NLO corrections to dijet production in DIS in the Color Glass Condensate

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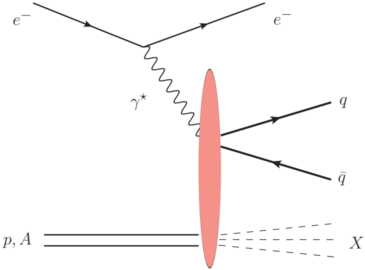
Motivations

- Inclusive dijet (or dihadron) production in DIS at small x:
  - $\Rightarrow$ probe of the saturated regime of QCD
  - $\Rightarrow$ access to the Weizsäcker-Williams gluon distribution
  - $\Rightarrow$ and to the quadrupole correlator of Wilson lines



- Reliable QCD prediction requires to account for NLO corrections.
- Systematic determination of the theoretical uncertainties.

Some existing results on NLO dijet in the CGC
(NLO impact factor)

- Fully inclusive DIS (= structure functions)
  - computed in light-cone perturbation theory.
    Beuf, 1708.06557, Hänninen, Lappi, Paatelainen, 1711.08207
  - or using an OPE approach
    Balitsky, Chirilli, 1207.3844
  - numerical results available, fits to HERA data.
    Beuf, Hänninen, Lappi, Mäntysaari, 2007.01645
- Exclusive dijet
  Boussarie, Grabovsky, Szymanowski, Wallon. 1606.00419 - 1905.07371
  - color correlators in momentum space
- Other related processes
  - dijet+photon Roy, Venugopalan, 1911.04530
  - dijet production in p-A collisions Iancu, Mulian, 2009.11930
  - exclusive J/Ψ Mäntysaari, Penttala, 2104.02349

  See also yesterday's talks by Jani Penttala and Yair Mulian

Goal of this presentation and main results

- Full NLO computation in the Regge limit $s \gg Q^2 \gg \Lambda_{\mathrm{QCD}}$, with completely general kinematics of the dijet system.
- Both longitudinal and transverse $\gamma^\star$.

Two main results

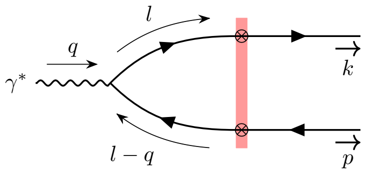
- *Conceptual*: **cancellation of the divergences** UV, slow (aka rapidity) and collinear. Proof of JIMWLK factorization for a process with non-trivial final state.
- *Practical*: **numerically tractable** expressions for the NLO **impact factor** (in the spirit of the results for inclusive DIS).

Outline of my talk

- Overview of the formalism and sketch of the calculation.
- Cancellation of the UV and IR divergences
- Present final formulas relevant for the NLO impact factor.

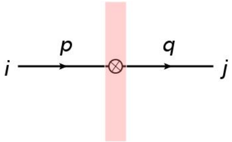
Dipole picture and CGC effective field theory

- We work in the dipole picture of DIS.



- Multiple gluon interactions with the target resummed via path-ordered Wilson lines $V(\boldsymbol{x}_\perp)$ = "shock-wave formalism".
- CGC effective vertex:

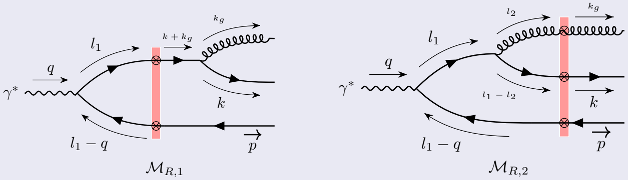
$\mathcal{T}^{q}_{ij}(q,p) \equiv$



$$= (2\pi)\delta(q^- - p^-)\gamma^- \int \mathrm{d}^2\boldsymbol{x}_\perp e^{-i(\boldsymbol{q}_\perp - \boldsymbol{p}_\perp)\boldsymbol{x}_\perp} V_{ij}(\boldsymbol{x}_\perp)$$

The diagrams we need to compute (1/2)

Real diagrams

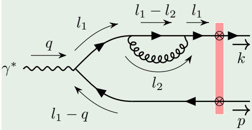


$+(q \longleftrightarrow \bar{q})$ $\quad$ $+(q \longleftrightarrow \bar{q})$

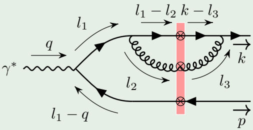
- Already computed by Ayala, Hentschinski, Jalilian-Marian, Tejeda-Yeomans, 1701.07143 using spinor helicities techniques.
- We recover their results.

The diagrams we need to compute (2/2)

Self-energies



$+(q \longleftrightarrow \bar{q})$



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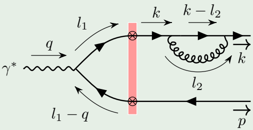
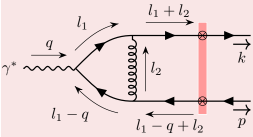
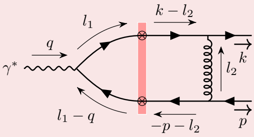
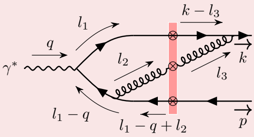


$+(q \longleftrightarrow \bar{q})$

Vertex corrections





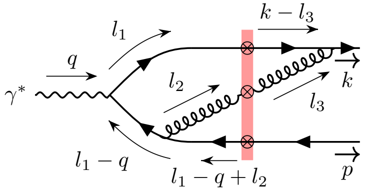


$+(q \leftrightarrow \bar{q})$

Advantages of covariant perturbation theory

- We use standard **covariant** perturbation theory (not LCPT).
- Setting up the calculation is very easy, standard QFT techniques are applicable.
- Example: dressed vertex correction

$$\mathcal{M}^{\lambda}_{\mathrm{V},3} = \int \frac{\mathrm{d}^4 l_1}{(2\pi)^4}\frac{\mathrm{d}^4 l_2}{(2\pi)^4}\frac{\mathrm{d}^4 l_3}{(2\pi)^4}[\bar{u}(k)(ig\gamma^{\mu}t^{a}) \overbrace{S^0(k-l_3)}^{\text{free propagator}} \mathcal{T}^q(k-l_3,l_1)S^0(l_1)$$
$$(-ie\not{\epsilon}(\lambda,q))S^0(l_1-q)(ig\gamma^{\nu}t^{b})S^0(l_1-q+l_2)\underbrace{\mathcal{T}^q(l_1-q+l_2,-p)}_{\text{CGC vertex}}v(p)]$$
$$\times\, G^{0,ac}_{\mu\rho}(l_3)\mathcal{T}^{g,\rho\sigma}_{cd}(l_3,l_2)G^{0,db}_{\sigma\nu}(l_2)$$



Recovering instantaneous LCPT diagrams

- The amplitude has the following structure

$$\mathcal{M}^\lambda = \frac{ee_f q^-}{\pi} \int_{\boldsymbol{x}_\perp, \boldsymbol{y}_\perp, \boldsymbol{z}_\perp} e^{-i\boldsymbol{k}_\perp \boldsymbol{x}_\perp - i\boldsymbol{p}_\perp \boldsymbol{y}_\perp + \ldots} \overbrace{\mathcal{C}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp, \boldsymbol{z}_\perp)}^{\text{color structure}} \underbrace{\mathcal{N}^\lambda(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp, \boldsymbol{z}_\perp)}_{\text{perturbative factor}}$$

- Computing explicitly the perturbative factor is the tough part!

$$\mathcal{N}^\lambda(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp, \boldsymbol{z}_\perp) = \int_{l_1, l_2, \ldots} \frac{\overbrace{N^\lambda(l_1, l_2, \ldots)}^{\text{Dirac structure}} \delta(l_1^- - \ldots) \ldots e^{i\boldsymbol{l}_{1\perp}\cdot(\ldots) + i\boldsymbol{l}_{2\perp}\cdot(\ldots) + \ldots}}{[l_1^2 + i\epsilon]\,[(q-l_1)^2 + i\epsilon]\,[l_2^2 + i\epsilon]\ldots}$$

- Using standard Dirac algebra,

$$N^\lambda(l_1, l_2, \ldots) = N^\lambda_{\mathrm{reg}}(\boldsymbol{l}_{1\perp}, \boldsymbol{l}_{2\perp}, \ldots) + \overbrace{(l_1 - q)^2}^{\text{cut the propagator}} N^\lambda_{\mathrm{ins,a}}(\boldsymbol{l}_{1\perp}, \boldsymbol{l}_{2\perp}, \ldots) + \ldots$$

- The cut propagators correspond to **instantaneous diagrams** in LCPT!

Cancellation of the divergences

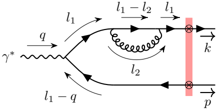
Cancellation of the divergences

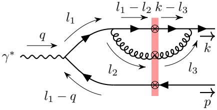
What kind of divergence do we get?

- UV (short distance) divergences
  - internal momentum goes to ∞ or $|\boldsymbol{z}_\perp - \boldsymbol{x}_\perp| \to 0$.
  - we use dim. reg. in the transverse plane to extract the UV pole of each diagram if any.
- Rapidity divergence, "slow gluon" when $k_g^- \to 0$.
- Soft divergence $k_g^\mu \to 0$: subset of the slow divergence.
- Collinear divergence, $z_q \boldsymbol{k}_{\perp g} - z_g \boldsymbol{k}_\perp \to 0$ or $z_{\bar{q}} \boldsymbol{k}_{\perp g} - z_g \boldsymbol{p}_\perp \to 0$.

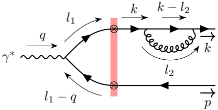
Cancellation of UV divergences

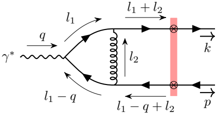
- Massless quark + universality of quark electric charge ⇒ no need for UV renormalization.
- UV divergence cancels between free self-energy before shock-wave and dressed self energy





- The free self-energies after SW turn UV divergence of the free vertex correction before shock-wave into IR





Remaining $\frac{2}{\varepsilon_{\mathrm{IR}}}$ pole canceled by the real corrections.

$+(q \leftrightarrow \bar{q})$

$\propto \left(\frac{2}{\varepsilon_{\mathrm{IR}}} - \frac{2}{\varepsilon_{\mathrm{UV}}}\right)$

$\propto \frac{2}{\varepsilon_{\mathrm{UV}}}$

Cancellation of rapidity divergences

- Rapidity divergence is regularized with a longitudinal momentum cut-off Λ^-.
- The slow gluon phase space is divided using a factorization scale k_f^-.
- One shows then that

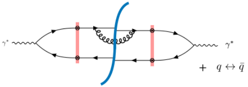
$$\mathrm{d}\sigma_{\mathrm{NLO}}^{\gamma^\star \to q\bar{q}+X} = \alpha_s \ln\left(\frac{k_f^-}{\Lambda^-}\right) \underbrace{\mathcal{H}_{\mathrm{JIMWLK}} \otimes \mathrm{d}\sigma_{\mathrm{LO}}^{\gamma^\star \to q\bar{q}+X}}_{\text{action of JIMWLK on the LO x-section}} + \overbrace{\text{finite}}^{\Lambda^- \to 0}$$

- Thus, the Λ^- dependence of the NLO impact factor is canceled by the JIMWLK evolution of the LO cross-section from Λ^- to k_f^-.

Cancellation of collinear divergences

- $1/\varepsilon$ pole from the free vertex correction before shock-wave remains:

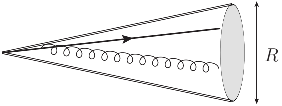
$$\mathrm{d}\sigma^{\gamma^*_\lambda \to q\bar{q}+X} = \frac{\alpha_s C_F}{\pi}\left(\ln\left(\frac{k^-}{k_f^-}\right) + \ln\left(\frac{p^-}{k_f^-}\right) - \frac{3}{2}\right) \times \frac{2}{\varepsilon} \times \mathrm{d}\sigma_{\mathrm{LO}}^{\gamma^*_\lambda + A \to q\bar{q}+X}$$



$$+ \int_{k_f^-} \frac{\mathrm{d}k_g^-}{k_g^-} \frac{\mathrm{d}^2\boldsymbol{k}_{\perp,g}}{(2\pi)^2}\, \mathrm{d}\sigma^{\gamma^*_\lambda \to q\bar{q}g+X}\, \mathcal{J}_R(k^\mu, p^\mu, k_g^\mu) + \text{finite}$$

$\rightarrow$

- The pole cancels for IRC safe observable only $\Rightarrow$ jets.
  - $\mathcal{J}_R(k^\mu, p^\mu, k_g^\mu)$ "recombines" two partons into one jet if they lie inside the same cone of opening angle R.



- The slow gluon phase space is $k_g^- < k_f^-$ is excluded.

Cancellation of collinear divergences

- $1/\varepsilon$ pole from the free vertex correction before shock-wave remains:

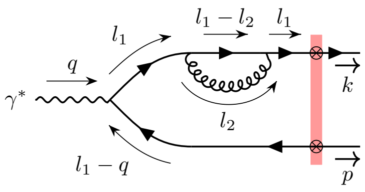
$$\mathrm{d}\sigma^{\gamma_\lambda^* \to q\bar{q}+X} = \frac{\alpha_s C_F}{\pi}\left(\ln\left(\frac{k^-}{k_f^-}\right) + \ln\left(\frac{p^-}{k_f^-}\right) - \frac{3}{2}\right) \times \frac{2}{\varepsilon} \times \mathrm{d}\sigma_{\mathrm{LO}}^{\gamma_\lambda^*+A\to q\bar{q}+X}$$
$$-\frac{\alpha_s C_F}{\pi}\left(\ln\left(\frac{k^-}{k_f^-}\right) + \ln\left(\frac{p^-}{k_f^-}\right) - \frac{3}{2}\right) \times \frac{2}{\varepsilon} \times \mathrm{d}\sigma_{\mathrm{LO}}^{\gamma_\lambda^*+A\to q\bar{q}+X}$$
$$+ \text{finite}$$

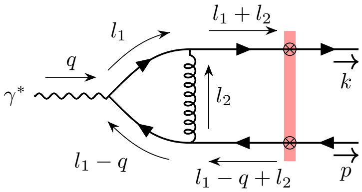
- The integration in $4-\varepsilon$ dimension over the gluon when it belongs to the q or $\bar{q}$ jet exactly cancels the virtual pole.

Some final results

Final expressions (examples)

Free gluon propagators before shock-wave

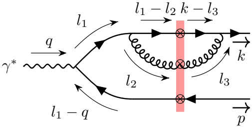




- Identical to the one-loop QCD corrections to the $\gamma^\star \to q\bar{q}$ light-cone wave-function.
- We recover the results obtained by Beuf, 1606.00777Hanninen, Lappi,Paatelainen 1711.08207.

Dressed self-energy

Longitudinal polarization



- Finite piece extracted following the method in 1711.0820.

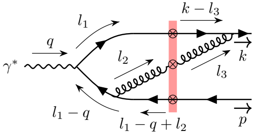
$$\mathcal{M}_{\rm SE}^{\lambda=0} = \frac{ee_f q^-}{\pi} \int \mathrm{d}^2\boldsymbol{x}_\perp \mathrm{d}^2\boldsymbol{y}_\perp e^{-i\boldsymbol{k}_\perp \boldsymbol{x}_\perp - i\boldsymbol{p}_\perp \boldsymbol{y}_\perp} (-2)(z_q z_{\bar{q}})^{3/2} Q \delta_{\sigma,-\bar{\sigma}} \frac{\alpha_s}{\pi^2} \int_0^{z_q} \frac{\mathrm{d}z_g}{z_g} \left[1 - \frac{z_g}{z_q} + \frac{z_g^2}{2z_q^2}\right]$$
$$\times \int \frac{\mathrm{d}^2\boldsymbol{z}_\perp}{\boldsymbol{r}_{zx}^2} \Bigg\{ e^{-i\frac{z_g}{z_q}\boldsymbol{k}_\perp \boldsymbol{r}_{zx}} K_0\left(\bar{Q}\sqrt{\boldsymbol{R}_{\rm SE}^2 + \omega_{\rm SE}\boldsymbol{r}_{zx}^2}\right) \left[t^a V(\boldsymbol{x}_\perp) V^\dagger(\boldsymbol{z}_\perp) t_a V(\boldsymbol{z}_\perp) V^\dagger(\boldsymbol{y}_\perp) - t^a t_a\right]$$
$$- e^{-\frac{\boldsymbol{r}_{zx}^2}{\boldsymbol{r}_{xy}^2 e^{\gamma_E}}} K_0\left(\bar{Q} r_{xy}\right) \left[C_F V(\boldsymbol{x}_\perp) V^\dagger(\boldsymbol{y}_\perp) - t^a t_a\right] \Bigg\} + \mathcal{M}_{\rm UV}^{\lambda=0}$$

- UV divergent piece in dim. reg.

$$\mathcal{M}_{\rm UV}^{\lambda=0} = \frac{ee_f q^-}{\pi} \mu^{-2\varepsilon} \int \mathrm{d}^{2-\varepsilon}\boldsymbol{x}_\perp \mathrm{d}^{2-\varepsilon}\boldsymbol{y}_\perp e^{-i\boldsymbol{k}_\perp \boldsymbol{x}_\perp - i\boldsymbol{p}_\perp \boldsymbol{y}_\perp} [V(\boldsymbol{x}_\perp) V^\dagger(\boldsymbol{y}_\perp) - 1] \mathcal{N}_{{\rm LO},\varepsilon}^{\lambda=0}(\boldsymbol{r}_{xy})$$
$$\times \frac{\alpha_s C_F}{2\pi} \left\{ \left(2\ln\left(\frac{z_q}{z_0}\right) - \frac{3}{2}\right)\left(\frac{2}{\varepsilon} + \ln(e^{\gamma_E}\pi\mu^2 \boldsymbol{r}_{xy}^2)\right) - \frac{1}{2} \right\}$$

Dressed vertex correction

Longitudinal polarization



$$\mathcal{M}_{\mathrm{V}}^{\lambda=0} = \frac{ee_f q^-}{\pi} \int \mathrm{d}^2\boldsymbol{x}_\perp \mathrm{d}^2\boldsymbol{y}_\perp \mathrm{d}^2\boldsymbol{z}_\perp e^{-i\boldsymbol{k}_\perp \boldsymbol{x}_\perp - i\boldsymbol{p}_\perp \boldsymbol{y}_\perp} [t^a V(\boldsymbol{x}_\perp) V^\dagger(\boldsymbol{z}_\perp) t_a V(\boldsymbol{z}_\perp) V^\dagger(\boldsymbol{y}_\perp) - t^a t_a]$$
$$\times \frac{\alpha_s}{\pi^2} 2(z_q z_{\bar{q}})^{3/2} Q \delta_{\sigma,-\bar{\sigma}} \int_0^{z_q} \frac{\mathrm{d}z_g}{z_g} e^{-iz_g \boldsymbol{k}_\perp / z_q \cdot \boldsymbol{r}_{zx}} \left(1 + \frac{z_g}{z_{\bar{q}}}\right)\left(1 - \frac{z_g}{z_q}\right) K_0\left(\Delta_{\mathrm{V},3}\sqrt{\boldsymbol{R}_V^2 + \omega_{\mathrm{V}} \boldsymbol{r}_{zy}^2}\right)$$
$$\times \left\{\left[1 - \frac{z_g}{2z_q} - \frac{z_g}{2(z_{\bar{q}} + z_g)}\right] \frac{\boldsymbol{r}_{zx} \cdot \boldsymbol{r}_{zy}}{\boldsymbol{r}_{zx}^2 \boldsymbol{r}_{zy}^2} + i\sigma \left[\frac{z_g}{2z_q} - \frac{z_g}{2(z_{\bar{q}} + z_g)}\right] \frac{\boldsymbol{r}_{zx} \times \boldsymbol{r}_{zy}}{\boldsymbol{r}_{zx}^2 \boldsymbol{r}_{zy}^2}\right\}$$

- UV finite
- Compact expression!

Conclusion

- Proof of UV and IR finiteness of the dijet cross-section within the CGC effective field theory.
- In particular: proof of JIMWLK factorization of the rapidity divergence for a process with non-trivial final state.
- We have obtained a numerically tractable NLO impact factor $\Rightarrow$ reach $\alpha_s^3 \ln(1/x)$ accuracy when combined with existing results for NLO BK-JIMWLK.
- Future directions:
  - Clarify the importance of Sudakov-type contributions, by considering the back to back limit of our result. See also next talk by Piotr Kotko
  - Numerical work for EIC predictions e.g. azimuthal correlations.