Theory and Phenomenology of the Odderon

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Outline

Odderon: the landscape

- A lot is known about the perturbative Odderon evolution: LO, NLO, different numbers of Reggeized gluon, unitarity corrections
- Little is known about magnitude of the Odderon coupling to the proton, until recently we had only upper limits. The uncertainties come from the non-pertubative nature of this coupling and moderate experimental input
- In the nonperturbative domain — very little theoretical insight
- Experiment: the first strongly positive data on the Odderon just have been presented

Obtained results:

Theory > Phenomenology > Experiment

Odderon — the beginning

- Motivated by rising total hadronic cross sections and the Regge pole model
- Introduced in 1973 by Leszek Łukaszuk i Basarab Nicolescu, the name given in 1975 by Nicolescu et al.
- Origin: the general Pomeranchuk theorem

$$\frac{\sigma_{p\bar{p}}}{\sigma_{pp}} \to 1 \qquad \text{for } s \to \infty$$

- The Froissart–Martin bound:

$$\sigma_{pp} < A \log^2 s$$

leaves some space for slowly growing with s or constant Odderon cross-sections, in fact $\sigma_{pp} - \sigma_{p\bar{p}} < C \log s$.

Note: the asymptotic theorems are always based on unitarity of the S-matrix, so the perturbative calculations, like BFKL and BKP may apparently violate them.

Odderon, S-matrix, Regge model

- In the Regge pole model the Reggeons are described by poles in the complex j-plane. This picture may be questionned within QCD (e.g. Regge cuts and multiple dense systems of poles are found in pQCD), but this still provides useful guidelines.

- The Regge pole exchange amplitudes split into two cathegories that differ by the Reggeon signature $\eta = \pm 1$

$$A^{(\eta)}(s,t) = \beta(t)\Gamma(-\alpha(t))\left[\eta + e^{i\pi\alpha(t)}\right]\left(\frac{s}{s_0}\right)^{\alpha(t)}$$

  where the function $\alpha(t)$ is the Regge trajectory, often assumed to be linear, $\alpha(t) = \alpha(0) + \alpha' t$, with the intercept $\alpha(0) \sim 1$ for P and O

Odderon, S-matrix, Regge model

- The signature factors $\eta + e^{i\pi\alpha(t)}$ determine the leading part of the Reggeon complex phase,

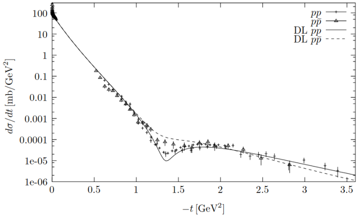
$$A_P \sim \left(\frac{-is}{s_0}\right)^{\alpha_{\mathbf{P}}(t)-1}, \qquad A_O \sim \mp i \left(\frac{-is}{s_0}\right)^{\alpha_{\mathbf{O}}(t)-1},$$

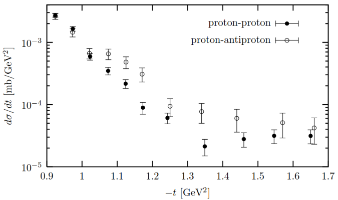
- Hence, the Pomeron scattering amplitude T (where $S = 1 + iT$), is close to imaginary (but with a possible real correction), and the Odderon scattering amplitude T is close to real.

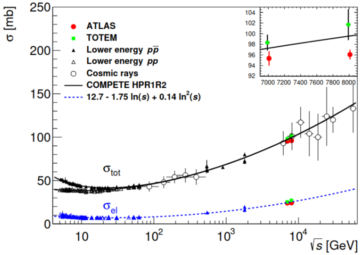
- Note also that the ratio $\rho(t) = \Re A(s,t)/\Im A(s,t)$, related to the famous $\rho = \rho(0)$ parameter may significantly depend on t.

Odderon in $d\sigma/dt$ in pp and $p\bar{p}$

First hints of a difference in $d\sigma/dt$ in the dip region coming from ISR (1985) at $\sqrt{s} = 52.8$ GeV







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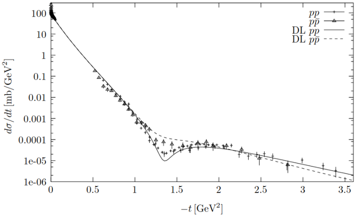
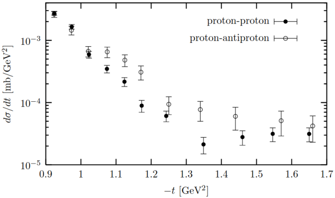
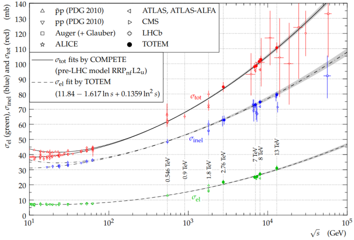
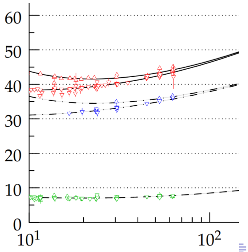


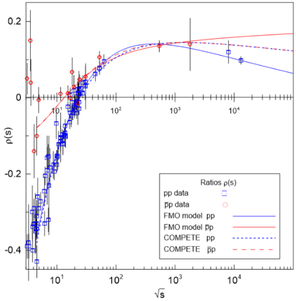






Odderon, Pomeron and ρ

- Tension of σ_{tot} and the ρ parameter in pp, $p\bar{p}$ revealed by TOTEM measurements nicely explained with a maximal Odderon.
- The model was proposed in 1991 by P. Gauron, B. Nicolescu, E. Leader, and the fits to LHC data were done by E. Martynow and B. Nicolescu in 2017.
- The maximal Odderon is a stretched scenario, as pointed out by Khoze, Martin and Ryskin, 2018



$\sqrt{s}$ (TeV)	σ_{tot}^{pp} (mb)		ρ^{pp}	
	TOTEM	FMO	TOTEM	FMO
2.76	84.7± 3.3	83.66	-	0.123
7	98.6± 2.2 98.0± 2.5	98.76	-	0.109
8	101.5± 2.1 102.9± 2.3	101.09	0.12 ± 0.03	0.106
13	110.6± 3	109.92	0.098±0.01	0.0976

Odderon in QCD: the lowest order

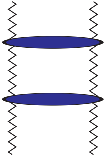
- Gluon field transformation under the charge conjugation:

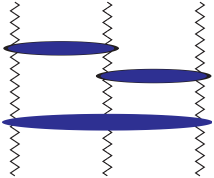
$$\hat{A}_\mu \to \hat{A}_\mu^C = C\hat{A}_\mu C^{-1} = -\hat{A}_\mu^T$$

- The minimal number of elementary gluons in the C-odd, color neutral state is 3
- The symmetric color tensor d^{abc} carries negative C-parity: Odderon sector
- The antisymmetric color f^{abc} tensor with three gluons carries the positive C-parity
- Interestingly enough, the Odderon color tensor d^{abc} corresponds to the universal form of the cubic Casimir operator of SU(N_c), $d^{abc}T^aT^bT^c$

Odderon in QCD: the BKP evolution

- The description of the Odderon exchange in pQCD is rooted in the BFKL framework, that is generalized to the case of more than two exchanged gluons
- AT LL the same ingredients: Reggeized gluons with the same trajectory as in the BFKL equation and the interaction BFKL kernels. Proposed by J. Bartels; J. Kwieciński, M. Praszałowicz, and also T. Jaroszewicz, in 1980.





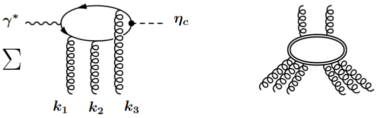
- Interactions between Reggeons at LL: all pairwise interactions with the BFKL interaction kernels
- At LL accuracy the BKP evolution equation obeys conformal symmetry in the transverse plane

Odderon solutions: Janik–Wosiek, towards integrability and complexity

- The conformal symmetry (or SL(2,C) invariance) of the BKP equation greatly simplifies the solution of the Odderon problem
- Works of L. Lipatov (1993) and L. Faddeev, G. Korchemsky show that the LL BKP equation leads to integrable problem, related to XXX Heisenberg spin chain with non-compact SL(2,C) spins. Solutions with a generalized Bethe ansatz
- R. Janik, J. Wosiek (1998) solved the Baxter equation (complex analysis + semi-analytic approach) for operator $\hat{q}_3$
- The Odderon intercept: $\alpha_O(t=0) = 1 - 0.24717\frac{N_c\alpha_s}{\pi}$ — flat dependece of cross section on energy
- Extension of this technique to higher number of Reggeons [G. Korchemsky, J. Kotański, A. Manashov]
- Recent work by A. Sabio-Vera and G. Chachamis (2018) — MC analysis of open spin chains. An intriguing observation of graph "complexity democracy" found.

Odderon solutions: Bartels-Lipatov-Vacca, 1999

- Origin of the idea: the Pomeron–Odderon–Odderon vertex obtained by J. Bartels and C. Ewerz and $\gamma \to \eta_c$ impact factor: in both cases the odderon solutions are spanned on two points in the transverse space

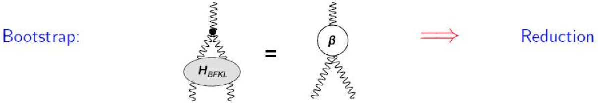


$$E_3^{(n,\nu)}(\vec{k}_1, \vec{k}_2, \vec{k}_3) = A \cdot \sum_{(123)} \frac{(\vec{k}_1 + \vec{k}_2)^2}{\vec{k}_1^2 \vec{k}_2^2} E^{(n,\nu)}(\vec{k}_1 + \vec{k}_2, \vec{k}_3)$$

- The energy evolution of such state is described by the BFKL equation, however the odd C-parity projection implies the odd value of conformal spin
- The maximal intercept $1 + \omega_O$ is reached for $n = 1$ and $\nu = 0$: $\omega_O = 0$

d-Reggeon and bootstrap equations

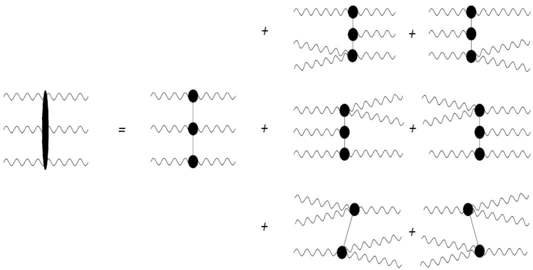
- Gluon Reggeons are gluon compounds that are pointlike in the transverse space, resum to all orders (LL, NLL, ...) virtual corrections. Rapidity evolution leads to gluon Regge trajectory. They obey bootstrap equation:



- The f and d tensors are related to Reggeon signatures. BLV Odderon may be understood by introducing gluonic the d-Reggeon – an octet state with the opposite signature to f-Reggeon
- The d-Reggeon is a solution of the same bootstrap equation that generates usual gluon f-Reggeon, but with a different initial condition. At the lowest order two elementary gluons at the same transverse position are necessary to bulid the d-Reggeon, but as for the f-Reggeon they resum contributions of the elementary gluon number up to infinity
- The BFKL interaction kernel in the (fd) system preserves the (fd) structure

Odderon at NLO

- The NLO corrections to the BKP equation were computed by J. Bartels, V. Fadin, L. Lipatov and G.P. Vacca in 2012.
- Ingredients: the NLO gluon Regge trajectory, the BFKL NLO interaction kernels for $2 \to 2$ Reggeized gluon scatterings
- A new piece: connected $3 \to 3$ Reggeized gluon scattering, at LO



Odderon intercept beyond NLO

There are strong hints that the BLV Odderon intercept stays at $\alpha_0 = 1$ beyond LO (in contrast to the BFKL Pomeron)

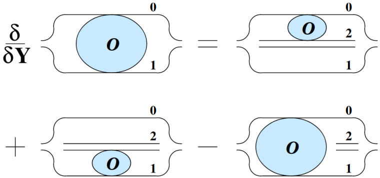
- A. Staśto, 2009: conjecture that the BLV odderon intercept stays at $\alpha_O = 1$ at all orders made on the basis of the ω expansion approach, and evaluation of the NLO BFKL kernel for conformal spin $n = 1$
- J. Bartels, C. Contreras, G.P. Vacca, 2019; G.P. Vacca, M.A. Braun, 2020 — inclusion of the running coupling effects does not change the BLV intercept
- Hints coming from the AdS/CFT correspondence — a relation to exchange of $B_{\mu\nu}$ field in SUGRA
- The intercept may be, however, pushed below $\alpha_0 = 1$ by unitarity corrections

Odderon in AdS/CFT

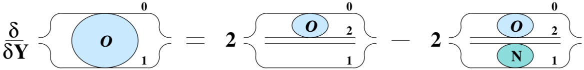
- The high energy scattering in Yang–Mills theories may be analyzed for the strong gauge coupling and in the large N_c limit using the AdS/CFT correspondence. In this limit, the YM amplitudes are obtained using weak coupling limit of string theory — the gravity (or supergravity)
- Pioneering this path, R. Janik and R. Peschanski (1999) evaluated the SuperYM high energy scattering amplitudes using $\mathcal{N} = 4$ SUGRA interactions between two string worldsheets
- Intercepts: 2 for graviton exchange ($\sim$ Pomeron), 0 for a scalar and dilaton exchange and 1 for SUGRA $B_{\mu\nu}$ antisymmetric field, with the scattering amplitude characteristic to a vector particle ($\sim$ Odderon),
- A more detailed analysis of the Odderon mediated scattering in AdS/CFT performed by E. Avsar, Y. Hatta, T. Matsuo in 2009. The baryons and antibaryons represented by D5-branes wrapped on S^5 of $AdS_5 \times S^5$
- The Reggeized B-field leads to series of Reggeons, leading $\alpha_0 = 1$ (decoupling?), then $\alpha_0 = 1 - 1/2\sqrt{\lambda}$. No unitarity corrections included.
- Surprisingly, the total pp cross-section found to be larger than $p\bar{p}$

Odderon from color dipoles

The BLV Odderon evolution has a natural interpretation in the Mueller's color dipole model [Kovchegov, Szymanowski, Wallon, 2003]. With a C-odd initial condition for color dipole scattering amplitude, the dipole cascade does not change the overall C-parity



Straightforward to include unitarity corrections



BLV Odderon with unitarity corrections

The evolution of C-odd dipole operator:

$$O(\vec{x},\vec{y}) = \frac{1}{2iN_c}\mathrm{Tr}\left(V^{\dagger}_{\vec{x}}V_{\vec{y}} - V^{\dagger}_{\vec{y}}V_{\vec{x}}\right)$$

The Odderon evolution equation couples to the Pomeron evolution equation:

$$\frac{\partial N(\boldsymbol{x},\boldsymbol{y};\tau)}{\partial\tau} = \frac{\bar{\alpha}_s}{2\pi}\int d^2z\,\frac{(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{z}-\boldsymbol{y})^2}\left[N(\boldsymbol{x},\boldsymbol{z};\tau)+N(\boldsymbol{z},\boldsymbol{y};\tau)-N(\boldsymbol{x},\boldsymbol{y};\tau)\right.$$

$$\left.-N(\boldsymbol{x},\boldsymbol{z};\tau)N(\boldsymbol{z},\boldsymbol{y};\tau)+O(\boldsymbol{x},\boldsymbol{z};\tau)O(\boldsymbol{z},\boldsymbol{y};\tau)\right],$$

$$\frac{\partial O(\boldsymbol{x},\boldsymbol{y};\tau)}{\partial\tau} = \frac{\bar{\alpha}_s}{2\pi}\int d^2z\,\frac{(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{z}-\boldsymbol{y})^2}\left[O(\boldsymbol{x},\boldsymbol{z};\tau)+O(\boldsymbol{z},\boldsymbol{y};\tau)-O(\boldsymbol{x},\boldsymbol{y};\tau)\right.$$

$$\left.-O(\boldsymbol{x},\boldsymbol{z};\tau)N(\boldsymbol{z},\boldsymbol{y};\tau)-N(\boldsymbol{x},\boldsymbol{z};\tau)O(\boldsymbol{z},\boldsymbol{y};\tau)\right],$$

The results were confirmed and generalized by [Hatta, Iancu, Itakura and McLerran 2005] beyond the large N_c limit within CGC / JIMWaLK framework

Impact of unitarity corrections on the Odderon

- The Pomeron amplitude is strongly leading and self-unitarizes itself, the impact of the Odderon amplitude on the Pomeron may be neglected
- The Odderon amplitude in the dense gluon regime is strongly affected – mostly absorbed by the Pomeron
- The leading behavior of C-even and C-odd scattering amplitudes may be estimated using the saddle point approach in the Mellin space related to the scale
- Approximate analytical results [LM, 2005] for functions

  $\Phi(\vec{k}, Y) = \int \frac{d^2\vec{r}}{r^2} N(\vec{r}, Y) e^{-i\vec{k}\vec{r}}$ and $\Psi(\vec{k}, Y) = \int \frac{d^2\vec{r}}{r^2} O(\vec{r}, Y) e^{-i\vec{k}\vec{r}}$
- The results: $Q_S(Y) \simeq Q_0 e^{2.4\alpha_s Y}$

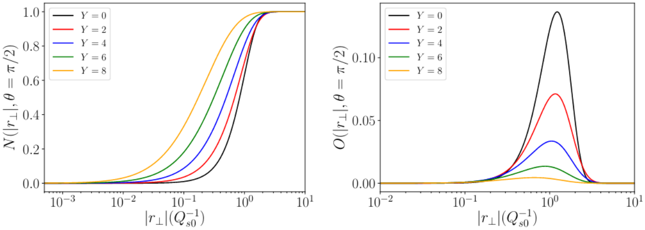
  Saturation: $\Phi(\vec{k}, Y) \simeq \log(Q_s(Y)/k)$, $\Psi(\vec{k}, Y) \simeq k/Q_s(Y) \exp(-4.1\bar{\alpha}_s Y)$

  Linear: $\Phi(\vec{k}, Y) \simeq (k/Q_s(Y))^{-1.26}$, $\Psi(\vec{k}, Y) \simeq (k/Q_s(Y))^{-2.08} \exp(-4.1\bar{\alpha}_s Y)$

Solving Odderon evolution with unitarity corrections

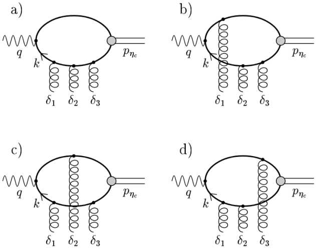
- In the saturation regime the Odderon amplitude quite strongly decreases with energy.
- In addition the Odderon amplitude decreases faster by $\sim (k^2)^{1/2}$ towards zero for $k^2 \to 0$ than the Pomeron
- No geometric scaling for the Odderon, strong decrease with Y saturation region

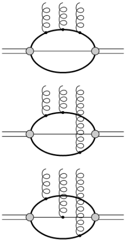
Recent numerical solution for $N(r, Y)$ and $O(r, Y)$ [X. Yao, Y. Hagiwara, Y. Hatta, 2018]



The perturbative Odderon couplings

- At lowest order couplings of two generic Odderon may be computed using C-odd projections of the dipole and baryon impact factors

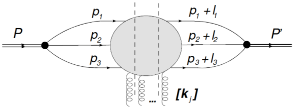


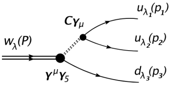


- The dipole corresponds to Bartels–Lipatov–Vacca Odderon, and the baryon couples also to Janik–Wosiek Odderon
- The uncertainties for the Odderon–proton coupling are rather large. The coupling has the origin in the non-perturbative region, a high power of strong coupling constant enters and the unknown details of the shape matter

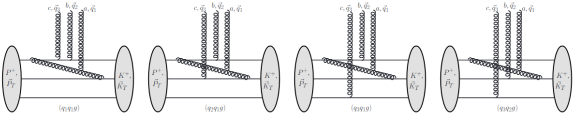
News in modeling and computing the couplings

- The proton is a complex particle with spin. The description requires a well guided modeling. Example: using Ioffe baryon current and Borel transform in description of a baryon projectile in high energy scattering [J. Bartels, LM, 2007]





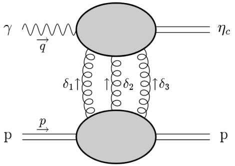
- Similar problem addressed in the CGC framework [A. Dumitru, V. Skokov, T. Stebel, 2020], , in addition a higher Fock stated included in the baryon wave function [A. Dumitru, R. Paatelainen, 2021]

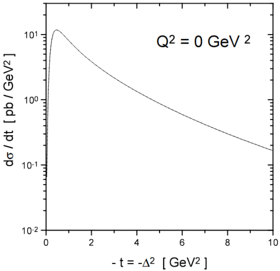


- The spin dependent Odderon coupling [L. Szymanowski, J. Zhou, 2016]

Odderon in exclusive processes: η_c photoproduction

- Proposal to search for the Odderon in the perturbative domain: exclusive η_c at HERA [J. Czyżewski, J. Kwieciński, M. Sadzikowski, LM (1996)]

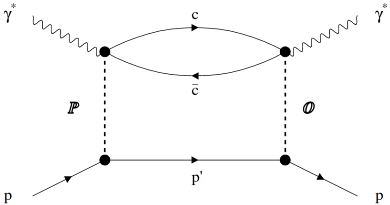




- The cross-section was estimated to be about 10 pb (taking no enhacement due to evolution). Difficult to measure: dominant measurable decay channel $\eta_c \to \rho\rho \to \pi\pi\pi\pi$, only about 2%
- Photo- and electroproduction of C-even mesons η_c, a_2, π_0 analyzed by [R. Engel, D.Yu. Ivanov, R. Kirschner, L. Szymanowski, 1997]

Odderon from charge asymmetry in diffractive charm photoproduction

- The Odderon couples much weaker than the Pomeron, so its effects are supposed to be largest in processes sensitive to Pomeron–Odderon interference, $\mathcal{A}_P^* \mathcal{A}_O + c.c.$, where the amplitude enters linearly
- Proposal by S. Brodsky, C. Merino, J. Rathsman (1999) to study forward–backward charge asymmetry in open charm photoproduction at HERA
- Exclusive or diffractive charm photoproduction: $c\bar{c}$ forward-backward charge asymmetry



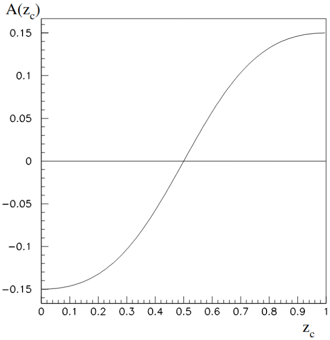
- Competition from the photon exchange amplitude at small t

Odderon from charge asymmetry — predictions

- The resulting charm distribution asymmetry

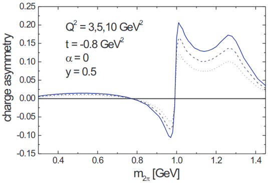
$$A(t=0, z_c) \sim \frac{g^{\mathcal{O}}_{pp'}}{g_{pp'}}^{\mathcal{P}} \frac{2z_c - 1}{z_c^2 + (1-z_c)^2}$$

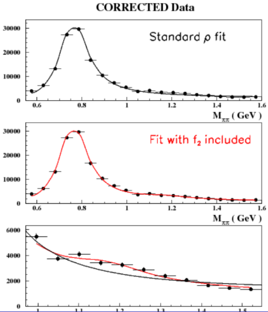
- Assuming $g^{\mathcal{O}}_{pp'}/g^{\mathcal{P}}_{pp'} = 0.1$ and $W = 100$ GeV at HERA



More ideas to search the Odderon in *ep* collisions

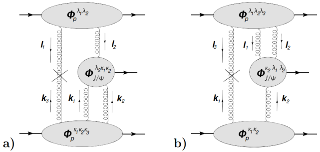
- Charge asymmetry and spin asymmetries in diffractive $\pi^+\pi^-$ electroproduction [P. Hägler, B. Pire, L. Szymanowski, O. Teryaev, 2002]
- Searches for diffractive π^0 electophotoproduction:
  Predicted $\sigma(\gamma^* p \to \pi^0 N^*) \simeq 200$ nb.
  H1 found $\sigma < 39$ nb
- Searches for diffractive f_2 production:
  Predicted $\sigma(\gamma^* p \to \pi^0 N^*) \simeq 21$ nb.
  ZEUS measured $\sigma = 3$ nb.
  There is, however, a photon exchange contribution.
- Implications for Odderon searches in ultraperipheral collisions

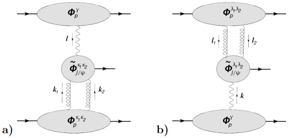




Odderon in exclusive hadroproduction of heavy vector mesons

The exclusive vector meson hadrodproduction $J/\psi, \Upsilon$ may be realized by Pomeron–Odderon or Pomeron-photon fusion [A. Schäfer, L. Mankiewicz, O. Nachtmann, 1991; A. Bzdak, L. Szymanowski, J.-R. Cudell, LM, 2007]





$d\sigma^{\rm corr}/dy$	J/ψ		Υ	
	odderon	photon	odderon	photon
Tevatron	0.3–1.3–5 nb	0.8–5–9 nb	0.7–4–15 pb	0.8–5–9 pb
LHC	0.3–0.9–4 nb	2.4–15–27 nb	1.7–5–21 pb	5–31–55 pb

In *pp* collisions Pomeron–photon fusion dominates, except of larger |t| region

Spin dependent Odderon

- In the last decade a strong a fruitful connection has been established between the QCD Odderon and the Sivers function in the small x domain, [Jian Zhou, 2013]
- The Sivers function, $f_{1T}^{\perp}(x, \vec{k}_T^2)$, is a TMD in a transversely polarized hadron

$$f(x, \vec{k}_T; \vec{S}_T) = f_1(x, \vec{k}_T^2) - \frac{(\vec{P} \times \vec{k}_T) \cdot \vec{S}_T}{M|\vec{P}||\vec{S}|} f_{1T}^{\perp}(x, \vec{k}_T^2)$$

- In collinear approximation the Sivers function is related to twist three quark–gluon and tri-gluon Sterman–Qiu functions. At very small x the tri-gluon part has no power suppression with x w.r.t. the unpolarized distribution and it should dominate. The Sivers function is explicitly related to the Odderon amplitude in a polarized hadron
- Clearly the Sivers function is related to spatial antisymmetric part of matter distribution, and parton orbital momentum, as

$$-\frac{(\vec{P} \times \vec{k}_T) \cdot \vec{S}_T}{M|\vec{P}||\vec{S}|} f_{1T}^{\perp}(x, \vec{k}_T^2) = \frac{1}{2}\left[f(x, \vec{k}_T; \vec{S}_T) - f(x, \vec{k}_T; -\vec{S}_T)\right]$$

Sivers function and the Single Spin Asymmetries

- The Sivers function drives single spin asymmetries (SSA) in DIS scattering of a transversaly polarized hadron, the hadron spin direction breaks the azimuthal symmetry of the SIDIS
- Also drives spin asymmetries in polarized hadron scattering e.g. $pp^{\uparrow} \to h\, X$, with $h = \pi, D$, $pp^{\uparrow} \to \mathrm{jet}\, X$, $\mathrm{pp}^{\uparrow} \to \gamma\, X$, $pp^{\uparrow} \to \mathrm{jet}\,\mathrm{jet}\, X$, etc.
- For the EIC, one of the most promising observables seems to be open charm production off a transversaly polarized proton, $ep^{\uparrow} \to e'\, c\bar{c}\, X$
- Interestingly enough, the Sivers function and the Odderon may be measured in forward exclusive processes on unpolarized target [R. Boussarie, Y. Hatta, L. Szymanowski, S. Wallon, 2019]. Without explicit polarization vector, the spin direction leads to asymmetry of matter distribution in proton.

Conclusions

- The C-odd glue-dominated exchange is necessarily present in QCD
- The theory of QCD Odderon is very well developed and interesting
- The still unknown are high energy asymptotics of the Odderon exchange and its coupling to the proton
- Results from pertubative QCD and hints from string theory — super Yang–Mills correspondence suggest that $\alpha_O \leq 1$
- Absorptive corrections strongly affect the Odderon: the "C-even brother" suppresses the Odderon exchange amplitudes
- Lessons from HERA and LHC: it is quite hard to measure Odderon in exclusive processes. Path that may be explored: measurements of charge and spin asymmetries in exclusive processes and polarized hadron scattering
- The TOTEM results are major step towards better understanding the Odderon