Toward an in-medium effective field theory for the nuclear shell model

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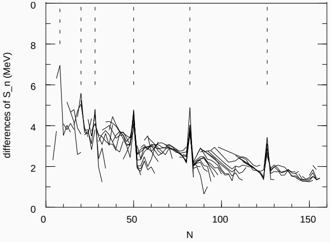
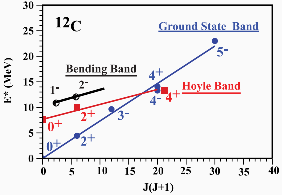
Atomic nuclei exhibit many emergent phenomena:

shell structure

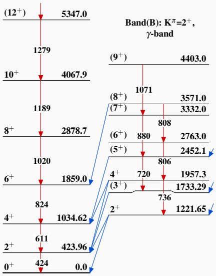


$$\Delta S_n = 2B(N,Z) - B(N-1,Z) - B(N+1,Z)$$

clustering

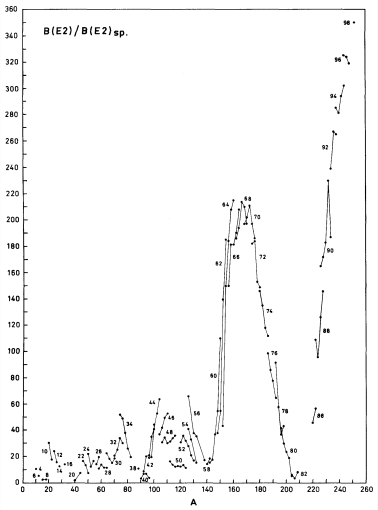


rotational and vibrational motions



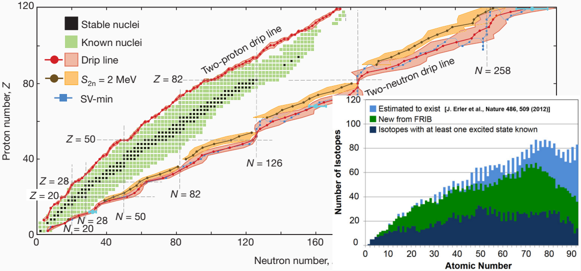
^{76}Kr

deformation

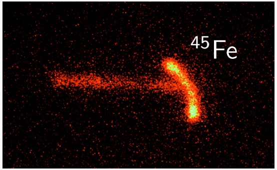


B. A. Brown, *Lecture Notes in Nuclear Structure Physics*, ENSDF database,
A. Bohr and B. R. Mottelson, *Nuclear Structure, Vol. 2: Nuclear Deformations*,
D. J. Marín-Lámbarri *et al.*, Phys. Rev. Lett. **113**, 012502 (2014).

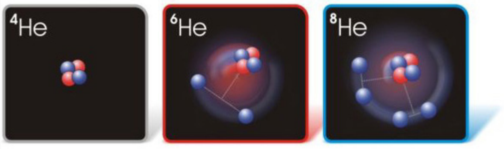
Near the drip-lines: continuum-related emergent phenomena



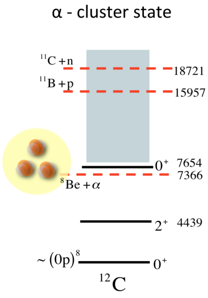
exotic decay modes



halos, Borromean systems



near-threshold clustering



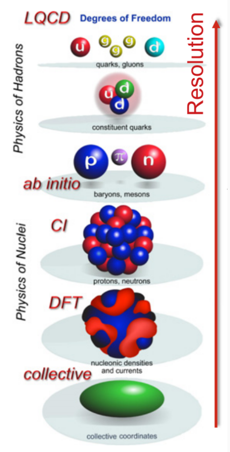
Low-energy virtual states, many-body resonances...

J. Erler *et al.*, Nature **486**, 509 (2012), K. Miernik *et al.*, Phys. Rev. Lett. **99**, 192501 (2007).
Figures stolen from A. Gade and M. Płoszajczak.

Exotic emergent phenomena:

→ provide a unique way to probe properties of nuclear matter,

→ motivation for the exploration of the drip lines.



In the language of **effective field theory** (EFT):

- An effective separation of scale lies behind each emergent phenomenon.
- There are "natural" degrees of freedom that efficiently describe properties of the system.

In the language of **renormalization group** (RG):

- There are low- and high-resolution descriptions of the system related by RG transformations preserving observables.

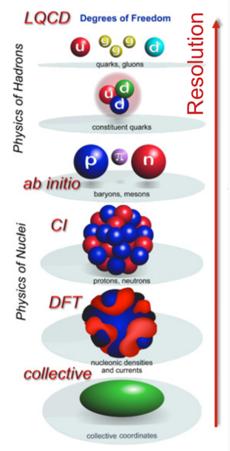
Reformulation of fundamental questions in a more precise way:

- Origin of emergent phenomena (conditions)
  → Identifying and quantifying effective scale separations.
- Understand emergent phenomena "microscopically"
  → How to connect low- and high-resolution descriptions of the system

W. Nazarewicz, J. Phys. G **43**, 044002 (2016).

One of the goals of nuclear theory:

→ Describe consistently all nuclear properties from nucleons as degrees of freedom.



Two main strategies:

- ***Ab initio* approaches.** Promise to describe all many-nucleon phenomena as exactly as possible with nuclear forces as the only input.
- **Density functional theory.** Could be exact if the "true" energy density functional is found.

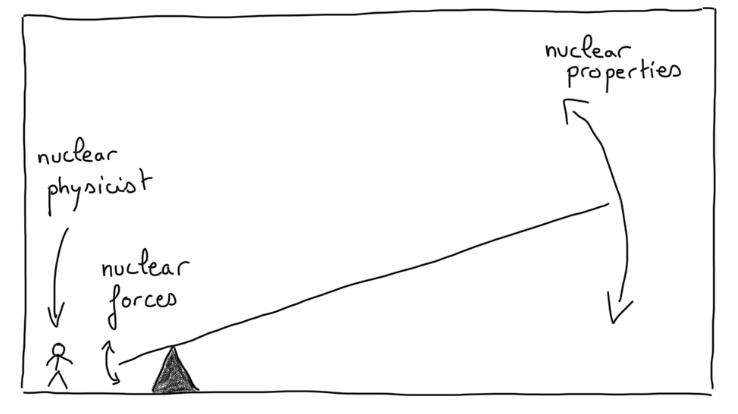
→ Great tools to study nuclear forces, minimal input, no ambiguity about the proper degrees of freedom.

→ Do not provide the insight that identifying emergent scales explicitly gives, only checks a posteriori that the information about emergent phenomena has been properly encoded into nuclear forces.

Known issue: nuclei are strongly correlated Fermi systems made of a few up to over 250 nucleons, and small changes in nuclear forces can and do produce large changes on binding energies, especially for medium-mass and heavy nuclei.

W. Nazarewicz, J. Phys. G **43**, 044002 (2016).

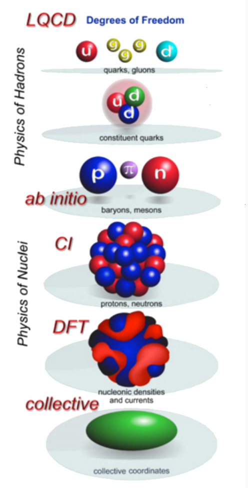
To make my point clear:

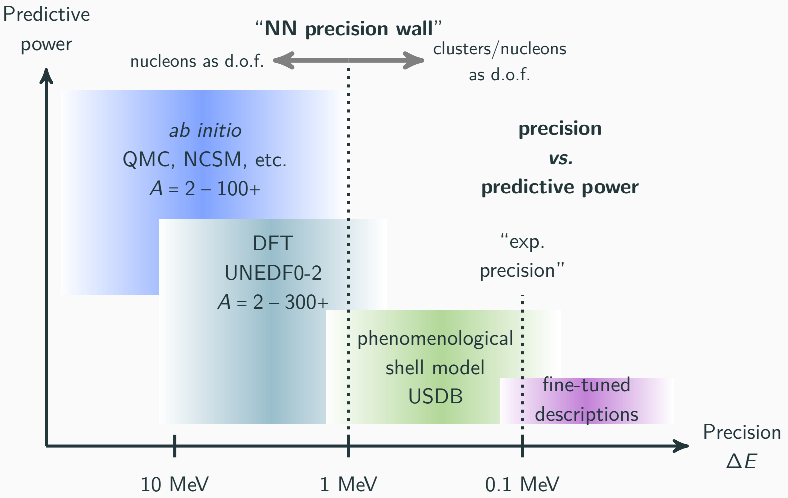


What precision can we get on nuclear properties given input forces and their uncertainties?

What is the importance of emergent phenomena in the answer?

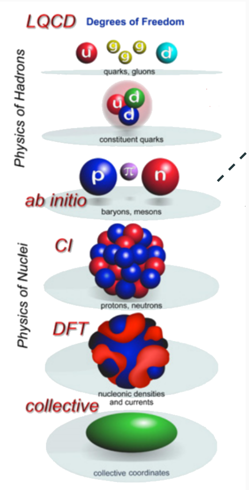
The nucleon-nucleon (NN) precision wall:

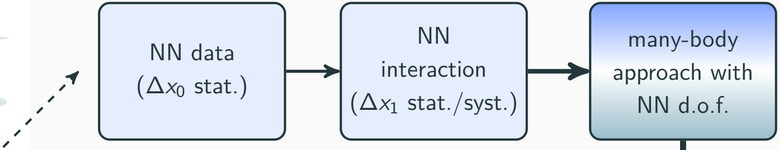


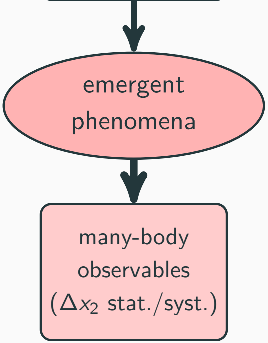


S. Binder *et al.*, PRC **93**, 044002 (2016), PRC **98**, 014002 (2018); M. Piarulli *et al.*, PRL **120**, 052503 (2018), E. Epelbaum *et al.*, PRC **99**, 024313 (2019); R. Navarro Pérez *et al.*, PRC **97**, 054304 (2018)

Origin of the NN precision wall:







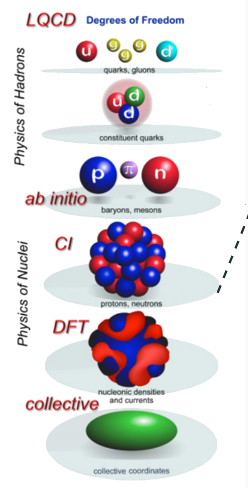
Claim 1: Emergent phenomena magnify details of the nuclear interaction making small stat. uncertainties a problem.

Similar situation in complex system theory:

small Δinput $\rightarrow$ large Δoutput.

Practical limit on *ab initio* precision?

Origin of the NN precision wall: a way out?



NN, cluster-nucleon, many-body data (Δx_0 stat.) → NN, cluster-nucleon interaction, (Δx_1 stat./syst.) → many-body approach with cluster-nucleon d.o.f.

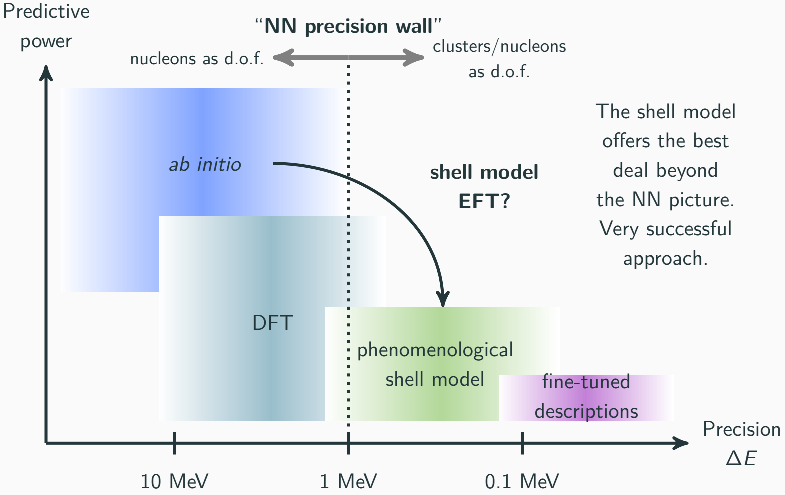
emergent phenomena

many-body observables (Δx_2 stat./syst.)

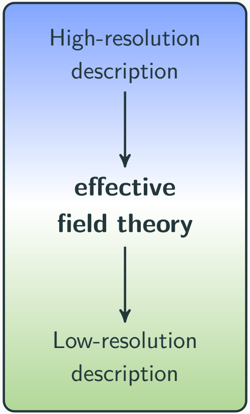
Claim 2: By using natural d.o.f., phenomenological approaches "bypass" some emergent phenomena, increasing their precision in exchange of an important loss in predictive power.

→ Connection lost with the NN picture.
Can we recover it?

Connecting the two pictures using EFTs



The shell model offers the best deal beyond the NN picture. Very successful approach.



Making the nuclear shell model more fundamental is not a new idea:

→ Essentially three strategies were employed in the past.

1) Nuclear shell model approach as an effective theory. Never extended beyond ^{3}He.

W. C. Haxton *et al.*, Phys. Rev. Lett. **84**, 5484 (2000)

2) Shell-model interactions from NN forces. The trade-off between precision and predictive power is not used, mostly an extension of *ab initio* theory.

S. Bogner *et al.*, Phys. Rev. C **65**, 051301(R) (2002),
J. D. Holt *et al.*, Nucl. Phys. A **733**, 153 (2004),
L. Coraggio *et al.*, Phys. Rev. C **2007**, 024311 (2007),

S. K Bogner *et al.*, Phys. Rev. Lett. **113**, 142501 (2014),
G. R. Jansen *et al.*, Phys. Rev. Lett. **113**, 142502 (2014),
S. R. Stroberg *et al.*, Phys. Rev. Lett. **118**, 032502 (2017)

3) Core and valence degrees of freedom, and EFT for the core-valence and valence-valence interactions.

- Halo systems (3-body):

J. Rotureau *et al.*, Few-Body Syst. **54**, 725 (2013),
C. Ji and *et al.*, Phys. Rev. C **90**, 044004 (2014)

- Nuclear shell model (*):

L. Huth *et al.*, Phys. Rev. C **98**, 044301 (2018)

D. J. Dean *et al.*, Prog. Part. Nucl. Phys. **53**, 419 (2004) S. R. Stroberg *et al.*, Ann. ReV. Nucl. Part. Sci. **69** (2019)

Toward an in-medium EFT for the nuclear shell model

→ When does it make sense to assume an effective core?

Scale separation measure in 3-body halo EFT:

$$\alpha_{\text{halo-EFT}} = \frac{S_{A-A_c}}{\min[E_x(A_c), S_{n,p}(A_c)]} \approx \frac{\overline{E}_{\text{remove all valence nucleons}}}{\overline{E}_{\text{remove/excite a nucleon from the core}}} << 1$$

- Too restrictive to account for the shell model past achievements.

Traditional prescription:
- Large $S_{n/p}(A_c)$.
- Spherical and coupled to 0^+.
- Inert core.

→ Closed-shell nuclei.

Tentative shell model scale separation measure:

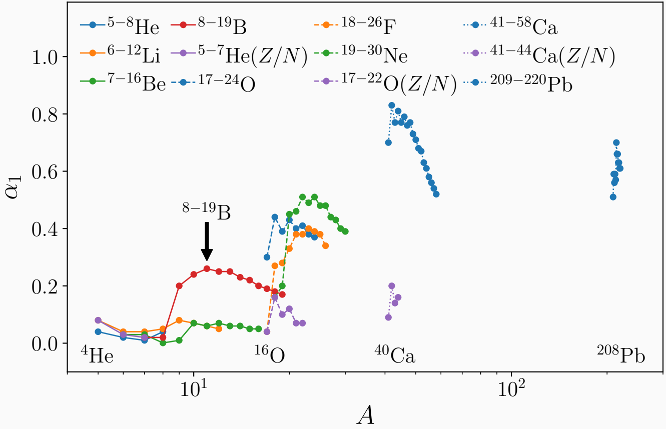
$$\alpha_1 = \frac{\mathcal{S}_A}{S_1(A_c)} \approx \frac{\overline{E}_{\text{remove a valence nucleon}}}{\overline{E}_{\text{remove a nucleon from the core}}} < 1$$

- $\mathcal{S}_A = S_{A-A_c}/(A-A_c)$ and $S_1(A_c) = [S_n(A_c) + S_p(A_c)]/2$.
- The effective core is not really "inert", small excitations are allowed.

C. A. Bertulani *et al.*, Nucl. Phys. A **712**, 37 (2002), R. Higa, Few-Body Syst. **50**, 251 (2011),
E. Ryberg *et al.*, Phys. Rev. C **89**, 014325 (2014), C. Ji *et al.*, Phys. Rev. C **90**, 044004 (2014),
J. Rotureau *et al.*, Few-Body Syst. **54**, 725 (2013)

The effective core scale separation

→ Some "surprises".



- Light and proton-rich nuclei have good scale separations.
- The scale separation improves along isotopic chains.
- Very good for "exotic" cores.

→ Suggests a special regime (link with halo EFT)

Data from the ENSDF database.

In-medium valence-space shell model interaction from EFT

→ What can we say about the valence-space interaction?

$$\alpha_1 = \frac{\mathcal{S}_A}{S_1(A_c)} \quad \rightarrow \quad \frac{p_A}{p_1} = \frac{\overline{p}_{\text{remove a valence nucleon}}}{\overline{p}_{\text{remove a nucleon from the core}}} \approx \frac{60 - 120\text{MeV}}{115 - 193\text{MeV}}$$

- **Remark 1:** $p_1 \sim m_\pi \approx 135$ MeV → pionless EFT!
- **Remark 2:** $p_1 \sim m_\pi >> p_c$ where p_c is the minimal momentum required to resolve details of the core. One has $p_c \approx \hbar/(2r_c) \sim 23 - 50$ MeV, with $r_c = 1.25 A_c^{\frac{1}{3}}$ and $A_c = 4 - 40$.

Consequence:

→ Regime of halo structures reached for $0 < p < p_c$.

→ Regime of low-lying collective excitations reached for $p_c < p < p_A$ (*i.e.* $r_c > r >> 1.4$ fm).

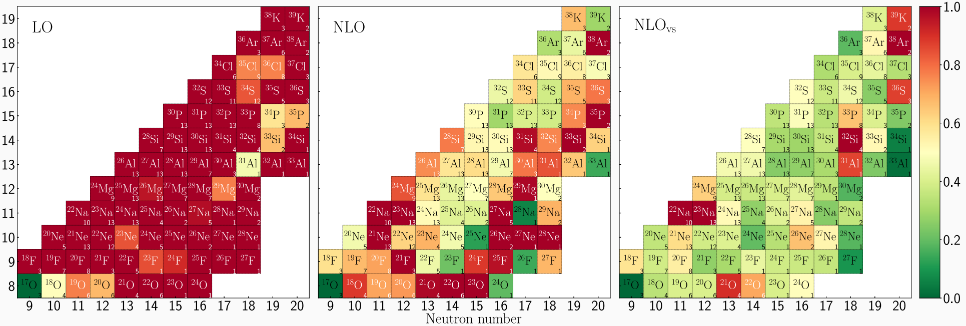
→ Regime of shell-model structures (only) reached for $p_A < p < p_1$.

Convention: The low end of the momentum range given by p_c defines the halo EFT limit.

S. Weinberg, Nucl. Phys. B **363**, 3 (1991), C. Ordóñez *et al.*, Phys. Rev. Lett. **72**, 1982 (1994),
C. Ordóñez *et al.*, Phys. Rev. C **53**, 2086 (1996), E. Epelbaum *et al.*, Rev. Mod. Phys. **81**, 1773 (2009)

Some reasons to be optimistic

→ Pionless EFT including core-valence terms and a long-range contribution in the *sd* space.



- Precision on binding energy of about 500 keV at NLO_{vs}.
- Long-range contribution probably useless (see previous arguments).
- Small model space and monople correction used.
- Pionless EFT used as an in-medium interaction implicitely.

L. Huth *et al.*, Phys. Rev. C **98**, 044301 (2018)

In-medium valence-space shell model interaction from EFT

→ How to make the interaction explicitely in-medium?

For $p < p_1 \sim m_\pi$ the effective valence-space interaction ≈ pionless EFT, but it is not explicitely an in-medium interaction.

Landau's Fermi liquid theory:

- Effective theory at the surface of an infinite Fermi system (around $\boldsymbol{p}_F$).
- Assumes that individual momenta of fermions satisfy $\boldsymbol{p}_i = \boldsymbol{p}_F + \delta\boldsymbol{p}_i$ with $\delta p_i << p_F$.

→ Problems: $\boldsymbol{p}_F \sim 250$ MeV and interaction formulated using relative momenta $\boldsymbol{p} = \boldsymbol{p}_i - \boldsymbol{p}_j$.

Solution: Expand relative momenta around a finite momentum $\boldsymbol{p}_0$.

- We have $\boldsymbol{p} \to \boldsymbol{p} - \boldsymbol{p}_0$ (idem $\boldsymbol{p}'$) and so $\boldsymbol{q} = \boldsymbol{p}' - \boldsymbol{p} \to \boldsymbol{q}$ and $\boldsymbol{k} = (\boldsymbol{p}' + \boldsymbol{p})/2 \to \boldsymbol{k} - \boldsymbol{p}_0$.
- Example: $\boldsymbol{k}^2$:

$$(k - p_0)^2 = k^2 - 2kp_0 + p_0^2 \sim Q^2 \left(1 + \frac{p_0}{Q} + \left(\frac{p_0}{Q}\right)^2\right)$$

Introduce a new power counting where $p_0 \sim c_{\mathrm{dim}}\sqrt{p_1} \sim p_c$!

L. D. Landau, Sov. Phys. JETP **3**, 920 (1957), L. D. Landau, Sov. Phys. JETP **5**, 101 (1957), L. D. Landau, Sov. Phys. JETP **8**, 70 (1959), R. Shankar, Rev. Mod. Phys. **66**, 129 (1994)

Pionless EFT around a finite momentum

→ Average contributions around $\boldsymbol{p}_0$.

$$\hat{V}^{(\mu=0)} = \hat{V}^{(\nu=0)} = C_S + C_T \boldsymbol{\sigma}_1.\boldsymbol{\sigma}_2$$

$$\hat{V}^{(\mu=1)} = (C_2 + C_4 \boldsymbol{\sigma}_1.\boldsymbol{\sigma}_2)\boldsymbol{p}_0^2 + C_7(\boldsymbol{\sigma}_1.\boldsymbol{p}_0)(\boldsymbol{\sigma}_2.\boldsymbol{p}_0)$$

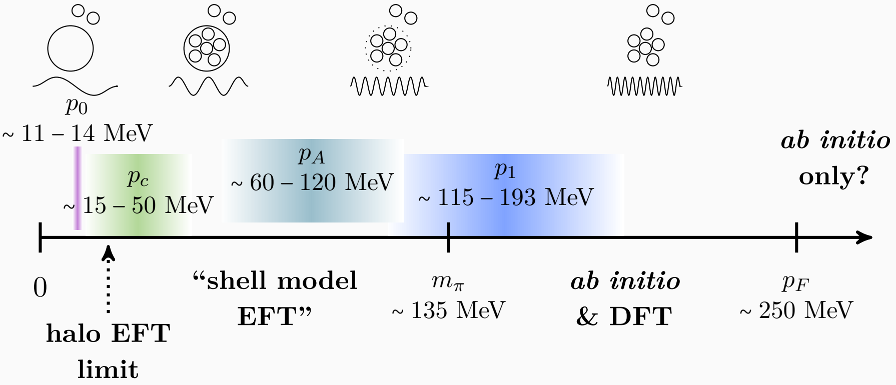
$$\hat{V}^{(\mu=3/2)} = -2(C_2 + C_4 \boldsymbol{\sigma}_1.\boldsymbol{\sigma}_2)\boldsymbol{k}.\boldsymbol{p}_0 - i\frac{C_5}{2}(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2).(\boldsymbol{q}\times\boldsymbol{p}_0) - C_7\left[(\boldsymbol{\sigma}_1.\boldsymbol{p}_0)(\boldsymbol{\sigma}_2.\boldsymbol{k}) + (\boldsymbol{\sigma}_1.\boldsymbol{k})(\boldsymbol{\sigma}_2.\boldsymbol{p}_0)\right]$$

$$\begin{aligned}\hat{V}^{(\mu=2)} = {} & C_1\boldsymbol{q}^2 + C_2\boldsymbol{k}^2 + (C_3\boldsymbol{q}^2 + C_4\boldsymbol{k}^2)\boldsymbol{\sigma}_1.\boldsymbol{\sigma}_2 \\ & + i\frac{C_5}{2}(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2).(\boldsymbol{q}\times\boldsymbol{k}) \\ & + C_6(\boldsymbol{\sigma}_1.\boldsymbol{q})(\boldsymbol{\sigma}_2.\boldsymbol{q}) + C_7(\boldsymbol{\sigma}_1.\boldsymbol{k})(\boldsymbol{\sigma}_2.\boldsymbol{k}) \\ & + D_2\boldsymbol{p}_0^4 + D_{14}\boldsymbol{p}_0^2(\boldsymbol{\sigma}_1.\boldsymbol{p}_0)(\boldsymbol{\sigma}_2.\boldsymbol{p}_0)\end{aligned}$$

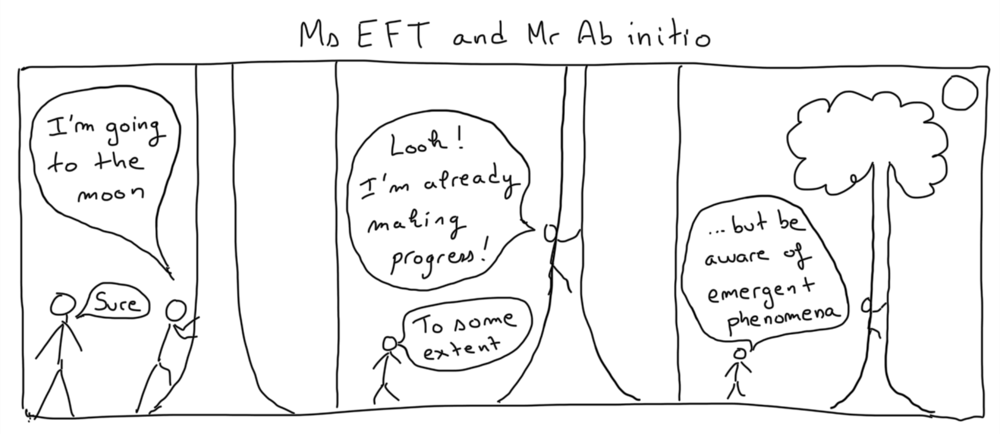
	LO $\nu = 0$	NLO $\nu = 2$	N2LO $\nu = 4$	N3LO $\nu = 6$
$\mu = 0$	x			
$\mu = 1/2$				
$\mu = 1$		x		
$\mu = 3/2$		x		
$\mu = 2$		x	x	
$\mu = 5/2$			x	
$\mu = 3$			x	x
$\mu = 7/2$			x	x
$\mu = 4$			x	x
$\mu = 9/2$				x
$\mu = 5$				x
$\mu = 11/2$				x
$\mu = 6$				x

In-medium valence-space shell model interaction from EFT

Shell model EFT can be seen as pionless EFT re-expanded around the halo-EFT limit.



Take home message:



Thank you for your attention!

Michigan State University:

- J. Rotureau.
- H. Hergert.
- S. Bogner.

Coordinate space representation

- Simplest choice: $\boldsymbol{p}_0 = p_0 \boldsymbol{u_r}$ along the relative postion of two nucleons.

$$\hat{V}^{(\mu=0)}(r) = (C_S + C_T \boldsymbol{\sigma}_1.\boldsymbol{\sigma}_2)\delta^{(3)}(\boldsymbol{r})$$

$$\hat{V}^{(\mu=1)}(r) = (C_2 + C_4 \boldsymbol{\sigma}_1.\boldsymbol{\sigma}_2)p_0^2\delta^{(3)}(\boldsymbol{r}) + C_7 p_0^2(\boldsymbol{\sigma}_1.\boldsymbol{u_r})(\boldsymbol{\sigma}_2.\boldsymbol{u_r})\delta^{(3)}(\boldsymbol{r})$$

$$\hat{V}^{(\mu=3/2)}(r) = -i(C_2 + C_4 \boldsymbol{\sigma}_1.\boldsymbol{\sigma}_2)p_0 \nabla_{\boldsymbol{r}}\delta^{(3)}(\boldsymbol{r})$$

$$\hat{V}^{(\mu=2)}(r) = \hat{V}^{(\nu=2)}(r) + D_2 p_0^4\delta^{(3)}(\boldsymbol{r}) + D_{14} p_0^4(\boldsymbol{\sigma}_1.\boldsymbol{u_r})(\boldsymbol{\sigma}_2.\boldsymbol{u_r})\delta^{(3)}(\boldsymbol{r})$$

- In the interaction channel $c = (T = 1, S = 0, L = 0)$:

$$\hat{V}^{(\mu=0,c)}(r) = (C_S - 3C_T)\delta^{(3)}(\boldsymbol{r})$$

$$\hat{V}^{(\mu=1,c)}(r) = (C_2 - 3C_4 - \frac{1}{3}C_7)p_0^2\delta^{(3)}(\boldsymbol{r})$$

$$\hat{V}^{(\mu=3/2,c)}(r) = (C_2 - 3C_4)p_0 \nabla_{\boldsymbol{r}}\delta^{(3)}(\boldsymbol{r})$$

$$\hat{V}^{(\mu=2,c)}(r) = \hat{V}^{(\nu=2,c)}(r) + (D_2 - D_{14})p_0^4\delta^{(3)}(\boldsymbol{r})$$

Two extra terms below NLO ($\nu = 2$). We largely recovered the phenomenological interaction at $\mu = 3/2$.

Very preliminary (incomplete) results

- **LO** ($\mu = 0$), $c = (T = 1, S = 0, L = 0)$, ^{6}He, E in MeV and Γ in keV, LECs not renormalized.

r_c (fm)	$C_S - 3C_T$	$E(0^+)$	$E(2^+)$	$\Gamma(2^+)$
0.6	-600	-0.987	1.329	708
0.8	-795	-0.977	1.250	549
1.0	-1010	-0.984	1.172	422
1.2	-1250	-0.967	1.098	327
1.4	-1525	-0.965	1.029	253
1.6	-1835	-0.969	0.969	198
1.8	-2180	-0.975	0.912	159
		-0.972	0.824	113

But for $r_c = 1.2$ fm:

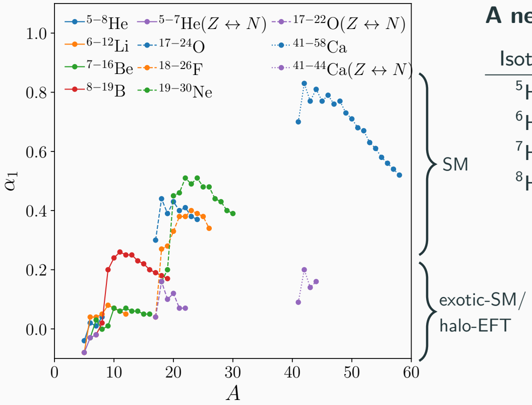
	E_{th}	E_{exp}	Γ_{th}	Γ_{exp}
^{7}He	$E(3/2^-) = -0.186$	-0.527	$\Gamma(3/2^-) = 144$	150
^{7}He	$E(1/2^-) = 0.785$	$\sim$ 0.84	$\Gamma(3/2^-) = 2777$	$\sim$ 2150
^{8}He	$E(0^+) = -2.592$	-3.10	$\Gamma(0^+) < 0$	
^{8}He	$E(2^+) = 0.723$	0.0	$\Gamma(2^+) = 1002$	600

- **N2LO** ($\mu = 3/2$): $r_c = 1.2$ fm, $C_S - 3C_T = -1137$, $C_2 - 3C_4 = 742$, $C_7 = 365$, $p_0 = 0.9886$. ^{8}He only at 2p2h.

	E_{th}	E_{exp}	Γ_{th}	Γ_{exp}
^{6}He	$E(0^+) = -1.05$	-0.972		
^{7}He	$E(3/2^-) = -0.427$	-0.527	$\Gamma(3/2^-) = 188$	150
^{8}He	$E(0^+) = -3.03$	-3.10	$\Gamma(0^+) < 0$	
^{8}He	$E(2^+) = 0.210$	0.0	$\Gamma(2^+) = 584$	600

The interaction is still being implemented and tested in the *sp* space (continuum). Only ^{6}He g.s. optimized.

The problem of the core: scales in the shell-model picture



A new shell-model scale at the drip lines?

Isotope	α_1	Isotope	α_1	Isotope	α_1
^{5}He	-0.04	^{15}C	0.08	^{25}O	-0.05
^{6}He	0.02	^{16}C	0.19	^{26}O	-0.02
^{7}He	0.01	^{17}C	0.14		
^{8}He	0.04	^{18}C	0.18		
		^{19}C	0.15		
		^{20}C	0.16		
		^{21}C	0.14		
		^{22}C	0.12		

By design $\alpha_1 \propto E/A$ for large A.

Data from the ENSDF database.