Phenomenology of the QCD phase transition

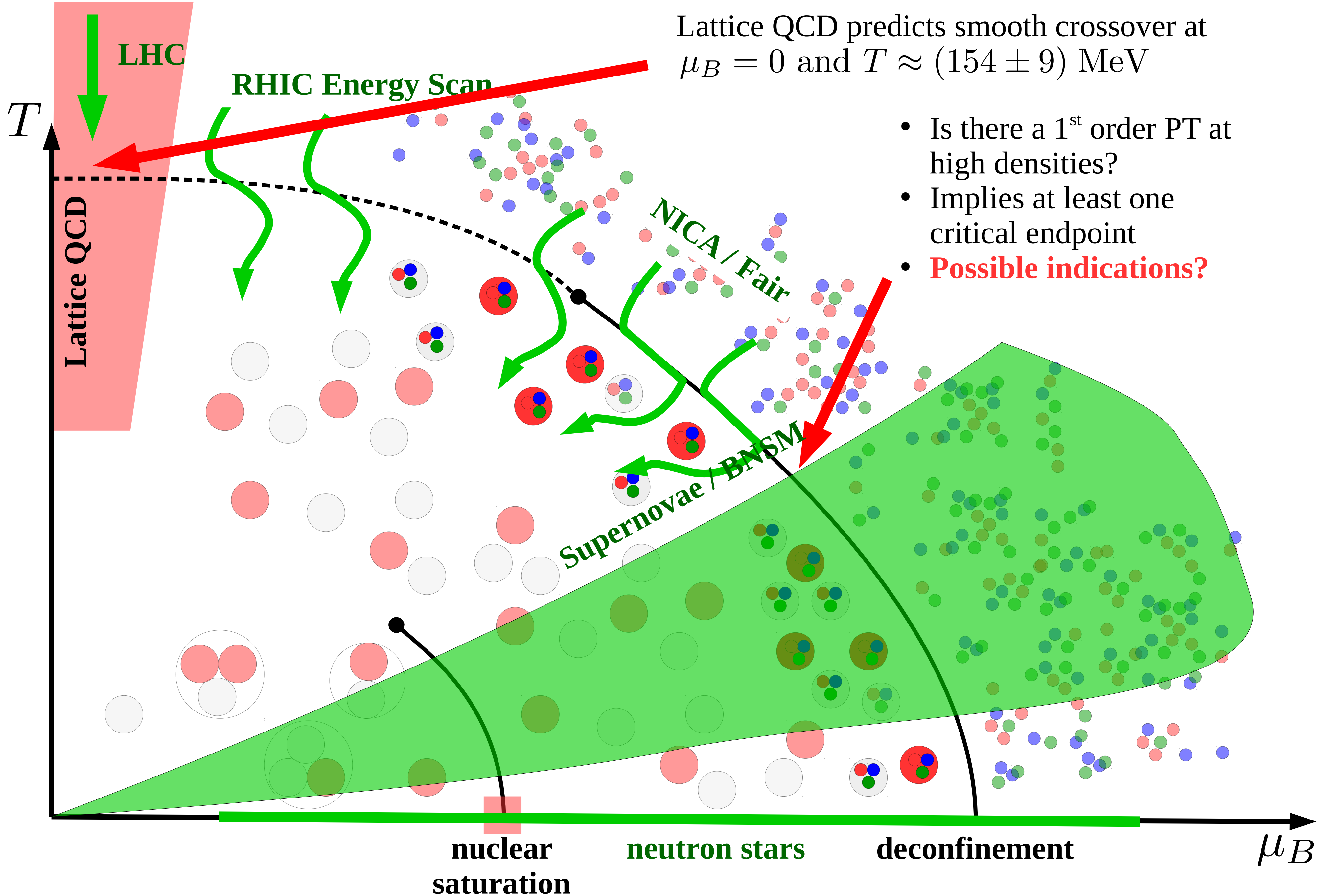
Niels-Uwe Friedrich Bastian
University of Wroclaw, Institute of Theoretical Physics

Trento, 14th of October 2019

Uniwersytet Wrocławski

NARODOWE CENTRUM NAUKI

Possibility of 1st order PT at high densities



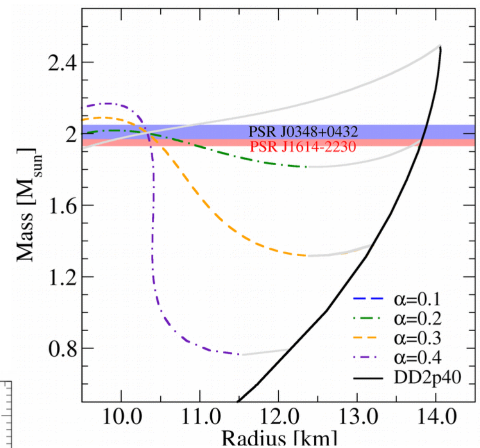
Lattice QCD predicts smooth crossover at $\mu_B = 0$ and $T \approx (154 \pm 9)$ MeV

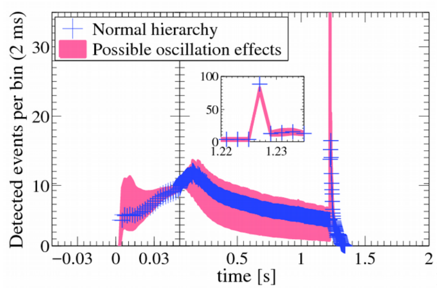
- Is there a 1st order PT at high densities?
- Implies at least one critical endpoint
- **Possible indications?**

Outline

Possible signals of 1st – order phase transitions.

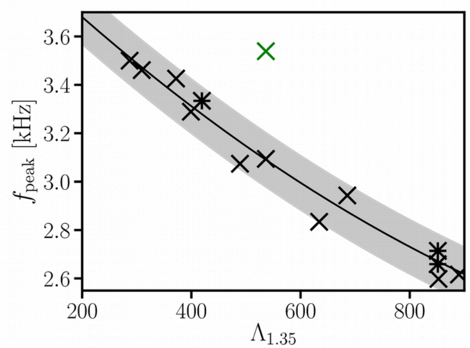
Neutron star configurations

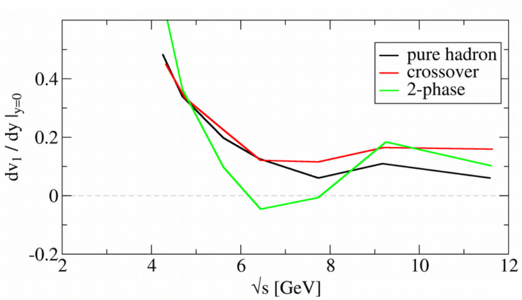




Supernova explosions of 50Ms stars

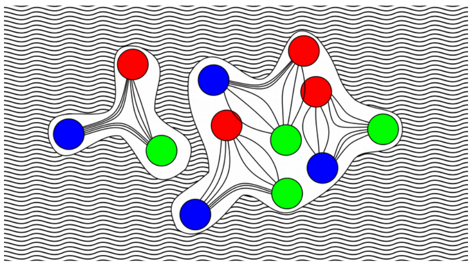
Binary neutron star mergers





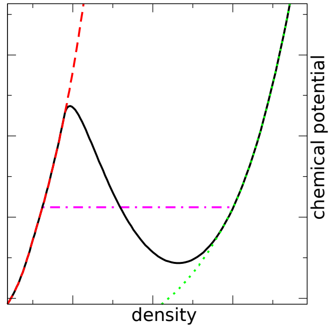
Heavy-Ion Collisions

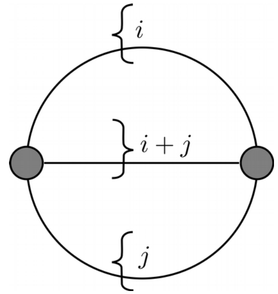
Unified description of the equation of state.



density functional theory

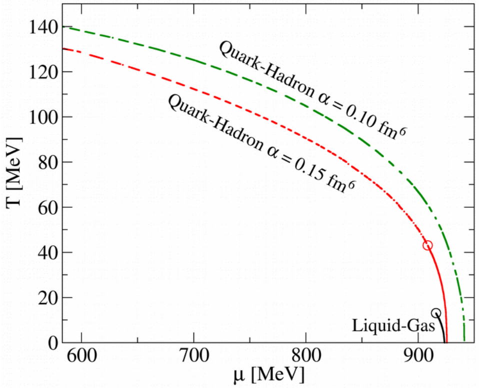
construction of phase transitions





Phi-derivable formalism

current status



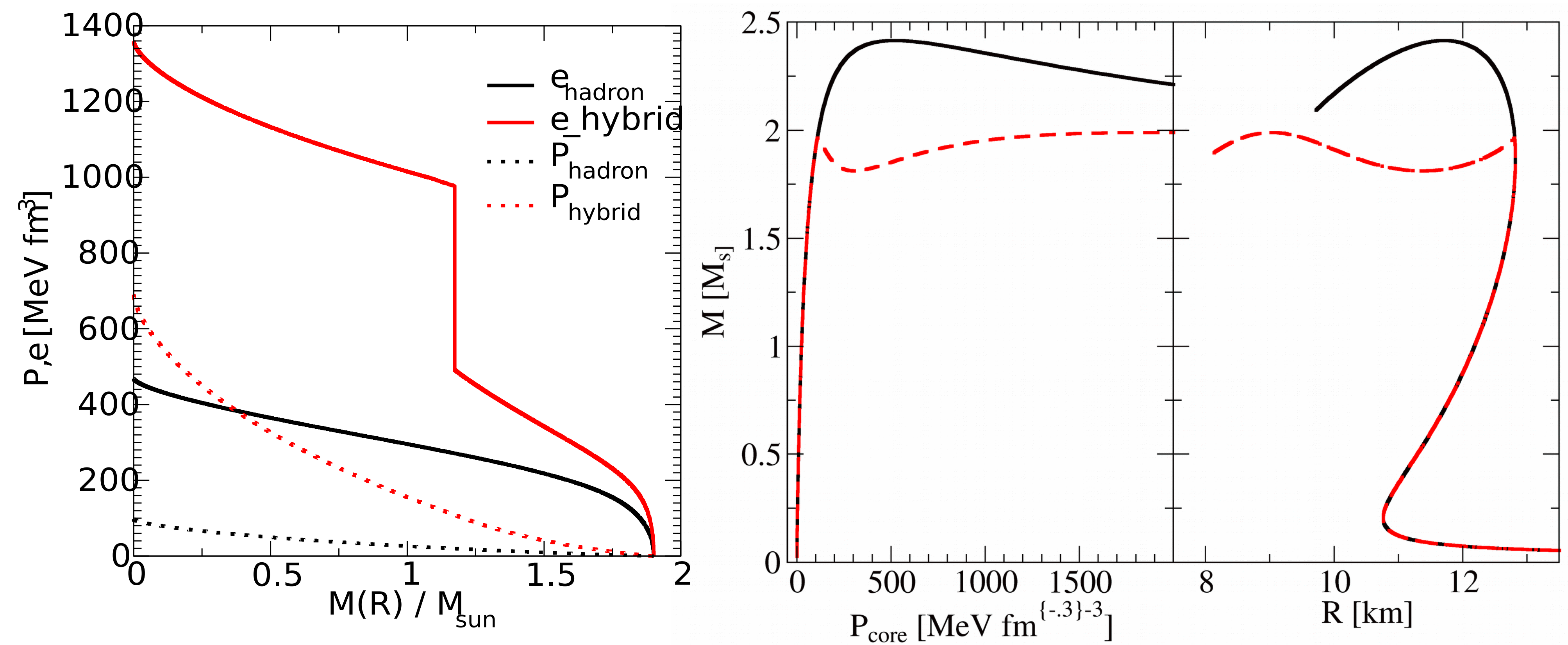
1^{st} order PT – Neutron stars

Tolman-Oppenheimer-Volkoff equations

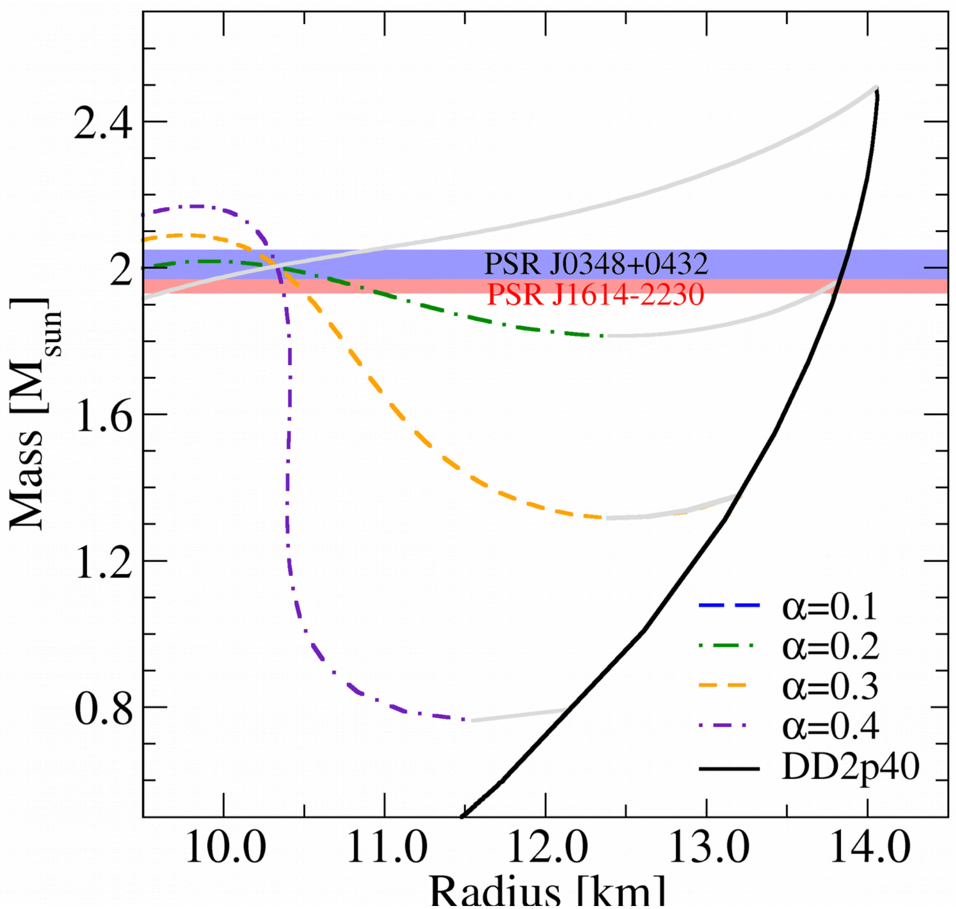
$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{Gm(r)\epsilon(r)}{r^2}\frac{[1+P(r)/\epsilon(r)]\left[1+4\pi r^3 P(r)/m(r)\right]}{1-2Gm(r)/r} \qquad \frac{\mathrm{d}m}{\mathrm{d}r} = 4\pi\epsilon(r)r^2$$

Needs an equation of state and the boundary condition of core density/pressure/energy density

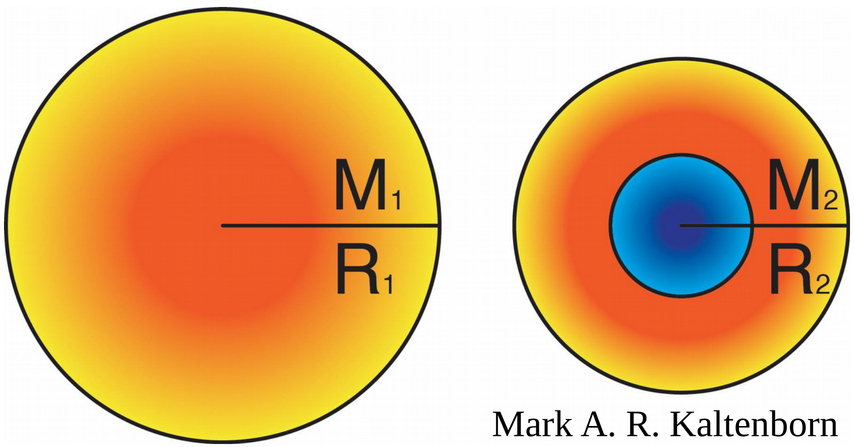
$$\epsilon = \epsilon(P) \qquad\qquad n_0,\ P_0,\ \epsilon_0$$



1st order PT – Neutron stars



- Star configurations with same masses, but different radii

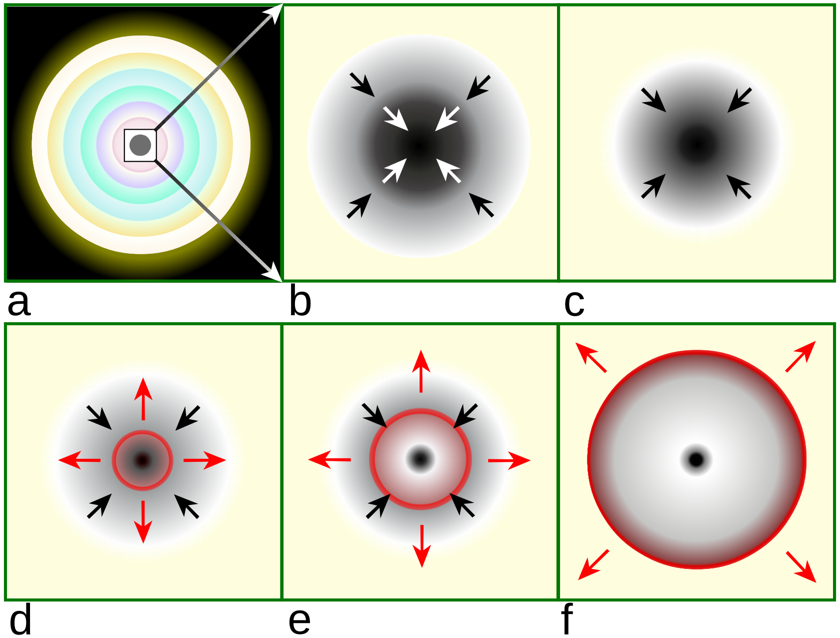


- **New class of EOS, that features high mass twins**
- NASA NICER mission: radii measurements ~ 0.5 km
- Existence of twins implies 1st order phase-transition and hence a critical point

M. Kaltenborn, **NUFB**, D. Blaschke, PRD 96, 056024 (2017)

Massive stars (~ 8 Ms)

- Sequential burning stages of light elements
- Onion structure with iron core (1.4 Ms)
- Gravitational collapse
- Bounce shock through stiffness of EOS
- Mainly neutrino heating drives shock wave

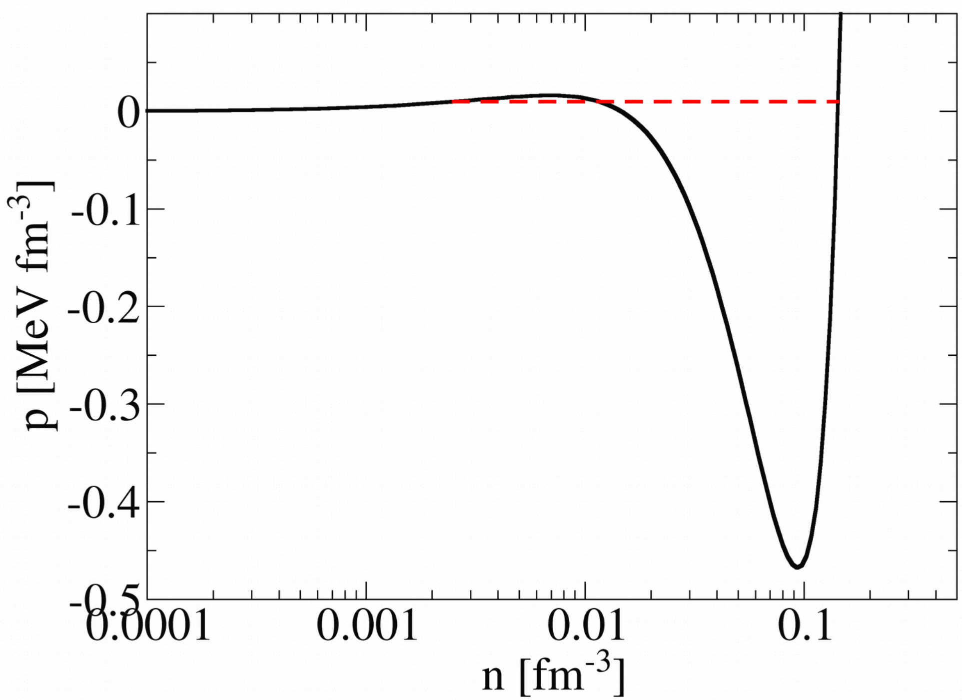


Super-massive stars (~ 50 Ms)

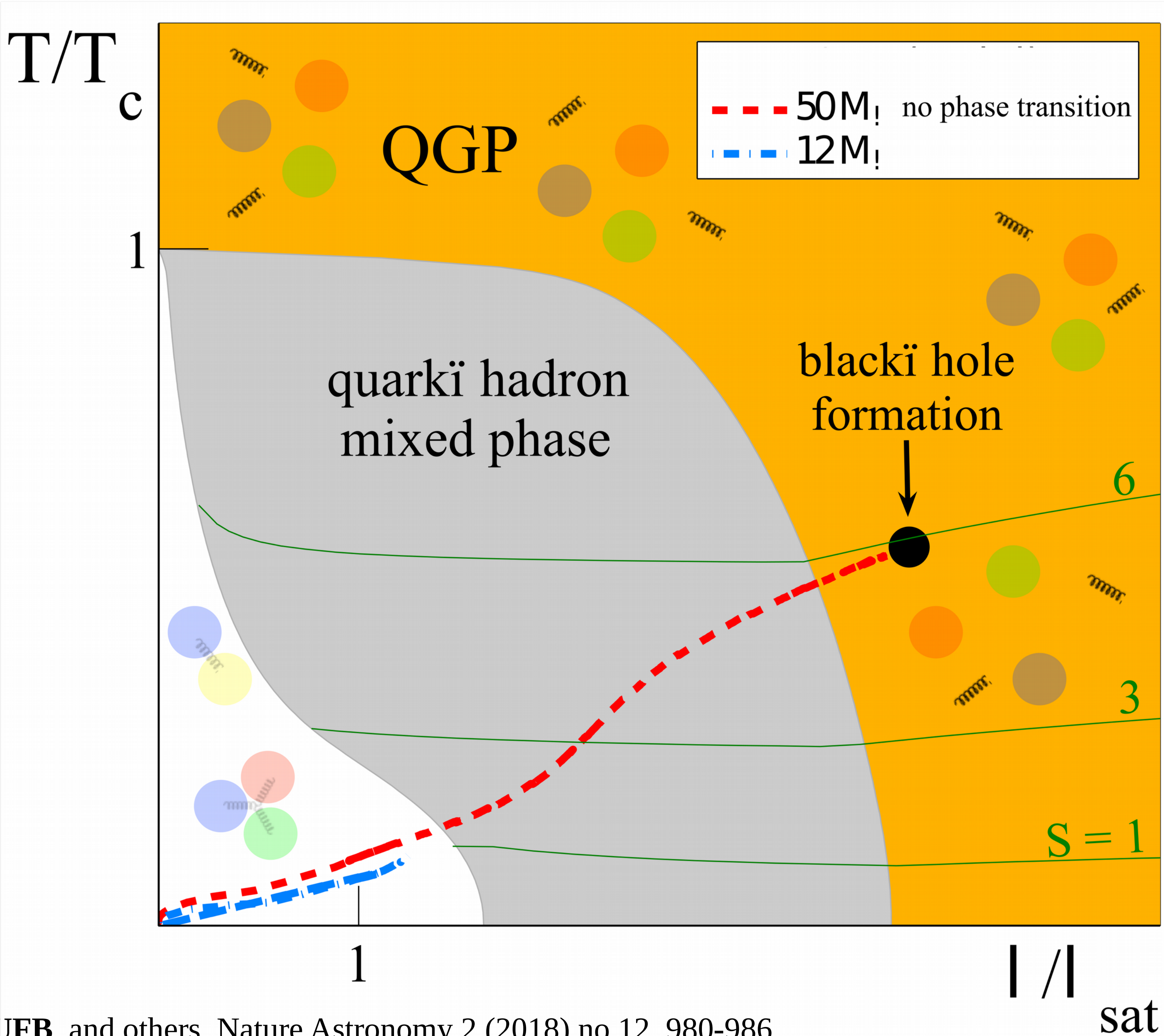
- Can not be explained by canonical models
- Have observational evidences
- One of biggest uncertainties:

high density EOS

how about a quark-hadron PT?

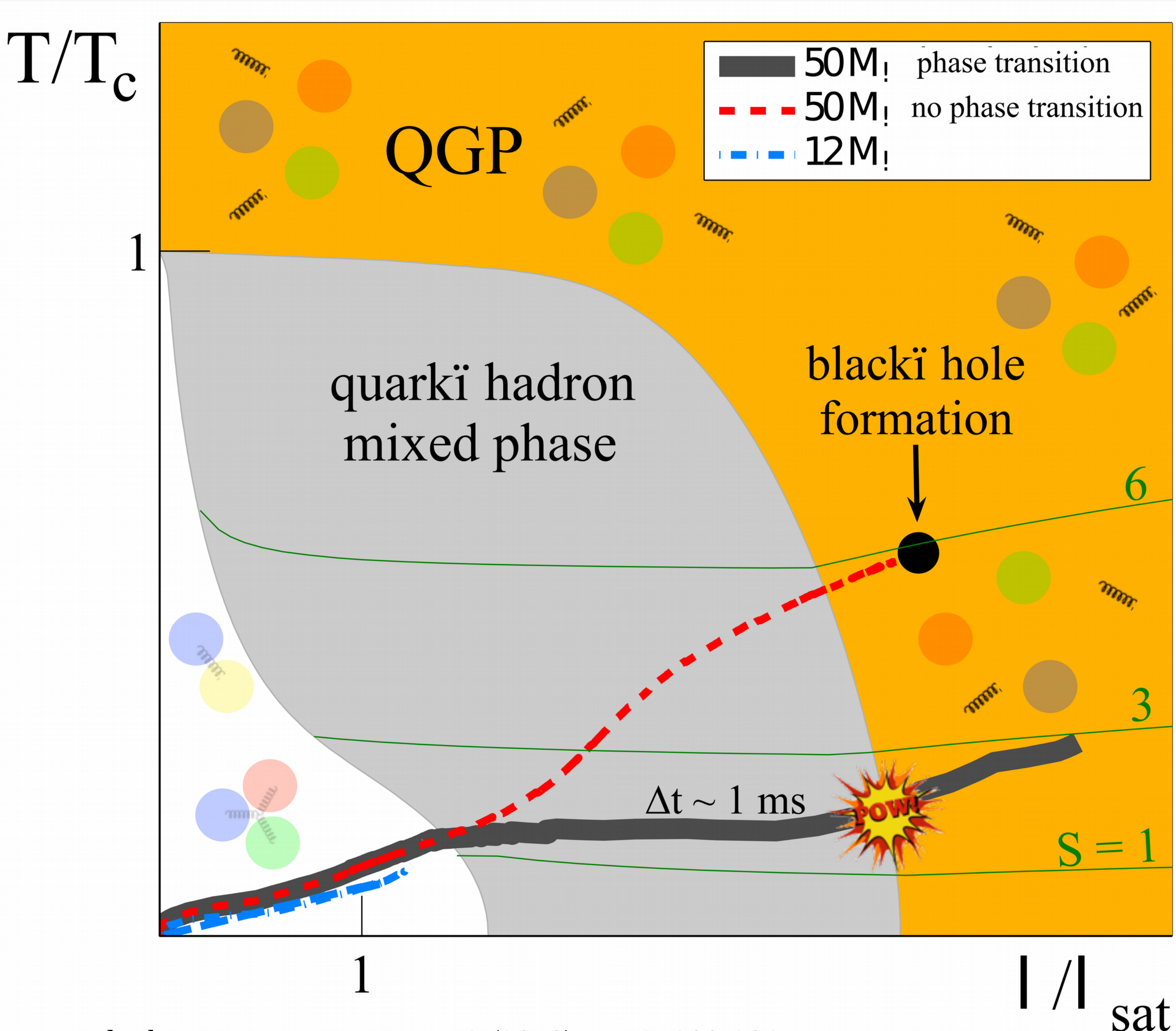


1st order PT – Supernovae of massive stars

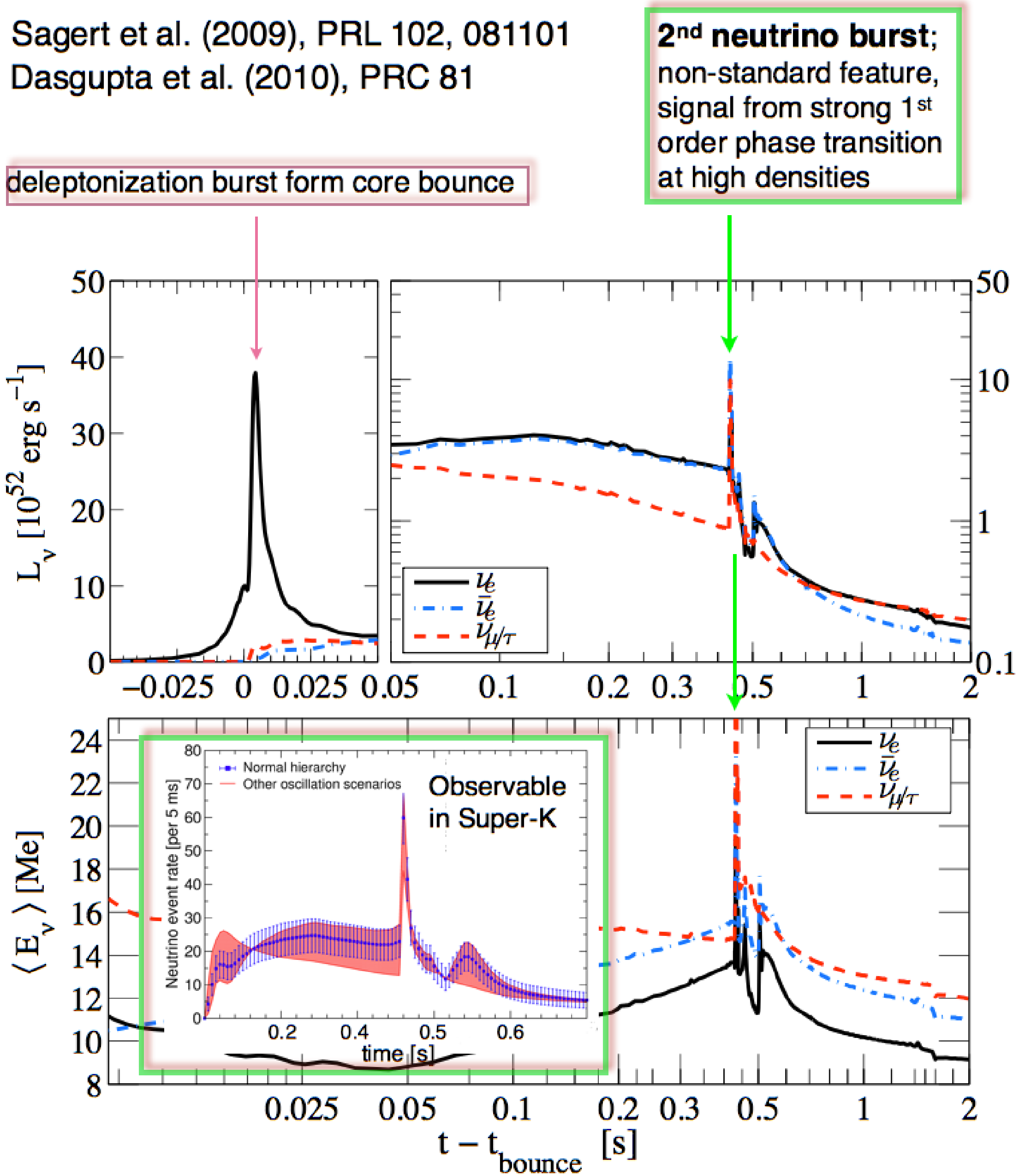


T. Fischer, **NUFB**, and others. Nature Astronomy 2 (2018) no.12, 980-986

1^{st} order PT – Supernovae of massive stars



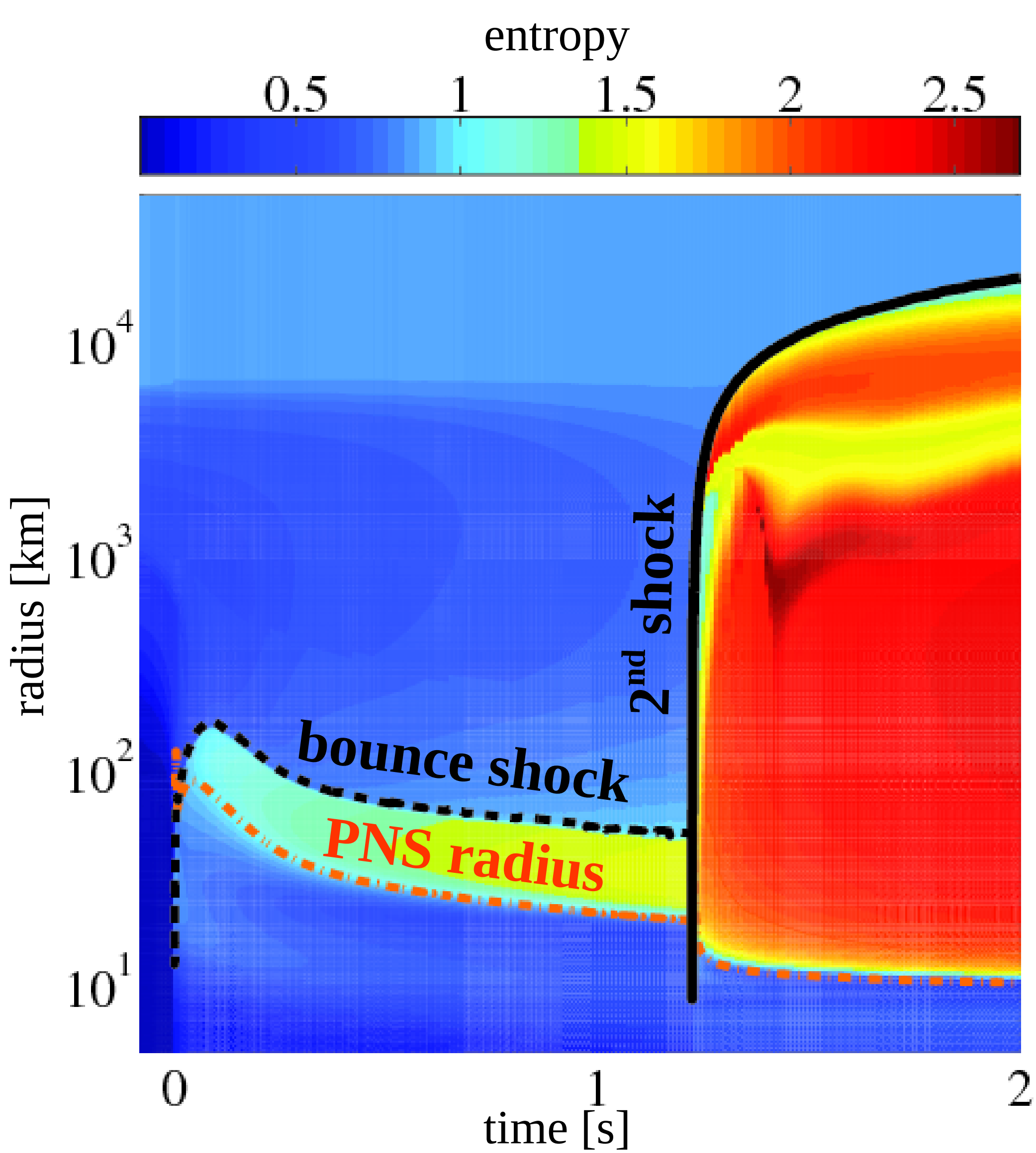
T. Fischer, **NUFB**, and others. Nature Astronomy 2 (2018) no.12, 980-986



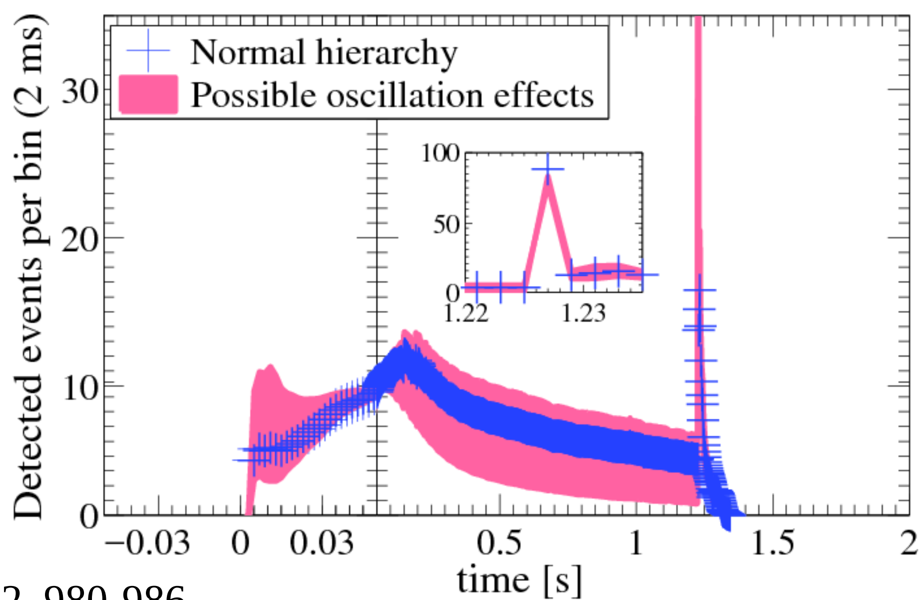
- Core-collapse supernovae as laboratories to probe the state of matter?
- Evidence for exotic states of matter: non-standard behavior of neutrino fluxes/energies (?)
- Additional neutrino outburst(s) due to high-density phase transition
- All flavors, unlike deleptonization burst
- Associated millisecond features observable with current neutrino detectors
- Structure of neutrino signal contains information about details of phase transition

Calculations are outdated due to known constraints!

1st order PT – Supernovae of massive stars

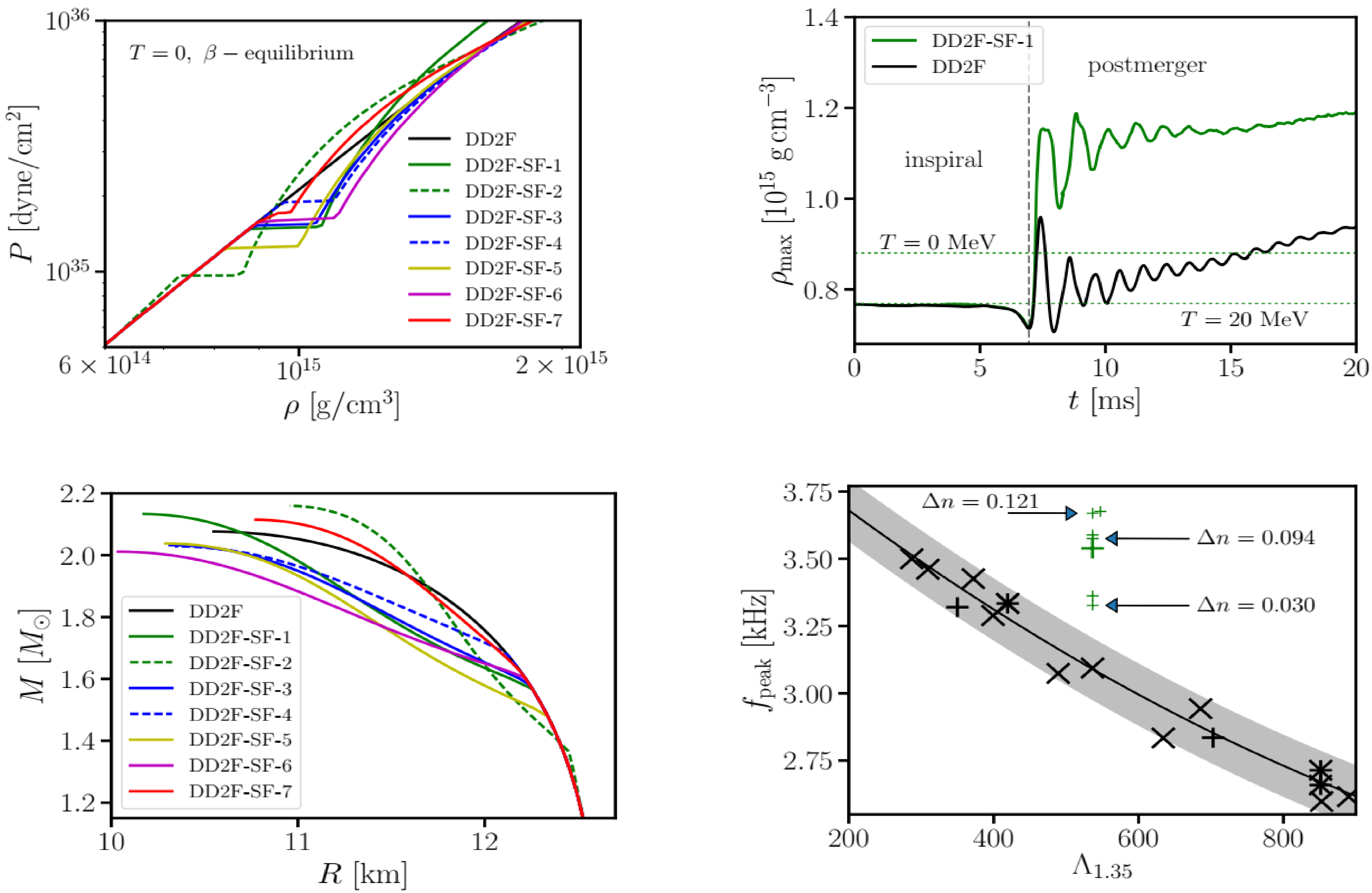


- EoS constructed under consideration of all constraints which are important in astrophysics
- Phase transition releases latent heat to explode "very" massive stars
- Remnant: $2M_\odot$ neutron stars (with quark core) at birth
- Neutrino signal measurable
- Energetic explosion, but almost no nickel



T. Fischer, **NUFB**, and others. Nature Astronomy 2 (2018) no.12, 980-986

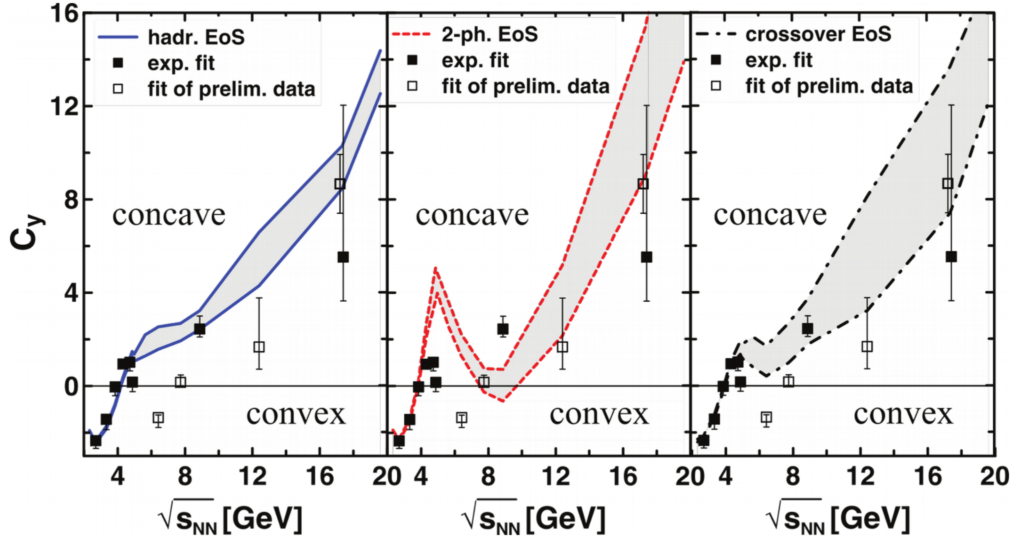
1^{st} order PT - Neutron star merger



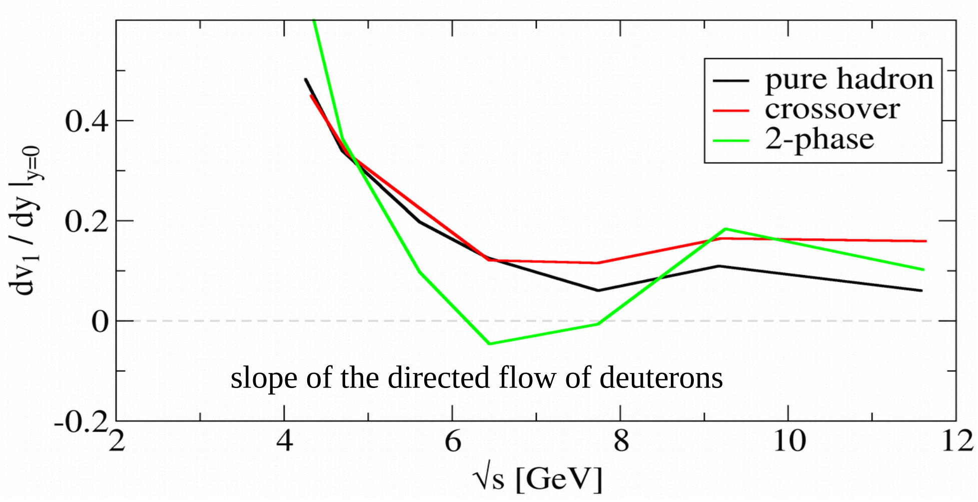
A. Bauswein, **NUFB**, and others, Phys.Rev.Lett. 122 (2019) no.6, 061102

1st order PT – Heavy Ion Collisions

strong signal (wiggle) in the baryon stopping signal [1]



Anti-flow of clusters occur [2]

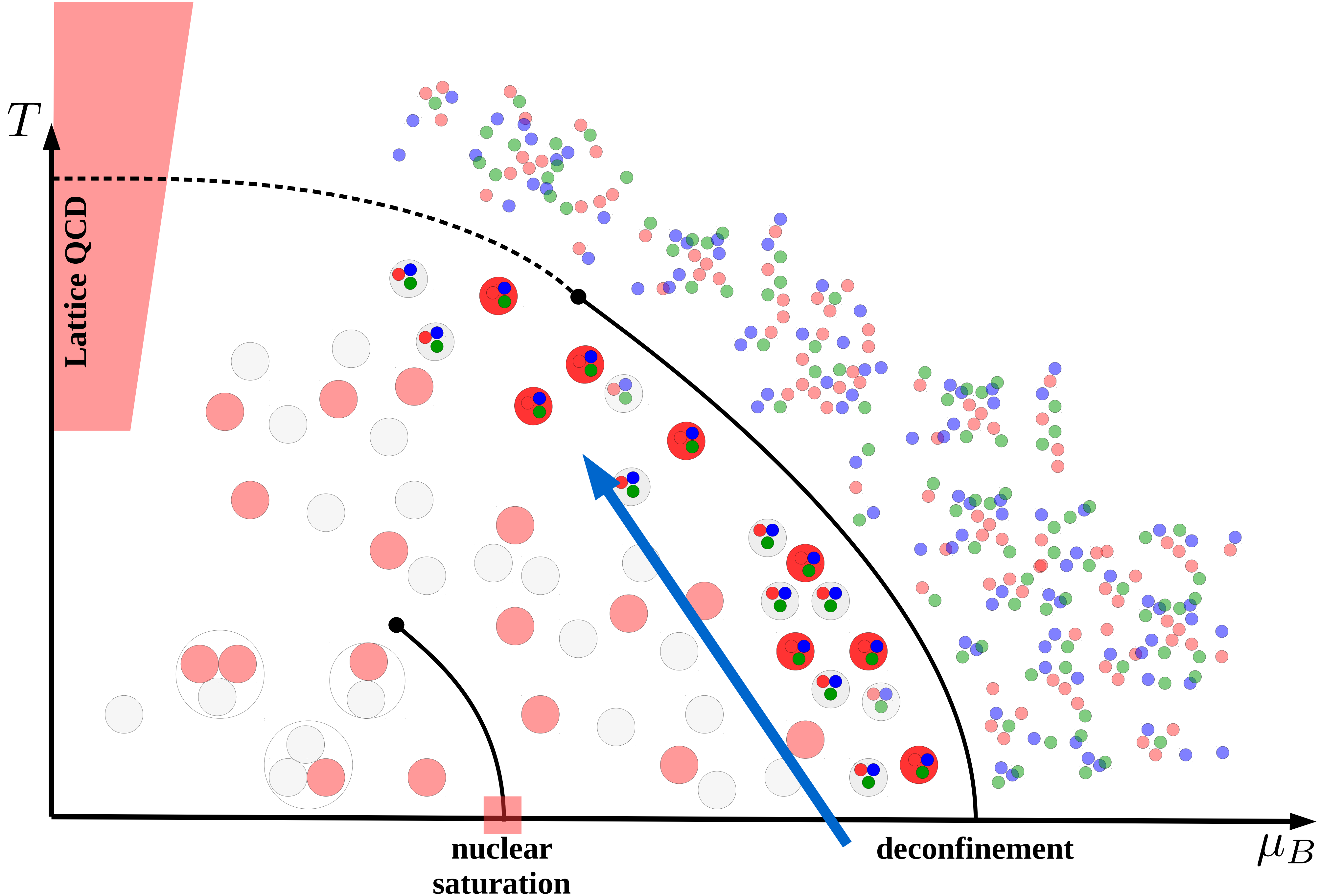


- Application of the SFM to HIC is ongoing work

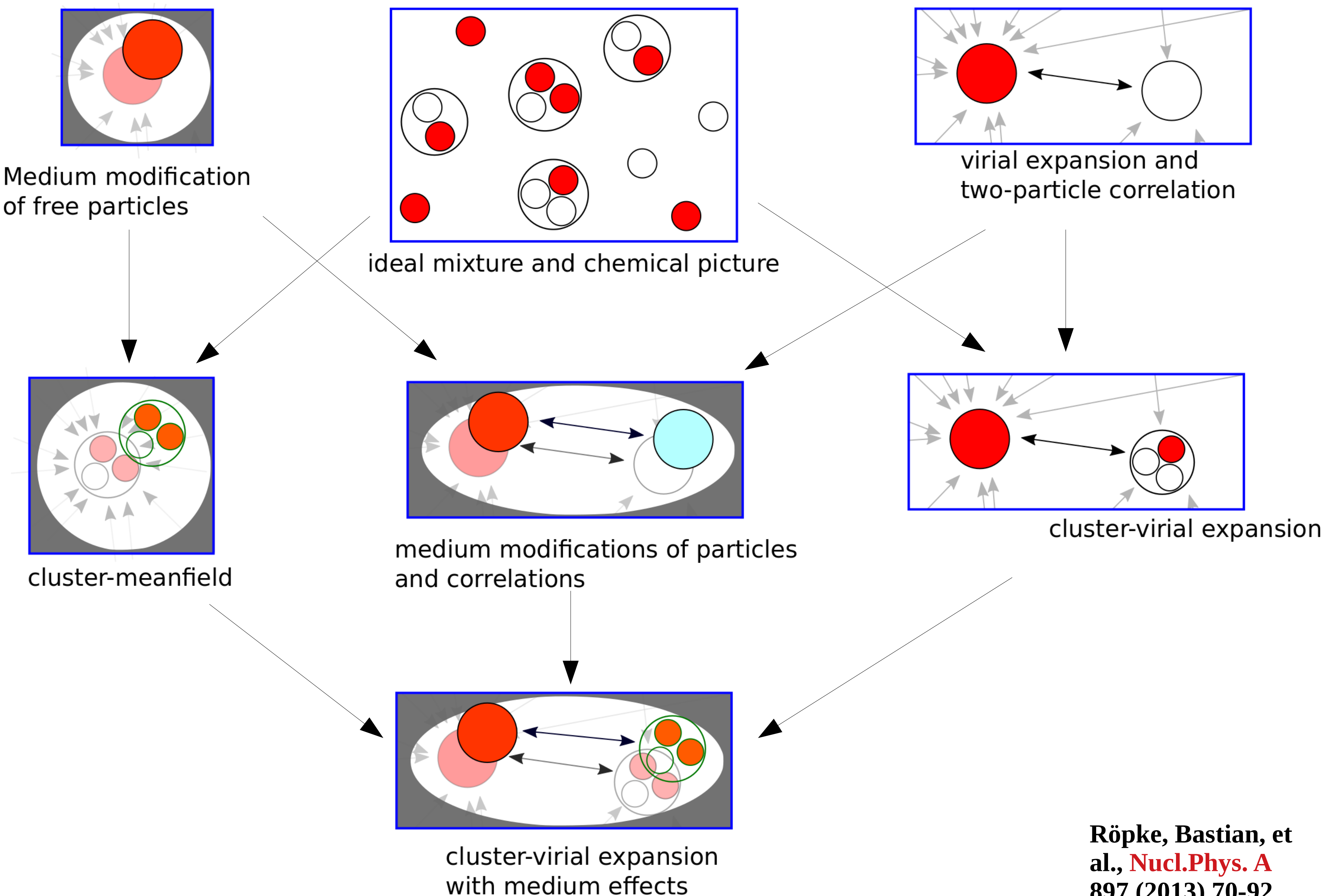
[1] Yu. B. Ivanov, PRC 87, 064904 (2013)

[2] **NUFB**, P. Batyuk, D. Blaschke, and others, Eur.Phys.J. A52 (2016) no.8, 244

Model for everything?



Methodology



Röpke, Bastian, et al., Nucl.Phys. A 897 (2013) 70-92

Relativistic density functionals

Starting with free fermion Lagrangian plus an interaction term, which depends on quark currents

$$\mathcal{L}_{\text{eff}} = \underbrace{\bar{q}\left(\imath\gamma^\mu\partial_\mu - m\right)q}_{\mathcal{L}_{\text{free}}} - U(\bar{q}q, \bar{q}\gamma^\mu q)$$

Mean field → linear dependence of U on densities is important! → expansion around expectation values

$$U(\bar{q}q, \bar{q}\gamma^\mu q) = U(n_\mathrm{S}, n_\mathrm{V}) + \Sigma_\mathrm{S}(\bar{q}q - n_\mathrm{S}) + \Sigma_\mathrm{V}(\bar{q}\gamma^\mu q - n_\mathrm{V}) + \ldots$$

derivatives

$$\mathcal{L}_{\text{eff}} \approx \underbrace{\bar{q}\left(\gamma^\mu(\imath\partial_\mu - \Sigma_\mathrm{V}) - (m + \Sigma_\mathrm{S})\right)q}_{\mathcal{L}_{\text{quasi}}} - \Theta(n_\mathrm{S}, n_\mathrm{V})$$

⬇

$$P = g\int\frac{\mathrm{d}^3p}{(2\pi)^3}\left[\ln(1 + e^{-\beta(\sqrt{p^2 - M^2} - \tilde{\mu})}) + \text{a.p.}\right] - \Theta$$

with

$$n_s = \langle\bar{q}q\rangle\ ,\ n_v = \langle\bar{q}\gamma^0 q\rangle \qquad M = m + \Sigma_\mathrm{S}\ ,\ \tilde{\mu} = \mu - \Sigma_\mathrm{V}$$

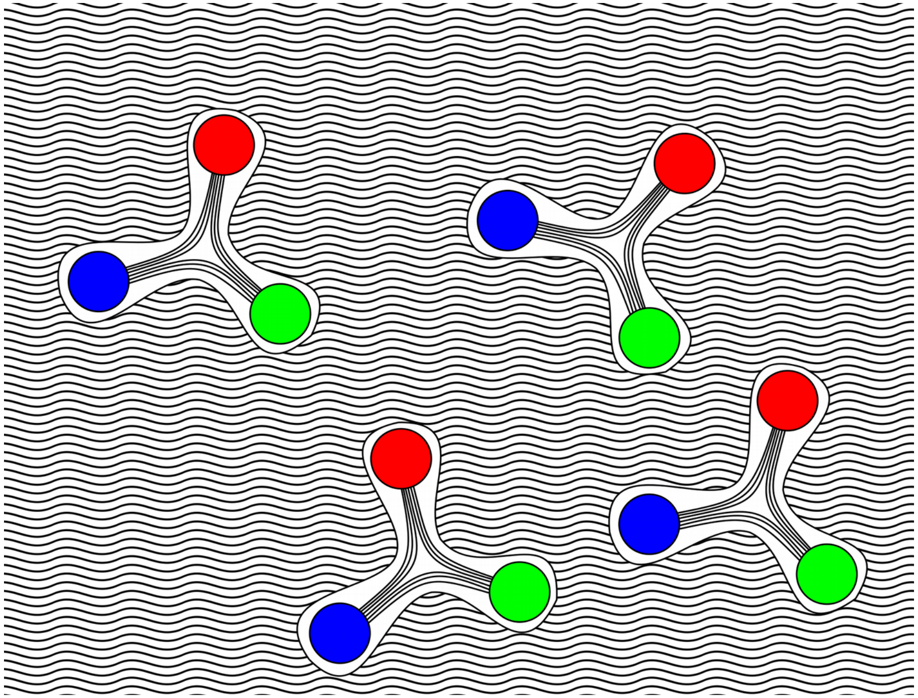
M. Kaltenborn, **NUFB**, D. Blaschke. Phys. Rev. D 2017, 96, 056024

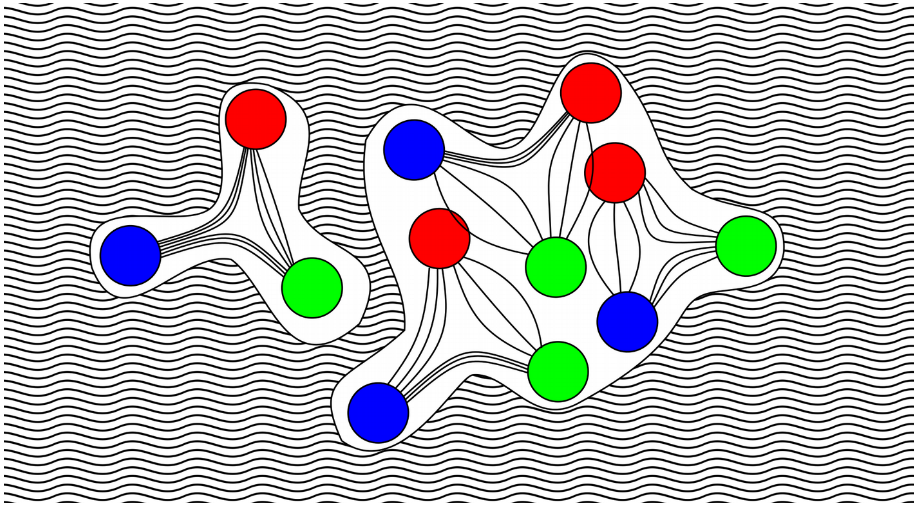
Density functional approach: Stringflip model

Low density

- Color field lines compressed by dual Meissner effect
- String-potential

$$V(r) = \sigma r \sim n^{-1/3}$$





High density

- Dual superconducting vacuum occupied by hadrons
- Pressure on field lines reduced
- Effective string-tension reduced

$$\sigma = \Phi \sigma_0$$

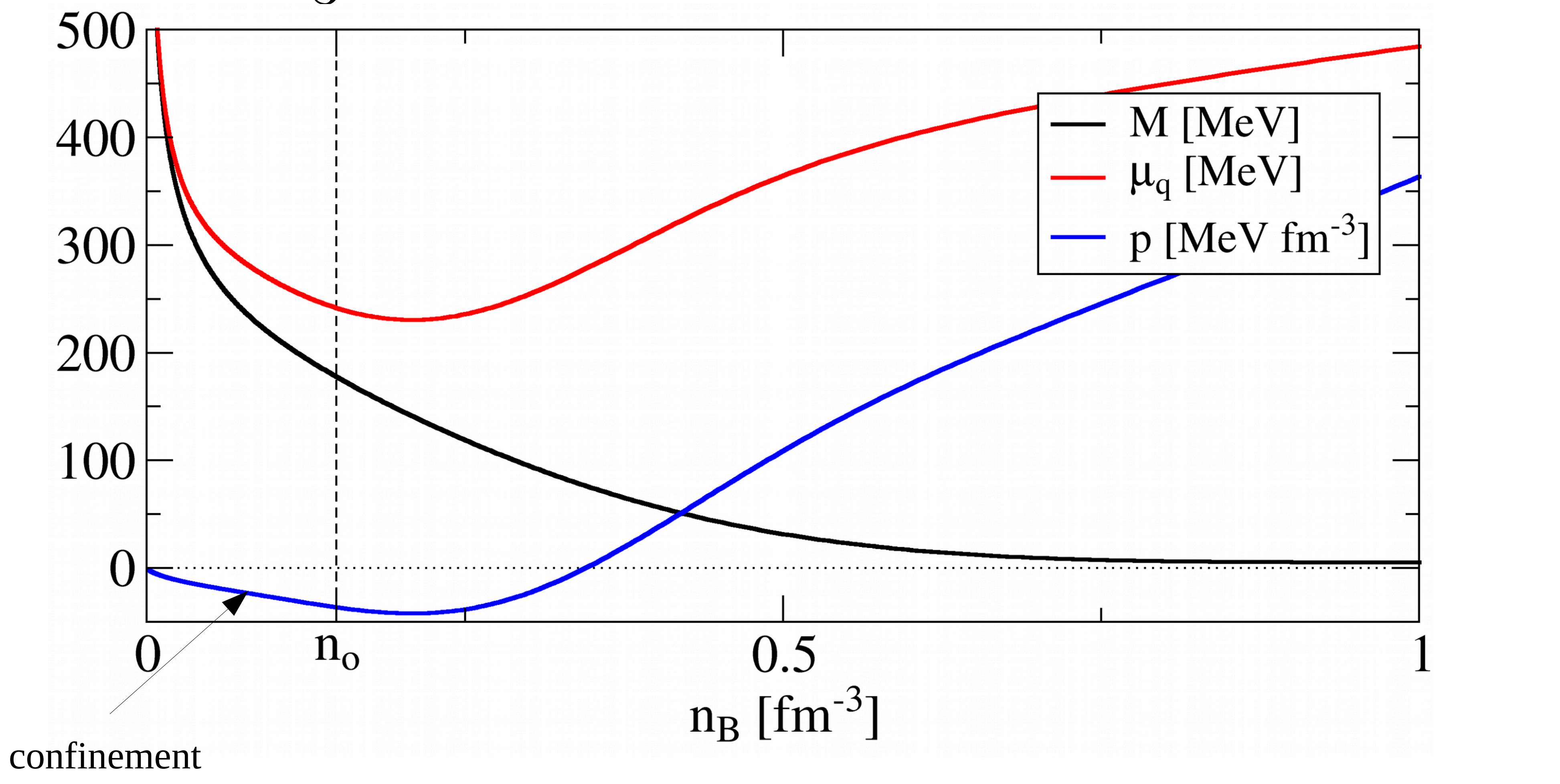
G. Ropke, et. al., Phys.Rev. D34 (1986) 3499-3513
M. Kaltenborn, **NUFB**, D. Blaschke, PRD 96, 056024 (2017)

$$U^{\mathrm{SF}}(n_{\mathrm{S}}, n_{\mathrm{V}}) = D(n_{\mathrm{V}}) n_{\mathrm{S}}^{2/3}$$

Stringflip model – effective mass

Mean-field model

$$M_i = m_i + \frac{2}{3} D \cdot (n^{\mathrm{s}})^{-1/3} \qquad D = D_0 \mathrm{e}^{-\alpha(n-n_0)^2}$$



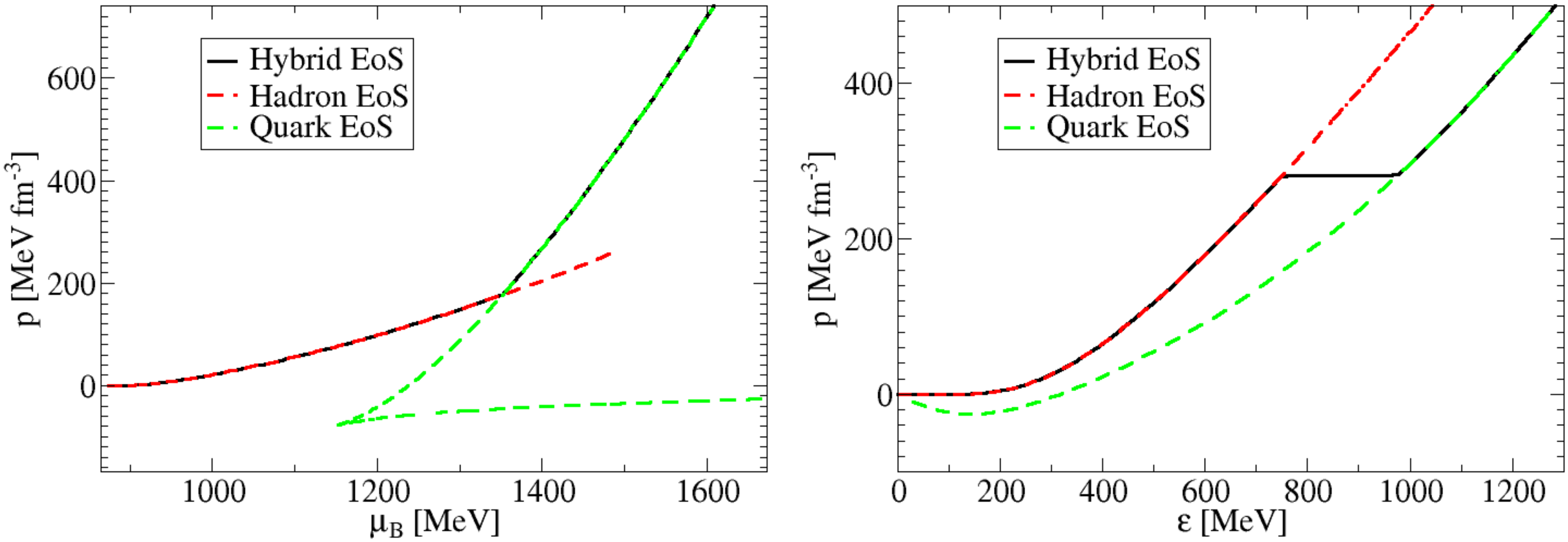
M. Kaltenborn, **NUFB**, D. Blaschke, PRD 96, 056024 (2017)

2-phase approach

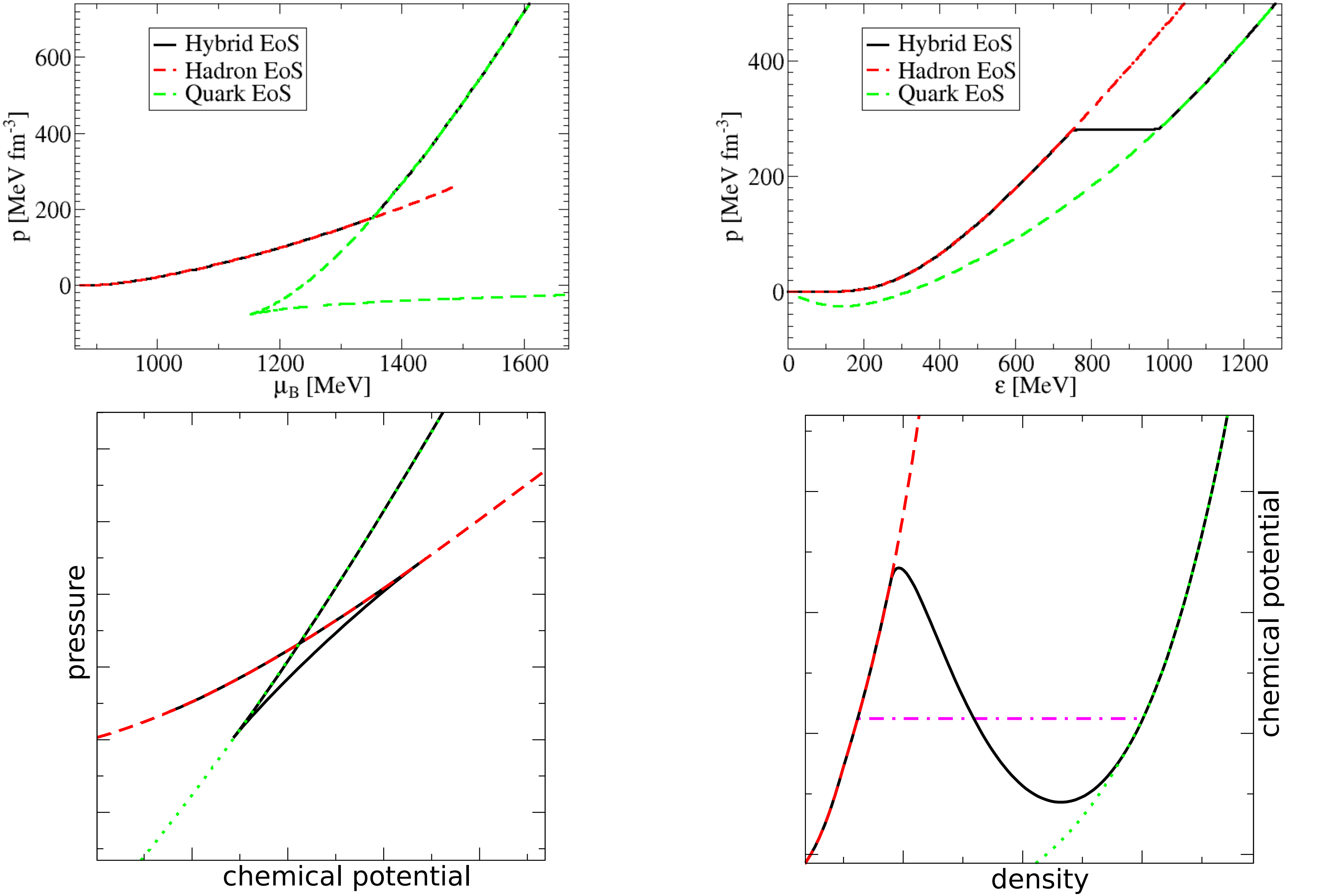
old

- Two independent models for hadrons and quarks
- Match while fulfilling Gibbs condition for thermal, mechanical and chemical phase equilibrium

$$T^{\mathrm{H}} = T^{\mathrm{Q}} \quad p^{\mathrm{H}} = p^{\mathrm{Q}} \quad \mu^{\mathrm{H}} = \mu^{\mathrm{Q}}$$



Two-phase approach vs van der Waals wiggle



Cluster expansion

Generating functional formalism by Baym and Kadanoff [1,2]

$$\Omega = -\mathrm{Tr}\ \ln(-G_1^{-1}) - \mathrm{Tr}\Sigma_1 G_1 + \Phi \qquad \text{With} \qquad \Sigma_1(1,1') = \frac{\delta\Phi}{\delta G_1(1,1')}.$$

Can be generalized for a consistent cluster expansion[3]

$$\Omega = \sum_{l=1}^{A} \Omega_l = \sum_{l=1}^{A} \left\{ c_l \left[\mathrm{Tr}\ln\left(-G_l^{-1}\right) + \mathrm{Tr}\left(\Sigma_l\ G_l\right)\right] + \sum_{\substack{i,j \\ i+j=l}} \Phi[G_i, G_j, G_{i+j}] \right\}$$

with

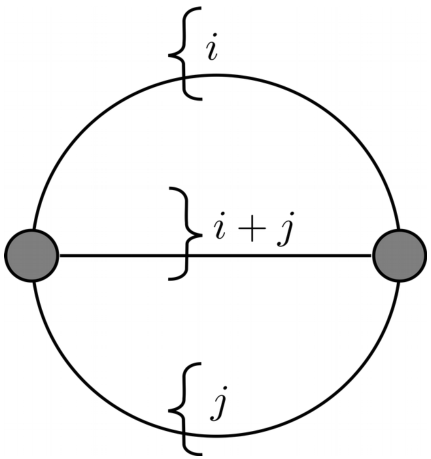
$$\Sigma_A(1\ldots A, 1'\ldots A', z_A) = \frac{\delta\Phi}{\delta G_A(1\ldots A, 1'\ldots A', z_A)}$$

Always sustains full Dyson equation and thermodynamic stability

$$G_A^{-1} = G_A^{0\ -1} - \Sigma_A^{-1} \qquad \frac{\partial\Omega}{\partial G_A} = 0$$

Reduction on generalized sunset diagrams is recommended

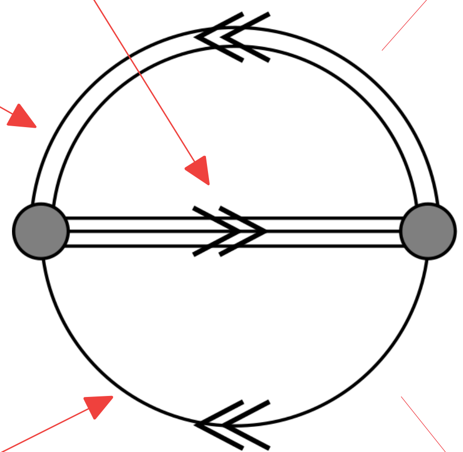
$$\Phi[G_i, G_j, G_{i+j}] =$$



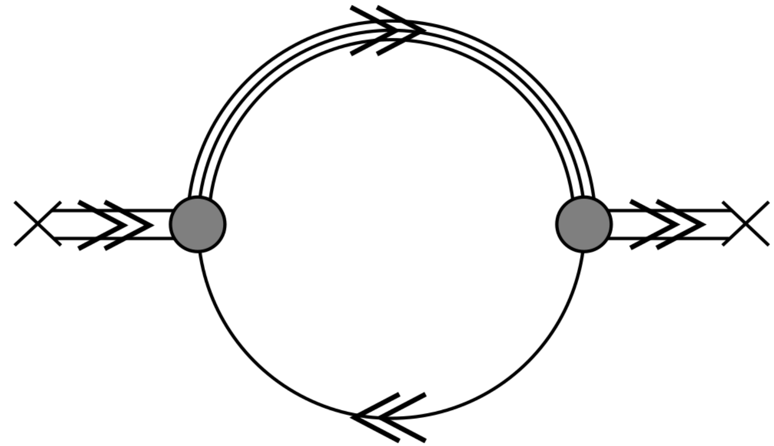
[1]Baym, G.; Kadanoff, L.P. Phys. Rev. 1961, 124, 287–299.
[2]Baym, G. Phys. Rev. 1962, 127, 1391–1401.
[3]**NUFB**, and others, Universe 2018, 4(6), 67

$$\Sigma_A = \frac{\delta \Phi}{\delta G_A}$$

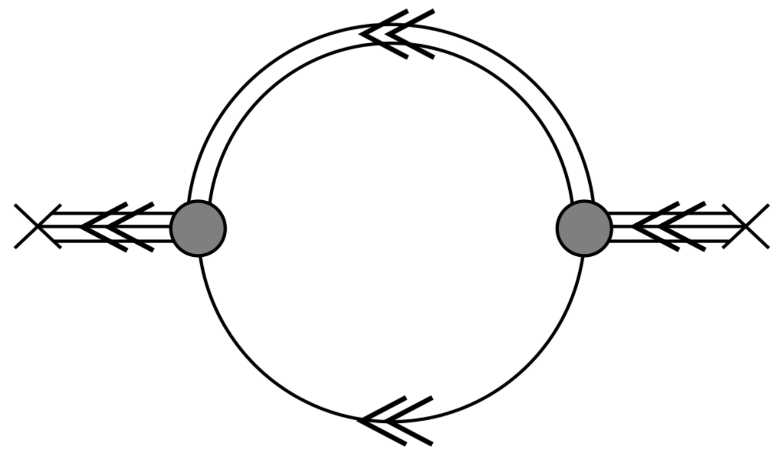
$$\Phi =$$



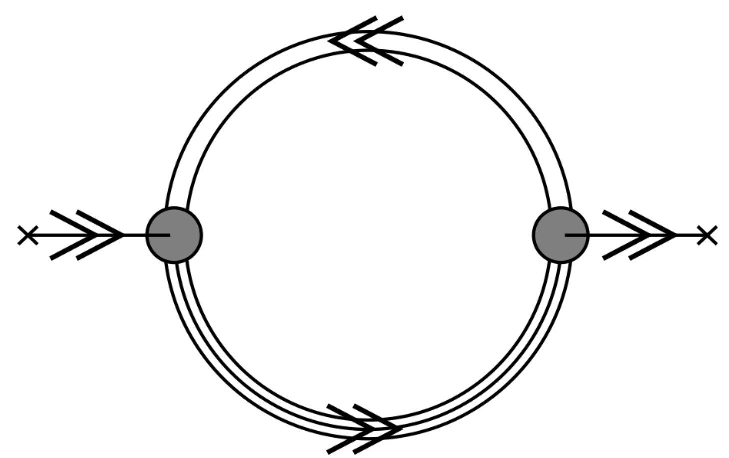
$$\Sigma_{\mathrm{D}} = \frac{\delta \Phi}{\delta G_{\mathrm{D}}} =$$



$$\Sigma_{\mathrm{B}} = \frac{\delta \Phi}{\delta G_{\mathrm{B}}} =$$



$$\Sigma_{\mathrm{Q}} = \frac{\delta \Phi}{\delta G_{\mathrm{Q}}} =$$



$$G_A^{-1} = {G_A^0}^{-1} - \Sigma_A^{-1}$$

Analogy to density functional approach

Phi-derivable approach

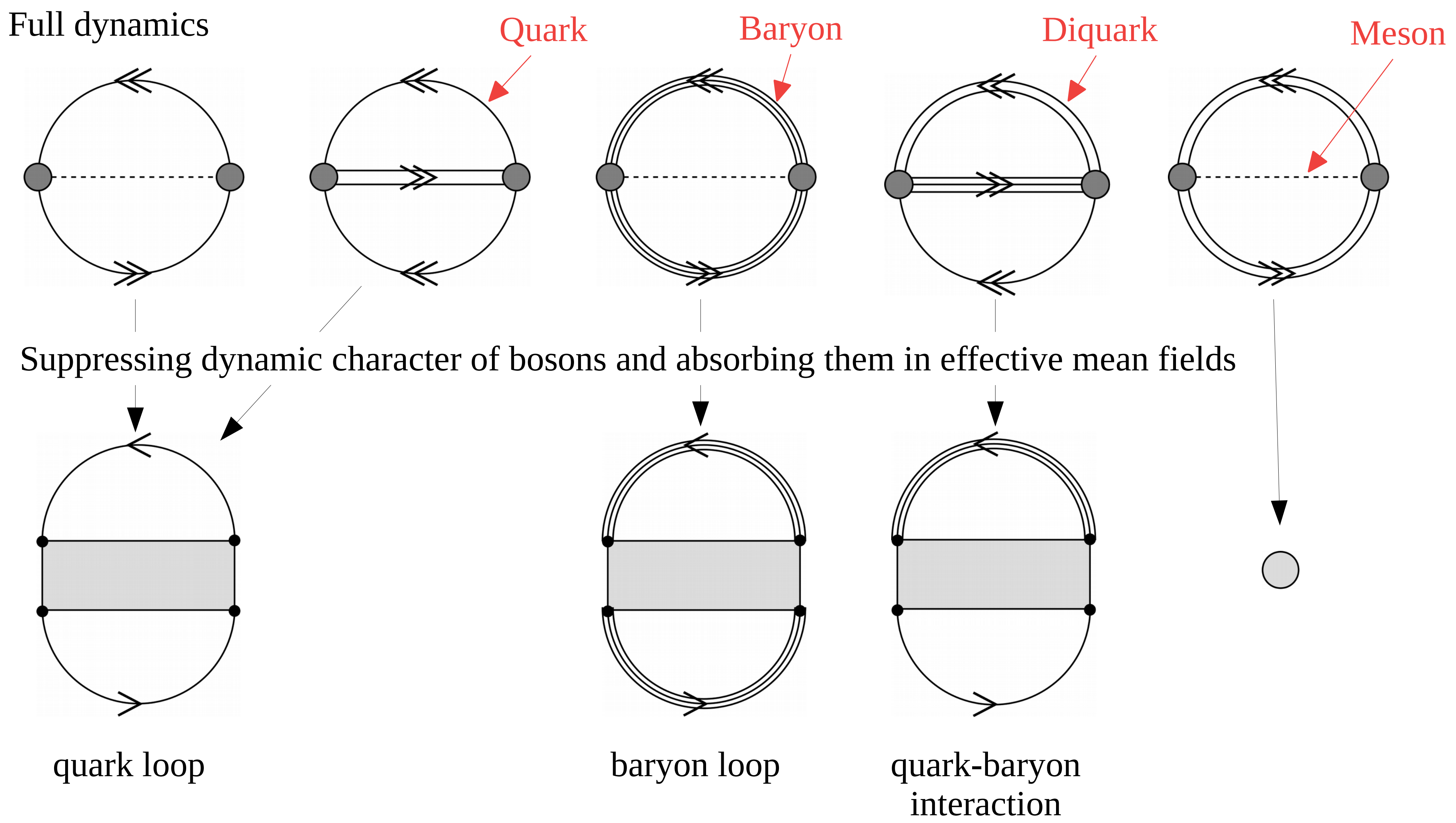
$$\Omega = -\mathrm{Tr}\ \ln(-G_1) - \mathrm{Tr}\Sigma_1 G_1 + \Phi[G_1]$$

Density functional approach

$$\Omega = \Omega^{\mathrm{quasi}} - n_{\mathrm{s}}\Sigma_{\mathrm{s}} - n_{\mathrm{v}}\Sigma_{\mathrm{v}} + U(n_{\mathrm{s}}, n_{\mathrm{v}})$$

First step: cluster expansion on basis of densities instead of Green functions (local limit)

The Quark-Diquark-Meson-Baryon Model



→ **Real self energies → quasi particles**

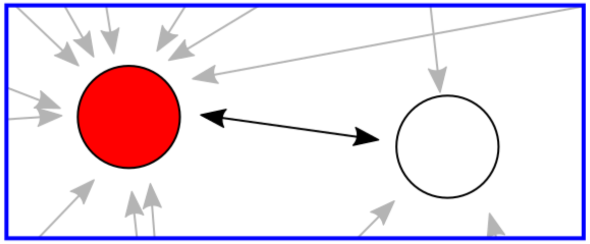
$$\delta := \arctan \frac{\mathrm{Im}\,\Sigma}{\mathrm{Re}\,\Sigma} = n\pi$$

Generalized Beth-Uhlenbeck

Cluster expansion

$$n_\mathrm{u} = n_\mathrm{u}^\mathrm{free} + 2n_\mathrm{p}^\mathrm{free} + 1n_\mathrm{n}^\mathrm{free}$$

$$n_\mathrm{d} = n_\mathrm{d}^\mathrm{free} + 1n_\mathrm{p}^\mathrm{free} + 2n_\mathrm{n}^\mathrm{free}$$



Chemical equilibrium

$$\mu_i = B_i\mu_\mathrm{B} + C_i\mu_\mathrm{C}$$

Generalized Beth-Uhlenbeck formula

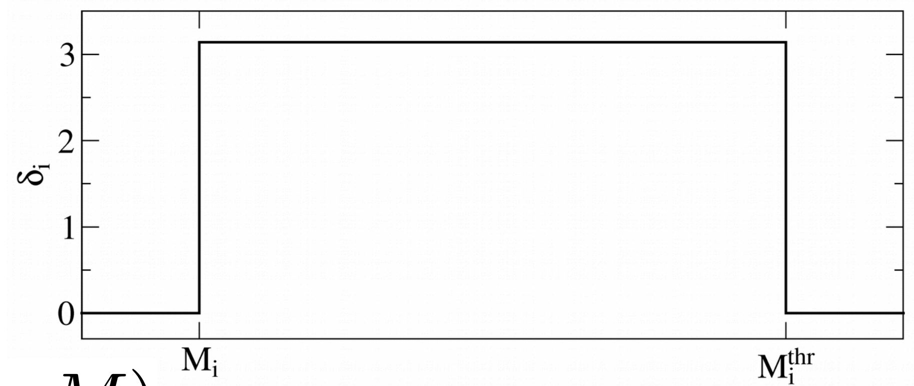
$$n_i^\mathrm{free} = g_i \int \frac{\mathrm{d}^3p}{(2\pi)^3} \int \frac{\mathrm{d}E}{2\pi} f_i(E_i) 2\sin^2\delta_i(E) \frac{\mathrm{d}\delta_i(E)}{\mathrm{d}E}$$

Substitution: $E_\mathrm{i} = \sqrt{p^2 + (m_i + S_i)^2} + V_i$

$$n_i^\mathrm{free} = g_i \int \frac{\mathrm{d}^3p}{(2\pi)^3} \int \frac{\mathrm{d}M}{2\pi} f_i\left(\sqrt{p^2+M^2} + V_i\right) 2\sin^2\delta_i(M) \frac{\mathrm{d}\delta_i(M)}{\mathrm{d}M}$$

Analogy to density functional approach

$$n_i^{\rm free} = g_i \int \frac{d^3p}{(2\pi)^3} \int \frac{dM}{2\pi} f_i\left(\sqrt{p^2+M^2}+V_i\right) 2\sin^2\delta_i(M) \frac{d\delta_i(M)}{dM}$$



$$\delta_{i={\rm u,d}}(M) = \pi\,\Theta(M-M_i)$$

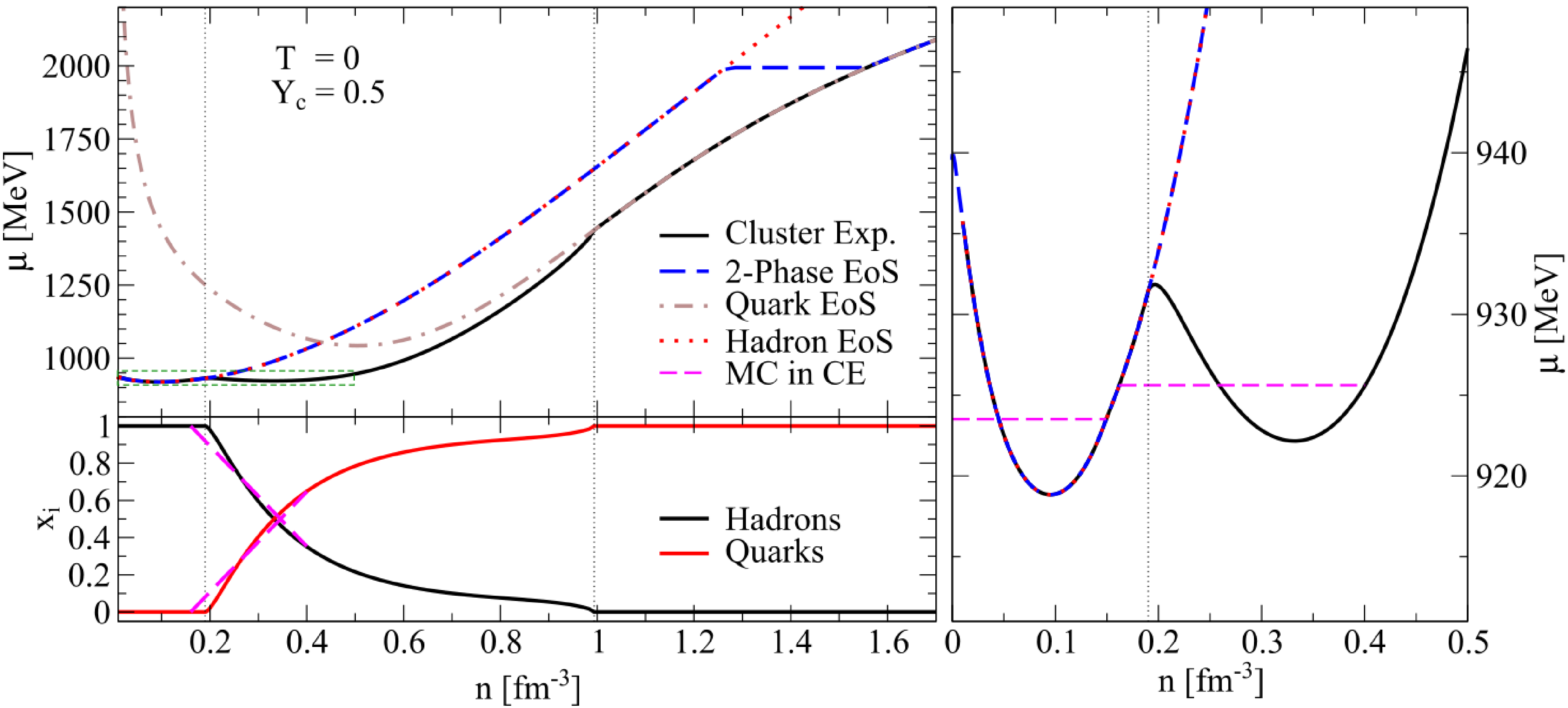
$$\delta_{i={\rm p,n}}(M) = \pi\,\Theta(M-M_i)\,\Theta(M_i^{\rm th}-M)$$

$$n_{i={\rm p,n}} = g_i \int \frac{d^3p}{(2\pi)^3} \Big[f_i(\sqrt{p^2+M_i^2}+V_i) - f_i(\sqrt{p^2+(M_i^{\rm th})^2}+V_i) \Big] \Theta(M_i^{\rm thr}-M_i)$$

$$= \left(n_{\rm N}^{\rm qu} - n_{\rm q}^{\rm thr}\right) \Theta(M_i^{\rm thr}-M_i)$$

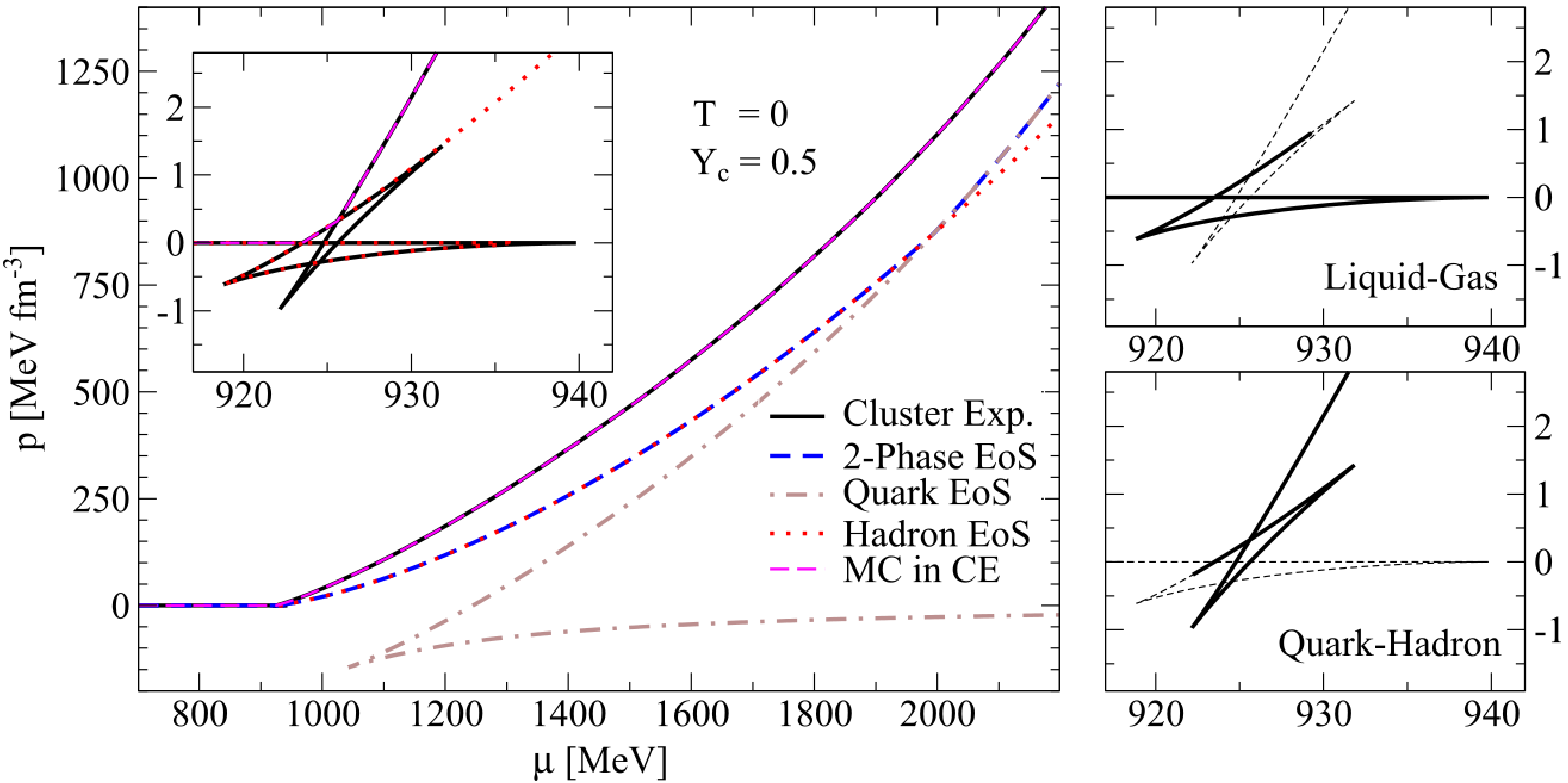
$$M_{\rm p}^{\rm thr} = 2M_{\rm u} + 1M_{\rm d}$$
$$M_{\rm n}^{\rm thr} = 1M_{\rm u} + 2M_{\rm d}$$

Cluster-expansion of Quarks

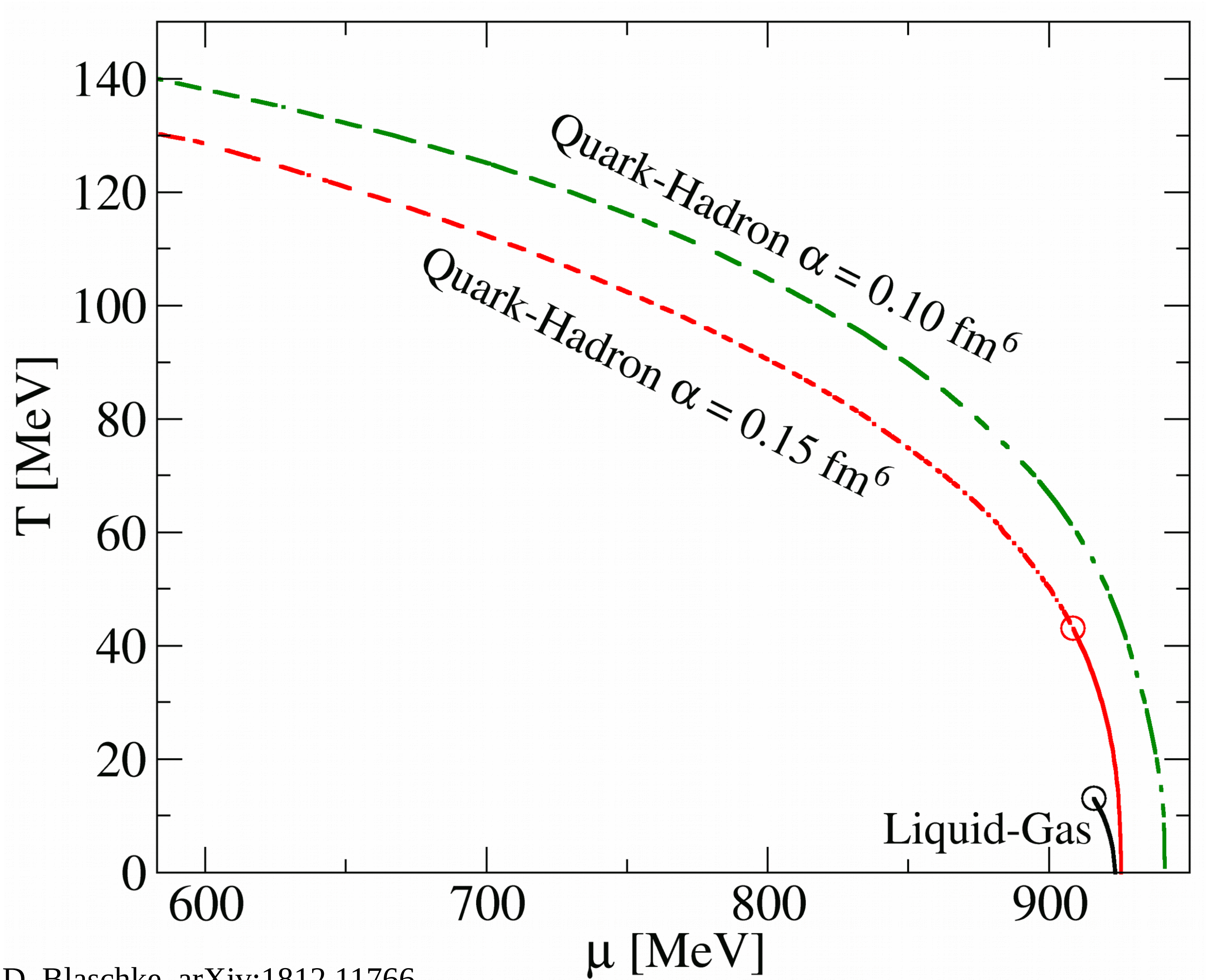


NUFB, D. Blaschke, arXiv:1812.11766

Cluster-expansion of Quarks

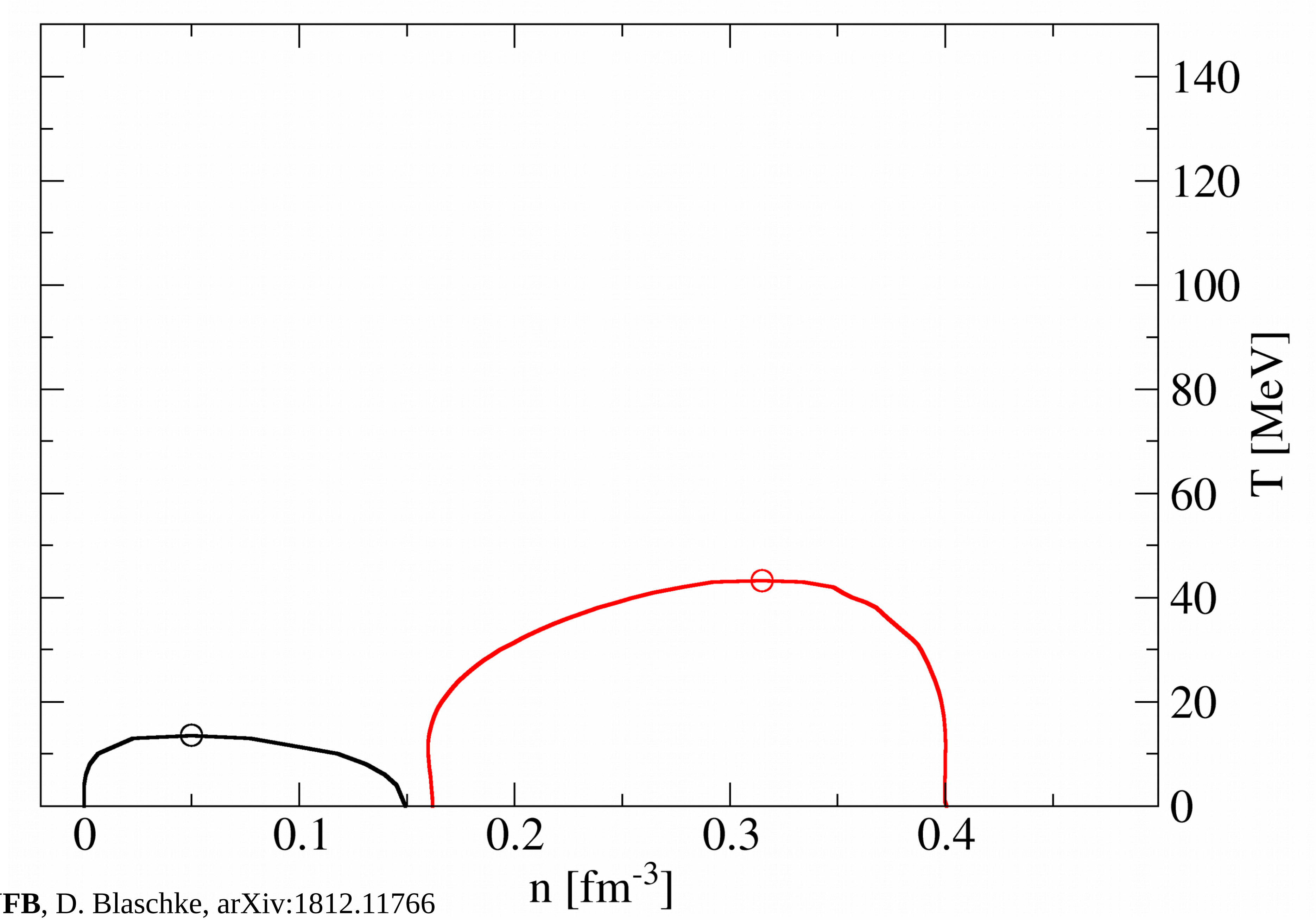


NUFB, D. Blaschke, arXiv:1812.11766



NUFB, D. Blaschke, arXiv:1812.11766

Cluster-expansion

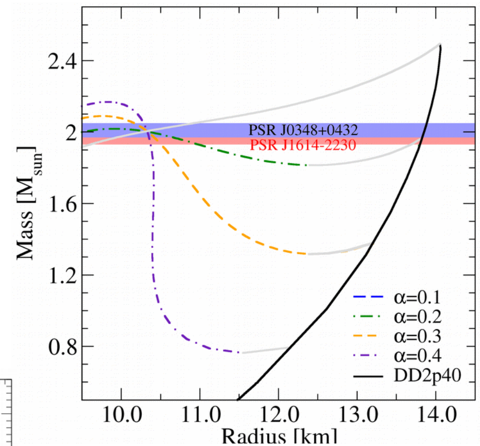


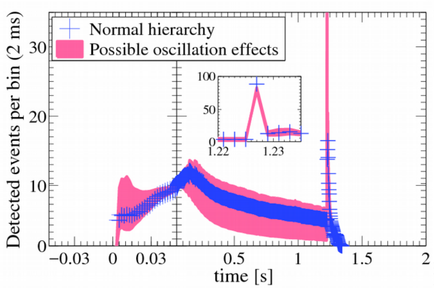
NUFB, D. Blaschke, arXiv:1812.11766

~~Outline~~ Summary

Possible signals of 1st – order phase transitions.

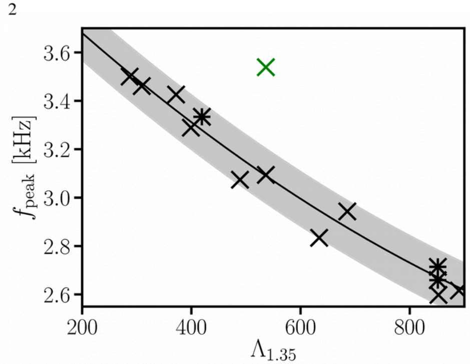
Neutron star configurations

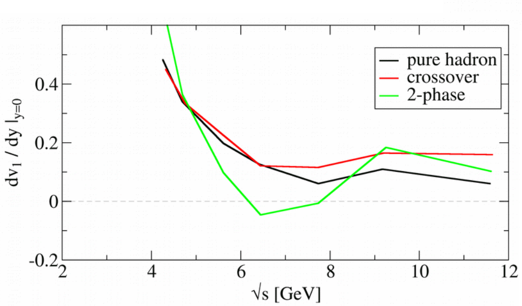




Supernova explosions of 50Ms stars

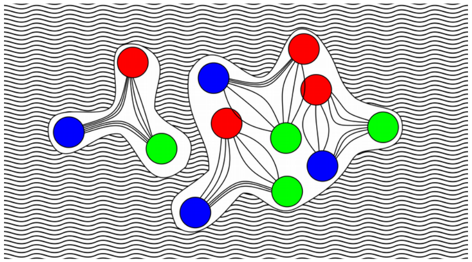
Binary neutron star mergers





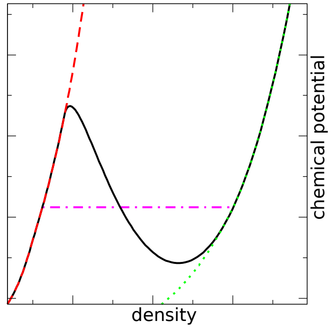
Heavy-Ion Collisions

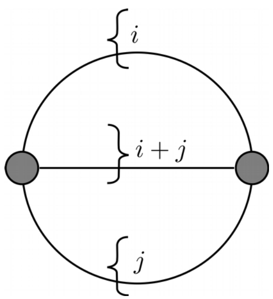
Unified description of the equation of state.



density functional theory

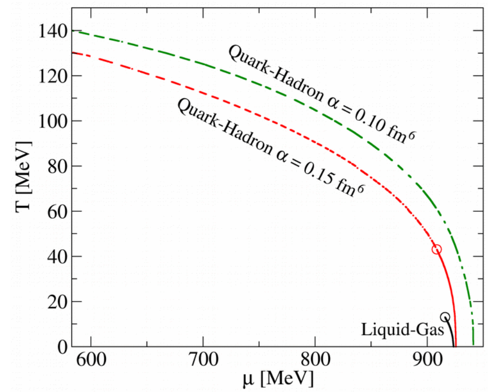
construction of phase transitions





Phi-derivable formalism

current status



Last Slide

Conclusions

- Possible scenarios are explored in which a 1st order phase transition is detectable in
  - neutron star configurations
  - neutrino signals of supernova explosions
  - Gravitational wave signal of binary neutron star mergers
  - Flow data of heavy-ion collision experiments
- Astrophysical objects and HIC collisions are based on the same physics of strongly interacting many-particle systems
- Hadrons are bound states of quarks and should be treated as such
- A cluster virial expansion within the Beth-Uhlenbeck formalism can be derived from the PHI-derivable approach
- Initial reduction to mean field already results in a consistent description of Quark-Hadron phase transition

Outlook

- Density functional with chiral physics
- Reproduction of Lattice results
- Continuum contributions and substructure effects
- Cluster mean field

Thank you!

Collaboration

- Tobias Fischer, David Blaschke, Andreas Bauswein, Stefan Typel, Gerd Röpke, Yuri Ivanov, Diana Alvear Terrero