Bayesian analysis for extracting properties of the nuclear EoS models

Hovik Grigorian

Computational Physics and IT Division,
A. Alikhanyan National Science Laboratory (AANL)
Yerevan State University

ECT* Trento Workshop on
"The first compact star merger event – Implications for nuclear and particle physics"
18 October 2019, Trento

The research was carried out under financial support of the Russian Science Foundation (project #17-12-01427)

Collaborators

Alexander Ayriyan (LIT & AANL),
David Blaschke (UW & JINR & MEPhI),
Dmitry Voskresensky (MEPhI & JINR),
Konstantin Maslov (MEPhI & JINR)
David-Edwin Alvarez-Castillo (JINR),
Nobutoshi Yasutake (Chiba Institute of Technology),
Vahagn Abgaryan (JINR & AANL)

Bayesian analysis for extracting properties of the nuclear EoS models

- Outline

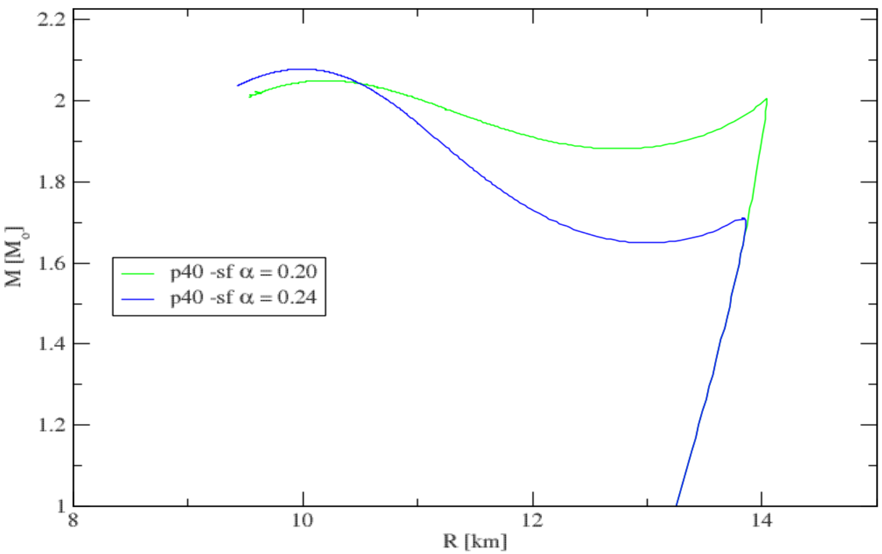
  I. Motivation

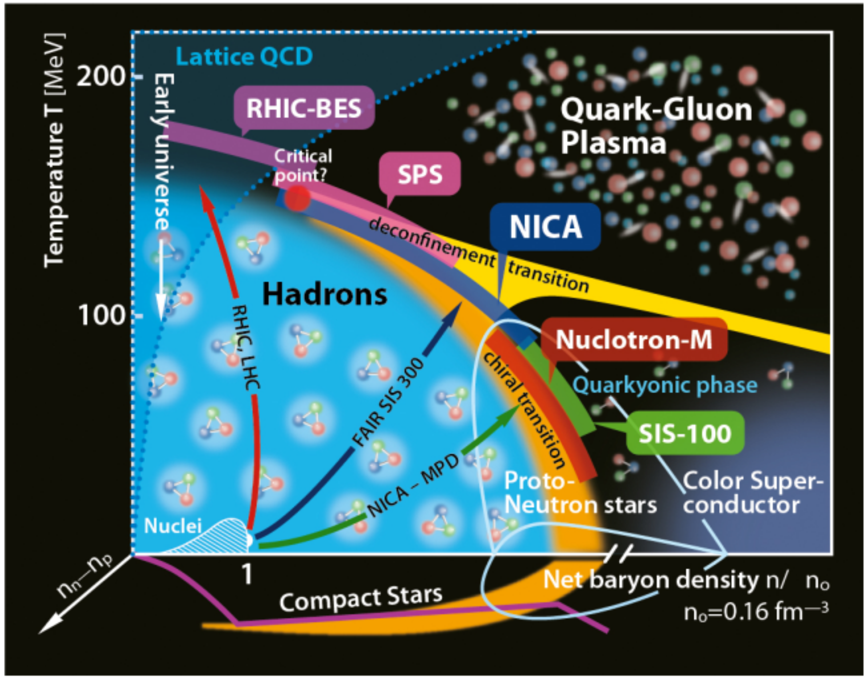
  II. Mixed phase construction for cold and dense nuclear matter

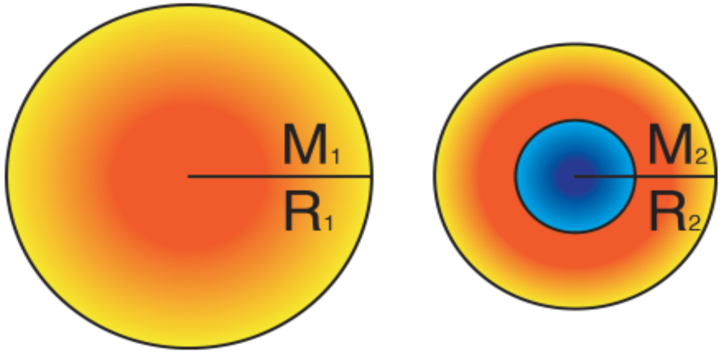
  III. Bayesian analysis for extracting properties of the nuclear equation of state from observational data

  IV. Conclusions

Motivation : What if we have twins ?

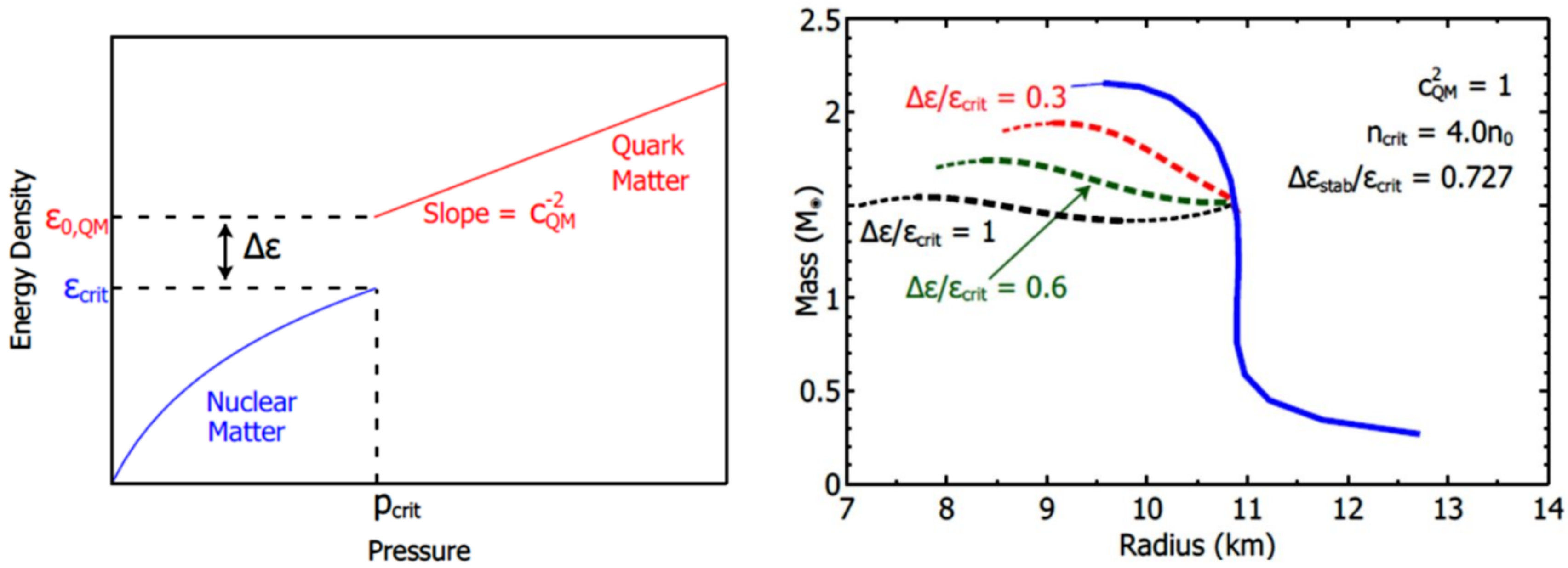






- Does hybrid neutron star exist?
- Does NS twin exist?
- Does CEP exist on QCD phas diagram?

Neutron star mass-radius relation



Seidov criterion for instability:

$$\frac{\Delta\varepsilon}{\varepsilon_{crit}} \geq \frac{1}{2} + \frac{3}{2}\frac{P_{crit}}{\varepsilon_{crit}}$$

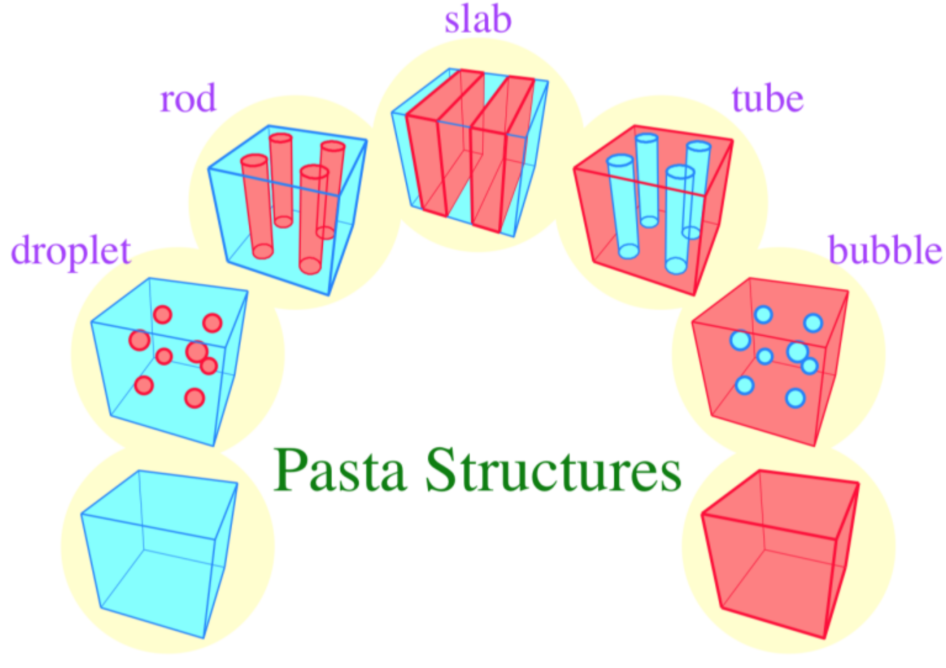
Credit: Mark G. Alford, Sophia Han, and Madappa Prakash. Phys. Rev. D 88, 083013 (2013)

Finite-size effects in mixed phase

Coulomb interaction **vs** **Surface tension**

Coulomb interaction: Tends to break up the like-charged regions into smaller ones

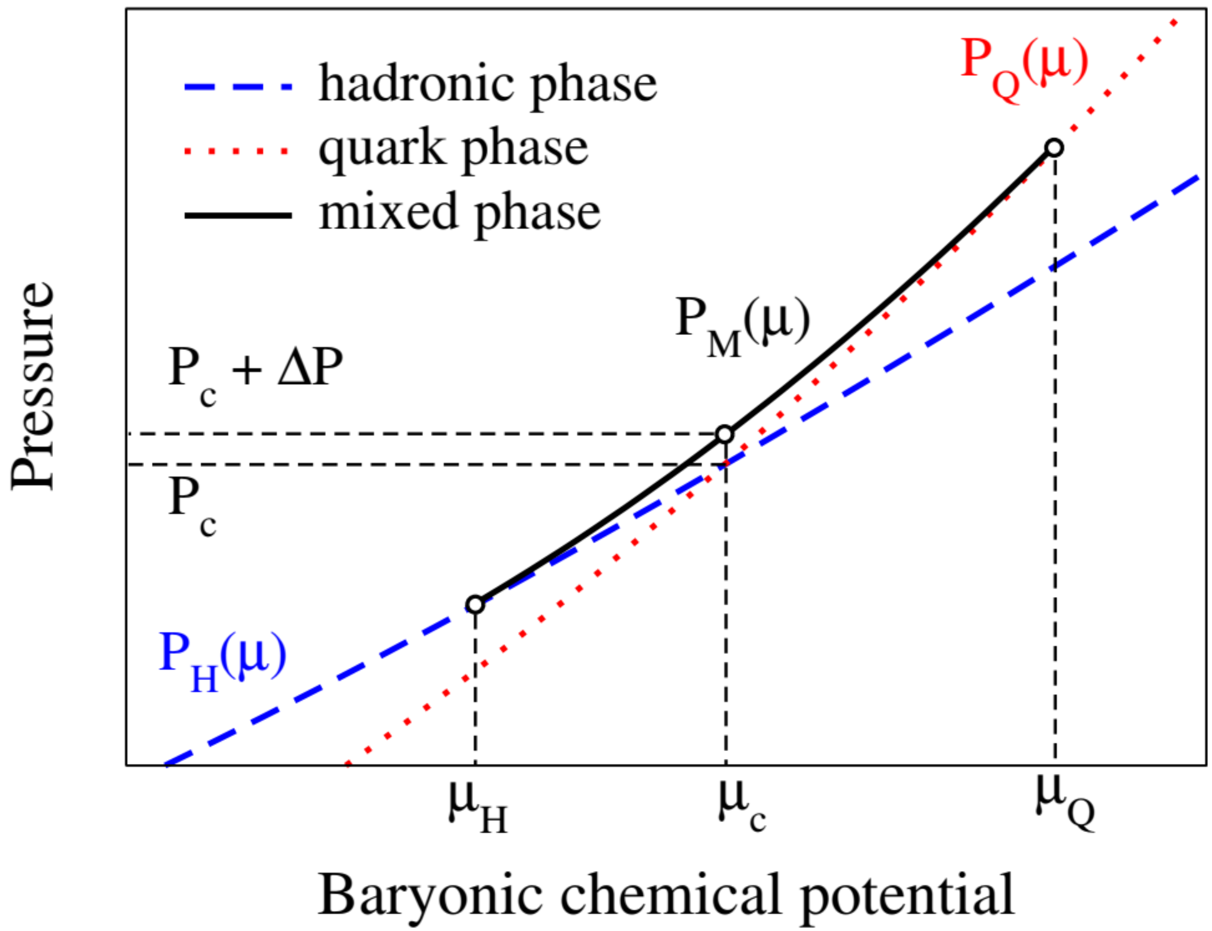
Surface tension: Requires minimization of the surface



The surface tension σ is unknown and used as free parameter.

Yasutake, Maruyama, Tatsumi, Phys. Rev. D80 (2009) 123009

Mimicking the Pasta phase.



Schematic representation of the interpolation function $P_M(\mu)$, it has to go though three points: $P_H(\mu_H)$, $P_c + \Delta P$ and $P_Q(\mu_Q)$.

The Interpolation Method

$$P_M(\mu) = \sum_{q=1}^{N} \alpha_q (\mu - \mu_c)^q + (1 + \Delta_P) P_c$$

where Δ_P is a free parameter representing additional pressure of the mixed phase at μ_c.

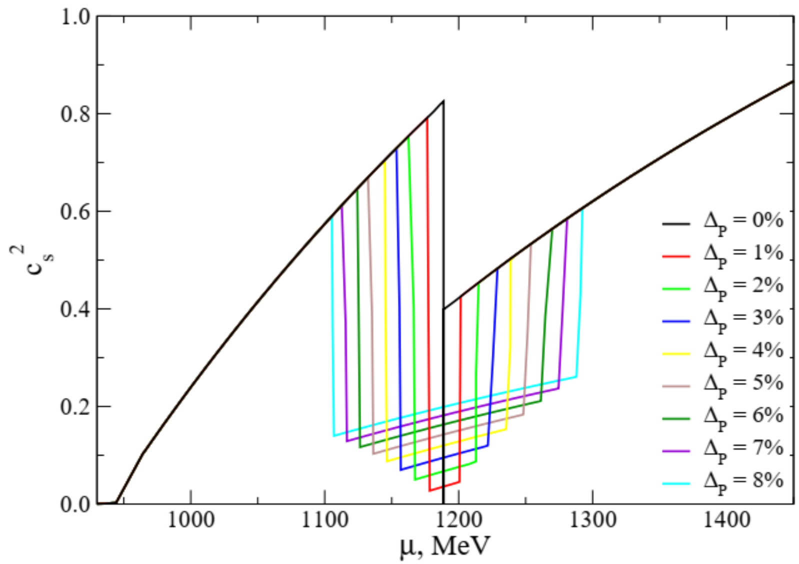
$$P_H(\mu_H) = P_M(\mu_H) \qquad P_Q(\mu_Q) = P_M(\mu_Q)$$

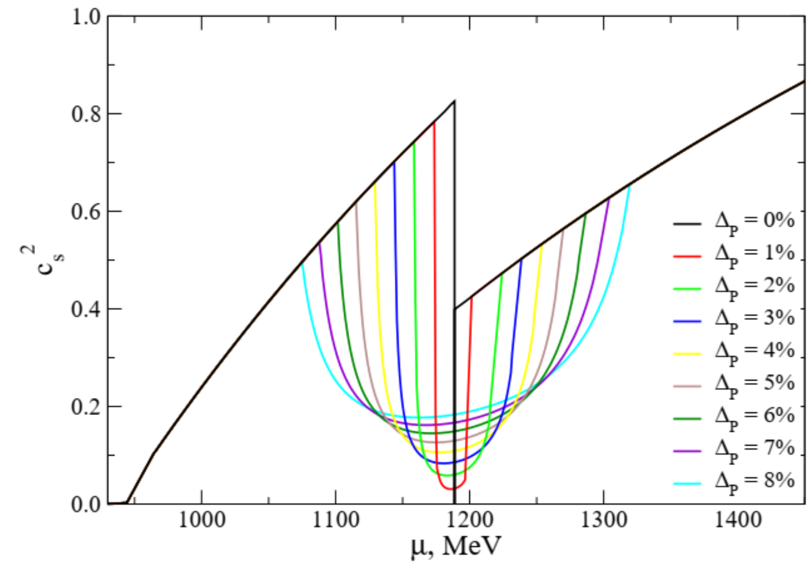
$$\frac{\partial^q}{\partial \mu^q} P_H(\mu_H) = \frac{\partial^q}{\partial \mu^q} P_M(\mu_H) \qquad \frac{\partial^q}{\partial \mu^q} P_Q(\mu_Q) = \frac{\partial^q}{\partial \mu^q} P_M(\mu_Q)$$

where $q = 1, 2, \ldots, k$. All $N + 2$ parameters (μ_H, μ_Q and α_q, for $q = 1, \ldots, N$) can be found by solving the above system of equations, leaving one parameter (ΔP) as a free one.

Ayriyan and Grigorian, *EPJ Web Conf.* **173**, 03003 (2018)
Abgaryan, Alvarez-Castillo, Ayriyan et al. *Universe* **4(9)**, 94 (2018)

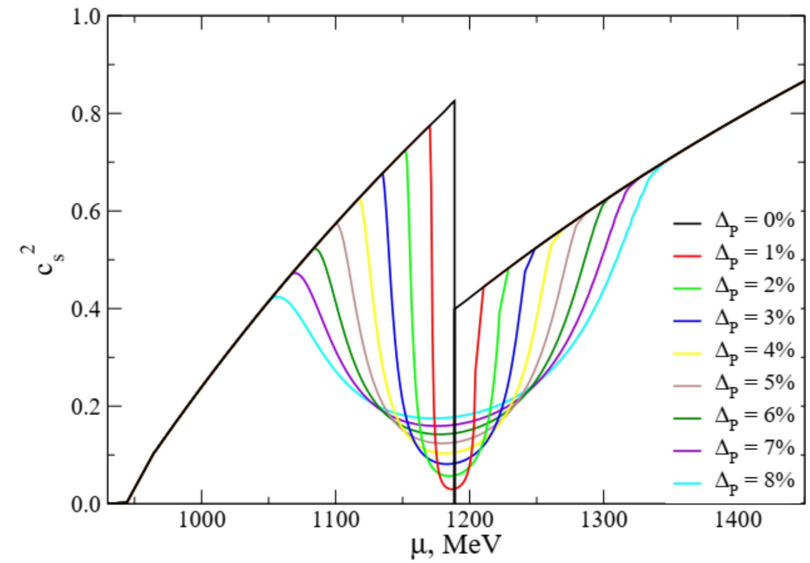
The Interpolation Method



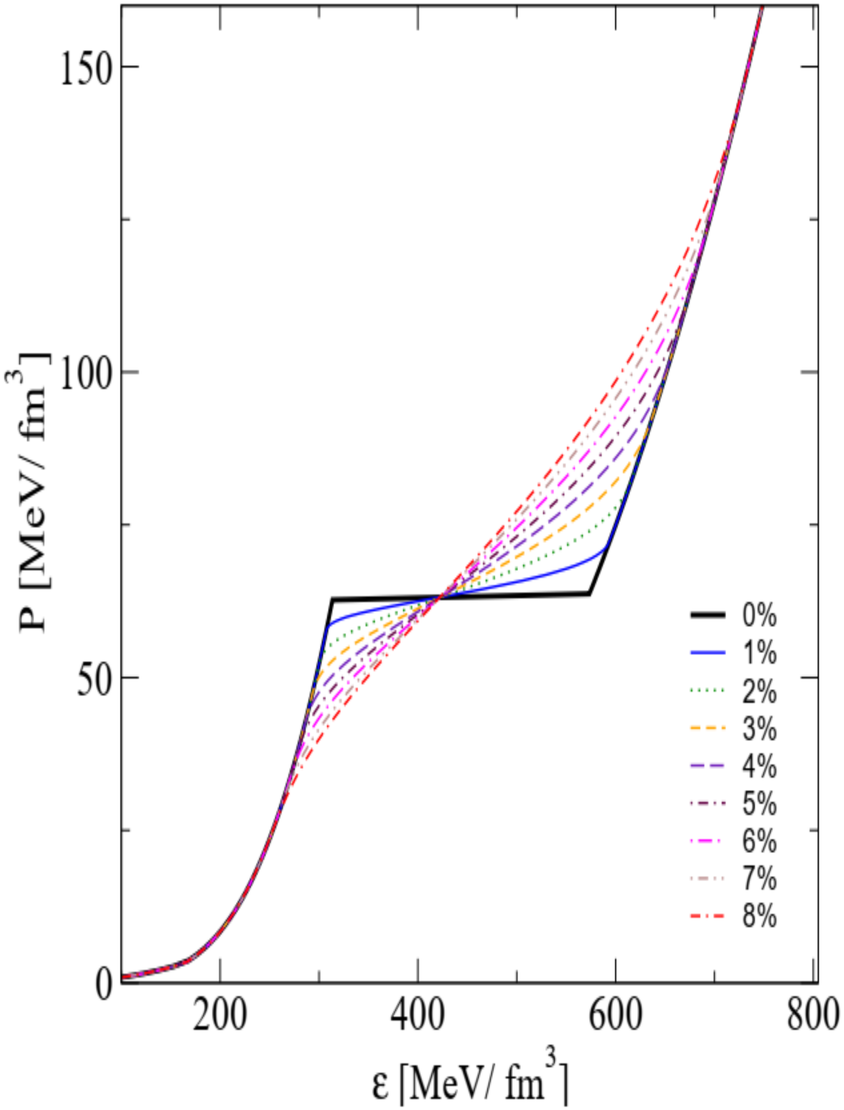


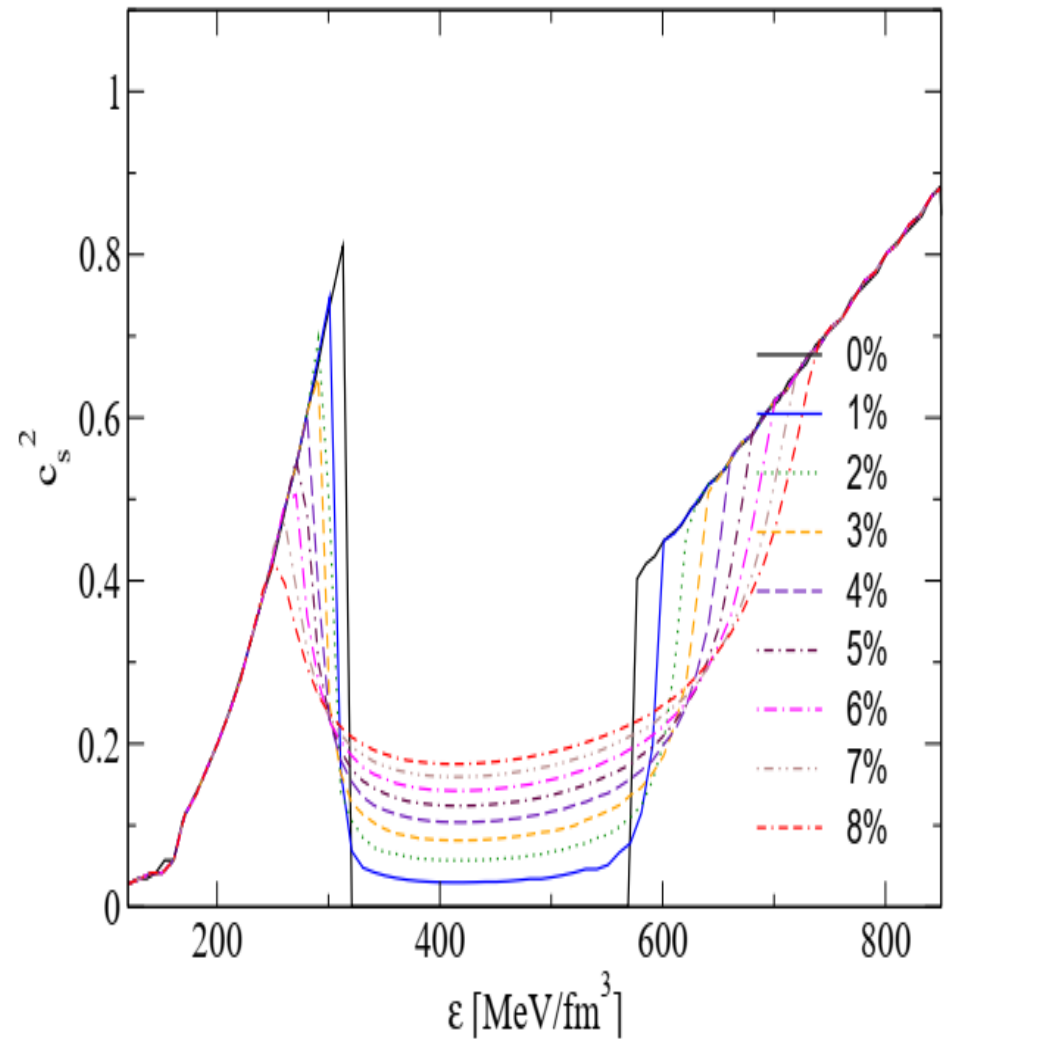
The squared speed vs chemical potential given by the interpolation with $k = 1$ (upper left) $k = 2$ (upper right) and $k = 3$ (right).

Abgaryan, Alvarez-Castillo, Ayriyan, Blaschke and Grigorian. Universe 4(9) (2018), 94

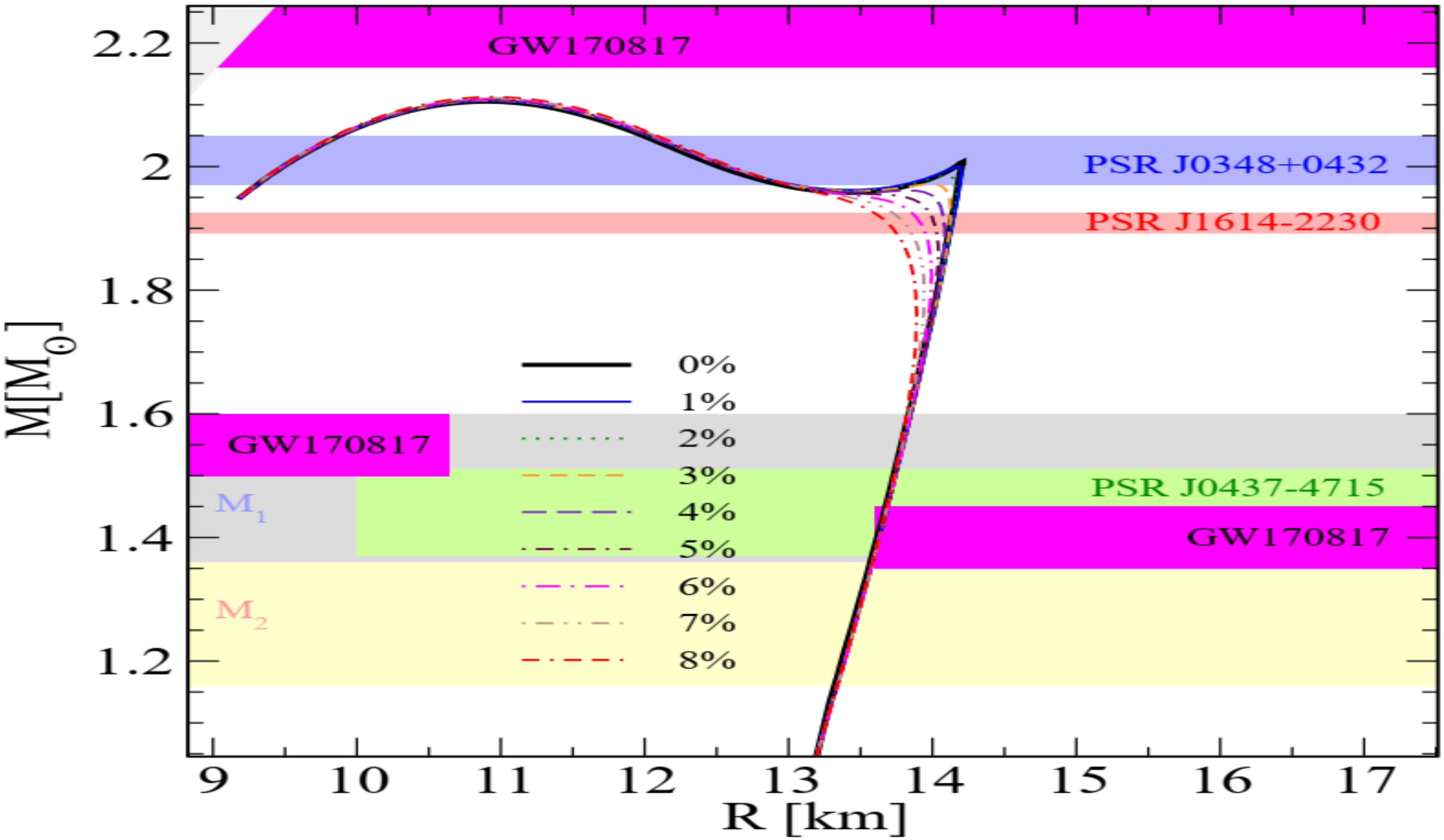


The results of pasta mimicking





The results of pasta effects

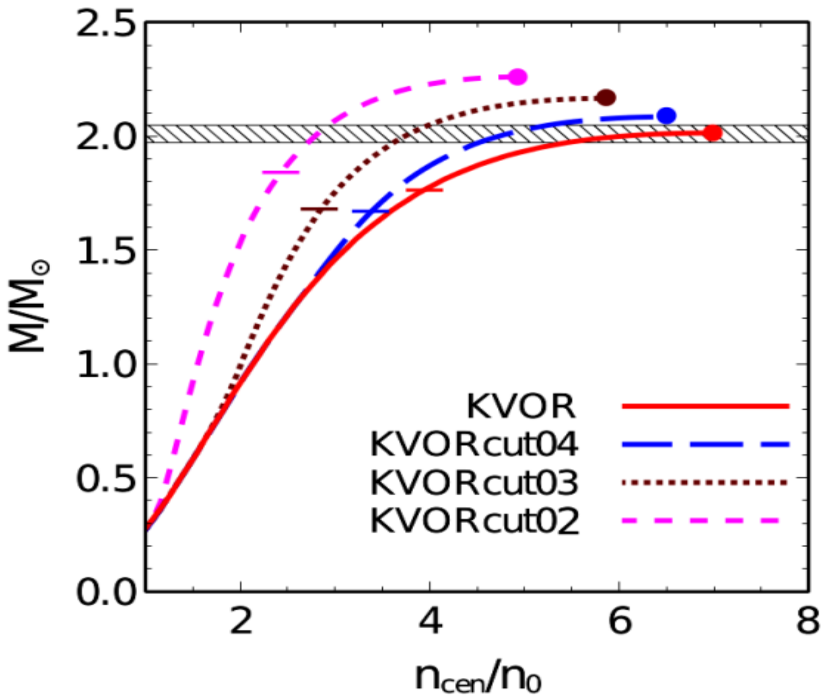


Third family robust against Δ_P up to around 5%!

Abgaryan, Alvarez-Castillo, Ayriyan et al. Universe 4(9), 94 (2018)

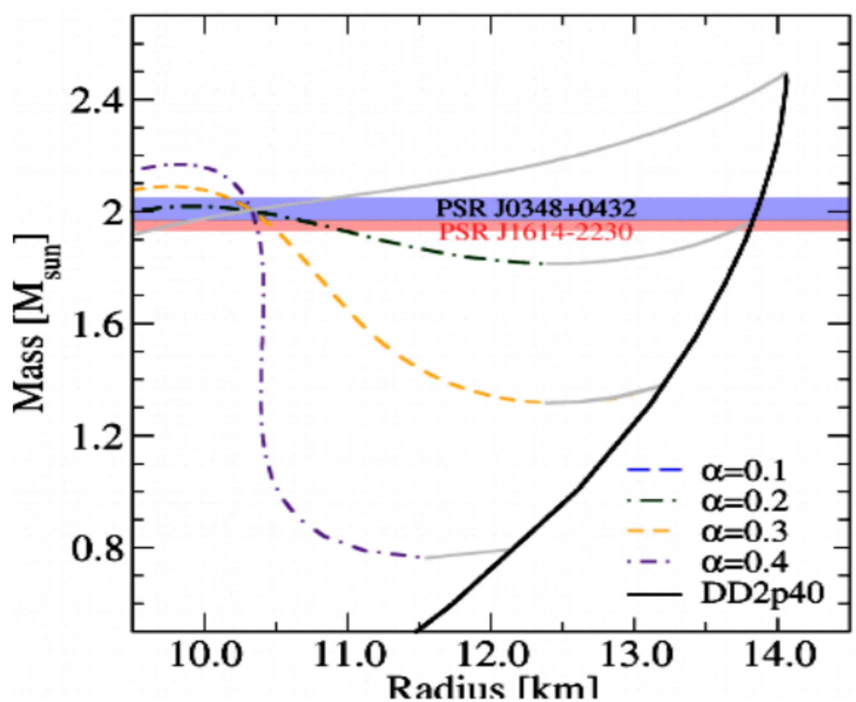
The realistic hadron and quark matter models

The hadron EoS model KVOR with modification of stiffness



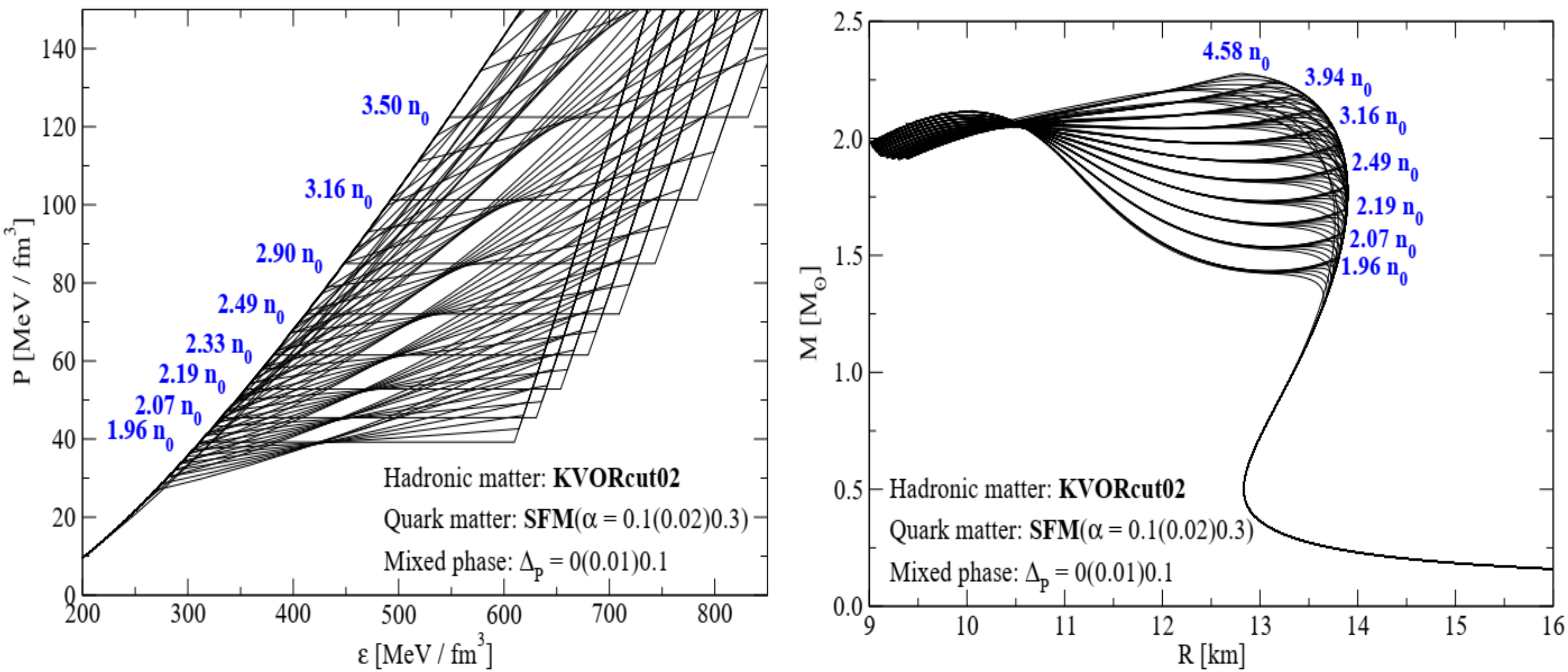
Maslov, Kolomeitsev, Voskresensky, Nucl.Phys. A950 (2016)
Kolomeitsev & Voskresensky, Nuc. Phys. A 759 (2005)

The quark EoS model SFM with available volume fraction parameter



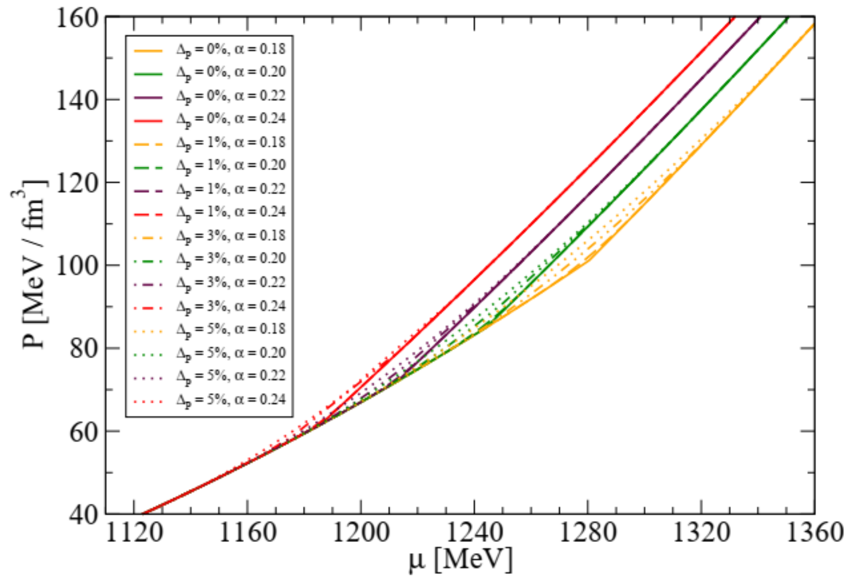
Kaltenborn, Bastian, Blaschke, Phys. Rev. D 96, 056024 (2017)

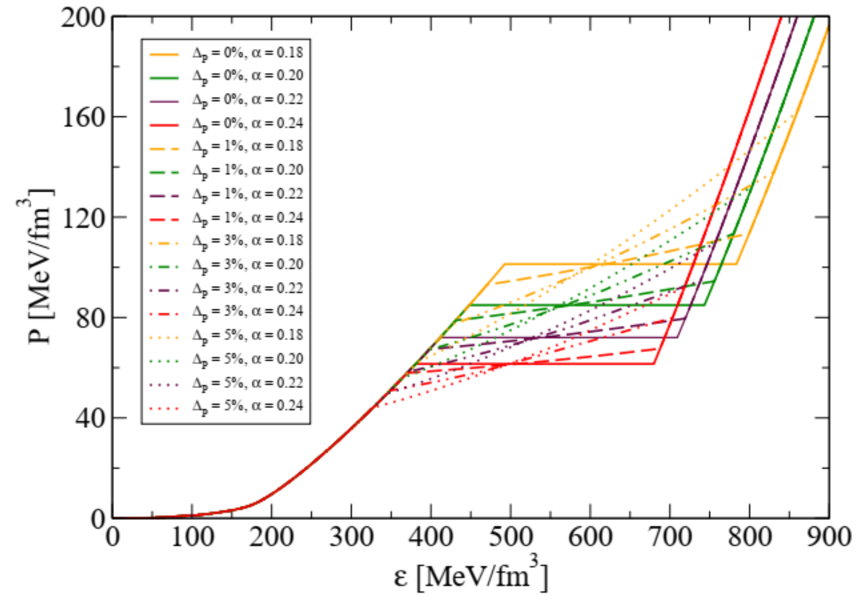
Robustness of third family solutions

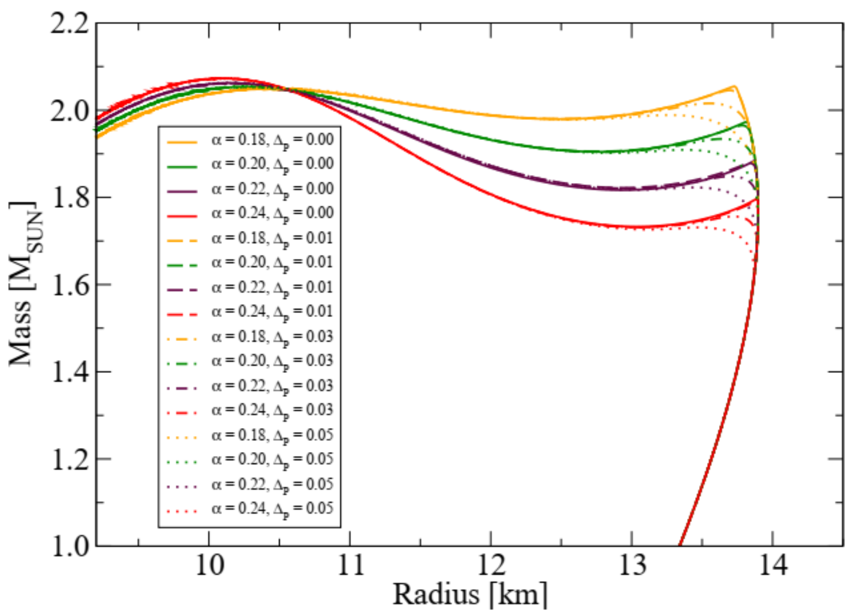


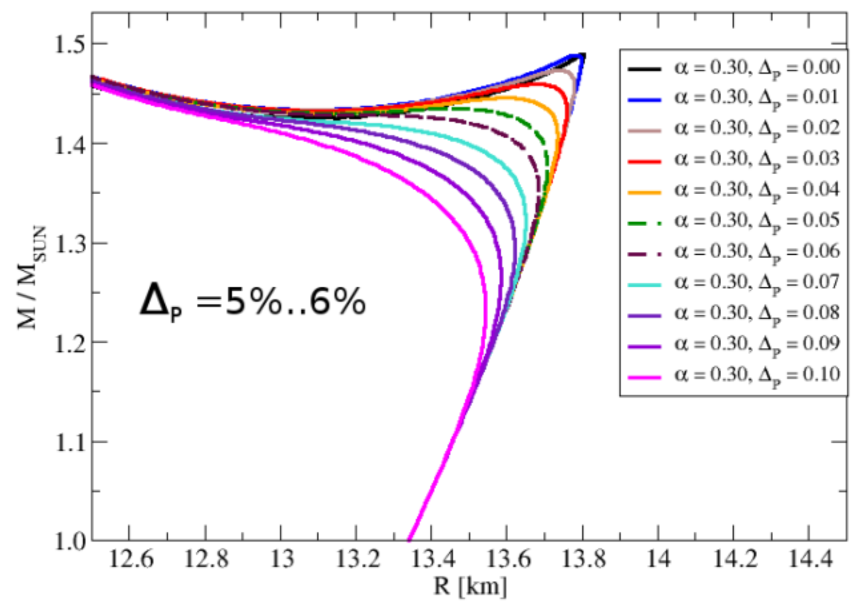
Ayriyan, Bastian, Blaschke, Grigorian, Maslov, Voskresensky. PRC 97, 045802 (2018)

Robustness of third family solutions

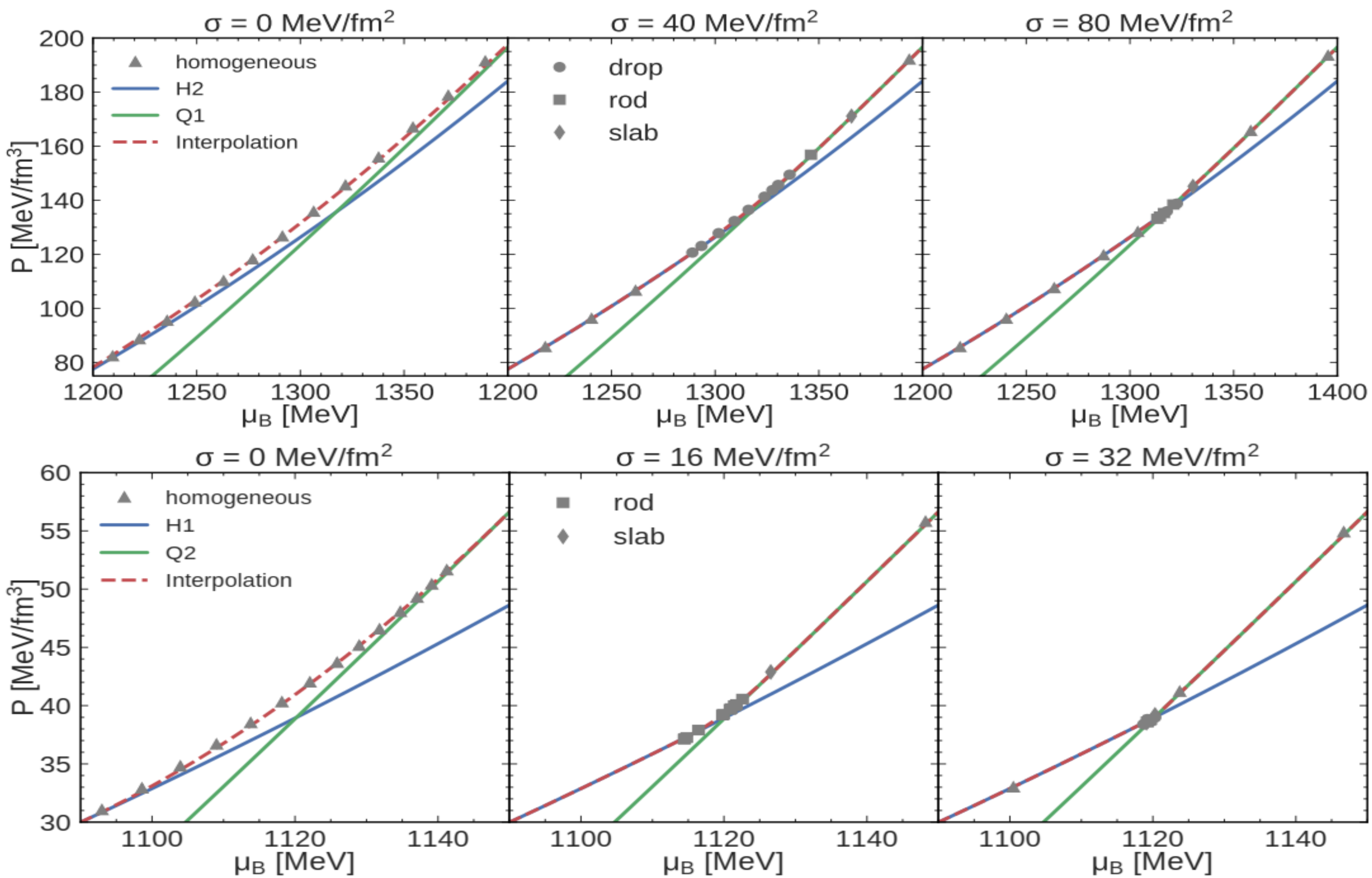




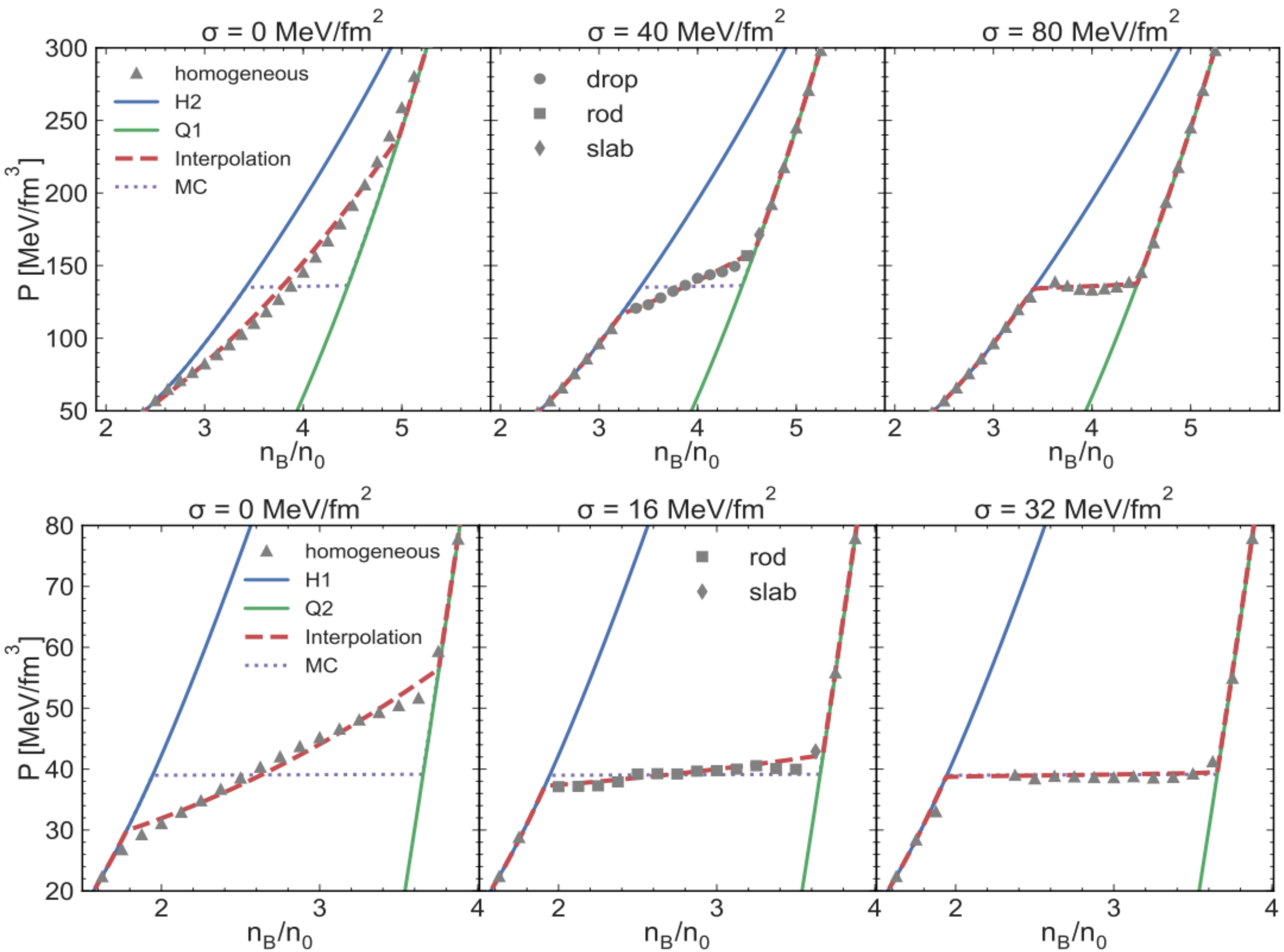




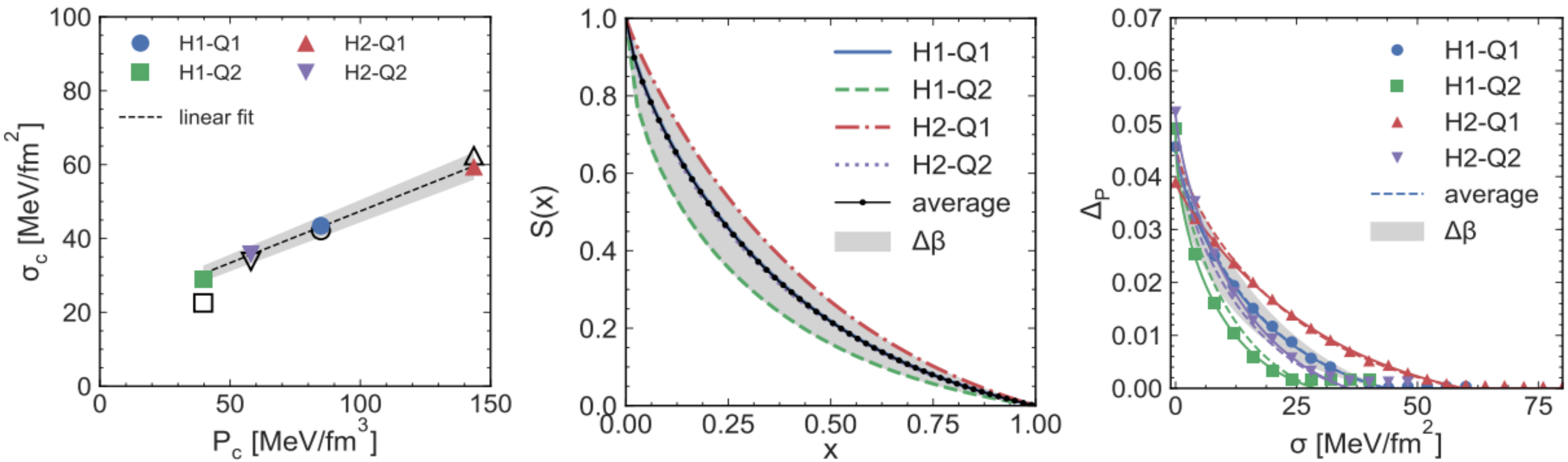
Dependence on surface tension



Dependence on surface tension



Dependence on surface tension



$\sigma_c = d\,(P_c - P_0) + \sigma_0$: $\quad d = 0.45 \pm 0.02$ fm,

$P_0 = 40$ MeV/fm^3,

and $\sigma_0 = 31.6 \pm 1.19$ MeV/fm^2

$\Delta_P(\sigma) = \Delta_P(0) S(\sigma/\sigma_c;\beta)$: $\quad \overline{\beta} = 0.64$

$S(x;\beta) = e^{-x}(1 - x^{\beta})\theta(1 - x)$

Maslov, Yasutake, Blaschke, Ayriyan, Grigorian, Maruyama, Tatsumi, Voskresensky. PRC100, 025802 (2019)

Bayesian method

Bayesian analysis is a statistical paradigm that shows the most expected hypotheses using probability statements and current knowledge.
One of the most frequent case is analysis of probable values of model parameters.

Bayes' theorem:

Likelihood Prior

$$p(H_1 \mid D, I) = \frac{p(D \mid H_1, I)\, p(H_1 \mid I)}{p(D \mid I)}$$

Posterior Evidence

Prior: knowledge before experiment (logically)
Likelihood: Probability for data if the hypothesis was true
Posterior: Probability that the hypothesis is true given the data
Evidence: normalization; important for model comparison

Generally, maximum likelihood (parameters which maximize the probability for data) **does not** give the most likely parameters!!!

Bayesian method

Formulation of set of models (set of hypothesis):
π_i here $i = 0..N-1$

↓

Finding the *a priori* probabilities of the models:
$P(\pi_i) = 1/N \quad \text{for} \quad \forall i = 0..N-1$

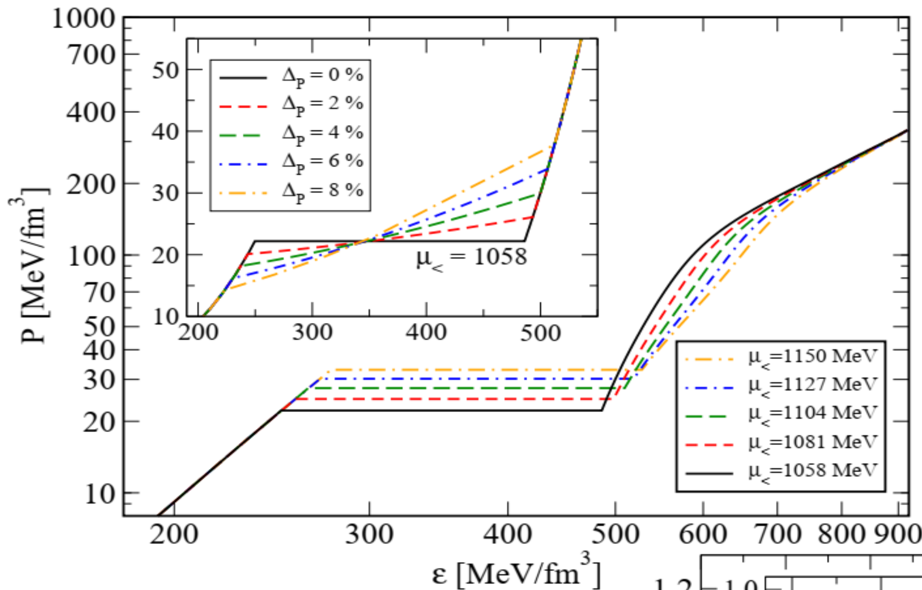
↓

Calculating the coditional probabilities of the events:
$P(E\,|\overrightarrow{\pi}_i) = \prod_\alpha P(E_\alpha\,|\overrightarrow{\pi}_i),$
where α is the index of the observational constraints.

↓

Calculating the *a posteriori* probabilities of the models:
$$P(\overrightarrow{\pi}_i\,|E) = \frac{P(E|\overrightarrow{\pi}_i)P(\overrightarrow{\pi}_i)}{\sum_{j=0}^{N-1} P(E|\overrightarrow{\pi}_j)P(\overrightarrow{\pi}_j)}$$

Model EoS for Hybrid NS

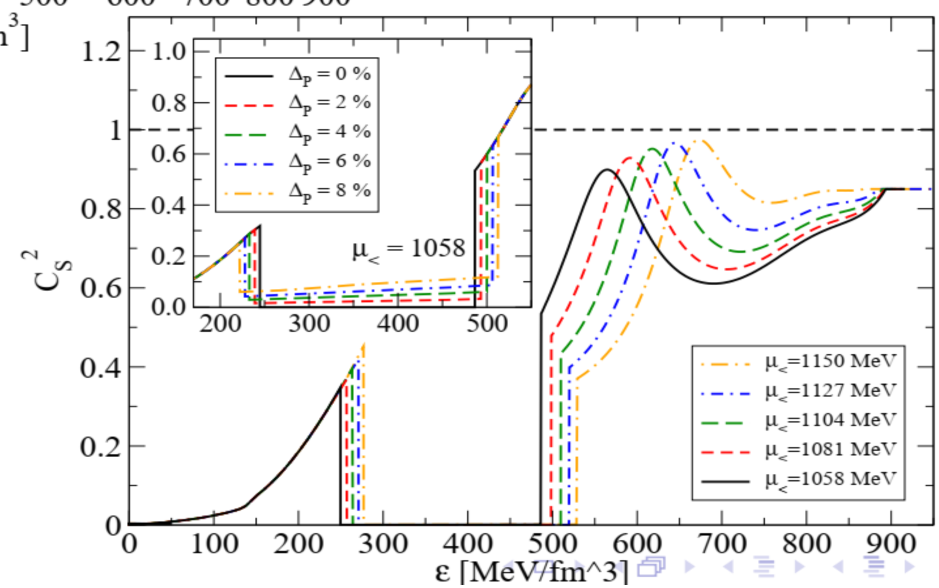


DD2 with excluded volume plus color superconducting two-flavor quark matter, described within a nonlocal covariant chiral quark model.

$$P(\mu) = P(\mu; \eta(\mu), B(\mu)) = -\Omega^{MFA}(\eta(\mu)) - B(\mu)$$

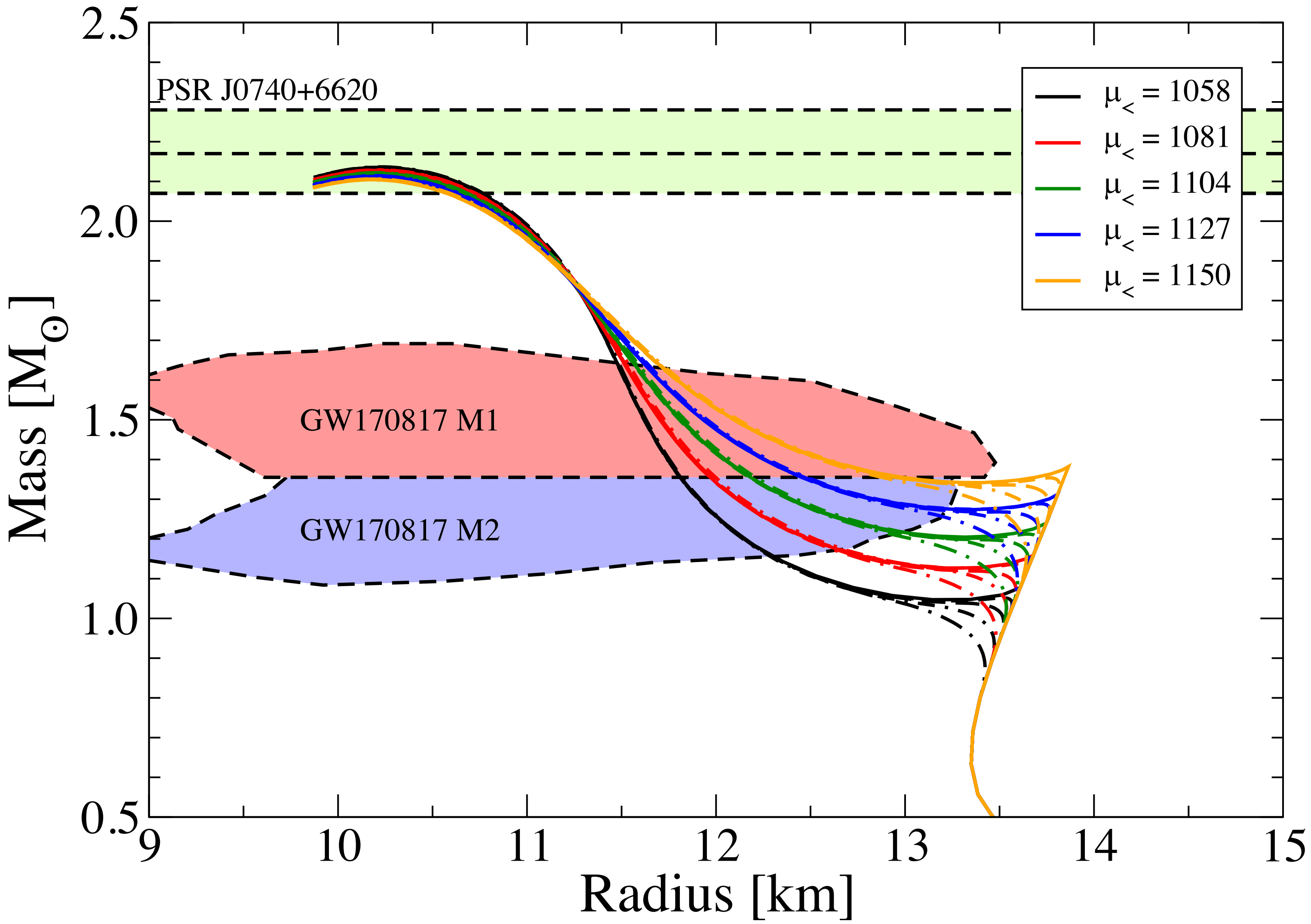
$$\Omega^{MFA} = \frac{\bar{\sigma}^2}{2G_S} + \frac{\bar{\Delta}^2}{2H} - \frac{\bar{\omega}^2}{2G_V} - \frac{1}{2}\int \frac{d^4p}{(2\pi)^4} \ln\det\left[\, S^{-1}(\bar{\sigma}, \bar{\Delta}, \bar{\omega}, \mu_{fc})\right]$$

$$\frac{d\Omega^{MFA}}{d\bar{\Delta}} = 0\,, \quad \frac{d\Omega^{MFA}}{d\bar{\sigma}} = 0\,, \quad \frac{d\Omega^{MFA}}{d\bar{\omega}} = 0\,.$$

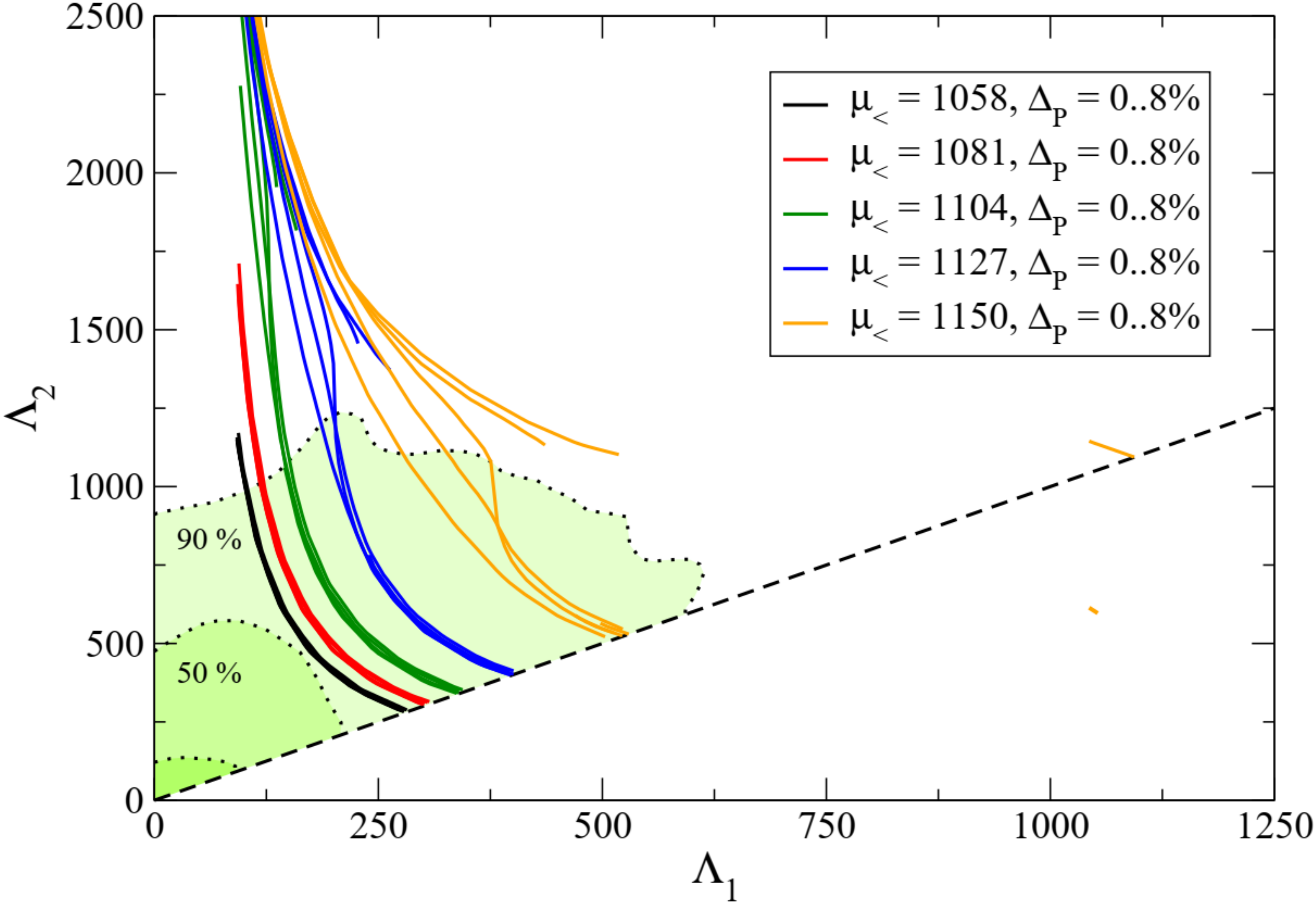


Alvarez-Castillo, Blaschke, Grunfeld, and Pagura, PRD99, 063010 (2019)

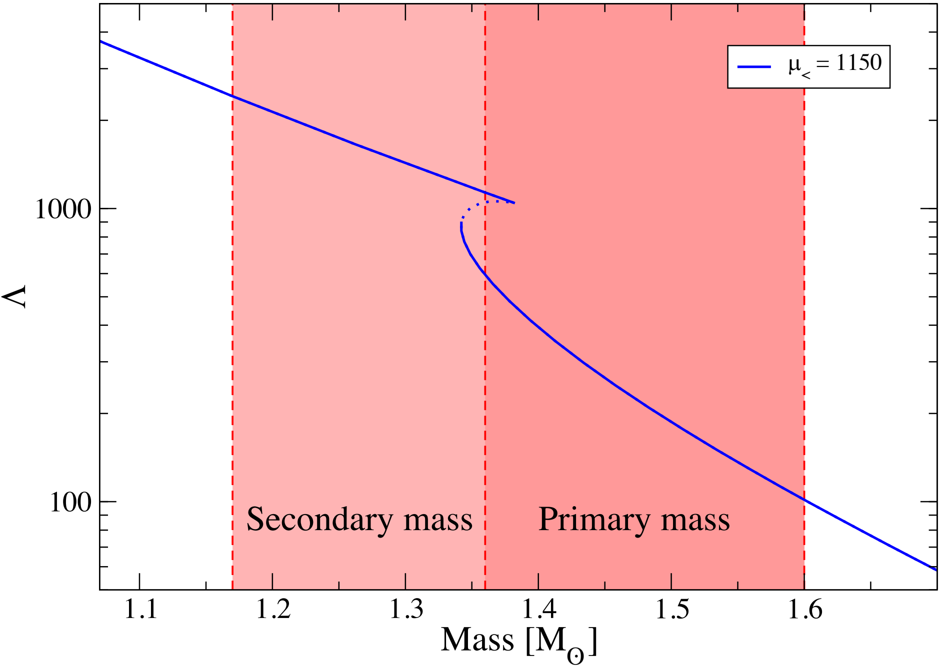
Stabile NS Configurations

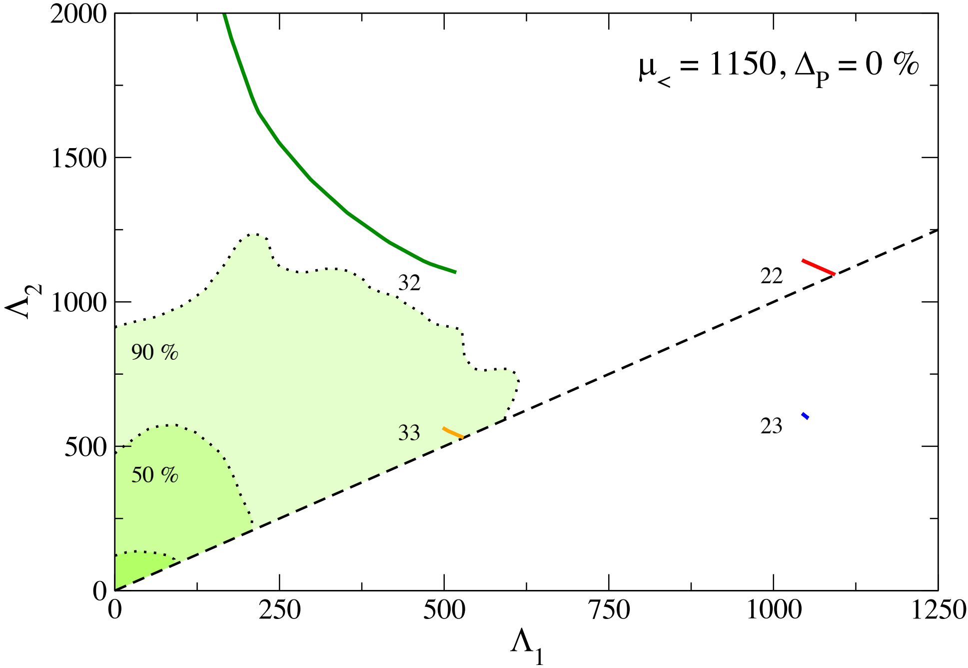


Λ_1- Λ_2 diagram and observational constraints



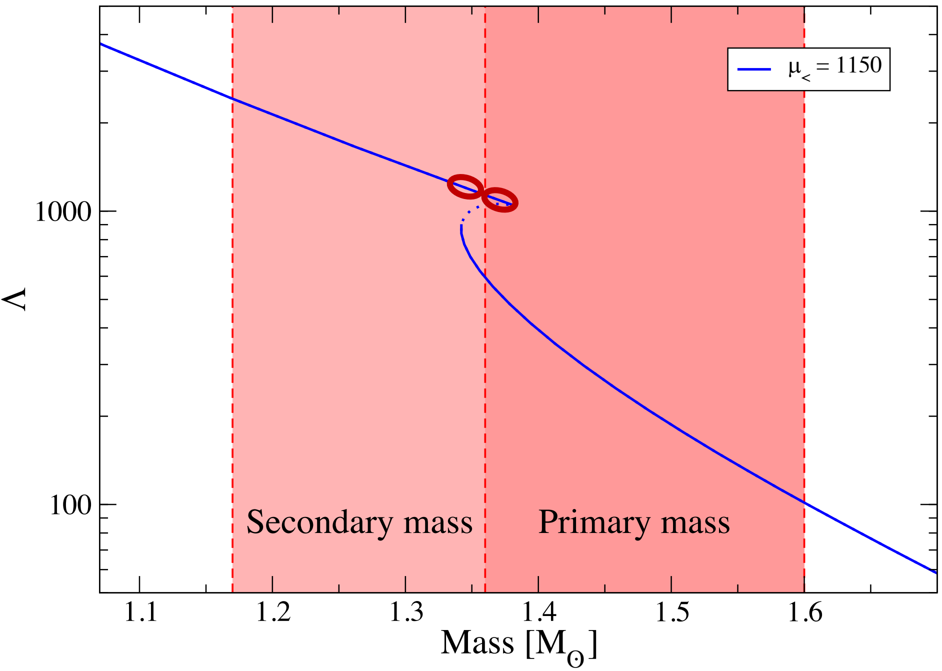
Lambda-Lambda diagram: Hybrid EoS NS – NS merging

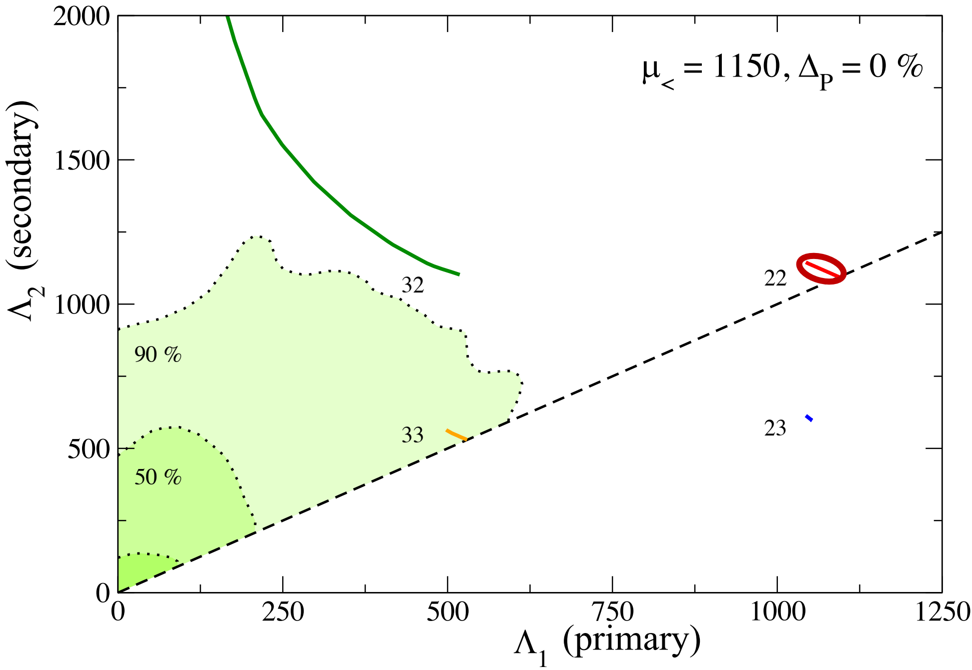




D. Alvarez-Castillo, D. Blaschke, G. Grunfeld, V. Pagura
Phys. Rev. D 99, 063010 (2019) - arXiv: 1805.04105

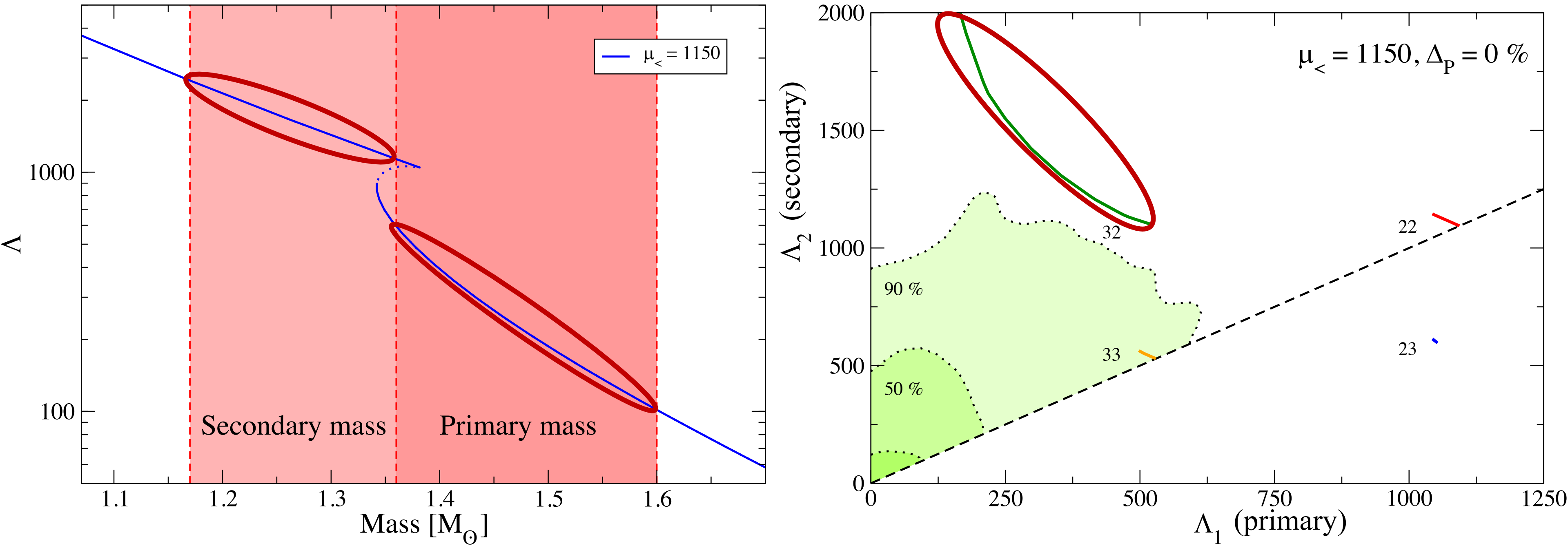
Hadron - Hadron





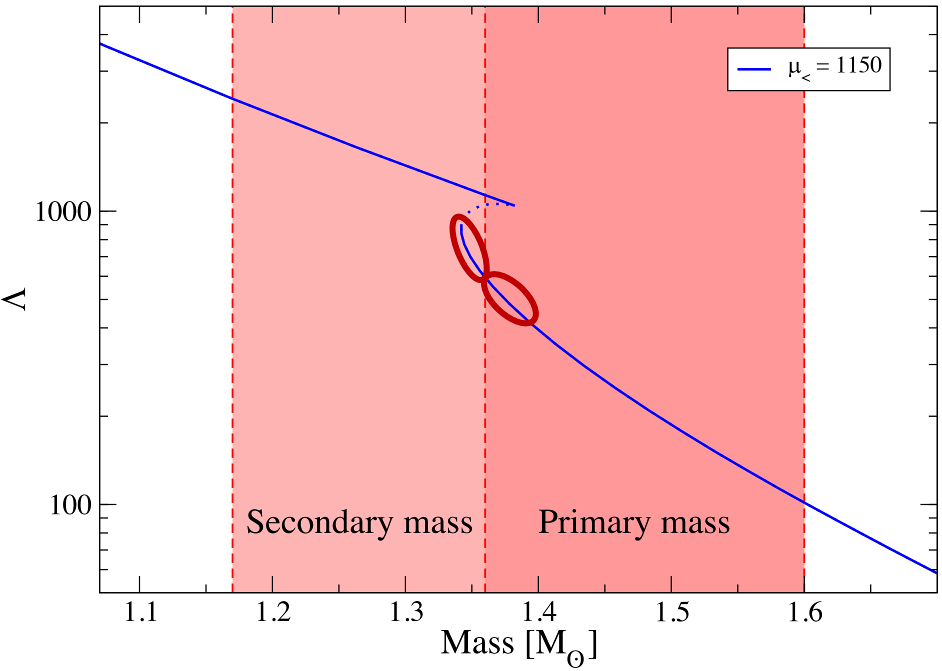
D. Alvarez-Castillo, D. Blaschke, G. Grunfeld, V. Pagura
Phys. Rev. D 99, 063010 (2019) - arXiv: 1805.04105

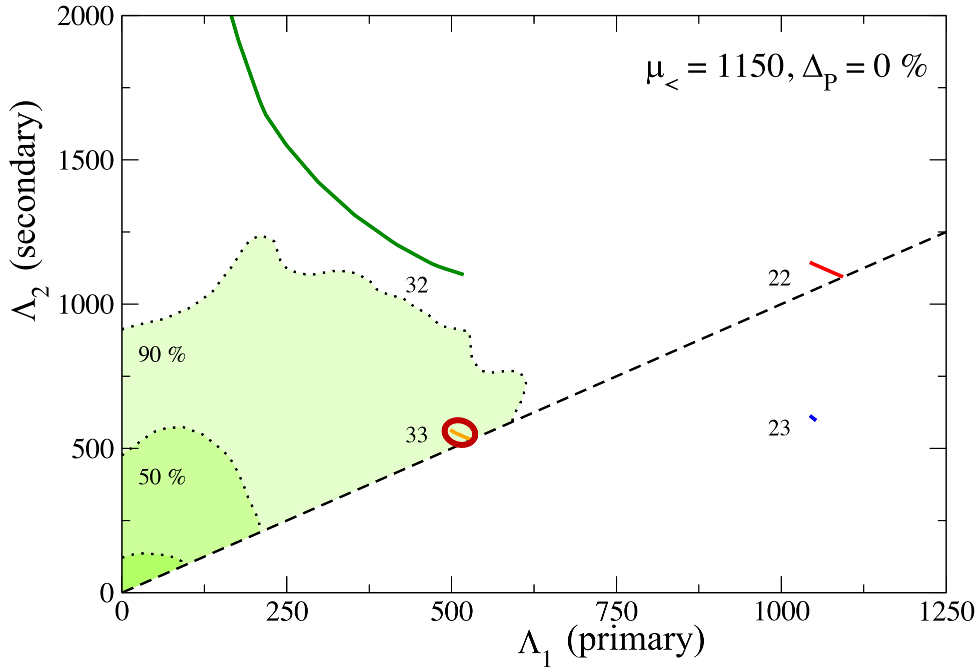
Hybrid - Hadron



D. Alvarez-Castillo, D. Blaschke, G. Grunfeld, V. Pagura
Phys. Rev. D 99, 063010 (2019) - arXiv: 1805.04105

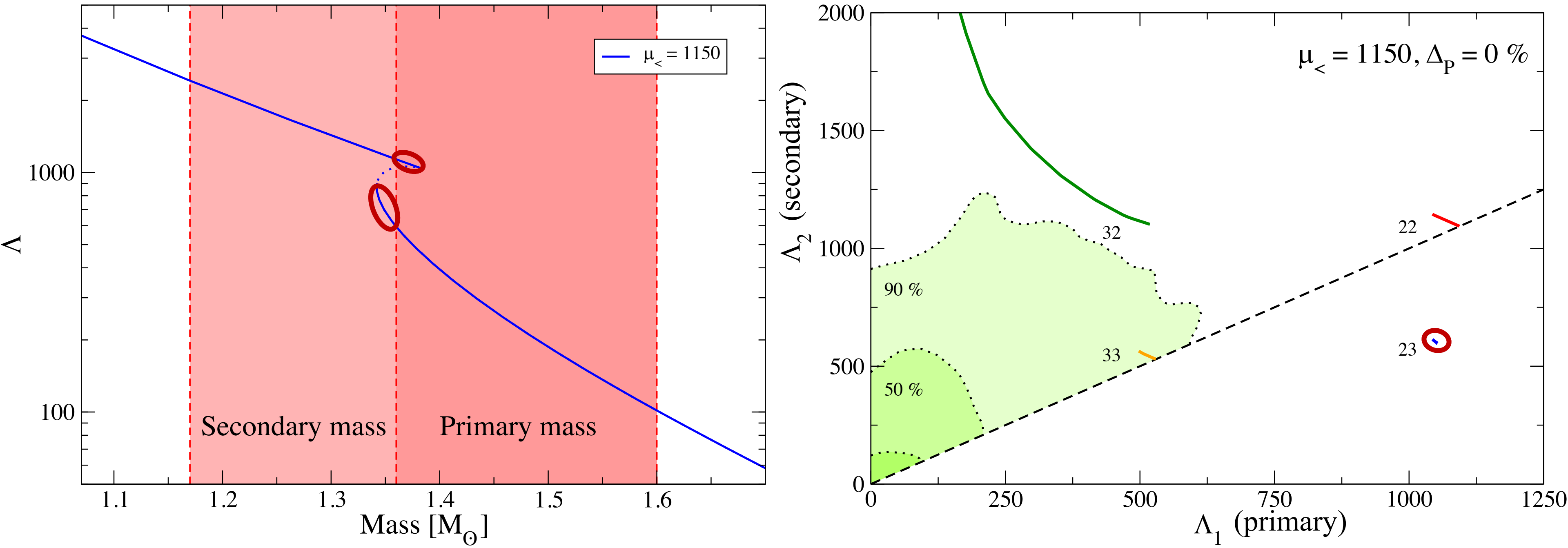
Hybrid - Hybrid





D. Alvarez-Castillo, D. Blaschke, G. Grunfeld, V. Pagura
Phys. Rev. D 99, 063010 (2019) - arXiv: 1805.04105

Hadron - Hybrid



The same phenomena were found in Montana, Tolos, Hanauske, Rezzolla. PRD99, 103009 (2019) for polytropic models

More interesting results have been achieved by Prof. Armen Sedrakian for triplet of compact stars produced by the fourth family.

The region $\Lambda_2 < \Lambda_1$ was called unphysical at Abbott *et al.* PRL121 (2018).

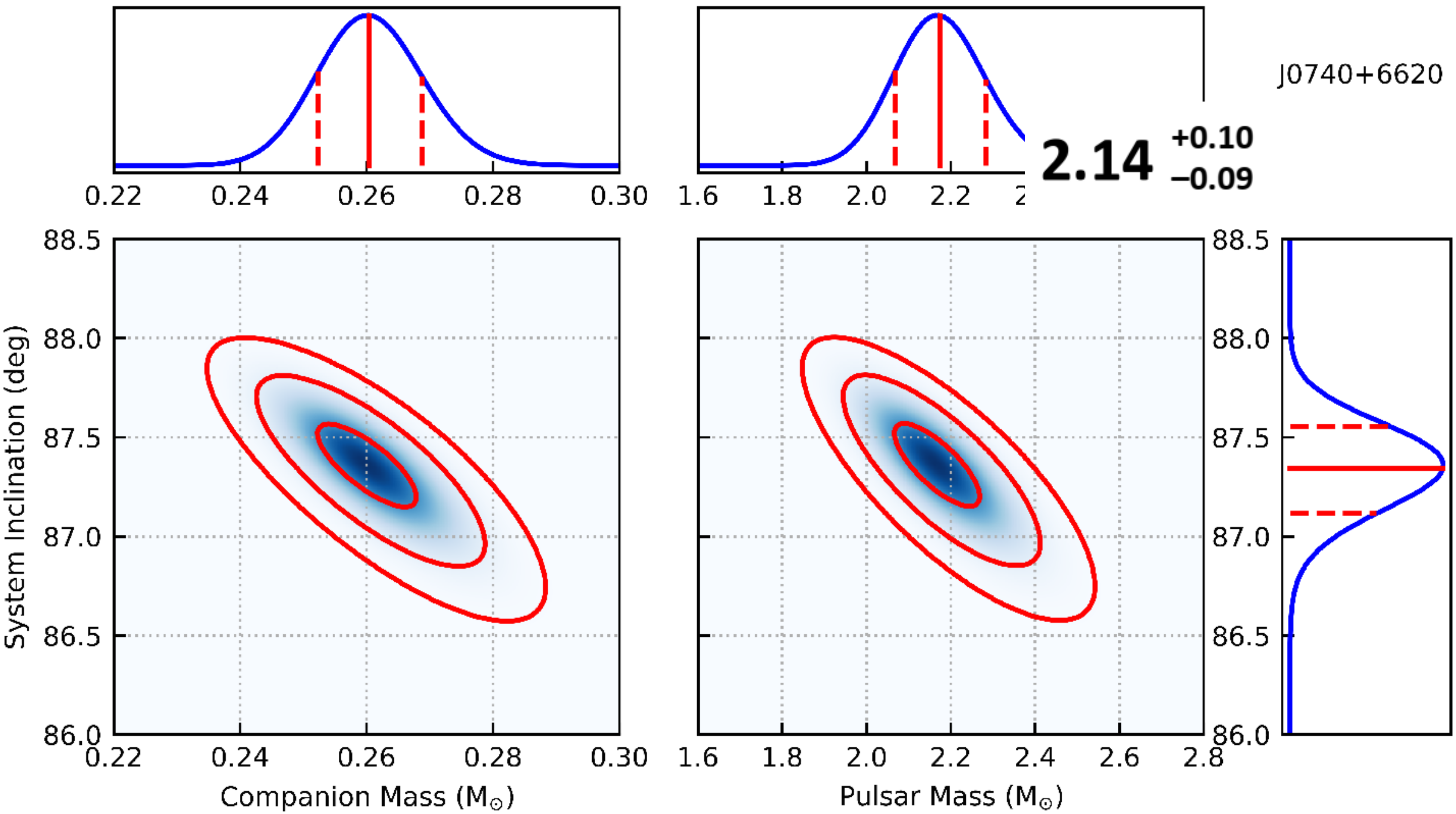
The parameter set for EoS model

The set of parameters of models could be represented in the parameter space with introduction of the vector of parameters, each vector is one fixed model from considered types of EoS model and transition construction:

$$\vec{\pi}_i = \left\{ \mu_{<(j)}, \Delta_{P(k)} \right\},$$

where $i = 0..N-1$ and $i = N_2 \times j + k$ and $j = 0..N_1 - 1$, $k = 0..N_2 - 1$ and N_1 and N_2 are number of values of model parameters $\mu_<$ and Δ_P correspondingly.

Mass constraint

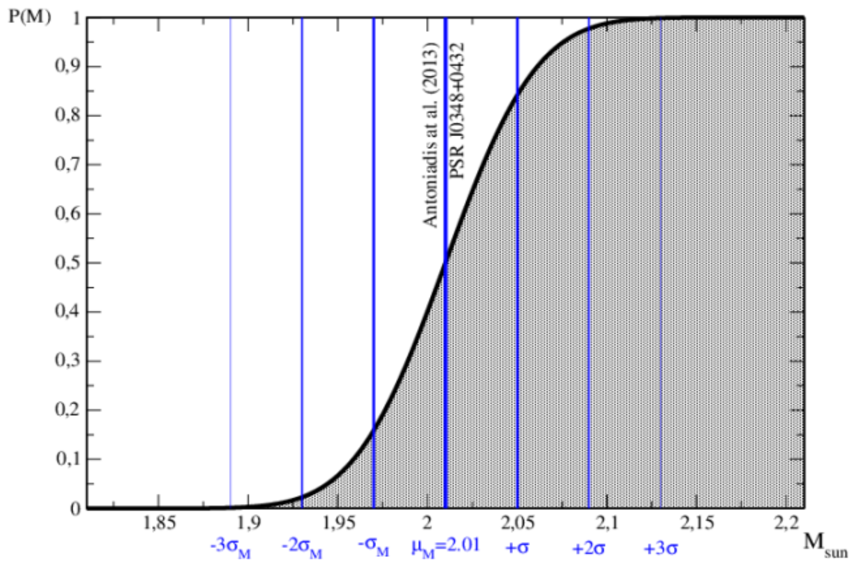


Cromartie et. al. Nature Astronomy (2019),
doi: 10.1038/s41550-019-0880-2

Likelihood of a EoS model for the mass constraint

$$P\left(E_M \left|\pi_i\right.\right) = \Phi(M_i, \mu_A, \sigma_A)$$

here M_i is maximum mass of the given by π_i, and $\mu_A =$ **2.14** $\mathrm{M}_\odot$ and $\sigma_A = 0.105\ \mathrm{M}_\odot$ is the mass measurement of PSR J0740+6620 $\mathbf{2.14}^{+0.10}_{-0.09}$ $\mathrm{M}_\odot$ [Cromartie *et al.*, arXiv:1904.06759 (2019)].

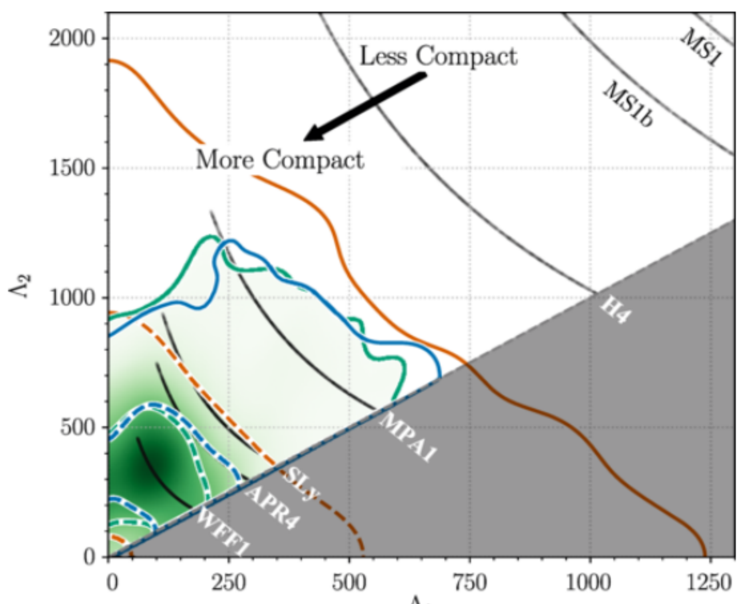


Note, that here we replace previously used mass measurement for two solar mass pulsar J0348+0432 $2.01^{+0.04}_{-0.04}\ \mathrm{M}_\odot$ [Antoniadis *et al.*, Science **340**, 6131 (2013)].

Likelihood for the Λ1-Λ2 constraint

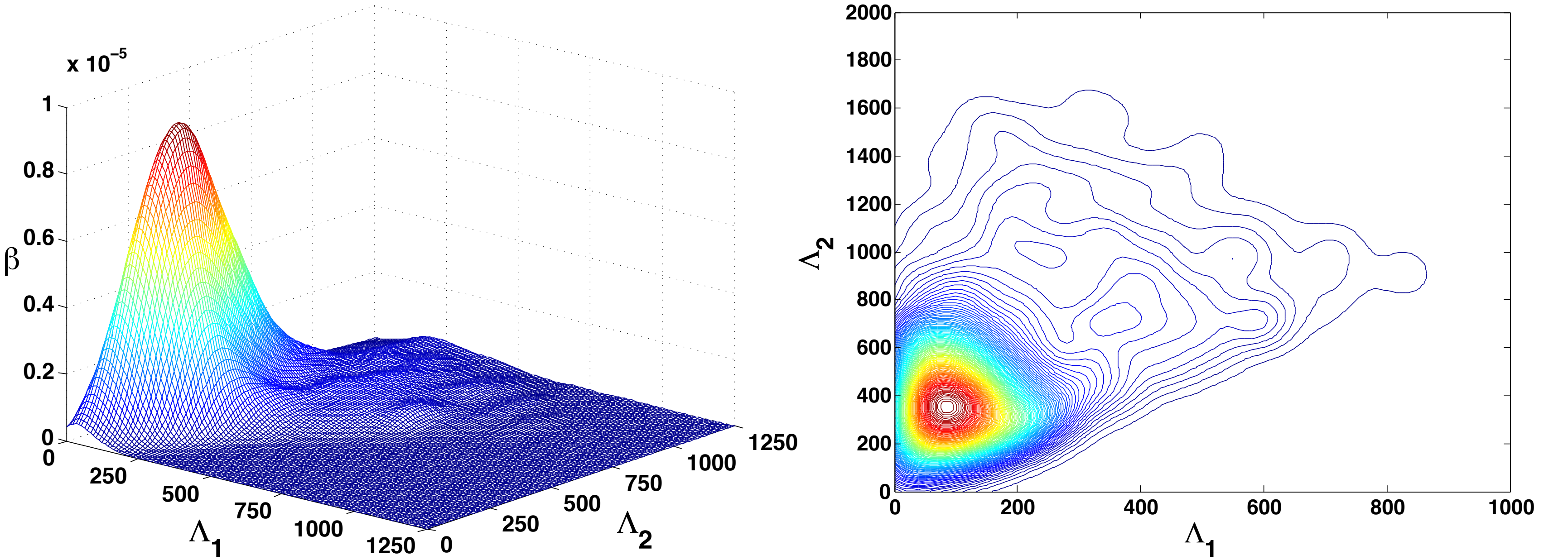
$$P\left(E_{GW}\,|\pi_i\right) = \int_{l_{22}} \beta(\Lambda_1(\tau),\Lambda_2(\tau))d\tau + \int_{l_{23}} \beta(\Lambda_1(\tau),\Lambda_2(\tau))d\tau + \int_{l_{32}} \beta(\Lambda_1(\tau),\Lambda_2(\tau))d\tau + \int_{l_{33}} \beta(\Lambda_1(\tau),\Lambda_2(\tau))d\tau,$$

where l_{ps} are the length of the line at Λ_1–Λ_2, the indecies p and s determine to which family of compact stars the GW170817 components belong. The parameter τ is, for instance, central density of a star.



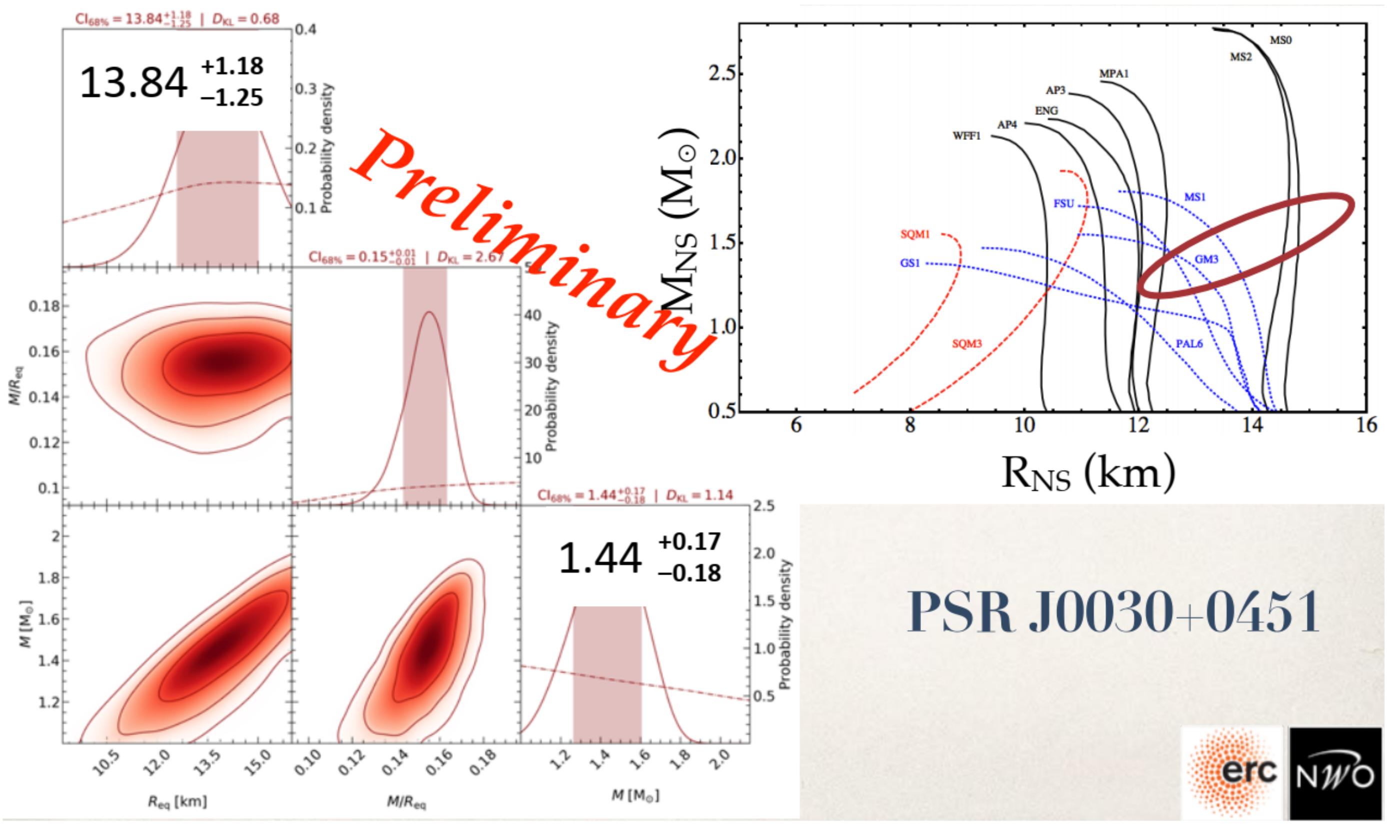
Credt: Abbott *et al.* PRL121 (2018)

$\beta(\Lambda_1, \Lambda_2)$



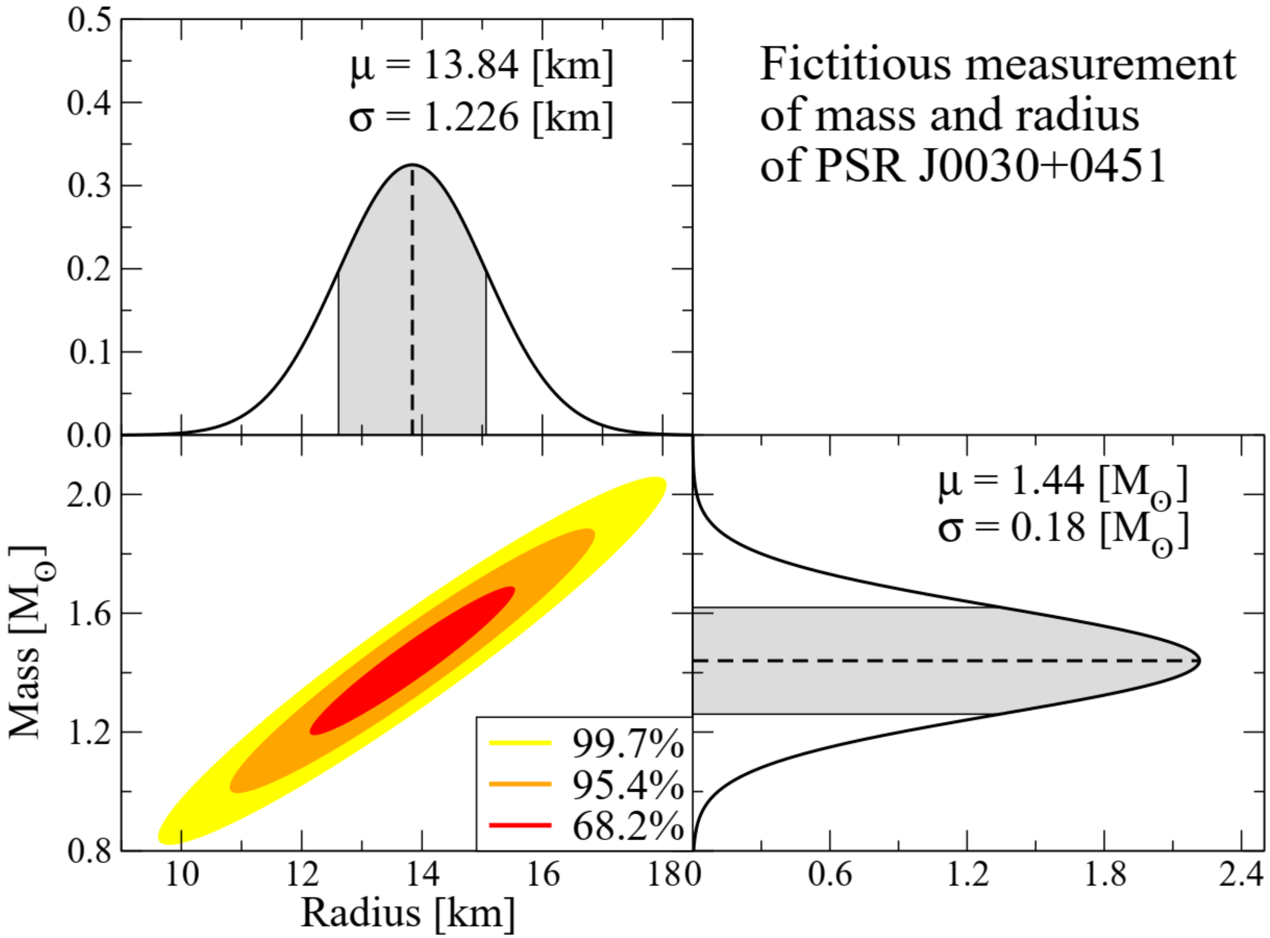
The PDF $\beta(\Lambda_1, \Lambda_2)$ has been reconstructed by the method Gaussian kernel density estimation with Λ_1–Λ_2 data given at LIGO web-page `https://dcc.ligo.org/LIGO-P1800115/public`.

Likelihood of a model for the fictitious *M*-*R* constraint

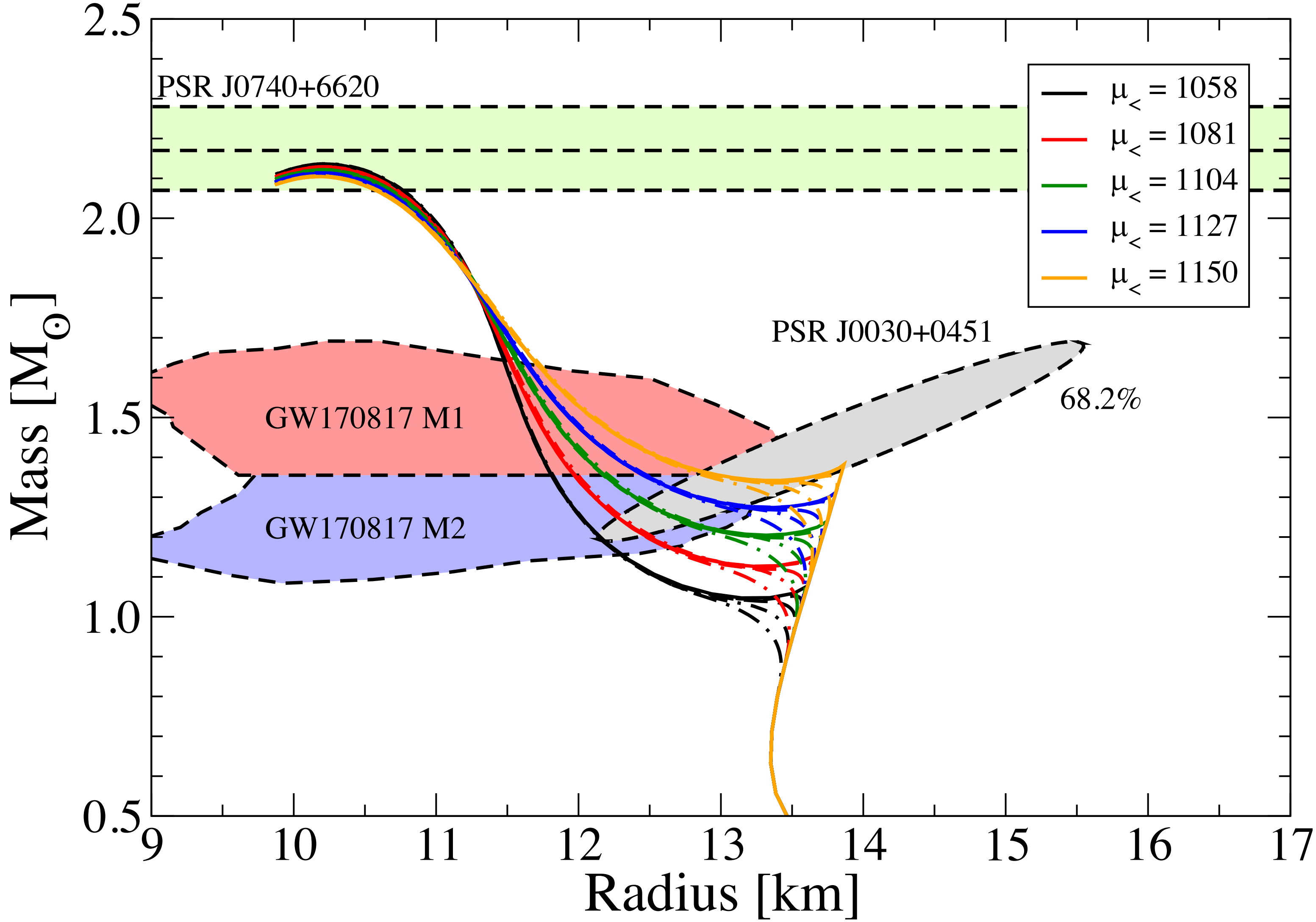


[Guillot. Talk at the Workshop "NSs and their environments", (April 8, 2019)]

Likelihood of a model for the fictitious *M-R* constraint



The fictitious *M*-*R* constraint



Likelihood fictitious measurements

The fictitious M-R measurement has been implemented, inspired by the preliminary results of NICER observation of M-R of the PSR J0030+0451.

$$P\left(E_{MR}\left|\pi_i\right.\right) = \int_{l_2} \mathcal{N}(\mu_R,\, \sigma_R,\, \mu_M,\, \sigma_M, \rho) d\tau + \int_{l_3} \mathcal{N}(\mu_R,\, \sigma_R,\, \mu_M,\, \sigma_M, \rho) d\tau,$$

where $\mu_R = 13.84$, $\sigma_R = 1.2276$, $\mu_M = 1.44$, $\sigma_M = 0.18$, and the correlation parameter $\rho = 0.9566$, winch corresponds to 8^{o} of th ellipse rotation. l_2 and l_3 are length of the lines at M-R diagram of the second and third families correspondingly.

The total likelihood and posteriori probability of the model parameters

The full likelihood for the given π_i can be calculated as a product of all likelihoods, since the considered constraints are independent of each other

$$P\left(E\,|\,\overrightarrow{\pi}_i\right) = \prod_m P\left(E_m\,|\,\overrightarrow{\pi}_i\right).$$

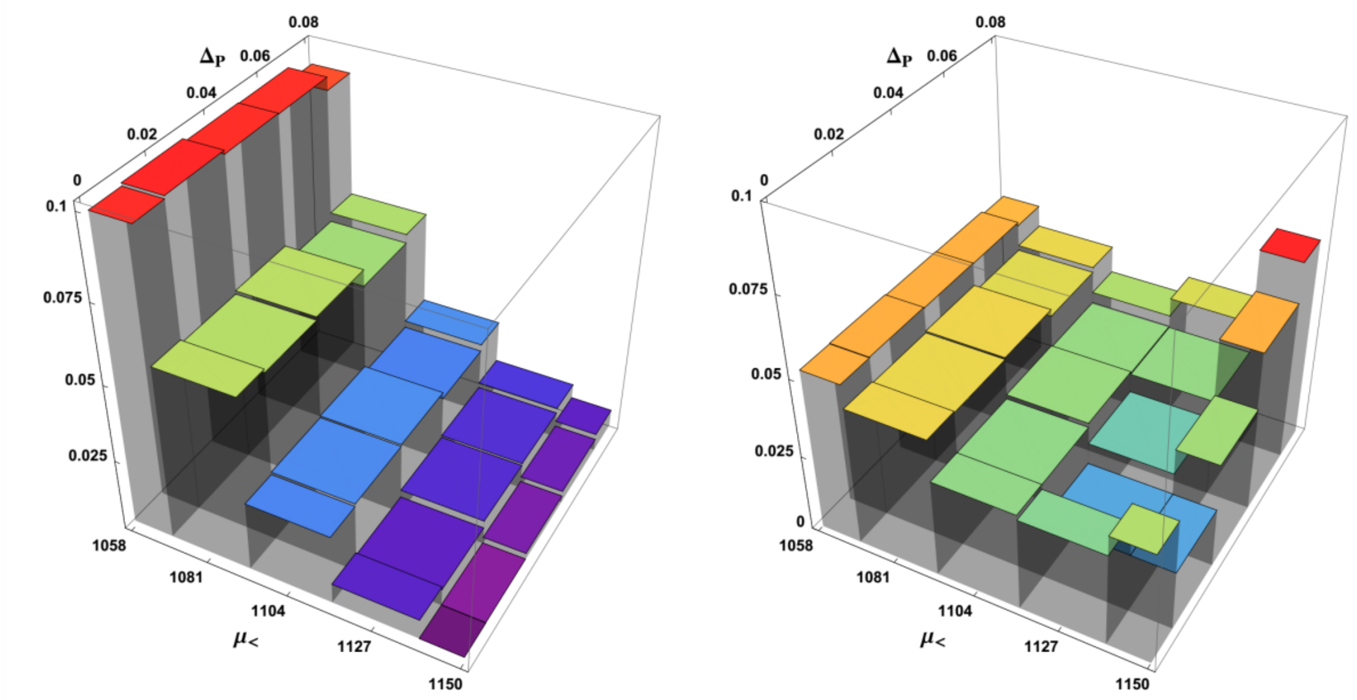
where m is index of the constraints.

The posterior distribution of models on parameter diagram is given by Bayes' theorem

$$P\left(\overrightarrow{\pi}_i\,|\,E\right) = \frac{P\left(E\,|\,\overrightarrow{\pi}_i\right) P\left(\overrightarrow{\pi}_i\right)}{\sum\limits_{j=0}^{N-1} P\left(E\,|\,\overrightarrow{\pi}_j\right) P\left(\overrightarrow{\pi}_j\right)},$$

where $P\left(\overrightarrow{\pi}_j\right)$ is a prior distribution of a models taken to be uniform: $P\left(\overrightarrow{\pi}_j\right) = 1/N$.

Results with and without fictitious measurements



Ayriyan, Alvarez-Castillo, Blaschke, Grigorian. In preparation.

Conclusions

The mixed phase interpolation method is very simple and well describes quark-hadron pasta phase for any given surface tension value.

The third family survives mixed phase effects for the pasta phase for the considered EoS models.

$\Lambda_1 - \Lambda_2$ relation from GW170817 favours softer EoS and hybrid stars with strong first order phase transitions (even with no third family due to the mixed phase).

The region $\Lambda_2 < \Lambda_1$ has physical meaning in case of low-mass twins, when heavier companion belongs to the second family and the lighter one to the third family.

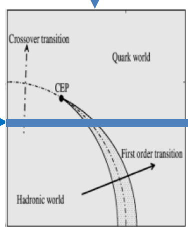
If NICER approves the "fictitious radius measurement" it will support late onset for the considered models.

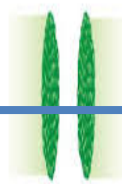
Simulations of Heavy Ion Collisions

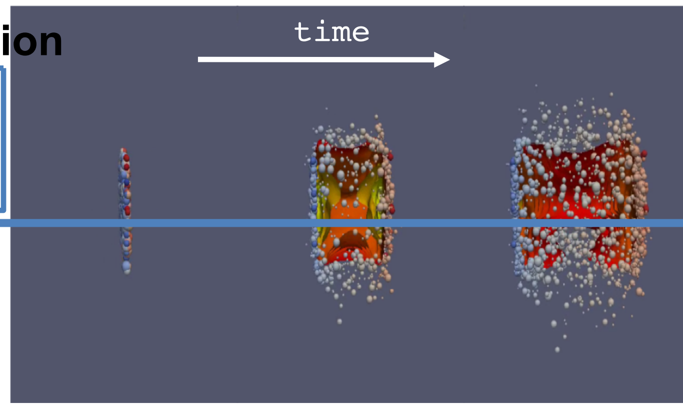
Bayesian analysis

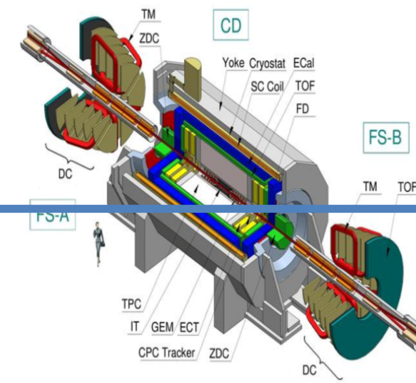
Data analysis

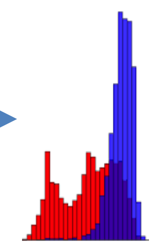
Parameter optimization











Parametrized EoS → Initial state → hydrodynamic evolution → Particlization → Event simulation at MPD detector | MPD roo

Development of EoS models

3-fluid hydro

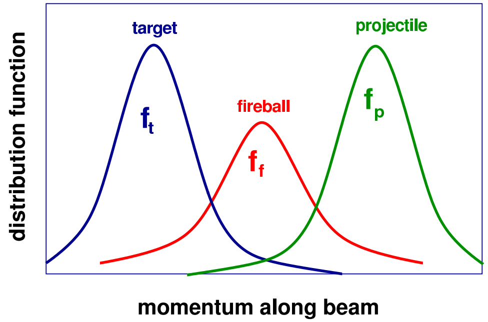
adapt the procedure from existing hybrid model

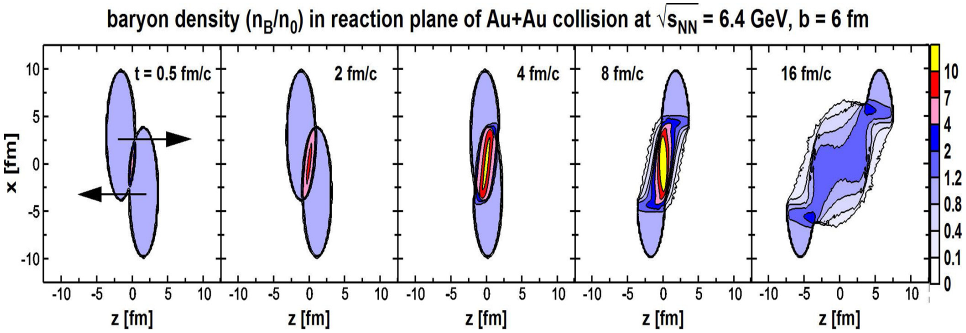
GEANT, MPD

Physical analys of simulated da

Multifluid Dynamic of Heavy Ion Collisions

Produced particles populate mid-rapidity $\Rightarrow$ fireball fluid





Target-like fluid: $\partial_\mu J_t^\mu = 0$ $\qquad$ $\partial_\mu T_t^{\mu\nu} = -F_{tp}^\nu + F_{ft}^\nu$

Leading particles carry bar. charge $\qquad$ exchange/emission

Projectile-like fluid: $\partial_\mu J_p^\mu = 0,$ $\qquad$ $\partial_\mu T_p^{\mu\nu} = -F_{pt}^\nu + F_{fp}^\nu$

Fireball fluid: $J_f^\mu = 0,$ $\qquad$ $\partial_\mu T_f^{\mu\nu} = F_{pt}^\nu + F_{tp}^\nu - F_{fp}^\nu - F_{ft}^\nu$

Baryon-free fluid $\qquad$ Source term $\qquad$ Exchange

The source term is delayed due to a formation time τ

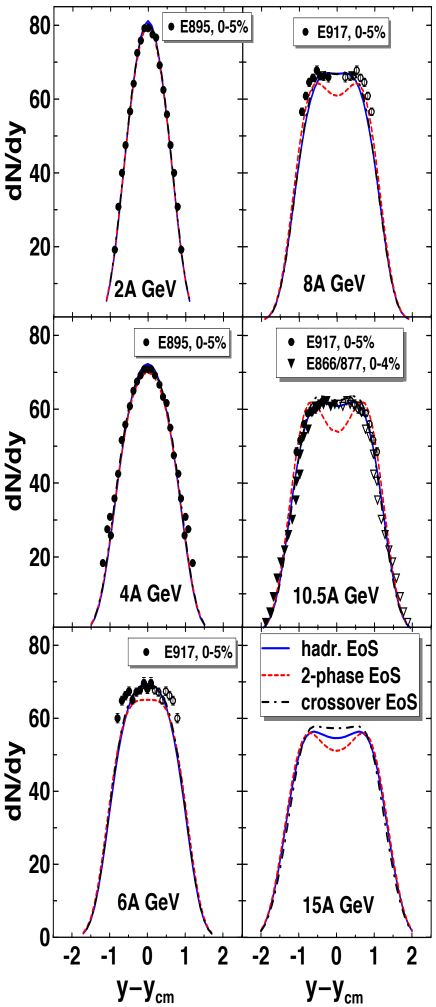
Total energy-momentum conservation:

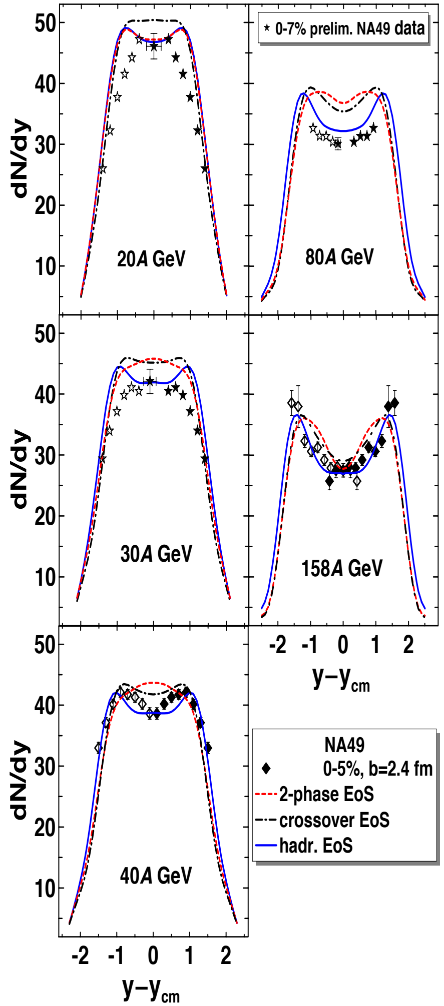
$$\partial_\mu (T_p^{\mu\nu} + T_t^{\mu\nu} + T_f^{\mu\nu}) = 0$$

Au+Au collision at $\sqrt{s_{NN}}$ = 6.4 GeV (Elab = 20A GeV) with impact parameter b = 6 fm

Yu. B. Ivanov, V. N. Russkikh, V. D. Toneev. Fluid Dynamics

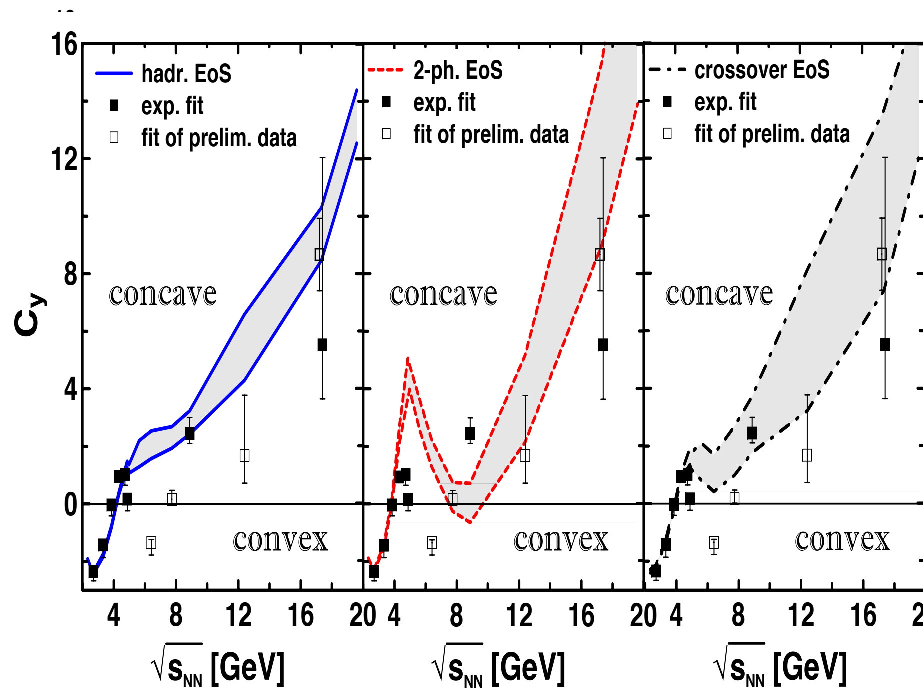
Rapidity Distribution & Curvature at Midrapidity





A reduced curvature of the spectrum at midrapidity

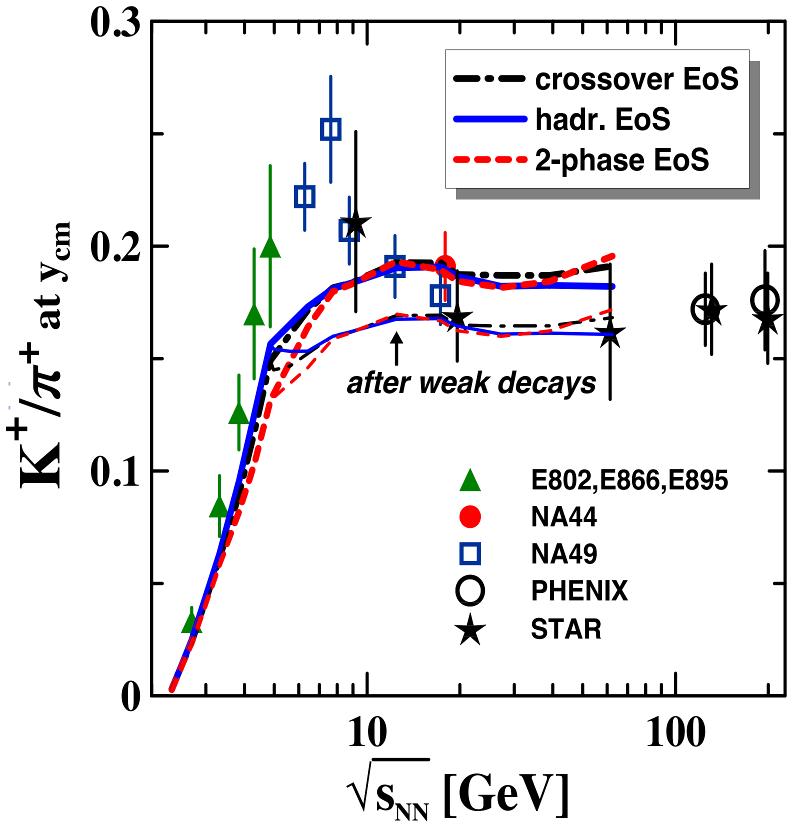
$$C_y = \left(y_{cm}^3 \frac{d^3N}{dy^3}\right)_{y=y_{cm}} \Big/ \left(y_{cm} \frac{dN}{dy}\right)_{y=y_{cm}} = (y_{cm}/w_s)^2 \left(\sinh^2 y_s - w_s \cosh y_s\right)$$

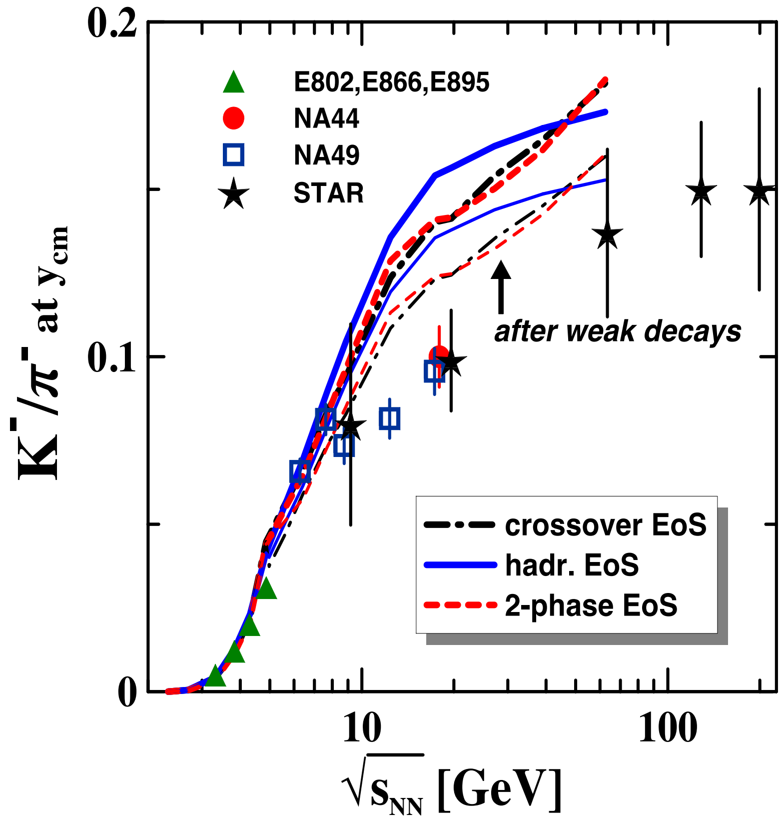


Yu. B. Ivanov. Phys. Lett. B 690(4), 3

P. Batyuk et al. Phys Rev C94, 04491

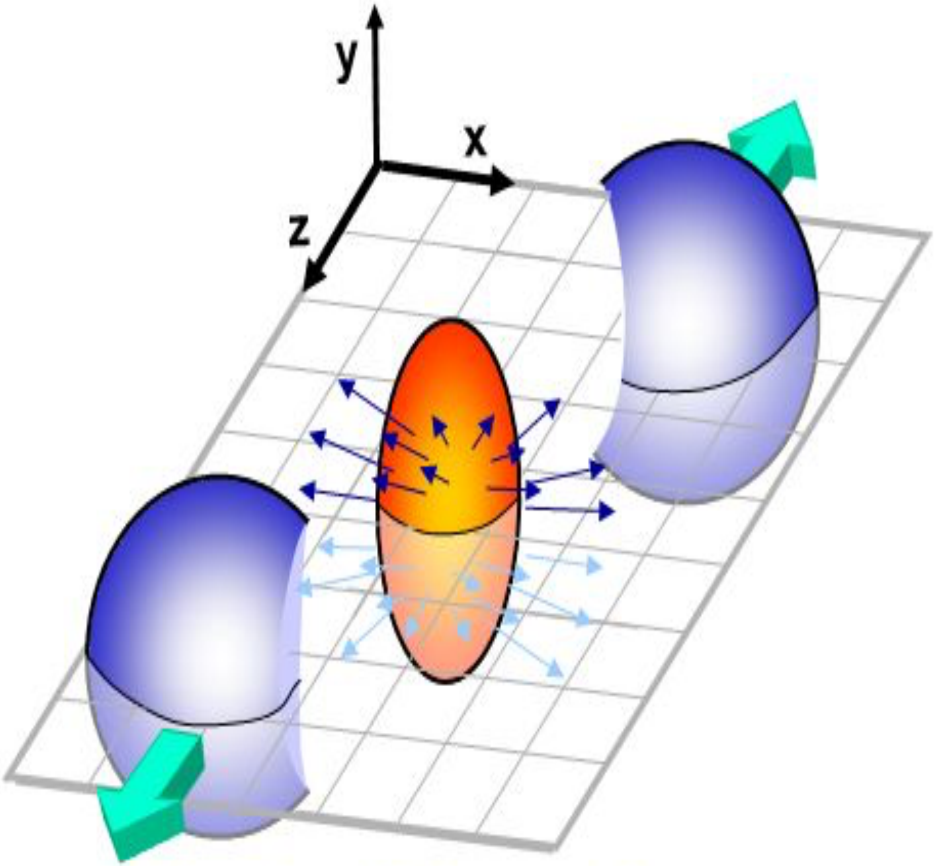
Hadron Ratios of Midrapidity





P. Batyuk et al. Phys Rev C94, 044917 (2016)

Flows



Credit: M. Oldenburg

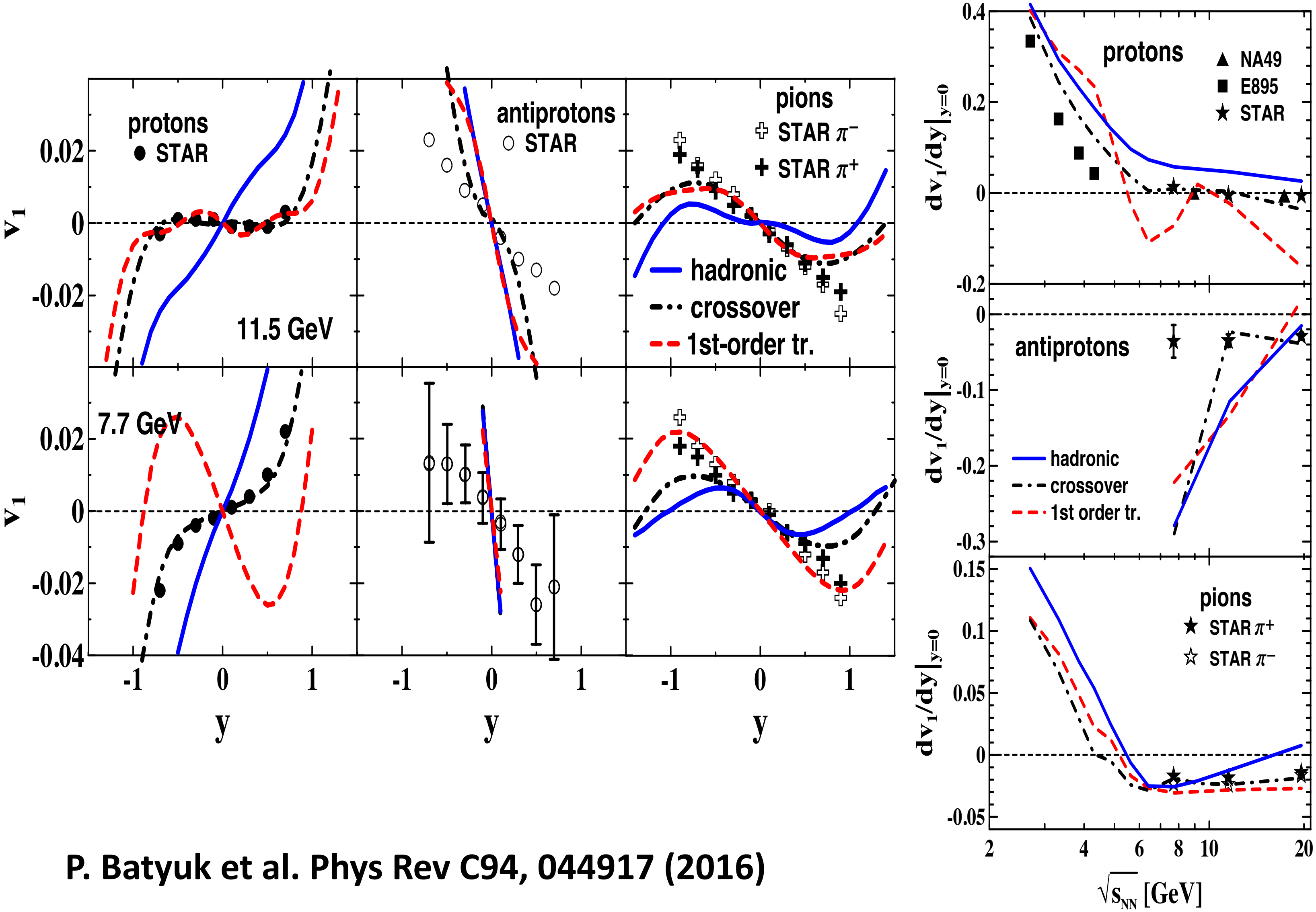
Fourier transformation of azimuthal
particle distribution in momentum
Space yields coefficients of different order

$$v_n = \langle \cos n \cdot \phi \rangle$$

$$\phi = \operatorname{atan} \frac{p_y}{p_x}$$

- v_1: "directed flow"
- v_2: "elliptic flow"

Directed Flow



P. Batyuk et al. Phys Rev C94, 044917 (2016)

Summary

- Varying EoS models
  - Formulation and solution of the optimization problem for definition of free hydrodynamic parameters
  - Development of hybrid EoS model construction
- Bayesian analysis for finding the best model parameter regions
  - Heavy ion simulation with different EoS models (parameters)
  - Collecting suitable experimental data around NICA energy range
  - Formulation and performing the Bayesian analysis
- Simulation of heavy ion collision at MPD detector
  - HIC simulation with the best models
  - Physical analysis of the simulated data within MPDroot
  - Comparing the results with the various experimental data

References

A. Ayriyan, D. Alvarez-Castillo, D. Blaschke and H. Grigorian, *Bayesian Analysis for Extracting Properties of the Nuclear Equation of State from Observational Data including Tidal Deformability from GW170817*, **Universe** 5, 61 (2019), doi 10.3390/universe5020061

K. Maslov, N. Yasutake, A. Ayriyan, D. Blaschke, H. Grigorian, T. Maruyama, T. Tatsumi, D. N. Voskresensky. *Hybrid equation of state with pasta phases, and third family of compact stars* **Physical Review C** 100, 025802 (2019), doi 10.1103/PhysRevC.100.025802

V. Abgaryan, D. Alvarez-Castillo, A. Ayriyan, D. Blaschke and H. Grigorian. *Two Novel Approaches to the Hadron-Quark Mixed Phase in Compact Stars*. **Universe** 4(9), 94 (2018), doi 10.3390/universe4090094

A. Ayriyan, N.-U. Bastian, D. Blaschke, H. Grigorian, K. Maslov, and D. N. Voskresensky. *Robustness of third family solutions for hybrid stars against mixed phase effects*. **Physical Review C** 97, 045802 (2018), doi 10.1103/PhysRevC.97.045802

A. Ayriyan and H. Grigorian. *Model of the Phase Transition Mimicking the Pasta Phase in Cold and Dense Quark-Hadron Matter*. **European Physical Journal WoC** 173, 03003 (2018), doi 10.1051/epjconf/201817303003

D. Alvarez-Castillo, A. Ayriyan, S. Benic, D. Blaschke, H. Grigorian and S. Typel, *New class of hybrid EoS and Bayesian M-R data analysis*, **European Physical Journal A** 52, 69 (2016), doi 10.1140/epja/i2016-16069-2