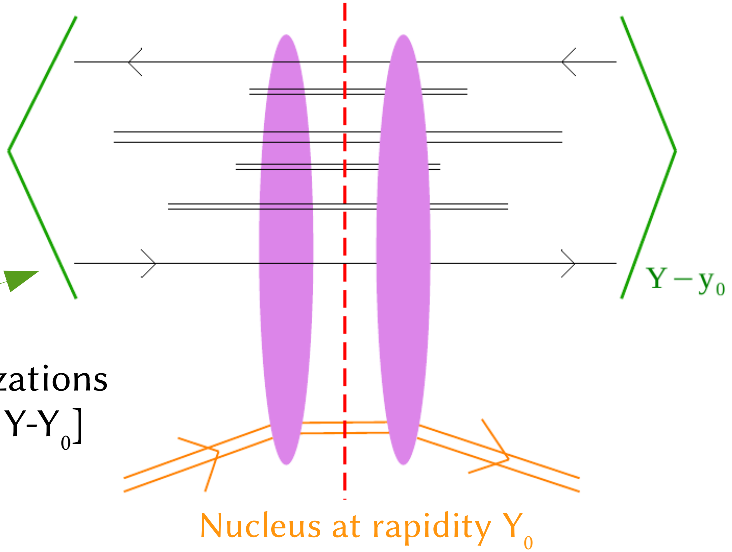
Probabilistic picture (I) : Cross-sections from S-matrix element

Chosen frame: *Nucleus boosted to Y_0*
Onium of size r evolved to $\tilde{Y}_0 = Y - Y_0$

Total rapidity Y



[Average over all possible realizations of (stochastic) dipole state at Y-Y_0]



$$\sigma_{tot} = 2\langle 1 - S(\{r_i\}, Y_0)\rangle_{Y - Y_0}$$

Total cross-section

$$\sigma_{diff} = \langle [1 - S(\{r_i\}, Y_0)]^2\rangle_{Y - Y_0}$$

Diffractive cross-section with minimal gap Y_0

Probabilistic picture (II) : S-matrix element for an event

For a dipole realization with density $n(r_i)$:

Each is S-matrix for the scattering of an dipole of size r_i off a nucleus boosted to Y_0 (solves BK equation)

$$S(\{r_i\},Y_0) = \prod_i S(r_i,Y_0)$$

$$= \prod_{x'}[S(x',Y_0)]^{n(x')dx'} \underset{dx'\to 0}{=} \exp\Big[-\underbrace{\int dx'\, n(x')\ln[1/S(x',Y_0)]}_{I}\Big]$$

Log variable: $x \equiv \ln[1/(r^2 Q_A^2)]$

For an initial dipole smaller than the inverse nuclear saturation scale [$r << 1/Q_S(Y)$]: relevant configurations contain small dipoles x' such that $S(x', Y_0) \simeq 1$

$$\Rightarrow \quad I \simeq \int dx'\, n(x')\big[1 - S(x',Y_0)\big]$$

Overlap of the dipole density and the dipole scattering amplitude ($T = 1 - S$)

Probabilistic picture (III) : Cross-sections of a small dipole

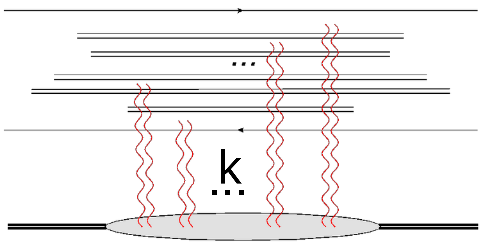
Total cross-section

$$\sigma_{tot} = 2\langle 1-e^{-I}\rangle_{Y-Y_0}$$

$$= 2\sum_{k\geqslant 1}\left\langle \frac{I^k}{k!}e^{-I}\right\rangle_{Y-Y_0}$$

$$\equiv 2\sum_{k\geqslant 1} w_k$$

w_k : proba. that k dipoles effectively scatter



Diffractive cross-section with minimal gap Y_0

$$\sigma_{diff} = \langle [1-e^{-I}]^2\rangle_{Y-Y_0}$$

$$= 2\sum_{k\ even\geqslant 2}\left\langle \frac{I^k}{k!}e^{-I}\right\rangle_{Y-Y_0}$$

$$\equiv 2\sum_{k\ even\geqslant 2} w_k$$

diffractive cross-section
~ proba. of even number of participating dipoles

Remaining issue: **How to average over relevant dipole configurations ??**

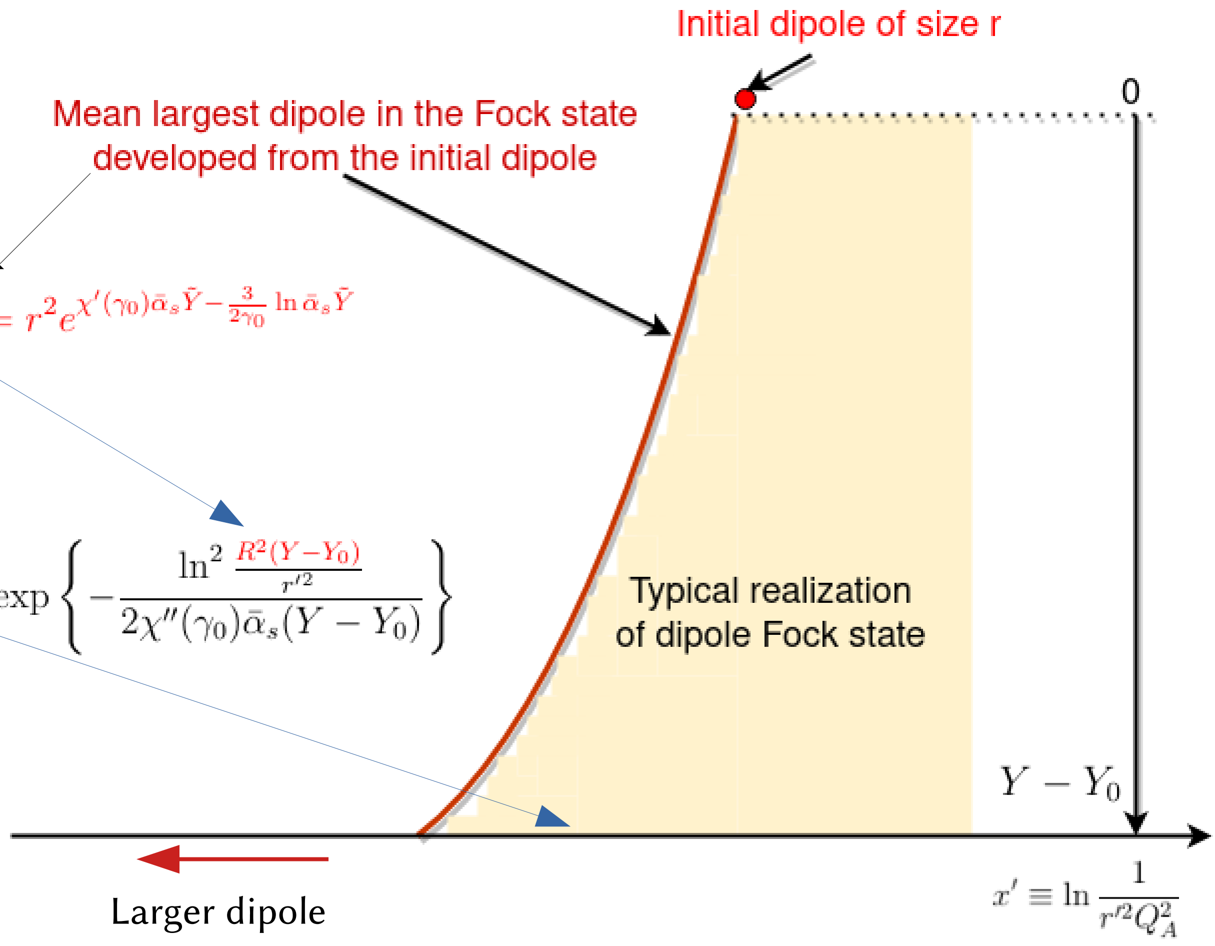
Phenomenological model (I): Deterministic mean-field evolution

$$R^2(\tilde{Y}) = r^2 e^{\chi'(\gamma_0)\bar{\alpha}_s \tilde{Y} - \frac{3}{2\gamma_0}\ln \bar{\alpha}_s \tilde{Y}}$$

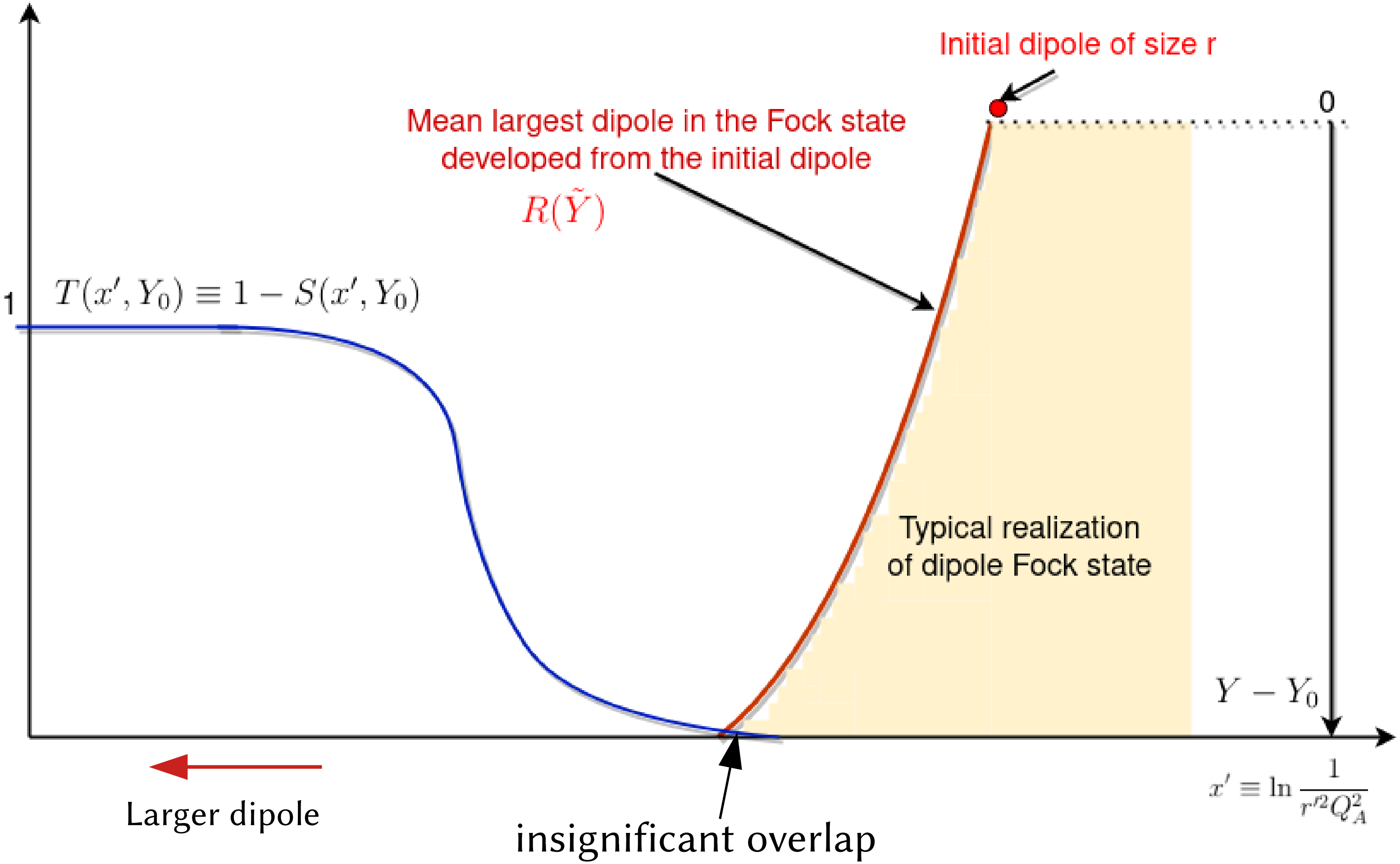
Mean-field density at Y-Y$_0$ (solves BFKL)

$$n(r') = \text{const} \times \ln \frac{R(Y-Y_0)}{r'} \times \left[\frac{R(Y-Y_0)}{r'}\right]^{2\gamma_0} \exp\left\{-\frac{\ln^2 \frac{R^2(Y-Y_0)}{r'^2}}{2\chi''(\gamma_0)\bar{\alpha}_s(Y-Y_0)}\right\}$$

$$\left(\begin{array}{c} \chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma) \\ \gamma_0\chi'(\gamma_0) = \chi(\gamma_0) \\ \gamma_0 \simeq 0.63 \end{array}\right)$$



Phenomenological model (I): Deterministic mean-field evolution



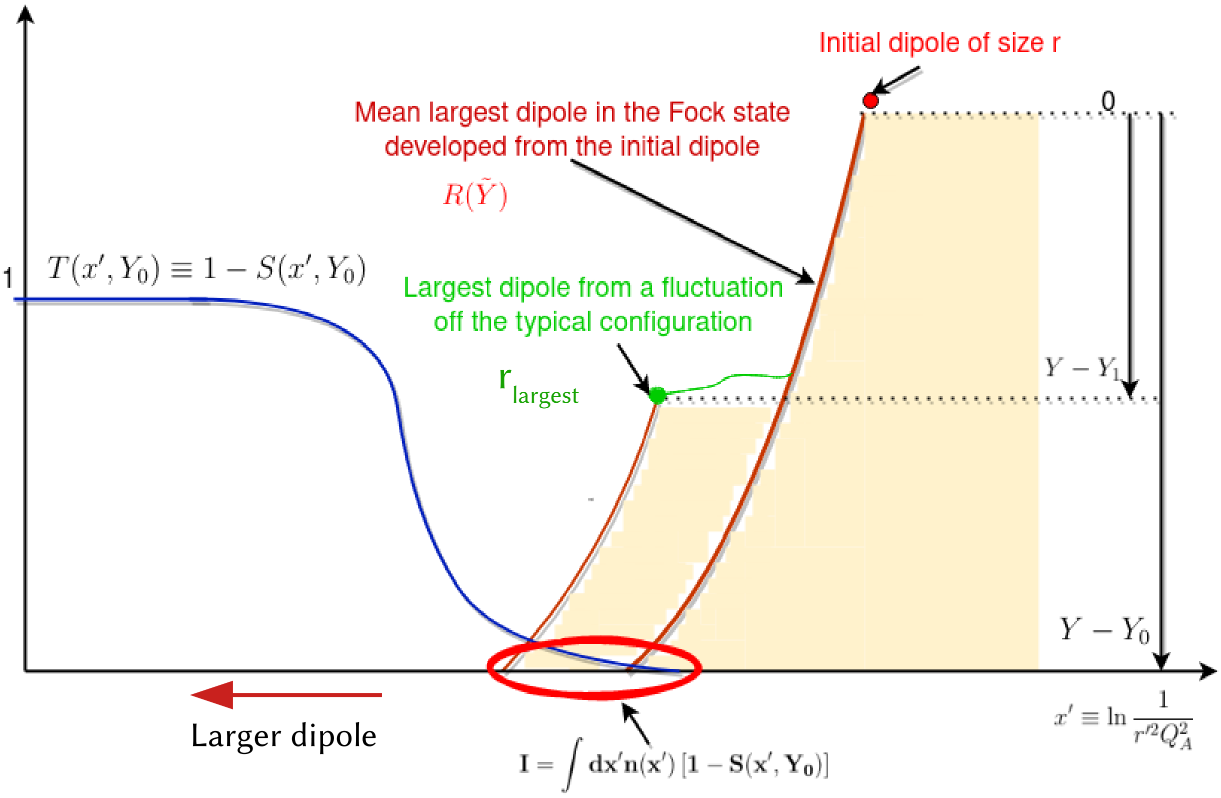
Phenomenological model (II): Large-dipole fluctuation

$$\langle \ldots \rangle_{Y-Y_0} = \int dY_1 \int d\,r_{largest}\, p(r_{largest}, Y-Y_1) \ldots$$

Fluctuation is the largest dipole at Y-Y_1 far from its mean value

Proba. density for a fluctuation creating $r_{largest}$ (solve BK)

$$p(r_{\rm largest}, Y-Y_1) = {\rm const} \times \ln \frac{r_{\rm largest}}{R(Y-Y_1)} \times \left(\frac{r_{\rm largest}}{R(Y-Y_1)}\right)^{2\gamma_0} \exp\left\{-\frac{\ln^2 \frac{r^2_{\rm largest}}{R^2(Y-Y_1)}}{2\chi''(\gamma_0)\bar{\alpha}_s(Y-Y_1)}\right\}$$



Phenomenological model = "mean-field" evolution + 1 single fluctuation

Analytical asymptotics of diffraction: Results

Recall:

$$w_k = \left\langle \frac{I^k}{k!} e^{-I} \right\rangle_{Y-Y_0}, \qquad \sigma_{tot} = 2\sum_{k\geq 1} w_k, \qquad \sigma_{tot} = 2\sum_{k\geq 2, even} w_k$$

Weights:

$$w_1 = c \ln \frac{1}{r^2 Q_S^2(Y)} \left[r^2 Q_S^2(Y)\right]^{\gamma_0}$$

$$w_{k\geq 2} = \frac{c}{\gamma_0} \frac{1}{k(k-1)} \left[1 + \sqrt{\frac{2}{\pi\chi''(\gamma_0)}} \frac{\ln\left[1/r^2 Q_S^2(Y)\right]}{\sqrt{\bar{\alpha}_s Y_0}}\right] \left[r^2 Q_S^2(Y)\right]^{\gamma_0}$$

$$Q_s^2(\tilde{Y}) = Q_A^2 e^{\chi'(\gamma_0)\bar{\alpha}_s \tilde{Y} - \frac{3}{2\gamma_0} \ln \bar{\alpha}_s \tilde{Y}}$$

$$1 \ll \ln^2 \frac{1}{r^2 Q_s^2(Y)} \ll \bar{\alpha}_s Y$$

$$\frac{w_{k\geq 2}}{w_2} = \frac{2}{k(k-1)}$$

Events involving many participating dipoles are not rare!!

Diffractive cross-section for a minimal gap Y_0:

$$\frac{\sigma_{diff}}{\sigma_{tot}} = \frac{\ln 2}{\gamma_0} \left(\frac{1}{\ln\left[1/r^2 Q_S^2(Y)\right]} + \sqrt{\frac{2}{\pi\chi''(\gamma_0)}} \frac{1}{\sqrt{\bar{\alpha}_s Y_0}} \right)$$

Rapidity-gap distribution :

$$\Pi(r, Y; Y_{gap} = Y_0) = \frac{1}{\sqrt{\bar{\alpha}_s}} \frac{\ln 2}{\gamma_0 \sqrt{2\pi\chi''(\gamma_0)}} \left[\frac{Y}{Y_0(Y - Y_0)}\right]^{3/2} \exp\left(-\frac{\ln^2\left[r^2 Q_S^2(Y)\right]}{2\chi''(\gamma_0)\bar{\alpha}_s (Y - Y_0)} \right)$$

Diffraction at electron-ion colliders

Diffractive dissociation of virtual photon

Diffractive cross section with minimal gap Y_0 :

$$\left(\frac{\sigma_{diff}}{\sigma_{tot}}\right)^{\gamma^* A} = \frac{\int d^2r \int_0^1 dz \sum_{p=L,T;f} |\psi_p^f(r,z,Q^2)|^2 \left[\mathbf{1} - \mathbf{2S(r,Y)} + \mathbf{S_D(r,Y;Y_0)}\right]}{\int d^2r \int_0^1 dz \sum_{p=L,T;f} |\psi_p^f(r,z,Q^2)|^2 \; \mathbf{2}\left[\mathbf{1} - \mathbf{S(r,Y)}\right]}$$

Rapidity gap distribution:

$$\mathfrak{R}^{\gamma * A} = -\frac{\partial}{\partial Y_0}\left(\frac{\sigma_{diff}}{\sigma_{tot}}\right)^{\gamma^* A}$$

BK equation for S and S_D:

$$\partial_Y \mathfrak{S}_r = \int d^2 r_1 K(r, r_1, r_2)\left[\mathfrak{S}_{r_1}\mathfrak{S}_{r_2} - \mathfrak{S}_r\right]$$

Kernel K:

+ Fixed coupling: $K^{fc} = \frac{\bar{\alpha}_s}{2\pi}\frac{r^2}{r_1^2 r_2^2}$

+ Running coupling:

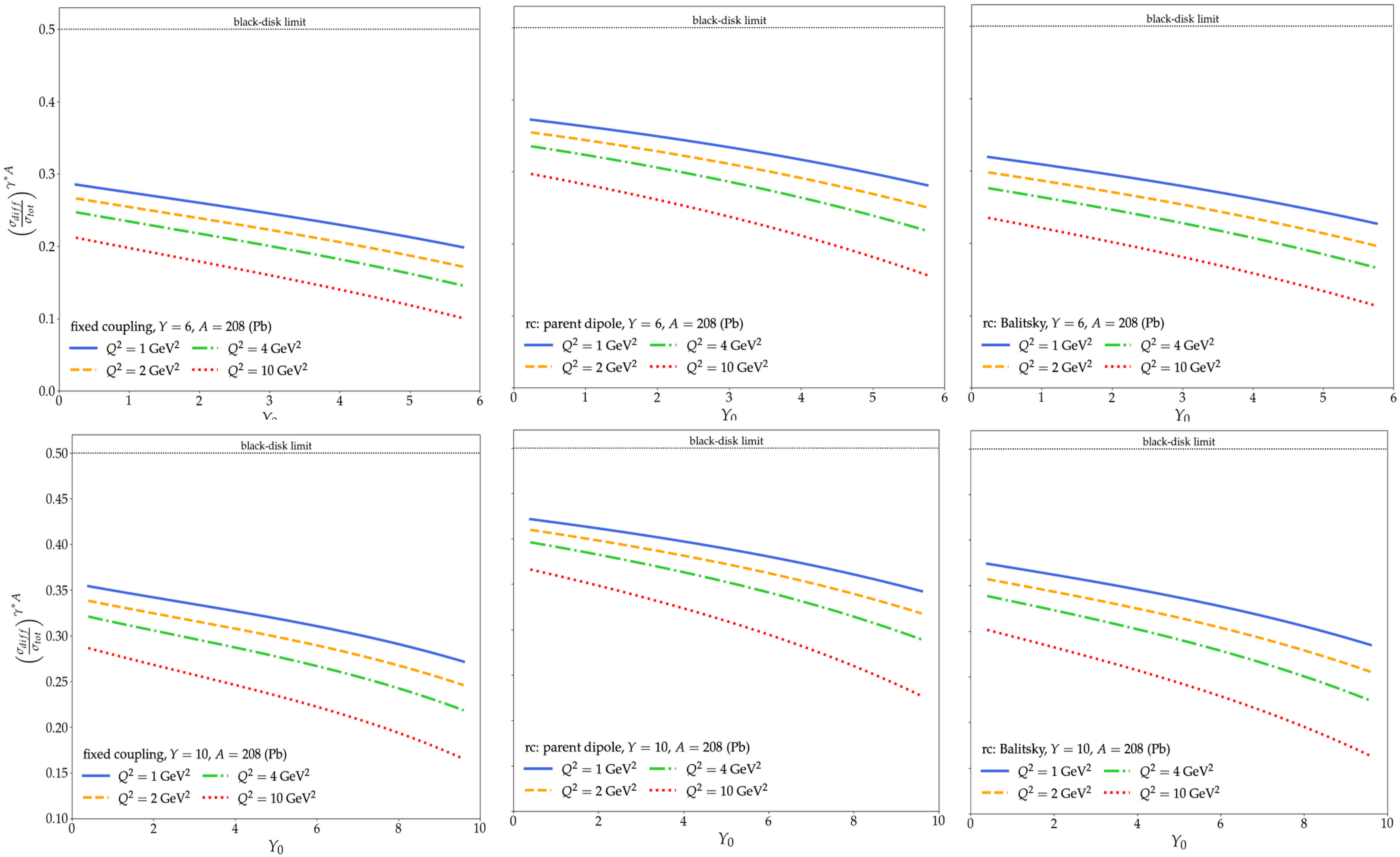
$$K^{pd} = \frac{\bar{\alpha}_s(r^2)}{2\pi}\frac{r^2}{r_1^2 r_2^2} \quad \text{(parent dipole presc.)}$$

$$K^{Bal} = \frac{\bar{\alpha}_s(r^2)}{2\pi}\left[\frac{r^2}{r_1^2 r_2^2} + \frac{1}{r_1^2}\left(\frac{\bar{\alpha}_s(r_1^2)}{\bar{\alpha}_s(r_2^2)} - 1\right) + \frac{1}{r_2^2}\left(\frac{\bar{\alpha}_s(r_2^2)}{\bar{\alpha}_s(r_1^2)} - 1\right)\right] \quad \text{(Balitsky presc.)}$$

Kinematics accessible at BNL-EIC and LHeC:

- Rapidity: $Y = 6, 10$ ($x \approx 2\text{x}10^{-3}, 5\text{x}10^{-5}$, resp.)
- Photon virtuality: $Q^2 = 1 - 10$ GeV^2

Result (1/3): Diffractive scattering with a minimal gap

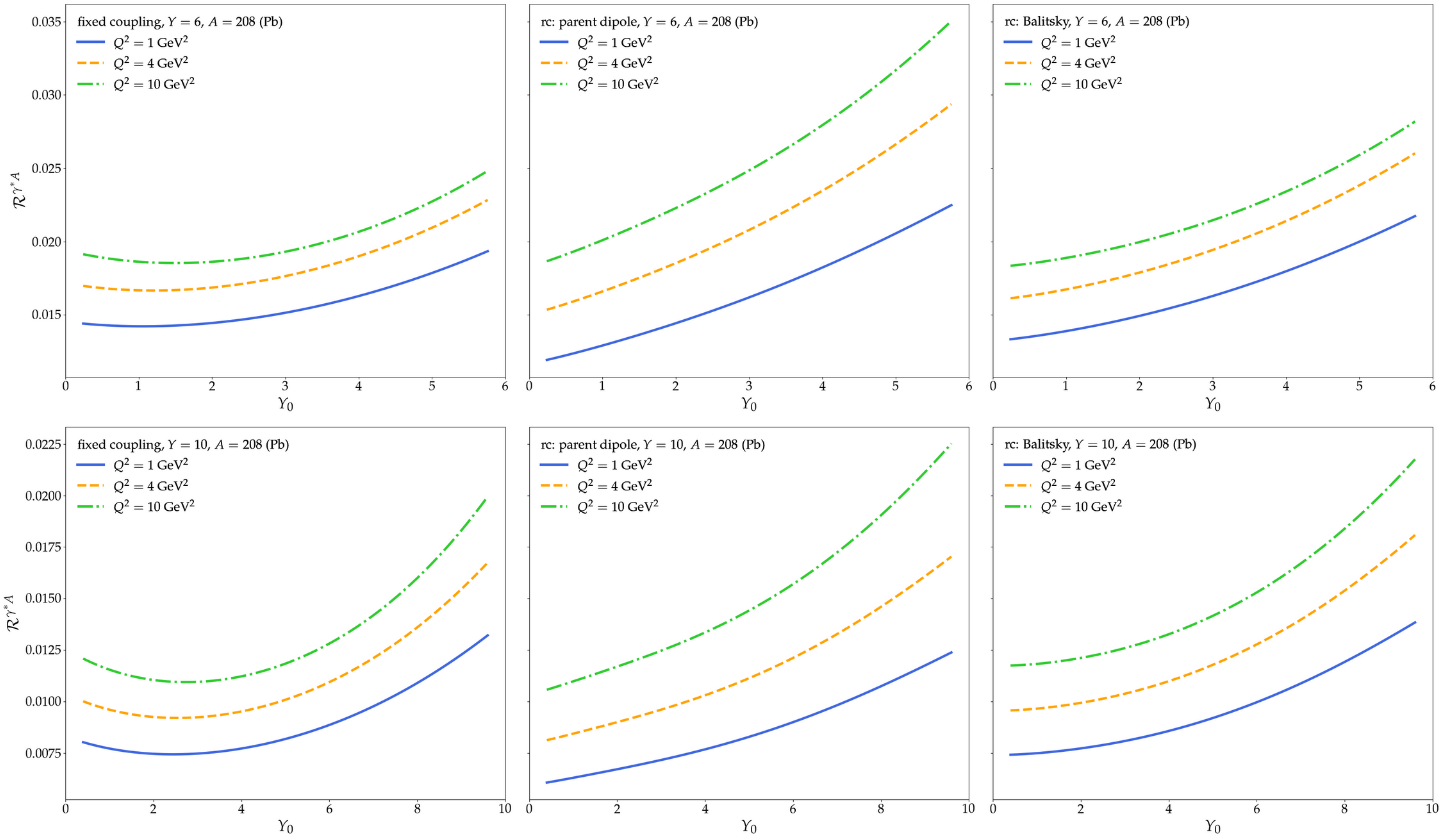


Diffractive cross section:

$$\left(\frac{\sigma_{diff}}{\sigma_{tot}}\right)^{\gamma^* A}(Q^2, Y, Y_0)$$

Y_0: minimal rapidity gap

Result (2/3): Rapidity gap distribution



$$\mathfrak{R}^{\gamma^* A} = -\frac{\partial}{\partial Y_0}\left(\frac{\sigma_{diff}}{\sigma_{tot}}\right)^{\gamma^* A}$$

(Y_0: rapidity gap)

Conclusions

i. The **parameter-free** expressions of ***the asymptotic*** ***rapidity gap distribution*** for small dipole-nucleus scattering

$$\Pi(r,Y;Y_{gap}=Y_0)=\frac{1}{\sqrt{\bar{\alpha}_s}}\frac{\ln 2}{\gamma_0\sqrt{2\pi\chi''(\gamma_0)}}\left[\frac{Y}{Y_0(Y-Y_0)}\right]^{3/2}\exp\left(-\frac{\ln^2\left[r^2Q_S^2(Y)\right]}{2\chi''(\gamma_0)\bar{\alpha}_s(Y-Y_0)}\right)$$

$\longrightarrow$ Diffraction is due to large-dipole fluctuation in the course of the QCD evolution of the dipole Fock state.

$\longrightarrow$ Multiple exchanges are typical.

ii. Diffractive DIS at EIC/LHeC is studied: Predictions for the rapidity gap distribution and diffractive-to-total scattering ratio.

- Diffrerent scenarios are discussed

Outlook:

i. Analytical study of diffraction with running-coupling corrections

ii. Determination of sub-asymptotic (finite-Y) corrections