# BFKL studies


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# Outline

- pQCD for hard processes: when $Q \gg \Lambda_{QCD}$:
  scale $Q$ is provided: either by large mass (Higgs boson production)
  or by some large transverse momentum $Q = k_\perp \gg \Lambda_{QCD}$ (high $k_\perp$ jets, hadron production)
- due to confinement even for hard process in Bjorken limit $s \sim Q^2 \gg \Lambda^2_{QCD}$
  we have both pQCD and nonperurbative QCD effects.
- QCD/collinear factorization:
  Observable – convolution of PDFs (parton densities) and hard cross section
  $\hat{O} \sim \int dx\ f_i(x, \mu_F) \times C_i(x, Q/\mu_F)$; $i =$ quark, gluon
  DGLAP evolution equation: $\frac{\partial}{\partial \mu_F} f_i(x, \mu_F) = \int K_{i,j}(x, y) f_j(y, \mu_F)$
- similar for fragmentation functions: $D_i^h(x, \mu_F)$
- hard cross sections $C_i$ and DGLAP kernels $K_{i,j}$ – calculable in pQCD (powers of $\alpha_s$)
  $f_i(x, \mu_0)$ and $D_i^h(x, \mu_0)$ -nonperturbative inputs at some low scale $\mu_0 > \Lambda_{QCD}$

- QCD/collinear factorization: resummation of large terms
  $\sim \sum_n \left[ c_n \, \alpha_s^n \ln^n Q^2 + d_n \, \alpha_s^{n+1} \ln^n Q^2 + e_n \, \alpha_s^{n+2} \ln^n Q^2 \ldots \right]$
- Bjorken $s \sim Q^2 \gg \Lambda^2_{QCD}$ vesrus Regge $s \gg Q^2 \gg \Lambda^2_{QCD}$ limits for hard processes
- in pQCD we have both large logs of $Q^2$ and energy $s$:
  $\sim \sum c_n \alpha_s^n \ln^n Q^2$ and $\sim \sum k_n \alpha_s^n \ln^n s$
- resummation of large energy logs: BFKL approach
  BFKL equation – evolution in energy
- large energy logs are related with gluon radiation in MRK
- basic concept of BFKL: gluon Reggeization
  High energy asymptotic of amplitudes with gluon quantum numbers in $t-$ channel is given by simple Regge pole in the complex angular momentum plane
  $A(s,t) \sim \left(\frac{s}{-t}\right)^{\omega(t)}$; $\omega(t)$- gluon Regge trajectory
- other amplitudes are build in terms of these Reggezied gluon interactions
- what resummation DGLAP or BFKL is more important for phenomenology?

In general, search for BFKL effects had these general drawbacks:

- ◇ too low $\sqrt{s}$ or rapidity intervals among tagged particels in the final state
- ◇ too inclusive observables, other approaches can fit them

Advent of LHC:

- → higher energies ↔ larger rapidity intervals
- → unique opportunity to **test pQCD in the high-energy limit**
- → disentangle applicability region of energy-log resummation (**BFKL approach**)

[V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975, 1976, 1977)]
[Y.Y. Balitskii, L.N. Lipatov (1978)]

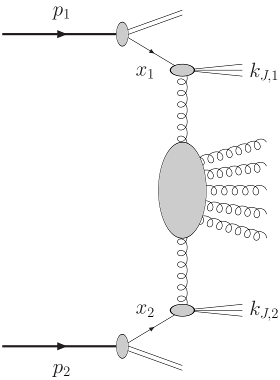
Last years:

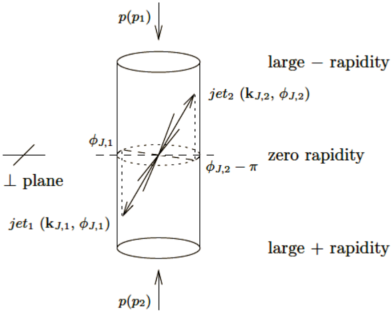
hadroproduction of two jets featuring high transverse momenta and well separed in rapidity, so called **Mueller–Navelet jets**...

- ◇ ...possibility to define *infrared-safe* observables...
- ◇ ...and constrain the PDFs...
- ◇ ...theory vs experiment

[talk of Cristian Baldenegro]

# Mueller–Navelet jets




Pictures from
[D. Colferai, F. Schwennsen, L. Szymanowski, S. Wallon (2010)]

...large jet transverse momenta: $\vec{k}_{J,1}^{\,2} \sim \vec{k}_{J,2}^{\,2} \gg \Lambda_{\rm QCD}^2$

...large rapidity gap between jets (high energies) $\Rightarrow$ $\Delta y = \ln \frac{x_{J,1} x_{J,2} s}{|\vec{k}_{J,1}||\vec{k}_{J,2}|}$

# How could we further and deeply probe BFKL?

**Study a less inclusive two-body final state...**
*instead of dijet production one can study similar processes:*

- $J/\Psi$ production, $p + p \to J/\Psi + X + Jet$
- forward Drell-Yan, $p + p \to \gamma^*(- > \mu^+\mu^-) + X + Jet$
- Higgs boson production, $p + p \to \mathrm{Higgs} + \mathrm{X} + \mathrm{Jet}$
- inclusive production of a pair of charged light hadrons well separated in rapidity $p + p \to h_1 + X + h_2$ or $p + p \to h_1 + X + Jet$

**I will speak about our recent results for ...**

- Higgs-plus-Jet production [F. Celiberto, D.I. , M. Mohammed, A. Papa (2021)]
- $\Lambda$ barion [F. Celiberto, D.I. , A. Papa (2020)] and $\Lambda_c$ barion [F. Celiberto, M/ Fucilla, D.I. , A. Papa (2021)] production

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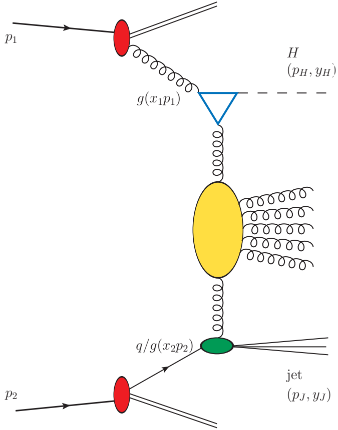
# Higgs-plus-jet production

Inclusive process: proton + proton $\rightarrow$ Higgs + X + jet *...LHC physics!*



# Outline

# The BFKL resummation

pQCD, **semi-hard processes**: $s \gg Q^2 \gg \Lambda^2_{\rm QCD}$

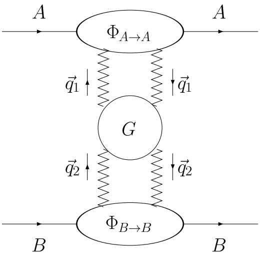
total cross section for $A + B \to X$: $\sigma_{AB}(s) = \frac{\mathcal{I}m_s\left(\mathcal{A}^{AB}_{AB}\right)}{s}$ $\Leftarrow$ **optical theorem**

- ◇ Pomeron channel: $t = 0$ + singlet colour representation in the $t$-channel
- ◇ Regge limit: $s \simeq -u \to \infty$, $t$ not growing with $s$

- **BFKL resummation**:

  **leading logarithmic approximation (LLA)**: $\alpha_s^n (\ln s)^n$

  **next-to-leading logarithmic approximation (NLA)**: $\alpha_s^{n+1} (\ln s)^n$




- $\mathrm{Im}_s\left(\mathcal{A}^{AB}_{AB}\right)$ factorization:

  convolution of the **Green's function** of two interacting Reggeized gluons with the **impact factors** of the colliding particles.

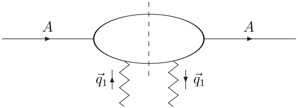
$$\mathrm{Im}_s\left(\mathcal{A}\right) = \frac{s}{(2\pi)^{D-2}}\int\frac{d^{D-2}q_1}{\vec{q}_1^{\,2}}\Phi_A(\vec{q}_1, s_0)\int\frac{d^{D-2}q_2}{\vec{q}_2^{\,2}}\Phi_B(-\vec{q}_2, s_0)\int\limits_{\delta-i\infty}^{\delta+i\infty}\frac{d\omega}{2\pi i}\left(\frac{s}{s_0}\right)^{\omega} G_\omega(\vec{q}_1, \vec{q}_2)$$

- **Green's function** is **process-independent**

  $\longrightarrow$ determined through the **BFKL equation**
  [Ya.Ya. Balitsky, V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975), NLA: V.S. Fadin, L.N. Lipatov (1998)]]

- **Impact factors** are **process-dependent**

  $\longrightarrow$ known in the NLA just for few processes

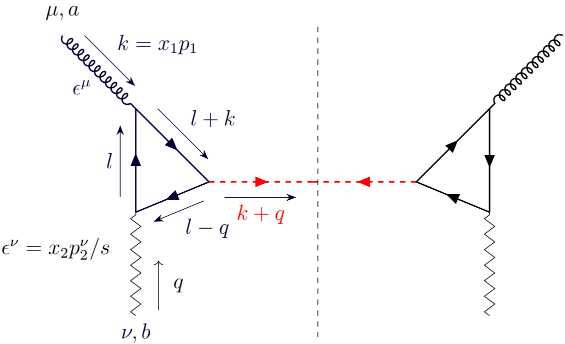



* In this work we use LLA impact factor for tagged Higgs production

- NLA Higgs IF was calculated recently in momentum representationand in large $M_t$ limit: [M. Hentschinski, K. Kutak, and A. van Hameren (2021)]

# Outline

# Higgs impact factor




The amplitude for the gluon scattering off a Reggeon to produce a Higgs particle.
For the impact factor/cross section we have to make a square of it.

$$V^{(0)}_{g\to H}(\vec{q}) = \frac{\alpha_s^2}{v^2}\frac{\vec{q}^{\,2}|\mathcal{F}(\vec{q}^{\,2})|^2}{128\pi^2\sqrt{N_c^2-1}}$$

with $V^{(0)}_{g\to H}(\vec{q})|_{\vec{q}=0} \to 0$, so that the infra-red finiteness of the BFKL amplitude is preserved. Here $v$ is the electroweak vacuum expectation value parameter, $v^2 = 1/(G_F\sqrt{2})$, and

$$\mathcal{F}(\vec{q}^{\,2}) = 4\int_0^1 dy\int_0^{1-y} dx\,\frac{1-4xy}{1-\left(\frac{M^2_{H,\perp}}{M_t^2}\right)xy+\left(\frac{\vec{q}^{\,2}}{M_t^2}\right)y(1-y)}$$

We confirm for $\mathcal{F}(\vec{q}^{\,2})$, the results obtained earlier [V. Del Duca, W. Kilgore, C. Oleari, C.R. Schmidt, D. Zeppenfeld, [2003]]:

$$\mathcal{F}(\vec{q}^{\,2}) = \frac{-4M_t^2}{M_{H,\perp}^2}\left\{-2-\left(\frac{2\vec{q}^{\,2}}{M_{H,\perp}^2}\right)\left[\sqrt{z_1}\,\mathcal{W}(z_1)-\sqrt{z_2}\,\mathcal{W}(z_2)\right]\right.$$
$$\left.+\frac{1}{2}\left(1-\frac{4M_t^2}{M_{H,\perp}^2}\right)\left[\mathcal{W}(z_1)^2-\mathcal{W}(z_2)^2\right]\right\} ,$$

with $\vec{q}$ the transverse component of the four-vector $q$, $M_{H,\perp}=\sqrt{M_H^2+|\vec{q}|^2}$ the Higgs-boson transverse mass, $z_1 = 1-4M_t^2/M_H^2$, $z_2 = 1+4M_t^2/\vec{q}^{\,2}$, and the root $\sqrt{z_1} = i\sqrt{|z_1|}$ is taken for negative values of $z_1$. Furthermore, we have

$$\mathcal{W}(z) = \begin{cases} -2i\arcsin\dfrac{1}{\sqrt{1-z}}\,, & z<0\,; \\ \ln\dfrac{1+\sqrt{z}}{1-\sqrt{z}} - i\pi\,, & 0<z<1\,; \\ \ln\dfrac{1+\sqrt{z}}{\sqrt{z}-1}\,, & z>1\,. \end{cases}$$

In the large-top mass limit, our LO impact factor reads

$$V_{g\to H}^{(0)}(\vec{q}) = \frac{\alpha_s^2}{v^2}\,\frac{\vec{q}^{\,2}}{72\pi^2\sqrt{N_c^2-1}}\,.$$

we need the projection of the impact factors onto the eigenfunctions of the leading-order BFKL kernel, to get their so called $(n, \nu)$-representation.

$$\frac{dV^{(0)}_{p\to H}(\nu, n)}{dx_H d^2\vec{p}_H} = \frac{\alpha_s^2}{v^2}\frac{|\mathcal{F}(\vec{p}_H^{\,2})|^2}{128\pi^3\sqrt{2(N_c^2-1)}}\left(\vec{p}_H^{\,2}\right)^{i\nu-1/2} f_g(x_H) e^{in\phi_H} ,$$

where $\phi_H$ denotes the azimuthal angle of the vector $\vec{p}_H$.

For the jet IF we have:

$$\frac{d\Phi^{(0)}_J(\nu, n)}{dx_J d^2\vec{p}_J} = 2\alpha_s\sqrt{\frac{C_F}{C_A}}(\vec{p}_J^{\,2})^{i\nu-3/2}\left(\frac{C_A}{C_F}f_g(x_J) + \sum_{a=q\bar{q}} f_a(x_J)\right)e^{in\phi_J} ,$$

where $\phi_J$ denotes the azimuthal angle of the vector $\vec{p}_J$.

# Outline

1. Motivation
2. Higgs-plus-jet production
   - BFKL resummation
   - Forward-Higgs LO impact factor
   - BFKL cross section and azimuthal coefficients
   - Born cross section
   - Phenomenology
3. $\Lambda$ and $\Lambda_c$ barion production
   - $\Lambda$ impact factors
   - $\Lambda$ production phenomenology
4. Conclusions & Outlook

Kinematics (Sudakov decomposition):

$$p_H = x_H p_1 + \frac{M_{H,\perp}^2}{x_H s} p_2 + p_{H\perp} \ , \quad p_{H\perp}^2 = -|\vec{p}_H|^2$$

$$p_J = x_J p_2 + \frac{|\vec{p}_J|}{x_J s} p_1 + p_{J\perp} \ , \quad p_{J\perp}^2 = -|\vec{p}_J|^2$$

$M_{H,\perp} = \sqrt{M_H^2 + |\vec{p}_H|^2}$ is the Higgs-boson transverse mass.

The longitudinal-momentum fractions, $x_{H,J}$, for the Higgs and jet are related to the corresponding rapidities in the center-of-mass frame via the relations

$$y_H = \frac{1}{2} \ln \frac{x_H^2 s}{M_{H,\perp}^2} \ , \qquad y_J = \frac{1}{2} \ln \frac{|\vec{p}_J|^2}{x_J^2 s} \ , \qquad dy_{H,J} = \pm \frac{dx_{H,J}}{x_{H,J}}$$

As for the rapidity distance, one has

$$\Delta Y = y_H - y_J = \ln \frac{x_H x_J s}{M_{H,\perp} |\vec{p}_J|} \ .$$

QCD collinear factorization:

$$\frac{d\sigma}{dx_H dx_J d^2\vec{p}_H d^2\vec{p}_J} = \sum_{i,j=q,\bar{q},g} \int_0^1 dx_1 \int_0^1 dx_2 \; f_i\left(x_1, \mu_{F_1}\right) \; f_j\left(x_2, \mu_{F_2}\right) \frac{d\hat{\sigma}_{i,j}\left(\hat{s}, \mu_{F_{1,2}}\right)}{dx_H dx_J d^2\vec{p}_H d^2\vec{p}_J}$$

We calculate partonic cross section $d\hat{\sigma}_{i,j}$ using BFKL resummation.

The BFKL cross section can be presented as the Fourier series of the so-called *azimuthal coefficients*, $\mathcal{C}_n$

$$\frac{d\sigma}{dy_H dy_J \, d|\vec{p}_H| \, d|\vec{p}_J| d\varphi_H d\varphi_J} = \frac{1}{(2\pi)^2} \left[\mathcal{C}_0 + \sum_{n=1}^{\infty} 2\cos(n\varphi)\, \mathcal{C}_n\right]$$

where $\varphi = \varphi_H - \varphi_J - \pi$, with $\varphi_{H,J}$ the Higgs and the jet azimuthal angles.

$$\mathcal{C}_n \equiv \int_0^{2\pi} d\varphi_H \int_0^{2\pi} d\varphi_J \cos(n\varphi) \frac{d\sigma}{dy_H dy_J \, d|\vec{p}_H| \, d|\vec{p}_J| d\varphi_H d\varphi_J}$$

$$= \frac{e^{\Delta Y}}{s} \frac{M_{H,\perp}}{|\vec{p}_H|}$$

$$\times \int_{-\infty}^{+\infty} d\nu \left(\frac{x_J x_H s}{s_0}\right)^{\bar{\alpha}_s(\mu_{R_c})\left\{\chi(n,\nu)+\bar{\alpha}_s(\mu_{R_c})\left[\bar{\chi}(n,\nu)+\frac{\beta_0}{8N_c}\chi(n,\nu)\left[-\chi(n,\nu)+\frac{10}{3}+4\ln\left(\frac{\mu_{R_c}}{\sqrt{\vec{p}_H\vec{p}_J}}\right)\right]\right]\right\}}$$

$$\times \left\{\alpha_s^2(\mu_{R_1}) c_H(n,\nu,|\vec{p}_H|,x_H)\right\} \left\{\alpha_s(\mu_{R_2})[c_J(n,\nu,|\vec{p}_J|,x_J)]^*\right\}$$

$$\times \left\{1 + \alpha_s(\mu_{R_1}) \frac{c_H^{(1)}(n,\nu,|\vec{p}_H|,x_H)}{c_H(n,\nu,|\vec{p}_H|,x_H)} + \alpha_s(\mu_{R_2}) \left[\frac{c_J^{(1)}(n,\nu,|\vec{p}_J|,x_J)}{c_J(n,\nu,|\vec{p}_J|,x_J)}\right]^*\right\}$$

where $\bar{\alpha}_s \equiv N_c/\pi\,\alpha_s$, with $N_c$ the QCD color number, $\beta_0 = \frac{11}{3}N_c - \frac{2}{3}n_f$ the first coefficient in the expansion of the QCD $\beta$-function ($n_f$ is the active-flavor number)

*In red: LLA BFKL kernel $\chi(n,\nu)$ and IFs*
*in blue: NLA parts of BFKL kernel $\bar{\chi}(n,\nu)$ and IFs*

$c^{(1)}_{H,J}(n,\nu,|\vec{p}_{H,J}|,x_{H,J})$ are the NLO impact-factor corrections.
We include "universal" contributions to the NLO Higgs impact factor, which can be expressed through the corresponding LO impact factor, and related with the scales variation:

$$\alpha_s c^{(1)}_H(n,\nu,|\vec{p}_H|,x_H) \to \bar{\alpha}_s \tilde{c}^{(1)}_H(n,\nu,|\vec{p}_H|,x_H)\;,$$

with

$$\tilde{c}^{(1)}_H(n,\nu,|\vec{p}_H|,x_H) = c_H(n,\nu,|\vec{p}_H|,x_H)\left\{\frac{\beta_0}{4N_c}\left(2\ln\frac{\mu_{R_1}}{|\vec{p}_H|}+\frac{5}{3}\right)+\frac{\chi(n,\nu)}{2}\ln\left(\frac{s_0}{M^2_{H,\perp}}\right)\right.$$
$$+\frac{\beta_0}{4N_c}\left(2\ln\frac{\mu_{R_1}}{M_{H,\perp}}\right)$$
$$\left.-\frac{1}{2N_c f_g(x_H,\mu_{F_1})}\ln\frac{\mu^2_{F_1}}{M^2_{H,\perp}}\int_{x_H}^1\frac{dz}{z}\left[P_{gg}(z)f_g\left(\frac{x_H}{z},\mu_{F_1}\right)+\sum_{a=q,\bar{q}}P_{ga}(z)f_a\left(\frac{x_H}{z},\mu_{F_1}\right)\right]\right\}\,.$$

And similar formula for jet IF, $\tilde{c}^{(1)}_J(n,\nu,|\vec{p}_J|,x_J)$...

$\chi(n,\nu)$- BFKL, $P_{gg}, P_{ga}$- DGLAP LO kernels our master formula for the azimuthal coefficients:

$$\mathcal{C}_n = \frac{e^{\Delta Y}}{s} \frac{M_{H,\perp}}{|\vec{p}_H|}$$

$$\times \int_{-\infty}^{+\infty} d\nu \left( \frac{x_J x_H s}{s_0} \right)^{\bar{\alpha}_s(\mu_{R_c}) \left\{ \chi(n,\nu) + \bar{\alpha}_s(\mu_{R_c}) \left[ \bar{\chi}(n,\nu) + \frac{\beta_0}{8N_c} \chi(n,\nu) \left[ -\chi(n,\nu) + \frac{10}{3} + 4 \ln \left( \frac{\mu_{R_c}}{\sqrt{\vec{p}_H \vec{p}_J}} \right) \right] \right] \right\}}$$

$$\times \left\{ \alpha_s^2(\mu_{R_1}) c_H(n, \nu, |\vec{p}_H|, x_H) \right\} \left\{ \alpha_s(\mu_{R_2}) [c_J(n, \nu, |\vec{p}_J|, x_J)]^* \right\}$$

$$\times \left\{ 1 + \bar{\alpha}_s(\mu_{R_1}) \frac{\tilde{c}_H^{(1)}(n, \nu, |\vec{p}_H|, x_H)}{c_H(n, \nu, |\vec{p}_H|, x_H)} + \bar{\alpha}_s(\mu_{R_2}) \left[ \frac{\tilde{c}_J^{(1)}(n, \nu, |\vec{p}_J|, x_J)}{c_J(n, \nu, |\vec{p}_J|, x_J)} \right]^* \right\} .$$

The renormalization scales ($\mu_{R_{1,2,c}}$) and the factorization ones ($\mu_{F_{1,2}}$) can, in principle, be chosen arbitrarily, since their variation produces effects beyond the NLO. It is however advisable to relate them to the physical hard scales of the process. We chose to fix them differently from each other, depending on the subprocess to which they are related: $\mu_{R_1} \equiv \mu_{F_1} = C_\mu M_{H,\perp}$, $\mu_{R_2} \equiv \mu_{F_2} = C_\mu |\vec{p}_J|$, $\mu_{R_c} = C_\mu \sqrt{M_{H,\perp} |\vec{p}_J|}$, where $C_\mu$ is a variation parameter introduced to gauge the effect of a change of the scale.

# Outline

# Born cross section

The Born contribution corresponds to the so-called *two-gluon* approximation, which describes the back-to-back emission of the Higgs and of the jet with no additional gluon radiation.

$$\frac{d\sigma^{\rm Born}(|\vec{p}_H|,\Delta Y,s)}{d|\vec{p}_H|d\Delta Y} = \pi\frac{e^{\Delta Y}}{s} M_{H,\perp} \int_{y_H^{\rm min}}^{y_H^{\rm max}} dy_H \int_{y_J^{\rm min}}^{y_J^{\rm max}} dy_J\, \delta\left(y_H - y_J - \Delta Y\right)$$
$$\times\, \alpha_s^2(\mu_{R_1})\,\frac{1}{v^2}\,\frac{|\mathcal{F}(\vec{p}_H^{\,2})|^2}{128\pi^3\sqrt{2(N_c^2-1)}} f_g(x_H,\mu_{F_1})$$
$$\times \alpha_s(\mu_{R_2})\, 2\sqrt{\frac{C_F}{C_A}}\left(\frac{C_A}{C_F} f_g(x_J,\mu_{F_2}) + \sum_{a=q,\bar{q}} f_a(x_J,\mu_{F_2})\right) .$$

Our numerical calculation in the Born limit at $\Delta Y = 3$ is in fair agreement with DelDuca et all, 2003.

# Outline

# Azimuthal correlations and $p_T$-distribution

The azimuthal-angle coefficients *integrated* over the phase space for two final-state particles, while the rapidity interval, $\Delta Y$, between the Higgs boson and the jet is kept fixed:

$$C_n(\Delta Y, s) = \int_{p_H^{\rm min}}^{p_H^{\rm max}} d|\vec{p}_H| \int_{p_J^{\rm min}}^{p_J^{\rm max}} d|\vec{p}_J| \int_{y_H^{\rm min}}^{y_H^{\rm max}} dy_H \int_{y_J^{\rm min}}^{y_J^{\rm max}} dy_J \, \delta\left(y_H - y_J - \Delta Y\right) \mathcal{C}_n$$

The goal – realistic kinematic cuts adopted by the current experimental analyses at the LHC, we constrain the Higgs emission inside the rapidity acceptances of the CMS barrel detector, *i.e.* $|y_H| < 2.5$, while we allow for a larger rapidity range of the jet, which can be detected also by the CMS endcaps, namely $|y_J| < 4.7$.
Furthermore, three distinct cases for the final-state transverse momenta are considered:

a) **symmetric** configuration, suited to the search of pure BFKL effects, where both the Higgs and the jet transverse momenta lie in the range: 20 GeV $< |\vec{p}_{H,J}| <$ 60 GeV;

b) **asymmetric** selection, typical of the ongoing LHC phenomenology, where the Higgs transverse momentum runs from 10 GeV to $2M_t$, where the jet is tagged inside its typical CMS configuration, from 20 to 60 GeV;

c) **disjoint windows**, which allows for the maximum exclusiveness in the final state: 35 GeV $< |\vec{p}_J| <$ 60 GeV and 60 GeV $< |\vec{p}_H| < 2M_t$.

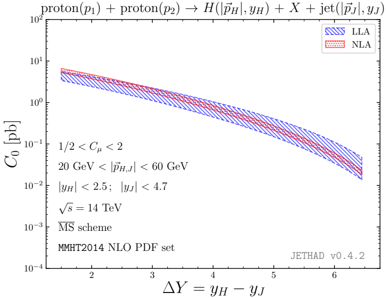
We study the $\varphi$-averaged cross section (*alias* the $\Delta Y$-distribution), $C_0(\Delta Y, s)$, the azimuthal-correlation moments, $R_{n0}(\Delta Y, s) = C_n / C_0 \equiv \langle \cos n\varphi \rangle$ and their ratios, $R_{nm} = C_n / C_m$ as functions of the Higgs-jet rapidity distance, $\Delta Y$.

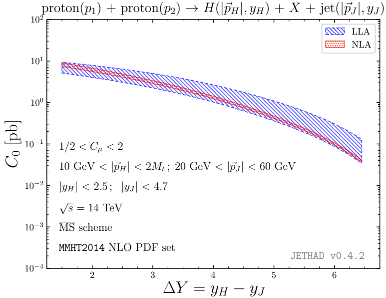
The second observable of our interest is the $|\vec{p}_H|$-distribution for a given value of $\Delta Y$:

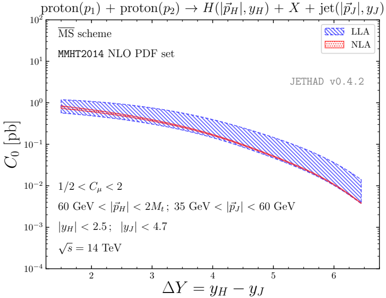
$$\frac{d\sigma(|\vec{p}_H|, \Delta Y, s)}{d|\vec{p}_H| d\Delta Y} = \int_{p_J^{\rm min}}^{p_J^{\rm max}} d|\vec{p}_J| \int_{y_H^{\rm min}}^{y_H^{\rm max}} dy_H \int_{y_J^{\rm min}}^{y_J^{\rm max}} dy_J \, \delta \left( y_H - y_J - \Delta Y \right) \mathcal{C}_0 \,,$$

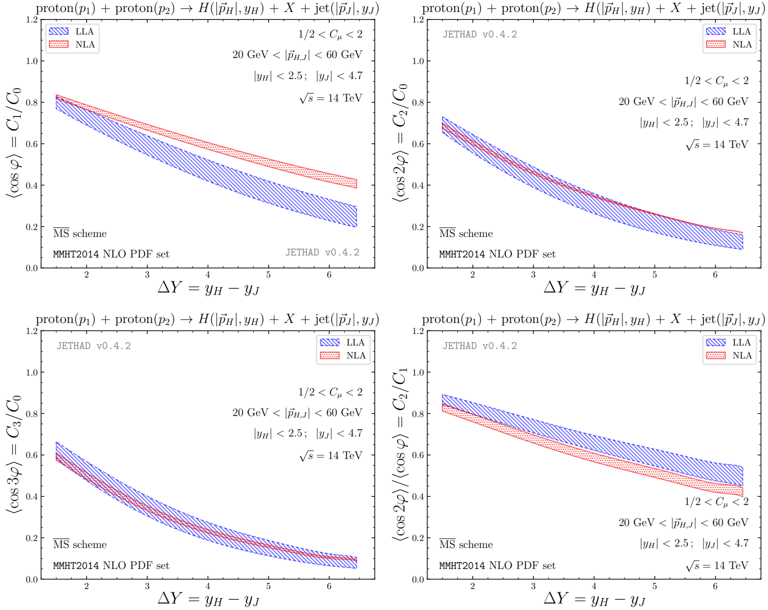
the Higgs and jet rapidity ranges being given above and 35 GeV $< |\vec{p}_J| <$ 60 GeV.

## Phenomenology



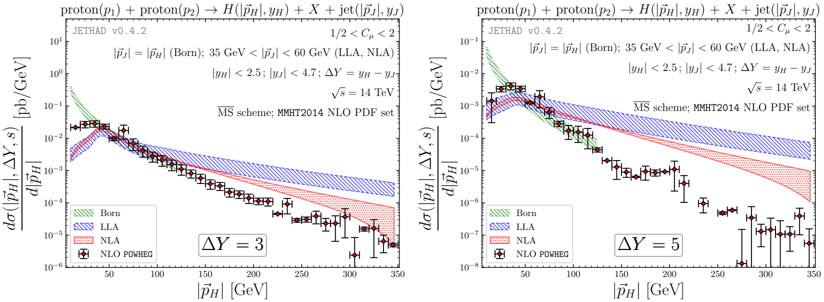



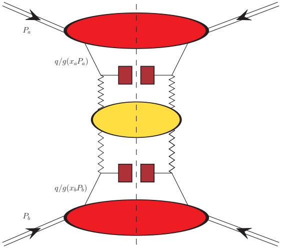

proton($p_1$) + proton($p_2$) → $H(|\vec{p}_H|, y_H)$ + X + jet($|\vec{p}_J|, y_J$)
LLA
NLA
$1/2 < C_\mu < 2$
20 GeV < $|\vec{p}_{H,J}|$ < 60 GeV
$|y_H| < 2.5$; $|y_J| < 4.7$
$\sqrt{s}$ = 14 TeV
$\overline{\rm MS}$ scheme
MMHT2014 NLO PDF set
JETHAD v0.4.2
$\langle \cos\varphi \rangle = C_1/C_0$
$\langle \cos 2\varphi \rangle = C_2/C_0$
$\langle \cos 3\varphi \rangle = C_3/C_0$
$\langle \cos 2\varphi \rangle / \langle \cos\varphi \rangle = C_2/C_1$
$\Delta Y = y_H - y_J$

# $|\vec{p}_H|-$ distribution. BFKL vs NLO DGLAP

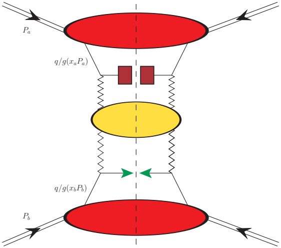
NLO DGLAP [J.M. Campbell, R.K. Ellis, R. Frederix, P. Nason, C. Oleari and C. Williams (2012), K. Hamilton, P. Nason, C. Oleari andG. Zanderighi (2013)]. We used POWHEG method to obtain NLO DGLAP results for our kinematics.



# $\Lambda$ and $\Lambda_c$ barion production




a) Double $\Lambda_c$




b) $\Lambda_c$ + Jet

a) the double $\Lambda_c$ and of b) the $\Lambda_c$ + jet production. Red blobs stand for proton collinear PDFs, whereas bordeaux rectangles denote baryon collinear FFs and green arrows refer to the jet selection function. The BFKL gluon Green's function, represented by the yellow central blob, is connected to impact factors through Reggeon (zigzag) lines.

# Outline

Here BFKL formulae for observable are similar to discussed above Higgs-plus-Jet production process, the difference is in form of $\Lambda$- impact factors.
It is given by the convolution of PDFs and $\Lambda$ FFs. At LO:

$$c_\Lambda(n,\nu,|\vec{p}\,|,x) = 2\sqrt{\frac{C_F}{C_A}}(|\vec{p}\,|^2)^{i\nu-1/2}\int_x^1 \frac{\mathrm{d}z}{z}\left(\frac{z}{x}\right)^{2i\nu-1}$$

$$\times\left[\frac{C_A}{C_F} f_g(z) D_g^\Lambda\left(\frac{x}{z}\right) + \sum_{a=q,\bar{q}} f_a(z) D_a^\Lambda\left(\frac{x}{z}\right)\right] .$$

NLO IFs were calculated:
for hadron production– [D.I. and A. Papa, (2012)]
for jet in small-cone-approximation – [D.I. and A. Papa, (2012)]

# Outline

- Another point is that we have lower values of the kinematic hard scales. NLA BFKL suffers from severe instabilities, due to large NLO corrections in the BFKL kernel and in impact factors in size and opposite in sign with respect to the LO.
- We adopted in our NLA BFKL studies in order to tame this instability the use of the Brodsky–Lepage–Mackenzie (BLM) optimization method for the renormalization scale fixing.
- BLM method: the *optimal* renormalization-scale value, labeled as $\mu_R^{\rm BLM}$, is the value of $\mu_R$ that cancels the non-conformal, $\beta_0$-dependent part of a given observable. A suitable procedure allows us to remove all non-conformal contributions that appear both in the NLA BFKL kernel and in the NLO non-universal impact factors of high-energy distributions. Its application makes $\mu_R^{\rm BLM}$ dependent on the kind of the process and its energy and thus on $\Delta Y$.
- For $\Lambda_c$ FFs we use recent KKSS19 NLO set [B. A. Kniehl, G. Kramer, I. Schienbein, and H. Spiesberger (2020)]
For light hadron FFs – AKK08 NLO set [S. Albino, B. A. Kniehl, and G. Kramer (2008)]
For PDFs – MMHT14 NLO set [L. Harland-Lang, A. Martin, P. Motylinski, and R. Thorne (2015)]

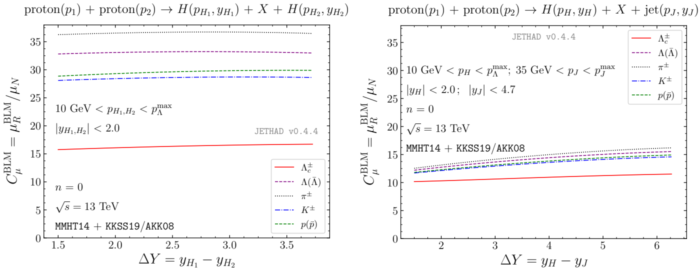
Our observables:

$$C_n = \int_{y_1^{\rm min}}^{y_1^{\rm max}} {\rm d}y_1 \int_{y_2^{\rm min}}^{y_2^{\rm max}} {\rm d}y_2 \int_{p_1^{\rm min}}^{p_1^{\rm max}} {\rm d}|\vec{p}_1| \int_{p_2^{\rm min}}^{p_2^{\rm max}} {\rm d}|\vec{p}_2| \; \delta(\Delta Y - y_1 + y_2) \; \mathcal{C}_n \left(|\vec{p}_1|, |\vec{p}_2|, y_1, y_2\right) \; .$$

- In order to match realistic LHC configurations, we allow the rapidity of $\Lambda$ baryons to be in the range from $-2.0$ and 2.0,
- their transverse momentum goes from 10 GeV to $p_H^{\rm max} \simeq 21.5$ GeV. The $p_H^{\rm max}$ value is constrained by the energy-scale lower cutoff on the FF sets.
- for the jet, we consider standard CMS configurations: $|y_J| < 4.7$ and 35 GeV $< p_J <$ 60 GeV.
- since $p_H > M_c$ and $M_b$, we use for $\Lambda_c$ production VFNS with $n_f = 5$.

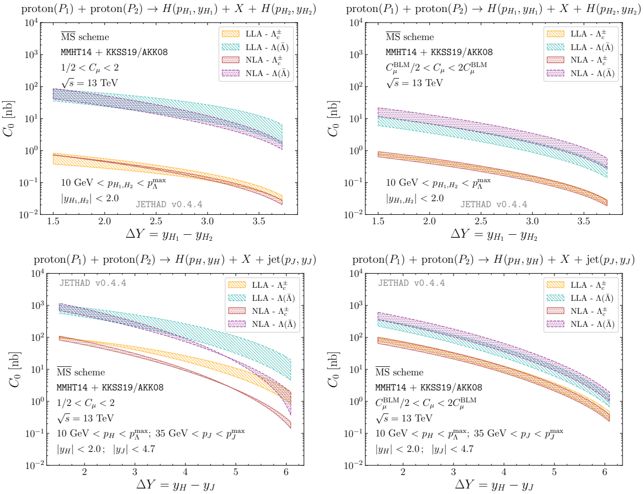
## BLM- scales:

- Kinematic scale: $\mu_N \equiv \sqrt{m_{1\perp} m_{2\perp}}$, where $m_{1\perp} = \sqrt{|\vec{p}_1|^2 + m_{\Lambda_c}^2}$, with $m_{\Lambda_c} = 2.286$ GeV, and $m_{2\perp} = \sqrt{|\vec{p}_2|^2 + m_{\Lambda_c}^2}$ in the double $\Lambda_c$ production, while $m_{2\perp}$ coincides with the jet $p_T$ in the other channel.
- $C_\mu^{\rm BLM} \equiv \mu_R^{\rm BLM} / \mu_N$.
- We set $\mu_F = \mu_R$ everywhere, as assumed by most of the existent PDF parameterizations.




BLM scales for the double $\Lambda_c$ (left) and the $\Lambda_c$ plus jet (right) production as functions of the rapidity separation, $\Delta Y$, for $n = 0$, and for $\sqrt{s} = 13$ TeV. Predictions for $\Lambda_c$ emissions are compared with configurations where other hadron species are tagged: $\Lambda(\bar\Lambda)$, $\pi^\pm$, $K^\pm$, and $p(\bar p)$.

## Cross sections: BLM vs natural scale choice predictions




$C_0$, as a function of $\Delta Y$, in the double $\Lambda_c$ (upper) and in the $\Lambda_c$ plus jet channel (lower), at natural scales (left) and after BLM optimization (right).

# Conclusions (for Higgs-plus-Jet)...

- We have proposed the inclusive hadroproduction of a Higgs boson and of a jet featuring high transverse momenta and separated by a large rapidity distance as a new diffractive semi-hard channel to probe the BFKL resummation.
- Statistics for cross sections differential in rapidity, tailored on different configurations for transverse-momentum ranges at CMS, is encouraging.
- At variance with previous analyses, where other kinds of final states were investigated, cross sections and azimuthal correlations for the Higgs-jet production exhibit quite a fair stability under higher-order corrections. Future analyses are needed in order to gauge the feasibility of precision calculations of the same observables.
- We have extended our study to distributions differential in the Higgs transverse momentum, providing evidence that a high-energy treatment is valid and can be afforded in the region where Higgs $p_T$ and the jet one are of the same order.
- This process enriches the selection of semi-hard reactions that can be used as probes of the QCD in the high-energy limit, and in particular of the BFKL resummation mechanism, in the kinematic ranges of the LHC and of future hadronic colliders.

# Outlook...

- An obvious extension of our Higgs-plus-Jet work consists in the full NLA BFKL analysis, including a NLO jet impact factor, with a realistic implementation of the jet selection function, and the NLO Higgs impact factor in $(n, \nu)-$ representation, when available.
- Our calculation of Higgs NLO IF is in progress...
- In $\Lambda_c\Lambda_c$ and $\Lambda_c\mathit{Jet}$ baryon production channels our results give a promising statistics. Also, we discovered that the tag of $\Lambda_c$ particles allows us to dampen the instabilities of the NLA BFKL series, this resulting in a partial stabilization of resummed distributions under higher-order logarithmic corrections. It would be very exciting to compare our results with possible future LHC experiments.