# Compact stars with QCD phase transition and GW170817

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# Outline

# Equilibria of compact objects

150 The Equilibrium and Stability of Fluid Configurations

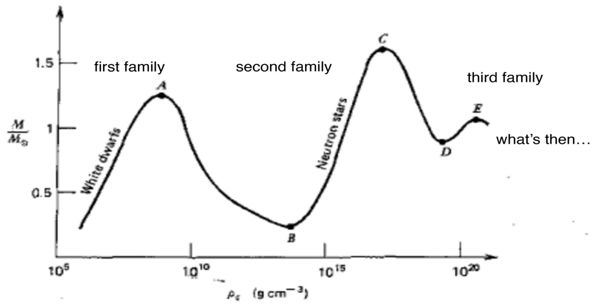



Figure 6.2 Schematic diagram showing the turning points in the mass versus central density diagram for equilibrium configurations of cold matter.

S. Shapiro, S. Teukolsky, "Black holes, White dwarfs and Neutron Stars"

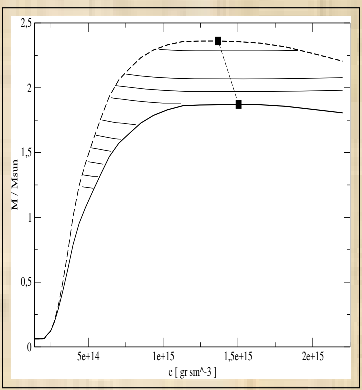
-*White dwarfs* -first family, $M \leq 1.5M_\odot$, [S. Chandrasekhar, L. Landau (1930-32)]
-*Neutron Stars* - second family, $M \leq 2M_\odot$, [Oppenhimer-Volkoff (1939)]
-*Hybrid Stars* - third family, $M \leq 2M_\odot$, [Gerlach (1968), Glendenning-Kettner (2000)]
- *Fourth Family?* M. Alford and A. Sedrakian, Phys. Rev. Lett. 119, 161104 (2017).

- Einstein's field equations:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = -8\pi T_{\mu\nu},$$

- Energy-momentum tensor:

$$T_{\mu\nu} = -P(r)g_{\mu\nu} + [P(r) + \epsilon(r)]\, u_\mu u_\nu$$




TOV equations:

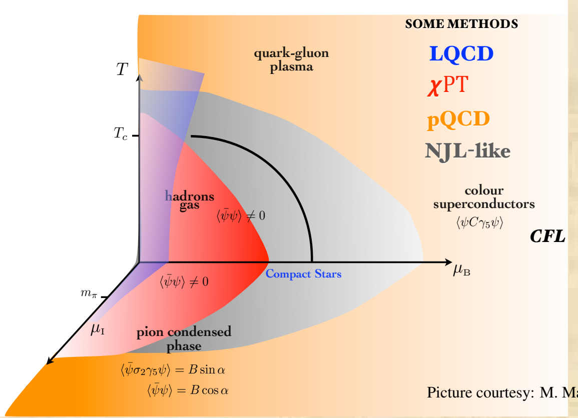
$$\frac{dP(r)}{dr} = -\frac{G\epsilon(r)M(r)}{c^2r^2}\left(1+\frac{P(r)}{\epsilon(r)}\right)\left(1+\frac{4\pi r^3P(r)}{M(r)c^2}\right)\left(1-\frac{2GM(r)}{c^2r}\right)^{-1}.$$

$$M(r) = 4\pi\int_0^r r^2\epsilon(r)dr.$$

The Lagrangian of QCD is written for $\psi_q = (\psi_{qR}, \psi_{qG}, \psi_{qB})^T$ as

$$\mathcal{L}_{QCD} = \underbrace{\bar{\psi}^i_q (i\gamma^\mu)(D_\mu)_{ij}\psi^j_q - m_q \bar{\psi}^i_q \psi_{qi}}_{\text{quarks}} - \underbrace{\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}}_{\text{gluons(Yang-Mills)}} ,$$

where $\underbrace{(D_\mu)_{ij} = \delta_{ij}\partial_\mu - i g_s t^a_{ij} A^a_\mu}_{\text{covarinat derivative}}$, and $\underbrace{F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu - 2q(A^\mu \times A^\nu)}_{\text{gluonic field (Yang-Mills) field tensor}}$




Picture courtesy: M. Mannarelli

# Color-superconductivity within the NJL model

$$\mathcal{L}_{NJL} = \underbrace{\bar\psi(i\gamma^\mu\partial_\mu - \hat m)\psi}_{\text{quarks}} + \underbrace{G_V(\bar\psi i\gamma^\mu\psi)^2}_{\text{vector}} + \underbrace{G_S\sum_{a=0}^{8}[(\bar\psi\lambda_a\psi)^2 + (\bar\psi i\gamma_5\lambda_a\psi)^2]}_{\text{scalar}-\text{pseudoscalar}}$$

$$+ \underbrace{G_D\sum_{\gamma,c}[\bar\psi^a_\alpha i\gamma_5\epsilon^{\alpha\beta\gamma}\epsilon_{abc}(\psi_C)^b_\beta][(\bar\psi_C)^r_\rho i\gamma_5\epsilon^{\rho\sigma\gamma}\epsilon_{rsc}\psi^s_\sigma]}_{\text{pairing}}$$

$$- \underbrace{K\left\{\det_f[\bar\psi(1+\gamma_5)\psi] + \det_f[\bar\psi(1-\gamma_5)\psi]\right\}}_{\text{t}'\text{Hooft interaction}},$$

- quarks: $\psi^a_\alpha$, color $a = r, g, b$, flavor ($\alpha = u, d, s$); mass matrix: $\hat m = \mathrm{diag}_f(m_u, m_d, m_s)$;
- other notations: $\lambda_a$, $a = 1, ..., 8$, $\psi_C = C\bar\psi^T$ and $\bar\psi_C = \psi^T C$, $C = i\gamma^2\gamma^0$.

Parameters of the model:

- $G_S$ the scalar coupling and cut-off $\Lambda$ are fixed from vacuum physics
- $G_D$ is the di-quark coupling $\simeq 0.75G_S$ (via Fierz) but free to change
- $G_V$ and $\rho_{\mathrm{tr}}$ are treated as free parameters

# QCD interactions pairing interactions and gaps

$$\Delta \propto \langle 0|\psi^a_{\alpha\sigma}\psi^b_{\beta\tau}|0\rangle$$

- Symmetric in space wave function (isotropic interaction)
- Antisymmetry in colors $a, b$ for attraction
- Antisymmetry in spins $\sigma, \tau$ (Cooper pairs as spin-0 objects)
- Antisymmetry in flavors $\alpha, \beta$

2SC phase:

Low densities, large $m_s$ (strange quark decoupled)

$$\Delta(2SCs) \propto \Delta\epsilon^{ab3}\epsilon_{\alpha\beta} \qquad \delta\mu \ll \Delta,$$

Crystalline or gapless phases:

Intermediate densities, large $m_s$ (strange quark decoupled)

$$\Delta(\text{cryst.}) \propto \epsilon_{\alpha\beta}\Delta_0\, e^{i\vec{Q}\cdot\vec{r}} \qquad \delta\mu \geq \Delta,$$

CFL phase:

High densities nearly massless $u, d, s$ quarks

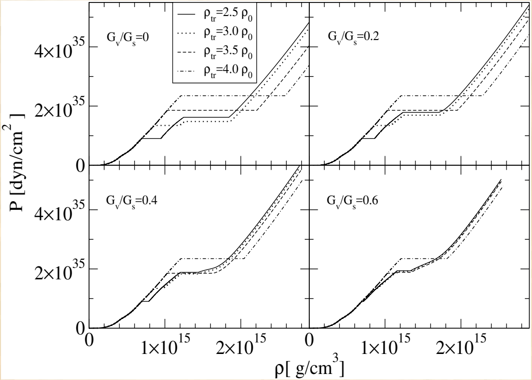
$$\Delta(CFL) \propto \langle 0|\psi^a_{\alpha L}\psi^b_{\beta L}|0\rangle = -\langle 0|\psi^a_{\alpha R}\psi^b_{\beta R}|0\rangle = \Delta\epsilon^{abC}\Delta\epsilon_{\alpha\beta C}.$$

# EOS including (hyper)nuclear, 2SC and CFL phases of matter

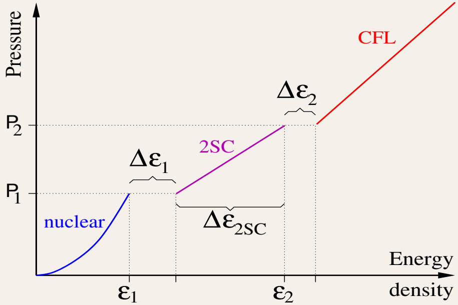
Choose Maxwell (large surface tension) or Glendenning (low surface tension) constructions. Matching condition for Maxwell is simply

$$P_N(\mu_B) = P_Q(\mu_B),$$

i.e., with low-density nuclear and high-density quark phases



# Synthetic equations of state with constant speed of sound




- Instead of full NJL-model EoS with 2SC-CFL transition use synthetic EoS
- Realistic DD-ME2 EoS below the deconfinement (Colucci-Sedrakian EoS)
- Parametrize synthetic EoS via Constant Speed of Sound (CSS) parameterization (Alford-Han-Prakash 2013), also Haensel-Zdunik (2012).

# Relativistic DFT theory

$$
\begin{aligned}
\mathcal{L} &= \underbrace{\sum_B \bar{\psi}_B\Big[\gamma^\mu\Big(i\partial_\mu - g_{\omega BB}\omega_\mu - \frac{1}{2}g_{\rho BB}\boldsymbol{\tau}\cdot\boldsymbol{\rho}_\mu\Big) - (m_B - g_{\sigma BB}\sigma)\Big]\psi_B}_{\text{baryonic contribution}} \\
&+ \underbrace{\frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2}_{\text{scalar mesons}} \\
&- \underbrace{\frac{1}{4}\omega^{\mu\nu}\omega_{\mu\nu} + \frac{1}{2}m_\omega^2\omega^\mu\omega_\mu - \frac{1}{4}\boldsymbol{\rho}^{\mu\nu}\boldsymbol{\rho}_{\mu\nu} + \frac{1}{2}m_\rho^2\boldsymbol{\rho}^\mu\cdot\boldsymbol{\rho}_\mu}_{\text{vector mesons}} \\
&+ \underbrace{\sum_\lambda \bar{\psi}_\lambda(i\gamma^\mu\partial_\mu - m_\lambda)\psi_\lambda}_{\text{leptons}} - \frac{1}{4}F^{\mu\nu}F_{\mu\nu},
\end{aligned}
$$

- $B$-sum is over the baryonic octet $B \equiv p, n, \Lambda, \Sigma^{\pm,0}, \Xi^{-,0}$
- Meson fields include $\sigma$ meson, $\boldsymbol{\rho}_\mu$-meson and $\omega_\mu$-meson
- Leptons include electrons, muons and neutrinos for $T \neq 0$

# Fixing the couplings: nucleonic sector

$$g_{iN}(\rho_B) = g_{iN}(\rho_0)h_i(x), \qquad i = \sigma, \omega, \qquad h_i(x) = a_i \frac{1 + b_i(x + d_i)^2}{1 + c_i(x + d_i)^2}$$

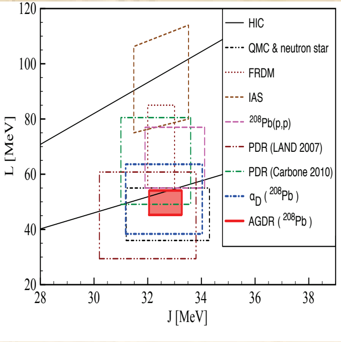
$$g_{\rho N}(\rho_B) = g_{\rho N}(\rho_0)\exp[-a_\rho(x - 1)].$$

DD-ME2 parametrization of D. Vretenar, P. Ring et al. Phys. Rev. C 71, 024312 (2005).

| | $\sigma$ | $\omega$ | $\rho$ |
|---|---|---|---|
| $m_i$ [MeV] | 550.1238 | 783.0000 | 763.0000 |
| $g_{Ni}(\rho_0)$ | 10.5396 | 13.0189 | 3.6836 |
| $a_i$ | 1.3881 | 1.3892 | 0.5647 |
| $b_i$ | 1.0943 | 0.9240 | — |
| $c_i$ | 1.7057 | 1.4620 | — |
| $d_i$ | 0.4421 | 0.4775 | — |

Total number of parameters 8: boundary conditions on $h(x)$ at $x = 1$.

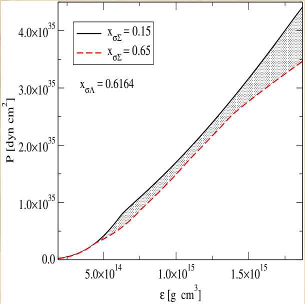
- saturation density $\rho_0 = 0.152 \text{ fm}^{-3}$
- binding energy per nucleon $E/A = -16.14$ MeV,
- incompressibility $K_0 = 250.90$ MeV,
- symmetry energy $J = 32.30$ MeV,
- symmetry energy slope $L = 51.24$ MeV,
- symmetry incompressibility $K_{sym} = -87.19$ MeV

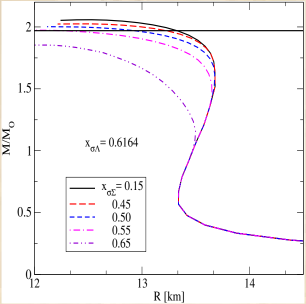



$$K_0 = k_F^2 \frac{\partial E/A}{\partial k^2}\Big|_{k=k_F} = 9\rho_0^2 \frac{\partial^2 E/A}{\partial \rho^2}\Big|_{\rho=\rho_0}, \qquad S(\rho) = \frac{1}{2}\frac{\partial^2 \epsilon/\rho}{\partial \delta^2}\Big|_{\delta=0}.$$

$$S(\rho) = J + L\left(\frac{\rho-\rho_0}{3\rho_0}\right) + \frac{1}{2}K_{\rm sym}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2.$$

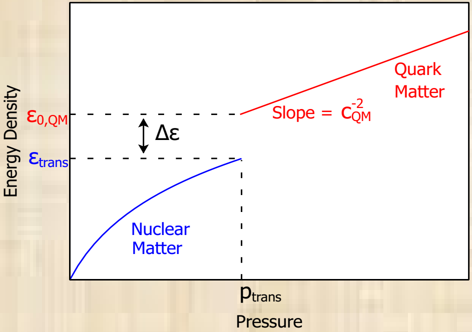
# Equation of state and (hyper)nuclear stars




Zero temperature equations of state of hypernuclear matter for fixed $x_{\sigma\Lambda} = 0.6164$ and a range of values $0.15 \leq x_{\sigma\Sigma} \leq 0.65$. These values generate the shaded area, which is bound from below by the softest EoS (dashed red line) corresponding to $x_{\sigma\Sigma} = 0.65$ and from above by the hardest EoS (solid line) corresponding to $x_{\sigma\Sigma} = 0.15$.

# CSS parameterization



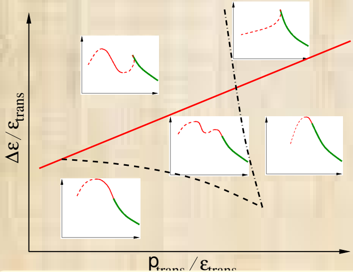

$$\varepsilon(p) = \begin{cases} \varepsilon_{NM}(p) & p < p_{trans} \\ \varepsilon_{NM}(p_{trans}) + \Delta\varepsilon + c_{QM}^{-2}(p - p_{trans}) & p > p_{trans} \end{cases},$$

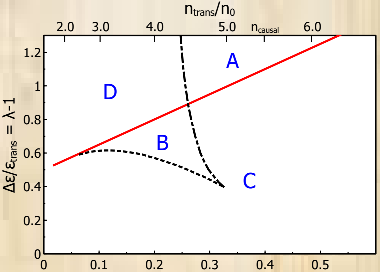
M. G. Alford, S. Han, M. Prakash, Phys. Rev. D 88, 083013 (2013).

J. Zdunik and P. Haensel A and A 551, A61, (2013).

# Phase diagram in *M*-*R* space

Phase diagram for hybrid star branches in the mass-radius relation of compact stars. The left panel shows schematically the possible topological forms of the mass-radius relation in each region of the diagram.

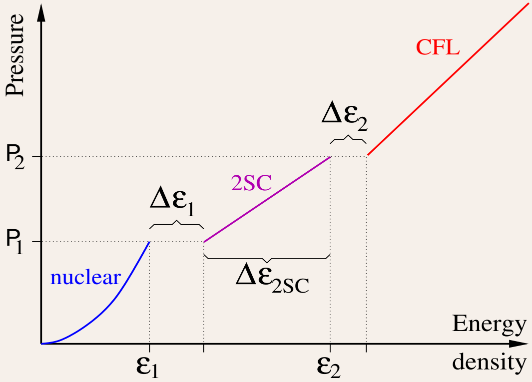



$$\frac{\Delta\varepsilon_{\rm crit}}{\varepsilon_{\rm trans}} = \frac{1}{2} + \frac{3}{2}\frac{p_{\rm trans}}{\varepsilon_{\rm trans}} .$$

M. G. Alford, S. Han, M. Prakash, Phys. Rev. D 88, 083013 (2013).

# EoS with sequential phase transitions




Parameters of the models:

$$(\epsilon_1, P_1) \quad \Delta\varepsilon_1, \quad \Delta\varepsilon_{2SC} \quad (\varepsilon_2, P_2) \quad \Delta\varepsilon_2$$

Note that there are five independent parameters.

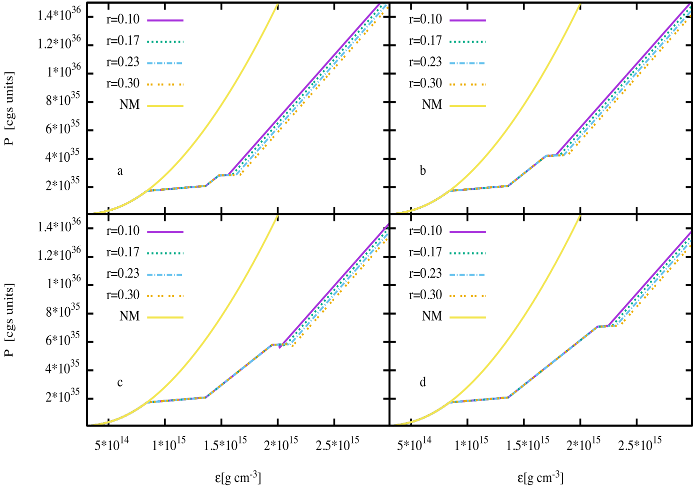
## The EOS is analytically given

$$P(\varepsilon) = \begin{cases} P_1, & \varepsilon_1 < \varepsilon < \varepsilon_1 + \Delta\varepsilon_1 \\ P_1 + s_1\left[\varepsilon - (\varepsilon_1 + \Delta\varepsilon_1)\right], & \varepsilon_1 + \Delta\varepsilon_1 < \varepsilon < \varepsilon_2 \\ P_2, & \varepsilon_2 < \varepsilon < \varepsilon_2 + \Delta\varepsilon_2 \\ P_2 + s_2\left[\varepsilon - (\varepsilon_2 + \Delta\varepsilon_2)\right], & \varepsilon > \varepsilon_2 + \Delta\varepsilon_2\ . \end{cases}$$

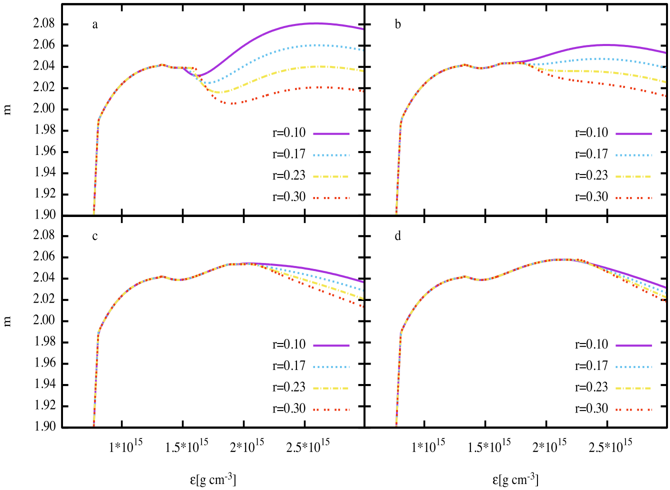
## Need to specify:

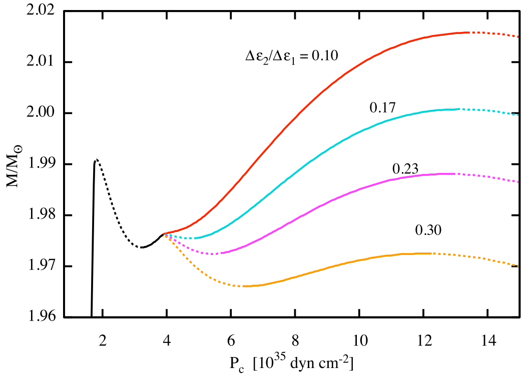
- the two speeds of sounds: $s_1$ and $s_2$
- the point of transition from NM to QM $\varepsilon_1, P_1$
- the magnitude of the first jump $\Delta\varepsilon_1$
- the size of the 2SC phase, i.e, the second transition point $\varepsilon_2, P_2$
- the size of the second jump $\Delta\varepsilon_2$

# Varying parameters of EoS with sequential phase transition



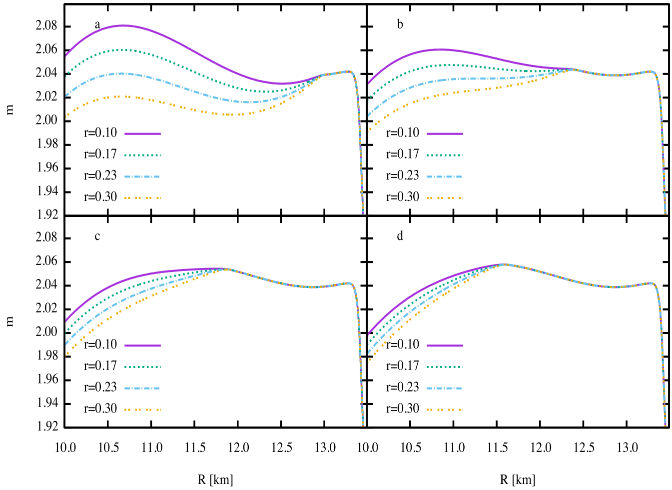
# .... and resulting topologies of sequences

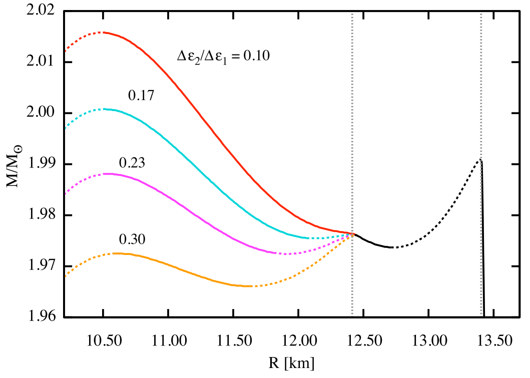





The stellar mass as a function of the star's central pressure for four different values of $\Delta\varepsilon_2$. The other parameters of the EOS are fixed at $P_1 = 1.7 \times 10^{35}$ dyn cm$^{-2}$, $s_1 = 0.7$, $\Delta\varepsilon_{2SC}/\varepsilon_1 = 0.27$, $\Delta\varepsilon_1/\varepsilon_1 = 0.6$, and $s_2 = 1$. The vertical dotted lines mark the two phase transitions at $P_1$ and $P_2$. Stable branches are solid lines, unstable branches are dashed lines. We see the emergence of separate 2SC and CFL hybrid branches along with the occurrence of triplets.

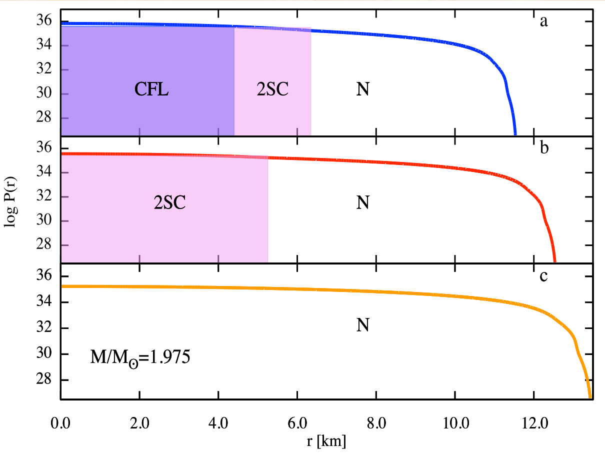
# .... and resulting topologies of mass-radius relations






The *M*-*R* relations for the parameter values defined above . We have fixed the properties of the nuclear $\rightarrow$ 2SC transition and the speed of sound in 2SC and CFL matter. For the 2SC $\rightarrow$ CFL transition we have fixed the critical pressure and we vary the energy-density discontinuity $\Delta\varepsilon_2$. The separate 2SC and CFL hybrid branches are clearly visible, along with the occurrence of triplets.

# Profiles of triplets stars (same mass)



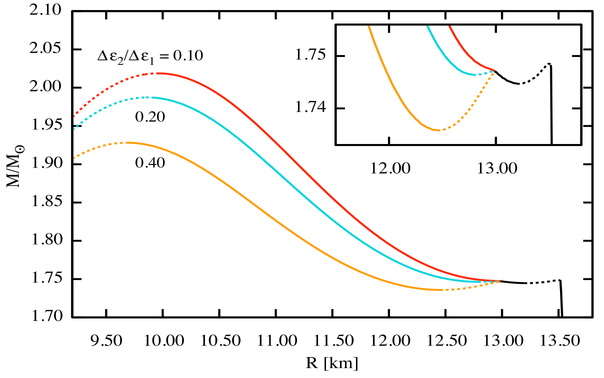

The profiles (here the log of pressure as a function of the internal radius) of the three members of a triplet with masses $M = 1.975\ \mathrm{M}_{\odot}$. Here "$N$" means the nuclear phase. The parameter values are as above, with $\Delta\varepsilon_2/\Delta\varepsilon_1 = 0.23$.

# Stability conditions for our models

| $\Delta\varepsilon_2/\Delta\varepsilon_1$ | $\Delta\varepsilon_1/\varepsilon_1$ 0.4 | 0.5 | 0.6 | 0.7 |
|---|---|---|---|---|
| 0.1 | $s, s$ | $s, s$ | $\underbrace{us, s}_{\text{N-2SC}}$ | $\underbrace{u, us}_{\text{N-CFL}}$ |
| 0.2 | $s, s$ | $s, s$ | $\underbrace{us, us}_{\text{triplet}}$ | $\underbrace{u, us}_{\text{N-CFL}}$ |
| 0.3 | $s, s$ | $s, s$ | $\underbrace{us, us}_{\text{N-2SC;N-CFL}}$ | $\underbrace{u, us}_{\text{N-CFL}}$ |
| 0.4 | $s, s$ | $\underbrace{s, us}_{\text{2SC-CFL}}$ | $\underbrace{us, u}_{\text{N-2SC}}$ | $u, u$ |
| 0.5 | $s, s$ | $\underbrace{s, us}_{\text{2SC-CFL}}$ | $\underbrace{us, u}_{\text{N-2SC}}$ | $u, u$ |

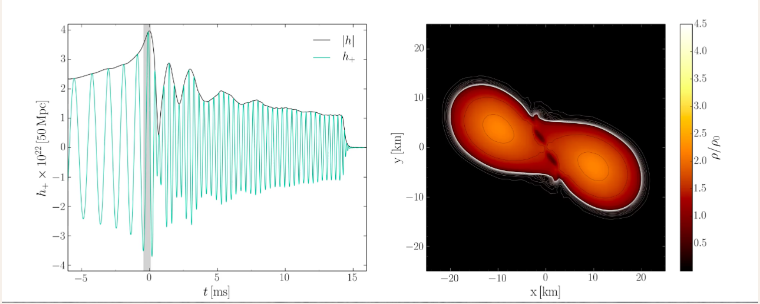
In each entry stable/unstable branches are referred by $s/u$, the 2SC and CFL phases are separated by comma, and the pressure increases from left to right. The presence of twin hybrid configurations or triplet configurations is marked by the underbraces with information about the involved phases ("N" means nuclear).

# Lower mass triplets




- Low-mass triplets via early transition NM$\rightarrow$ QM
- Still 2-solar mass members possible but only with the NM-2SC-CFL composition

## GW170817: First gravitational waves from a neutron star merger (Ligo-Virgo-Collaboration)

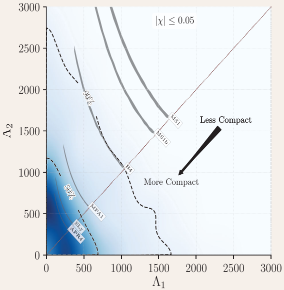


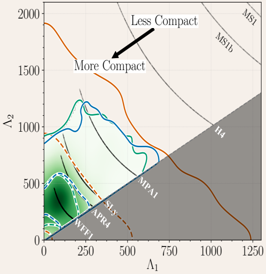

The associated EM events observed by over 70 observatories :

- + 2sec gamma ray burst is detected
- +10 h 52 min bright source in optical
- +11 h 36 min infrared emission; +15 h ultraviolet
- +9 days X-rays; +16 days radio

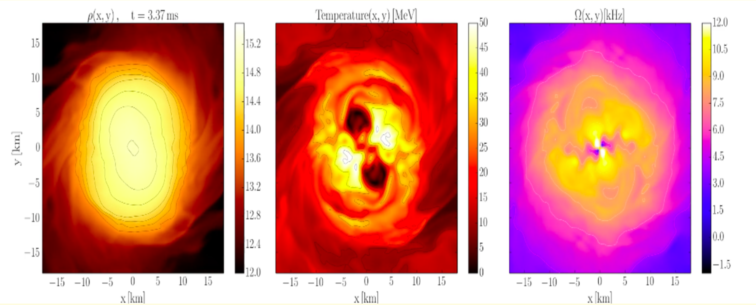
TABLE I. Source properties for GW170817: we give ranges encompassing the 90% credible intervals for different assumptions of the waveform model to bound systematic uncertainty. The mass values are quoted in the frame of the source, accounting for uncertainty in the source redshift.

| | Low-spin priors ($\|\chi\| \leq 0.05$) | High-spin priors ($\|\chi\| \leq 0.89$) |
|---|---|---|
| Primary mass $m_1$ | 1.36–1.60 $M_\odot$ | 1.36–2.26 $M_\odot$ |
| Secondary mass $m_2$ | 1.17–1.36 $M_\odot$ | 0.86–1.36 $M_\odot$ |
| Chirp mass $\mathcal{M}$ | $1.188^{+0.004}_{-0.002} M_\odot$ | $1.188^{+0.004}_{-0.002} M_\odot$ |
| Mass ratio $m_2/m_1$ | 0.7–1.0 | 0.4–1.0 |
| Total mass $m_{\rm tot}$ | $2.74^{+0.04}_{-0.01} M_\odot$ | $2.82^{+0.47}_{-0.09} M_\odot$ |
| Radiated energy $E_{\rm rad}$ | $> 0.025 M_\odot c^2$ | $> 0.025 M_\odot c^2$ |
| Luminosity distance $D_{\rm L}$ | $40^{+8}_{-14}$ Mpc | $40^{+8}_{-14}$ Mpc |
| Viewing angle $\Theta$ | $\leq 55°$ | $\leq 56°$ |
| Using NGC 4993 location | $\leq 28°$ | $\leq 28°$ |
| Combined dimensionless tidal deformability $\tilde{\Lambda}$ | $\leq 800$ | $\leq 700$ |
| Dimensionless tidal deformability $\Lambda(1.4 M_\odot)$ | $\leq 800$ | $\leq 1400$ |



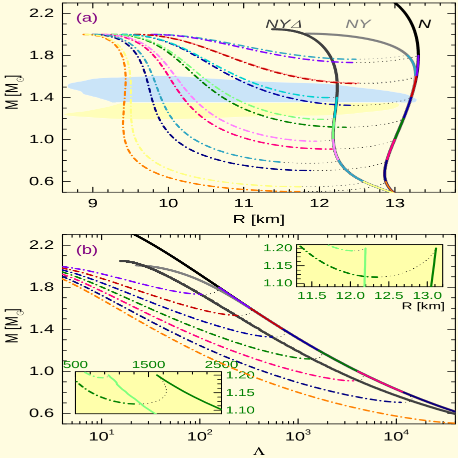



## New nuclear physics labora tories

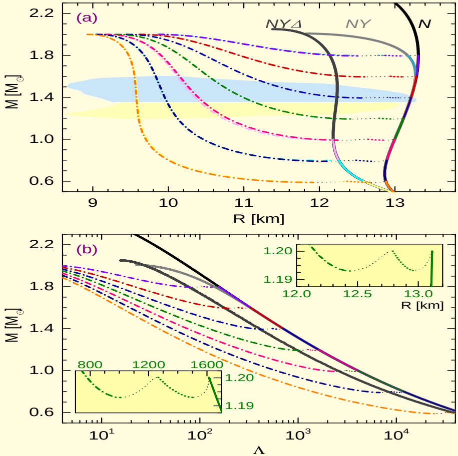



pictures courtesy: J. Pappenfort

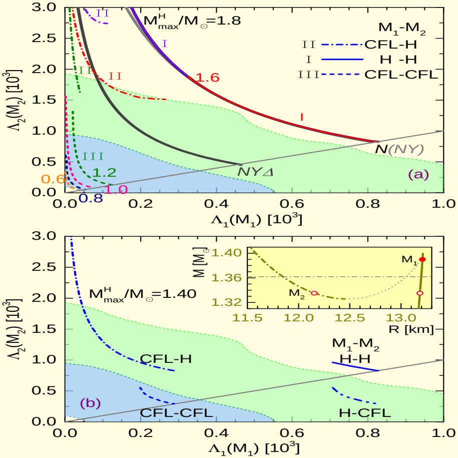
- extreme high temperatures $\sim$ 100 MeV
- supra-nuclear densities $\sim 5 \times n_s$
- high and differential rotation rates



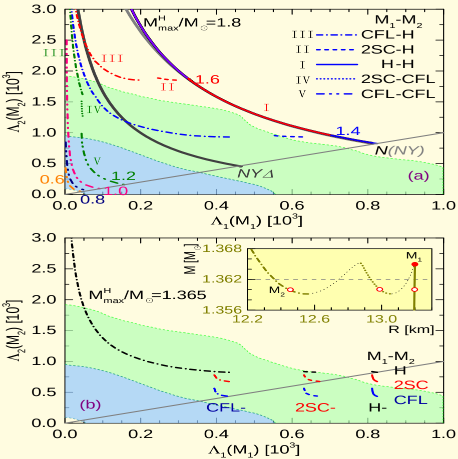

(a) Mass-radius relation for hybrid stars with a single QCD phase translation, with different hadronic envelopes. (b) Mass-deformability relation for stars featuring nucleonic envelopes. The inset shows the results for the case $M^{\rm H}_{\rm max}/M_{\odot} = 1.20$. (J.-J- Li et al, in preparation.)




(a) Mass-radius relation for hybrid stars with a single QCD phase translation, with different hadronic envelopes. (b) Mass-deformability relation for stars featuring nucleonic envelopes. The inset shows the results for the case $M^{\rm H}_{\rm max}/M_\odot = 1.20$. (J.-J- Li et al, in preparation.)



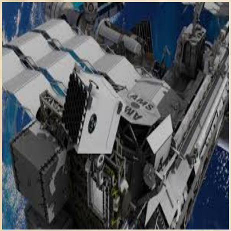

a) Tidal deformabilities of compact objects in the binary with chirp mass $\mathcal{M} = 1.186 M_\odot$ (b) Prediction by an EoS with maximal hadronic mass $M^H_{max} = 1.365 M_\odot$. The inset shows the mass-radius relation around the phase transition region. The circles $M_2$ are two possible companions for circle $M_1$, generating two points in the $\Lambda_1$-$\Lambda_2$ curves while one point is located below the diagonal line. (J.-J- Li et al, in preparation.)

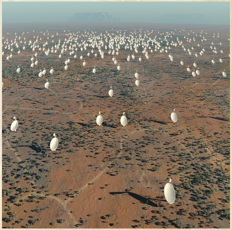



The case of double phase transition a) Tidal deformabilities of compact objects in the binary with chirp mass $\mathcal{M} = 1.186 M_{\odot}$ (b) Prediction by an EoS with maximal hadronic mass $M^{\rm H}_{\rm max} = 1.365 M_{\odot}$. The inset shows the mass-radius relation around the phase transition region. The circles $M_2$ are two possible companions for circle $M_1$, generating two points in the $\Lambda_1$-$\Lambda_2$ curves while one point is located below the diagonal line. (J.-J- Li et al, in preparation.)

**Near future experimental advances:**

- NICER (X-ray studies of neutron stars)
- LIGO-VIRGO (Gravitational waves from BNS and pulsars)
- SKA (radio timing of pulsars)





**Theory questions:**

- Dense QCD phases: static and dynamic properties
- Astrophysical properties of compact stars with quark phases
- Triplets and twins
- Gravity wave and QCD

*Thank you for your attention!*