Froissaron and Maximal Odderon model approach
Theory and Phenomenology

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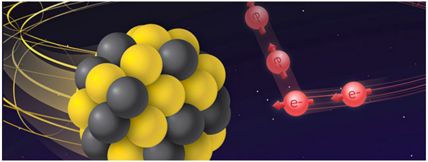


Outline

1. Introduction
   - Maximality of strong interaction
   - The main strict results for elastic scattering amplitude
   - History of Odderon and Maximal Odderon
2. TOTEM data on elastic pp and $\bar{p}p$ collisions
   - Total cross sections σ_t and ratios ρ
3. The amplitudes in FMO model
   - Froissaron and Maximal Odderon at $t = 0$
   - Extended FMO model at $t = 0$
   - Froissaron and Maximal Odderon at $t \neq 0$
4. Description of the data at $t \neq 0$) in the FMO model
5. Conclusion

Outline

Froissaron and Maximal Odderon are the model realization of

Maximality principle for the strong interaction:

The strong interactions at high energies are as strong as the main assumptions of the S-matrix theory and asymptotic theorems allow them to be (G. Chew (1962) for simple Regge poles).
The cross sections at high energies should saturate the asymptotic bounds in their functional form.

Froissaron realises principle of maximality for the total hadron cross-sections.

Maximal Odderon realises the maximal growth of the difference of particle-particle and antipartice-partice cross sections.

It was in 70-s, ISR experiments had confirmed the fast growth of σ_{tot}.

No experimental evidences for rising $\Delta\sigma_t$. The first hint at difference in $d\sigma_{pp}/dt$ and $d\sigma_{\bar{p}p}/dt$ in the dip region has been found later, in 1980.

Outline

The main asymptotic ($s \to \infty$ or $s \gg m^2$) theorems

- Froissart bound: $\sigma_t(s) \leq C \ln^2(s/s_0)$
  Łukaszuk-Martin: $C \leq \dfrac{\pi}{m_\pi^2}$

- Pomeranchuk theorems:
  1.) $\sigma_t(s) \to \text{const} \quad \Rightarrow |\sigma_t^{ab} - \sigma_t^{\bar{a}b}| \to 0$
  2.) $\sigma_t(s) \to \infty \quad \Rightarrow \quad \sigma_t^{ab}/\sigma_t^{\bar{a}b} \to 1$

- Difference of ab and $\bar{a}b$ total cross sections:
  Eden-Cornelli
  $\sigma_{\bar{a}b}^{ab}(s) \sim C \ln^\alpha(s/s_0) \pm D \ln^\beta(s/s_0),$
  $\alpha \leq 2, \quad \text{then} \quad \beta \leq \alpha/2 + 1$
  $|\Delta\sigma| \leq 2D \ln^{\alpha/2}(s/s_0)$

In FMO (Froissaron + Maximal Odderon) model:
$\sigma_t(s) \propto \ln^2(s/s_0), \quad |\Delta\sigma_t(s)| \propto \ln(s/s_0)$

- Auberson-Kinoshita-Martin theorem:
  If $\sigma_{tot} \propto \ln^2(s/s_0)$ then
  $|t| \lesssim C/\ln^2(s/s_0) \Rightarrow A(s,t) = s \ln^2(-is/s_0) f(\tau),$
  $\tau = R \ln(-is/s_0)\sqrt{-t}$

- Amplitudes satisfy the dispersion relations.

  Definitions for $\binom{pp\to pp}{\bar{p}p\to\bar{p}p}$:

$$F_{\bar{p}p}^{pp}(z_t,t) = F_+(z_t,t) \pm F_-(z_t,t), \quad z_t = 1 + \frac{2s}{t-4m^2},$$

$$F_\pm(-z_t,t) = \pm F_\pm(z_t,t)$$

Derivative form of the Dispersion Relations (DDR) at $s \gg 4m^2$

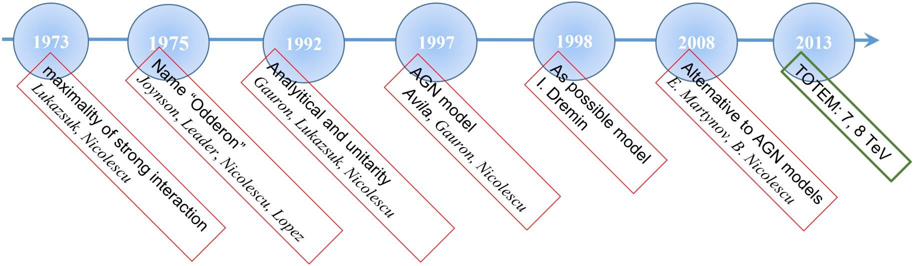
$$\mathrm{Re}[F_+(z_t,t)/s] = \left[\frac{\pi}{2}\frac{\partial}{\partial\xi} + \cdots\right]\mathrm{Im}[F_+(z_t,t)/s], \quad \xi = \ln(s/s_0)$$

FMO model: $\mathrm{Im}[F_+(z_t,0)/s] \propto \xi^2 \quad \Rightarrow \quad \mathrm{Re}[F_+(z_t,0)/s] \propto \xi$

$$\frac{\pi}{2}\frac{\partial}{\partial\xi}\mathrm{Re}[F_-(z_t,t)/s] = -\left[1 - \frac{1}{3}\left(\frac{\pi}{2}\frac{\partial}{\partial\xi}\right)^2 + \cdots\right]\mathrm{Im}[F_-(z_t,t)/s]$$

FMO model: $\mathrm{Im}[F_-(z_t,0)/s] \propto \xi \quad \Rightarrow \quad \mathrm{Re}[F_-(z_t,0)/s] \propto \xi^2$

Outline



L. Łukaszuk and B. Nicolescu, Letter al Nuovo Cimento 8 (1973) 405.

D. Joynson, E. Leader, B. Nicolescu, C. Lopez, Il Nuovo Cimento 30A (1975) 345.

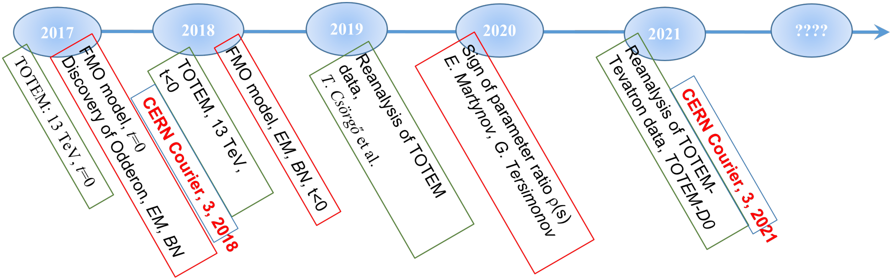
The name "Froissaron" was suggested by K. Ter-Martirosyan and M. Dubovikov in NP B124 (1977) 163.

In the later papers the F and MO models were modified, improved, compared with the data.

R. Avila, P. Gauron, B. Nicolescu, Eur. Phys. J. C 49 (2007) 581, in this paper the low values of ρ_{pp} at LHC (but with too high σ_t) were predicted.

I. Dremin, Soviet Physics Uspekhi, 31(1988)462. Qualitative consideration of F and MO.

E.M., Phys. Rev. D 76 (2007) 074030; E. M., B. N., Eur. Phys. J. C 56(2008) 57.



TOTEM, Eur. Phys. J. C 79, (2019)103;Eur. Phys. J. C (2019) 79:785.

TOTEM,Eur. Phys. J. C 79 (2019) 861.

E. Martynov, B. Nicolescu, Eur.Phys.J.C 79 (2019) 6, 461.

E. Martynov, G. Tersimonov, Phys.Rev.D 100 (2019) 11, 1140398.

T. Csörgő et al., arXiv:1912.11968v3 [hep-ph], E ur. Phys. J. 81 C (2021) 180.

TOTEM-D0, arXiv:2012.03981v2 [hep-ex] .

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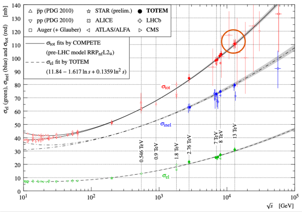
TOTEM forward scattering data at 13 TeV

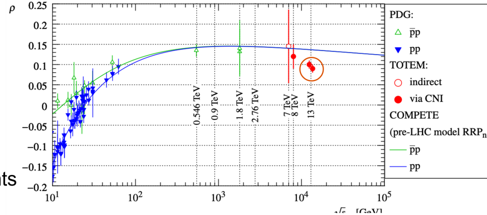
$$\sigma_t^{pp} = 110.6 \pm 3 mb, \qquad \rho^{pp} = 0.098 \pm 0.01\,(0.1 \pm 0.01)$$

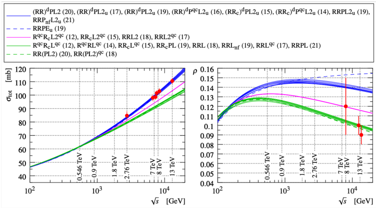
Figures with the TOTEM measurements from the talk of S. Giani at RRB meeting at CERN (October 2017);

Eur. Phys. J. C 79 (2019)103 and COMPETE fit predictions (up to 2002).

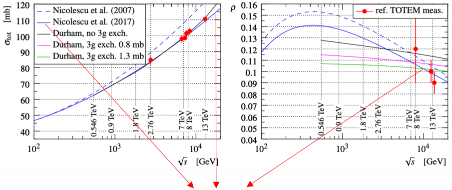


Results of the measurements





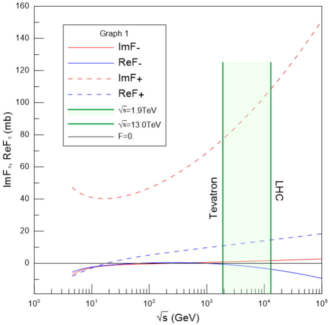
Predictions of COMPETE (no Odderon)



E. Martynov, B. Nicolescu, Did TOTEM experiment discover the Odderon? Phys. Lett, (2018) B 778, 414–418.

The set of COMPETE models cover almost all the models used for σ_t and ρ without Odderon, but they fail to describe simultaneously the 13 TeV TOTEM data for σ_t and ρ

Then, this fact we considered as the clear evidence in favor of existence of Odderon



When our paper was already in HEP ArXiv we asked ourselves "Why odderon was neglected in the most part of the models? Why the various estimations allowed to do that? " Searching an answer we plotted the partial contributions to imaginary and real parts of amplitudes at $t = 0$.

Only at LHC energies the Re$F^{MO}(s)$ (solid blue line) becomes to be visible!

Outline

How to describe the total cross section rising as $\xi = \ln^2(s/s_0)$, $s_0 = 1\text{GeV}^2$?

It would seem that one can assume that the partial amplitude is a contribution of three-fold pole with a linear trajectory

$$\phi(\omega.t) = \frac{\beta(\omega, t)}{(\omega - \alpha' t)^3}, \quad \omega = j - 1, \quad \Rightarrow \sigma_t(s) \propto \xi^2$$

However, the integral of $d\sigma_{el}(s,t)/dt$ over t has the effective value $t_{eff} \sim 1/\xi$ and $\sigma_{el}(s) \propto t_{eff} \cdot \xi^4 \propto \xi^3$. Violation of unitarity! .

More correct partial amplitude should have another form.

Assumption from the beginning:

Froissaron and Maximal Odderon as inputs are not single j-poles.

$$\phi(\omega, t) = \frac{\beta(\omega, t)}{[\omega^m - rt/t_0]^n} \quad \text{where } t_0 = 1\,\text{GeV}^2 \quad \text{and} \begin{cases} mn \leq m + 1, \\ mn \leq 3. \end{cases}$$

If $\sigma_{el}/\sigma_t \to$ const at $s \to \infty$ then $n = 1 + 1/m$. Furthermore, if $\sigma_t \propto \xi$ (pole pomeron mode) then $m = 1$ and $n = 2$. In the Froissaron term (or tripole pomeron) $m = 2$ and $n = 3/2$. Then $\sigma_t \propto \sigma_{el} \propto \xi^2$. The same arguments are valid for Maximal Odderon.

FMO model at $t = 0$

Forward pp and $\bar{p}p$ amplitudes contain contributions of Froissaron, Maximal Odderon, Standard Pomeron and Odderon with intercepts $\alpha_{P,O}(0) = 1$ and effective crossing-even and crossing-odd secondary reggeons:.

$$A^{pp}_{\bar{p}p}(z) = A^F_+(z) + A^R_+(z) \pm \left(A^{MO}_-(z) + AF^R_-(z)\right),$$

$$z = |z_t(t=0)| = (s-2m^2)/2m^2,$$

$$A^R_\pm(z) = -\begin{pmatrix}1\\ i\end{pmatrix} C^R_\pm(-iz)^{\alpha_\pm(0)},$$

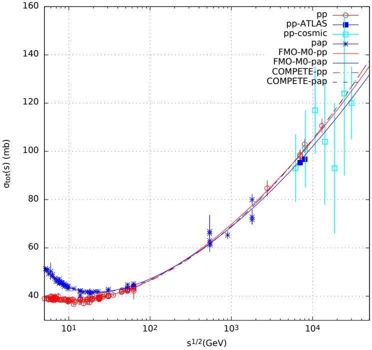
$$A^F_+(z) = i(s-2m^2)[H_1\ln^2(-iz) + H_2\ln(-iz) + H_3],$$

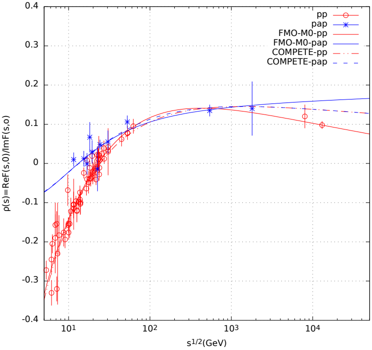
$$A^{MO}_-(z) = (s-2m^2)[O_1\ln^2(-iz) + O_2\ln(-iz) + O_3].$$

The standard Pomeron and Odderon (with intercepts equal to 1) are not shown because their contributions to $A^{pp}_{\bar{p}p}(z)$ at $t = 0$ are constants and they are absorbed in F_3, O_3 which as well are constant. These terms would be distinguished at $t \neq 0$.

Description of the forward pp and $\bar{p}p$ data on $\sigma_t(s)$ and $\rho(s)$ in FMO model at

$$5\,\text{GeV} \le \sqrt{s} \le 13\,\text{TeV}$$





E. Martynov, B. Nicolescu, Phys. Lett. B778 (2018) 414.

Curves (dashed lones) of the best COMPETE model without Odderon (J.R. Cudell, ..., E. Martynov, B. Nicolescu et al., COMPETE Collaboration, Phys. Rev. Lett. 89 (2002) 201801) are shown for a comparison.

Outline

Extended FMO model at $t = 0$

We have considered as well the models of "Froissaron" and "Maximal Odderon" in a generalized form with free β_F , β_O including standard Pomeron and Odderon with free intercepts $\alpha_{P,O}(0) \leq 1$.

$$A_+^F(z) = i(s - 2m^2)[H_1 \ln^{\beta_F}(-iz) + H_2 \ln^{\beta_F - 1}(-iz) + H_3],$$

$$A_-^{MO}(z) = (s - 2m^2)[O_1 \ln^{\beta_O}(-iz) + O_2 \ln^{\beta_O - 1}(-iz) + O_3],$$

where $\beta_O = \beta_F/2 + 1 - \delta_{MO} \geq 0, \quad \delta_{MO} \geq 0.$

$$A_\pm^R(z) = \begin{pmatrix} P, R_+ \\ O, R_- \end{pmatrix} = -\begin{pmatrix} 1 \\ i \end{pmatrix} C_\pm^R(-iz)^{\alpha_\pm(0)}.$$

The aim was to verify a stability of the choise of Froissaron and Maximal Odderon as solution uniquly describe the LHC data. Which values of parameters β_F, β_{MO} and Pomeron and Odderon intercepts $\alpha_P(0), \alpha_O(0)$ are the best for agreement of the FMO model with experimental data on $\sigma_t(s)$ and $\rho(s)$ for pp and $\bar{p}p$ interactions.

The main results obtained in this fit and analysis are the following

1. β_F and β_{MO}, if they are free, come back to the saturation values $\beta_F = \beta_O = 2$.

2. Pomeron intercept comes back to 1 from any input value < 1.

3. More details conserning the standard odderon (the single j-pole with an intercept < 1) are given in our paper

E. Martynov, B. Nicolescu, Evidence for maximality of strong interactions from LHC forward data, Phys. Lett. B786 (2018) 207.

4. If we put $O_1 = 0$ (no Maximal Odderon) description of the data is failed and $\chi^2/\mathrm{ndf} \approx 1.07(O_1 \neq 0) \Rightarrow 1.18(O_1 = 0)$.

5. Thus, we confirm that the principle of maximality of strong interaction works. The Froissaron and Maximal Odderon are neccessary for succesful description of the forward LHC data.

Outline

FMO model at $t \neq 0$

Let me come back for the moment to the simple reasoning and assumption in order to choose the generalization of the FMO model for $t \neq 0$.
We suppose that in agreement with the structure of the singularity of $\phi_\pm(\omega, t)$ at $\omega^2 - r_\pm^2 t/t_0 = 0$ the functions $\tilde{\beta}_\pm(\omega, t)$ depend on ω through the variable $\kappa_\pm = (\omega^2 - r_\pm^2 t/t_0)^{1/2}$ and it can be expanded in powers of $\kappa_\pm$ at small $\kappa_\pm$

$$\phi_\pm(\omega, t) = \begin{pmatrix} i \\ 1 \end{pmatrix} \frac{\beta(\omega, t)}{[\omega^2 - r_\pm^2 t/t_0]^{3/2}} = \begin{pmatrix} i \\ 1 \end{pmatrix} \frac{\tilde{\beta}_{1\pm}(t) + \kappa_\pm \tilde{\beta}_{2\pm}(t) + \kappa_\pm^2 \tilde{\beta}_{3\pm}(t)}{\kappa_\pm^3}.$$

There is correspondence to AKM theorem.
At $s \to \infty$ the main contribution to amplitude $A(z_t, t)$ comes from the region $\omega \sim 1/\xi, \quad \omega \sim r_\pm\sqrt{-t/t_0}, \quad \kappa_\pm \to 0.$

FMO model at $t \neq 0$

Choice of Froissaron and Maximal Odderon at $t \neq 0$

Amplitude of antiproton-proton scattering

$$A_{pp}(s,t) = A_{+}(s,t) + A_{-}(s,t)$$

amplitude of antiproton-proton scattering

$$A_{\bar{p}p}(s,t) = A_{+}(s,t) - A_{-}(s,t)$$

$$A_{\pm}(-z_t,t) = \pm A_{\pm}(z_t,t)$$

In the FMO model crossing-even and crossing-odd terms of amplitudes are defined as sums of the asymptotic and subasymptotac contributions $A^{F}(s,t)$ (Froissaron), $A^{MO}(s,t)$ (Maximal Odderon) shown in the expansion at the previous slide. Other contributions are presented at the next slides.

Froissaron and Maximal Odderon

$$\frac{-1}{iz}A^F(z_t,t) = H_1\xi^2\frac{2J_1(r_+\tau\xi)}{r_+\tau\xi}\Phi^2_{H,1}(t)$$
$$+H_2\xi\frac{\sin(r_+\tau\xi)}{r_+\tau\xi}\Phi^2_{H,2}(t) + H_3J_0(r_+\tau\xi)\Phi^2_{H,3}(t),$$
$$\Phi_{H,i}(t) = \exp(b_i^H q_+), \quad i = 1.2, 3$$
$$\tau = \sqrt{(-t/t_0)},\ t_0 = 1\mathrm{Gev}^2, \quad q_+ = 2m_\pi - \sqrt{4m_\pi^2 - t}.$$

$$\frac{1}{z}A^{MO}(z_t,t) = O_1\xi^2\frac{2J_1(r_-\tau\xi)}{r_-\tau\xi}\Phi^2_{O,1}(t)$$
$$+O_2\xi\frac{\sin(r_-\tau\xi)}{r_-\tau\xi}\Phi^2_{O,2}(t) + O_3J_0(r_-\tau\xi)\Phi^2_{O,3}(t),$$
$$\Phi_{O,i}(t) = \exp(b_i^O q_-), \quad i = 1, 2, 3$$
$$q_- = 3m_\pi - \sqrt{9m_\pi^2 - t}.$$

Here the vertex functions $\Phi(t)$ are chosen in a none pure exponential form with the lowest thresholds $2m_\pi^2$ and $3m_\pi^2$ for Froissaron and Maximal Odderon correspondingly.

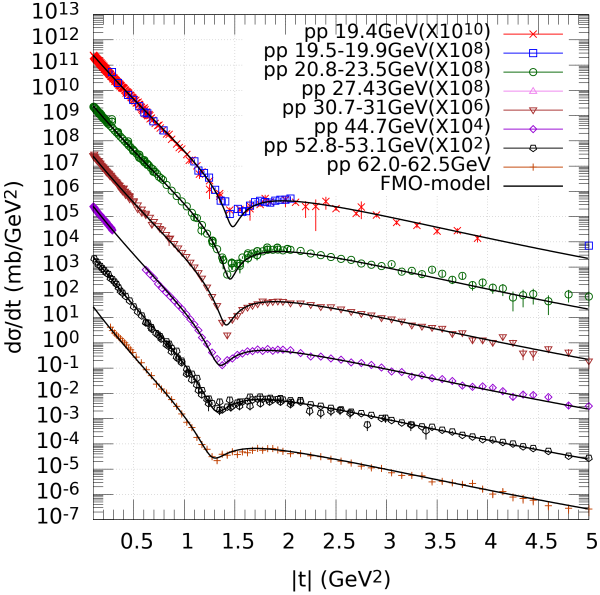
All other terms of the amplitudes are added in the standard parametrization. "Hard" pomeron, A^{hP}, and odderon, A^{hO}, are decreasing at high t as t^{-4}.

Full amplitudes have the form

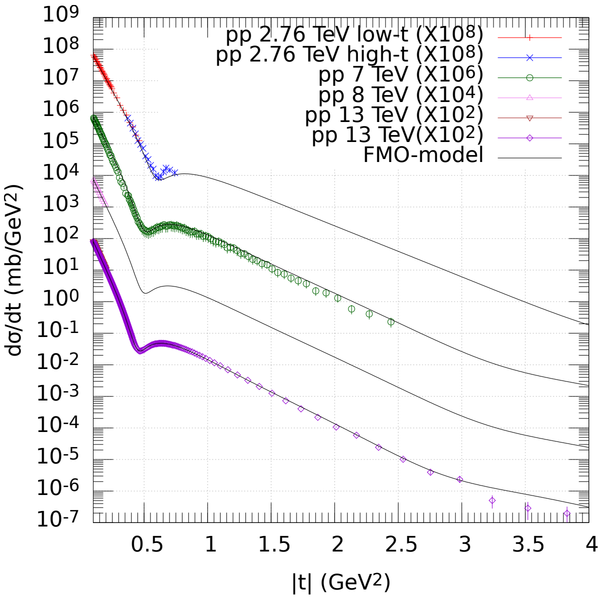
$$A_{+}(z_t, t) = A^F + A^P + A^{R_+} + A^{PP} + A^{OO} + A^{hP}$$

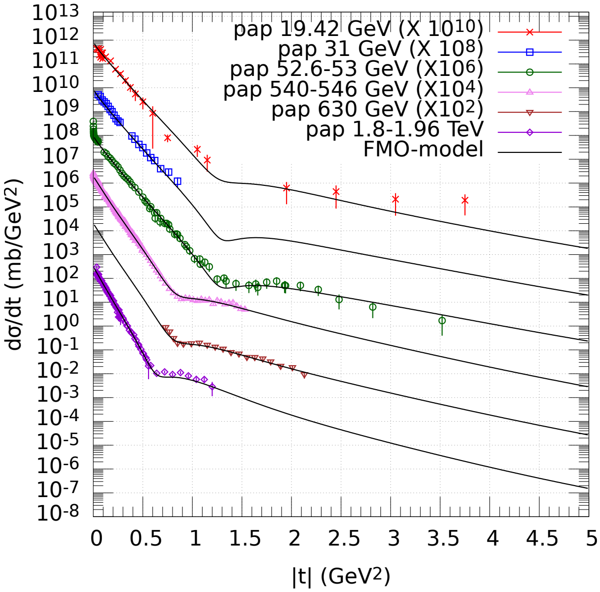
$$A_{-}(z_t, t) = A^{MO} + A^O + A^{R_-} + A^{PO} + A^{hO}.$$

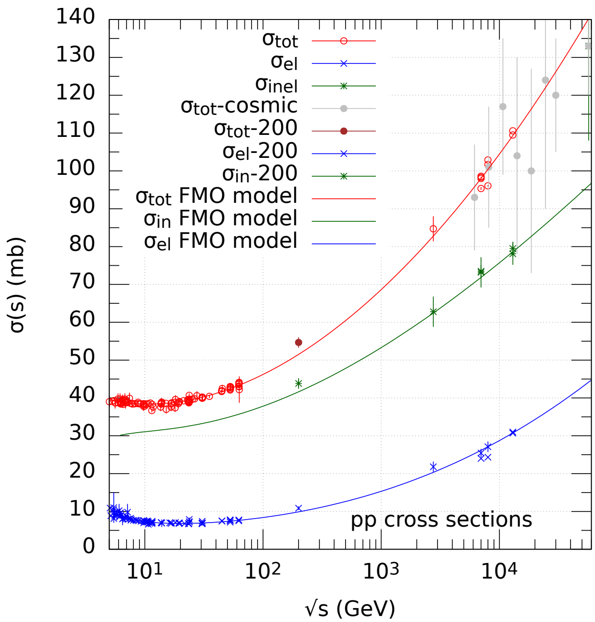
- Froissaron and Maximal Odderon are important in whole kinematic region. However they start to dominate in TeV region.
- Standard Pomeron and Odderon are important in GeV region. The cuts tune the $d\sigma/dt$ region around dip-bump structure and in the second diffraction cones.
- A role of "hard" terms is not clear enough, however. They help to obtain a good description of the considered data (despite that we consider region where $|t|/s, z_t \lesssim 1$).
- Secondary reggeons are negligible starting from energy about 100 GeV.



pp 19.4GeV(X10^10)
pp 19.5-19.9GeV(X10^8)
pp 20.8-23.5GeV(X10^8)
pp 27.43GeV(X10^8)
pp 30.7-31GeV(X10^6)
pp 44.7GeV(X10^4)
pp 52.8-53.1GeV(X10^2)
pp 62.0-62.5GeV
FMO-model
dσ/dt (mb/GeV^2)
|t| (GeV^2)


pp 2.76 TeV low-t (X10^8)
pp 2.76 TeV high-t (X10^8)
pp 7 TeV (X10^6)
pp 8 TeV (X10^4)
pp 13 TeV (X10^2)
pp 13 TeV(X10^2)
FMO-model
dσ/dt (mb/GeV^2)
|t| (GeV^2)
10^9
10^8
10^7
10^6
10^5
10^4
10^3
10^2
10^1
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
10^-7
0.5
1
1.5
2
2.5
3
3.5
4


pap 19.42 GeV (X 10^10)
pap 31 GeV (X 10^8)
pap 52.6-53 GeV (X10^6)
pap 540-546 GeV (X10^4)
pap 630 GeV (X10^2)
pap 1.8-1.96 TeV
FMO-model
dσ/dt (mb/GeV^2)
|t| (GeV^2)
10^13
10^12
10^11
10^10
10^9
10^8
10^7
10^6
10^5
10^4
10^3
10^2
10^1
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
10^-7
10^-8
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5



σ(s) (mb)
√s (GeV)
σtot
σel
σinel
σtot-cosmic
σtot-200
σel-200
σin-200
σtot FMO model
σin FMO model
σel FMO model
pp cross sections
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
10¹
10²
10³
10⁴

CONCLUSIONS

- When we say "discovery of Odderon" we do not mean glueball resonance with quantum numbers of odderon. Generally, glueball resonances, either of Pomeron or Odderon kind, can not be detected in high energy elastic scattering. In elastic processes Pomeron and Odderon manifests only as reggeons.
- The main properties of the Froissaron and Maximal Odderon model are determined by the hypothesis of maximality of strong interactions and follow from the rigorous theorems of the analytic S-matrix theory and theory of the complex Regge singularities. From the beginning Froissaron and Maximal Odderon are considered as more complicated singularities in the complex angular momentum plane than single Regge poles.
- The values of pp total cross section and the ratio ρ measured by TOTEM at 13TeV can be described and treated as Odderon effects only if the Maximal Odderon contribution to amplitude exists, i.e. parameter O_1 in MO contribution is not zero.
- Until now the FMO model is the only physical model (not just a parametrization) which does not violate the main unitarity bounds for physical observables and can qualitatively describe the data in a wide kinematic region.

Thank you for attention!

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