Neutron star phenomenology using the Dyson–Schwinger equations

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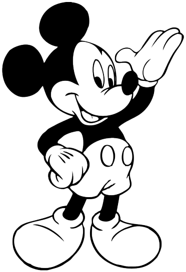
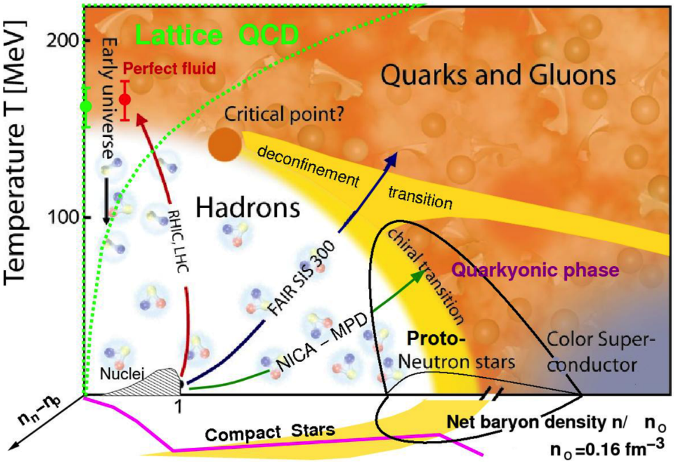
Overview

Motivation

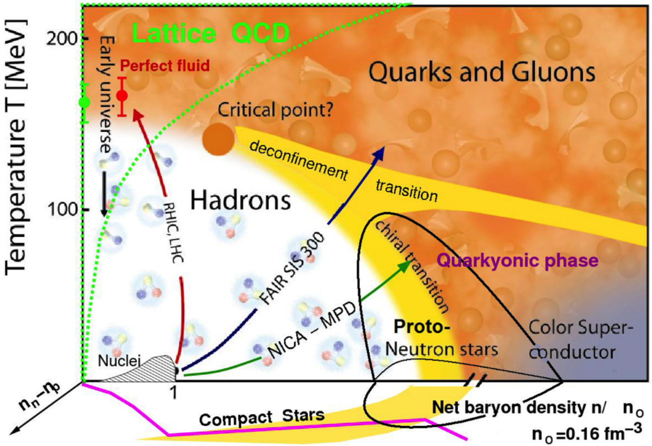
QCD phase diagram



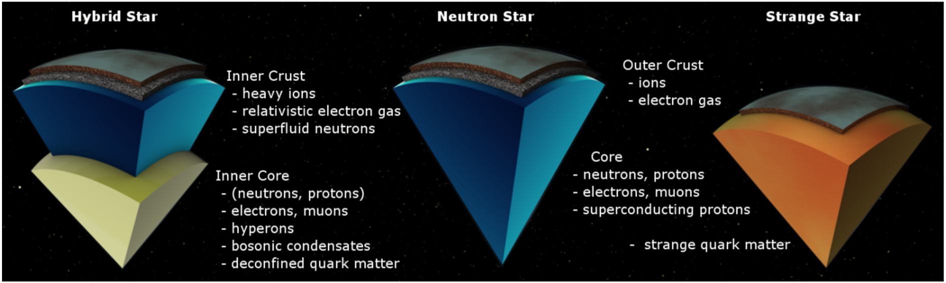


[1]Image retrieved from http://theor0.jinr.ru/twiki-cgi/view/NICA.

QCD phase diagram



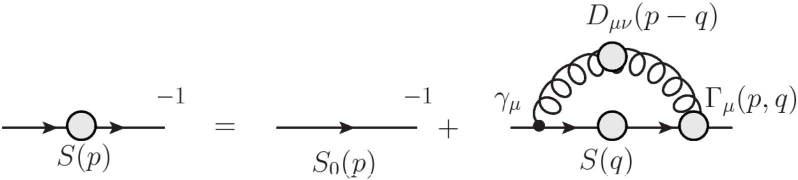
[1] Image retrieved from http://theor0.jinr.ru/twiki-cgi/view/NICA.



[2] Image courtesy of Thomas Klähn

The Dyson–Schwinger equations

The Quark Dyson–Schwinger equation



- One particle propagator in–medium

$$S^{-1}(p,\mu) = i\vec{\gamma}\vec{p} + i\gamma_4(p_4 + i\mu) + m + \Sigma(p,\mu)$$

- Self–energy term

$$\Sigma(p,\mu) = \int \frac{d^4q}{(2\pi)^4} g^2 D_{\rho\sigma}(p-q)\gamma^\rho \frac{\lambda^\alpha}{2} S(q)\Gamma^\sigma_\alpha(p,q)$$

Munczek–Nemirovsky model (M.C., T.Klähn)

- Quark DSE:

$$S^{-1}(p,\mu) = i\vec{\gamma}\vec{p} + i\gamma_4(p_4 + i\mu) + m + \Sigma(p,\mu)$$

- Interaction term:

$$\Sigma(p,\mu) = \int \frac{d^4q}{(2\pi)^4} g^2 D_{\rho\sigma}(p-q)\gamma^\rho \frac{\lambda^\alpha}{2} S(q)\Gamma^\sigma_\alpha(p,q)$$

- General form of the propagator:

$$S^{-1}(p,\mu) = i\bar{\gamma}\bar{p}A(p,\mu) + i\gamma_4\tilde{p}_4 C(p,\mu) + B(p,\mu)$$

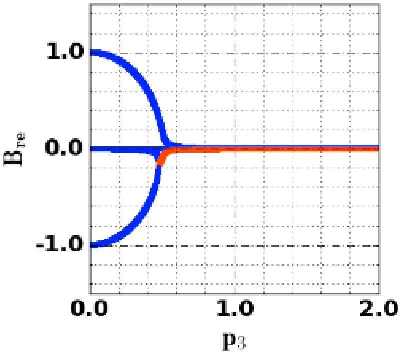
- The MN truncation:

$$g^2 D^{\rho\sigma}(k) = 3\pi^4\eta^2\delta^{\rho\sigma}\delta^{(4)}(k)$$

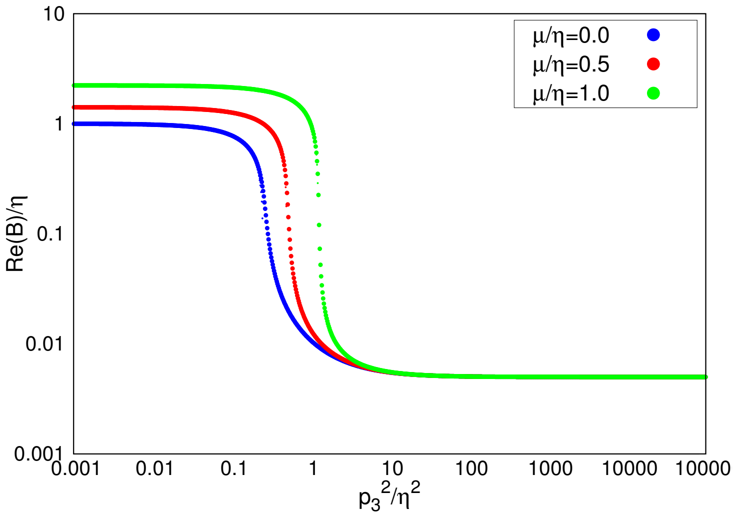
- Solution:

$$\tilde{p}^2 = \vec{p}^2 + (p_4 + i\mu)^2$$

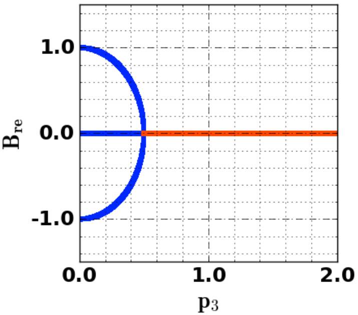
$$\begin{cases} A(p,\mu) = C(p,\mu) = \frac{2B(p,\mu)}{B(p,\mu)+m} \\ B^4 + mB^3 + B^2(4\tilde{p}^2 - m^2 - \eta^2) - mB(4\tilde{p}^2 + m^2 + 2\eta^2) - \eta^2 m^2 = 0 \end{cases}$$



[3] Cierniak, Klähn, Acta Phys.Polon.Supp. 10 (2017) 811



[3] Cierniak, Klähn, Acta Phys.Polon.Supp. 10 (2017) 811



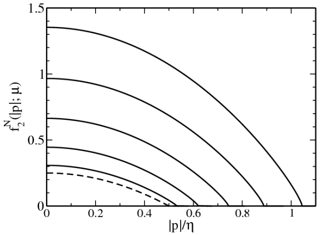
- DSE results ($m = 0$), Nambu–Goldstone phase

$$\begin{cases} A(p,\mu) = C(p,\mu) = 2 \\ B(p,\mu) = \pm\sqrt{\eta^2 - 4\tilde{p}^2} \end{cases}$$

- DSE results ($m = 0$), Wigner-Weyl phase

$$\begin{cases} A(p,\mu) = C(p,\mu) = \frac{1}{2}\left(1 + \sqrt{1 + \frac{2\eta^2}{\tilde{p}^2}}\right) \\ B(p,\mu) = 0 \end{cases}$$

[3] Cierniak, Klähn, Acta Phys.Polon.Supp. 10 (2017) 811

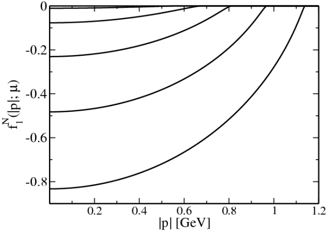


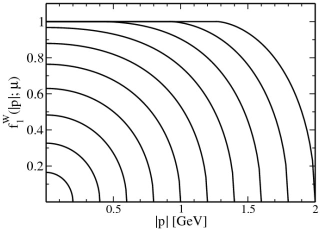
- Scalar density

$$f_2(p,\mu) = \frac{1}{4\pi}\int_{-\infty}^{\infty} dp_4\, Tr[S(p,\mu)]$$

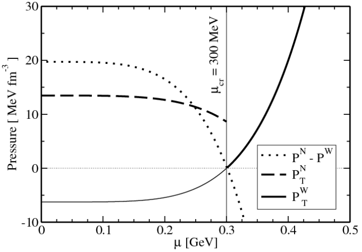
- Vector (particle number) density

$$f_1(p,\mu) = \int_{-\infty}^{\infty} dp_4\, Tr[-\gamma_4 S(p,\mu)]$$





[4] Klähn et al., Phys.Rev.C 82 (2010) 035801



- Pressure

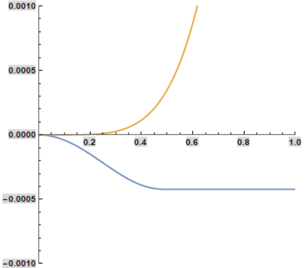
$$P(\mu) = P^0 + \int_0^\mu dz n_v(z)$$

- Strategy 1 [4]:

$$\begin{cases} B = \sqrt{\eta^2 - 4\tilde{p}^2} & Re(\tilde{p}^2) < \eta^2/4 \\ B = 0 & otherwise \end{cases}$$

- Strategy 2:

$$\begin{cases} B = \sqrt{\eta^2 - 4\tilde{p}^2} & (\tilde{p}^2 \in \mathbb{R}) < \eta^2/4 \\ B = 0 & otherwise \end{cases}$$



[4] Klähn et al., Phys.Rev.C 82 (2010) 035801

vBag (T.Klähn, T.Fischer, M.C.)

- Quark DSE:

$$S^{-1}(p,\mu) = i\vec{\gamma}\vec{p} + i\gamma_4(p_4 + i\mu) + m + \Sigma(p,\mu)$$

- Interaction term:

$$\Sigma(p,\mu) = \int \frac{d^4q}{(2\pi)^4} g^2 D_{\rho\sigma}(p-q)\gamma^\rho \frac{\lambda^\alpha}{2} S(q)\Gamma^\sigma_\alpha(p,q)$$

- General form of the propagator:

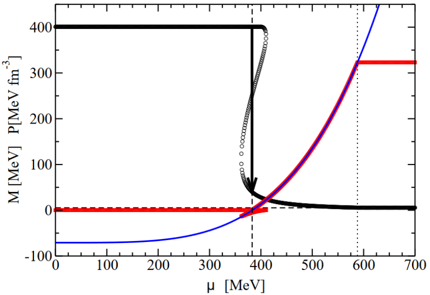
$$S^{-1}(p,\mu) = i\bar{\gamma}\bar{p}A(p,\mu) + i\gamma_4\tilde{p}_4 C(p,\mu) + B(p,\mu)$$

- The truncation:

$$g^2 D_{\rho\sigma}(p-q) = \delta_{\rho\sigma}\frac{1}{m_G^2}\Theta(\Lambda^2 - \vec{p}^2)$$

- Solution: $$\begin{cases} A(p,\mu) = 1 \\ B(p,\mu) = m + \frac{16N_c}{9m_G^2}\int_\Lambda \frac{d^4q}{(2\pi)^4}\frac{B(q,\mu)}{\vec{q}^2A^2(q,\mu)+\tilde{q}_4^2C^2(q,\mu)+B^2(q,\mu)} \\ \tilde{p}_4^2 C(p,\mu) = \tilde{p}_4 + \frac{8N_c}{9m_G^2}\int_\Lambda \frac{d^4q}{(2\pi)^4}\frac{\tilde{p}_4\tilde{q}_4C(q,\mu)}{\vec{q}^2A^2(q,\mu)+\tilde{q}_4^2C^2(q,\mu)+B^2(q,\mu)} \end{cases}$$

The chiral bag

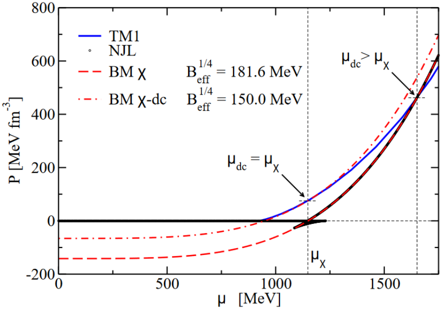


vBag EoS:

- $\mu_f = \mu_f^* + K_v n_{FG,f}(\mu_f^*)$
- $P_f(\mu_f) = P_{FG,f}(\mu_f^*) + \frac{K_v}{2} n_{FG,f}^2(\mu_f^*) - B_{\chi,f}$
- $P^Q = \sum P_f(\mu_f)$
- $\epsilon_f(\mu_f) = \epsilon_{FG,f}(\mu_f^*) + \frac{K_v}{2} n_{FG,f}^2(\mu_f^*) + B_{\chi,f}$
- $\epsilon^Q = \sum \epsilon_f(\mu_f)$
- $n_{v,f}(\mu_f) = n_{FG,f}(\mu_f^*)$

[5] Cierniak, Klähn, Fischer, Bastian, Universe 4 (2018) 2, 30

The (de)confinement bag

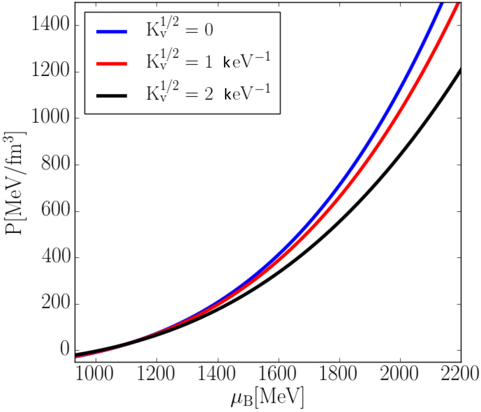


vBag EoS:

- $\mu_f = \mu_f^* + K_v n_{FG,f}(\mu_f^*)$
- $P_f(\mu_f) = P_{FG,f}(\mu_f^*) + \frac{K_v}{2} n_{FG,f}^2(\mu_f^*) - B_{\chi,f}$
- $P^Q = \sum P_f(\mu_f) + B_{dc}$
- $\epsilon_f(\mu_f) = \epsilon_{FG,f}(\mu_f^*) + \frac{K_v}{2} n_{FG,f}^2(\mu_f^*) + B_{\chi,f}$
- $\epsilon^Q = \sum \epsilon_f(\mu_f) + B_{dc}$
- $n_{v,f}(\mu_f) = n_{FG,f}(\mu_f^*)$

[6] Klähn, Fischer, Astrophys.J. 810 (2015) 2, 134

Vector repulsion

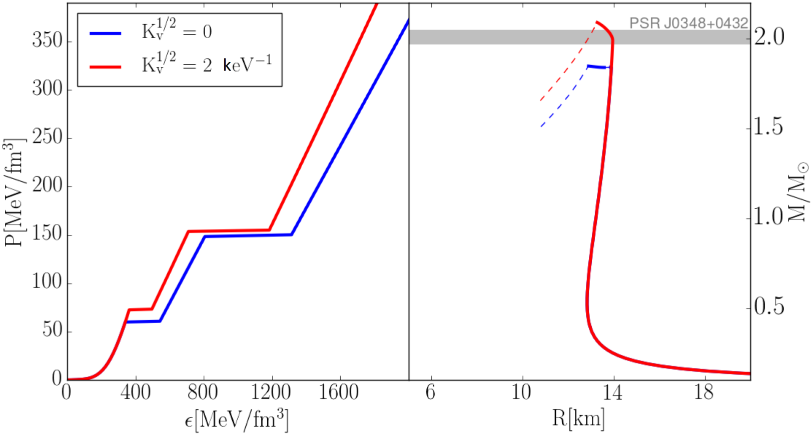


vBag EoS:

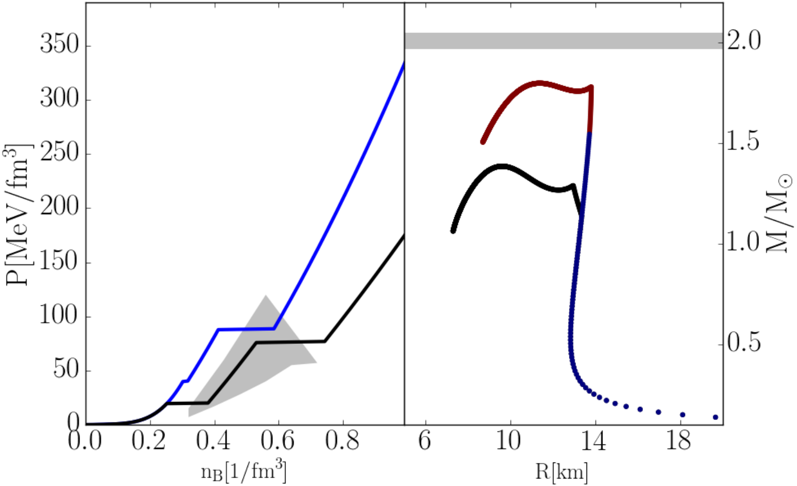
- $\mu_f = \mu_f^* + K_v n_{FG,f}(\mu_f^*)$
- $P_f(\mu_f) = P_{FG,f}(\mu_f^*) + \frac{K_v}{2} n_{FG,f}^2(\mu_f^*) - B_{\chi,f}$
- $P^Q = \sum P_f(\mu_f) + B_{dc}$
- $\epsilon_f(\mu_f) = \epsilon_{FG,f}(\mu_f^*) + \frac{K_v}{2} n_{FG,f}^2(\mu_f^*) + B_{\chi,f}$
- $\epsilon^Q = \sum \epsilon_f(\mu_f) + B_{dc}$
- $n_{v,f}(\mu_f) = n_{FG,f}(\mu_f^*)$

[5] Cierniak, Klähn, Fischer, Bastian, Universe 4 (2018) 2, 30

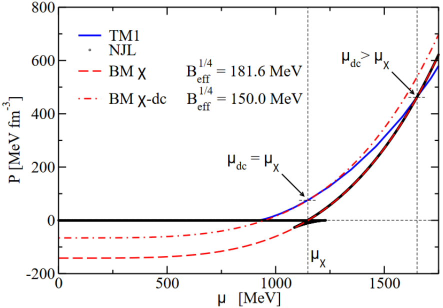
Mass–radius relation



[5] Cierniak, Klähn, Fischer, Bastian, Universe 4 (2018) 2, 30


P[MeV/fm³]
350
300
250
200
150
100
50
0
0.0
0.2
0.4
0.6
0.8
n_B[1/fm³]
6
10
14
18
R[km]
2.0
1.5
1.0
0.5
M/M_⊙

vBag at $T \neq 0$

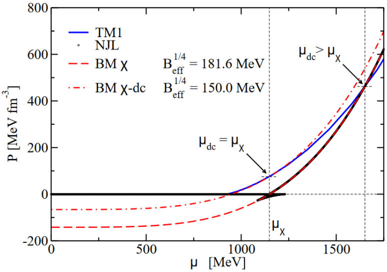


vBag EoS:

- $\mu_f = \mu_f^* + K_v n_{FG,f}(\mu^*)$
- $P_f(T, \mu_f) = P_{FG,f}(T, \mu_f^*) + \frac{K_v}{2} n_{FG,f}^2(\mu_f^*) - B_{\chi,f}$
- $P^Q = \sum P_f(T, \mu_f) + B_{dc}(T)$
- $\epsilon_f(T, \mu_f) = \epsilon_{FG,f}(T, \mu_f^*) + \frac{K_v}{2} n_{FG,f}^2(\mu_f^*) + B_{\chi,f}$
- $\epsilon^Q = \sum \epsilon_f(T, \mu_f^*) - B_{dc}(T) + T\frac{\partial B_{dc}(T)}{\partial T}$
- $n_f(\mu_f) = n_{FG,f}(\mu_f^*)$
- $s_f(T, \mu_f) = \left.\frac{\partial P_f(T,\mu_f)}{\partial T}\right|_{\mu_f}$
- $s(T, \mu_f) = \sum s_f(T, \mu_f) + \frac{\partial B_{dc}(T)}{\partial T}$
- $\mu_B = \mu_u + 2\mu_d$
- $n_B = \frac{\partial P}{\partial \mu_B}$

[6] Klähn, Fischer, Astrophys.J. 810 (2015) 2, 134
[7] Fischer, Klähn, Hempel, Eur.Phys.J. A52 (2016) 8, 225

vBag at $T \neq 0$ and $\mu_C \neq 0$

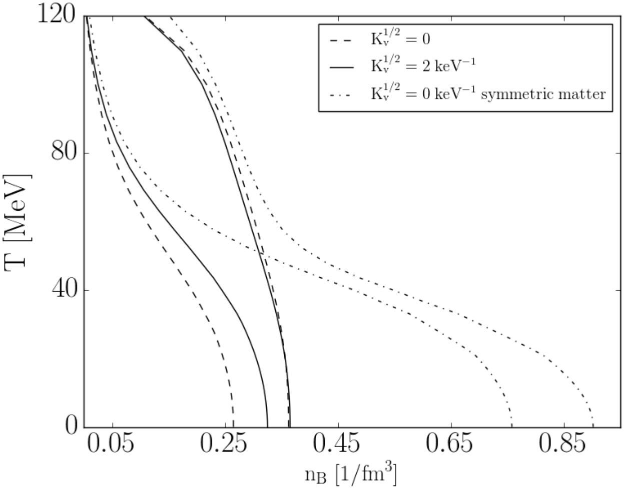


vBag EoS:

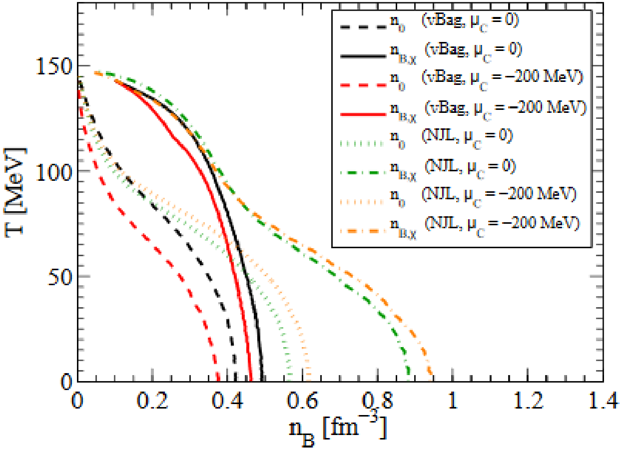
- $\mu_f = \mu_f^* + K_v n_{FG,f}(\mu^*)$
- $P_f(T, \mu_f) = P_{FG,f}(T, \mu_f^*) + \frac{K_v}{2} n_{FG,f}^2(\mu_f^*) - B_{\chi,f}$
- $P^Q = \sum P_f(T, \mu_f) + B_{dc}(T)$
- $\epsilon_f(T, \mu_f) = \epsilon_{FG,f}(T, \mu_f^*) + \frac{K_v}{2} n_{FG,f}^2(\mu_f^*) + B_{\chi,f}$
- $\epsilon^Q = \sum \epsilon_f(T, \mu_f^*) - B_{dc}(T) + T\frac{\partial B_{dc}(T)}{\partial T} + \mu_C \frac{\partial B_{dc}(T,\mu_C)}{\partial \mu_c}$
- $n_f(\mu_f) = n_{FG,f}(\mu_f^*)$
- $s_f(T, \mu_f) = \left.\frac{\partial P_f(T,\mu_f)}{\partial T}\right|_{\mu_f}$
- $s(T, \mu_f) = \sum s_f(T, \mu_f) + \frac{\partial B_{dc}(T)}{\partial T}$
- $\mu_B = \mu_u + 2\mu_d$
- $n_B = \frac{\partial P}{\partial \mu_B}$
- $\mu_c = \mu_u - \mu_d$

[8] Klähn, Fischer, Hempel, Astrophys.J. 836 (2017) 1, 89

Phase diagram

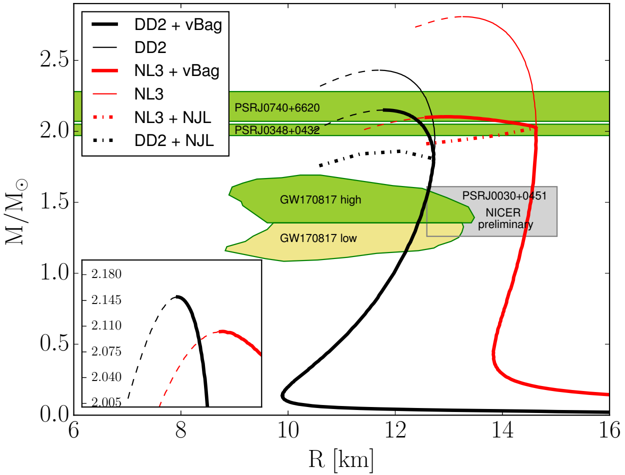


Phase diagram



[8] Klähn, Fischer, Hempel, Astrophys.J. 836 (2017) 1, 8

Phase diagram



[9] Cierniak et al., Universe 5 (2019) 8, 186

Conclusions and outlook

- Dyson–Schwinger equations are a useful tool for deriving dense matter properties for use in astrophysical studies
- The NJL model can be derived as truncations of the QCD DSE
- DSE can be used to derive hadrons as bound states of quarks
- So far no attempts have been made to study NS properties using a consistent DSE–derived hadron matter model