Constraining neutron star equation of state with GW170817

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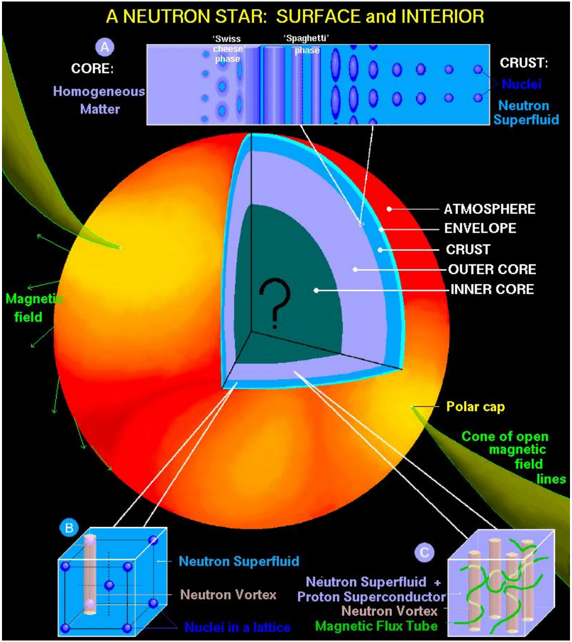
ECT* - European Center for Theoretical Studies in Nuclear Physics and Related Areas



courtesy Dany Page.

Outline

- Introduction.
- Equation of state in Density Dependent Hadron Field Theory.
- Constraining equation of state (low density part) with GW170817
- Probing high density part of equation of state with g-modes.
- Frequency shift of g-mode with companion tide
- Coupling of modes with tide and resulting instability (Static and non-static cases)

Introduction

- Neutron stars provide cosmic laboratory in which the phases of **cold, dense and strongly interacting** nuclear matter are realized.
- Exotic particles are supposed to be present in neutron star core at supranuclear densities but these exotic particles soften the equation of state and cannot explain maximum neutron star mass, this is called **hyperon puzzle**.
- Low density part of EOS can be constrained using tidal deformability limit obtained from GW170817.
- High density part of EOS can be probed via g-modes.
- Different oscillation modes reach different depths inside the star. Here g-modes can reach the core of the star and hence are important for unveiling the core of Star.

Equation of state

- Lorentz covariant theory of dense matter involving baryons and meson(σ, ω, ρ) is called **Walecka Model**.
- Density Dependent hadron Field Theory is the extension of Walecka Model in which phase transition from hadronic to antikaon condensed matter is considered.
- The hadronic phase is made of different species of the baryon octet along with electrons and muons making a uniform background.
- The Lagrangian density for baryons is:
-

$$\begin{aligned}\mathcal{L}_B &= \sum_{B=N,\Lambda,\Sigma,\Xi} \bar{\psi}(i\gamma_\mu\partial^\mu - m_B + g_{\sigma B}\sigma - g_{\omega B}\gamma_\mu\omega^\mu + g_{\rho B}\gamma_\mu t_B.\rho^\mu)\psi_B \\ &+ \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2) - \frac{1}{4}\omega_{\mu\nu}\omega\mu\nu \\ &+ \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\rho_{\mu\nu}.\rho^{\mu\nu} + \frac{1}{2}m_\rho^2\rho_\mu.\rho^\mu \end{aligned} \tag{1}$$

- Leptons are treated as non-interacting particles and described by the Lagrangian density:

$$\mathcal{L}_l = \sum_l \bar{\psi}_l (i\gamma_\mu \partial^\mu - m_l)\psi_l \tag{2}$$

- The $g_{\alpha B}(\hat{n})$ where $\alpha = \sigma, \omega, \rho$ specify the coupling strength of the mesons with baryons and are vector density-dependent. The density operator has the form, $\hat{n} = \sqrt{\hat{j}_\mu \hat{j}^\mu}$, where $\hat{j}_\mu = \bar{\psi}\gamma_\mu\psi$.
- An additional vector meson ϕ and a scalar meson σ^* are also included, they are important for the the hyperon-hyperon interaction only .

- Interaction among hyperons is represented by the Lagrangian density:

$$\begin{aligned}\mathcal{L}_{YY} &= \sum_B \bar{\psi}_B(g_{\sigma^* B}\sigma^* - g_{\phi B}\gamma_\mu \phi^\mu)\psi_B \\ &+ \frac{1}{2}(\partial_\mu \sigma^* \partial^\mu \sigma^* - m_{\sigma^*}^2 \sigma^{*2}) \\ &- \frac{1}{4}\phi_{\mu\nu}.\phi^{\mu\nu} + \frac{1}{2} m_\phi^2 \phi_\mu.\phi^\mu \end{aligned} \tag{3}$$

- Since the attractive hyperon-hyperon interaction mediated by σ^* meson is very weak , so We neglect the contribution of σ^* meson in this calculation.
- ϕ mesons do not couple with nucleons i.e. $g_{\phi N} = 0$.
- The total Lagrangian density $\mathcal{L} = \mathcal{L}_B + \mathcal{L}_I + \mathcal{L}_{YY}$.

Anti-kaon Condensate

- The Lagrangian density for (anti)kaons in the minimal coupling scheme is given by :

$$\mathcal{L}_K = D^*_\mu \bar{K} D^\mu K - m_K^{*2} \bar{K} K \tag{4}$$

where the covariant derivative
$D_\mu = \partial_\mu + ig_{\omega K}\omega_\mu + ig_{\rho K}\tau_K.\rho_\mu + ig_{\phi K}\phi_\mu$ and effective mass
$m^*_K = m_K - g_{\sigma K}\sigma$

- For s-wave $(k = 0)$ $\bar{K}$ condensation, the in-medium energies of $\bar{K} \equiv (K^-, \bar{K}^0)$ are given by:

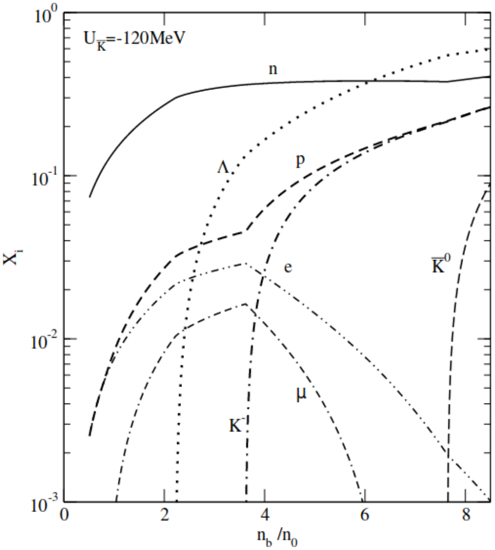
$$\omega_{K^-,\bar{K}^0} = m^*_K - g_{\omega K}\omega_0 - g_{\phi K}\phi_0 \mp g_{\rho K}\rho_{03} \tag{5}$$

The scalar and vector densities of antikaons are given by:

$$n_{K^-,\bar{K}^0} = 2(\omega_{K^-,K^0} + g_{\omega K}\omega_0 + g_{\phi K}\phi_0 \pm g_{\rho K}\rho_{03})\bar{K}K \tag{6}$$

- The requirement of chemical equilibrium fixes the onset condition of antikaon condensations in neutron star matter.

$\mu_n - \mu_p = \mu_{K^-} = \mu_e \; , \mu_{\bar{K}^0} = 0$



Credit: Prasanta char et al.

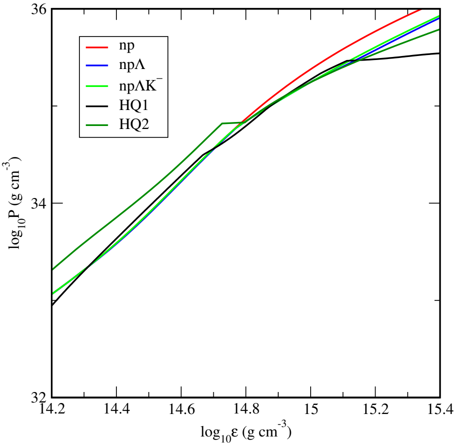
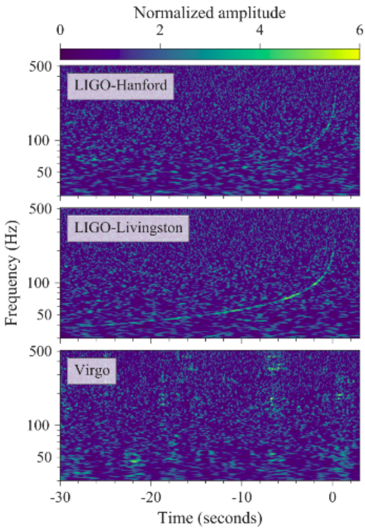


Figure: Pressure versus energy is plotted for different compositions of matter.

GW170817



Ligo-Virgo Collaboration

- A hypermassive neutron star was believed to have formed initially, as evidenced by the large amount of ejecta (much of which would have been swallowed by an immediately forming black hole).
- The lack of evidence for emissions being powered by neutron star spin-down, which would occur for longer-surviving neutron stars, suggest it collapsed into a black hole within milliseconds.
- GR simulations show that immediately after the NSNS merger the remnant is a differentially rotating NS that has undergone shock heating following contact. The configuration either settles into quasie-quilibrium in a couple of rotation periods, or under-goes promptcollapse.
- A combination of GW emission, magnetic winding, turbulent viscosity and neutrino cooling radiate away or redistribute some of its angular momentum,driving the core toward rigid rotation

M. Ruiz,S. L. Shapiro and A. Tsokaros P R D97,021501(R) (2018)

- The neutron star merger parameters given by GW170817 are given in table:

	Low-spin priors ($|\chi| \leq 0.05$)	High-spin priors ($|\chi| \leq 0.89$)
Primary mass m_1	1.36–$1.60\ M_\odot$	1.36–$2.26\ M_\odot$
Secondary mass m_2	1.17–$1.36\ M_\odot$	0.86–$1.36\ M_\odot$
Chirp mass $\mathcal{M}$	$1.188^{+0.004}_{-0.002} M_\odot$	$1.188^{+0.004}_{-0.002} M_\odot$
Mass ratio m_2/m_1	0.7–1.0	0.4–1.0
Total mass $m_{\rm tot}$	$2.74^{+0.04}_{-0.01} M_\odot$	$2.82^{+0.47}_{-0.09} M_\odot$
Radiated energy $E_{\rm rad}$	$> 0.025 M_\odot c^2$	$> 0.025 M_\odot c^2$
Luminosity distance D_L	40^{+8}_{-14} Mpc	40^{+8}_{-14} Mpc
Viewing angle Θ	$\leq 55°$	$\leq 56°$
Using NGC 4993 location	$\leq 28°$	$\leq 28°$
Combined dimensionless tidal deformability $\tilde{\Lambda}$	≤ 800	≤ 700
Dimensionless tidal deformability $\Lambda(1.4 M_\odot)$	≤ 800	≤ 1400

Credit: LIGO Scientific Collaboration

Neutron star merger parameters according to new analysis:

	Low-spin prior ($\chi \leq 0.05$)	High-spin prior ($\chi \leq 0.89$)
Binary inclination θ_{JN}	146^{+25}_{-27} deg	152^{+21}_{-27} deg
Binary inclination θ_{JN} using EM distance constraint [104]	151^{+15}_{-11} deg	153^{+15}_{-11} deg
Detector frame chirp mass $\mathcal{M}^{\mathrm{det}}$	$1.1975^{+0.0001}_{-0.0001} \mathrm{M}_\odot$	$1.1976^{+0.0004}_{-0.0002} \mathrm{M}_\odot$
Chirp mass $\mathcal{M}$	$1.186^{+0.001}_{-0.001} \mathrm{M}_\odot$	$1.186^{+0.001}_{-0.001} \mathrm{M}_\odot$
Primary mass m_1	$(1.36,\ 1.60)\ \mathrm{M}_\odot$	$(1.36,\ 1.89)\ \mathrm{M}_\odot$
Secondary mass m_2	$(1.16,\ 1.36)\ \mathrm{M}_\odot$	$(1.00,\ 1.36)\ \mathrm{M}_\odot$
Total mass m	$2.73^{+0.04}_{-0.01} \mathrm{M}_\odot$	$2.77^{+0.22}_{-0.05} \mathrm{M}_\odot$
Mass ratio q	$(0.73,\ 1.00)$	$(0.53,\ 1.00)$
Effective spin χ_{eff}	$0.00^{+0.02}_{-0.01}$	$0.02^{+0.08}_{-0.02}$
Primary dimensionless spin χ_1	$(0.00,\ 0.04)$	$(0.00,\ 0.50)$
Secondary dimensionless spin χ_2	$(0.00,\ 0.04)$	$(0.00,\ 0.61)$
Tidal deformability $\tilde{\Lambda}$ with flat prior	300^{+500}_{-190}(symmetric)/ 300^{+420}_{-230}(HPD)	$(0, 630)$

Credit: LIGO Scientific Collaboration

- Moving from the upper right corner to lower left corner the equation of state changes from stiffer to softer and the star changes from being less compact to more compact.

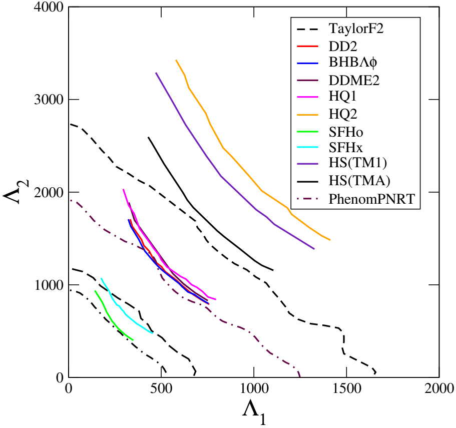
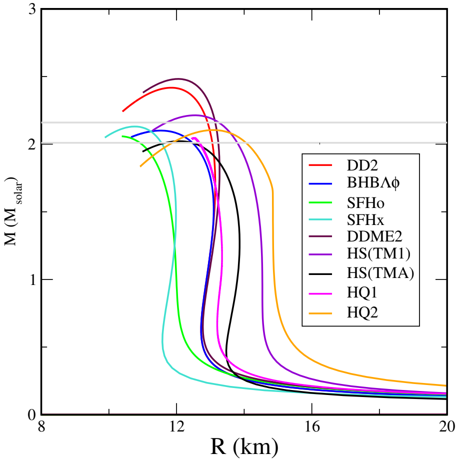


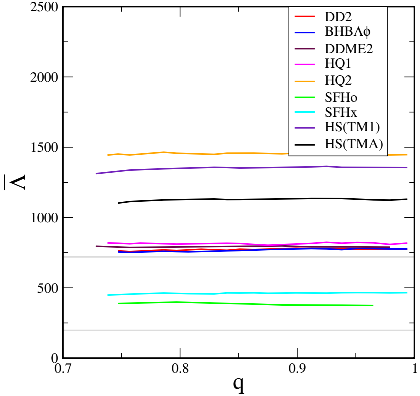
Figure: Tidal deformability of lower mass component versus tidal deformability of higher mass component with respect to 50% and 90% probability contours.


DD2
BHBΛϕ
SFHo
SFHx
DDME2
HS(TM1)
HS(TMA)
HQ1
HQ2
M (M_solar)
R (km)
0
1
2
3
8
12
16
20

- The dimensionless effective tidal deformability is defined as

$$\tilde{\Lambda} = \frac{16}{13}\frac{(m_1 + 12m_2)m_1^4\Lambda_1 + (m_2 + 12m_1)m_2^4\Lambda_2}{(m_1 + m_2)^5} \tag{7}$$

- The effective tidal deformability as a function of mass ratio $q = m_2/m_1$ is shown in figure below:



- Here I have taken (1.58, 1.18) mass combination of merger components.

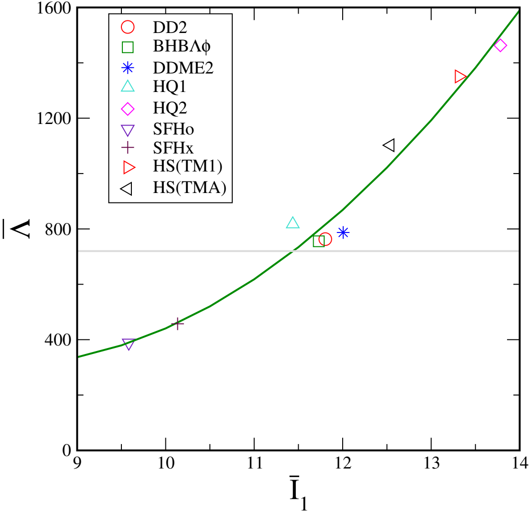


Figure: Effective Tidal deformability $\tilde{\Lambda}$ vs moment of inertia of heavier component $\bar{I}_1$.

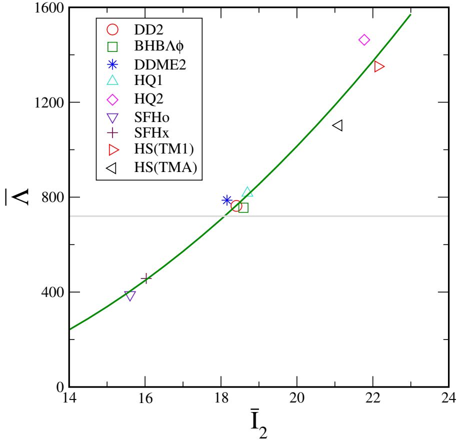
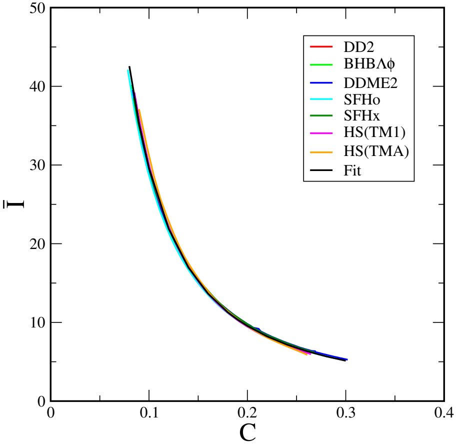


Figure: Effective Tidal deformability $\tilde{\Lambda}$ vs moment of inertia of lighter component $\bar{I}_2$.

- The values of I_1 and I_2 so obtained are $\sim 2.0 \times 10^{45} gcm^2$ and $\sim 1.2 \times 10^{45} gcm^2$,respectively.

- Tidal deformability constrains radii of merger components to be $\sim 13km$.



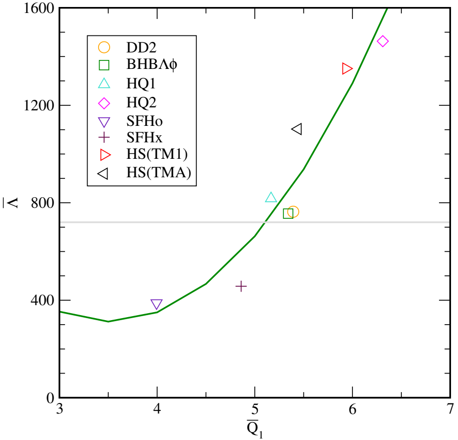


Figure: Effective Tidal deformability $\tilde{\Lambda}$ vs quadrupole moment of heavier component $\bar{Q}_1$.

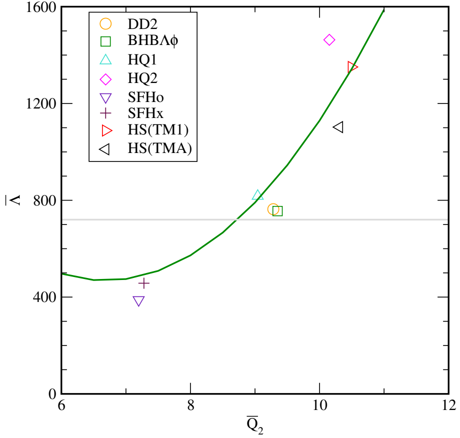


Figure: Effective Tidal deformability $\tilde{\Lambda}$ vs quadrupole moment of lower component $\bar{Q}_2$.

- Quadrupole moments are found to be in the range $0.29 - 0.30 \times 10^{43} gcm^2$.

Probing high density part of EOS with g-modes

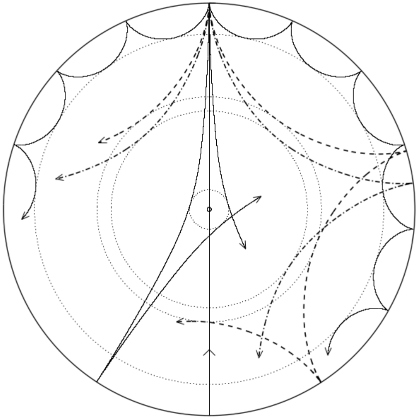
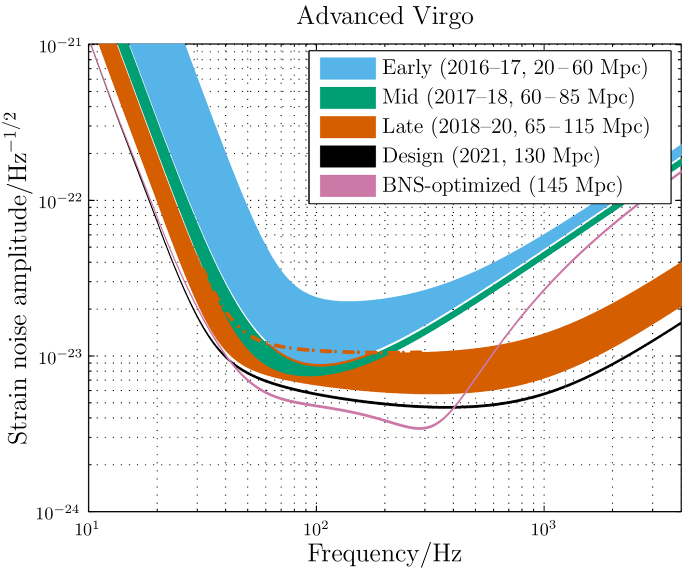
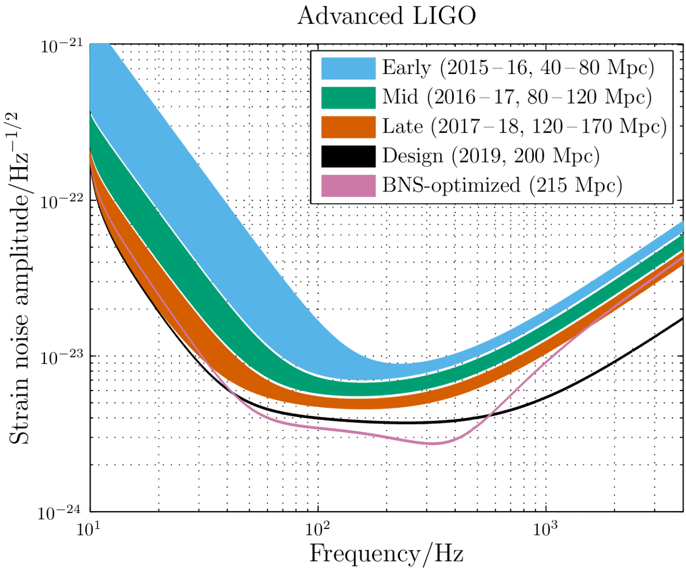


Figure: Different oscillation modes reach different depths inside the star. Here g-modes can reach the core of the star and hence are important for unveiling the core of Star



Credit: LIGO Scientific Collaboration



Credit: LIGO Scientific Collaboration

g-modes

- Using Newton's equation, continuity equation and Poisson equation, g-mode oscillation equations are given by

$$\frac{d\eta_r}{dr} = \left(\frac{gr}{c_s^2} - 2\right)\frac{\eta_r}{r} + \left[l(l+1) - \frac{\omega^2 r^2}{c_s^2}\right]\frac{\eta_\perp}{r},$$
$$\frac{d\eta_\perp}{dr} = \left(1 - \frac{\omega_{BV}^2}{\omega^2}\right)\frac{\eta_r}{r} + \left(\frac{\omega_{BV}^2 r}{g} - 1\right)\frac{\eta_\perp}{r}.$$

- Boundary conditions are

$$\eta_r = A\,l r^{l-1}, \qquad \eta_\perp = A\,r^{l-1} \qquad (r \to 0)$$

$$\begin{aligned}\Delta p &= (\delta + \boldsymbol{\xi}\cdot\nabla)p \\ &= (\omega^2 r\eta_\perp - g\eta_r)\rho = 0 \qquad (r = R)\end{aligned}$$

arXiv:1701.00418 [hep-ph]

- Stellar oscillations of g-mode originate from the buoyancy in the star and, thus, the eigen-frequency of g-mode is intimately linked to Brunt-Vaisala frequency ω_{BV}.
- The eigen frequencies of f-mode and p-mode are therefore related only with the sound speed of the stellar matter.
- Buoyancy frequency is given by

$$N^2 \equiv \mathfrak{g}^2 \left(\frac{1}{c_e^2} - \frac{1}{c_s^2} \right)$$

$$N^2 \approx -\frac{\mathfrak{g}^2}{c_e^4} \left(\frac{\partial P}{\partial Y_p} \right)_\rho \left(\frac{dY_p}{d\rho} \right)$$

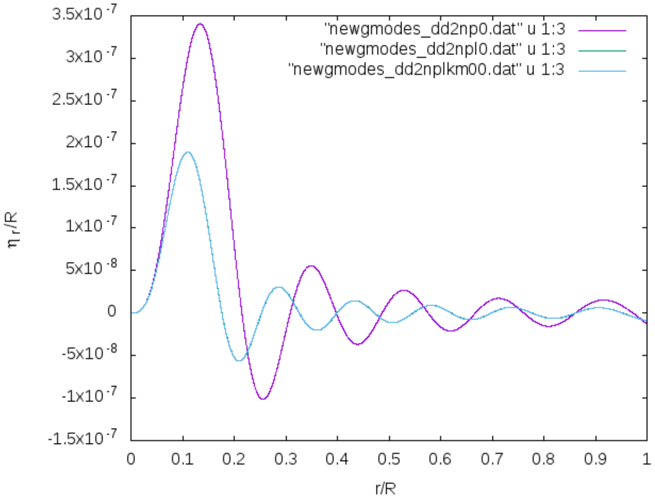


Figure: Radial function for g-mode with frequency 100Hz

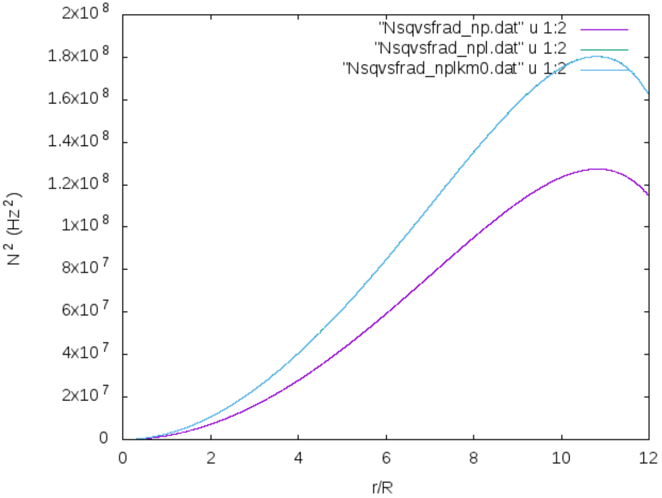


Figure: Radial Profile of Buoyancy Frequency inside neutron star for different EOSs

Shifting of g-mode frequency with external tide

- Lagrangian displacement due to a mode is given as

$$\vec{\xi}_a = \xi_r(r) Y_{l_a m_a}(\theta,\phi)\hat{r} + \xi_h(r)\left(\partial_\theta Y_{l_a m_a}(\theta,\phi)\hat{\theta} + \frac{1}{\sin\theta}\partial_\phi Y_{l_a m_a}(\theta,\phi)\hat{\phi}\right)$$

- Orthonormality of the basis is given by

$$\int d^3\vec{x}\rho\vec{\xi}^*_a\cdot\vec{\xi}_b = \frac{E_0}{\omega_a^2}\delta_{ab},$$

  where $E_0 = \frac{GM}{R^2}$ is the energy per unit mode amplitude.

- We can decompose a generic displacement to a fluid in the star into basis modes as

$$\vec{\gamma} = \sum_a \chi_a \vec{\xi}_a$$

- When a collection of basis modes are excited, they will evolve as per their contribution in total potential energy and couple nonlinearly with coupling constants:

$$\kappa_{abc} = -\frac{1}{2E_0}\int d^3\vec{x}\rho f_3\left(\vec{\xi_a},\vec{\xi_b},\vec{\xi_c}\right),$$
$$\kappa_{abcd} = -\frac{1}{6E_0}\int d^3\vec{x}\rho f_4\left(\vec{\xi_a},\vec{\xi_b},\vec{\xi_c},\vec{\xi_d}\right)$$

- where f_n represents the form of potential energy of modes at nth order.
- If we add a static tide to the potential energy , the original g-modes will be perturbed by ϵU and the equations of motion will change.

- Applying usual perturbation theory , the shifted g-mode frequency will be given by

$$
\begin{aligned}
\frac{\omega_-^2}{\omega_g^2} =& 1-\epsilon\left(U_{\bar{g}g}+\sum_a 2\kappa_{a\bar{g}g}\chi_a^{(1)}\right) \\
&-\epsilon^2\sum_{a,b}\left(2\kappa_{a\bar{g}g}\chi_a^{(2)}+3\kappa_{ab\bar{g}g}\chi_a^{(1)}\chi_b^{(1)}\right) \\
&-\epsilon^2\frac{\omega_p^2}{\omega_p^2-\omega_g^2}\left|U_{\bar{p}g}+\sum_a 2\kappa_{a\bar{p}g}\chi_a^{(1)}\right|^2+\mathcal{O}(\epsilon^3)
\end{aligned}
$$

In this equation pair of daughter modes p and g couple to tide χ.

arXiv:1307.2890 [astro-ph.HE]

- χ and U_{ab} are given by

$$\vec{\chi} = \sum_a \left(\epsilon \chi_a^{(1)} + \epsilon^2 \chi_a^{(2)} \right) \vec{\xi_a},$$

$$U_{ab} = -\frac{1}{E_0} \int d^3\vec{x} \rho \vec{\xi_a} \cdot \left(\vec{\xi_b} \cdot \nabla \right) \nabla U$$

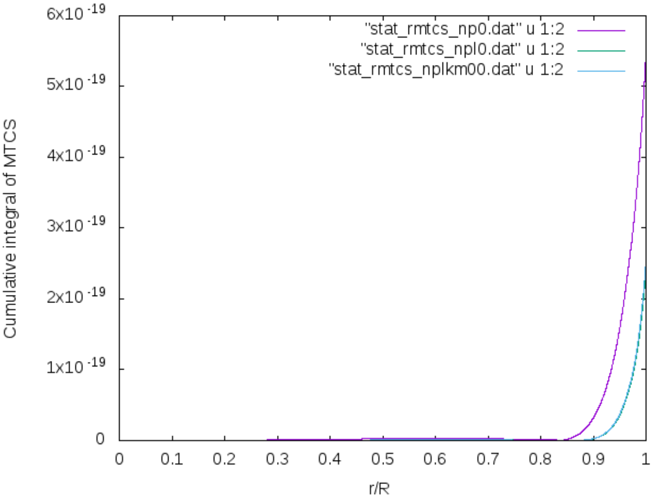
- χ is response of the neutron star to the static tide .
- Vector potential is given by

$$U_a = -\frac{1}{E_0} \int d^3x \rho \vec{\xi}_a^* \cdot \nabla U$$

- Three-mode coupling $\sum_a \kappa_{apg} \chi_a^{(1)} \sim \frac{\omega_p}{\omega_g}$ for higher order p and g modes can drive instability if no other cancellation. This is called non-resonant p-mode g-mode instability

- After VPT , the explicit four-mode coupling drops out and MTCS reduces to

$$\frac{\omega_-^2}{\omega_g^2} \approx 1 \pm \left[2\epsilon J_{gg}^{(1)} - \epsilon^2 \left(2\kappa_{gg\sigma} + V_{gg}\right)\right]$$



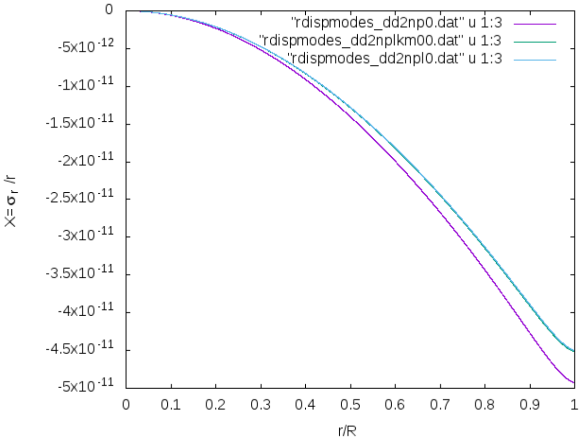


Figure: Profile of radial displacement measure in the neutron star

- σ_r represents the displacement of fluid element in the radially stretched star w.r.t to its position in unperturbed star.
- Static tidal force compresses the star. Soft EOS has smaller displacement because soft stars are denser and have higher inner pressure and hence more tightly bound.
- Coupling to tide tends to be stronger in stars with stiffer EOSs because stiffer EOSs have larger radii.
- MTCS strongly depends on large inhomogeneous terms which are inversely proportional to g . g is smaller for stiffer EOSs and hence stiffer EOSs give larger MTCS.
- Cancellation of three-mode and four-mode couplings is near exact in the core.
- Near crust-core interface , $X = \sigma_r / r$ inflates and hence near-exact cancellation is killed.

- Large intrinsic modal deformation g_r will ensure greater overlap with tidal deformation.
- Neutron star is assumed to be of fluid nature everywhere and crust and surface will not support g-modes.
- for fixed n,l_g, smaller N will give large g_r and hence larger MTCS.

$$g_r^2 \approx E_0/(\rho N^2)$$

- For static case instability is suppressed by cancellation between three-mode and four-mode couplings.

$$\frac{\omega_-^2}{\omega_g^2} \approx 1 - \left| 2\epsilon J_{gg}^{(1)} + \epsilon^2 |2\kappa_{gg\sigma} + V_{gg}| \right| > 0$$

Non-static Tide

- Tidal potential in case of non-static tide can be written as

$$U_{\text{full}} \approx -\omega_0^2 r^2 \sum_{m=-2}^{2} W_{2m} Y_{2m}(\theta, \phi) e^{-im\Omega t}$$

- Temporal variation doesn't preserve the volume of the star and resurrects instability.
- Non-static can play two parts. (a) Resonantly excite the eigen modes of the star and (b) Spoil the cancellation by changing the volume of the star.
- The perturbed g-mode frequency in case of non-static tide is

$$\frac{\omega_-^2}{\omega_g^2} = 1 - \epsilon \left| U_{gg} + 2\kappa_{\chi^{(1)} gg, I} + 2\kappa_{\chi^{(1)} gg, H} \right| + \mathcal{O}(\epsilon^2)$$

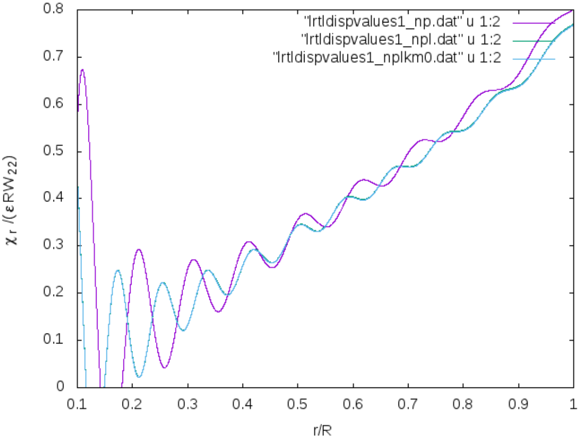
where the $O(\epsilon)$ term is no longer insignificant and drives the instability.

- The dynamic tide χ_r is given by the forced oscillation equations.

$$\frac{1}{r^2}\frac{d}{dr}(r^2\chi_r^{(1)}) - \frac{\mathfrak{g}}{c_s^2}\chi_r^{(1)} + \left(1 - \frac{l(l+1)c_s^2}{r^2(m\Omega)^2}\right)\frac{P'}{\rho c_s^2} - \frac{l(l+1)}{r^2(m\Omega)^2}\Phi' = -\epsilon\frac{l(l+1)\omega_0^2 W_{22}}{(m\Omega)^2},$$
$$\frac{1}{\rho}\frac{dP'}{dr} + \frac{\mathfrak{g}}{\rho c_s^2}P' + \left(N^2 - (m\Omega)^2\right)\chi_r^{(1)} + \frac{d\Phi'}{dr} = 2\epsilon\omega_0^2 r W_{22},$$
$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\Phi'}{dr}\right) - \frac{l(l+1)}{r^2}\Phi' - 4\pi G\rho\left(\frac{P'}{\rho c_s^2} + \frac{N^2}{\mathfrak{g}}\chi_r^{(1)}\right) = 0,$$

- χ_r represents the response of the fluid star to the external non-static tide.
- Non-linear coupling of non-static to normal modes results in shifting of frequency.

Unno, W., Osaki, Y., Ando, H., Saio, H., & Shibahashi, H. 1989, Nonradial oscillations of stars


"lrtldispvalues1_np.dat" u 1:2
"lrtldispvalues1_npl.dat" u 1:2
"lrtldispvalues1_nplkm0.dat" u 1:2
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
r/R
χ r /(ε RW22)

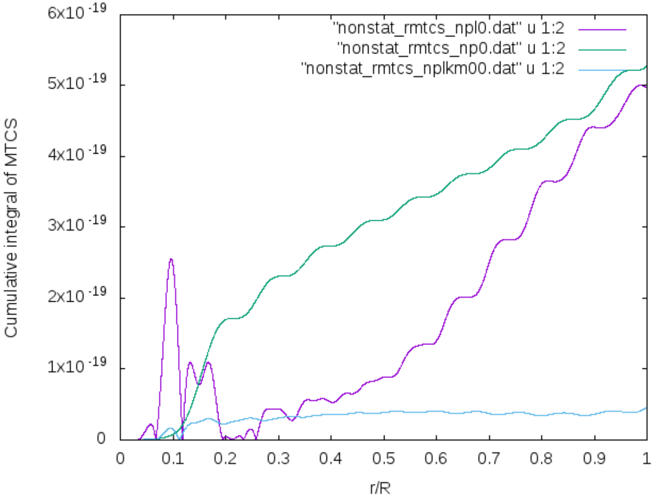


Figure: Cumulative integral of MTCS is ploted against fractional radius for different EOSs for dynamic tide.

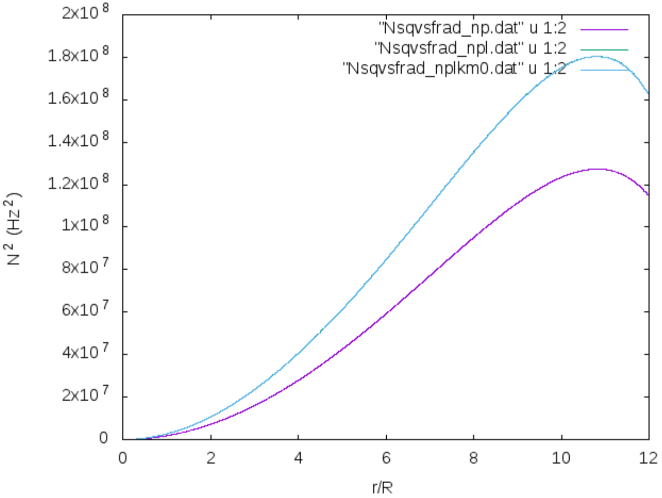


Figure: Radial Profile of Buoyancy Frequency inside neutron star for different EOSs

Important Factors affecting MTCS

- Onset of instability is possible only in case of non-static tide.
- The interdistance of the NSs in case of coalescing binary where instability is set on i.e. $\omega_- = 0$ is called threshold distance.
- Instability growth rate $\omega_g \sqrt(MTCS - 1)$ is not likely to be interrupted by collapse instability prior to merger because of tidal stabilization effect.
- Tidal stabilization effect may lead to three situations:
- (a) Instability may be terminated by dissipation and mode saturation.
- (b) Instability mode may continue to grow till merger.
- (c) Instability amplitude may grow to such an extent that it may tear apart neutron star before merger.
- Time interval between instability threshold and merger called Growth window is important because greater the growth window greater will be the amplitude growth of instability and hence greater chance of being detected.

- Smaller buoyancy frequency gives more intense mode-tide couplings whereas MTCS has a moderate dependence on stiffness and stiffness influences the tide through tidal deformation k_2.
- MTCS sensitiveness to EOS effects binary coalescences and in turn gives phase shift in GW Signal.
- If modes with with different EOSs give similar amplitudes , nevertheless they will reach saturation at different times and hence will imprint different phase shifts in the GW signal.
- If modes are weak still modal effects imprinted on orbital motion should be observable.
- Qualitative differences in EOS are expected to be printed on amplitude of GW signal.The accumulated phase , proportional to $A_{th}^{9/2}$, will be different for different EOSs because of different thesholds.

Limitations

- We have not included the details of whether the neutron star is spinning or non-spinning.
- Whether the neutron star has near-zero or finite temperature, that will affect the mode-tide couling strength.
- Buoyancy frequency will depend on whether to consider normal or superfluid neutron stars.
- Tidal dissipation allows for the transfer of orbital energy into oscillations and internal heating of the star.
- Dissipation is also not included which may have some effect on MTCS.

Summary

- We exploit the different equations of state involving **hyperons** and **Bose-Einstein condensate** of **antikaons** and **hadron to quark phase transition**.
- Multimessenger observations of GW170817 provided tidal deformability limits which I used to constraint EoS
- Moment of inertia and quadrupole moment of merger components are constrained using effective tidal deformability limit obtained from analysis of GW170817.
- Probing high density equation of state with g-modes.

THANK YOU