

Combining Electromagnetic and Gravitational Wave Observations to Determine the Nature of Dense Matter

Andrew W. Steiner

UTK/ORNL

June 16, 2021

Collaborators: **Mohammad Al-Mamun**, **Spencer Beloin**, Stefano Gandolfi, **Sophia Han**, Craig Heinke, Jacob Lange, Joonas Nättilä, Khorgolkhuu Odbadrakh, Richard O'Shaughnessy, Ingo Tews

- The space of all possible equations of state is uncountably infinite

TABLE I. A comparison of our posterior distributions for the radius of a $1.4 M_{\odot}$ NS in comparison to other results obtained in the literature.

Reference	$R_{1.4}$	Credible interval	Source
[17]	[10.5, 13.3]	90%	GW
[21]	[9.9, 13.6]	90%	GW
[22]	< 13.6	90%	GW
[23]	[9.4, 12.8]	90%	GW
[27]	[9.8, 13.2] ^a	90%	GW
[36]	[10.36, 12.78]	90%	GW
Model “a”	[11.30, 13.95]	95%	GW
Model “b”	[10.65, 13.09]	95%	GW
[28]	[8.9, 13.2]	90%	GW, merger remnant
[29]	[11.4, 13.2]	90%	GW, merger remnant
[30]	[10.4, 11.9]	90%	GW, merger remnant
[31]	[11.98, 12.76]	90%	GW, QLMXB
[32]	[10.5, 11.8]	90%	GW, QLMXB
[33]	[10.94, 12.72]	90%	GWs ^b , NICER
[34,35]	[10.85, 13.41]	90%	GWs, NICER
[36]	[11.91, 13.25]	90%	GW, NICER
[37]	[11.3, 13.3]	90%	GW, NICER
[41]	[12, 13]	90%	GWs, NICER
[41]	[10.0, 11.5]	90%	GWs, QLMXB, PRE
Model “c”	[11.21, 12.55]	95%	GW, QLMXB, PRE
Model “e”	[11.28, 12.58]	95%	GW, QLMXB, PRE, NICER

^aRadius measurement for the primary NS of the merger event.

^bGWs refer to the joint analysis of GW170817 and GW190425.

- The space of all possible equations of state is uncountably infinite
- There is no unambiguous procedure for listing, plotting, or analyzing the equations of state, and no such thing as "model-independent".

TABLE I. A comparison of our posterior distributions for the radius of a $1.4 M_{\odot}$ NS in comparison to other results obtained in the literature.

Reference	$R_{1.4}$	Credible interval	Source
[17]	[10.5, 13.3]	90%	GW
[21]	[9.9, 13.6]	90%	GW
[22]	< 13.6	90%	GW
[23]	[9.4, 12.8]	90%	GW
[27]	[9.8, 13.2] ^a	90%	GW
[36]	[10.36, 12.78]	90%	GW
Model "a"	[11.30, 13.95]	95%	GW
Model "b"	[10.65, 13.09]	95%	GW
[28]	[8.9, 13.2]	90%	GW, merger remnant
[29]	[11.4, 13.2]	90%	GW, merger remnant
[30]	[10.4, 11.9]	90%	GW, merger remnant
[31]	[11.98, 12.76]	90%	GW, QLMXB
[32]	[10.5, 11.8]	90%	GW, QLMXB
[33]	[10.94, 12.72]	90%	GWs ^b , NICER
[34,35]	[10.85, 13.41]	90%	GWs, NICER
[36]	[11.91, 13.25]	90%	GW, NICER
[37]	[11.3, 13.3]	90%	GW, NICER
[41]	[12, 13]	90%	GWs, NICER
[41]	[10.0, 11.5]	90%	GWs, QLMXB, PRE
Model "c"	[11.21, 12.55]	95%	GW, QLMXB, PRE
Model "e"	[11.28, 12.58]	95%	GW, QLMXB, PRE, NICER

^aRadius measurement for the primary NS of the merger event.

^bGWs refer to the joint analysis of GW170817 and GW190425.

- The space of all possible equations of state is uncountably infinite
- There is no unambiguous procedure for listing, plotting, or analyzing the equations of state, and no such thing as "model-independent".
- Statements like "most EOSs do not have a phase transition" have little meaning except with respect to some prior

TABLE I. A comparison of our posterior distributions for the radius of a $1.4 M_{\odot}$ NS in comparison to other results obtained in the literature.

Reference	$R_{1.4}$	Credible interval	Source
[17]	[10.5, 13.3]	90%	GW
[21]	[9.9, 13.6]	90%	GW
[22]	< 13.6	90%	GW
[23]	[9.4, 12.8]	90%	GW
[27]	[9.8, 13.2] ^a	90%	GW
[36]	[10.36, 12.78]	90%	GW
Model "a"	[11.30, 13.95]	95%	GW
Model "b"	[10.65, 13.09]	95%	GW
[28]	[8.9, 13.2]	90%	GW, merger remnant
[29]	[11.4, 13.2]	90%	GW, merger remnant
[30]	[10.4, 11.9]	90%	GW, merger remnant
[31]	[11.98, 12.76]	90%	GW, QLMXB
[32]	[10.5, 11.8]	90%	GW, QLMXB
[33]	[10.94, 12.72]	90%	GWs ^b , NICER
[34,35]	[10.85, 13.41]	90%	GWs, NICER
[36]	[11.91, 13.25]	90%	GW, NICER
[37]	[11.3, 13.3]	90%	GW, NICER
[41]	[12, 13]	90%	GWs, NICER
[41]	[10.0, 11.5]	90%	GWs, QLMXB, PRE
Model "c"	[11.21, 12.55]	95%	GW, QLMXB, PRE
Model "e"	[11.28, 12.58]	95%	GW, QLMXB, PRE, NICER

^aRadius measurement for the primary NS of the merger event.

^bGWs refer to the joint analysis of GW170817 and GW190425.

- The space of all possible equations of state is uncountably infinite
- There is no unambiguous procedure for listing, plotting, or analyzing the equations of state, and no such thing as "model-independent".
- Statements like "most EOSs do not have a phase transition" have little meaning except with respect to some prior
- You must have a prior distribution

TABLE I. A comparison of our posterior distributions for the radius of a $1.4 M_{\odot}$ NS in comparison to other results obtained in the literature.

Reference	$R_{1.4}$	Credible interval	Source
[17]	[10.5, 13.3]	90%	GW
[21]	[9.9, 13.6]	90%	GW
[22]	< 13.6	90%	GW
[23]	[9.4, 12.8]	90%	GW
[27]	[9.8, 13.2] ^a	90%	GW
[36]	[10.36, 12.78]	90%	GW
Model "a"	[11.30, 13.95]	95%	GW
Model "b"	[10.65, 13.09]	95%	GW
[28]	[8.9, 13.2]	90%	GW, merger remnant
[29]	[11.4, 13.2]	90%	GW, merger remnant
[30]	[10.4, 11.9]	90%	GW, merger remnant
[31]	[11.98, 12.76]	90%	GW, QLMXB
[32]	[10.5, 11.8]	90%	GW, QLMXB
[33]	[10.94, 12.72]	90%	GWs ^b , NICER
[34,35]	[10.85, 13.41]	90%	GWs, NICER
[36]	[11.91, 13.25]	90%	GW, NICER
[37]	[11.3, 13.3]	90%	GW, NICER
[41]	[12, 13]	90%	GWs, NICER
[41]	[10.0, 11.5]	90%	GWs, QLMXB, PRE
Model "c"	[11.21, 12.55]	95%	GW, QLMXB, PRE
Model "e"	[11.28, 12.58]	95%	GW, QLMXB, PRE, NICER

^aRadius measurement for the primary NS of the merger event.

^bGWs refer to the joint analysis of GW170817 and GW190425.

- The space of all possible equations of state is uncountably infinite
- There is no unambiguous procedure for listing, plotting, or analyzing the equations of state, and no such thing as "model-independent".
- Statements like "most EOSs do not have a phase transition" have little meaning except with respect to some prior
- You must have a prior distribution
- We are (still) in a data-starved regime

TABLE I. A comparison of our posterior distributions for the radius of a $1.4 M_{\odot}$ NS in comparison to other results obtained in the literature.

Reference	$R_{1.4}$	Credible interval	Source
[17]	[10.5, 13.3]	90%	GW
[21]	[9.9, 13.6]	90%	GW
[22]	< 13.6	90%	GW
[23]	[9.4, 12.8]	90%	GW
[27]	[9.8, 13.2] ^a	90%	GW
[36]	[10.36, 12.78]	90%	GW
Model "a"	[11.30, 13.95]	95%	GW
Model "b"	[10.65, 13.09]	95%	GW
[28]	[8.9, 13.2]	90%	GW, merger remnant
[29]	[11.4, 13.2]	90%	GW, merger remnant
[30]	[10.4, 11.9]	90%	GW, merger remnant
[31]	[11.98, 12.76]	90%	GW, QLMXB
[32]	[10.5, 11.8]	90%	GW, QLMXB
[33]	[10.94, 12.72]	90%	GWs ^b , NICER
[34,35]	[10.85, 13.41]	90%	GWs, NICER
[36]	[11.91, 13.25]	90%	GW, NICER
[37]	[11.3, 13.3]	90%	GW, NICER
[41]	[12, 13]	90%	GWs, NICER
[41]	[10.0, 11.5]	90%	GWs, QLMXB, PRE
Model "c"	[11.21, 12.55]	95%	GW, QLMXB, PRE
Model "e"	[11.28, 12.58]	95%	GW, QLMXB, PRE, NICER

^aRadius measurement for the primary NS of the merger event.
^bGWs refer to the joint analysis of GW170817 and GW190425.

- The space of all possible equations of state is uncountably infinite
- There is no unambiguous procedure for listing, plotting, or analyzing the equations of state, and no such thing as "model-independent".
- Statements like "most EOSs do not have a phase transition" have little meaning except with respect to some prior
- You must have a prior distribution
- We are (still) in a data-starved regime
- There is no optimal prior, only better data sets

Even the conditional probability, which is formally separate from the prior, often contains assumptions which are arguably prior assumptions; Steiner (2018)

TABLE I. A comparison of our posterior distributions for the radius of a $1.4 M_{\odot}$ NS in comparison to other results obtained in the literature.

Reference	$R_{1.4}$	Credible interval	Source
[17]	[10.5, 13.3]	90%	GW
[21]	[9.9, 13.6]	90%	GW
[22]	< 13.6	90%	GW
[23]	[9.4, 12.8]	90%	GW
[27]	[9.8, 13.2] ^a	90%	GW
[36]	[10.36, 12.78]	90%	GW
Model "a"	[11.30, 13.95]	95%	GW
Model "b"	[10.65, 13.09]	95%	GW
[28]	[8.9, 13.2]	90%	GW, merger remnant
[29]	[11.4, 13.2]	90%	GW, merger remnant
[30]	[10.4, 11.9]	90%	GW, merger remnant
[31]	[11.98, 12.76]	90%	GW, QLMXB
[32]	[10.5, 11.8]	90%	GW, QLMXB
[33]	[10.94, 12.72]	90%	GWs ^b , NICER
[34,35]	[10.85, 13.41]	90%	GWs, NICER
[36]	[11.91, 13.25]	90%	GW, NICER
[37]	[11.3, 13.3]	90%	GW, NICER
[41]	[12, 13]	90%	GWs, NICER
[41]	[10.0, 11.5]	90%	GWs, QLMXB, PRE
Model "c"	[11.21, 12.55]	95%	GW, QLMXB, PRE
Model "e"	[11.28, 12.58]	95%	GW, QLMXB, PRE, NICER

^aRadius measurement for the primary NS of the merger event.
^bGWs refer to the joint analysis of GW170817 and GW190425.

Goals of Our Combined Analysis

Goals of Our Combined Analysis

- See what information can be obtained with QLMXBs + PREs + GWs + NICER

Goals of Our Combined Analysis

- See what information can be obtained with QLMXBs + PREs + GWs + NICER
- Anchor our EOS model in up-to-date nuclear physics

Goals of Our Combined Analysis

- See what information can be obtained with QLMXBs + PREs + GWs + NICER
- Anchor our EOS model in up-to-date nuclear physics
 - Chiral effective theory + MBPT for neutron matter

Goals of Our Combined Analysis

- See what information can be obtained with QLMXBs + PREs + GWs + NICER
- Anchor our EOS model in up-to-date nuclear physics
 - Chiral effective theory + MBPT for neutron matter
 - Posteriors from experimental data for nuclear matter

Goals of Our Combined Analysis

- See what information can be obtained with QLMXBs + PREs + GWs + NICER
- Anchor our EOS model in up-to-date nuclear physics
 - Chiral effective theory + MBPT for neutron matter
 - Posteriors from experimental data for nuclear matter
- Show how our results depend on our prior assumptions

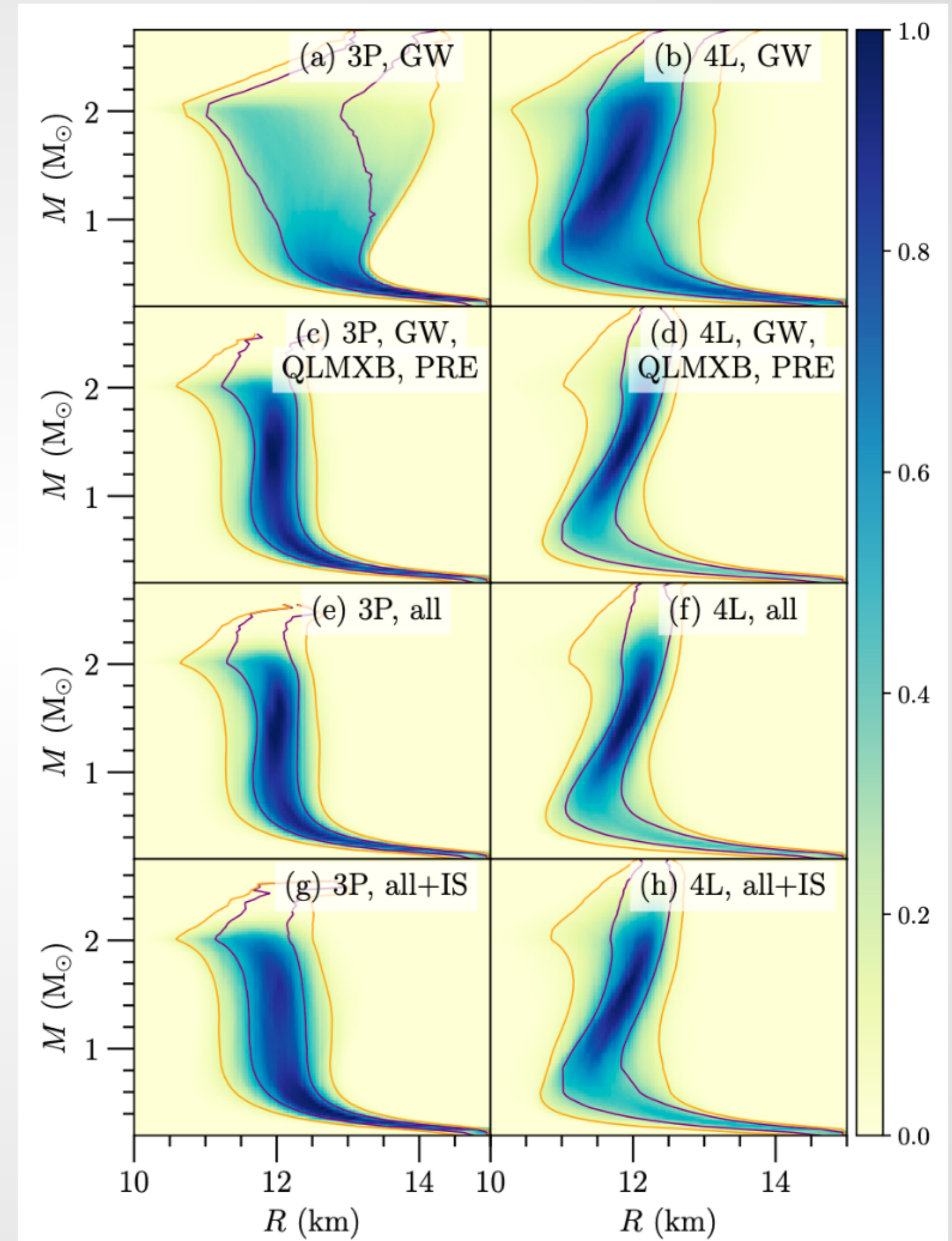
Goals of Our Combined Analysis

- See what information can be obtained with QLMXBs + PREs + GWs + NICER
- Anchor our EOS model in up-to-date nuclear physics
 - Chiral effective theory + MBPT for neutron matter
 - Posteriors from experimental data for nuclear matter
- Show how our results depend on our prior assumptions
 - E.g. polytropes vs. line-segments at high densities

Goals of Our Combined Analysis

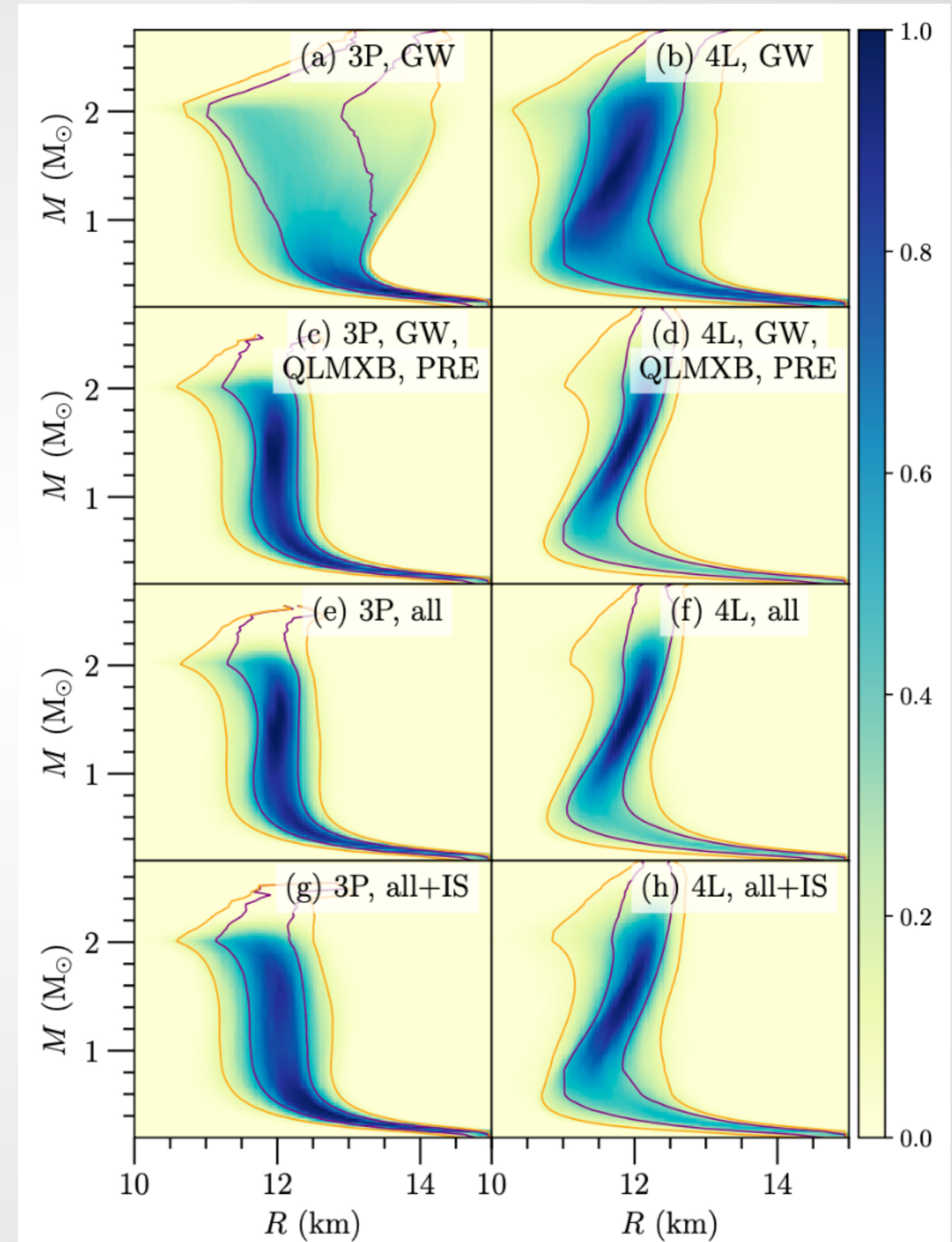
- See what information can be obtained with QLMXBs + PREs + GWs + NICER
- Anchor our EOS model in up-to-date nuclear physics
 - Chiral effective theory + MBPT for neutron matter
 - Posteriors from experimental data for nuclear matter
- Show how our results depend on our prior assumptions
 - E.g. polytropes vs. line-segments at high densities
- Attempt to determine if EM observations contain systematic uncertainties which are unaccounted for

Mass-Radius Results



Mass-Radius Results

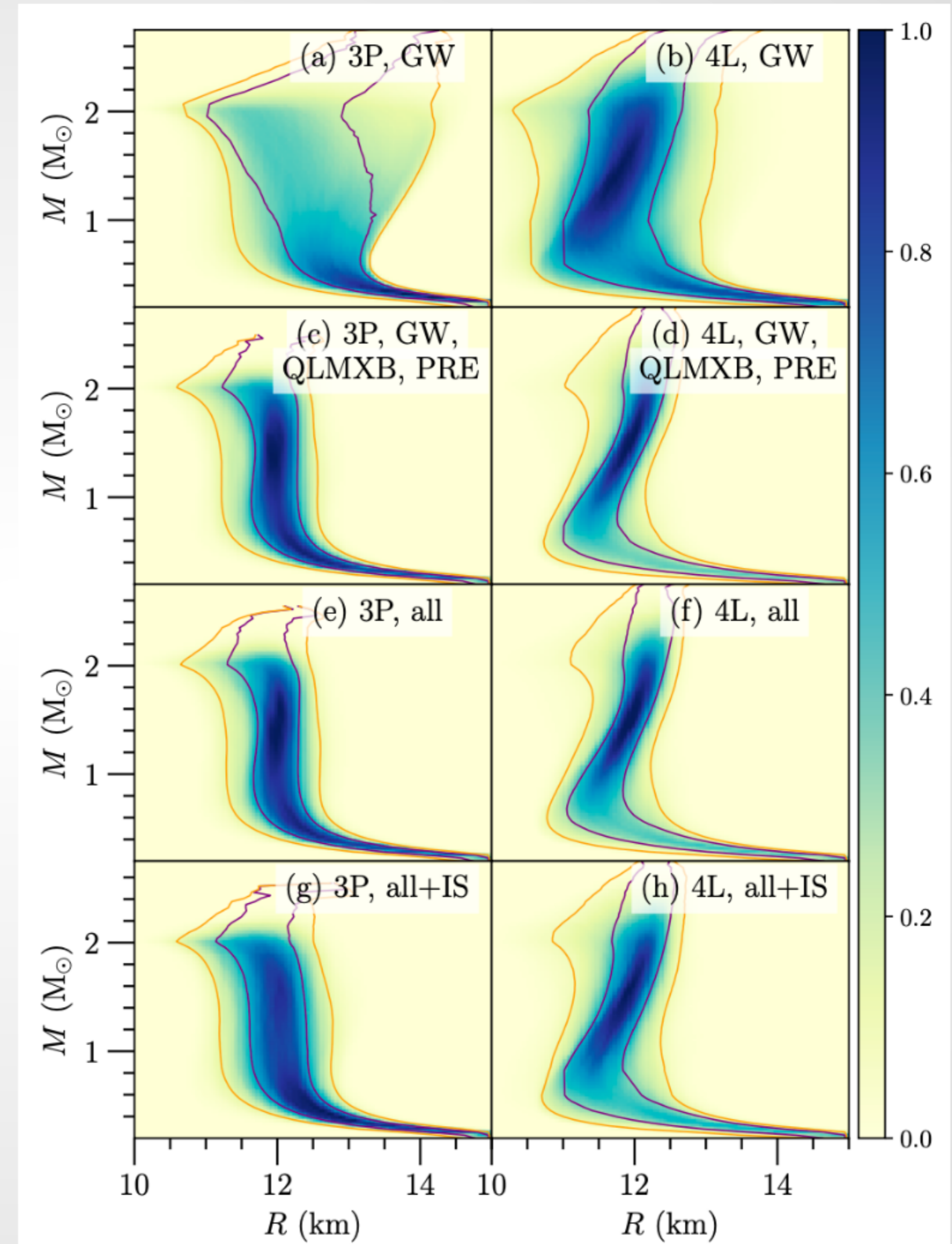
- Combined results show relatively good agreement on $R_{1.4}$, independent of the EOS prior polytropes (line segments):
 $11.18(11.12) < R_{1.4} < 12.75(12.45)$
(95% C.I.'s)



Mass-Radius Results

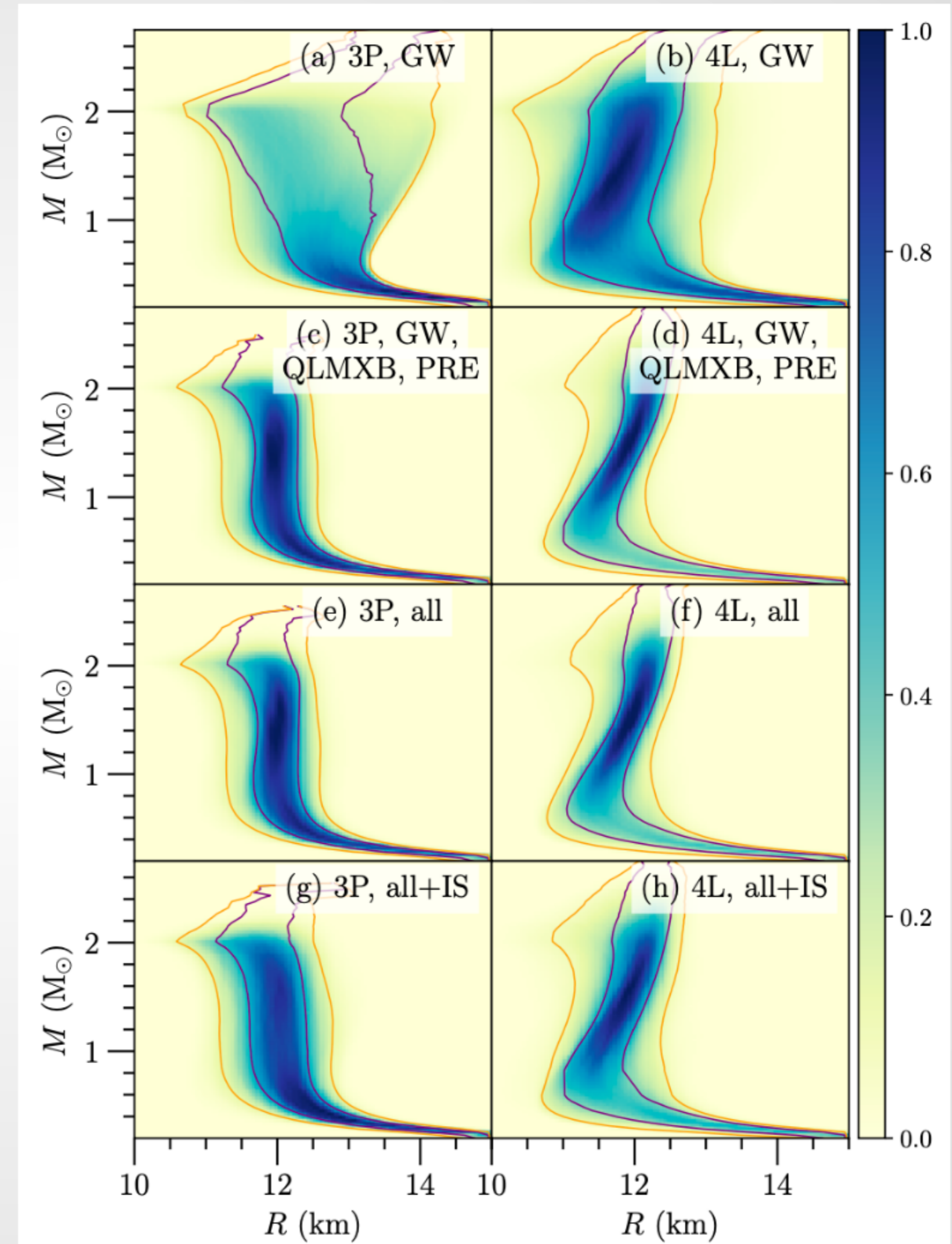
- Combined results show relatively good agreement on $R_{1.4}$, independent of the EOS prior polytropes (line segments):
 $11.18(11.12) < R_{1.4} < 12.75(12.45)$
(95% C.I.'s)

- Results for lower masses are more prior dependent
 $11.20(10.88) < R_{1.0} < 12.76(12.38)$
Dear Universe: please merge two $1 M_{\odot}$ NSs



Mass-Radius Results

- Combined results show relatively good agreement on $R_{1.4}$, independent of the EOS prior polytropes (line segments):
 $11.18(11.12) < R_{1.4} < 12.75(12.45)$
(95% C.I.'s)
- Results for lower masses are more prior dependent
 $11.20(10.88) < R_{1.0} < 12.76(12.38)$
Dear Universe: please merge two $1 M_{\odot}$ NSs
- Results for higher masses appear less prior dependent...
 $11.53(11.37) < R_{2.4} < 12.52(12.62)$



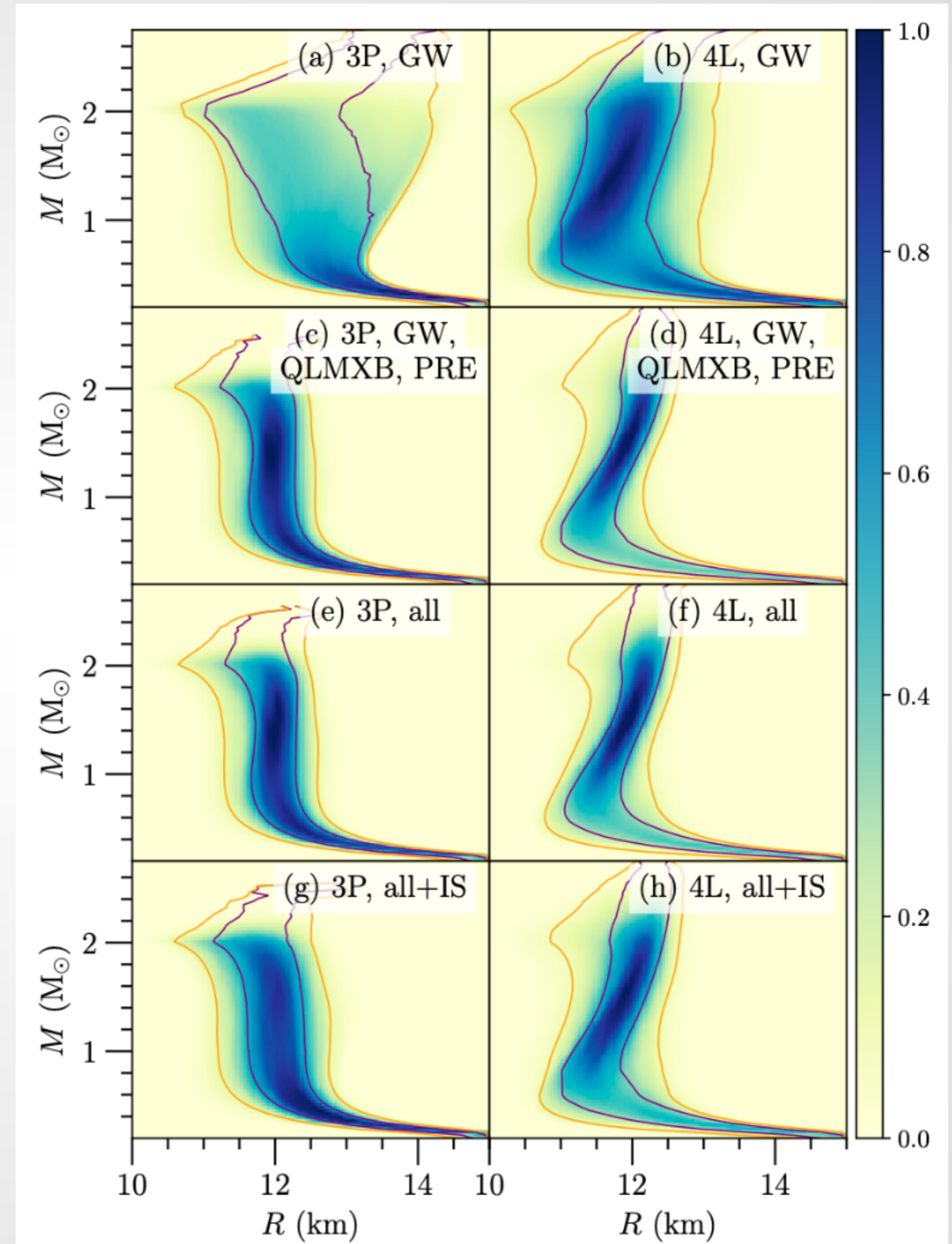
Mass-Radius Results

- Combined results show relatively good agreement on $R_{1.4}$, independent of the EOS prior polytropes (line segments):
 $11.18(11.12) < R_{1.4} < 12.75(12.45)$
(95% C.I.'s)

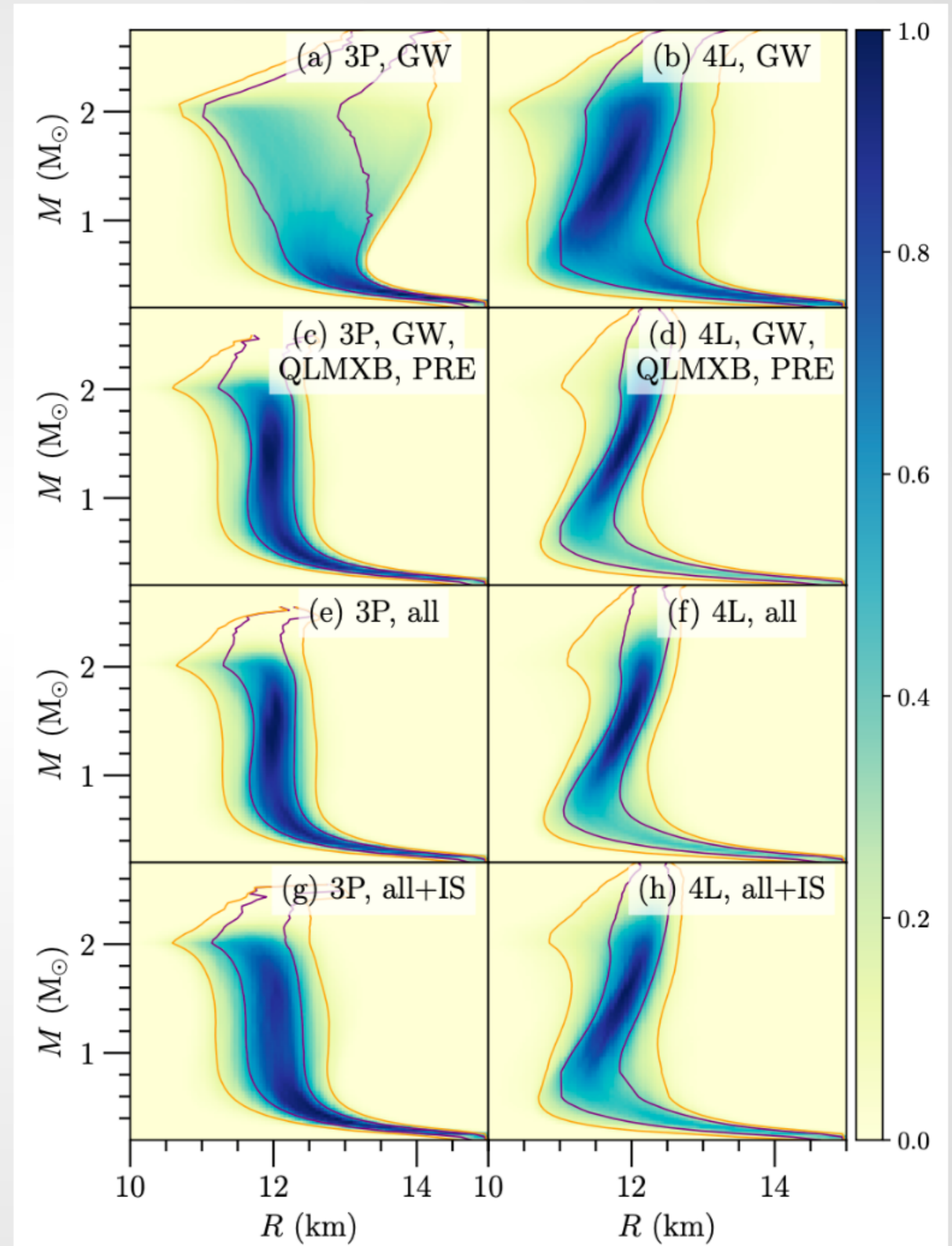
- Results for lower masses are more prior dependent
 $11.20(10.88) < R_{1.0} < 12.76(12.38)$
Dear Universe: please merge two $1 M_{\odot}$ NSs

- Results for higher masses appear less prior dependent...
 $11.53(11.37) < R_{2.4} < 12.52(12.62)$

- ...but maximum mass posterior distribution varies significantly
This is why NICER results on J0740 are so important



Assessing Systematics and J0030



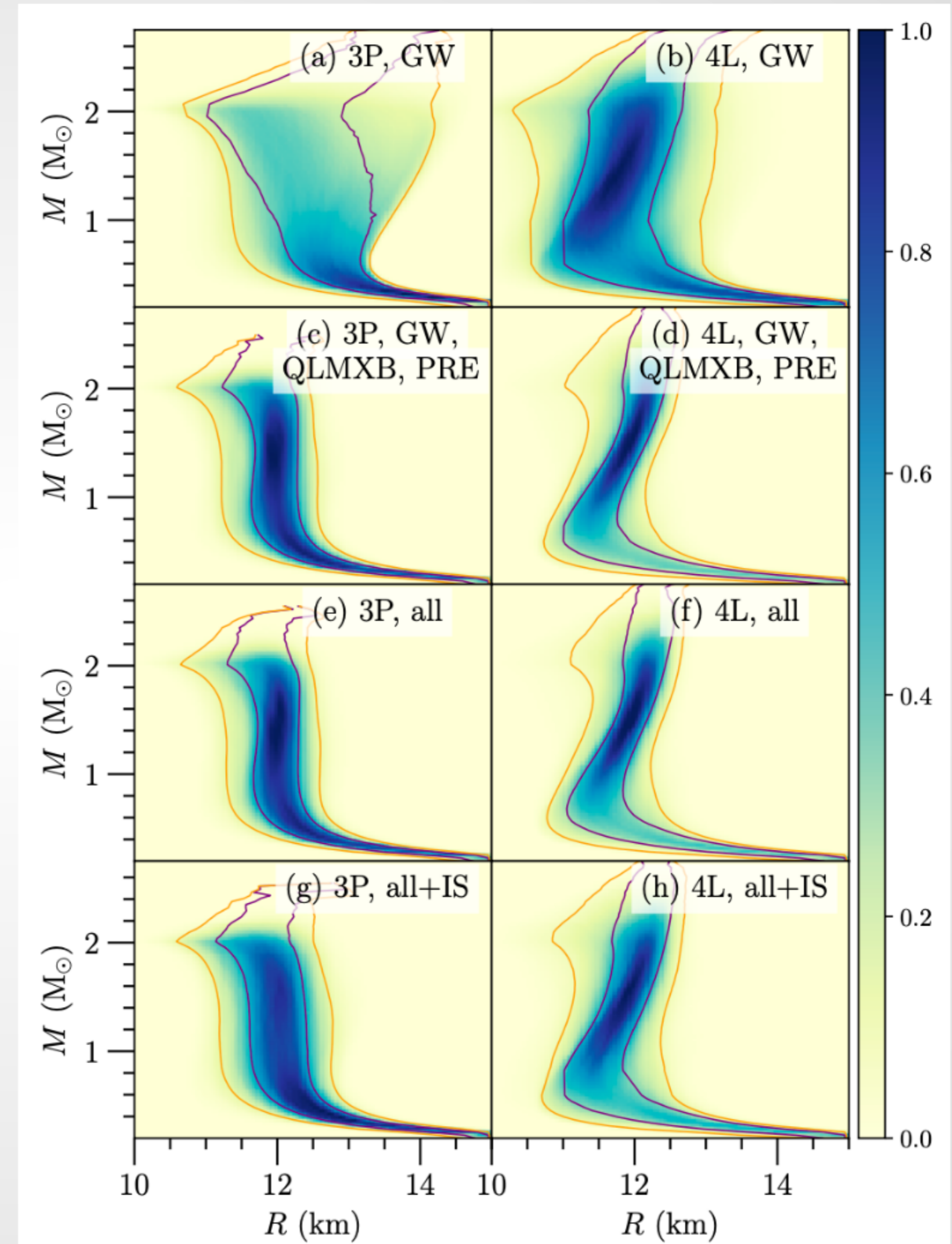
- Fold all EM observations with an independent uncertainty

$$D_{\text{IS}}(R, M, \sigma) = \frac{1}{\mathcal{N}} \int_{R_0}^{R_1} \int_{M_0}^{M_1} dR' dM' \mathcal{D}(R', M')$$

$$\times \exp \left[-\frac{1}{2} \left(\frac{R - R'}{\sigma_R} \right)^2 - \frac{1}{2} \left(\frac{M - M'}{\sigma_M} \right)^2 \right],$$

with

$$\sigma_M \propto \sigma_R$$



- Fold all EM observations with an independent uncertainty

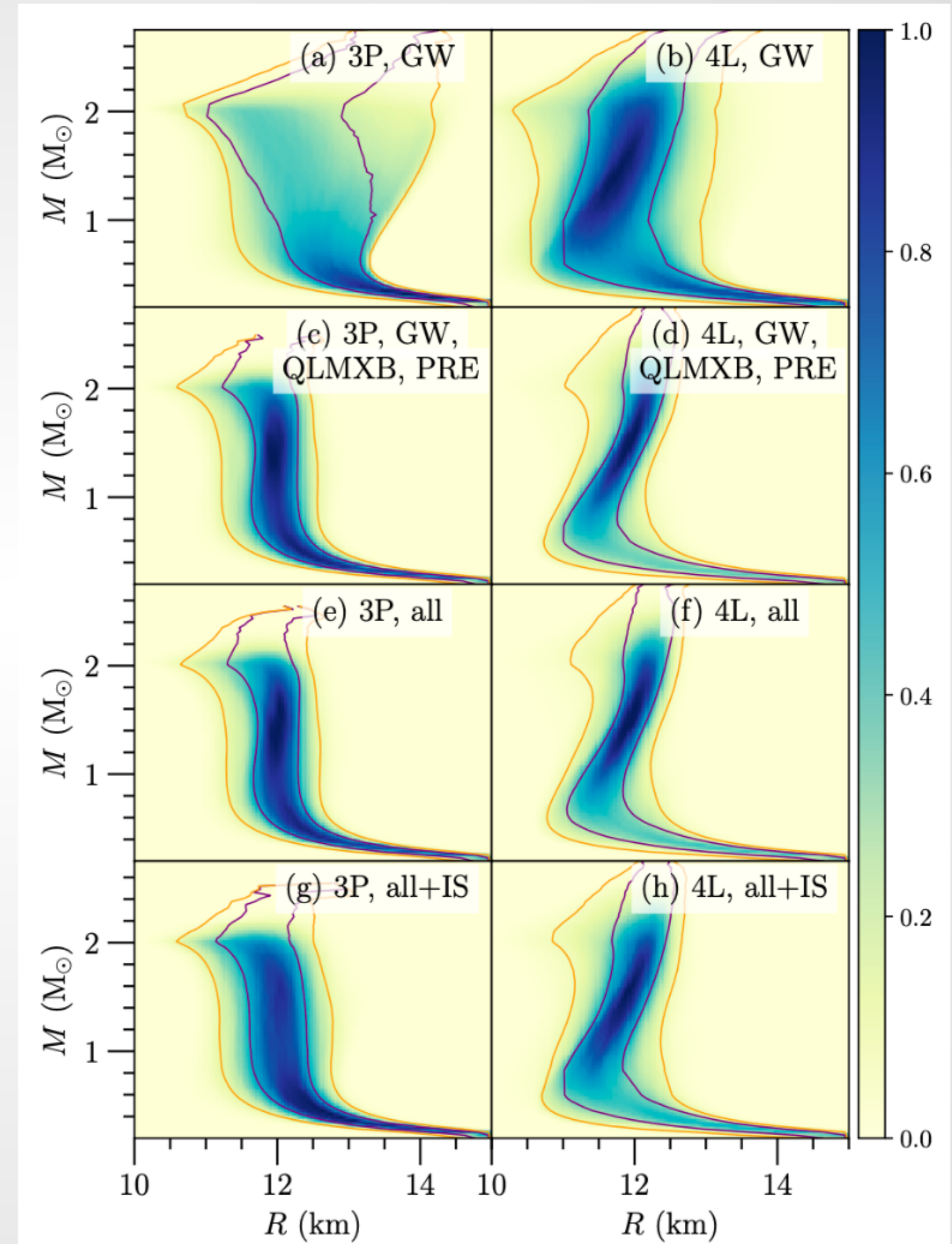
$$D_{\text{IS}}(R, M, \sigma) = \frac{1}{\mathcal{N}} \int_{R_0}^{R_1} \int_{M_0}^{M_1} dR' dM' \mathcal{D}(R', M') \times \exp \left[-\frac{1}{2} \left(\frac{R - R'}{\sigma_R} \right)^2 - \frac{1}{2} \left(\frac{M - M'}{\sigma_M} \right)^2 \right],$$

with

$$\sigma_M \propto \sigma_R$$

- Additional systematic uncertainty appears not to make a significant shift in M-R curves

Method fails to disprove the EM and GW data are inconsistent



Assessing Systematics and J0030

- Fold all EM observations with an independent uncertainty

$$\mathcal{D}_{\text{IS}}(R, M, \sigma) = \frac{1}{\mathcal{N}} \int_{R_0}^{R_1} \int_{M_0}^{M_1} dR' dM' \mathcal{D}(R', M') \\ \times \exp \left[-\frac{1}{2} \left(\frac{R - R'}{\sigma_R} \right)^2 - \frac{1}{2} \left(\frac{M - M'}{\sigma_M} \right)^2 \right],$$

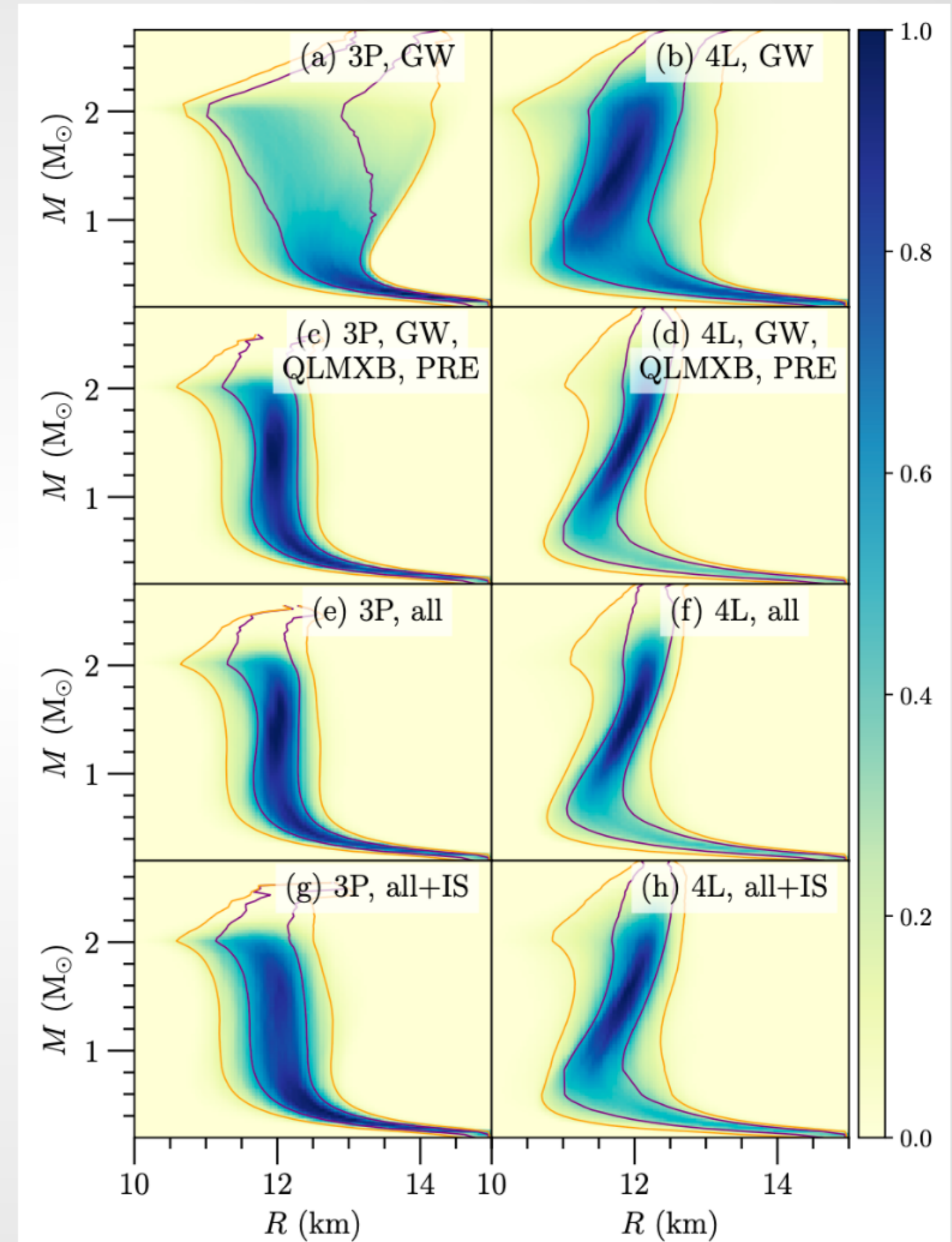
with

$$\sigma_M \propto \sigma_R$$

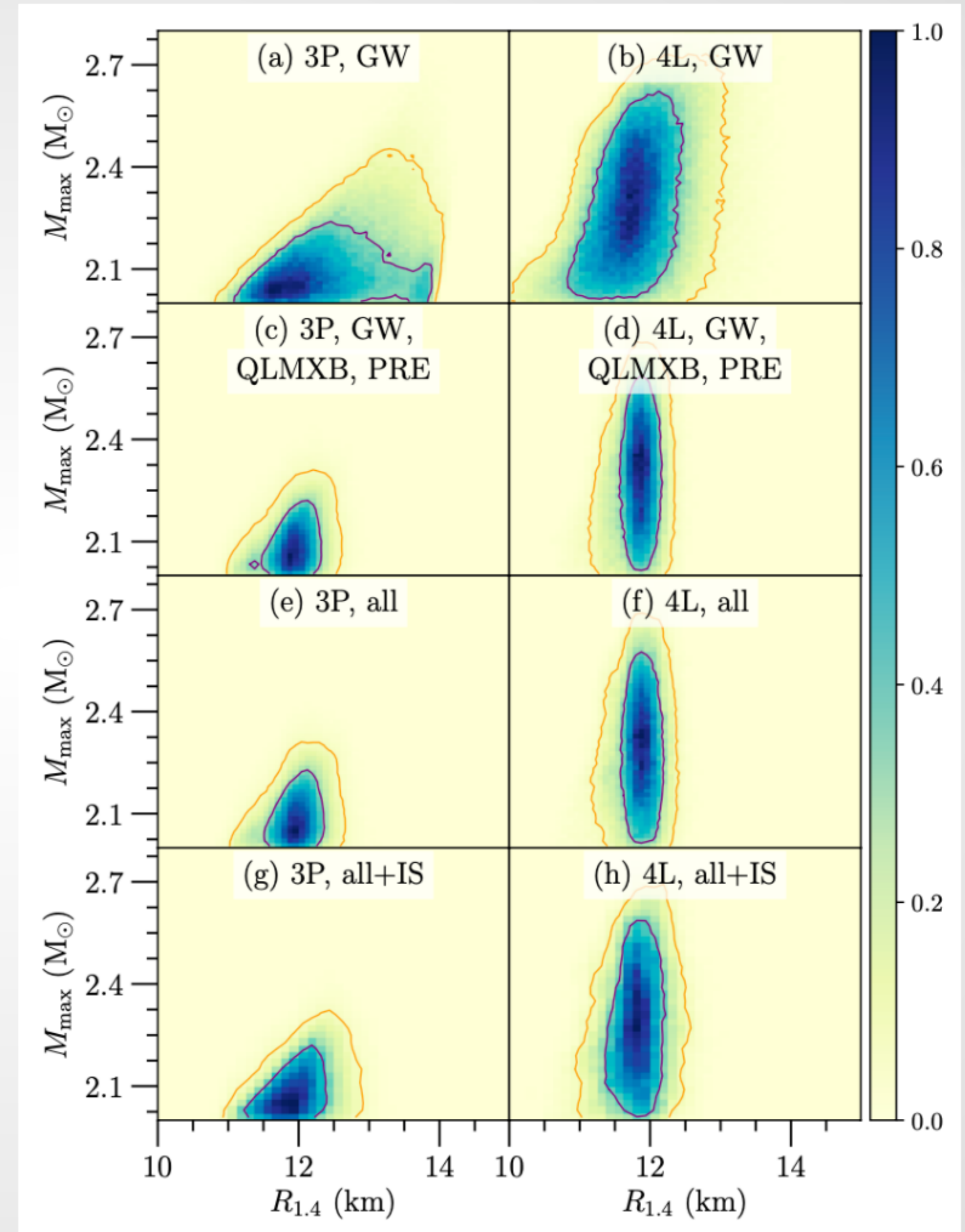
- Additional systematic uncertainty appears not to make a significant shift in M-R curves

Method fails to disprove the EM and GW data are inconsistent

- NICER's constraint from J 0030 makes only small changes



Equation of State Results and Maximum Mass



Al-Mamun et al. (2021)

Equation of State Results and Maximum Mass

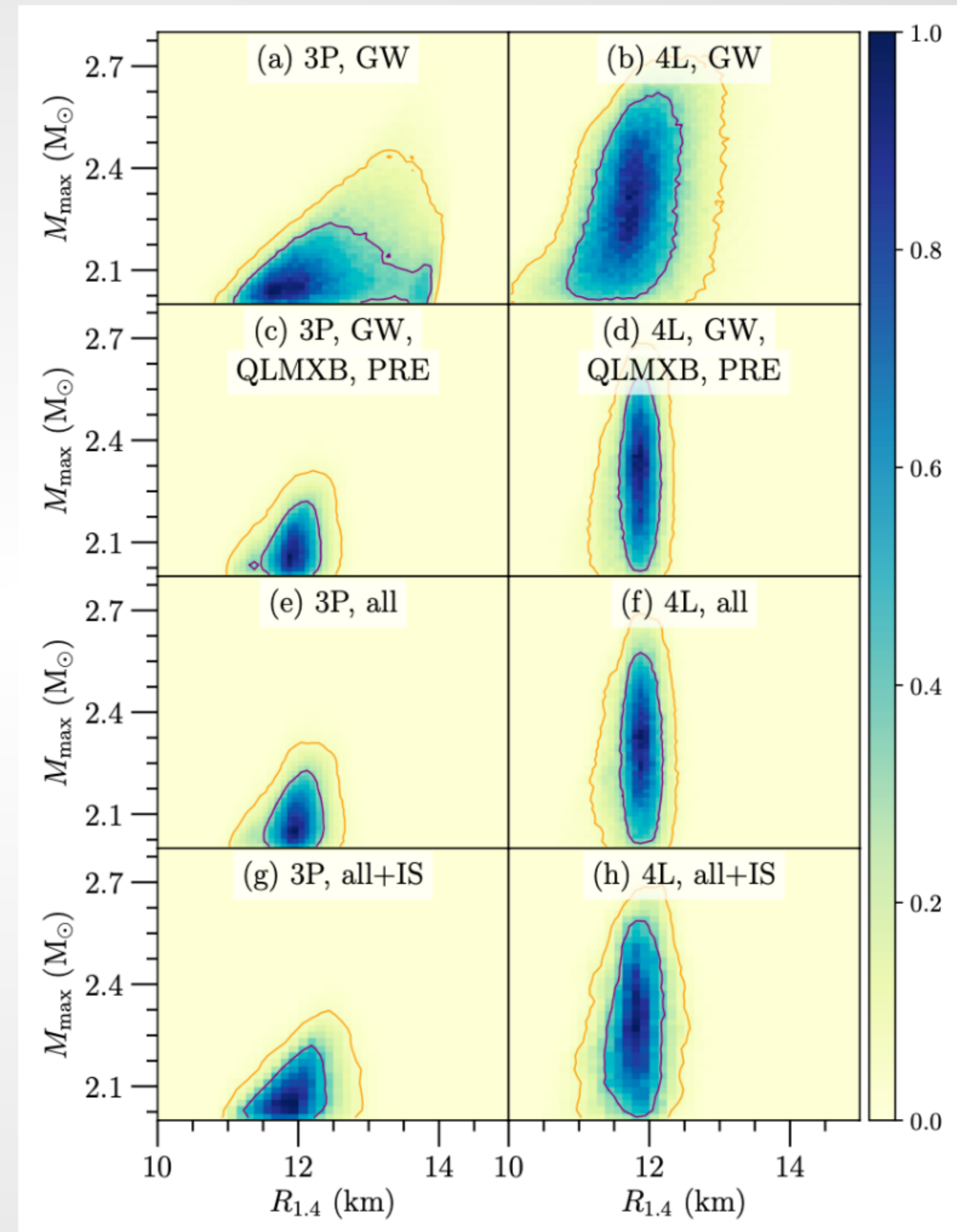
- Limits on pressure at fixed energy density [MeV/fm³] polytropes (line segments)

$$30.5(29.0) < P_{400} < 60.9(78.6)$$

$$144(153) < P_{700} < 226(314)$$

$$251(214) < P_{1000} < 413(588)$$

(95% C.I.'s)



Equation of State Results and Maximum Mass

- Limits on pressure at fixed energy density [MeV/fm³] polytropes (line segments)

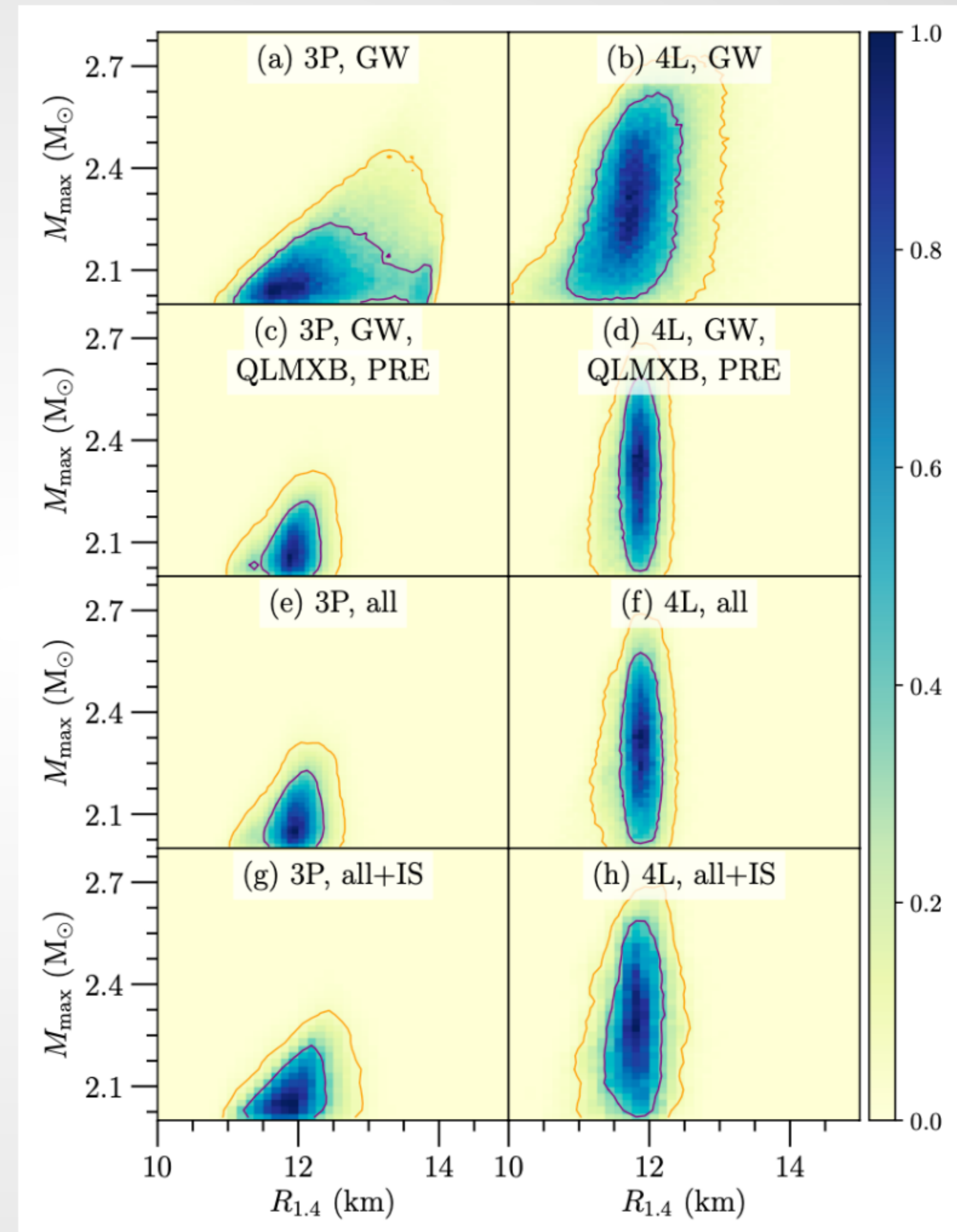
$$30.5(29.0) < P_{400} < 60.9(78.6)$$

$$144(153) < P_{700} < 226(314)$$

$$251(214) < P_{1000} < 413(588)$$

(95% C.I.'s)

- Phase transitions tend to give the ability to increase the pressure in one-region, and make up for it in another



Equation of State Results and Maximum Mass

- Limits on pressure at fixed energy density [MeV/fm³] polytropes (line segments)

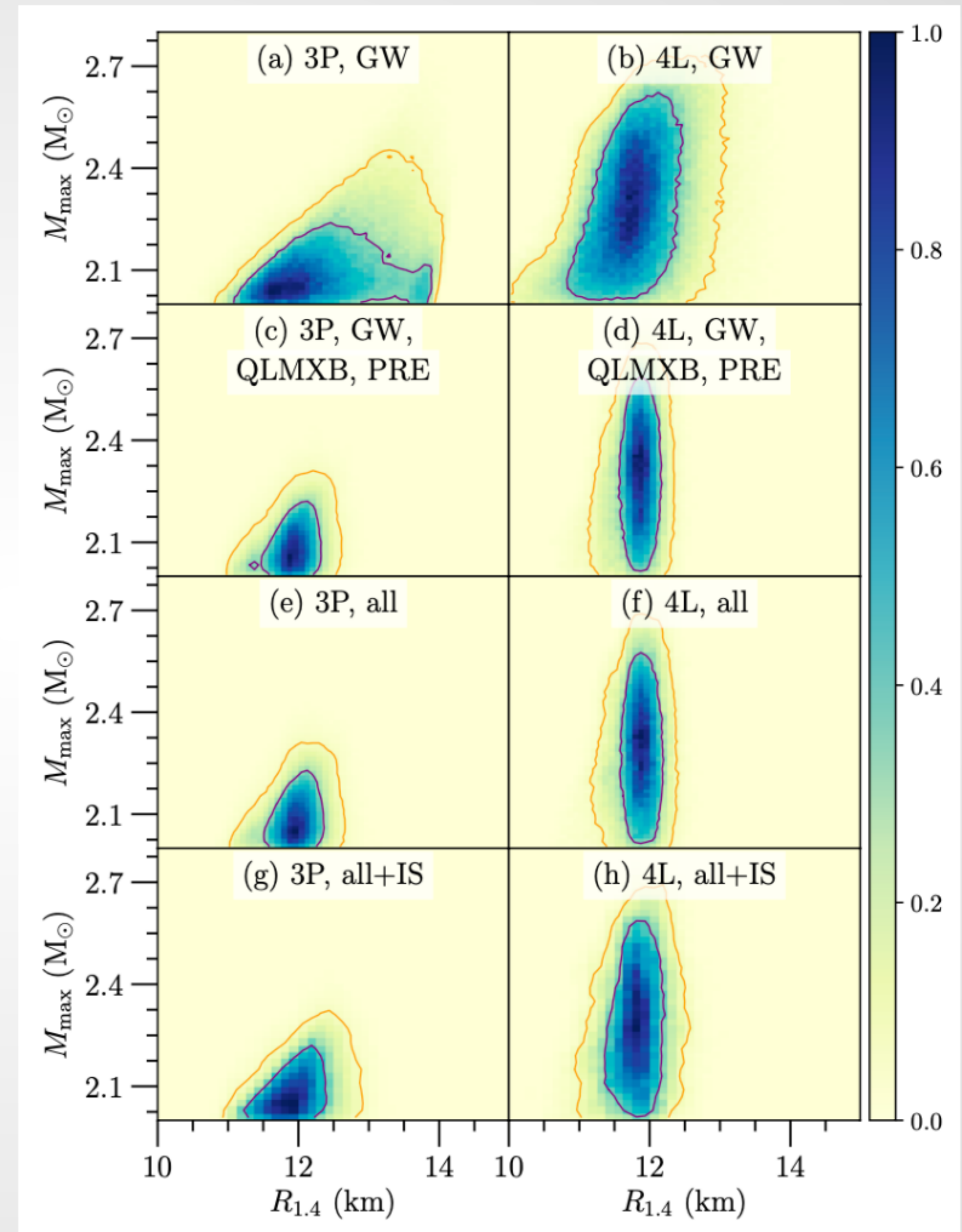
$$30.5(29.0) < P_{400} < 60.9(78.6)$$

$$144(153) < P_{700} < 226(314)$$

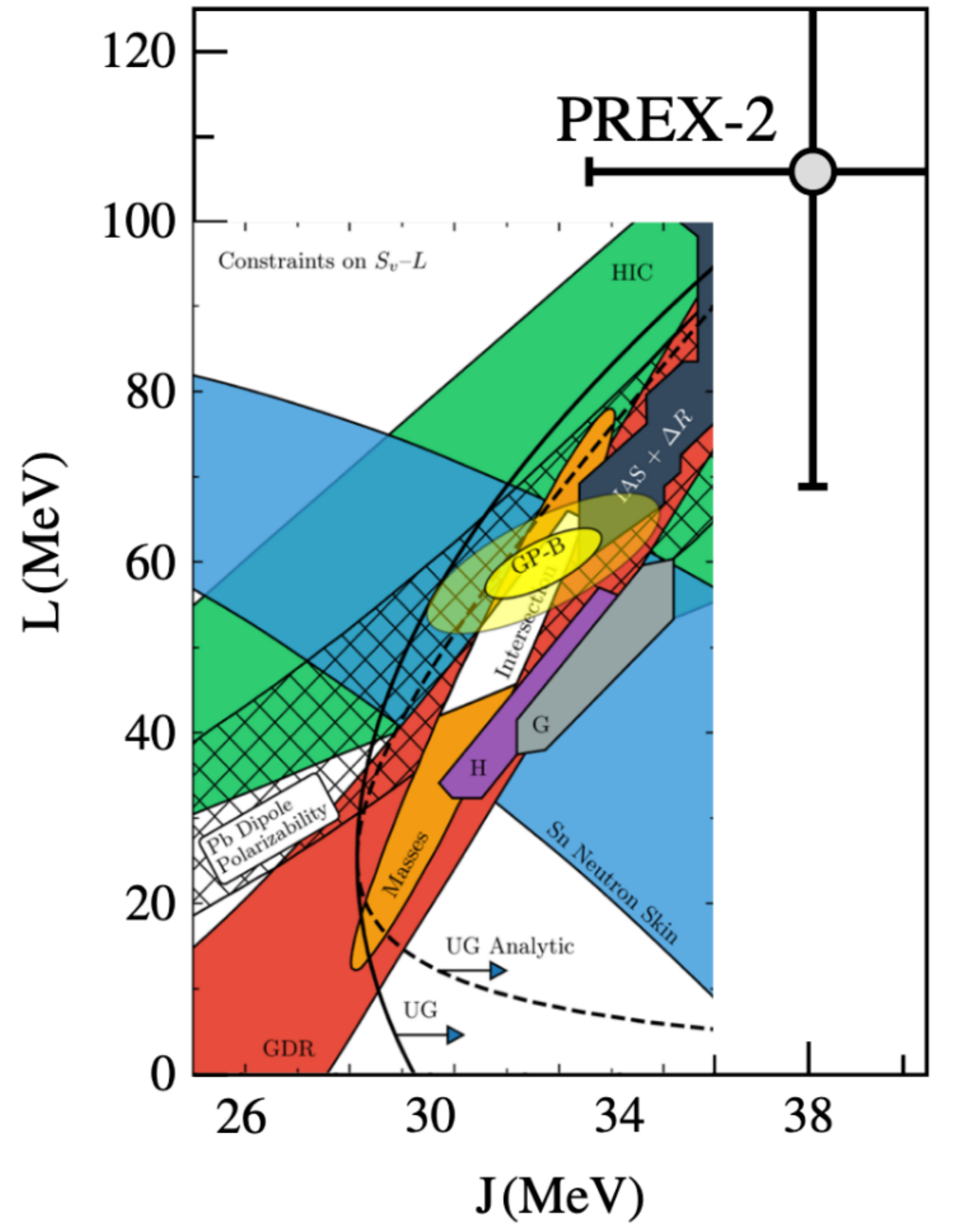
$$251(214) < P_{1000} < 413(588)$$

(95% C.I.'s)

- Phase transitions tend to give the ability to increase the pressure in one-region, and make up for it in another
- No connection between radius of a 1.4 solar mass NS and maximum mass



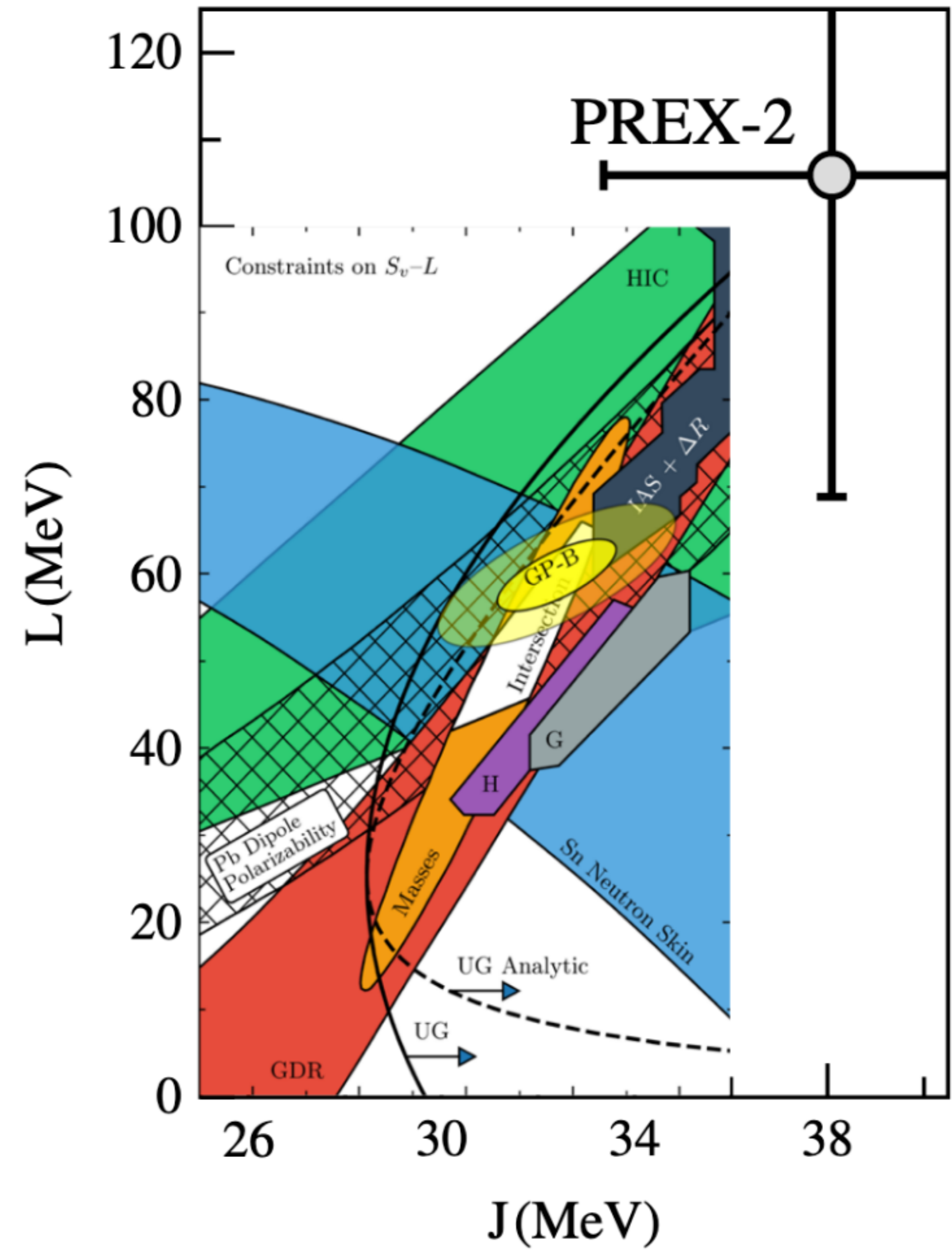
What about PREX II and NICER J0740?



Reed et al. (2021)

What about PREX II and NICER J0740?

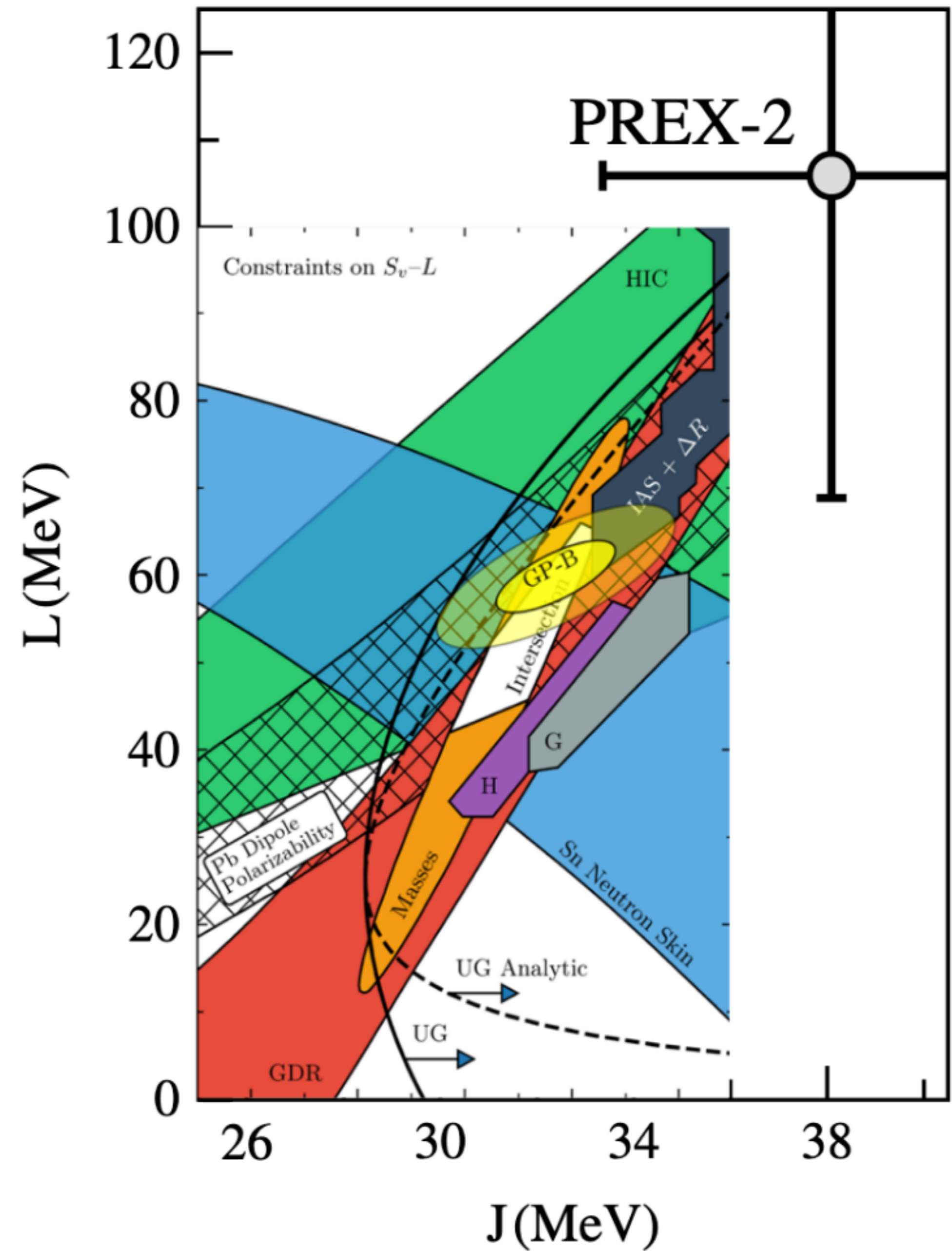
- Likely some tension between recent nuclear theory results, QLMXB observations and PREX II, J0740 results



Reed et al. (2021)

What about PREX II and NICER J0740?

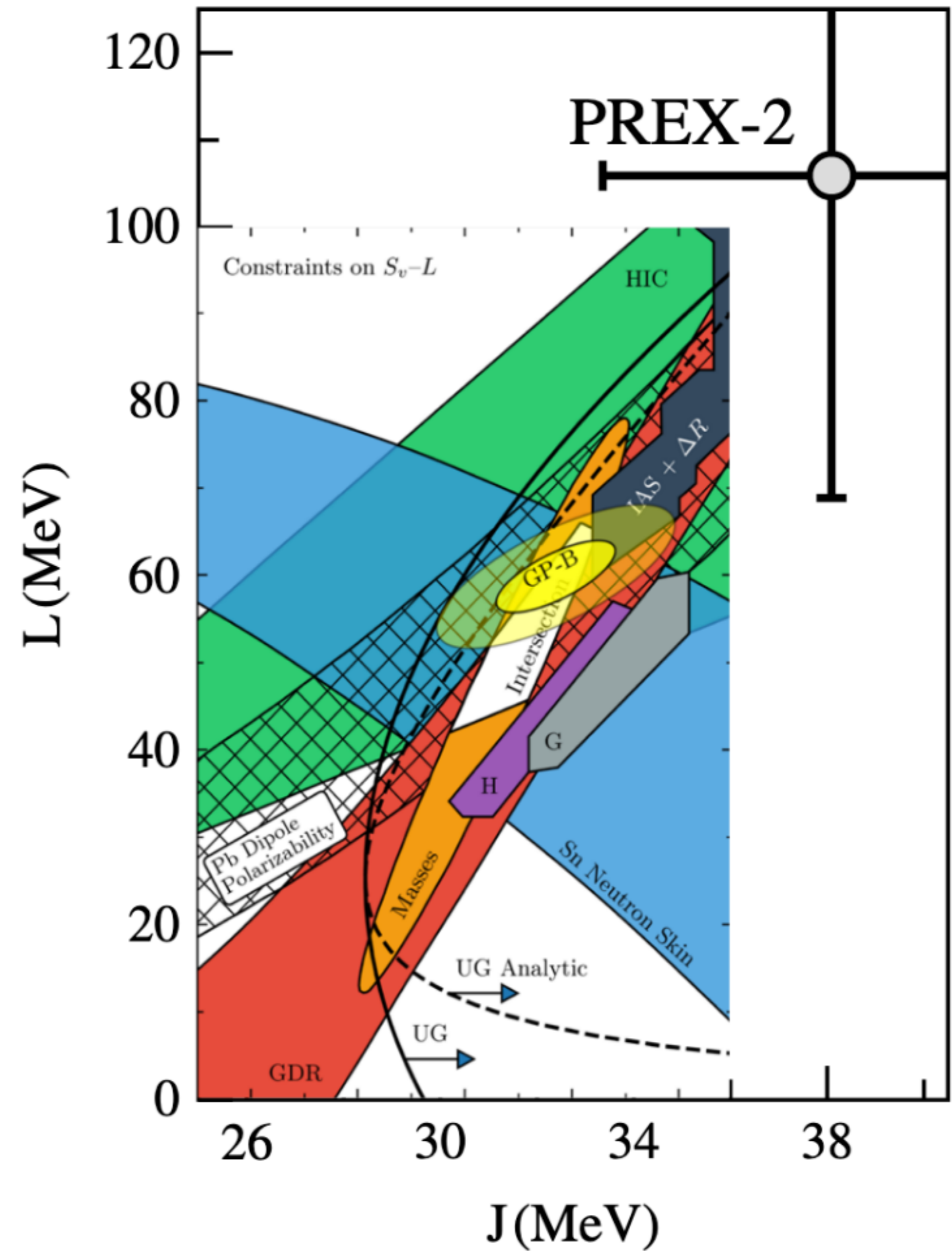
- Likely some tension between recent nuclear theory results, QLMXB observations and PREX II, J0740 results
- There could be systematics lurking in the QLMXB or NICER observations



Reed et al. (2021)

What about PREX II and NICER J0740?

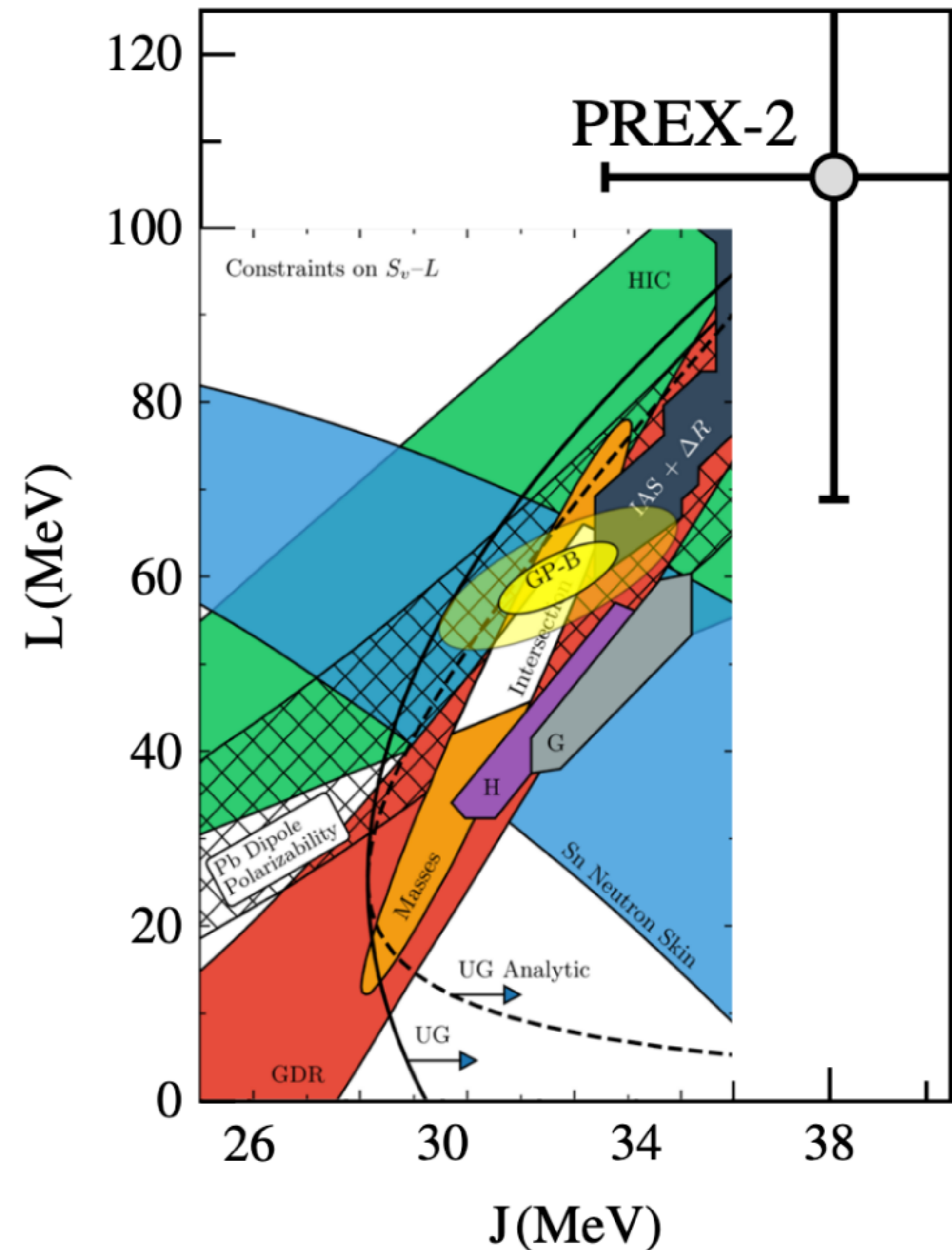
- Likely some tension between recent nuclear theory results, QLMXB observations and PREX II, J0740 results
- There could be systematics lurking in the QLMXB or NICER observations
- Some theoretical systematic?



Reed et al. (2021)

What about PREX II and NICER J0740?

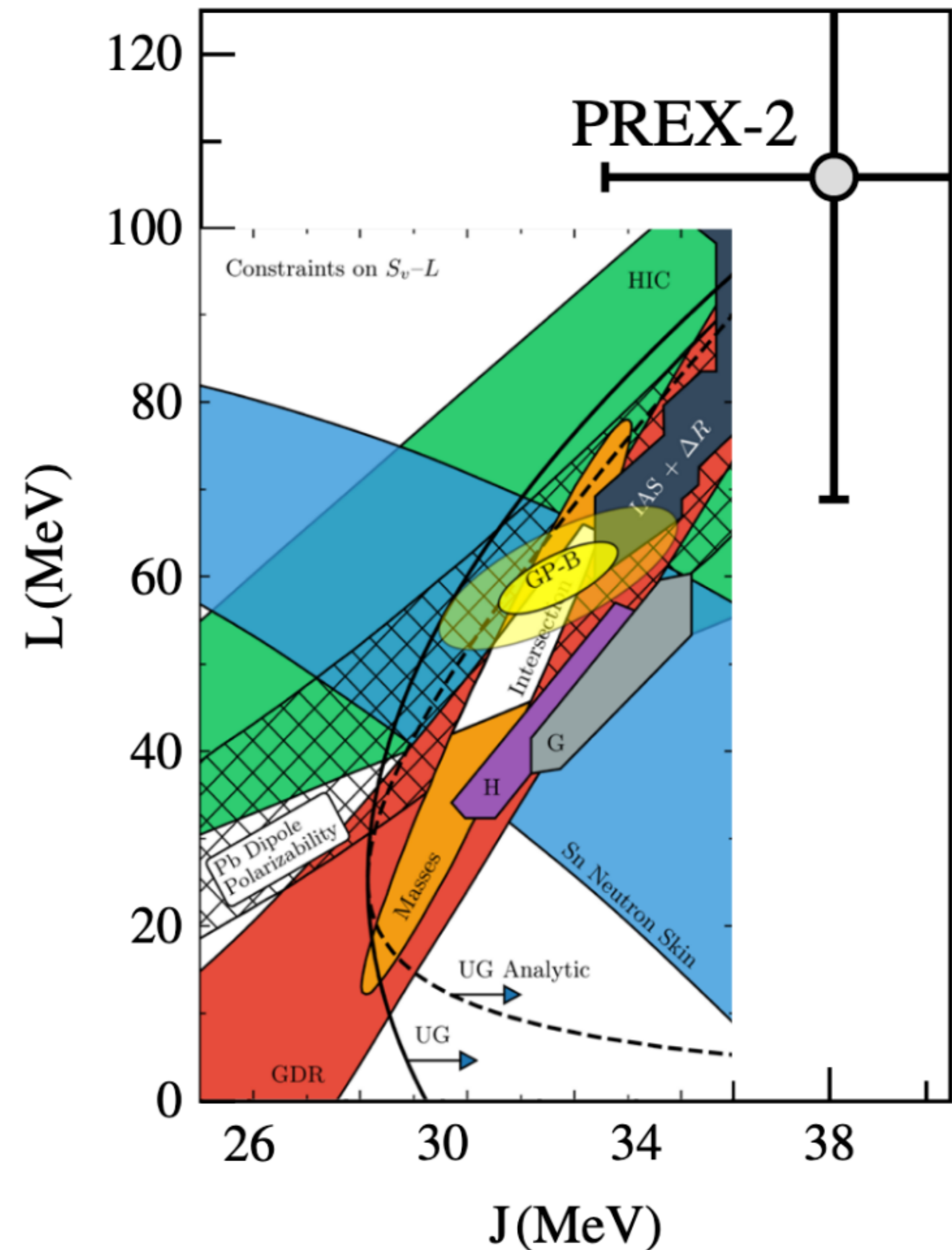
- Likely some tension between recent nuclear theory results, QLMXB observations and PREX II, J0740 results
- There could be systematics lurking in the QLMXB or NICER observations
- Some theoretical systematic?
- A PREX II systematic, failure of correlation between form factor and L



Reed et al. (2021)

What about PREX II and NICER J0740?

- Likely some tension between recent nuclear theory results, QLMXB observations and PREX II, J0740 results
- There could be systematics lurking in the QLMXB or NICER observations
- Some theoretical systematic?
- A PREX II systematic, failure of correlation between form factor and L
- Or they're all right, and we're just a little bit unlucky

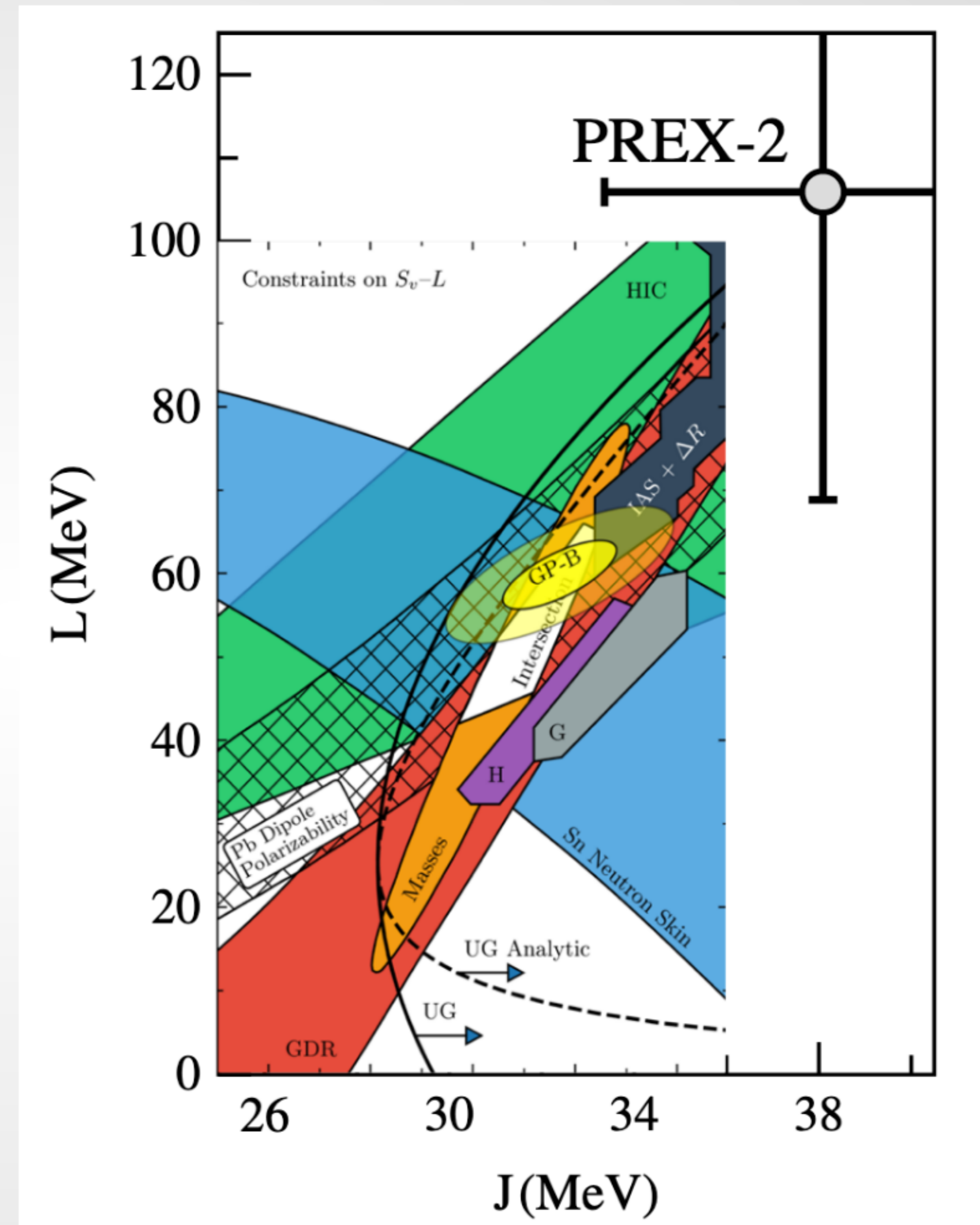


Reed et al. (2021)

What about PREX II and NICER J0740?

- Likely some tension between recent nuclear theory results, QLMXB observations and PREX II, J0740 results
- There could be systematics lurking in the QLMXB or NICER observations
- Some theoretical systematic?
- A PREX II systematic, failure of correlation between form factor and L
- Or they're all right, and we're just a little bit unlucky
- Lots of current theory work, but the best way to resolve this is more data

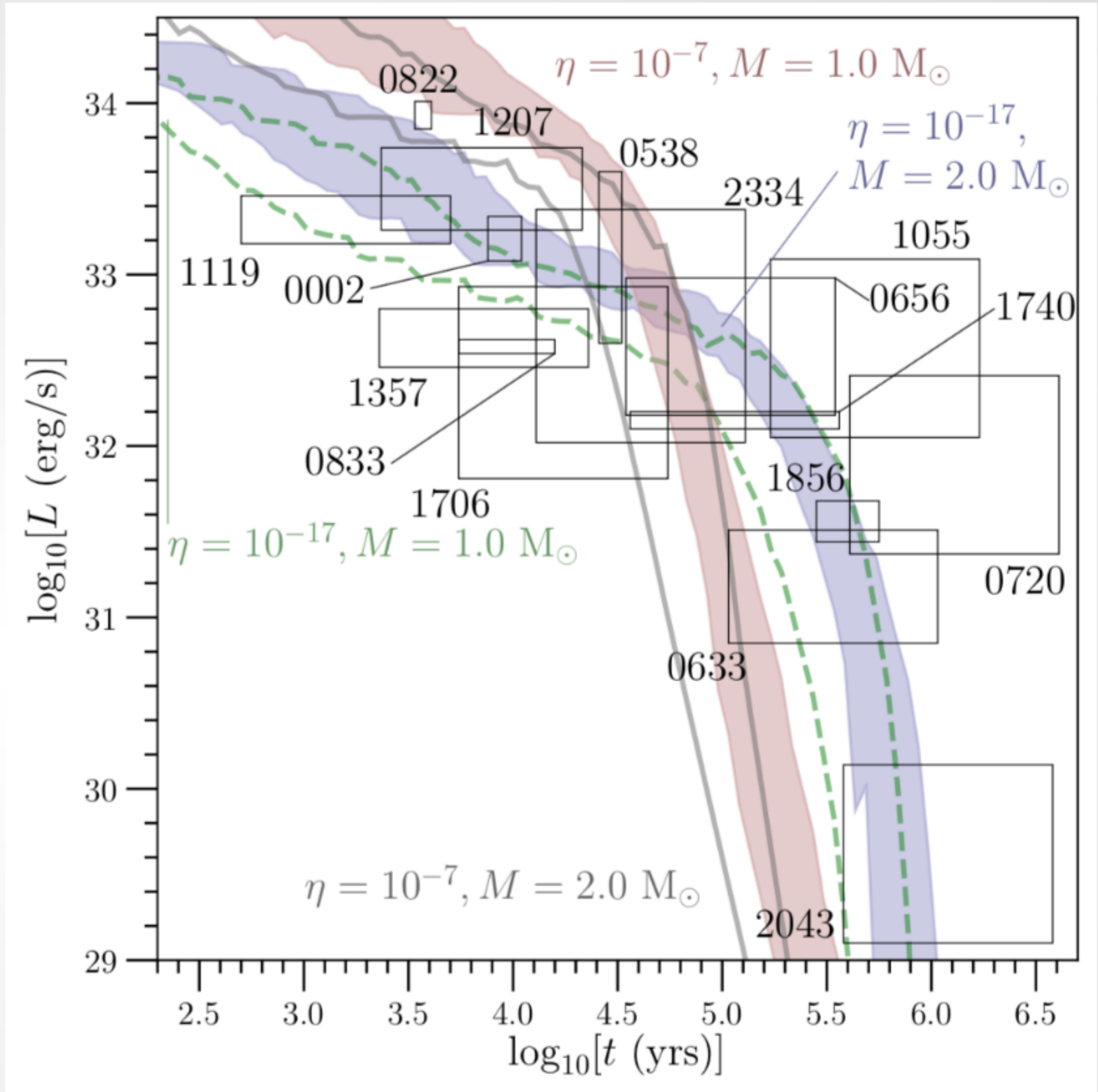
It would be nice to put more NICERs on the ISS...



Reed et al. (2021)

Thermal Emission from Isolated Neutron Stars

- After ~ 10 years, the neutron star is isothermal \Rightarrow one temperature = T
- $$C_V \frac{dT}{dt} = L_\nu + L_\gamma$$
- Assume only neutrons and protons
 - Age taken from, e.g., association with a supernova remnant



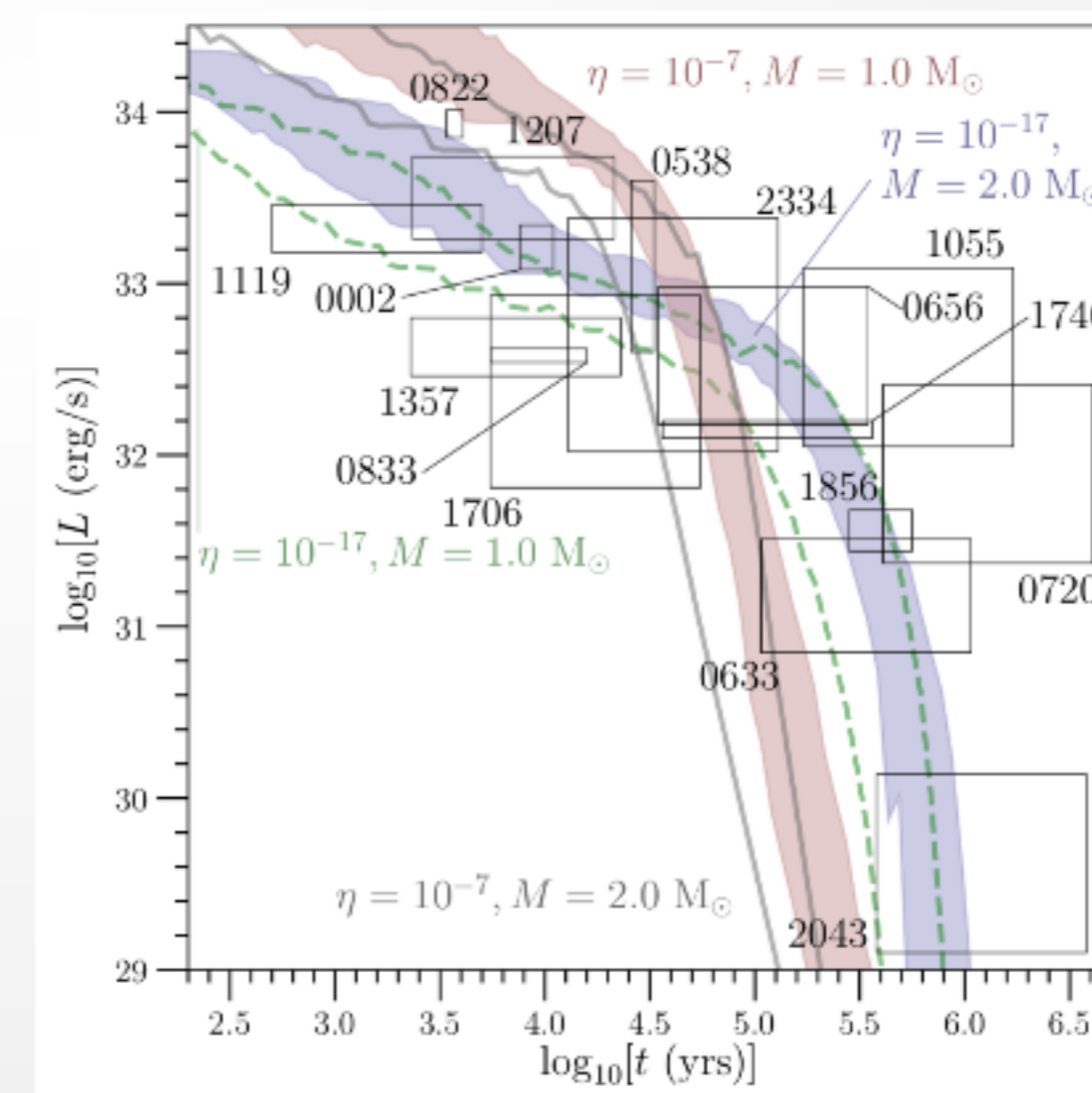
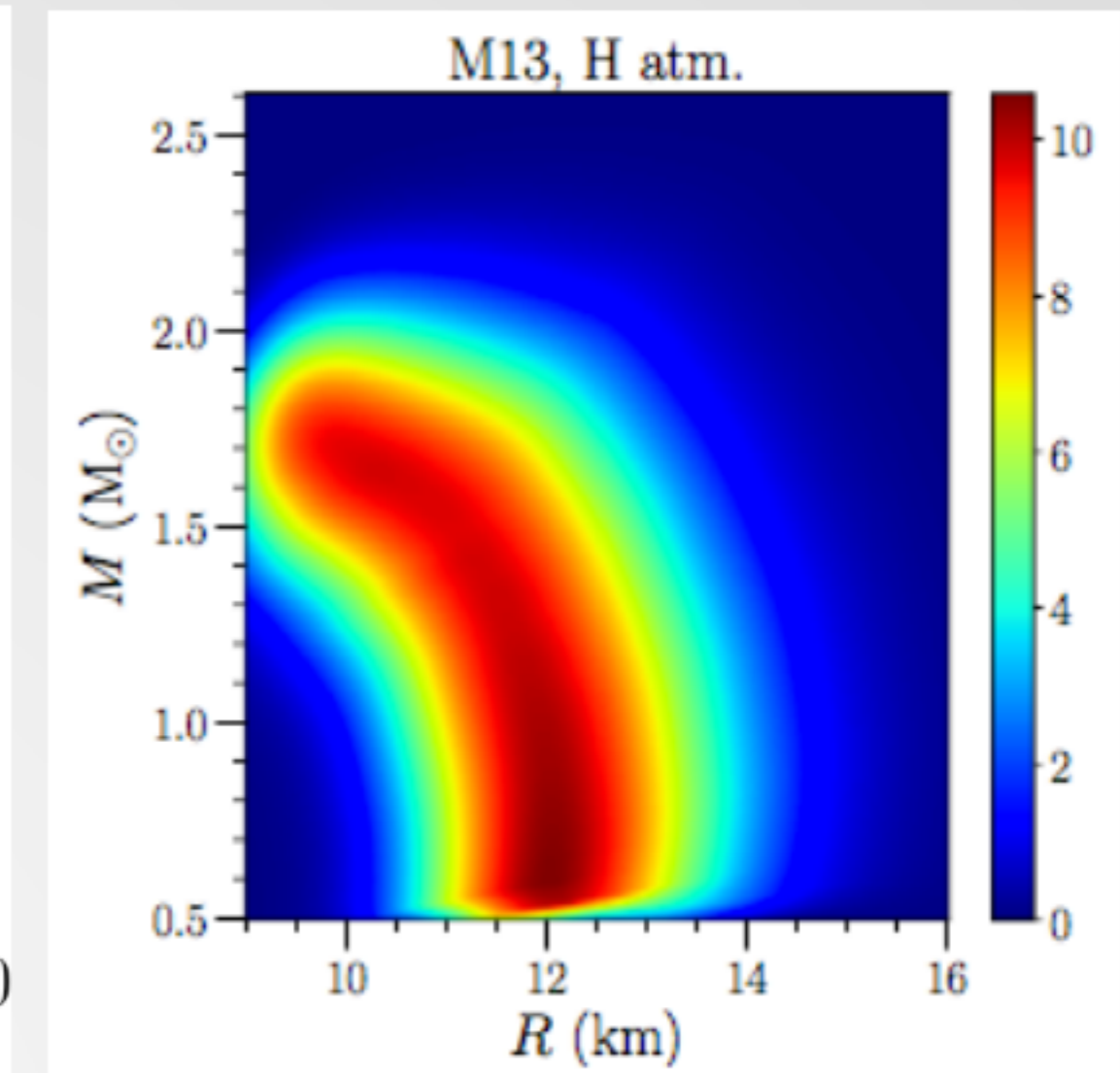
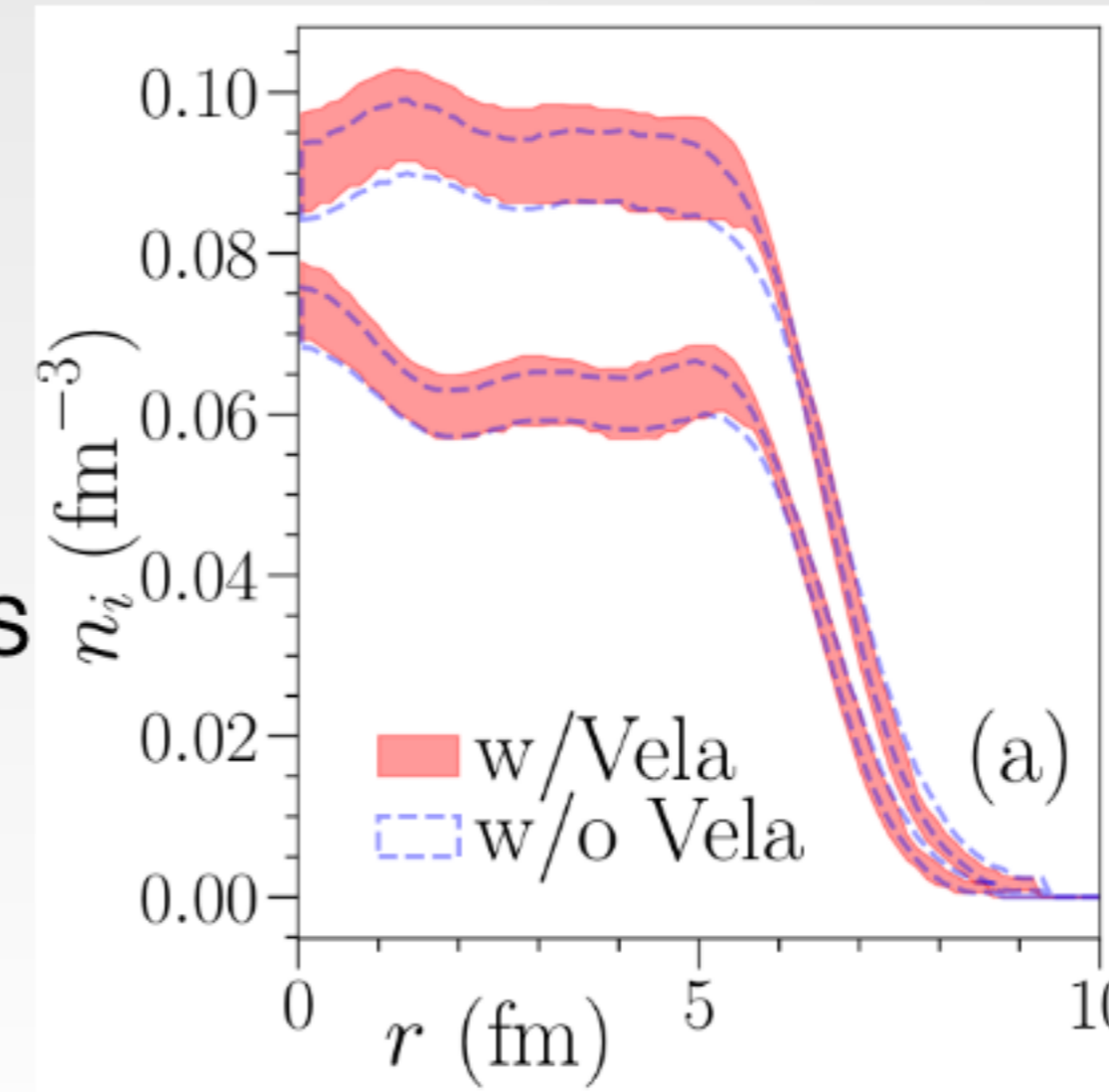
Beloin et al. (2019)

J0002-6246, i.e. the "cannonball pulsar"

First Large-Scale Bayesian Inference for NSs

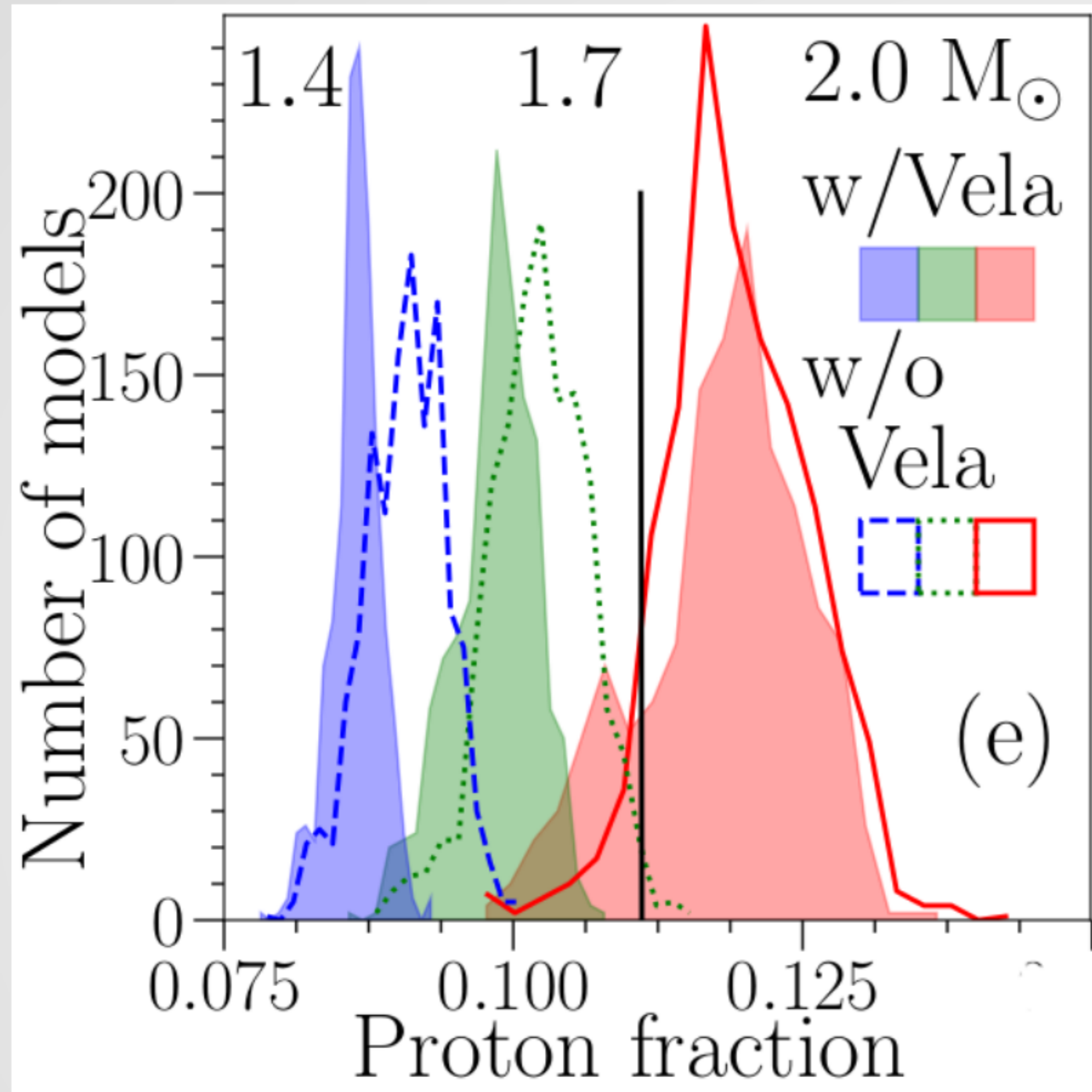
1. Include nuclear data (binding energies & charge radii of nuclei; Hartree approx.)
2. Neutron star mass and radius determinations from QLMXBs
3. Luminosity and age measurements of isolated neutron stars with $B < 10^{12}$ G

- Nuclear structure calculations
- Solution to TOV equations
- Simplified stellar evolution equations for neutron star cooling



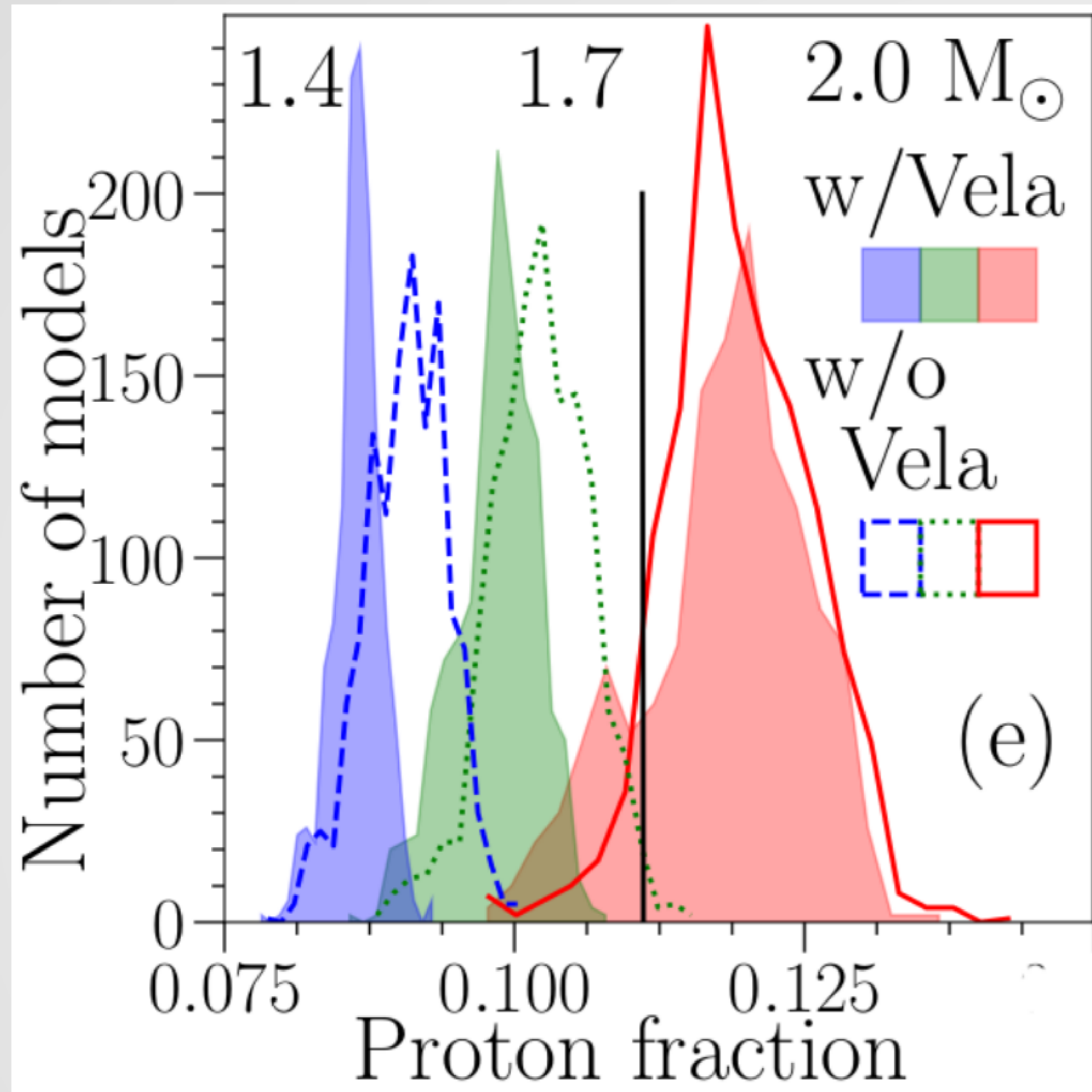
Beloin et al. (2019)

Now we have information on composition!



Beloin et al. (2019)

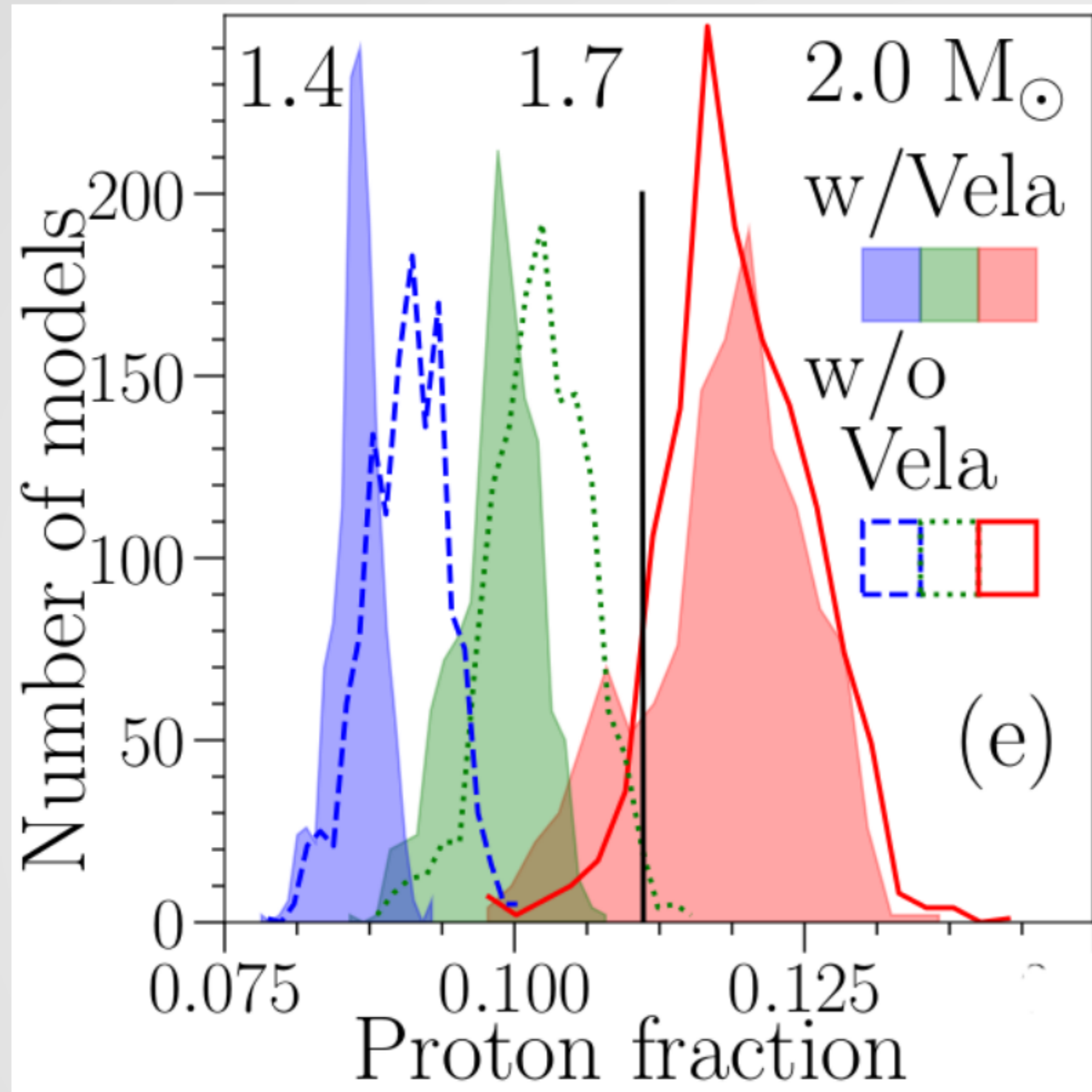
Now we have information on composition!



[Beloin et al. \(2019\)](#)

- Proton fraction is larger than 11% in the core of massive stars

Now we have information on composition!

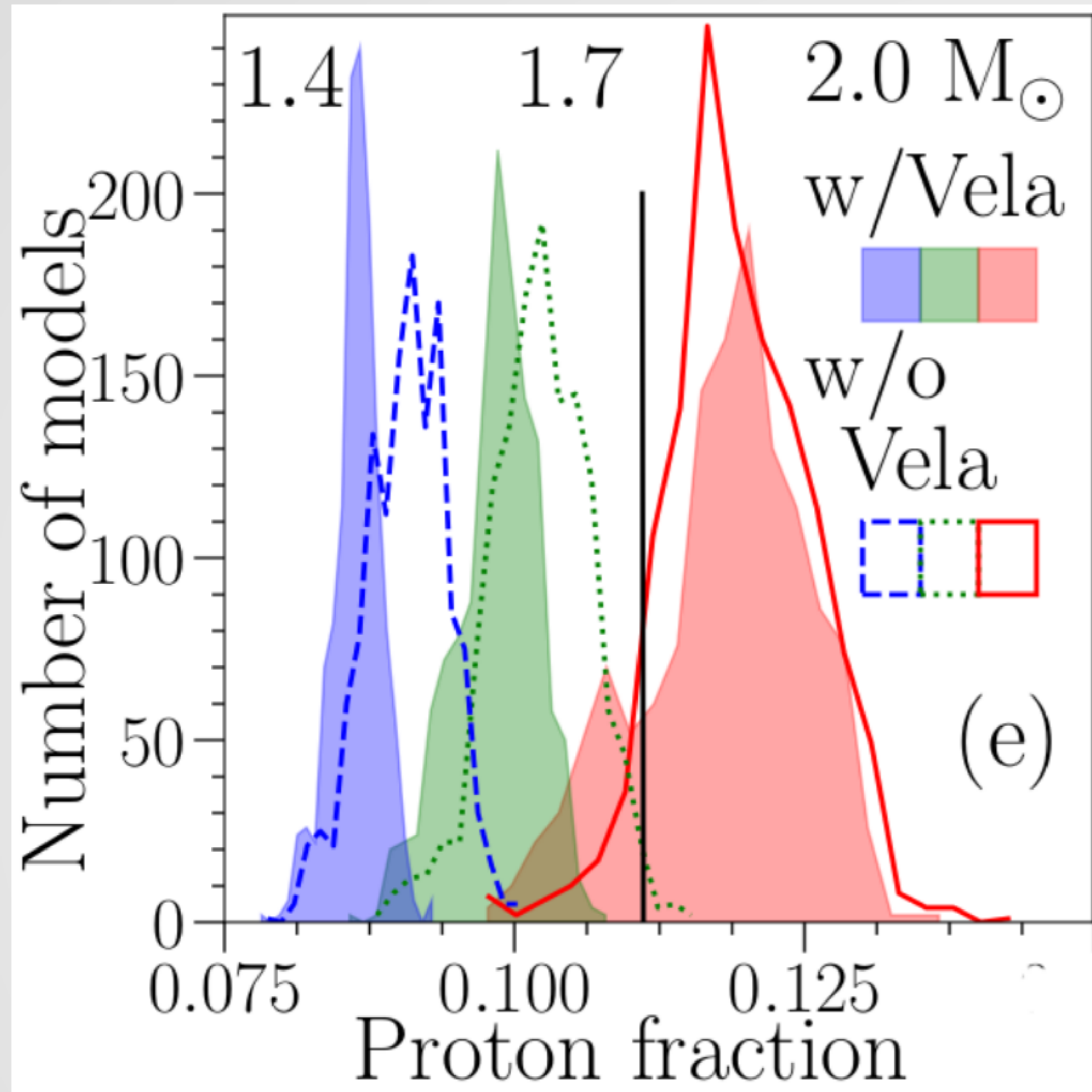


- This would imply a direct Urca process which would cool the stars quickly

[Beloin et al. \(2019\)](#)

- Proton fraction is larger than 11% in the core of massive stars

Now we have information on composition!

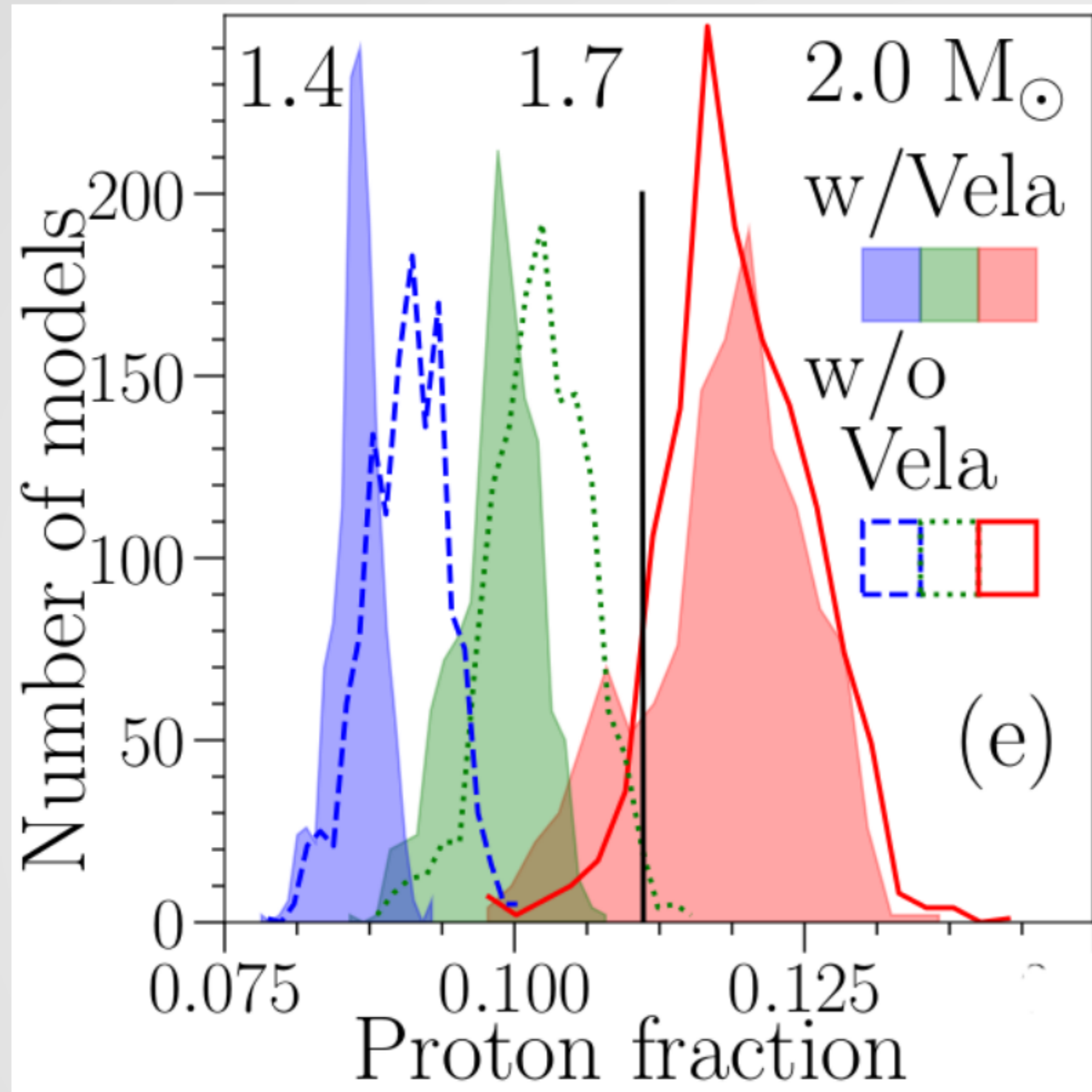


- This would imply a direct Urca process which would cool the stars quickly
- Constrains many of the stars to a small mass

[Beloin et al. \(2019\)](#)

- Proton fraction is larger than 11% in the core of massive stars

Now we have information on composition!

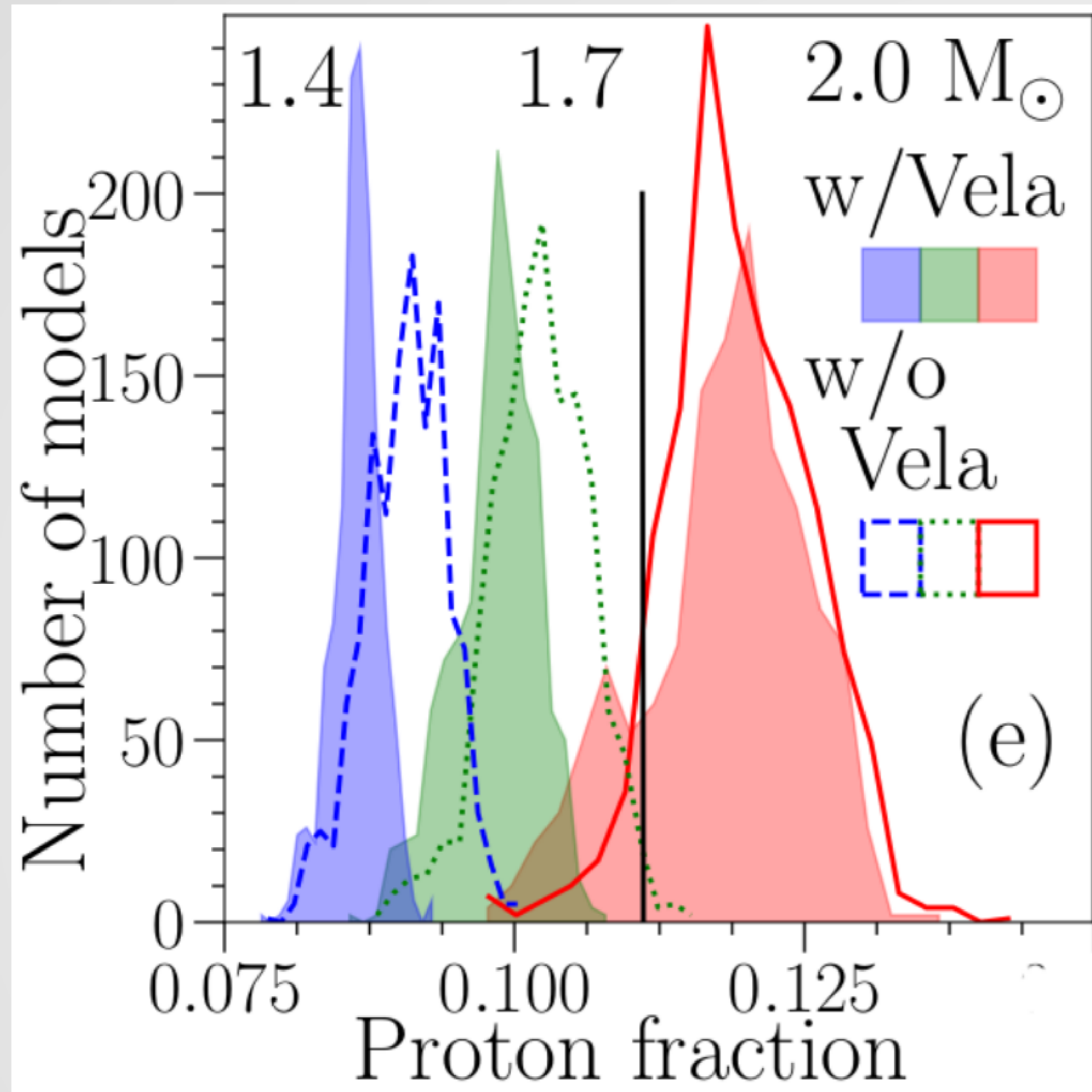


- This would imply a direct Urca process which would cool the stars quickly
- Constrains many of the stars to a small mass
- More likely way out: make a superfluid

[Beloin et al. \(2019\)](#)

- Proton fraction is larger than 11% in the core of massive stars

Now we have information on composition!



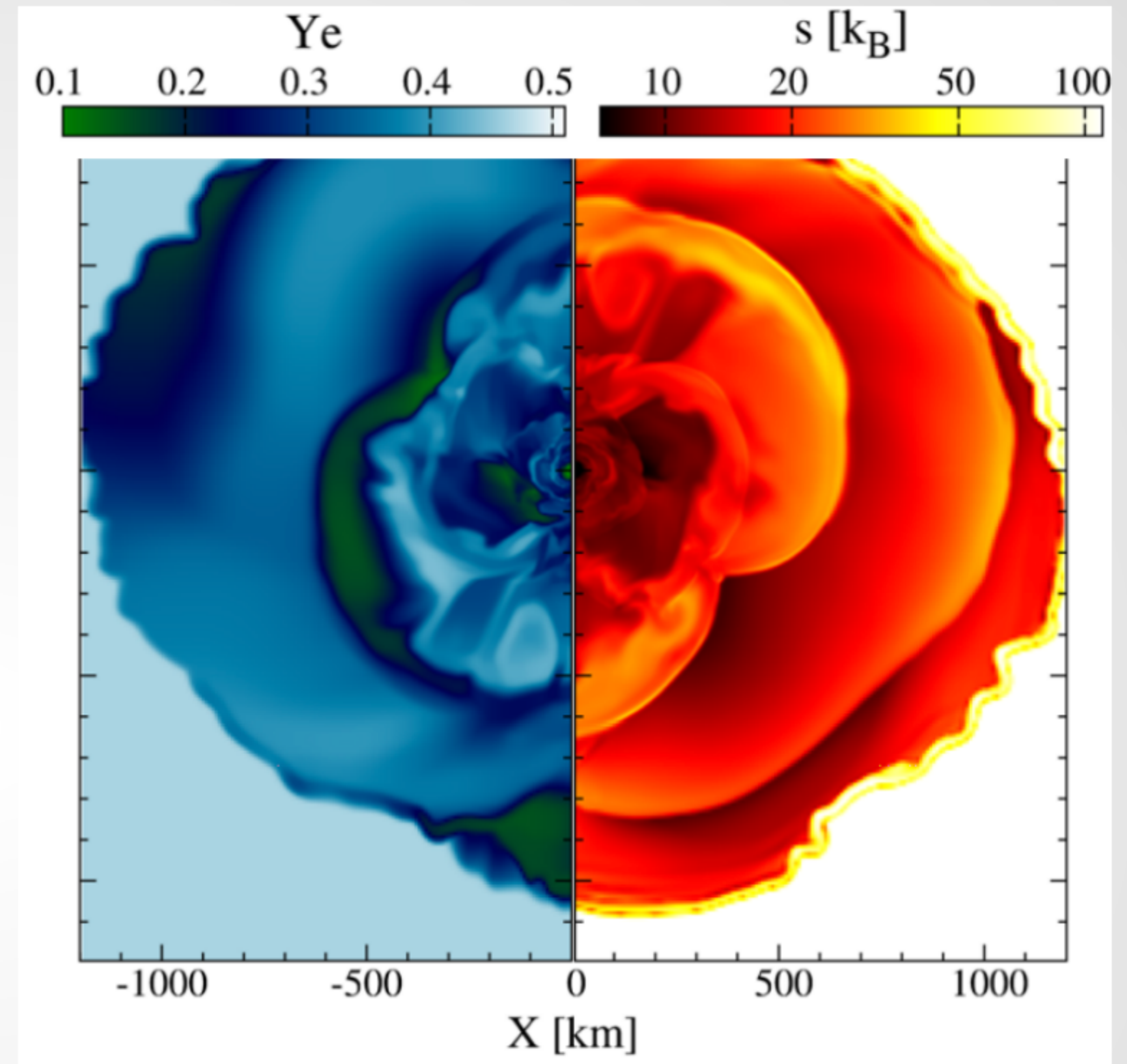
- This would imply a direct Urca process which would cool the stars quickly
- Constrains many of the stars to a small mass
- More likely way out: make a superfluid
- Neutron triplet superfluidity pervades the star

[Beloin et al. \(2019\)](#)

- Proton fraction is larger than 11% in the core of massive stars

Direct Relation to Neutron Star Mergers

- SFHo equation of state used in neutron star merger simulations
- Originally motivated by NS radius measurements
- R-process abundances are not strongly modified by equation of state changes
- However, amount of mass ejected significantly increased:
SFHo: $> 1.0 \times 10^{-2} M_{\odot}$
DD2: $< 2.1 \times 10^{-3} M_{\odot}$
TM1: $< 1.2 \times 10^{-3} M_{\odot}$



[Sekiguchi et al. \(2015\)](#)

- Improves ability of mergers to produce r-process elements!

- EM observations and GW observations combined constrain the radius of a $M = 1.4 M_{\odot}$ neutron star — prior dependence is weak
- But, prior still matters for low and high masses
- EM and GW observations seem to agree, but interesting future with PREX II and NICER's observation of J0740
- Combined analysis with NS cooling constrains composition
- There are important connections between the EOS and NS mergers: maximum mass and amount of mass ejected
- Exciting news just around the corner...