High-energy physics at ultra-cold temperatures, ECT*, Trento, June 2019

QUANTUM SIMULATION OF LATTICE GAUGE THEORIES

EREZ ZOHAR

Theory Group, Max Planck Institute of Quantum Optics (MPQ)

→ Racah Institute of Physics, Hebrew University of Jerusalem

האוניברסיטה העברית בירושלים
THE HEBREW UNIVERSITY OF JERUSALEM

Gauge Theories are challenging:

- Local symmetry → many constraints
- Involve non-perturbative physics
  - Confinement of quarks → hadronic spectrum
  - Exotic phases of QCD (color superconductivity, quark-gluon plasma)

→ Hard to treat experimentally (strong forces)

→ Hard to treat analytically (non perturbative)

→ Lattice Gauge Theory (Wilson, Kogut-Susskind…)

→ Lattice regularization in a gauge invariant way

Conventional LGT techniques

- Discretization of both space and time
- Monte Carlo computations on a Wick-rotated, Euclidean lattice

$$\left\langle \hat{A}\left(\hat{\Phi}\right)\right\rangle = \frac{\int \mathcal{D}\phi A(\phi) e^{iS_M}}{\int \mathcal{D}\phi e^{iS_M}} \xrightarrow[t\to -i\tau]{} \frac{\int \mathcal{D}\phi A(\phi) e^{-S_E}}{\int \mathcal{D}\phi e^{-S_E}} \equiv \int \mathcal{D}\phi A\left(\phi\right) p\left(\phi\right)$$

- **Very (very) successful for many applications, e.g. the hadronic spectrum**
- **Problems:**
  - **Real-Time evolution:**
    - Not available in Wick rotated, Euclidean spacetimes, used in conventional Monte-Carlo path integral LGT calculations
  - **Sign problem:**
    - Appears in several scenarios with fermions (finite density), represented by Grassman variables in a Wick-rotated, Euclidean spacetime

Quantum Information Methods for LGTs

- An active, rapidly growing research field

- Quantum Simulation for LGTs (around 8 years):
  - MPQ Garching & Tel Aviv University (Cirac, Reznik, EZ)
  - IQOQI Innsbruck, Bern, Trieste, Waterloo (Zoller, Wiese, Blatt, Dalmonte, Muschik)
  - Barcelona (Lewenstein, Tagliacozzo, Celi)
  - Heidelberg (Berges, Oberthaler, Jendrzejewski, Hauke …)
  - Iowa (Meurice)
  - Bilbao (Solano, Rico)
  - …

- Tensor Networks for LGTs (around 6 years):
  - MPQ Garching & DESY (Cirac, Jansen, Banuls, EZ…)
  - Ghent (Verstraete, Haegeman)
  - Barcelona (Lewenstein, Tagliacozzo, Celi)
  - IQOQI, Bern, Trieste, Ulm (Zoller, Wiese, Dalmonte , Montangero,…)
  - Iowa (Meurice)
  - Mainz (Orus)
  - …

- Quantum Computation for LGTs (relatively new):
  - Seattle (Kaplan, Savage)
  - Fermilab, …
  - Bilbao (Solano, Rico)
  - …

Quantum Simulation of LGTs

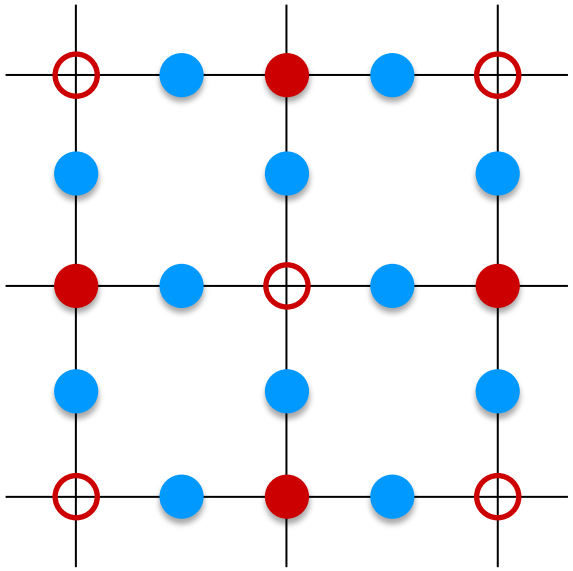
- **Real-Time evolution:**
  - Not available in Wick rotated, Euclidean spacetimes, used in conventional Monte-Carlo path integral LGT calculations
  - Exists by default in a real experiment done in a **quantum simulator**: prepare some initial state and the appropriate Hamiltonian (in terms of the simulator degrees of freedom), and let it evolve
- **Sign problem:**
  - Appears in several scenarios with fermions (finite density), represented by Grassman variables in a Wick-rotated, Euclidean spacetime
  - In real experiments, as those carried out by a **quantum simulator**, fermions are simply fermions, and no path integral is calculated:

    Nature does not calculate determinants.

LATTICE GAUGE THEORIES: THE HAMILTONIAN APPROACH

Hamiltonian LGTs

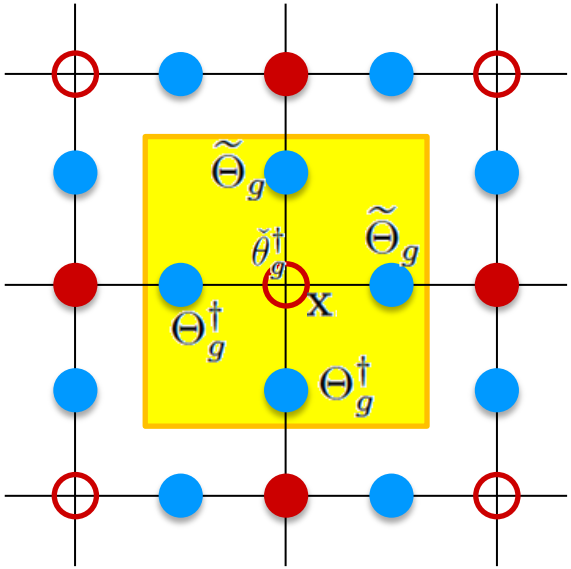
- **The lattice is spatial**: time is a continuous, real coordinate.
- **Matter particles** (fermions) – on the **vertices**.
- **Gauge fields** – on the lattice's **links**



Gauge Transformations

- Act on both the matter and gauge degrees of freedom.

- **Local** : a unique transformation (depending on a unique element of the **gauge group**) may be chosen for each site
- The states are **invariant under each local transformation separately**.



$$\hat{\Theta}_g(\mathbf{x}) = \prod_{k=1\ldots d} \left(\widetilde{\Theta}_g(\mathbf{x},k)\,\Theta_g^\dagger\left(\mathbf{x}-\hat{\mathbf{k}},k\right)\right)\breve{\theta}_g^\dagger(\mathbf{x})$$

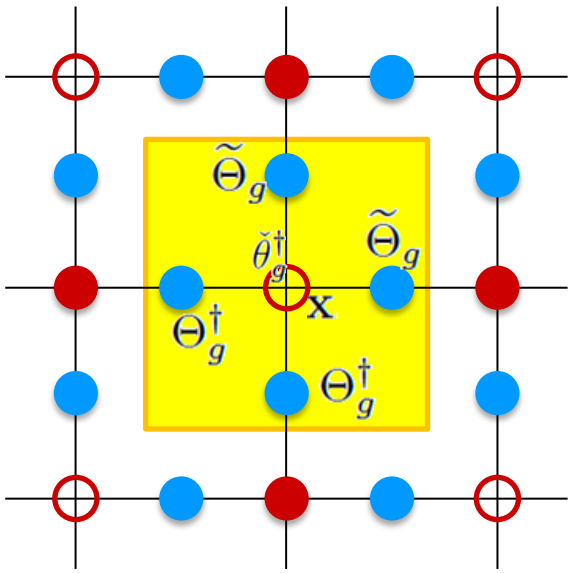
Symmetry → Conserved Charge

– **Transformation rules on the links**

$$\{|g\rangle\}_{g\in G}$$

$$\Theta_g\,|h\rangle = \left|hg^{-1}\right\rangle \qquad \Theta_g = e^{i\phi_a(g)R_a}$$

$$\widetilde{\Theta}_g\,|h\rangle = \left|g^{-1}h\right\rangle \qquad \widetilde{\Theta}_g = e^{i\phi_a(g)L_a}$$



– **Gauge Transformations:**

$$\hat{\Theta}_g(\mathbf{x}) = \prod_{k=1\ldots d}\left(\widetilde{\Theta}_g(\mathbf{x},k)\,\Theta_g^\dagger\left(\mathbf{x}-\hat{\mathbf{k}},k\right)\right)\check{\theta}_g^\dagger(\mathbf{x})$$

$$\hat{\Theta}_g(\mathbf{x})\,|\Psi\rangle = |\Psi\rangle \qquad \forall \mathbf{x}, g$$

– **Generators → Gauss law , left and right E fields:**

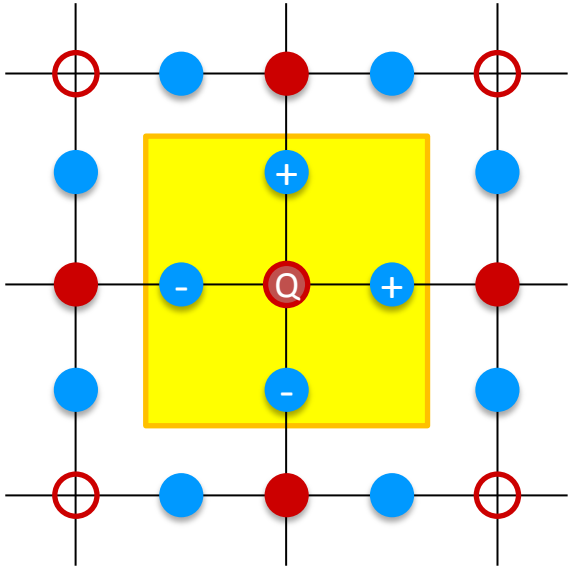
$$G_a(\mathbf{x}) = \sum_{k=1\ldots d}\left(L_a(\mathbf{x},k) - R_a\left(\mathbf{x}-\hat{\mathbf{k}},k\right)\right) - Q_a(\mathbf{x})$$

$$G_a(\mathbf{x})\,|\Psi\rangle = 0 \qquad [G_a(\mathbf{x}),H] = 0 \qquad \forall \mathbf{x}, a$$

Structure of the Hilbert Space

- Generators of gauge transformations (cQED):

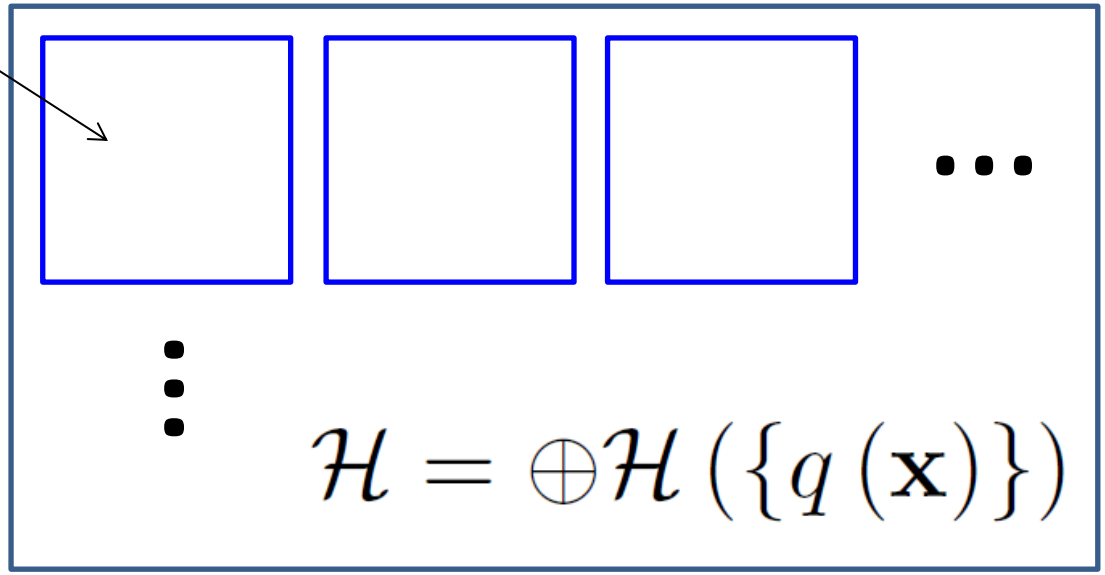
$$G(\mathbf{x}) = \mathrm{div} L(\mathbf{x}) - Q(\mathbf{x})$$
$$\equiv \sum_k \left(L_k(\mathbf{x}) - L_k(\mathbf{x} - \hat{\mathbf{e}}_k)\right) - Q(\mathbf{x})$$



Gauss' Law $G(\mathbf{x})|\psi\rangle = q(\mathbf{x})|\psi\rangle$

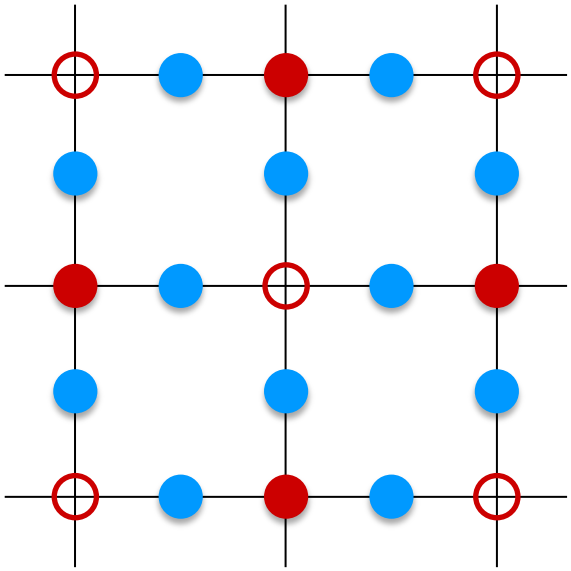
$$[G(\mathbf{x}), H] = 0 \quad \forall \mathbf{x}$$

Sectors with fixed Static charge configurations



Allowed Interactions

- Must preserve the symmetry – commute with the “Gauss Laws” (generators of symmetry transformations)

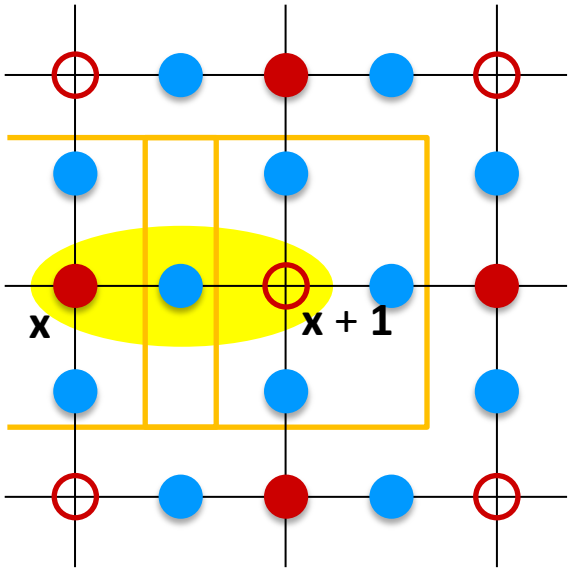


Allowed Interactions

- Must preserve the symmetry – commute with the "Gauss Laws" (generators of symmetry transformations)
- <u>First option</u>: Link (**matter**-**gauge**) interaction:

$$\psi_m^\dagger(\mathrm{x})\, U_{mn}(\mathrm{x}, k)\, \psi_n\left(\mathrm{x} + \hat{\mathrm{k}}\right)$$

  – A **fermion** hops to a **neighboring site**, and the **flux on the link in the middle changes** to preserve **Gauss laws on the two relevant sites**

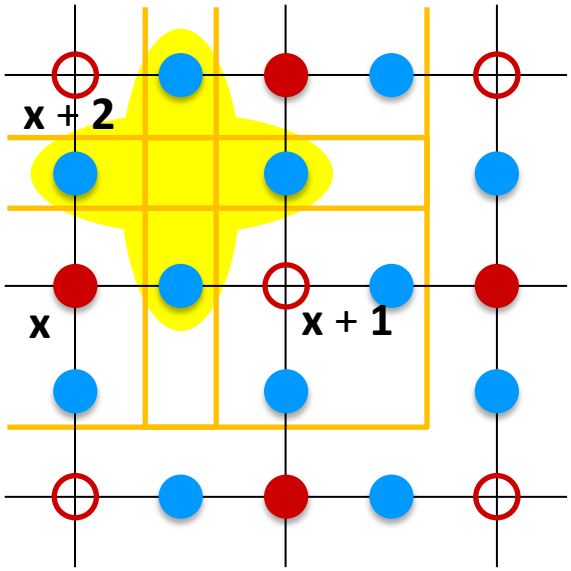


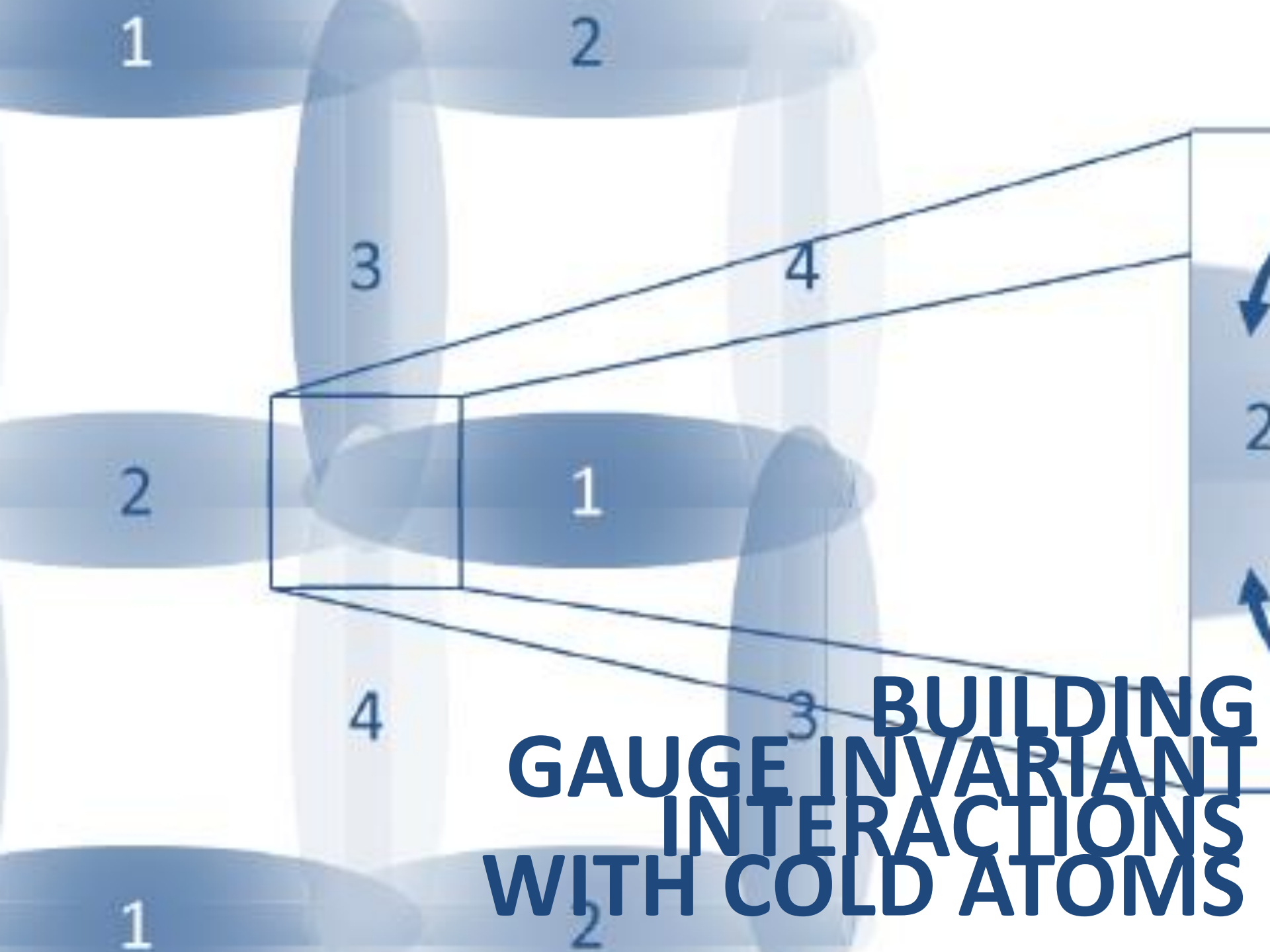
Allowed Interactions

- Must preserve the symmetry – commute with the "Gauss Laws" (generators of symmetry transformations)
- <u>Second option</u>: plaquette interaction:

$$\mathrm{Tr}\left(U(\mathrm{x},1)U(\mathrm{x}+\hat{1},2)U^{\dagger}(\mathrm{x}+\hat{2},1)U^{\dagger}(\mathrm{x},2)\right)$$

  - The **flux on the links of a single plaquette changes** such that the **Gauss laws on the four relevant sites** is preserved.
  - **Magnetic interaction**.





BUILDING GAUGE INVARIANT INTERACTIONS WITH COLD ATOMS

Basic Requirements from a LGT Q. Simulator

- Include both fermions (matter) and gauge fields

- Have Lorentz (relativistic) symmetry

- Manifest **Local** (Gauge) Invariance

E. Zohar, J. I. Cirac, B. Reznik, Phys. Rev. Lett. **110**, 055302 (2013)

E. Zohar, J. I. Cirac, B. Reznik, Rep. Prog. Phys. **79**, 014401 (2016)

Basic Requirements from a LGT Q. Simulator

- Include both fermions (matter) and gauge fields

  Use ultracold atoms in optical lattices: both bosonic and fermionic atoms may be trapped and manipulated.

- Have Lorentz (relativistic) symmetry

- Manifest **Local** (Gauge) Invariance

E. Zohar, J. I. Cirac, B. Reznik, Phys. Rev. Lett. **110**, 055302 (2013)

E. Zohar, J. I. Cirac, B. Reznik, Rep. Prog. Phys. **79**, 014401 (2016)

Basic Requirements from a LGT Q. Simulator

- Include both fermions (matter) and gauge fields

  Use ultracold atoms in optical lattices: both bosonic and fermionic atoms may be trapped and manipulated.

- Have Lorentz (relativistic) symmetry

  Simulate lattice gauge theory. Symmetry may be restored in a careful continuum limit.

- Manifest **Local** (Gauge) Invariance

E. Zohar, J. I. Cirac, B. Reznik, Phys. Rev. Lett. **110**, 055302 (2013)

E. Zohar, J. I. Cirac, B. Reznik, Rep. Prog. Phys. **79**, 014401 (2016)

Basic Requirements from a LGT Q. Simulator

- Include both fermions (matter) and gauge fields

  Use ultracold atoms in optical lattices: both bosonic and fermionic atoms may be trapped and manipulated.

- Have Lorentz (relativistic) symmetry

  Simulate lattice gauge theory. Symmetry may be restored in a careful continuum limit.

- Manifest **Local** (Gauge) Invariance on top of the natural global atomic symmetries (number conservation)

E. Zohar, J. I. Cirac, B. Reznik, Phys. Rev. Lett. **110**, 055302 (2013)

E. Zohar, J. I. Cirac, B. Reznik, Rep. Prog. Phys. **79**, 014401 (2016)

Basic Requirements from a LGT Q. Simulator

- Include both fermions (matter) and gauge fields

  Use ultracold atoms in optical lattices: both bosonic and fermionic atoms may be trapped and manipulated.

- Have Lorentz (relativistic) symmetry

  Simulate lattice gauge theory. Symmetry may be restored in a careful continuum limit.

- Manifest **Local** (Gauge) Invariance on top of the natural global atomic symmetries (number conservation)

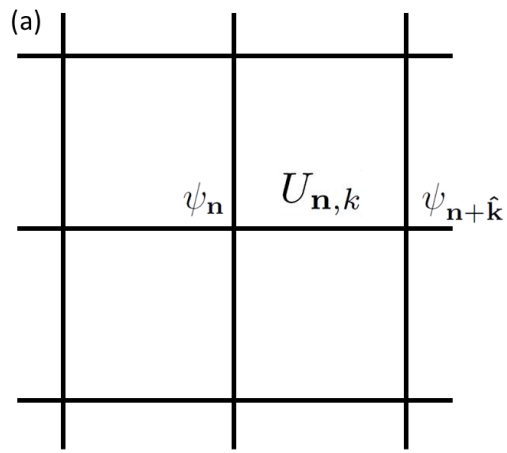
  Local (gauge) symmetries may be introduced to the atomic simulator using several methods.

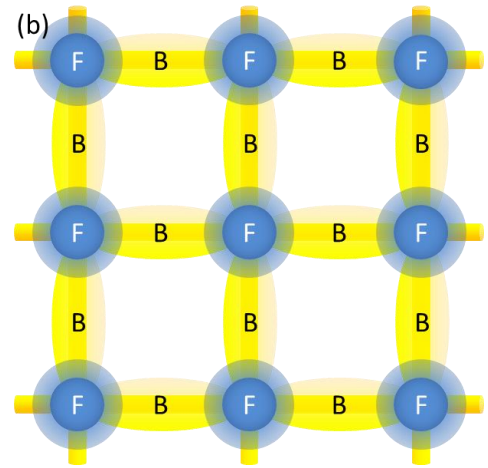
E. Zohar, J. I. Cirac, B. Reznik, Phys. Rev. Lett. **110**, 055302 (2013)

E. Zohar, J. I. Cirac, B. Reznik, Rep. Prog. Phys. **79**, 014401 (2016)

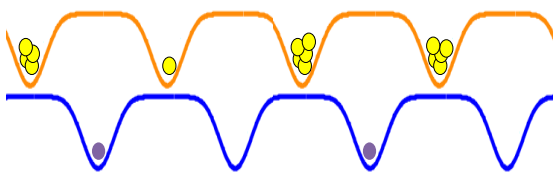
QS of LGTs with Ultracold Atoms in Optical Lattices

- **Fermionic** matter fields
- (Bosonic) gauge fields





Super-lattice:



Atomic internal (**hyperfine**) levels

0 (ultracold)

$$\mathbf{F} = \mathbf{I} + \cancel{\mathbf{L}} + \mathbf{S} \qquad \mathbf{F}^2|F, m_{\mathrm{F}}\rangle = F(F+1)|F, m_{\mathrm{F}}\rangle \qquad F_z|F, m_{\mathrm{F}}\rangle = m_{\mathrm{F}}|F, m_{\mathrm{F}}\rangle$$

$$\mathcal{H} = \sum_{\alpha,\beta} \Phi^\dagger_\alpha(\mathbf{x})\left(\delta^{\alpha\beta}\left(-\frac{\nabla^2}{2m} + V^\alpha_{\mathrm{op}}(\mathbf{x}) + V_{\mathrm{T}}(\mathbf{x})\right) + \Omega^{\alpha\beta}(\mathbf{x})\right)\Phi_\beta(\mathbf{x})$$
$$+ \sum_{\alpha,\beta,\gamma,\delta} \int \mathrm{d}^3x' \Phi^\dagger_\alpha(\mathbf{x}')\Phi^\dagger_\beta(\mathbf{x}) V_{\alpha\beta\gamma\delta}(\mathbf{x}-\mathbf{x}')\Phi_\gamma(\mathbf{x})\Phi_\delta(\mathbf{x}')$$

The atomic Hamiltonian (Hubbard) has a global symmetry

General form (after "overlap" Wannier integrations)

$$H = \sum_{\mathbf{m},\mathbf{n}} J_{\mathbf{m},\mathbf{n}} a^\dagger_{\mathbf{m}} a_{\mathbf{n}} + \sum_{\mathbf{m},\mathbf{n},\mathbf{k},\mathbf{l}} U_{\mathbf{m},\mathbf{n},\mathbf{k},\mathbf{l}} a^\dagger_{\mathbf{m}} a^\dagger_{\mathbf{n}} a_{\mathbf{k}} a_{\mathbf{l}}$$

Assuming nearest neighbor interactions

$$H = J \sum_{\langle \mathbf{m},\mathbf{n} \rangle} a^\dagger_{\mathbf{m}} a_{\mathbf{n}} + U \sum_{\mathbf{m}} N_{\mathbf{m}} \left(N_{\mathbf{m}} - 1 \right)$$

For many species

$$H = \sum_{\mathbf{m},\mathbf{n},\alpha,\beta} J^{\alpha,\beta}_{\mathbf{m},\mathbf{n}} a^\dagger_{\mathbf{m},\alpha} a_{\mathbf{n},\beta} + \sum_{\mathbf{m},\mathbf{n},\mathbf{k},\mathbf{l}} U^{\alpha,\beta,\gamma,\delta}_{\mathbf{m},\mathbf{n},\mathbf{k},\mathbf{l}} a^\dagger_{\mathbf{m},\alpha} a^\dagger_{\mathbf{n},\beta} a_{\mathbf{k},\gamma} a_{\mathbf{l},\delta}$$

Total number of particles is conserved (global symmetry): no <u>apparent</u> local symmetry

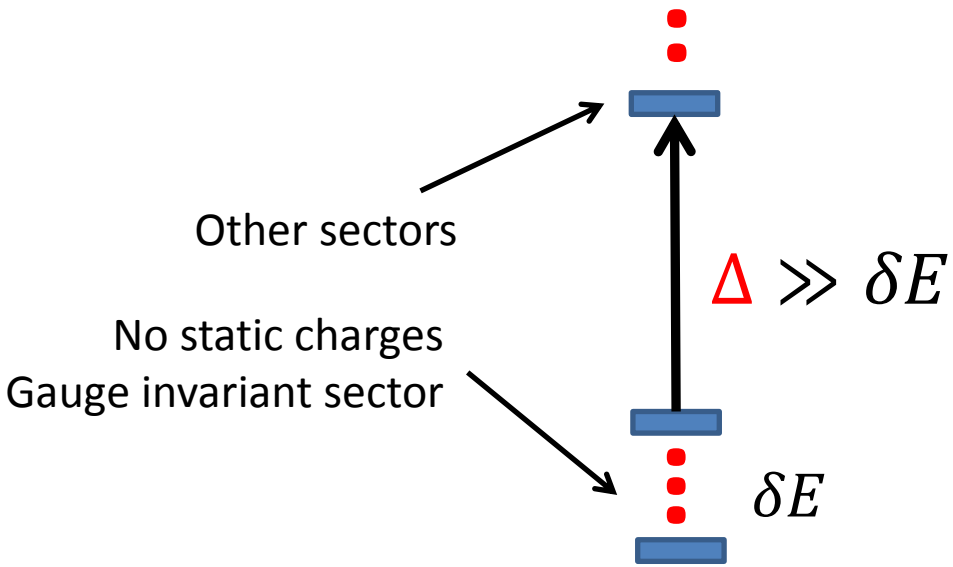
Analog Approach I: Effective Local Gauge Invariance

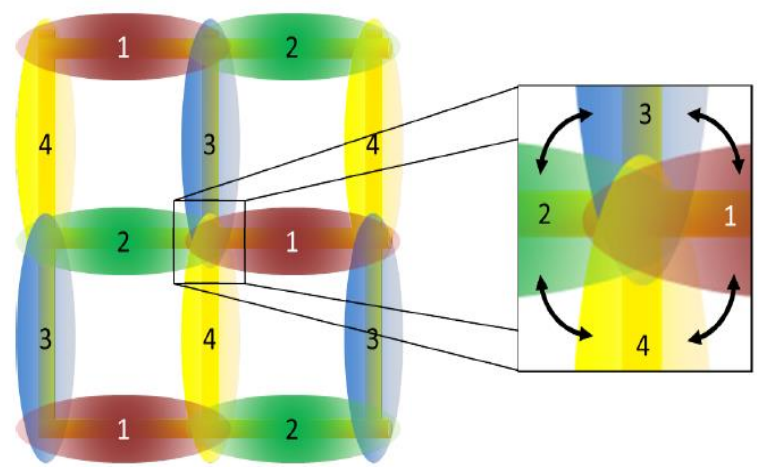
Gauss law is added to the Hamiltonian as a constraint (penalty term).

Leaving a gauge invariant sector of Hilbert space costs too much Energy.

Low energy sector with an effective gauge invariant Hamiltonian.

Emerging plaquette interactions (second order perturbation theory).





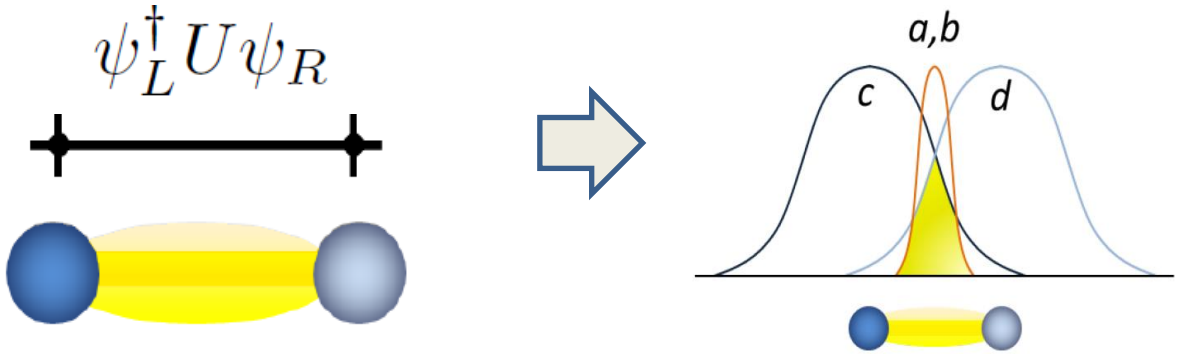
E. Zohar, B. Reznik, Phys. Rev. Lett. 107, 275301 (2011)

E. Zohar, J. I. Cirac, B. Reznik, Phys. Rev. Lett. 109, 125302 (2012)

E. Zohar, J. I. Cirac, B. Reznik, Phys. Rev. Lett. 110, 055302 (2013)

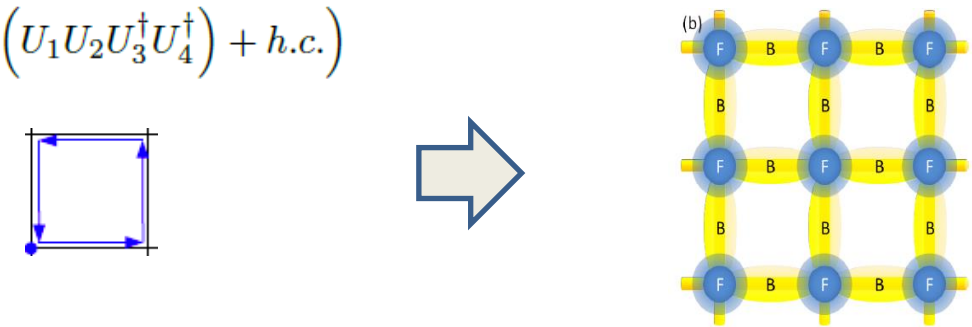
E. Zohar, J. I. Cirac, B. Reznik, Rep. Prog. Phys. 79, 014401 (2016)

Analog Approach II:
Atomic Symmetries → Local Gauge Invariance



- Links ↔ atomic scattering : gauge invariance is a <u>fundamental</u> symmetry

$$\sum_{\text{plaquettes}} \left(\mathrm{Tr}\left(U_1 U_2 U_3^\dagger U_4^\dagger \right) + h.c. \right)$$



- Plaquettes ↔ gauge invariant links ↔ virtual loops of ancillary fermions.

E. Zohar, J. I. Cirac, B. Reznik, Phys. Rev. Lett. **110**, 125304 (2013)

E. Zohar, J. I. Cirac, B. Reznik, Phys. Rev. A **88** 023617 (2013)

E. Zohar, J. I. Cirac, B. Reznik, Rep. Prog. Phys. **79**, 014401 (2016)

D. González Cuadra, **E. Zohar**, J. I. Cirac, New J. Phys. **19** 063038 (2017)

Realization of Link Interactions

$$\psi_L^\dagger U \psi_R$$

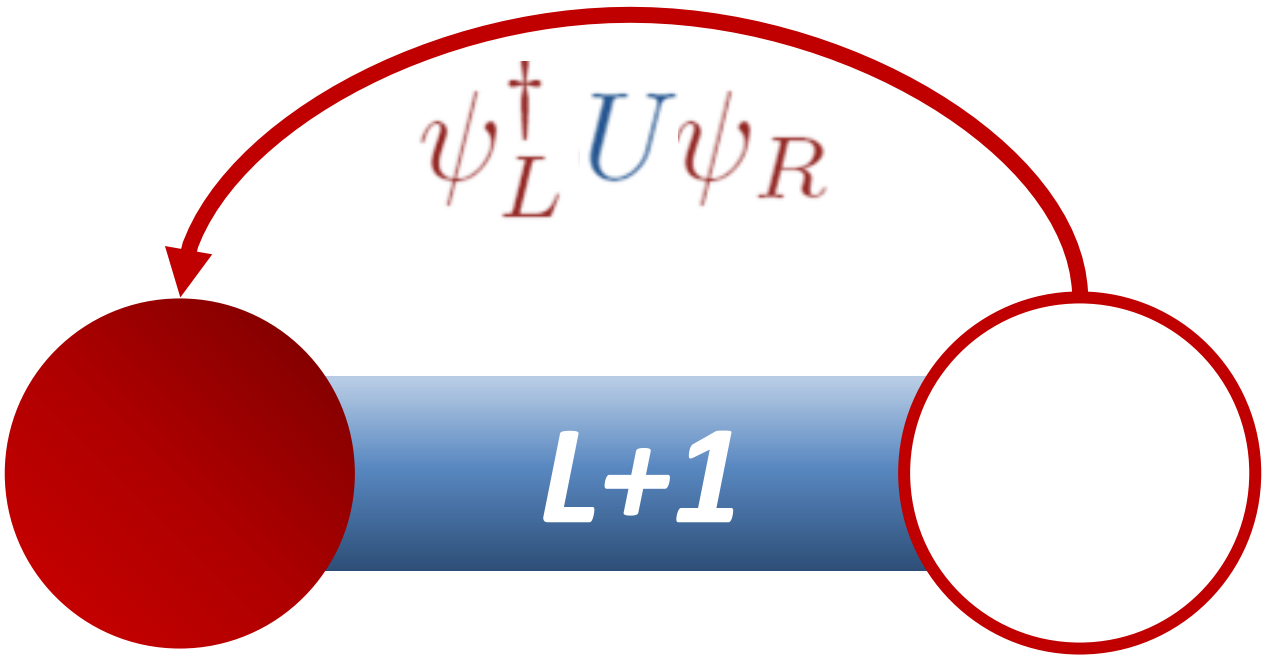
E. Zohar, J. I. Cirac, B. Reznik, Phys. Rev. Lett. **110**, 125304 (2013)

E. Zohar, J. I. Cirac, B. Reznik, Phys. Rev. A **88** 023617 (2013)

E. Zohar, J. I. Cirac, B. Reznik, Rep. Prog. Phys. **79**, 014401 (2016)

D. González Cuadra, **E. Zohar**, J. I. Cirac, New J. Phys. **19** 063038 (2017)

Realization of Link Interactions



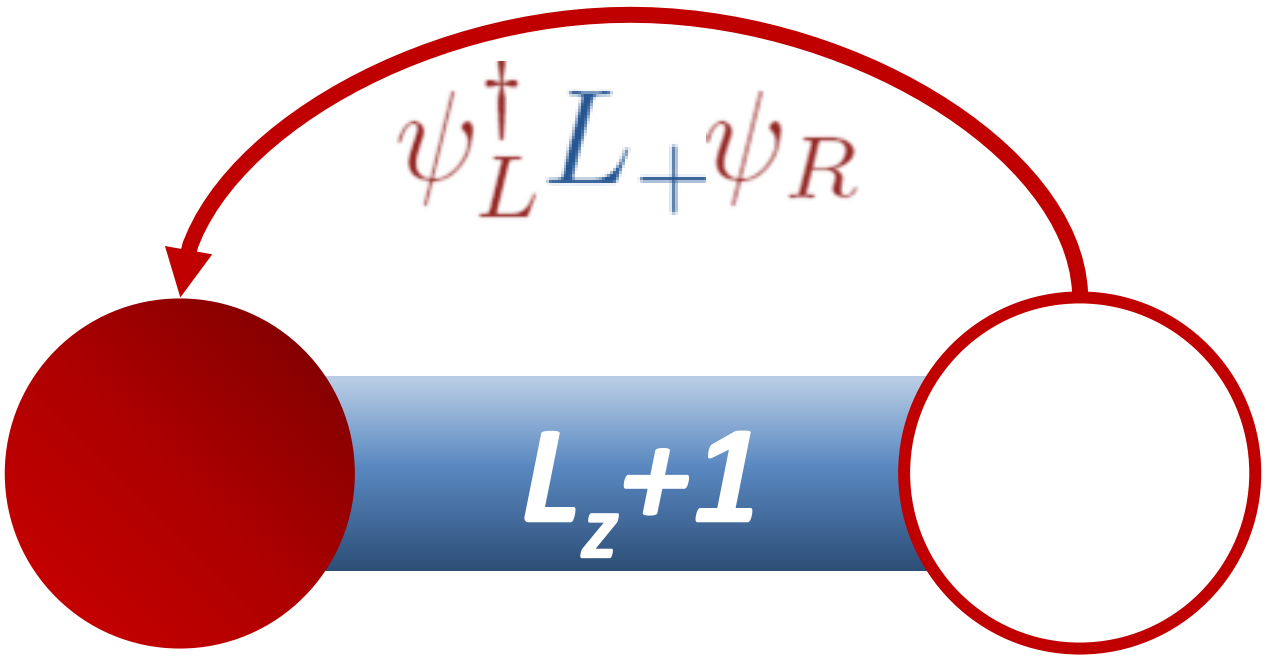
E. Zohar, J. I. Cirac, B. Reznik, Phys. Rev. Lett. **110**, 125304 (2013)

E. Zohar, J. I. Cirac, B. Reznik, Phys. Rev. A **88** 023617 (2013)

E. Zohar, J. I. Cirac, B. Reznik, Rep. Prog. Phys. **79**, 014401 (2016)

D. González Cuadra, **E. Zohar**, J. I. Cirac, New J. Phys. **19** 063038 (2017)

Realization of Link Interactions



E. Zohar, J. I. Cirac, B. Reznik, Phys. Rev. Lett. **110**, 125304 (2013)

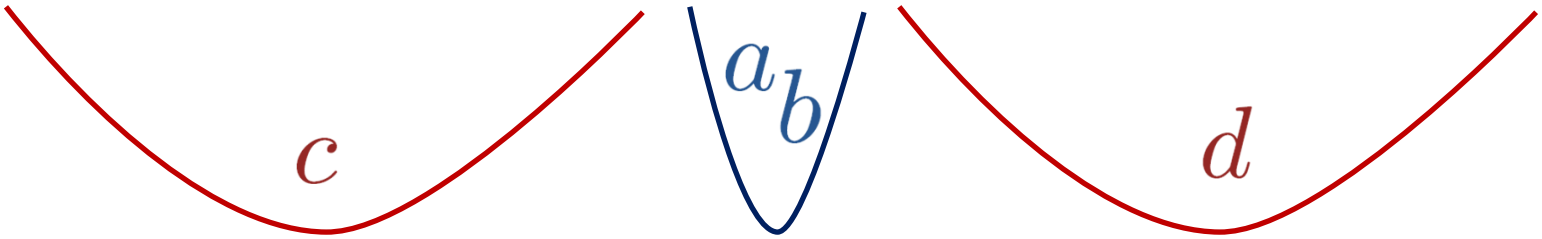
E. Zohar, J. I. Cirac, B. Reznik, Phys. Rev. A **88** 023617 (2013)

E. Zohar, J. I. Cirac, B. Reznik, Rep. Prog. Phys. **79**, 014401 (2016)

D. González Cuadra, **E. Zohar**, J. I. Cirac, New J. Phys. **19** 063038 (2017)

Superlattice Structure

$$\psi_L^\dagger U \psi_R \sim \psi_L^\dagger L_+ \psi_R = c^\dagger a^\dagger b d$$



Schwinger algebra

$$\ell = \frac{1}{2}\left(a^\dagger a + b^\dagger b\right)$$

$$L_+ = a^\dagger b \qquad L_- = b^\dagger a \qquad L_z = \frac{1}{2}\left(a^\dagger a - b^\dagger b\right)$$

E. Zohar, J. I. Cirac, B. Reznik, Phys. Rev. Lett. **110**, 125304 (2013)

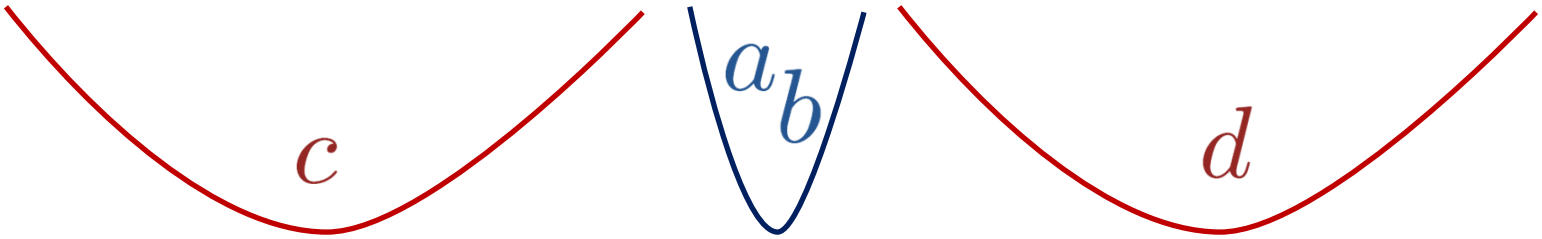
E. Zohar, J. I. Cirac, B. Reznik, Phys. Rev. A **88** 023617 (2013)

E. Zohar, J. I. Cirac, B. Reznik, Rep. Prog. Phys. **79**, 014401 (2016)

D. González Cuadra, **E. Zohar**, J. I. Cirac, New J. Phys. **19** 063038 (2017)

Superlattice Structure

$$\psi_L^\dagger U \psi_R \sim \psi_L^\dagger L_+ \psi_R = c^\dagger a^\dagger b d$$



Schwinger algebra

$$L_+ = a^\dagger b \sim e^{i(\phi_a - \phi_b)} \equiv e^{i\phi} = U$$

For large ℓ , $m \ll \ell$

E. Zohar, J. I. Cirac, B. Reznik, Phys. Rev. Lett. **110**, 125304 (2013)

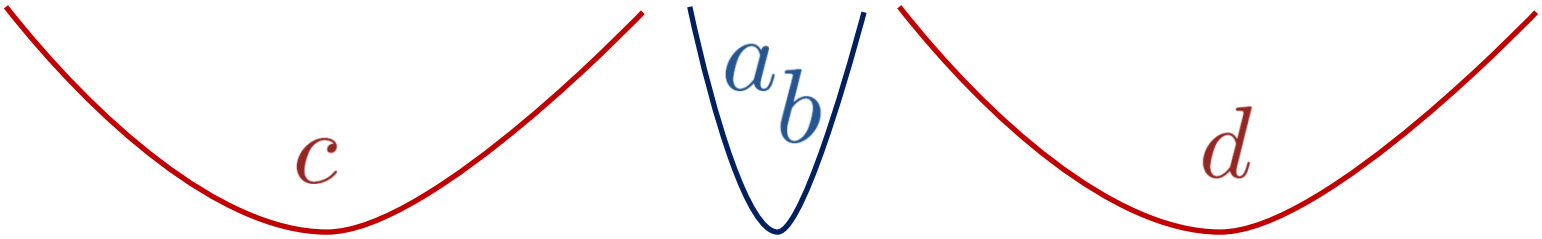
E. Zohar, J. I. Cirac, B. Reznik, Phys. Rev. A **88** 023617 (2013)

E. Zohar, J. I. Cirac, B. Reznik, Rep. Prog. Phys. **79**, 014401 (2016)

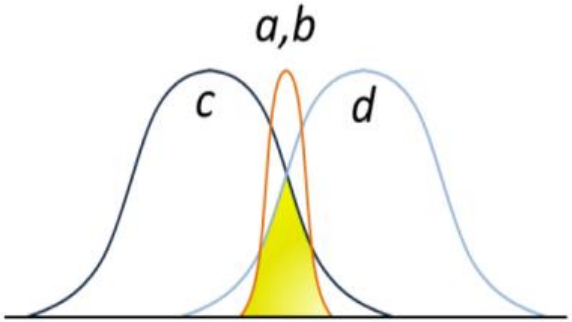
D. González Cuadra, **E. Zohar**, J. I. Cirac, New J. Phys. **19** 063038 (2017)

Superlattice Structure

$$\psi_L^\dagger U \psi_R \sim \psi_L^\dagger L_+ \psi_R = c^\dagger a^\dagger b d$$



- Narrow, deep bosonic wells → no tunneling, fixed number on link → fixed Schwinger representation: $\ell = \frac{1}{2}\left(a^\dagger a + b^\dagger b\right)$
- Staggered fermionic wells → no tunneling
- Only possible Hamiltonian terms: Scattering on the link: B-F – interaction, B-B – electric energy



E. Zohar, J. I. Cirac, B. Reznik, Phys. Rev. Lett. **110**, 125304 (2013)

E. Zohar, J. I. Cirac, B. Reznik, Phys. Rev. A **88** 023617 (2013)

E. Zohar, J. I. Cirac, B. Reznik, Rep. Prog. Phys. **79**, 014401 (2016)

D. González Cuadra, **E. Zohar**, J. I. Cirac, New J. Phys. **19** 063038 (2017)

Ultracold Atoms → S-wave scattering

- Described by scattering lengths a_F (tunable via Feshbach resonances)

- Different scattering channels governed by total hyperfine angular momentum – $\{F_T, m_T\}$, which is conserved in the collision

$$\sum_{\alpha,\beta,\gamma,\delta} \int \mathrm{d}^3x \mathrm{d}^3x' \Phi^\dagger_\alpha(\mathbf{x}')\Phi^\dagger_\beta(\mathbf{x}) V_{\alpha\beta\gamma\delta}(\mathbf{x}-\mathbf{x}')\Phi_\gamma(\mathbf{x})\Phi_\delta(\mathbf{x}')$$

$$V_{\alpha,\beta,\gamma,\delta}(\mathbf{x}-\mathbf{x}') = \frac{2\pi}{m}\delta^{(3)}(\mathbf{x}-\mathbf{x}')\sum_{F_\mathrm{T}} a_{F_\mathrm{T}}(P_{F_\mathrm{T}})_{\alpha,\beta,\gamma,\delta}$$

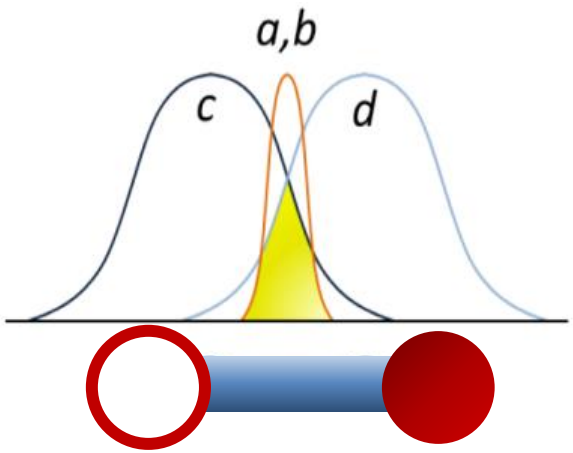
$$P_{F_\mathrm{T}} = \sum_{k=0}^{n-1} G_{F_\mathrm{T},k}(\mathbf{F_1}\cdot\mathbf{F_2})^k \qquad V_{\alpha,\beta,\gamma,\delta}(\mathbf{x}-\mathbf{x}') = \delta^{(3)}(\mathbf{x}-\mathbf{x}')\frac{g_k}{2}((\mathbf{F_1}\cdot\mathbf{F_2})^k)_{\alpha,\beta,\gamma,\delta}$$

Atomic symmetry → Gauge Invariance

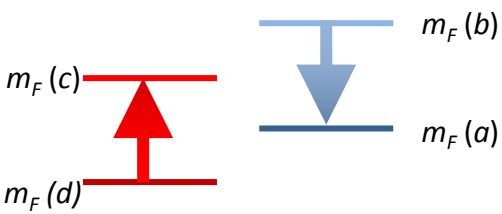
$$\sum_{\alpha,\beta,\gamma,\delta} \int d^3x d^3x' \Phi^\dagger_\alpha(\mathbf{x}')\Phi^\dagger_\beta(\mathbf{x})V_{\alpha\beta\gamma\delta}(\mathbf{x}-\mathbf{x}')\Phi_\gamma(\mathbf{x})\Phi_\delta(\mathbf{x}')$$

$$V_{\alpha,\beta,\gamma,\delta}(\mathbf{x}-\mathbf{x}') = \delta^{(3)}(\mathbf{x}-\mathbf{x}')\sum_{F_T} C_{F_T}\langle F_1, m_{F1}=\alpha; F_2, m_{F2}=\beta | F_T, M_F\rangle$$

$$\times \langle F_T, M_F | F_1, m_{F1}=\gamma; F_2, m_{F2}=\delta\rangle$$

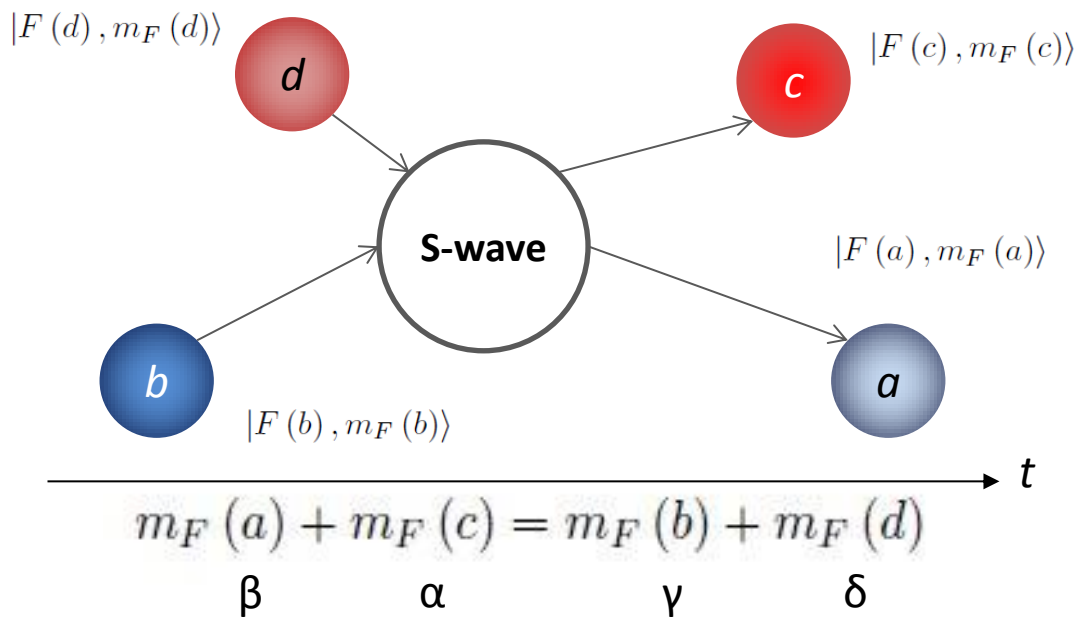


$$\psi_L^\dagger U \psi_R \sim \psi_L^\dagger L_+ \psi_R = c^\dagger a^\dagger b d$$



Fermionic atoms
- matter

Bosonic atoms
- Gauge field



$$m_F(a) + m_F(c) = m_F(b) + m_F(d)$$

β α γ δ

E. Zohar, J. I. Cirac, B. Reznik, Phys. Rev. Lett. **110**, 125304 (2013)

E. Zohar, J. I. Cirac, B. Reznik, Phys. Rev. A **88** 023617 (2013)

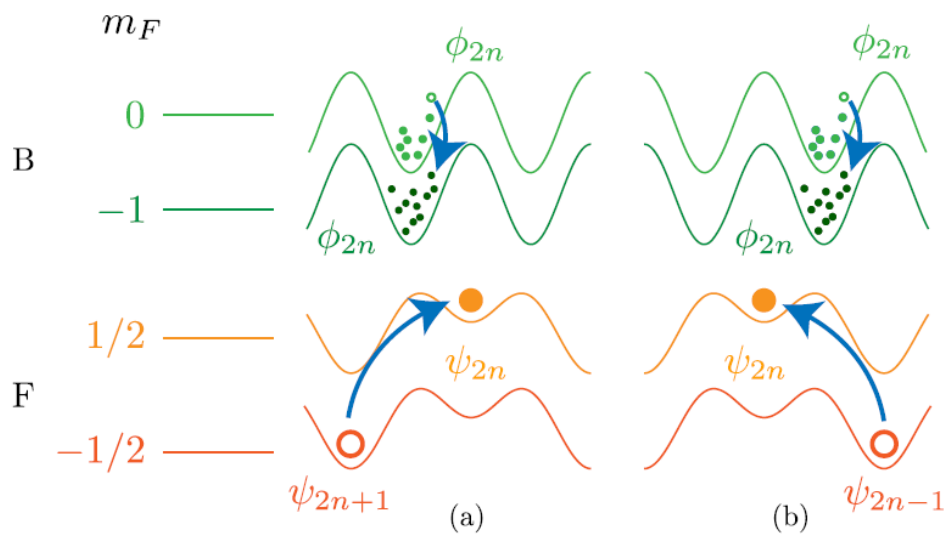
E. Zohar, J. I. Cirac, B. Reznik, Rep. Prog. Phys. **79**, 014401 (2016)

D. González Cuadra, **E. Zohar**, J. I. Cirac, New J. Phys. **19** 063038 (2017)

Heidelberg Omplementation

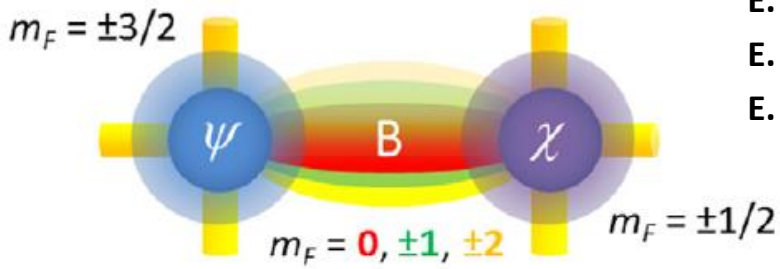
- **Kasper, Hebenstreit, Jendrzejewski, Oberthaler, Berges**
  NJP 19 023030 (2017) – very exciting results

- Matter: F = ½ ^{6}Li atoms
- Gauge field: F = 1 ^{23}Na atoms
- No Feshbach resonance!
- On the links, around 100 atomic bosons – very high electric field truncation (±50)



Generalizations

- Valid for any gauge group, including non-Abelian



E. Zohar, J. I. Cirac, B. Reznik, Phys. Rev. Lett. **110**, 125304 (2013)
E. Zohar, J. I. Cirac, B. Reznik, Phys. Rev. A **88** 023617 (2013)
E. Zohar, J. I. Cirac, B. Reznik, Rep. Prog. Phys. **79**, 014401 (2016)

- Truncation schemes for general groups (analogous to the Schwinger representation used in the abelian case)– possible as well)

$$U^{j}_{mm'} = \sum_{J,K} \sqrt{\frac{\dim(J)}{\dim(K)}} \langle JM\, jm | KN \rangle \langle KN' | JM'\, jm' \rangle\, a^{\dagger K}_{NN'} a^{J}_{MM'}$$

$$U^{j=1/2}_{mn} = \frac{1}{\sqrt{2}} \begin{pmatrix} |{++}\rangle\langle 0| + |0\rangle\langle {--}| & |{+-}\rangle\langle 0| - |0\rangle\langle {-+}| \\ |{-+}\rangle\langle 0| - |0\rangle\langle {+-}| & |0\rangle\langle {++}| + |{--}\rangle\langle 0| \end{pmatrix}$$

E. Zohar, M. Burrello, Phys. Rev. D. 91, 054506 (2015)

Further Dimensions → Plaquette Interactions

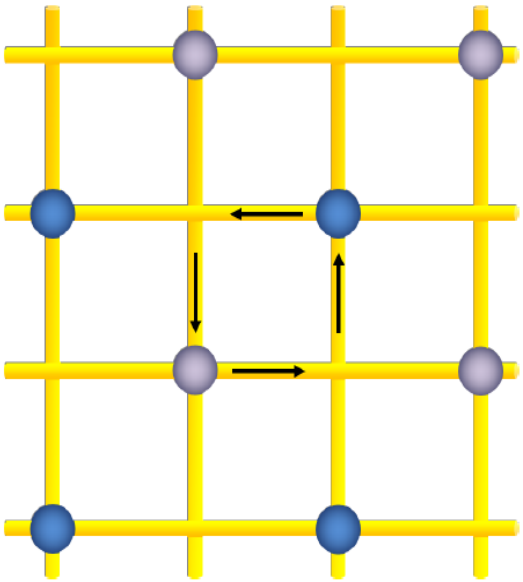
$$\sum_{\text{plaquettes}} \left(\mathrm{Tr}\left(U_1 U_2 U_3^\dagger U_4^\dagger \right) + h.c. \right)$$

1d elementary link interactions are **already gauge invariant**

Auxiliary fermions:

Heavy,
constrained to "sit"
at special vertices

- **Virtual processes → Weak**
- **Valid for any gauge group, once the link interactions are realized**



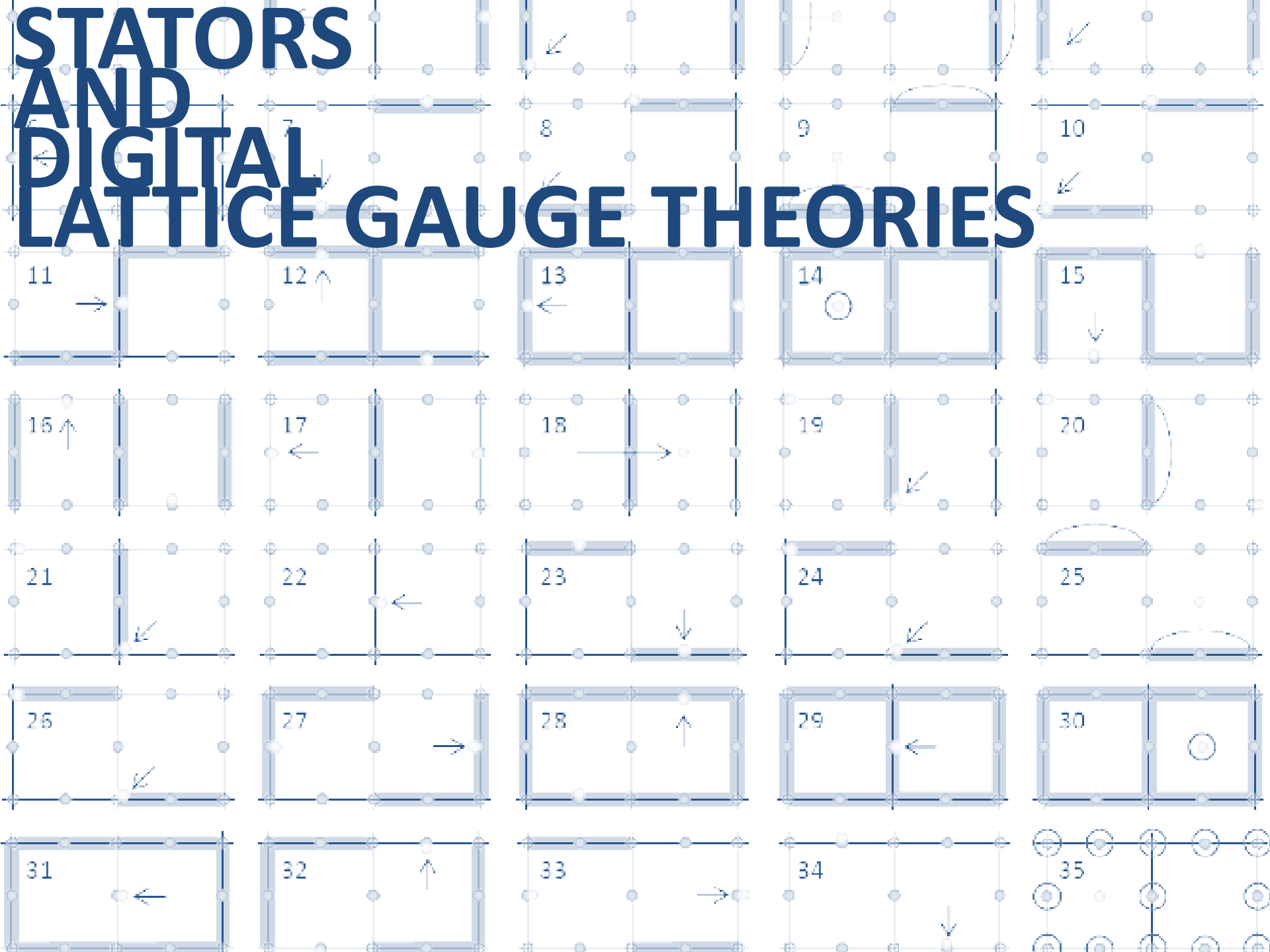
E. Zohar, J. I. Cirac, B. Reznik, Phys. Rev. Lett. 110, 125304 (2013)

E. Zohar, J. I. Cirac, B. Reznik, Phys. Rev. A 88 023617 (2013)

E. Zohar, J. I. Cirac, B. Reznik, Rep. Prog. Phys. 79, 014401 (2016)

D. González Cuadra**, E. Zohar**, J. I. Cirac, New J. Phys. 19 063038 (2017)

STATORS AND DIGITAL LATTICE GAUGE THEORIES



Stators

- "Half a state, half an operator" – operator in one Hilbert space, state in the other

$$S \in \mathcal{O}\left(\mathcal{H}_A\right) \times \mathcal{H}_B$$

- Created by a unitary interaction acting on an initial "control" state

$$S = U_{AB} \left|0_B\right\rangle$$

B. Reznik, Y. Aharonov, B. Groisman, Phys. Rev. A **65** 032312 (2002)

E. Zohar, J. Phys. A. 50 085301 (2017)

Eigenoperator Relations

- One may define stators such that

$$\Theta_B S = S \Theta_A$$

- For example,

$$S = \frac{1}{\sqrt{2}} \left(1_A \otimes |\uparrow_B\rangle + \sigma_A \otimes |\downarrow_B\rangle \right)$$

$$\sigma_{x,B} S = S \sigma_A$$

B. Reznik, Y. Aharonov, B. Groisman, Phys. Rev. A **65** 032312 (2002)

E. Zohar, J. Phys. A. 50 085301 (2017)

Eigenoperators and Dynamics

- In particular, if $H_B S = S H_A$

  we obtain that $e^{-iH_B t} S = S e^{-iH_A t}$

  which may help to generate effective dynamics.

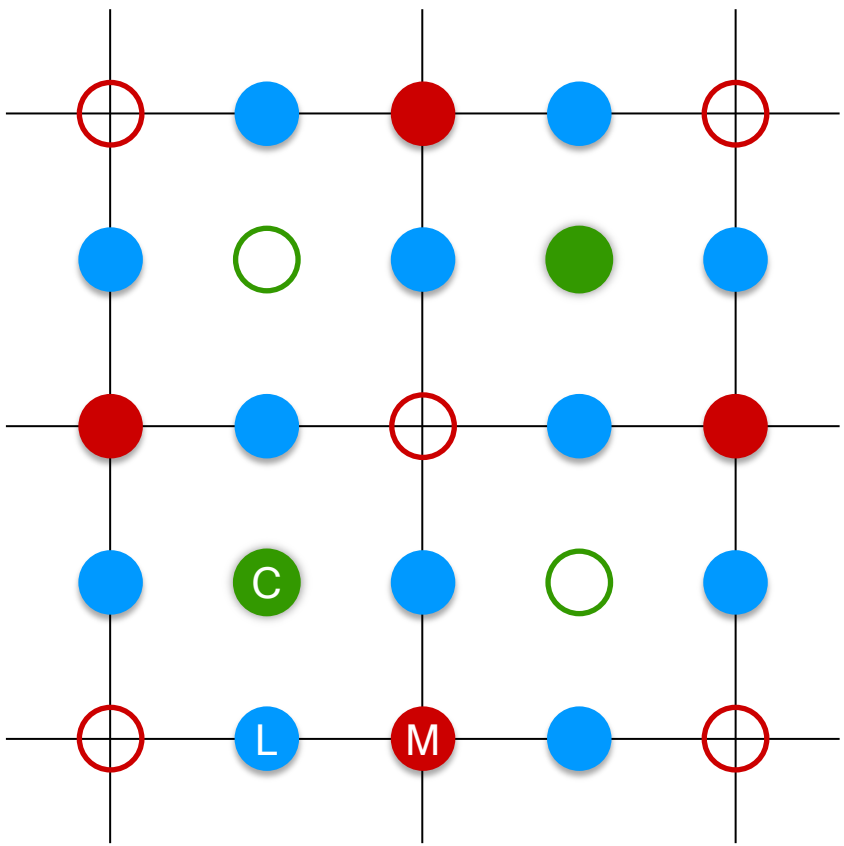
$$e^{-iH_B t} U_{AB} \left|\psi_A\right\rangle \left|0_B\right\rangle = e^{-iH_B t} S \left|\psi_A\right\rangle$$

$$e^{-iH_B t} U_{AB} \left|\psi_A\right\rangle \left|0_B\right\rangle = S e^{-iH_A t} \left|\psi_A\right\rangle = U_{AB} \left|0_B\right\rangle e^{-iH_A t} \left|\psi_A\right\rangle$$

$$e^{-iH_A t} \left|\psi_A\right\rangle = \left\langle 0_B\right| U_{AB}^{\dagger} e^{-iH_B t} U_{AB} \left|\psi_A\right\rangle \left|0_B\right\rangle$$

E. Zohar, J. Phys. A. 50 085301 (2017)

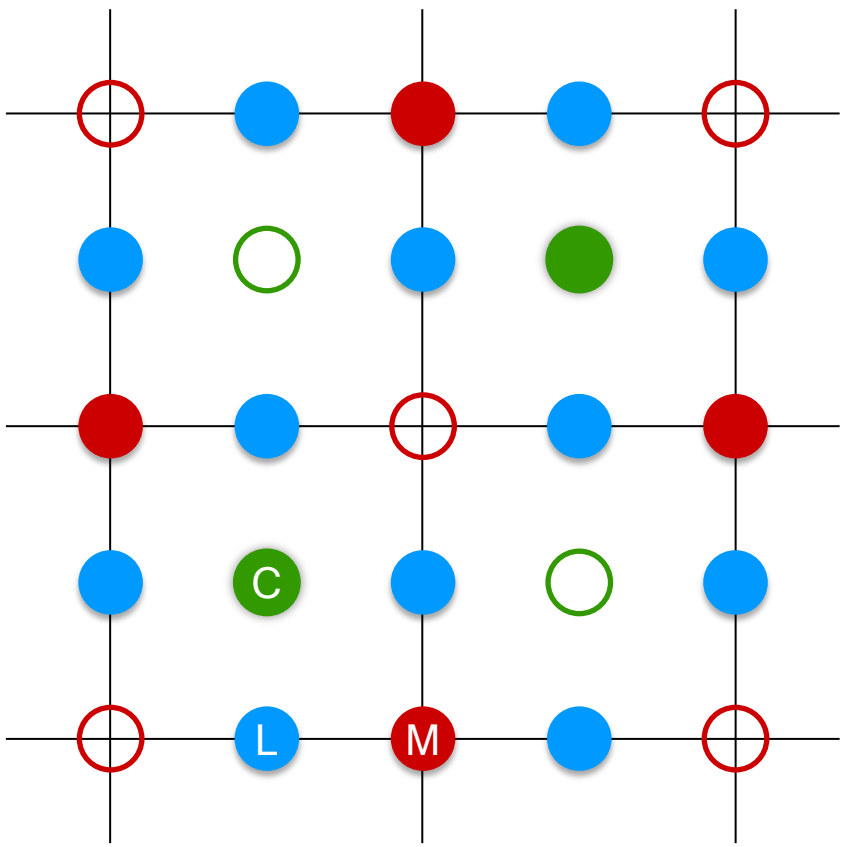
Lattice Gauge Theory with Stators



Matter Fermions
Link (Gauge) degrees of freedom
Control degrees of freedom

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)
E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. A. 95 023604 (2017)

Lattice Gauge Theory with Stators

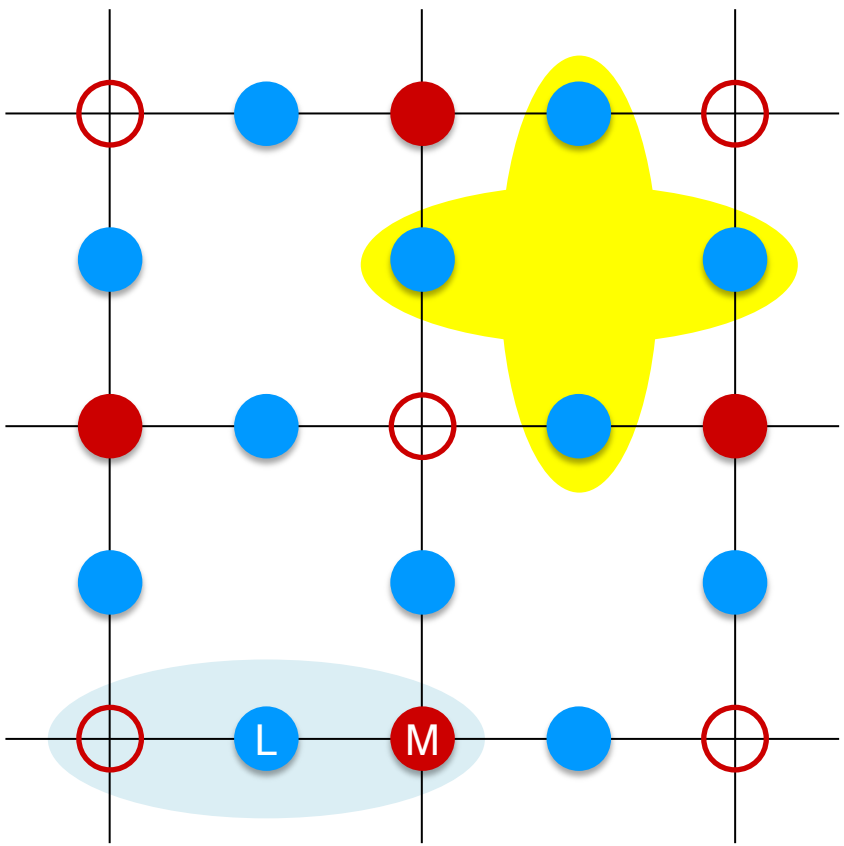


Matter Fermions
Link (Gauge) degrees of freedom
Control degrees of freedom

Stators are created and undone between the control and the physical degrees of freedom.

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)
E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. A. 95 023604 (2017)

Lattice Gauge Theory with Stators



The Z_N example:

- Plaquette interactions

$$Q(\mathbf{x},1)Q(\mathbf{x}+\hat{1},2)Q^\dagger(\mathbf{x}+\hat{2},1)Q^\dagger(\mathbf{x},2)+\text{H.c.}$$

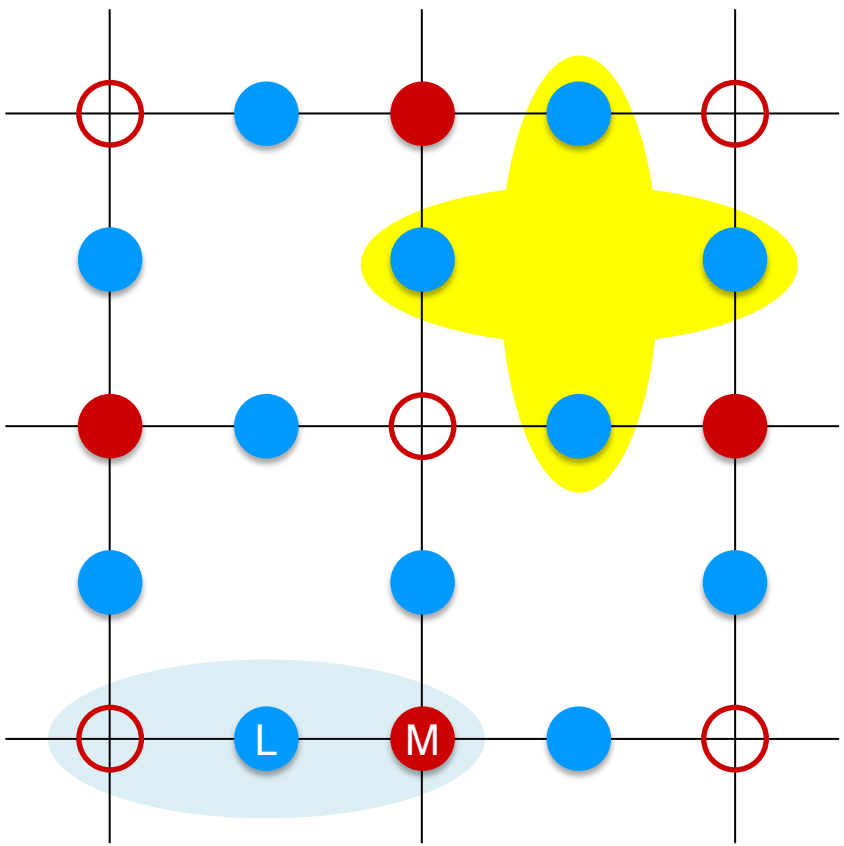
- Link interactions

$$\psi^\dagger(\mathbf{x})Q(\mathbf{x},k)\psi(\mathbf{x}+\hat{\mathbf{k}})$$

$$P^N = Q^N = 1,$$
$$PQP^\dagger = e^{i(2\pi/N)}Q,$$
$$Q|m\rangle = |m+1\rangle \text{ (cyclically)},$$
$$P|m\rangle = e^{i(2\pi/N)m}|m\rangle.$$

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)
E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. A. 95 023604 (2017)

Lattice Gauge Theory with Stators



The Z_2 example:

- Plaquette interactions

$$\sigma_x(\mathbf{x},1)\,\sigma_x(\mathbf{x}+\hat{\mathbf{1}},2)\,\sigma_x(\mathbf{x}+\hat{\mathbf{2}},1)\,\sigma_x(\mathbf{x},2)$$

- Link interactions

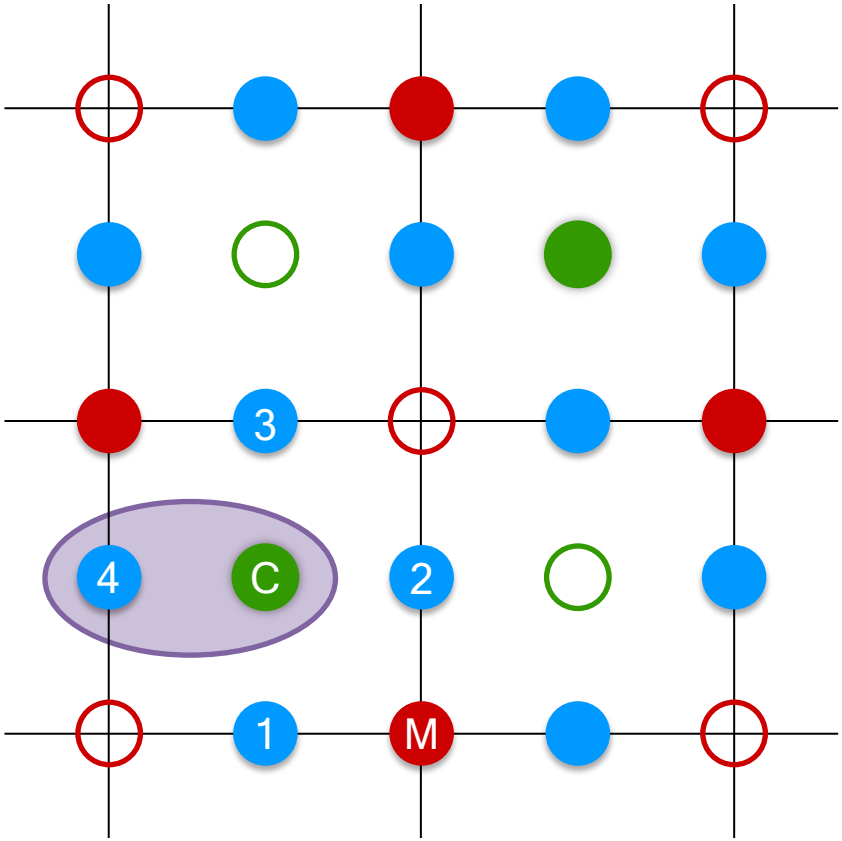
$$\psi^\dagger(\mathbf{x})\sigma_x(\mathbf{x},k)\psi(\mathbf{x}+\hat{\mathbf{k}})$$

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)

Plaquettes: Four-body Interactions

Stators: two-body interactions → four-body interactions

$\mathcal{U} = \mathcal{U}^\dagger = |\tilde{\uparrow}\rangle\langle\tilde{\uparrow}| + \sigma^x \otimes |\tilde{\downarrow}\rangle\langle\tilde{\downarrow}|$



$$|\widetilde{in}\rangle = \frac{1}{\sqrt{2}}\left(|\tilde{\uparrow}\rangle + |\tilde{\downarrow}\rangle\right)$$

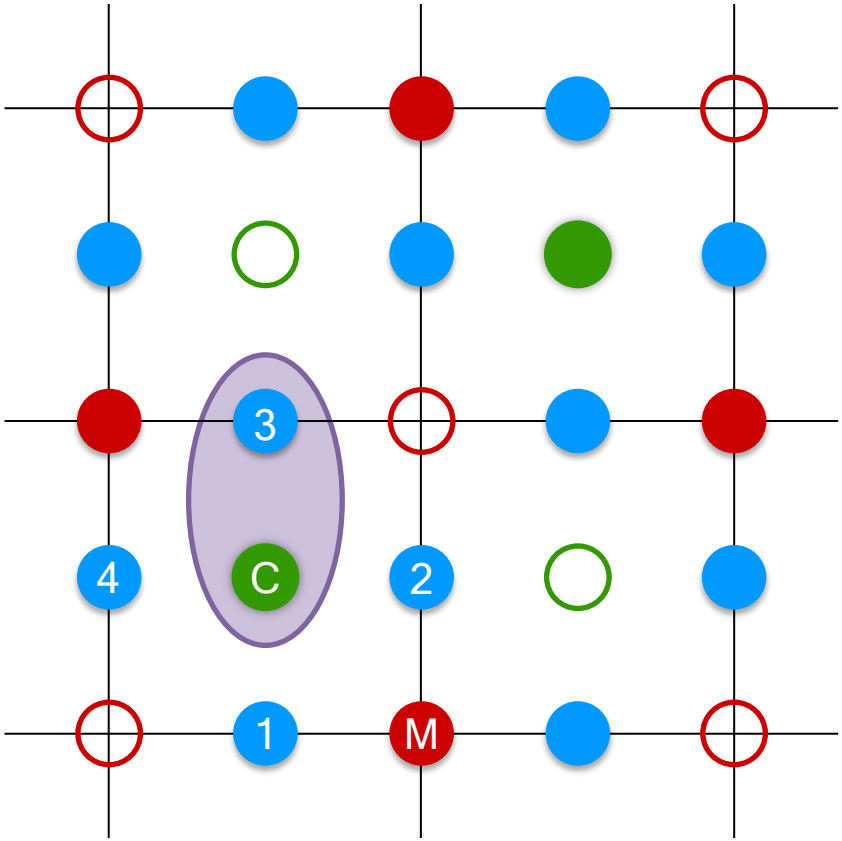
$$\mathcal{U}_4^\dagger |\widetilde{in}\rangle = \frac{1}{\sqrt{2}}\left(|\tilde{\uparrow}\rangle + \sigma_4^x \otimes |\tilde{\downarrow}\rangle\right)$$

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)

Plaquettes: Four-body Interactions

Stators: two-body interactions → four-body interactions

$\mathcal{U} = \mathcal{U}^\dagger = \left|\tilde{\uparrow}\right\rangle\left\langle\tilde{\uparrow}\right| + \sigma^x \otimes \left|\tilde{\downarrow}\right\rangle\left\langle\tilde{\downarrow}\right|$



$$\left|\widetilde{in}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\tilde{\uparrow}\right\rangle + \left|\tilde{\downarrow}\right\rangle\right)$$

$$\mathcal{U}_4^\dagger \left|\widetilde{in}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\tilde{\uparrow}\right\rangle + \sigma_4^x \otimes \left|\tilde{\downarrow}\right\rangle\right)$$

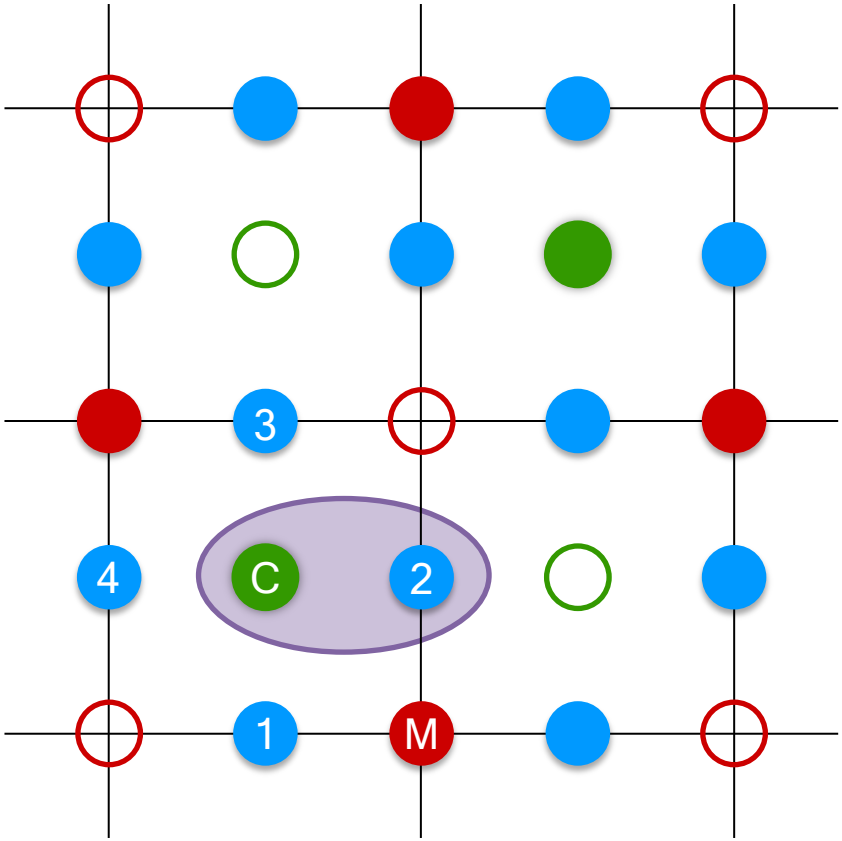
$$\mathcal{U}_3^\dagger \mathcal{U}_4^\dagger \left|\widetilde{in}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\tilde{\uparrow}\right\rangle + \sigma_3^x \sigma_4^x \otimes \left|\tilde{\downarrow}\right\rangle\right)$$

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)

Plaquettes: Four-body Interactions

Stators: two-body interactions → four-body interactions

$\mathcal{U} = \mathcal{U}^\dagger = \left|\tilde{\uparrow}\right\rangle\left\langle\tilde{\uparrow}\right| + \sigma^x \otimes \left|\tilde{\downarrow}\right\rangle\left\langle\tilde{\downarrow}\right|$



$$\left|\widetilde{in}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\tilde{\uparrow}\right\rangle + \left|\tilde{\downarrow}\right\rangle\right)$$

$$\mathcal{U}_4^\dagger \left|\widetilde{in}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\tilde{\uparrow}\right\rangle + \sigma_4^x \otimes \left|\tilde{\downarrow}\right\rangle\right)$$

$$\mathcal{U}_3^\dagger \mathcal{U}_4^\dagger \left|\widetilde{in}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\tilde{\uparrow}\right\rangle + \sigma_3^x \sigma_4^x \otimes \left|\tilde{\downarrow}\right\rangle\right)$$

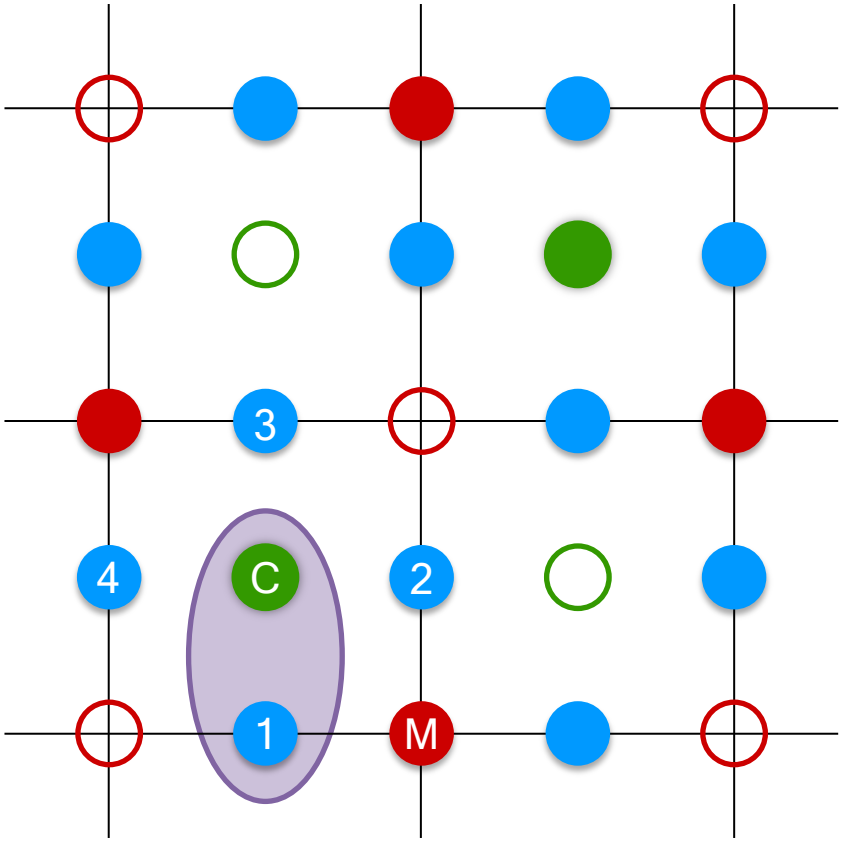
$$\mathcal{U}_2 \mathcal{U}_3^\dagger \mathcal{U}_4^\dagger \left|\widetilde{in}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\tilde{\uparrow}\right\rangle + \sigma_2^x \sigma_3^x \sigma_4^x \otimes \left|\tilde{\downarrow}\right\rangle\right)$$

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)

Plaquettes: Four-body Interactions

Stators: two-body interactions → four-body interactions

$\mathcal{U} = \mathcal{U}^\dagger = \left|\tilde{\uparrow}\right\rangle\left\langle\tilde{\uparrow}\right| + \sigma^x \otimes \left|\tilde{\downarrow}\right\rangle\left\langle\tilde{\downarrow}\right|$



$$\left|\widetilde{in}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\tilde{\uparrow}\right\rangle + \left|\tilde{\downarrow}\right\rangle\right)$$

$$\mathcal{U}_4^\dagger \left|\widetilde{in}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\tilde{\uparrow}\right\rangle + \sigma_4^x \otimes \left|\tilde{\downarrow}\right\rangle\right)$$

$$\mathcal{U}_3^\dagger \mathcal{U}_4^\dagger \left|\widetilde{in}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\tilde{\uparrow}\right\rangle + \sigma_3^x \sigma_4^x \otimes \left|\tilde{\downarrow}\right\rangle\right)$$

$$\mathcal{U}_2 \mathcal{U}_3^\dagger \mathcal{U}_4^\dagger \left|\widetilde{in}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\tilde{\uparrow}\right\rangle + \sigma_2^x \sigma_3^x \sigma_4^x \otimes \left|\tilde{\downarrow}\right\rangle\right)$$

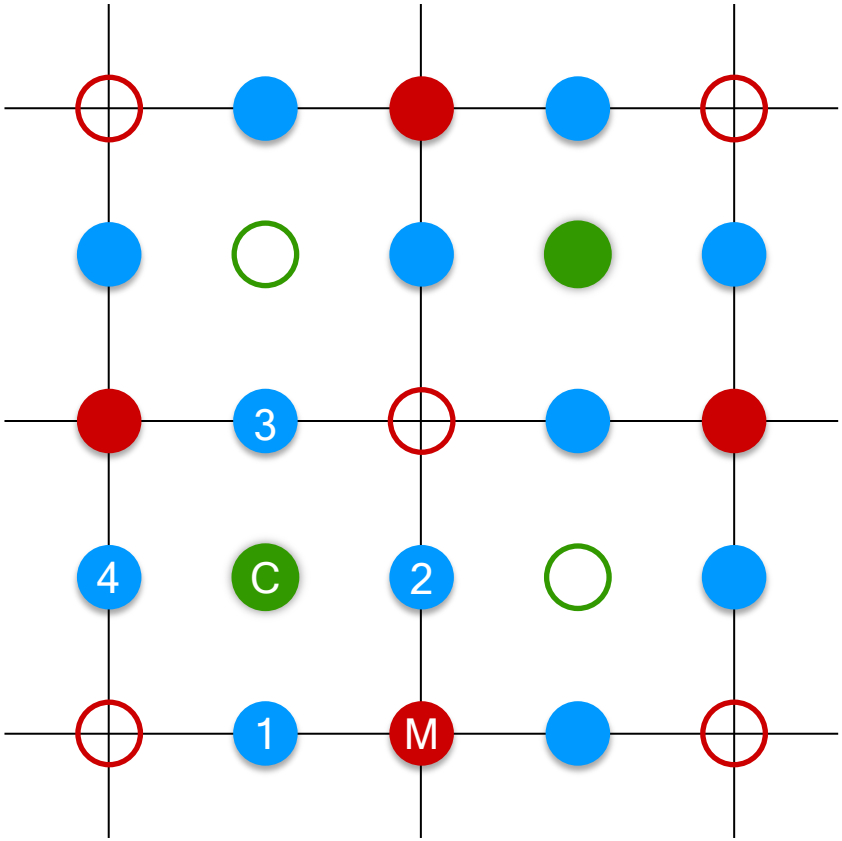
$$\mathcal{U}_1 \mathcal{U}_2 \mathcal{U}_3^\dagger \mathcal{U}_4^\dagger \left|\widetilde{in}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\tilde{\uparrow}\right\rangle + \sigma_1^x \sigma_2^x \sigma_3^x \sigma_4^x \otimes \left|\tilde{\downarrow}\right\rangle\right)$$

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)

Plaquettes: Four-body Interactions

Stators: two-body interactions → four-body interactions

$\mathcal{U} = \mathcal{U}^\dagger = \left|\tilde{\uparrow}\right\rangle\left\langle\tilde{\uparrow}\right| + \sigma^x \otimes \left|\tilde{\downarrow}\right\rangle\left\langle\tilde{\downarrow}\right|$



$$\left|\widetilde{in}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\tilde{\uparrow}\right\rangle + \left|\tilde{\downarrow}\right\rangle\right)$$

$$\mathcal{U}_4^\dagger\left|\widetilde{in}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\tilde{\uparrow}\right\rangle + \sigma_4^x \otimes \left|\tilde{\downarrow}\right\rangle\right)$$

$$\mathcal{U}_3^\dagger\mathcal{U}_4^\dagger\left|\widetilde{in}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\tilde{\uparrow}\right\rangle + \sigma_3^x\sigma_4^x \otimes \left|\tilde{\downarrow}\right\rangle\right)$$

$$\mathcal{U}_2\mathcal{U}_3^\dagger\mathcal{U}_4^\dagger\left|\widetilde{in}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\tilde{\uparrow}\right\rangle + \sigma_2^x\sigma_3^x\sigma_4^x \otimes \left|\tilde{\downarrow}\right\rangle\right)$$

$$\mathcal{U}_1\mathcal{U}_2\mathcal{U}_3^\dagger\mathcal{U}_4^\dagger\left|\widetilde{in}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\tilde{\uparrow}\right\rangle + \sigma_1^x\sigma_2^x\sigma_3^x\sigma_4^x \otimes \left|\tilde{\downarrow}\right\rangle\right)$$

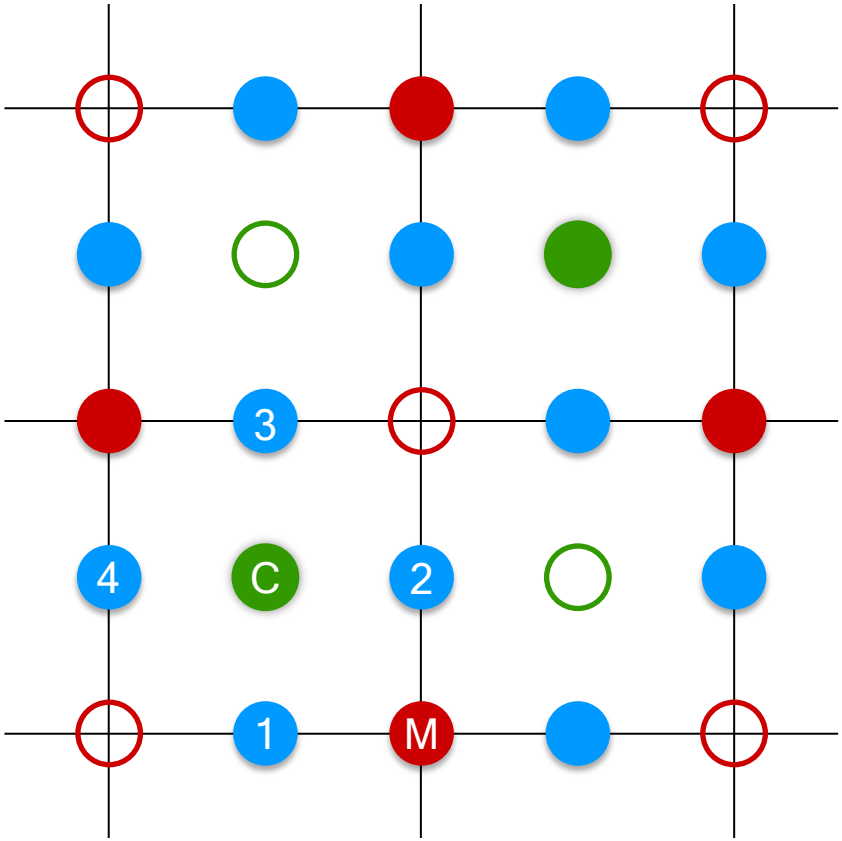
$$S_\square = \frac{1}{\sqrt{2}}\left(\left|\tilde{\uparrow}\right\rangle + \sigma_\square^x \otimes \left|\tilde{\downarrow}\right\rangle\right)$$

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)

Plaquettes: Four-body Interactions

Stators: two-body interactions → four-body interactions

$\mathcal{U} = \mathcal{U}^\dagger = \left|\tilde{\uparrow}\right\rangle\left\langle\tilde{\uparrow}\right| + \sigma^x \otimes \left|\tilde{\downarrow}\right\rangle\left\langle\tilde{\downarrow}\right|$



$$S_\square = \frac{1}{\sqrt{2}}\left(\left|\tilde{\uparrow}\right\rangle + \sigma^x_\square \otimes \left|\tilde{\downarrow}\right\rangle\right)$$

$$\tilde{\sigma}^x S_\square = S_\square \sigma^x_\square$$

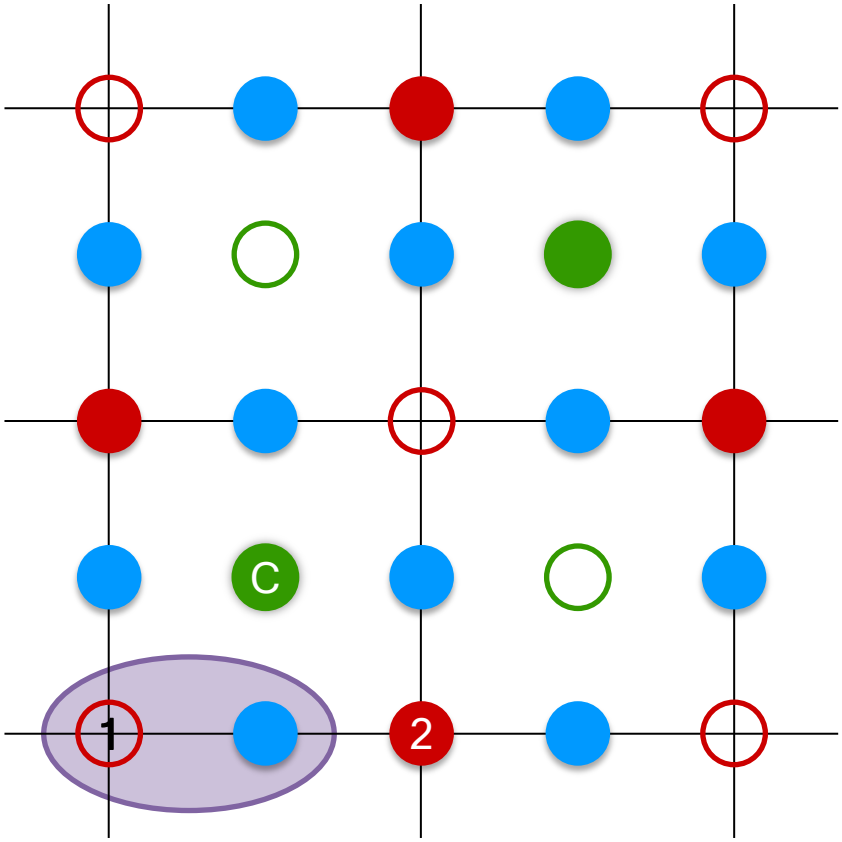
$$e^{-i\lambda\tilde{\sigma}^x\tau} S_\square = S_\square e^{-i\lambda\sigma^x_\square\tau}$$

$$\mathcal{U}_4\mathcal{U}_3\mathcal{U}_2^\dagger\mathcal{U}_1^\dagger e^{-i\lambda\tilde{\sigma}^x\tau}\mathcal{U}_1\mathcal{U}_2\mathcal{U}_3^\dagger\mathcal{U}_4^\dagger\left|\tilde{in}\right\rangle = \left|\tilde{in}\right\rangle e^{-i\lambda\sigma^x_\square\tau}$$

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)

Link Interactions

$$\mathcal{U}_W^\dagger = e^{-\psi^\dagger \psi \log \sigma^x}$$

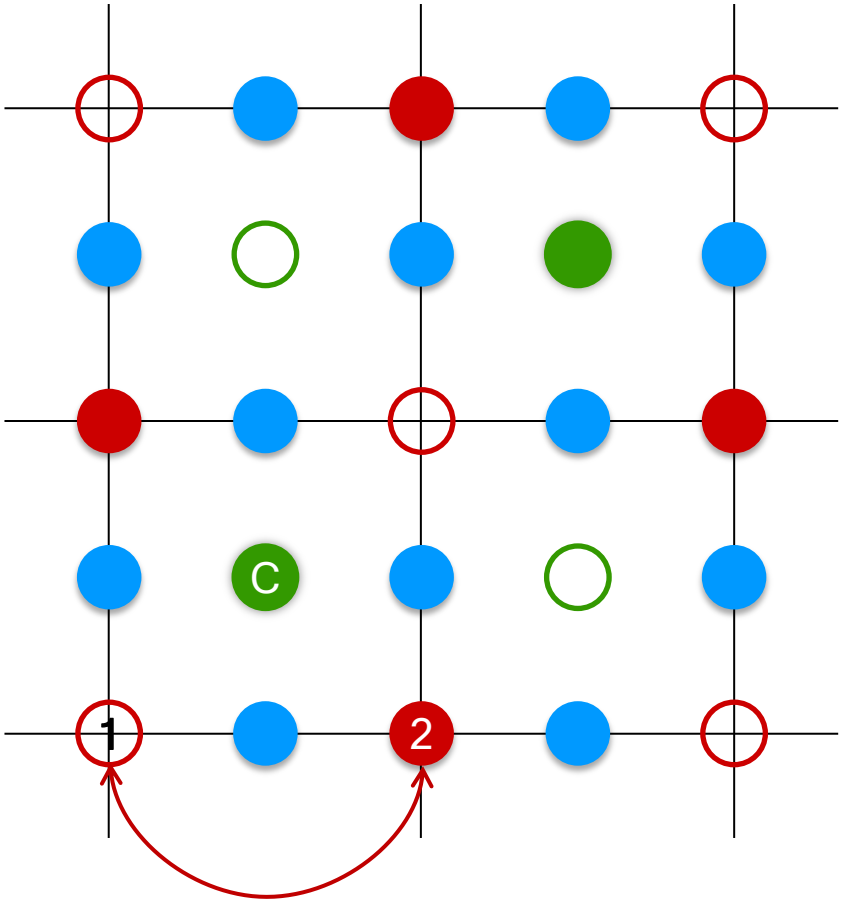


$\mathcal{U}_W^\dagger$

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)

Link Interactions

$$\mathcal{U}_W^\dagger = e^{-\psi^\dagger \psi \log \sigma^x}$$

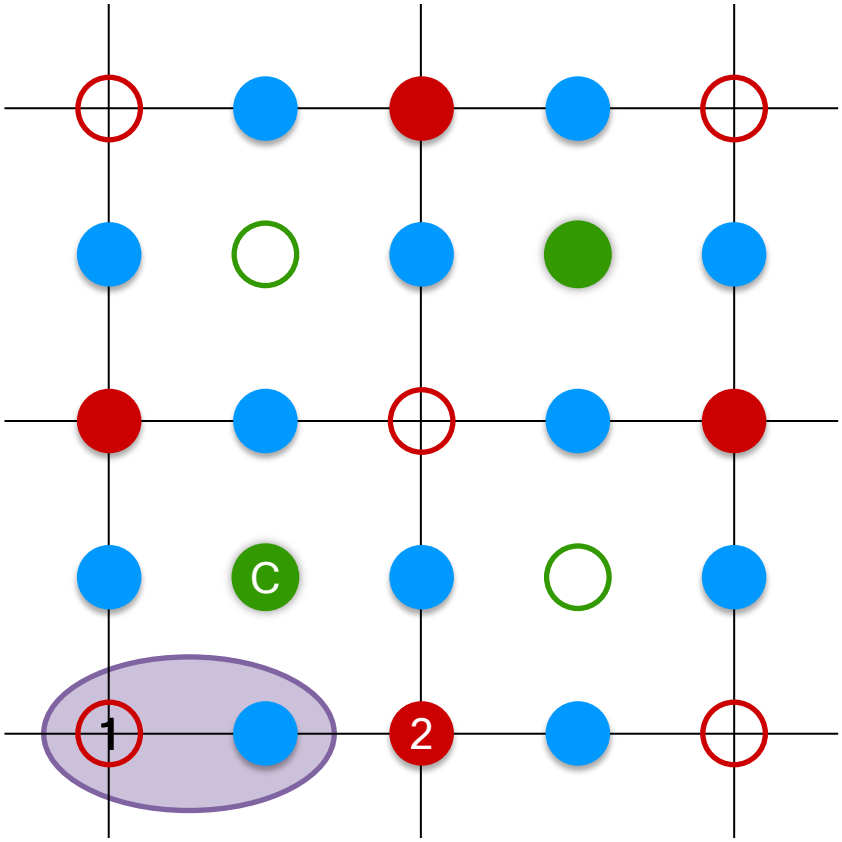


$$e^{-i\epsilon\left(\psi_1^\dagger \psi_2 + \psi_2^\dagger \psi_1\right)\tau} \mathcal{U}_W^\dagger$$

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)

Link Interactions

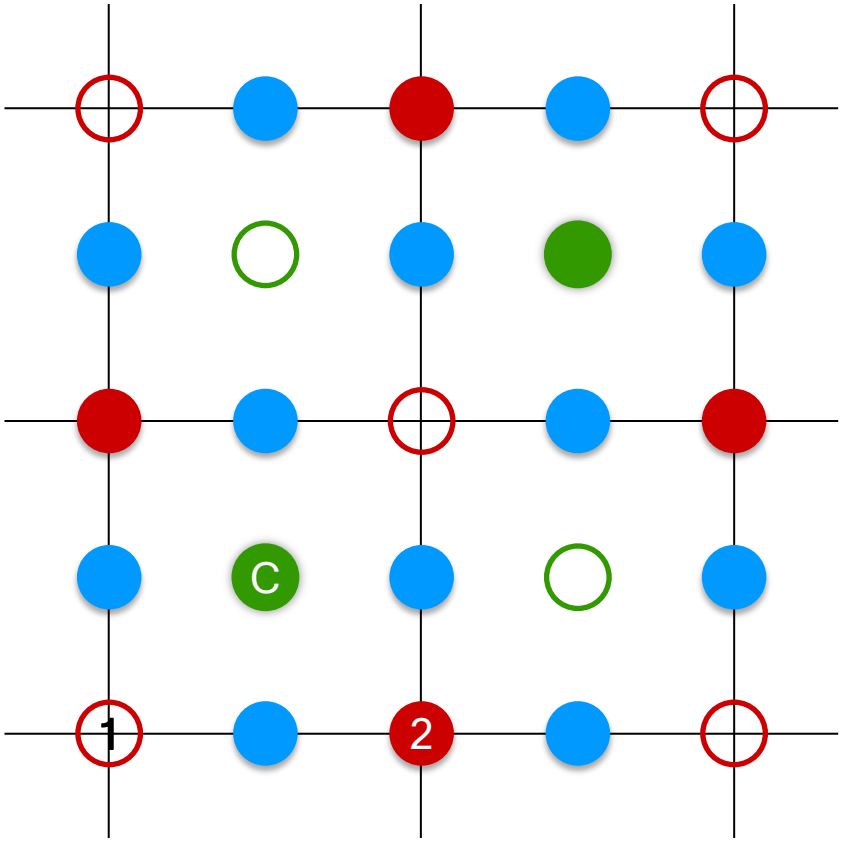
$$\mathcal{U}_W^\dagger = e^{-\psi^\dagger \psi \log \sigma^x}$$



$$\mathcal{U}_W e^{-i\epsilon\left(\psi_1^\dagger \psi_2 + \psi_2^\dagger \psi_1\right)\tau} \mathcal{U}_W^\dagger$$

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)

Link Interactions

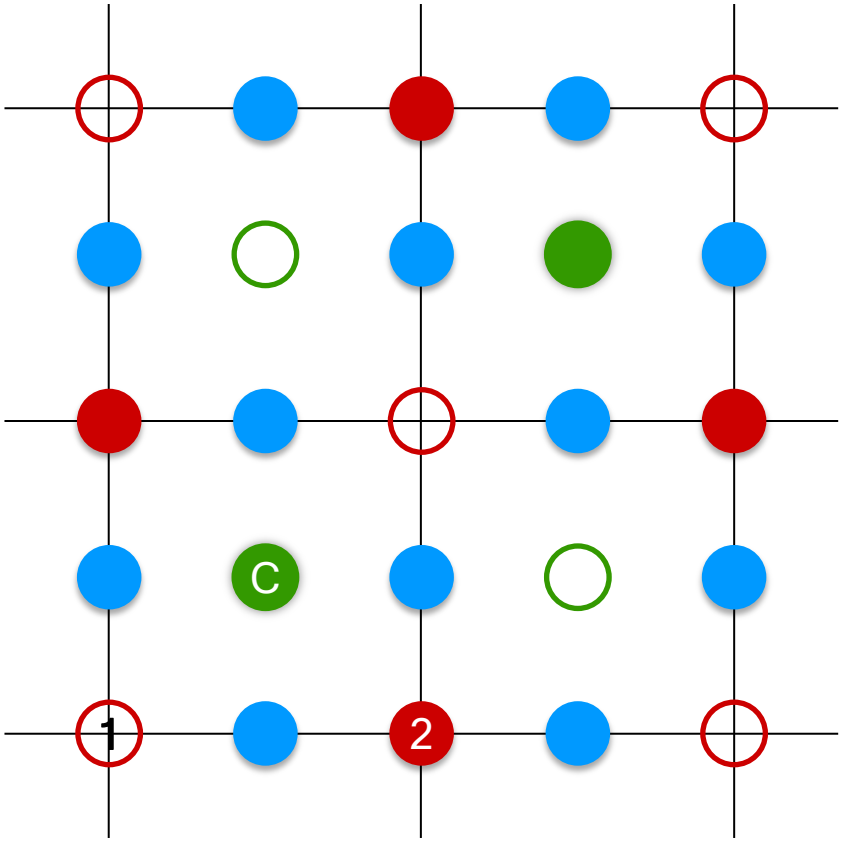


$$\mathcal{U}_W e^{-i\epsilon\left(\psi_1^\dagger \psi_2 + \psi_2^\dagger \psi_1\right)\tau} \mathcal{U}_W^\dagger$$

$$= e^{-i\epsilon\left(\psi_1^\dagger \sigma^x \psi_2 + \psi_2^\dagger \sigma^x \psi_1\right)\tau}$$

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)

Link Interactions



$$\mathcal{U}_W e^{-i\epsilon\left(\psi_1^\dagger\psi_2+\psi_2^\dagger\psi_1\right)\tau}\mathcal{U}_W^\dagger$$

$$= e^{-i\epsilon\left(\psi_1^\dagger\sigma^x\psi_2+\psi_2^\dagger\sigma^x\psi_1\right)\tau}$$

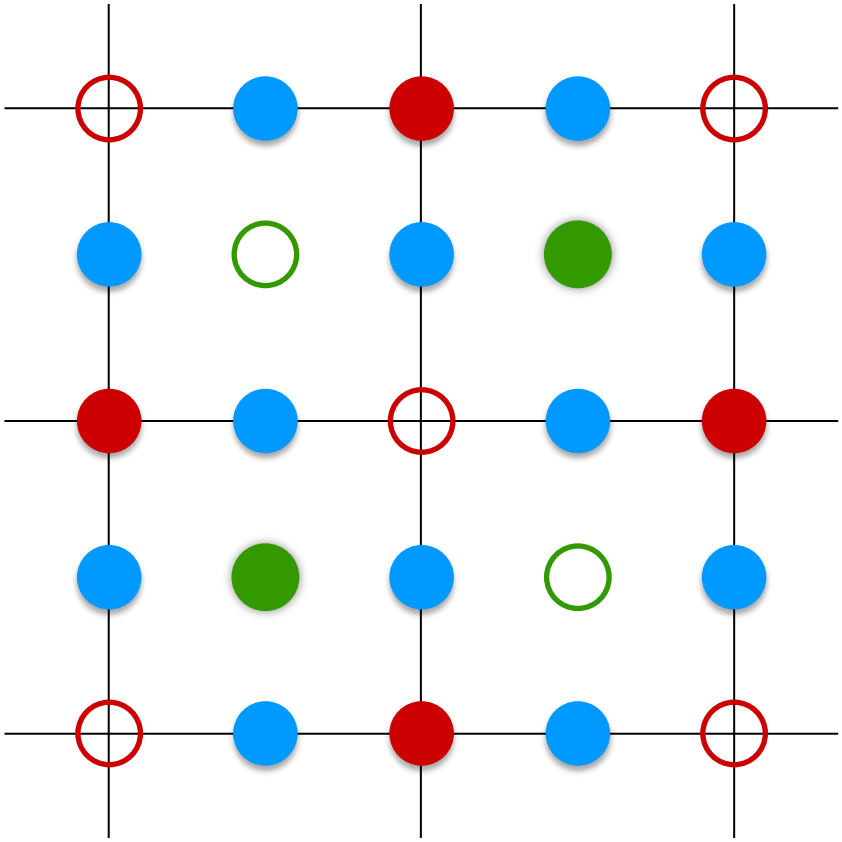
Global tunneling becomes
Locally Gauge invariant interaction

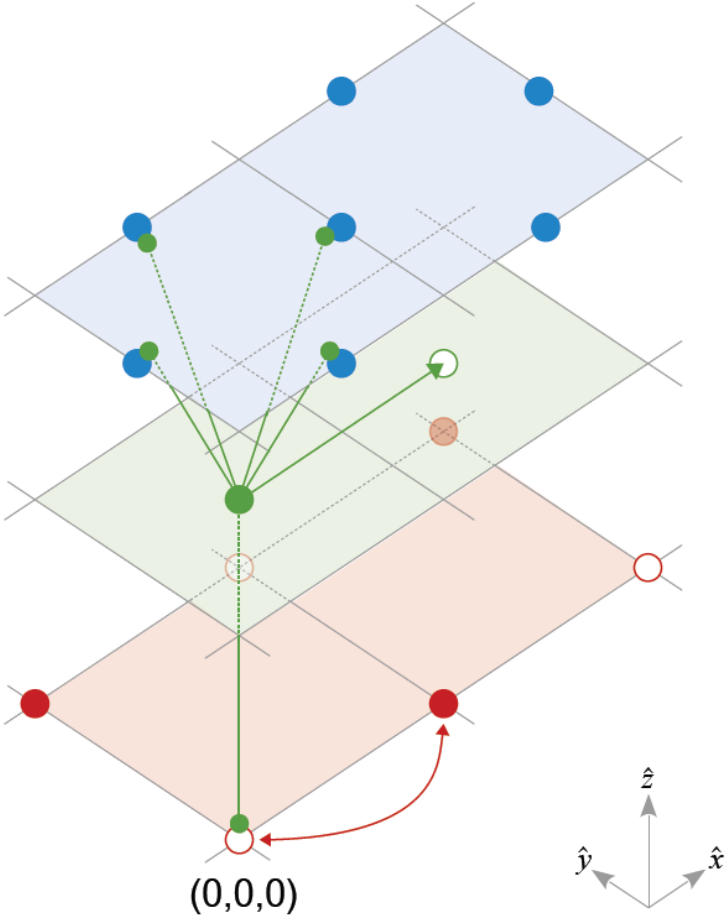
The interactions with the fermions are mediated through stators – interactions with the controls

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)

Realization

Three atomic layers:
The control atoms are movable

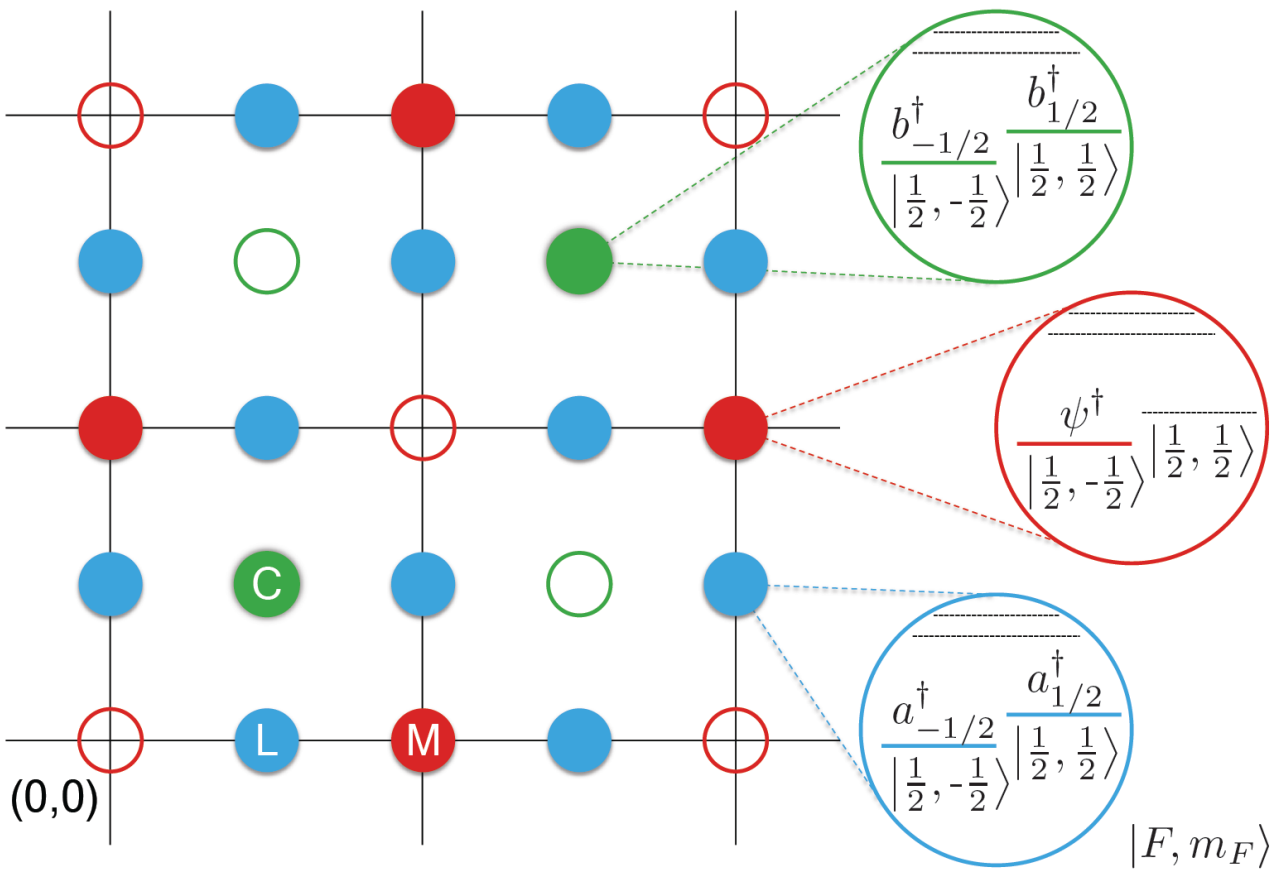




E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)

Realization

Use of hyperfine structure



E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)

Realization

- Local operations – Raman lasers

$$V_{\mathbf{n}}(\phi) = e^{-i\phi\sum_{\mathbf{x}}\mathbf{n}\cdot\sigma(\mathbf{x})}$$
$$\tilde{V}_{\mathbf{n}}(\phi) = e^{-i\phi\sum_{\mathbf{x}}\mathbf{n}\cdot\tilde{\sigma}(\mathbf{x})}$$

- Interactions – S-wave scattering, when the wavefunctions overlap

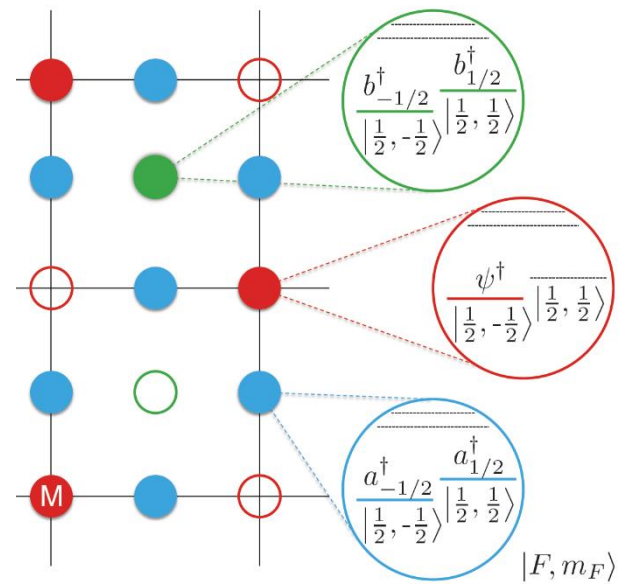
$$H_{ab} = f_0(t)(g_0\sum_{m,n}a_m^\dagger a_m b_n^\dagger b_n + g_1\mathbf{F}\cdot\tilde{\mathbf{F}})$$

$$H_{b\psi} = f_0'(t)(g_0'\psi^\dagger\psi\sum_m b_m^\dagger b_m + g_1'\psi^\dagger\psi\tilde{\sigma}_z)$$

In both cases, two channels: ½ x ½ = 0 + 1

$$g_0 = \pi(a_0 + 3a_1)/2\mu,\ g_1 = 2\pi(a_1 - a_0)/\mu$$

- Constraints:
  - Magnetic field in ***z*** direction + RWA
  - $\sum_m a_m^\dagger a_m = \sum_m b_m^\dagger b_m = 1$
- Careful design of the control movement (adiabaticity, overlap)



$$F^\alpha(\mathbf{x},k) = \tfrac{1}{2}\sigma^\alpha(\mathbf{x},k) = \tfrac{1}{2}a_m^\dagger(\mathbf{x},k)\,\sigma^\alpha_{mn}a_n(\mathbf{x},k)$$
$$\tilde{F}^\alpha(\mathbf{x}) = \tfrac{1}{2}\tilde{\sigma}^\alpha(\mathbf{x}) = \tfrac{1}{2}b_m^\dagger(\mathbf{x})\,\sigma^\alpha_{mn}b_n(\mathbf{x})$$

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)

Realization

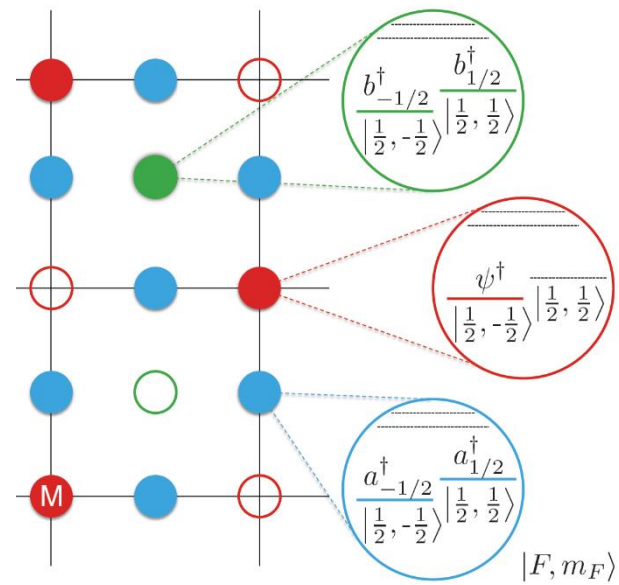
- Local operations – Raman lasers

$$V_{\mathbf{n}}(\phi) = e^{-i\phi\sum_{\mathbf{x}}\mathbf{n}\cdot\sigma(\mathbf{x})}$$
$$\tilde{V}_{\mathbf{n}}(\phi) = e^{-i\phi\sum_{\mathbf{x}}\mathbf{n}\cdot\tilde{\sigma}(\mathbf{x})}$$

  - Realize the local (non-interacting) terms of the Hamiltonian
  - Auxiliary operations (basis changes etc.)

- Interactions – S-wave scattering, when the wavefunctions overlap

$$\mathcal{U}_{\mathrm{ab}}(\phi) = e^{-4i\phi F_z \tilde{F}_z} = e^{-i\phi\sigma_z\tilde{\sigma}_z}$$
$$\mathcal{U}_{b\psi}(\phi) = e^{-i\phi'(\phi)\psi^\dagger\psi} e^{(-\phi/\pi)\psi^\dagger\psi\log\tilde{\sigma}_z}$$



$$F^\alpha(\mathbf{x},k) = \tfrac{1}{2}\sigma^\alpha(\mathbf{x},k) = \tfrac{1}{2}a^\dagger_m(\mathbf{x},k)\,\sigma^\alpha_{mn}a_n(\mathbf{x},k)$$
$$\tilde{F}^\alpha(\mathbf{x}) = \tfrac{1}{2}\tilde{\sigma}^\alpha(\mathbf{x}) = \tfrac{1}{2}b^\dagger_m(\mathbf{x})\,\sigma^\alpha_{mn}b_n(\mathbf{x})$$

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)

Realization – Plaquettes

Atomic collisions → Interactions

1. Move **all controls** to **link 4**
2. Move **all controls** to **link 3**
3. Move **all controls** to **link 2**
4. Move **all controls** to **link 1**

$$\mathcal{U}_{\mathrm{ab}}(\phi) = e^{-4i\phi F_z \tilde{F}_z} = e^{-i\phi\sigma_z\tilde{\sigma}_z}$$

$$V_{\mathbf{n}}(\phi) = e^{-i\phi\sum_{\mathbf{x}}\mathbf{n}\cdot\sigma(\mathbf{x})}$$

$$\mathcal{U} = \mathcal{U}^\dagger = \left|\tilde{\uparrow}\right\rangle\left\langle\tilde{\uparrow}\right| + \sigma^x \otimes \left|\tilde{\downarrow}\right\rangle\left\langle\tilde{\downarrow}\right|$$

$$V_y^\dagger\left(\frac{\pi}{4}\right)\mathcal{U}_{\mathrm{a_1b}}\left(\frac{\pi}{4}\right)\mathcal{U}_{\mathrm{a_2b}}\left(\frac{\pi}{4}\right)\mathcal{U}_{\mathrm{a_3b}}\left(\frac{\pi}{4}\right)\mathcal{U}_{\mathrm{a_4b}}\left(\frac{\pi}{4}\right)V_y\left(\frac{\pi}{4}\right)V_x\left(\frac{\pi}{4}\right)\tilde{V}_z\left(\frac{\pi}{4}\right)$$

5. Act locally on **all controls**

$$\tilde{V}_B = \tilde{V}_x\left(2\lambda_B\tau\right)$$

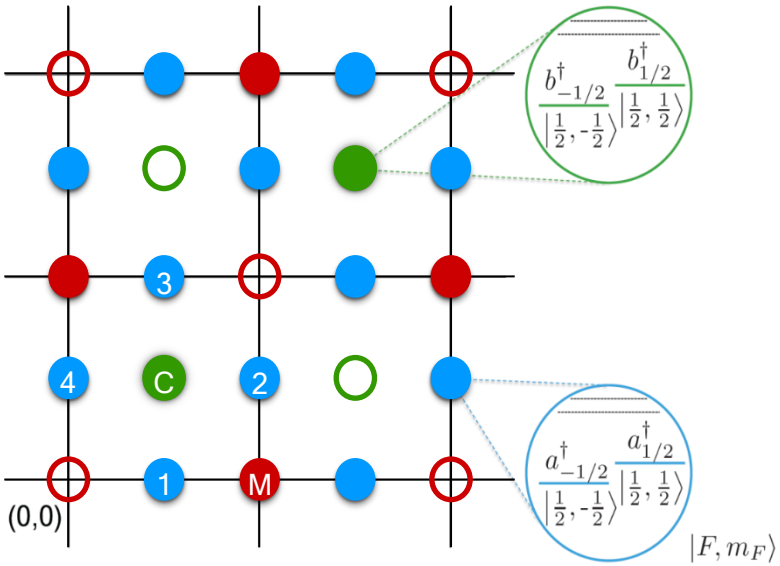
6. Undo steps 1-4, go to the other sublattice

$$S_\square = \frac{1}{\sqrt{2}}\left(\left|\tilde{\uparrow}\right\rangle + \sigma^x_\square \otimes \left|\tilde{\downarrow}\right\rangle\right)$$

$$\tilde{\sigma}^x S_\square = S_\square \sigma^x_\square$$

$$e^{-i\lambda\tilde{\sigma}^x\tau} S_\square = S_\square e^{-i\lambda\sigma^x_\square\tau}$$

$$\mathcal{U}_4\mathcal{U}_3\mathcal{U}_2^\dagger\mathcal{U}_1^\dagger e^{-i\lambda\tilde{\sigma}^x\tau}\mathcal{U}_1\mathcal{U}_2\mathcal{U}_3^\dagger\mathcal{U}_4^\dagger\left|\tilde{in}\right\rangle = \left|\tilde{in}\right\rangle e^{-i\lambda\sigma^x_\square\tau}$$



E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)

Realization – Links

Atomic collisions → Interactions

1. Move the **control** to the **link**
2. Move the **control** to the **left fermion**
3. Allow **fermions** to tunnel (reducing the potential barrier along the link)
4. Undo step 2
5. Undo step 1

$$\mathcal{U}_{b\psi}(\phi) = e^{-i\phi'(\phi)\psi^\dagger\psi} e^{(-\phi/\pi)\psi^\dagger\psi \log\tilde{\sigma}_z}$$

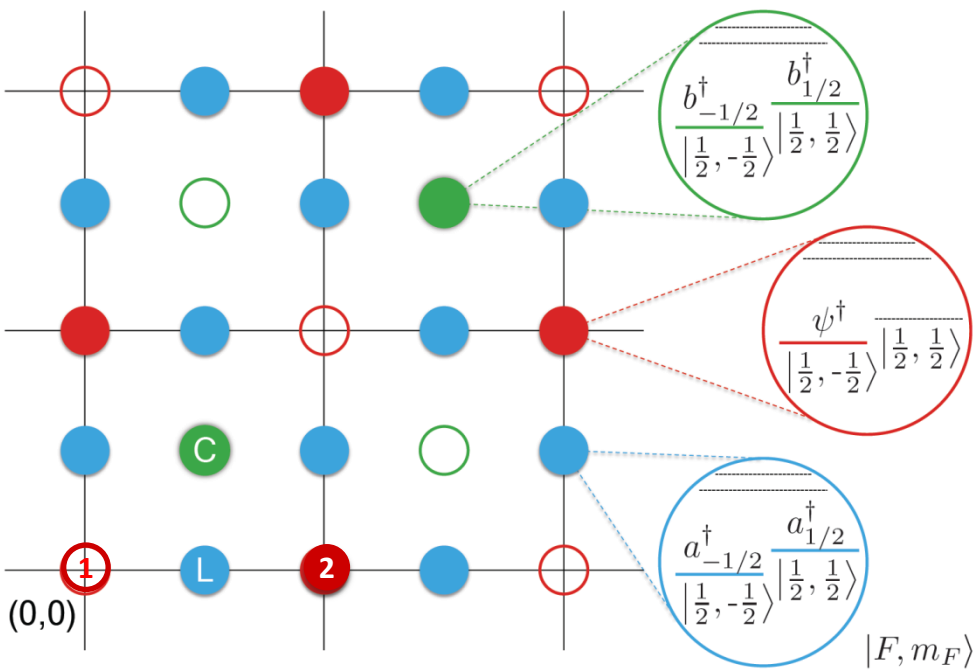
$$\tilde{V}_{\mathbf{n}}(\phi) = e^{-i\phi\sum_{\mathbf{x}}\mathbf{n}\cdot\tilde{\sigma}(\mathbf{x})}$$

$$\tilde{V}_y\left(\frac{\pi}{4}\right)\mathcal{U}_{b\psi}(\pi)\,\tilde{V}_y^\dagger\left(\frac{\pi}{4}\right) = e^{-i\phi'\psi^\dagger\psi}\mathcal{U}_W^\dagger = \mathcal{U}_W^\dagger e^{-i\phi'\psi^\dagger\psi}$$

$$\mathcal{U}_W^\dagger = e^{-\psi^\dagger\psi\log\sigma^x}$$

$$\mathcal{U}_W e^{-i\epsilon\left(\psi_1^\dagger\psi_2+\psi_2^\dagger\psi_1\right)\tau}\mathcal{U}_W^\dagger$$

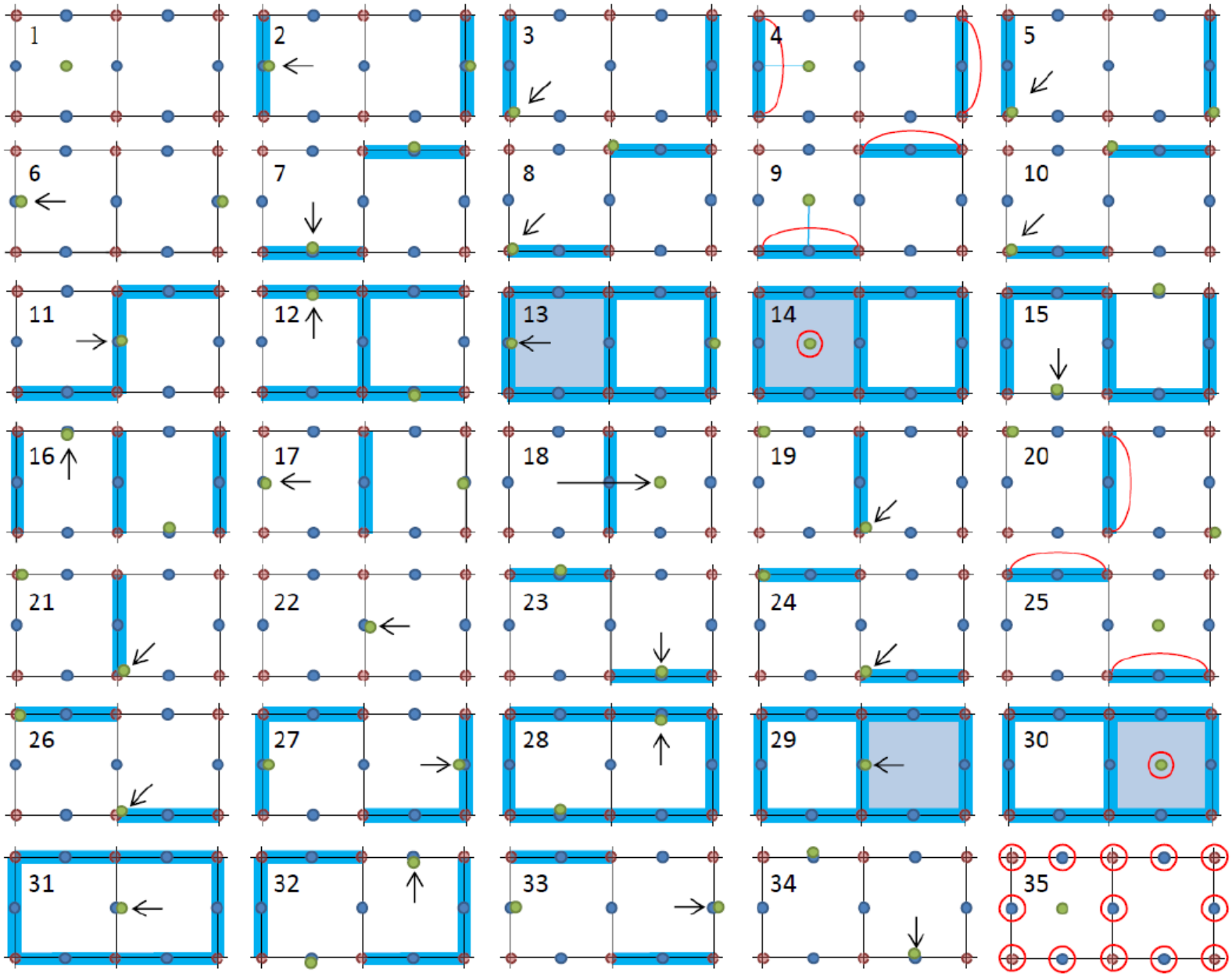
$$= e^{-i\epsilon\left(\psi_1^\dagger\sigma^x\psi_2+\psi_2^\dagger\sigma^x\psi_1\right)\tau}$$



"Rotate" **fermions** with respect to **gauge field** (The rotation parameter is an operator)

Method we apply for tensor constructions as well.

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)



E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. A. 95 023604 (2017)

Realization

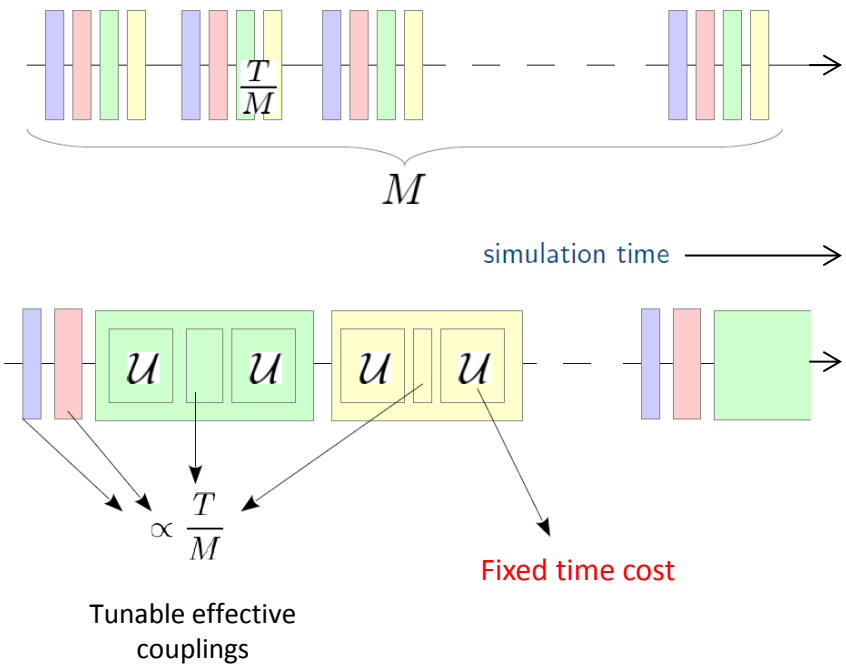
A bipartite single time step (two sublattices)

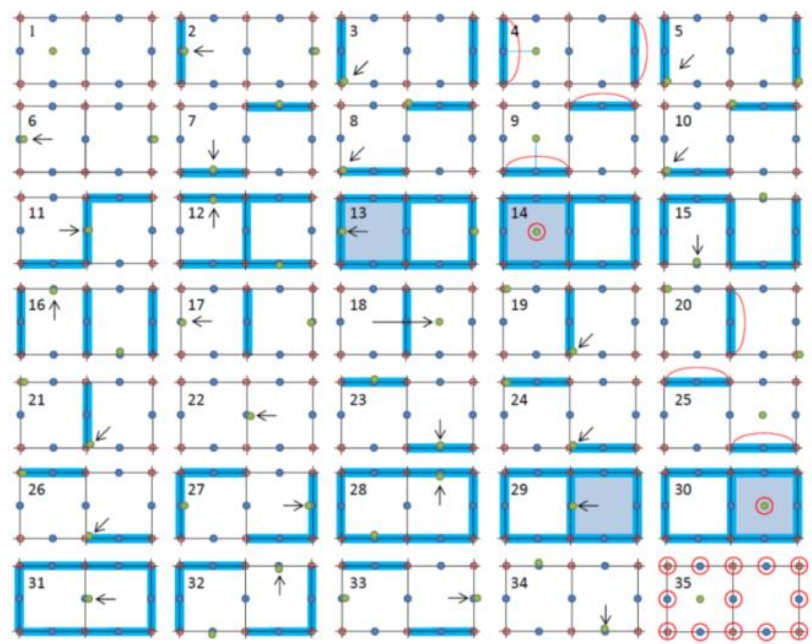
All plaquettes of a given parity are realized at once

Trotterized time evolution, of already gauge invariant pieces

(implementation errors can break the symmetry)

$$e^{-i\Sigma_j H_j T} = \lim_{M\to\infty} \left(\Pi_j e^{-iH_j \frac{T}{M}}\right)^M \qquad M \geqslant \frac{60 L^3 \lambda_{\max}^{3/2} T^{3/2}}{\sqrt{\epsilon}}$$



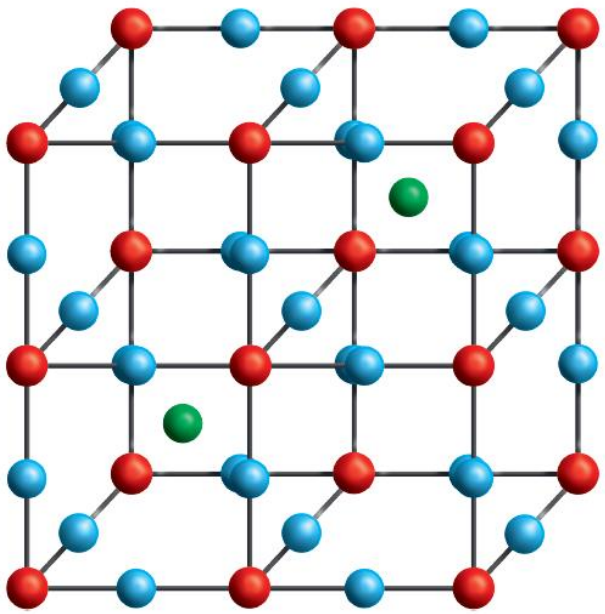


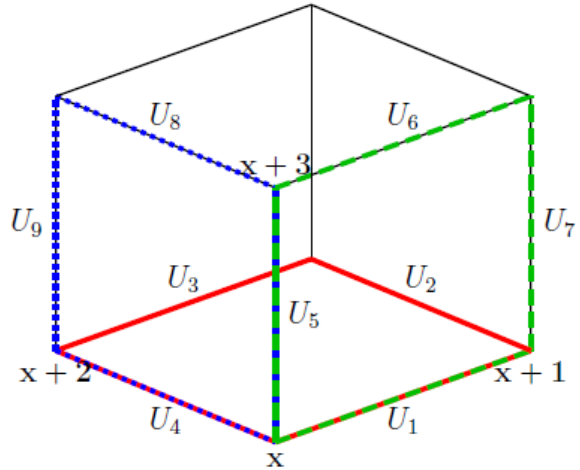
E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. Lett. 118 070501 (2017)

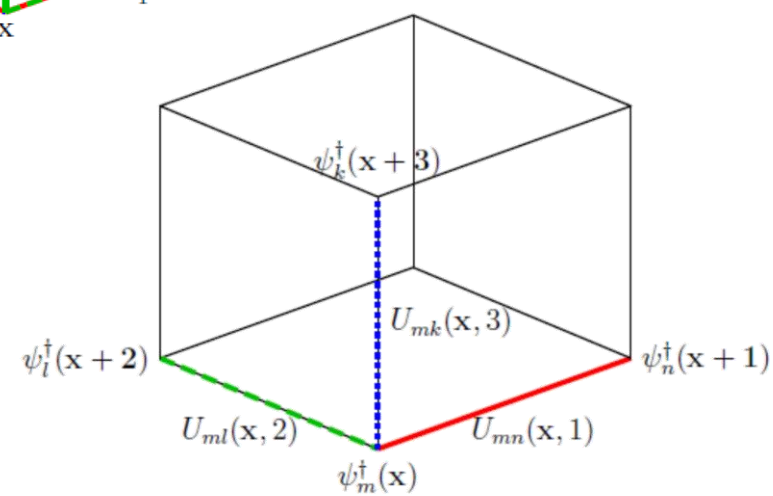
E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. A. 95 023604 (2017)

J. Bender**, E. Zohar**, A. Farace, J. I. Cirac, New J. Phys. 20 093001 (2018)

Realization in 3D





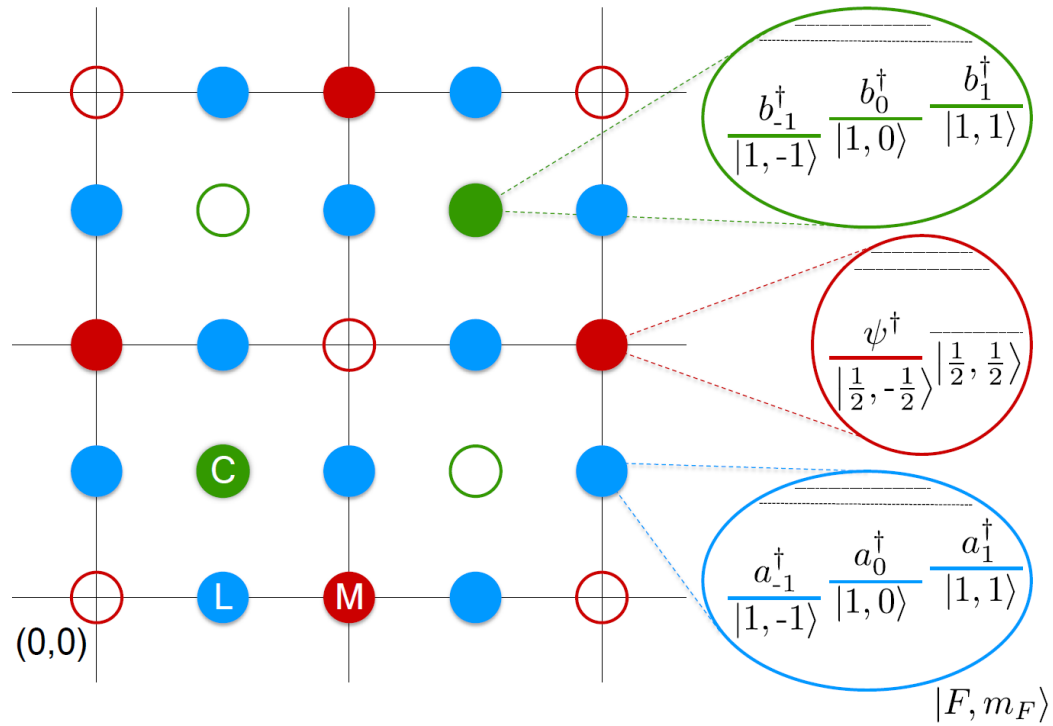


J. Bender**, E. Zohar**, A. Farace, J. I. Cirac, New J. Phys. 20 093001 (2018)

First generalization: Z_3

- Larger Hilbert spaces, more complicated interactions

$$P^3 = Q^3 = 1,$$
$$PQP^\dagger = e^{i(2\pi/3)}Q,$$
$$Q|m\rangle = |m+1\rangle \text{ (cyclically)},$$
$$P|m\rangle = e^{i(2\pi/3)m}|m\rangle.$$



$$|\widetilde{in}\rangle = \frac{1}{\sqrt{3}}\sum_{m=-1}^{1}|\widetilde{m}\rangle$$

$$\mathcal{U}_i = e^{i(3/2\pi)\ln Q_i \ln \widetilde{P}}$$

$$\mathcal{U}'_i(\mathbf{x}) = e^{i(3/2\pi)\ln P_i(\mathbf{x})\ln \widetilde{P}(\mathbf{x})}$$

$$S_{Q,i} \equiv \mathcal{U}_i|\widetilde{in}\rangle = \frac{1}{\sqrt{3}}\sum_{m=-1}^{1} Q_i^m \otimes |\widetilde{m}\rangle$$

$$F^z = -\frac{3i}{2\pi}\ln P \rightarrow \mathcal{U}' = e^{-i(2\pi/3)F_z\tilde{F}_z}$$

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. A. 95 023604 (2017)

First generalization: Z_3

- More scattering channels for the a-b interaction

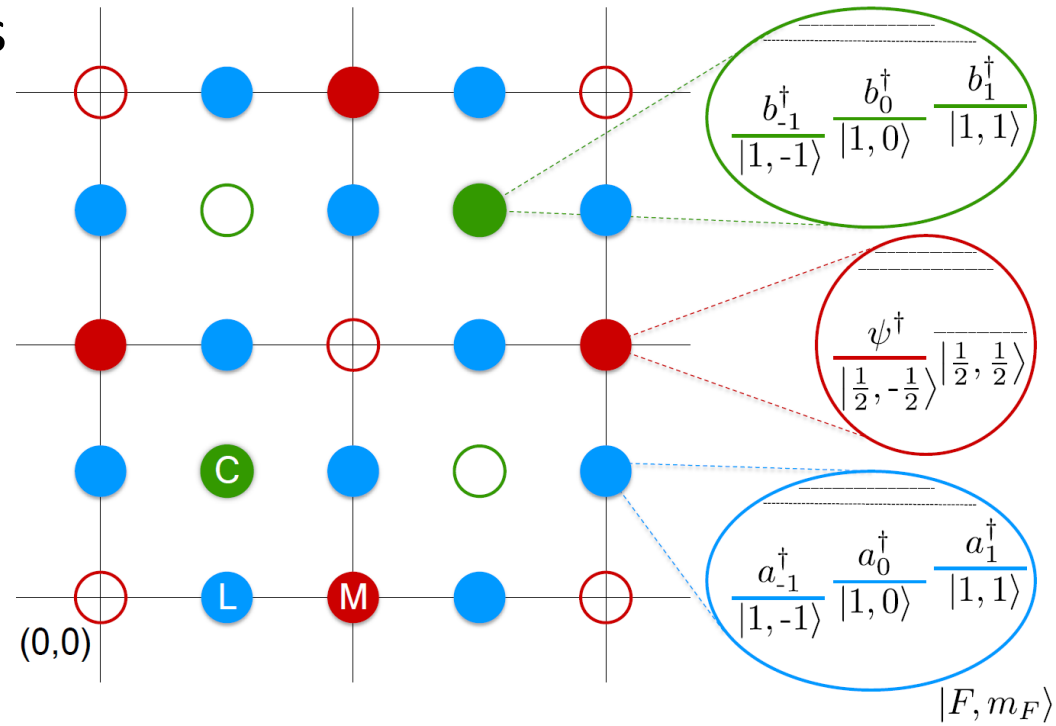
$$V_{\text{scat}}(\mathbf{x}) = \frac{2\pi}{\mu}\delta(\mathbf{x})\sum_{j=0}^{2} g_j(\vec{F}\cdot\vec{\tilde{F}})^j$$

$$g_0 = \tfrac{1}{3}(a_2 + 3a_1 - a_0)$$

$$g_1 = \tfrac{1}{2}(a_2 - a_1)$$

$$g_2 = \tfrac{1}{6}(a_2 - 3a_1 + 2a_0)$$

$$\mathcal{U}_{\text{scat}} = e^{-i\alpha\sum_{j=0}^{2} g_j(\vec{F}\cdot\vec{\tilde{F}})^j}$$

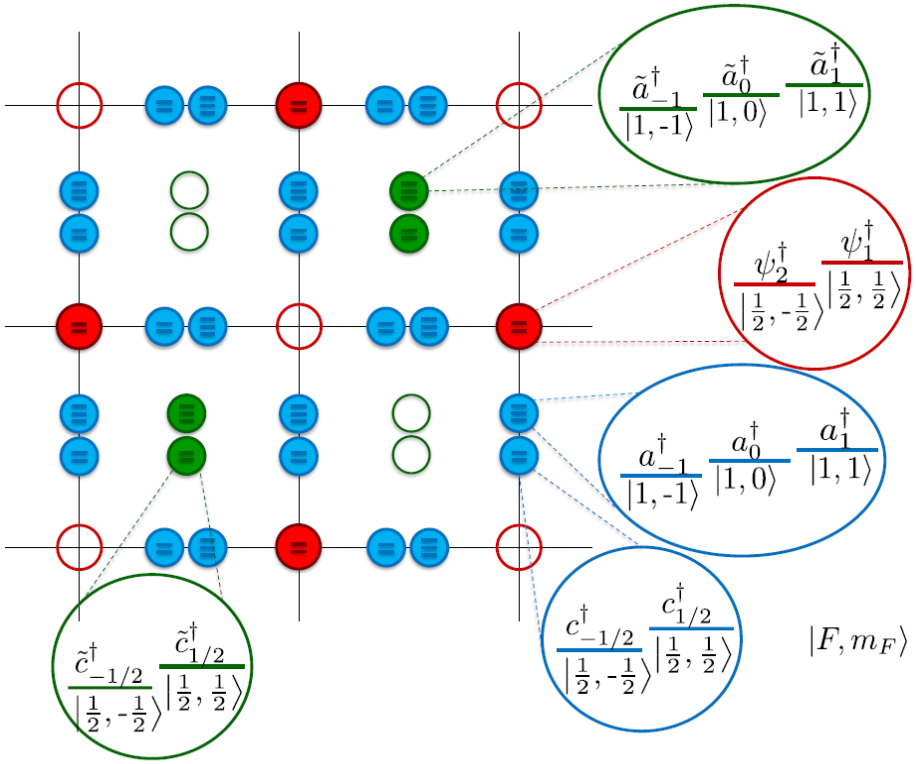


giving rise to undesired interactions – eliminated by using a magnetic field gradient which allows spatial separation of different levels.

- Interaction with the matter fermions – similar.

E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. A. 95 023604 (2017)

Second generalization: D_3



Dihedral group D_N:
$D_N = \{\theta^p r^m | p \in (0,1,2,..,N-1), m \in (0,1)\}$ with θ rotations around $\frac{2\pi}{N}$ and r reflection

D_3: symmetry group of the triangle, rotations around multiples of $\frac{2\pi}{3}$ and refelction $\rightarrow$ 6 elements

$$\mathcal{H}_{aux} \simeq \mathcal{H}_{link} \simeq \mathcal{H}_3 \otimes \mathcal{H}_2$$

J. Bender, **E. Zohar**, A. Farace, J. I. Cirac, New J. Phys. 20 093001 (2018)

Further generalization

Any gauge group

$$S = \int dg \, |g_A\rangle \langle g_A| \otimes |g_B\rangle$$

$$\left(U^j_{mn}\right)_B S = S \left(U^j_{mn}\right)_A$$

$$S_\square = \mathcal{U}_\square \left|\widetilde{in}\right\rangle \equiv \mathcal{U}_1 \mathcal{U}_2 \mathcal{U}_3^\dagger \mathcal{U}_4^\dagger \left|\widetilde{in}\right\rangle$$

$$\mathrm{Tr}\left(\widetilde{U^j} + \widetilde{U}^{j\dagger}\right) S_\square = S_\square \mathrm{Tr}\left(U_1^j U_2^j U_3^{j\dagger} U_4^{j\dagger} + H.c.\right)$$

Feasible for finite or truncated infinite groups

E. Zohar, J. Phys. A. 50 085301 (2017)
E. Zohar, A. Farace, B. Reznik, J. I. Cirac, Phys. Rev. A. 95 023604 (2017)

Is it necessary to use cold atoms?

- **Cold atoms** offer a **combination of fermionic and bosonic degrees of freedom**, which makes them useful for the quantum simulation of gauge theories with **fermionic matter in 2+1d and more**.

- Using systems that **do not offer fermionic degrees of freedom**, one can simulate
  - **Pure gauge theories** could be simulated using other architectures – e.g. trapped ions (Innsbruck), superconducting qubits (Bilbao),…
  - **1+1d gauge theories with matter**, using Jordan-Wigner transformations (like in the trapped ions **Innsbruck experiment**).
  - **Something else?!**

Do we really need fermions?

- Fermions are subject to a **global Z_2 symmetry** (parity superselection)
- If this symmetry is **local** (which happens naturally in a lattice gauge theory whose gauge group contains $\mathbf{Z}_2$ as a normal subgroup), it can be used for **locally transferring the statistics information to the gauge field**
- One is left with **hard-core bosonic matter (spins),** with **fermionic statistics taken care of by the gauge field**

$$\psi^{\dagger}(\mathbf{x}) = c(\mathbf{x})\,\eta^{\dagger}(\mathbf{x})$$

Majorana Fermion: Statistics

Hardcore Boson: Physics

E. Zohar, J. I. Cirac, Phys. Rev. B 98, 075119 (2018)

Do we really need fermions?

- With a **local unitary transformation** which is independent of the space dimension, one can remove the fermions from the Hamiltonian, and stay with **hard-core bosonic matter** and **electric field dependent signs** that preserve the fermionic statistics.

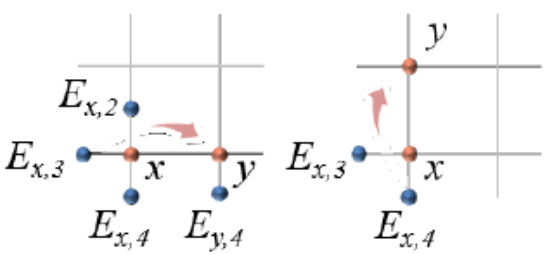
$$\epsilon \sum_{\mathbf{x},i=1,2} \left(\psi^{\dagger}(\mathbf{x})\, U(\mathbf{x},i)\, \psi(\mathbf{x}+\hat{\mathbf{e}}_i) + h.c\right)$$

$$\psi^{\dagger}(\mathbf{x}) = c(\mathbf{x})\, \eta^{\dagger}(\mathbf{x})$$

$$\epsilon \sum_{\mathbf{x},i=1,2} \left(\eta^{\dagger}(\mathbf{x})\, c(\mathbf{x})\, U(\mathbf{x},i)\, c(\mathbf{x}+\hat{\mathbf{e}}_i)\, \eta(\mathbf{x}+\hat{\mathbf{e}}_i) + h.c\right)$$

Unitary transformation

$$-i\epsilon \sum_{\mathbf{x},i=1,2} \left(\xi_i \sigma_+(\mathbf{x})\, U(\mathbf{x},i)\, \sigma_-(\mathbf{x}+\hat{\mathbf{e}}_i) + h.c\right)$$



$$\xi_h = e^{i\pi(E_{x,2}+E_{x,3}+E_{x,4}+E_{y,4})}$$

$$\xi_v = e^{i\pi(E_{x,3}+E_{x,4})}$$

E. Zohar, J. I. Cirac, Phys. Rev. B 98, 075119 (2018)

Do we really need fermions?

- This procedure opens the way for **quantum simulation of lattice gauge theories with fermionic matter in 2+1d and more**, even with **simulating systems that do not offer fermionic degrees of freedom**.

- In the U(N) case **the matter can be removed completely**!

E. Zohar, J. I. Cirac, Phys. Rev. B 98, 075119 (2018)
E. Zohar, J. I. Cirac, arXiv:1905.00652 [quant-ph], accepted to PRD

Summary

- **Lattice gauge theories** may be simulated by **ultracold atoms** in optical lattices. **Gauge invariance** may be obtained in several methods.

- Atomic interactions may be **mapped exactly to a gauge symmetry** in the ultracold limit, making the gauge invariance fundamental in some sense.

- Using **stators**, lattice gauge theories may be formulated in a **digital way**: two body interactions with **ancillary atoms may induce four body interactions**. This can be done with **ultracold atoms in a layered structure**; By doing that, plaquette interactions are **possibly stronger** than in the analog, perturbative approach.

- **Fermions** in lattice gauge theories may be **mapped to hard-core bosons**, opening the way for **other simulators, without fermionic degrees of freedom**.