# Odderons, Glueballs and Holography

## Saturation and Diffraction at the LHC and the EIC

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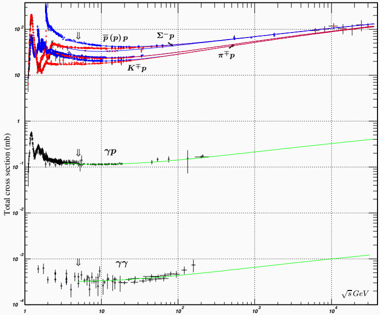



[**Brower**,**Djurić**,**Tan**'08]

## Regge behavior

for $s \gg \Lambda^2_{\rm QCD} \gg |t|$,

$$\mathcal{A}(s,t) \;\sim\; s^{\alpha(t)}$$

- the behavior shared by phenomenology and strings in flat space

## High energies, small angle

for $s \gg -t \gg \Lambda^2_{\rm QCD}$

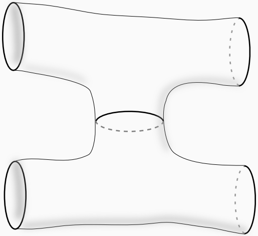
- strings are still Regge
- hadrons aren't

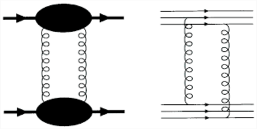
## Elastic, large angle

for $s \simeq -t \gg \Lambda^2_{\rm QCD}$, amplitudes decay

- as powers of $s$ for particles
- exponentially for strings

# Pomerons





## Soft pomeron

String theory in flat space explains Regge behavior in terms of exchange of a closed string (glueball)

- valid for $t > 0$ with

$$\alpha(t) \simeq \alpha_0 + \alpha' t$$

  – Regge trajectory of the graviton (containing $2^{++}$ glueball)

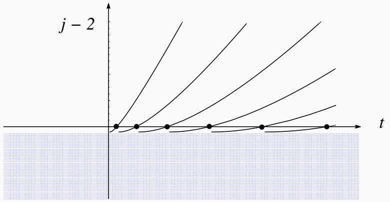
- single-pomeron exchange dominates for $N_c \to \infty$

## BFKL pomeron

In perturbation theory one can resum the leading log terms [**Lipatov**'76, **Kuraev** et al'77, **Balitsky** et al'78]

- valid at $-t \gg \Lambda^2_{\rm QCD}$ to ignore confinement
- hard BFKL pomeron

# Strings in AdS




[**Brower**,**Polchinski**,**Strassler**,**Tan**'06]

## AdS/CFT correspondence

Strings in anti de Sitter space are dual to gauge theories [**Maldacena**'97]

- type IIB strings in $AdS_5 \times S^5$ dual to $\mathcal{N} = 4$ supersymmetric Yang-Mills
- strong coulpling regime of SYM

$$N_c \to \infty\,, \qquad \lambda \equiv g^2_{\rm YM} N_c \to \infty$$

is accessible to classical gravity

## BFKL regime

hard scattering regime is accessible to strings in anti de Sitter space [**Polchinski**,**Strassler**'01'02]

- the additional dimension is interpreted as energy scale
- this scale corresponds to the transverse momentum

for large $N_c$ and large $\lambda$ analytical structure of the amplitudes is accessible [**Brower** et al'06]

- for $s \gg \Lambda^2_{\rm QCD}$ and any $t$

## $\mathcal{N} = 4$ SYM pomeron

BFKL approach can be applied to a conformal theory

- calculations up to NNNLA

# Odderon

## Phenomenology

Total cross sections of particle-particle and particle-anti-particle scattering

$$\sigma_{\mathrm{T}}^{\pm} \sim s^{j_0^{\pm}-1}$$

- $j_0^+ - j_0^- > 0$ can be fitted from the imaginary parts of amplitudes or differences in $\sigma_{\mathrm{T}}$

## *C*-odd closed string

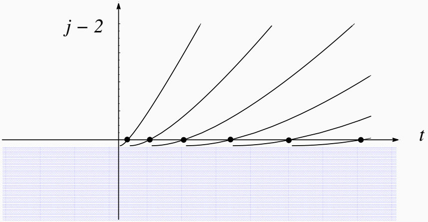
It is straightforward to generalize pomeron to *C*-odd string exchange [**Brower** *et al.*'08]

- pomeron: graviton Regge trajectory (perturbation of background $G_{MN}$)
- odderon: perturbation of $B_{MN}$ (Kalb-Ramond) field – spin 1 glueball

| *C* | weak coupling | strong coupling |
|---|---|---|
| + | $j_0^+ = 1 + (\log 2)\lambda/\pi^2 + O(\lambda^2)$ | $j_0^+ = 2 - 2/\sqrt{\lambda} + O(1/\lambda)$ |
| − | $j_0^- = 1 - 0.24717\lambda/\pi + O(\lambda^2)$ | $j_0^- = 1 - 8/\sqrt{\lambda} + O(1/\lambda)$ |

Table: Leading Regge singularities [**Brower**,**Djurić**,**Tan**'08].

# Glueballs




[**Brower**,**Polchinski**,**Strassler**,**Tan**'06]

## Coupling to gluons

A closed string can be sourced by the operator

$$\mathrm{Tr}\left[Pe^{i\oint d\sigma\partial_\sigma x_\mu A_\mu(x)} \pm Pe^{-i\oint d\sigma\partial_\sigma x_\mu A_\mu(x)}\right]$$

- picks even $(+)$ or odd $(-)$ $C$-parity
- perturbatively expanding obtains leading 2-gluon ($\mathcal{O}^{+}_{\mu\nu}$) or 3-gluon ($\mathcal{O}^{-}_{\mu\nu\rho}$) sources

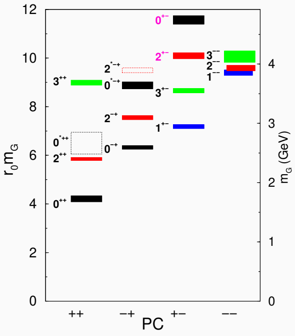
## Correlators and glueballs

- Pomeron/odderon propagators can be obtained from
$$\langle\mathcal{O}^{\pm}\mathcal{O}^{\pm}\rangle$$
correlators of local operators
- AdS/CFT computes them at strong coupling
$$\frac{\delta^n}{\delta J_1\cdots\delta J_n}\,e^{iS_{\mathrm{grav}}[J]}\Big|_{J=0}$$
- Structure of Regge trajectories can be recovered at strong coupling for pomerons and odderons
- glueball masses are given by the intersection with integer $j$

# Towards quantitative agreement

- Glueballs' Regge trajectories control the high energy scattering (in $\lambda \to \infty$, $N_c \to \infty$ picture)
- Are there any interesting quantitative predictions?
- New tools from recent progress in experiments, lattice data and string amplitudes
- I will focus on the example of the glueball spectrum

# Lattice results




[**Morningstar**,**Peardon**'99]

## Glueballs on a lattice

- spectrum of glueballs in pure glue $SU(3)$ Yang-Mills
- unquenched approximation (adding quarks)
- effective couplings and decay constants
- limit of large $N_c$

## OZI rule

- quarks do not like to mix with purely gluonic states

| $J^{PC}$ | $0^{++}$ | $2^{++}$ | $0^{++*}$ | $1^{+-}$ | $0^{-+}$ | $m_{2^{++}}/m_{0^{++}}$ |
|---|---|---|---|---|---|---|
| YM | 1710 | 2390 | 2670 | 2980 | 3640 | 1.40 |
| $QCD_3$ | 1795 | 2620 | 3760 | 3270 | 4490 | 1.46 |

Table: quenched Yang-Mills vs QCD with 3 flavors [**Gregory** *et al.*'12].

# Holographic limit

## Large $N_c$ results

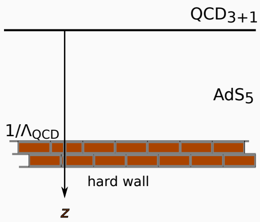
- Light dependence of masses on $N_c$ ($SU(N_c)$)

$$m \simeq m_\infty + \frac{c}{N_c^2}$$

- $N_c \to \infty$ is the limit where one expects holography to work

| $G$ | $SU(2)$ | $SU(3)$ | $SU(4)$ | $SU(6)$ | $SU(8)$ | $SU(\infty)$ |
|---|---|---|---|---|---|---|
| $m_{0^{++}}$ | 3.78 | 3.55 | 3.36 | 3.25 | 3.55 | 3.31 |
| $m_{2^{++}}/m_{0^{++}}$ | 1.44 | 1.35 | 1.45 | 1.46 | 1.32 | 1.46 |
| $G$ | $Sp(1)$ | $Sp(2)$ | $Sp(3)$ | $Sp(4)$ | – | $Sp(\infty)$ |
| $m_{2^{++}}/m_{0^{++}}$ | 1.41 | 1.41 | 1.48 | 1.41 | – | 1.47 |

Table: $0^{++}$ and $2^{++}$ mass in $SU(N_c)$ [**Lucini**,**Teper**,**Wenger**'04] and $Sp(N_c)$ [**Bennett** *et al.*'20]




## Wave equation

- Spin $\ell$ particle in the anti de Sitter ($AdS_5$) geometry

$$z^5\partial_z\left(\frac{1}{z^3}\partial_z\Psi\right)+z^2\partial^\mu\partial_\mu\Psi-L^2\ell(\ell+1)\Psi = 0$$

- Bessel equation, $\Psi = e^{ik\cdot x}z^2J_{\ell+2}(kz)$
- Introducing a cutoff (hard wall) at $z=1/\Lambda_{\rm QCD}$ leads to a discrete spectrum of $k^2=m^2$

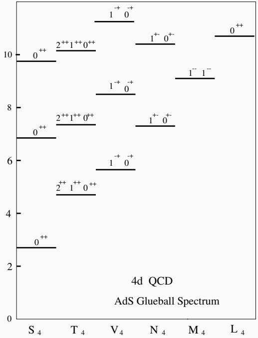
$$\text{Dirichlet}:\quad J_{\ell+2}\big(\frac{m^{(n)}_{\ell+2}}{\Lambda_{\rm QCD}}\big) = 0$$

## Regge trajectories

Regge trajectories $j_n(t)$ are found from a modified equation [**Brower** et al'06'08]

$$L^2\left(j_n-j^c\right)\Psi+\frac{2}{\sqrt{\lambda}}\left(z^5\partial_z\left(\frac{1}{z^3}\partial_z\Psi\right)+z^2\frac{t}{t_0}\Psi-L^2m^2_{\rm AdS}\Psi\right) = 0$$

# Witten's model




[**Brower**,**Mathur**,**Tan**'00]

## WM against the hard wall

- Type IIA string theory compactified on $AdS_6 \times S^4$ (D4 branes)
- further compactified on $S^1$ – breaking SUSY, introducing scale ($\Lambda_{\rm QCD}$)

| $J^{PC}$ | $SU(\infty)$ | $Sp(\infty)$ | BMT | $\text{BBC}_D$ | $\text{BBC}_N$ |
|---|---|---|---|---|---|
| $0^{++}$ | 1.63 | 1.63 | 1.63 | 1.63 | 1.63 |
| $2^{++}$ | 2.37 | 2.41 | 2.83 | 2.41 | 2.54 |
| $0^{-+}$ | – | 2.51 | 3.41 | – | – |
| $1^{+-}$ | – | 4.19 | 4.41 | – | – |
| $0^{++*}$ | 2.99 | 3.16 | 4.13 | 2.67 | 2.98 |
| $2^{++*}$ | – | – | 4.50 | 3.51 | 4.06 |
| $R$ | 1.46 | 1.47 | 1.74 | 1.48 | 1.56 |

[**Lucini**,**Teper**,**Wenger**'04]
[**Bennett** *et al.*'20]
[**Brower**,**Mathur**,**Tan**'00]
[**Boschi**,**Braga**,**Carrion**'05]

Table: lightest glueballs in WM and hard wall model normalized to the lightest mass on the lattice.

# Klebanov-Strassler model

## non-AdS/non-CFT

- type IIB string theory compactified on $AdS_5 \times S^3 \times S^2$ with warping (breaking of conformal symmetry)
- the compactification is compatible with $\mathcal{N} = 1$ SUSY
- the dual theory is $SU(N + M) \times SU(M)$ supersymmetric Yang-Mills theory with bifundamental matter multiplets.
- the theory has global $SU(2) \times SU(2)$, anomalous $U(1)_R$ and spontaneously broken $U(1)_B$ baryon symmetries

## Pros and Cons

- Flows to confined $SU(M)$ YM in IR
- lightest states are SYM $J^{PC}$ glueballs
- has additional fields and symmetries
- has massless particles (Goldstones of $U(1)_B$)

# **Spectrum** of the singlet sector

## Early contributions

- E. Caceres and R. Hernandez, Phys. Lett. B **504** (2001), 64-70
- S. S. Gubser, C. P. Herzog and I. R. Klebanov, JHEP **09** (2004), 036
- M. Berg, M. Haack and W. Mück, Nucl. Phys. B **736** (2006), 82-132; Nucl. Phys. B **789** (2008), 1-44

- A. Y. Dymarsky and D. Melnikov, JETP Lett. **84** (2006), 368-371; JHEP **05** (2008), 035
- M. K. Benna, A. Dymarsky, I. R. Klebanov and A. Solovyov, JHEP **06** (2008), 070
- A. Dymarsky, D. Melnikov and A. Solovyov, JHEP **05** (2009), 105

- I. Gordeli and D. Melnikov, JHEP **08** (2011), 082; arXiv: 1311.6537 [hep-th]

## Recent results

- D. Melnikov and C. R. Filho, JHEP **01** (2021) 024
- C. Rodrigues Filho, Braz. J. Phys. **51** (2021) no.3, 788

## $m^2$ in the singlet sector

- 6+2 scalar multiplets, graviton multiplet, 3 vector multiplets, 2 gravitino multiplets

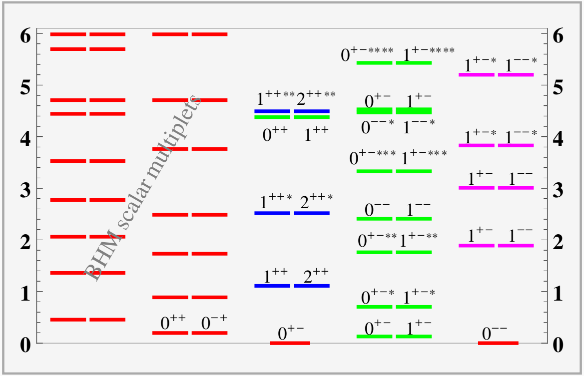



Figure: from [**Gordeli,DM**’13]

## Comparison with the lattice

- in the units of $2^{++}$ mass

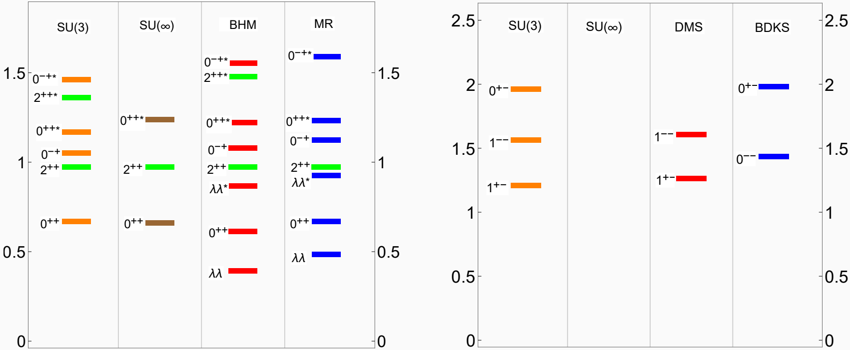



Figure: from [**DM**,**Rodrigues**'20]

## Holography vs lattice

- Klebanov-Strassler theory yields a very reasonable match for all (5) lightest states with spin < 2.
- Higher spin states cannot be captured by the classical gravity approximation
- Excited states are more susceptible to mixing

| $J^{PC}$ | MP | $SU(\infty)$ | BMT | $BBC_D$ | $BBC_N$ | BHM | MR |
|---|---|---|---|---|---|---|---|
| $0^{++}$ | 1.63 | 1.63 | 1.63 | 1.63 | 1.63 | 1.63 | 1.63 |
| $2^{++}$ | 2.26 | 2.38 | 2.83 | 2.41 | 2.54 | 2.54 | 2.33 |
| $R$ | 1.4 | 1.46 | 1.74 | 1.48 | 1.56 | 1.56 | 1.43 |
| $0^{++}$ | 2.51 | 2.99 | 4.13 | 2.67 | 2.98 | 3.18 | 2.94 |
| $2^{++}$ | – | – | 4.50 | 3.51 | 4.06 | 3.83 | 3.51 |

| $J^{PC}$ | $2^{++}$ | $0^{++}$ | $0^{-+}$ | $1^{+-}$ | $1^{--}$ | $0^{+-}$ |
|---|---|---|---|---|---|---|
| KS/MP | 1 | 1.01 | 1.07 | 1.03 | 1.05 | 1.01 |

# Odderon conclusions

- Two $C$-odd spin one glueballs corresponding to fluctuations of $B_{MN}$ and $C_{MN}$ (Neveu-Schwarz and Ramond forms in type IIB)
- Ground state mass predictions

$$\frac{m_{1^{+-}}}{m_{2^{++}}} = 1.30 \qquad \frac{m_{1^{--}}}{m_{2^{++}}} = 1.64$$

  in very good agreement with lattice calculations
- Operator dimensions $\Delta = 6$ ($m^2_{\mathrm{AdS}} = 15$),

$$\mathcal{O}_{\mu\nu} = \mathrm{Tr}\left[F_{\mu\nu}F^2\right]$$

  they actually mix and acquire anomalous dimensions
- no $m^2_{\mathrm{AdS}} = 0$ mode (no $j = 1$ cut at finite $\lambda$)

# Conclusions and Outlook

- String theory offers a unified picture of high energy scattering
- Corrections beyond the leading order in strong coupling are desirable in both conformal and non-conformal models
- Klebanov-Strassler theory is an appealing model of a confining gauge theory. Some of its predictions are valid beyond the holographic limit
- Is there any integrable structure beyond the KS model and can it be used for other interesting predictions?