Bulk viscosity of neutrino-trapped baryonic matter in neutron star mergers

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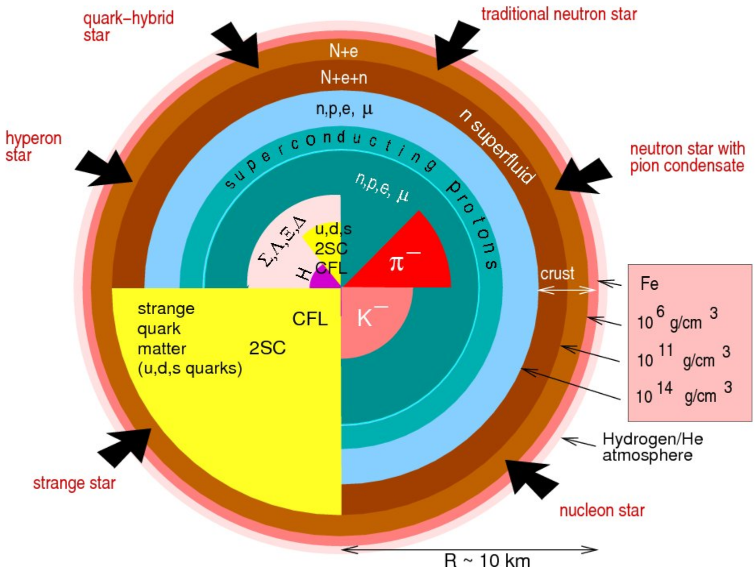
Outline

- Introduction & motivation
- Urca processes and bulk viscosity
- Numerical results
- Conclusions

The structure of a neutron star



Compact-star binaries

- Compact stars are natural laboratories which allow us to study the properties of nuclear matter under extreme physical conditions (strong gravity, strong magnetic fields, etc.).
- The recent detection of gravitational and electromagnetic waves originating from black hole or neutron star mergers motivates studies of compact binary systems.
- Such studies might place constraints on the properties of compact star parameters and contain useful information about the properties of extremely hot and dense matter.
- Various physical processes in the compact binary systems can be modelled in the framework of general-relativistic hydrodynamics simulations.
- Transport coefficients (viscosities, conductivities, etc.) are key inputs in hydrodynamic modelling of binary compact star mergers.
- The bulk viscosity might affect the hydrodynamic evolution of neutron star mergers by damping the density oscillations which can be detected from gravitational signals.

- Our aim is to study the bulk viscosity in dense baryonic matter for temperatures relevant to neutron star mergers and supernovas $T \geq 5$ MeV.
- At these temperatures neutrinos are trapped in matter, and the bulk viscosity arises from weak interaction (neutron decay and electron capture) processes.

Literature on bulk viscosity

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Urca process rates

- We consider a simple composition of baryonic matter consisting of neutrons, protons, electrons and neutrinos. The simplest weak-interaction processes are the following (direct) Urca processes

$$n \rightleftarrows p + e^- + \bar{\nu}_e \qquad (\textit{neutron decay process}) \qquad (1)$$

$$p + e^- \rightleftarrows n + \nu_e \qquad (\textit{electron capture process}) \qquad (2)$$

- In β-equilibrium the chemical potentials of particles obey the relation $\mu_n + \mu_\nu = \mu_p + \mu_e$. Out of β-equilibrium in general implies an imbalance

$$\mu_\Delta \equiv \mu_n + \mu_\nu - \mu_p - \mu_e \neq 0.$$

as a measure of deviation from β-equilibrium. The rate at which μ_Δ relaxes to zero is a measure of speed at which the constitution of matter adjusts to a change in pressure.

- The β-equilibration rate for the neutron decay is given by

$$\Gamma_{1p}(\mu_\Delta) = \int d\Omega \sum_{s_i} |\mathcal{M}_{Urca}|^2 f(p')\bar{f}(k')\bar{f}(k)\bar{f}(p)\,(2\pi)^4\,\delta^{(4)}(p+k+k'-p'),$$

with $\bar{f}(p) = 1 - f(p)$. Similar expressions can be written also for Γ_{1n}, Γ_{2p} and Γ_{2n}.

- The squared matrix element of Urca processes is

$$\sum_{s_i} |\mathcal{M}_{Urca}|^2 = 32G^2(k\cdot p')(p\cdot k') \simeq 32G^2 p_0 p_0' k_0 k_0'.$$

Density oscillations in neutron-star matter

- Consider now small-amplitude density oscillations in baryonic matter with frequency ω

$$n_B(t) = n_{B0} + \delta n_B(t), \quad n_L(t) = n_{L0} + \delta n_L(t), \quad \delta n_B(t), \, \delta n_L(t) \sim e^{i\omega t}.$$

- The baryon and lepton number conservation $\partial n_i / \partial t + \mathrm{div}\,(n_i \boldsymbol{v}) = 0$ implies

$$\delta n_i(t) = -\frac{\theta}{i\omega} n_{i0}, \quad i = \{B, L\}, \quad \theta = \mathrm{div}\,\boldsymbol{v}.$$

- The oscillations cause perturbations in particle densities $n_j(t) = n_{j0} + \delta n_j(t)$, due to which the chemical equilibrium of matter is disturbed leading to a small shift $\mu_\Delta = \delta\mu_n + \delta\mu_\nu - \delta\mu_p - \delta\mu_e$, which can be written as

$$\mu_\Delta = (A_{nn} - A_{pn})\delta n_n + A_{\nu\nu}\delta n_\nu - (A_{pp} - A_{np})\delta n_p - A_{ee}\delta n_e, \qquad A_{ij} = \left(\frac{\partial \mu_i}{\partial n_j}\right)_0 .$$

- The off-diagonal elements A_{np} and A_{pn} are non-zero because of the cross-species strong interaction between neutrons and protons.
- If the weak processes are turned off, then a perturbation conserves all particle numbers

$$\frac{\partial}{\partial t}\delta n_j(t) + \theta n_{j0} = 0, \qquad \delta n_j(t) = -\frac{\theta}{i\omega} n_{j0}.$$

Chemical balance equations and bulk viscosity

- Out of equilibrium the chemical equilibration rate to linear order in μ_Δ is given by

$$\Gamma_p - \Gamma_n = \lambda \mu_\Delta, \quad \lambda > 0.$$

- The rate equations which take into account the loss and gain of particles read as

$$\frac{\partial}{\partial t}\delta n_n(t) = -\theta n_{n0} - \lambda \mu_\Delta(t), \qquad \frac{\partial}{\partial t}\delta n_p(t) = -\theta n_{p0} + \lambda \mu_\Delta(t).$$

- Solving these equations we can compute the pressure out of equilibrium

$$p = p(n_j) = p(n_{j0} + \delta n_j) = p_0 + \delta p = p_{\rm eq} + \delta p',$$

where the non-equilibrium part of the pressure - the bulk viscous pressure, is given by

$$\Pi \equiv \delta p' = \sum_j \left(\frac{\partial p}{\partial n_j}\right)_0 \delta n_j' = \sum_{ij} n_{i0} A_{ij} \delta n_j'.$$

- The bulk viscosity is then identified from $\Pi = -\zeta\theta$

$$\zeta = \frac{C^2}{A}\frac{\lambda A}{\omega^2 + \lambda^2 A^2}$$

with susceptibilities $A = -\frac{1}{n_B}\left(\frac{\partial \mu_\Delta}{\partial x_p}\right)_{n_B}$ and $C = n_B\left(\frac{\partial \mu_\Delta}{\partial n_B}\right)_{x_p}$.

Low-temperature limit of bulk viscosity

- In chemical equilibrium the conditions of the detailed balance are satisfied

$$\Gamma_{1p} = \Gamma_{1n} \equiv \Gamma_1, \qquad \Gamma_{2p} = \Gamma_{2n} \equiv \Gamma_2.$$

- In the low-temperature limit (degenerate matter) we have the following results

$$\Gamma \equiv \Gamma_1 + \Gamma_2 = \frac{m^{*2} G^2}{12\pi^3} T^3 p_{Fe} p_{F\nu} (p_{Fe} + p_{F\nu} - |p_{Fn} - p_{Fp}|), \quad \lambda = \frac{\Gamma}{T}.$$

- The “beta-disequilibrium–proton-fraction” susceptibility is given by

$$A = \frac{\pi^2}{m^*} \left(\frac{1}{p_{Fn}} + \frac{1}{p_{Fn}} \right) + \frac{\pi^2}{p_{Fe}^2} + \frac{2\pi^2}{p_{F\nu}^2} + \left(\frac{g_\rho}{m_\rho} \right)^2.$$

- The “beta-disequilibrium–baryon-density” susceptibility reads

$$C = \frac{p_{Fn}^2 - p_{Fp}^2}{3m^*} + \frac{p_{F\nu} - p_{Fe}}{3} + \frac{n_n - n_p}{2} \left(\frac{g_\rho}{m_\rho} \right)^2 + n_B \frac{p_{Fn}^2 - p_{Fp}^2}{2m^{*2}} \left(\frac{g_\sigma}{m_\sigma} \right)^2.$$

Beta-equilibrated nuclear matter

We use the density functional theory approach to the nuclear matter, which is based on phenomenological baryon-meson Lagrangians of the type proposed by Walecka and others. The Lagrangian density of matter is written as $\mathcal{L} = \mathcal{L}_N + \mathcal{L}_L$, where

$$\begin{aligned}\mathcal{L}_N &= \sum_N \bar{\psi}_N \left[\gamma^\mu \left(i\partial_\mu - g_{\omega B}\omega_\mu - \frac{1}{2} g_{\rho N} \boldsymbol{\tau}\cdot\boldsymbol{\rho}_\mu\right) - (m_N - g_{\sigma N}\sigma)\right]\psi_N \\ &+ \frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{4}\omega^{\mu\nu}\omega_{\mu\nu} + \frac{1}{2}m_\omega^2\omega^\mu\omega_\mu - \frac{1}{4}\boldsymbol{\rho}^{\mu\nu}\boldsymbol{\rho}_{\mu\nu} + \frac{1}{2}m_\rho^2\boldsymbol{\rho}^\mu\cdot\boldsymbol{\rho}_\mu,\end{aligned}$$

The leptonic contribution is given by

$$\mathcal{L}_L = \sum_\lambda \bar{\psi}_\lambda (i\gamma^\mu\partial_\mu - m_\lambda)\psi_\lambda .$$

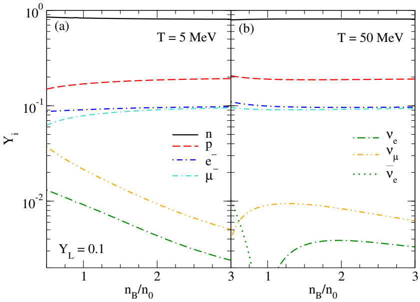
The pressure of baryonic matter is given by

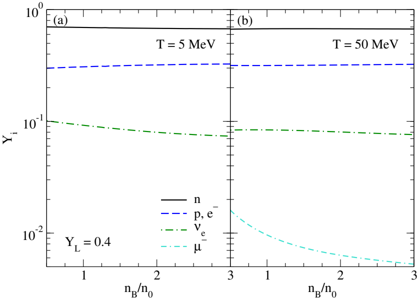
$$\begin{aligned}P_N = -\frac{m_\sigma^2}{2}\sigma^2 + \frac{m_\omega^2}{2}\omega_0^2 + \frac{m_\rho^2}{2}\rho_{03}^2 + \frac{g_\lambda}{3\pi^2}\sum_\lambda \int_0^\infty \frac{k^4\, dk}{(k^2+m_\lambda^2)^{1/2}}\left[f(E_k^\lambda - \mu_\lambda) + f(E_k^\lambda + \mu_\lambda)\right] \\ + \frac{1}{3}\sum_N \frac{2J_N+1}{2\pi^2}\int_0^\infty \frac{k^4\, dk}{(k^2+m_N^{*2})^{1/2}}\left[f(E_k^N - \mu_N^*) + f(E_k^N + \mu_N^*)\right],\end{aligned}$$

where $m_N^* = m_N - g_{\sigma N}\sigma$ and $\mu_N^* = \mu_N - g_{\omega N}\omega_0 - g_{\rho N}\rho_{03} I_3$ are the nucleon effective mass and effective chemical potentials, respectively.

Particle fractions in equilibrium

The particle fractions are found from β-equilibrium conditions $\mu_n + \mu_\nu = \mu_p + \mu_e$ and $\mu_\mu = \mu_e$, the charge neutrality condition $n_p = n_e + n_\mu$, the baryon number conservation $n_B = n_n + n_p$, and the lepton number conservation $n_l + n_{\nu_l} = n_L = Y_L n_B$.

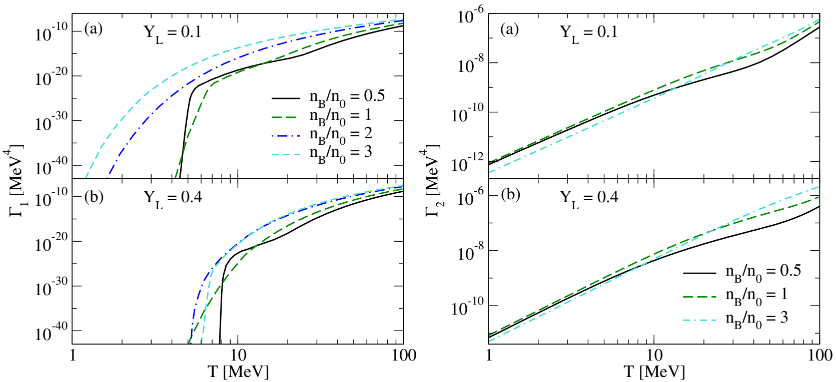




- We consider two cases: (i) $Y_L = 0.1$ for both flavors, typical for neutron star mergers; (ii) $Y_{L_e} = 0.4$ and $Y_{L_\mu} = 0$ typical for matter in supernovae and proto-neutron stars.
- The particle fractions are not sensitive to the temperature for the given value of Y_L.
- In the low-density and high-temperature regime the net neutrino density becomes negative, indicating that there are more anti-neutrinos than neutrinos in that regime.
- Merger matter has much smaller electron neutrino fraction than supernova matter.

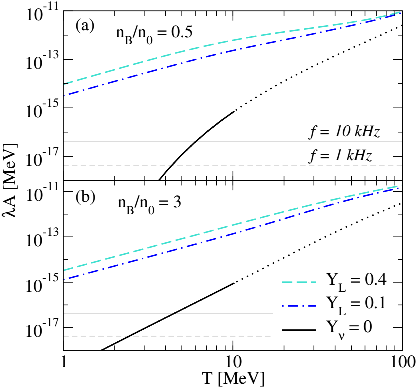
β-equilibration rates

$n \rightleftarrows p + e^- + \bar{\nu}_e$ $\qquad$ $p + e^- \rightleftarrows n + \nu_e$



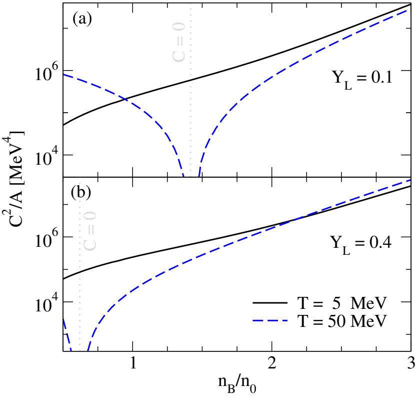
- The neutron decay rate Γ_1 is exponentially suppressed at low temperatures because of damping of anti-neutrino population in the degenerate matter.
- The electron capture rate Γ_2 has a finite low-temperature limit which is $\propto T^3$.
- In the regime of interest $\Gamma_1 \ll \Gamma_2$, therefore the electron capture process dominates in the β-equilibration and the bulk viscosity.

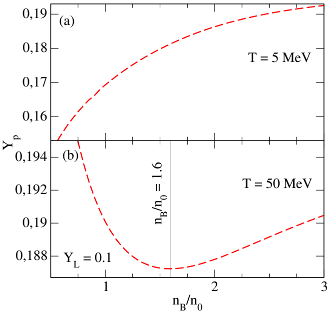
β-relaxation rate



- The β-relaxation rate λA determines the frequency at which the bulk viscosity reaches its resonant maximum ($\omega_{\text{max}} = \lambda A$).
- The relaxation rate is slowest in the neutrino-transparent case, and increases with the lepton fraction in the neutrino-trapped case.
- In neutrino-trapped matter $\lambda A \gg \omega$ for oscillation frequencies typical to neutron star mergers and supernovas $\quad\Rightarrow\quad$ the bulk viscosity takes the form $\zeta \approx C^2/(\lambda A^2)$.
- The neutrino-transparent matter instead features a relaxation rate which is comparable to the oscillation frequencies at typical temperatures $T \simeq 2 \div 7$ MeV.

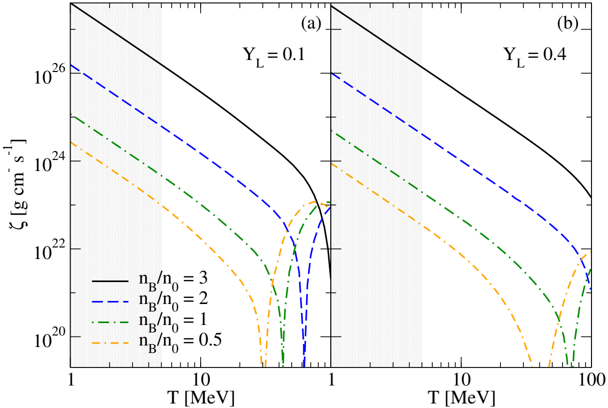
The susceptibility prefactor C^2/A





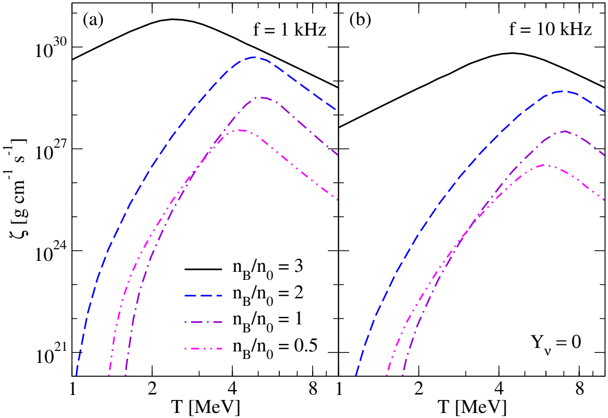
- The susceptibility A does not depend strongly on the density and temperature and has roughly the same order of magnitude $A \sim 10^{-3}$ MeV^{-2}.
- The susceptibility C increases with density and at high temperatures $T \gtrsim 30$ MeV crosses zero at certain values of density where the proton fraction attains a minimum.
- At this critical density the system is scale-invariant: it can be compressed and remain in beta equilibrium $\Rightarrow$ the bulk viscosity drops to zero at the critical point.

Bulk viscosity of neutrino-trapped matter



- The density dependence of the bulk viscosity follows that of the susceptibility C^2/A.
- The temperature dependence of ζ arises mainly from that of β-relaxation rate $\lambda A \propto T^2$.
- Bulk viscosity in independent of oscillation fequency and decreases as $\zeta \propto T^{-2}$ in the neutrino-trapped regime.
- This scaling breaks down at high temperatures $T \geq 30$ MeV where the bulk viscosity has sharp minimums $\zeta \to 0$ when the matter becomes scale-invariant.

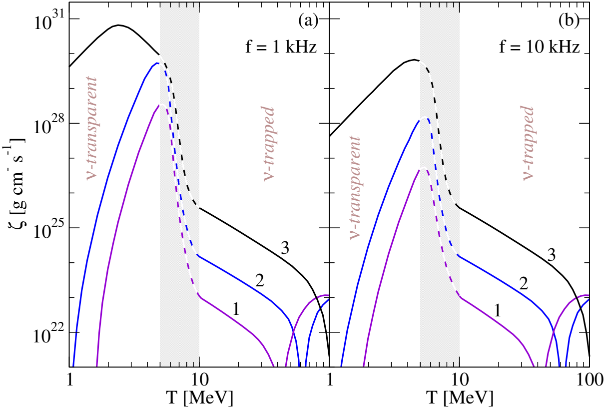
Bulk viscosity of neutrino-transparent matter



- The bulk viscosity in neutrino-transparent matter is frequency-dependent.
- The relaxation rate is slower for neutrino-transparent matter, and the resonant peak of the bulk viscosity occurs within its regime of validity.
- It attains its maximum value at temperature $T \simeq 2 \div 7$ MeV, where $\omega = \lambda A$.
- This is the temperature range which is relevant for neutron-star mergers. [1]

[1] M. G. Alford, *et al.*, On the importance of viscous dissipation and heat conduction in binary neutron-star mergers, 2017

Bulk viscosity of baryonic matter



- We interpolate our numerical results for the bulk viscosity between the neutrino-transparent and neutrino-trapped regimes in the interval $5 \leq T \leq 10$ MeV.
- The bulk viscosity in the neutrino transparent regime is larger, and drops by orders of magnitude as the matter enters the neutrino-trapped regime.
- The bulk viscosity attains its maximum at temperatures $T \simeq 2 \div 6$ MeV.

Estimation of oscillation damping timescale

- The energy density of baryonic oscilations with amplitude δn_B is

$$\epsilon = \frac{K}{2}\frac{(\delta n_B)^2}{n_B}.$$

- Coefficient K is the compressibility of nuclear matter

$$K = n_B \frac{\partial^2 \varepsilon}{\partial n_B^2}.$$

- The enegy dissipation rate per volume by bulk viscosity is

$$\frac{d\epsilon}{dt} = \frac{\omega^2 \zeta}{2}\left(\frac{\delta n_B}{n_B}\right)^2.$$

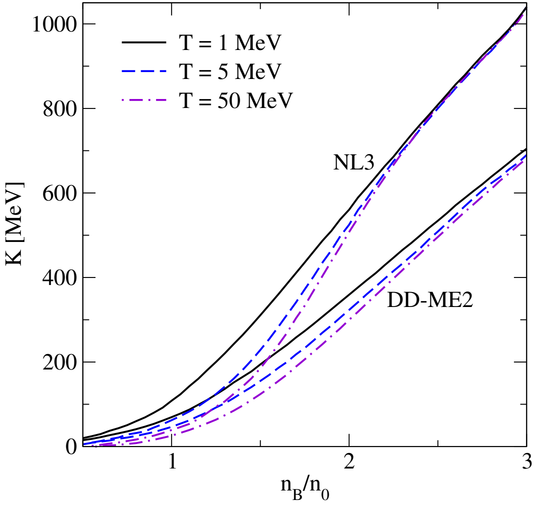
- The characteristic timescale required for dissipation is $\tau = \epsilon/(d\epsilon/dt)$

$$\tau = \frac{K n_B}{\omega^2 \zeta}.$$

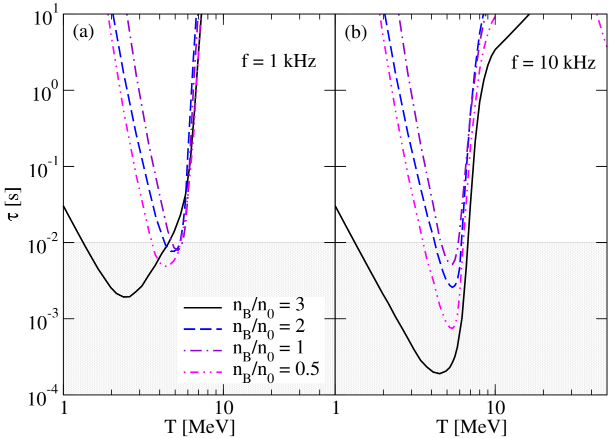
- In the high-frequency ($\omega \gg \lambda A$), low-frequency ($\omega \ll \lambda A$) limits and at the maximum of the bulk viscosity ($\omega = \lambda A$) we find

$$\tau_{\rm high} = \frac{K n_B}{\lambda C^2}, \qquad \tau_{\rm low} = \frac{\lambda^2 A^2}{\omega^2}\frac{K n_B}{\lambda C^2}, \qquad \tau_{\rm min} = 2\frac{K n_B}{\lambda C^2}.$$

Nuclear compressibility for two EoS

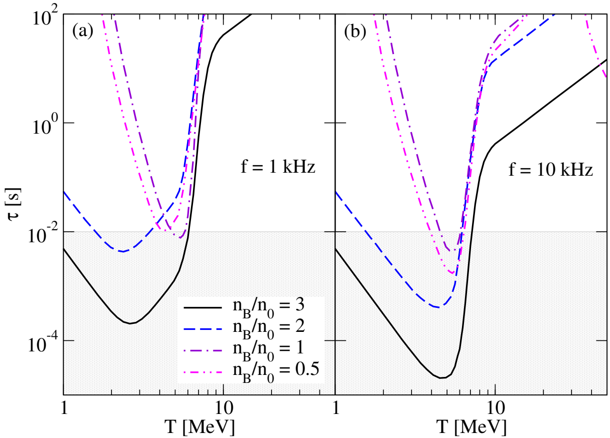


Oscillation damping timescale for DD-ME2 model



- Damping timescales are comparable to the merging timescales $\tau_{\rm merg} \simeq 10$ ms at temperatures $T \lesssim 7$ MeV where neutrinos are not trapped.
- Therefore, bulk viscosity will have its greatest impact on neutron star mergers in regions that are neutrino transparent rather than neutrino trapped.
- This also implies weak damping of gravitational waves emitted by the oscillations of the post-merger remnant in the high-temperature, neutrino-trapped phase of evolution.

Oscillation damping timescale for NL3 model



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- This also implies weak damping of gravitational waves emitted by the oscillations of the post-merger remnant in the high-temperature, neutrino-trapped phase of evolution.

THANK YOU FOR ATTENTION!