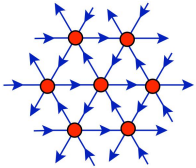
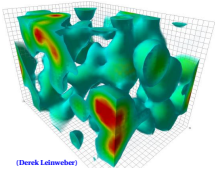
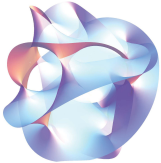
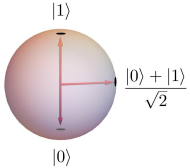
Supersymmetric lattice field theories

— Classical simulations and quantum opportunities —









David Schaich (University of Liverpool)

High-energy physics at ultra-cold temperatures

ECT* Trento 12 June 2019

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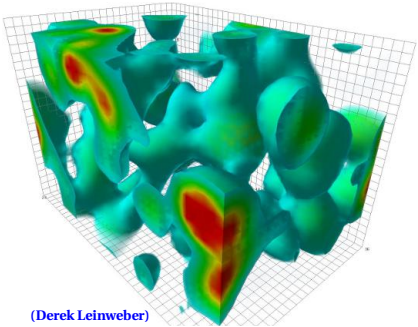
Motivations

Pursuing first-principles predictions for strongly coupled supersymmetric QFTs

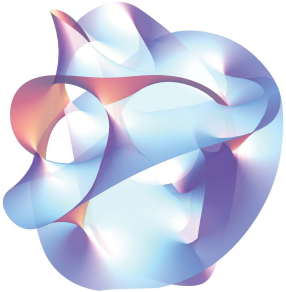
BSM

QFT



(Derek Leinweber)

Holography



Overview

Significant progress currently being made
in classical lattice studies of supersymmetric QFTs

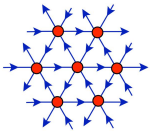
Opportunities for near-future quantum simulations

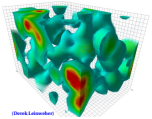
✓ Motivations

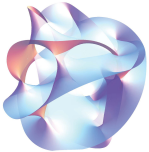
Recent highlights of lower-dim'l classical simulations

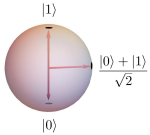
- Super-Yang–Mills (SYM) quantum mechanics
- Plane-wave matrix model
- (1+1)-dimensional SYM

Quantum opportunities









Supersymmetry must be broken on the lattice

Supersymmetry is a space-time symmetry,
adding spinor generators Q^{I}_{α} and $\overline{Q}^{\mathrm{I}}_{\dot{\alpha}}$ to translations, rotations, boosts

$\left\{ Q^{\mathrm{I}}_{\alpha}, \overline{Q}^{\mathrm{J}}_{\dot{\alpha}} \right\} = 2\delta^{\mathrm{IJ}} \sigma^{\mu}_{\alpha\dot{\alpha}} P_{\mu}$ broken in discrete space-time

$\longrightarrow$ relevant susy-violating operators

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$\longrightarrow$ relevant susy-violating operators

Easier to handle in lower dimensions $\longrightarrow$ active area for classical simulations

Super-Yang–Mills quantum mechanics

Holographic duality conjecture

Maximal SYM at non-zero temperature $\longleftrightarrow$ properties of stringy black holes

Ultimate simplification — compactify all spatial dimensions

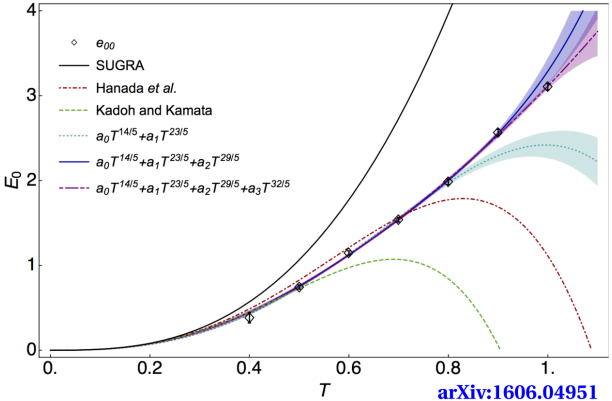
(9+1)d U(N) SYM $\longrightarrow$ quantum mechanics of interacting $N \times N$ matrices
(16 'fermionic', 9 'scalar', 1 'gauge')

Large-N limit of SYM QM $\longleftrightarrow$ strong-coupling limit of type IIA string theory
[Banks–Fischler–Shenker–Susskind, hep-th/9610043]

Testing holography with lattice super-Yang–Mills QM

Dual black hole energy from maximal SYM QM

Monte Carlo String/M-Theory Collaboration [1606.04951]



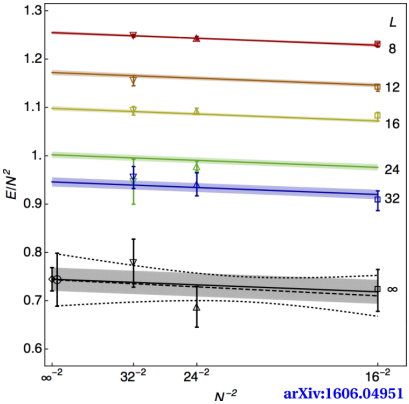
Approach leading-order curve at low temperatures

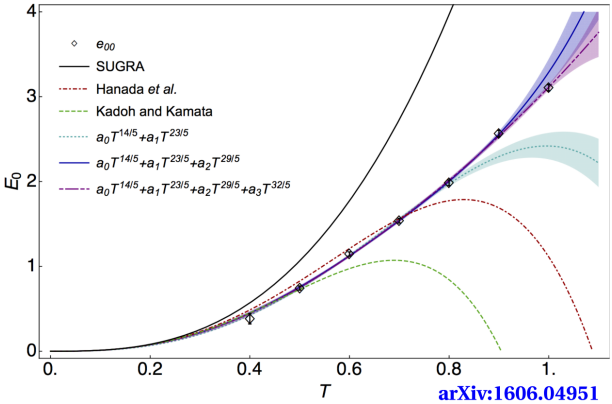
Predict higher-order quantum gravitational effects due to $T > 0$

Challenges for lattice super-Yang–Mills QM

Large-N continuum extrapolations under control

Low-temperature instability due to emergence of flat directions





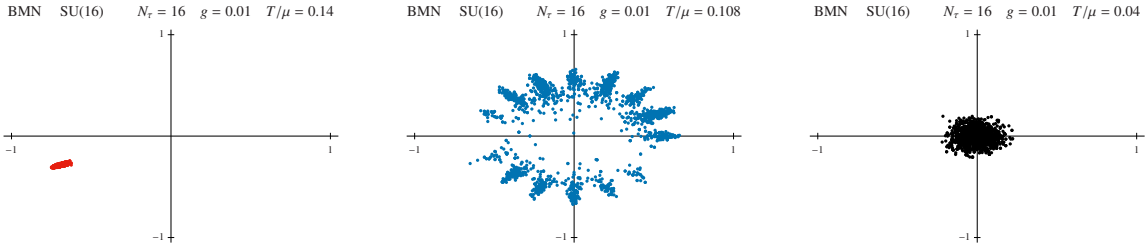
Plane-wave matrix model

Deform SYM QM to lift flat directions while preserving all 16 supersymmetries

[Berenstein–Maldacena–Nastase, hep-th/0202021]

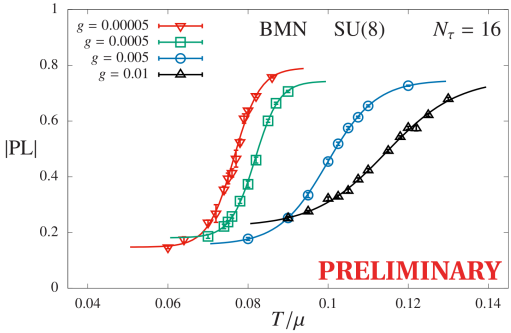
Deformation parameter $\mu \longrightarrow$ dimensionless coupling $g \equiv \lambda/\mu^3 = g_{YM}^2 N/\mu^3$
dimensionless temperature T/μ

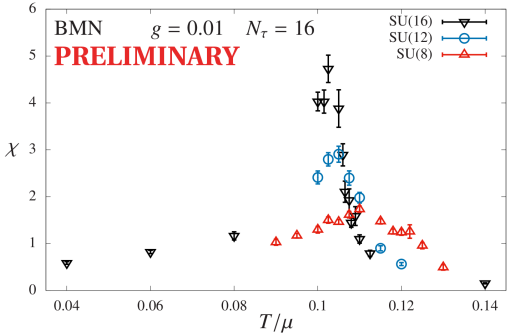
'Confinement' transition as T/μ decreases for fixed g



Plane-wave matrix model transition signals

Peaks in Polyakov loop susceptibility match change in its magnitude |PL|,
grow with size of SU(N) gauge group, comparing $N = 8, 12, 16$





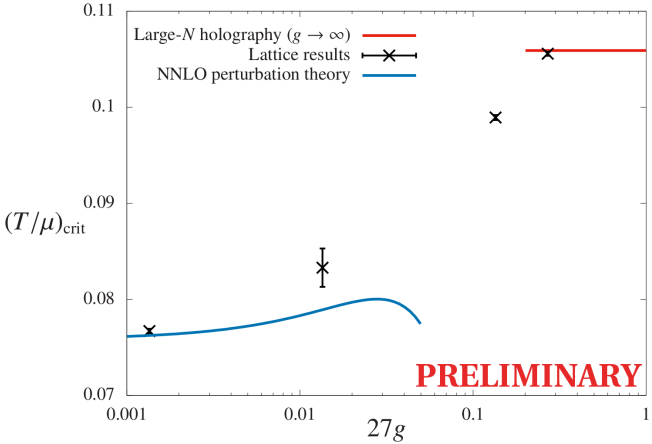
Plane-wave matrix model phase diagram

Goal: Interpolate between $g \to 0$ perturbation theory
and large-N $g \to \infty$ holographic prediction

For each fixed g, scan to find transition temperature $(T/\mu)_{\text{crit}}$

SU(N) gauge groups with $N = 8, 12, 16$

Lattice sizes $N_\tau = 8, 12, 16, 24$

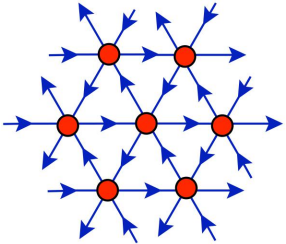


Beyond quantum mechanics

Higher dimensions $\longrightarrow$ more challenging fine-tuning

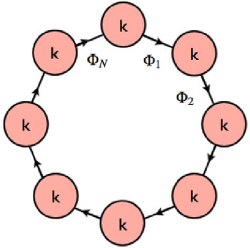
Address by preserving susy sub-algebra at non-zero lattice spacing

Equivalent constructions from topological twisting and deconstruction



Review:

arXiv:0903.4881



Need 2^{d+1} supersymmetries in $d+1$ dimensions

Topological twisting

Need 2^{d+1} supersymmetries in $d+1$ dimensions

$\longrightarrow$ global 'R' symmetry contains $\mathrm{SO}(d+1)_R$ subgroup

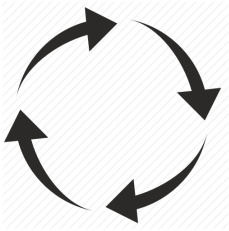
Replace spinors by objects with integer spin under 'twisted rotation group'

$$\mathrm{SO}(d+1)_{tw} \equiv \mathrm{diag}\left[\mathrm{SO}(d+1)_{\mathrm{euc}} \otimes \mathrm{SO}(d+1)_R\right]$$

Discrete space-time

Can preserve closed sub-algebra

$$\{\mathcal{Q}, \mathcal{Q}\} = 2\mathcal{Q}^2 = 0$$



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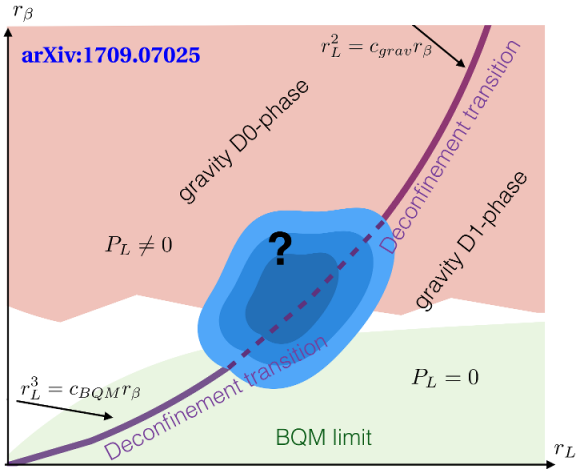
Discrete space-time

Can preserve closed sub-algebra

$$\{\mathcal{Q}, \mathcal{Q}\} = 2\mathcal{Q}^2 = 0$$

Expectations in (1+1) dimensions

Maximal SYM deconfined for any inverse temperature $r_\beta = 1/t$



For decreasing r_L at low t and large N

homogeneous black string (D1)
$\longrightarrow$ localized black hole (D0)

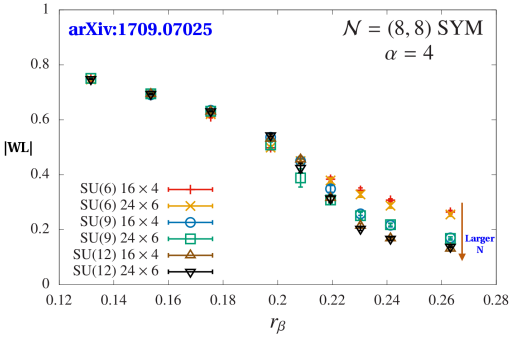
$\Updownarrow$

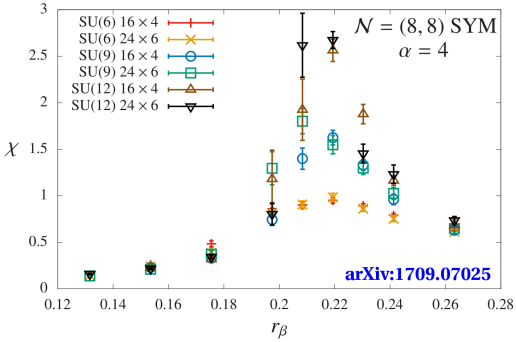
"spatial deconfinement"
signalled by Wilson line

(1+1)-dimensional SYM transition signals

Peaks in Wilson line susceptibility match change in its magnitude |WL|,
grow with size of SU(N) gauge group, comparing $N = 6, 9, 12$

Agreement for 16×4 vs. 24×6 lattices (aspect ratio $\alpha = r_L/r_\beta = 4$)

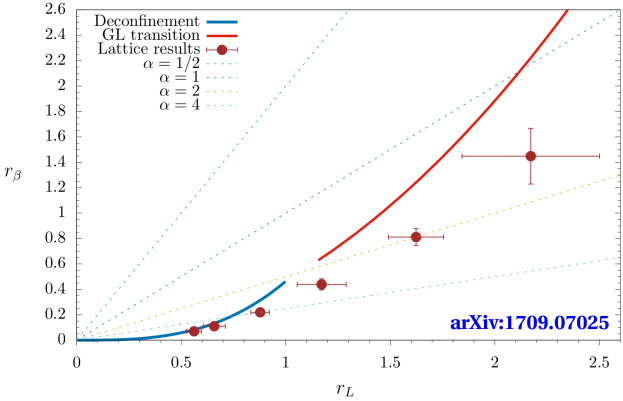




Phase diagram of (1+1)-dimensional SYM

Large $\alpha = r_L/r_\beta \gtrsim 4 \longrightarrow$ good agreement with high-temperature bosonic QM

Small $\alpha \lesssim 2 \longrightarrow$ harder to control uncertainties with $6 \leq N \leq 16$



Overall consistent with holography

Comparing multiple lattice sizes

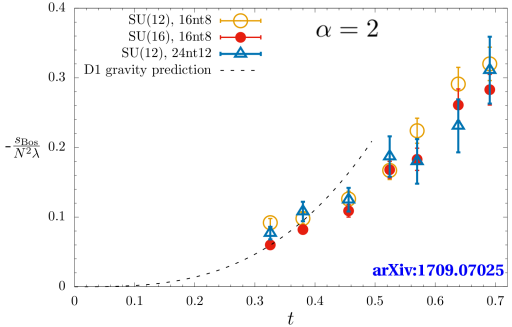
Controlled extrapolations are work in progress

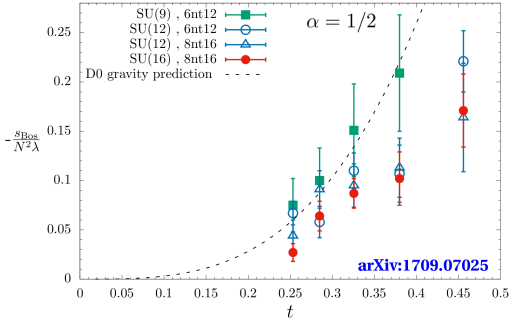
Dual black hole thermodynamics

Dual black hole energy from (1+1)-dimensional maximal SYM

$\propto t^3$ for large-r_L D1 phase $\propto t^{3.2}$ for small-r_L D0 phase

Lattice results consistent with holography for sufficiently low $t \lesssim 0.4$





From SYM to superQCD

[Catterall–Veernala, arXiv:1505.00467]

Preserve twisted supersymmetry sub-algebra in 1+1 or 2+1 dimensions

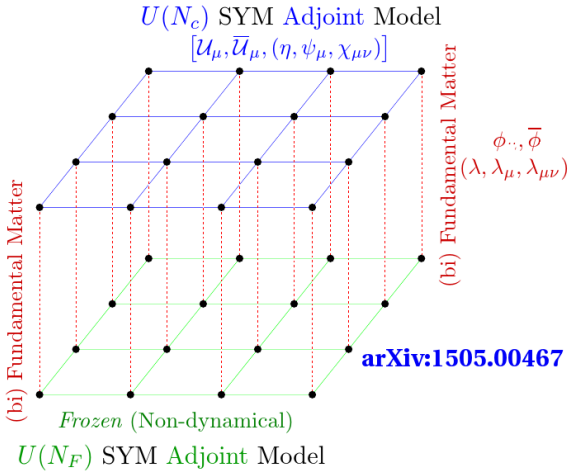
2-slice lattice SYM
with U(N) $\times$ U(F) gauge group

Adj. fields on each slice

Bi-fundamental in between

Decouple U(F) slice

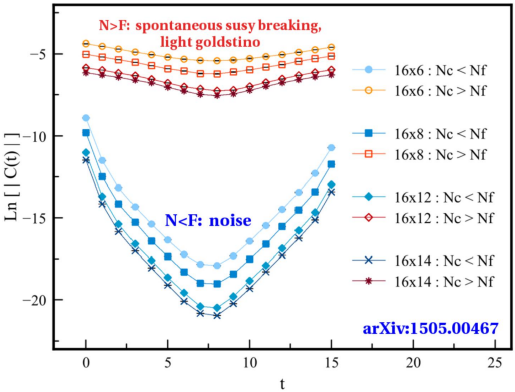
$\longrightarrow$ U(N) superQCD
with F fund. hypermultiplets

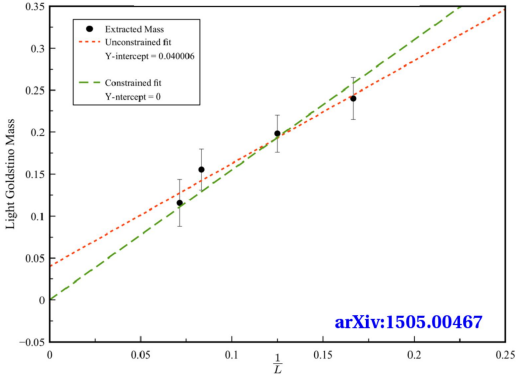


Dynamical susy breaking in (1+1)-dimensional lattice superQCD

U(N) superQCD with F fundamental hypermultiplets

Spontaneous susy breaking requires $N > F$

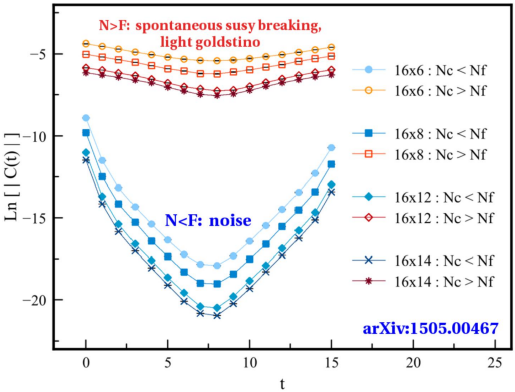


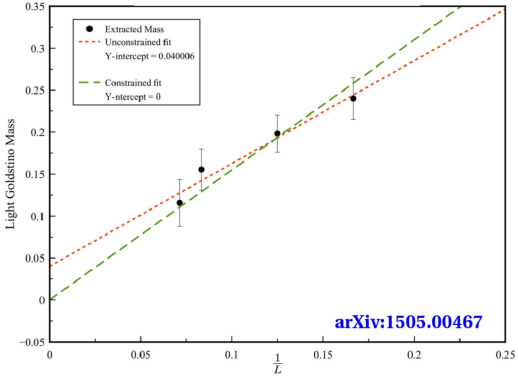


Dynamical susy breaking in (1+1)-dimensional lattice superQCD

U(N) superQCD with F fundamental hypermultiplets

Spontaneous susy breaking requires $N > F$ and zero Witten index





Quantum opportunities: Sign problems

All results above from phase-quenched rational hybrid Monte Carlo (RHMC)

$$\langle \mathcal{O} \rangle = \frac{\int [d\Phi] \;\; \mathcal{O} \;\; e^{-S_B} \;\; \mathrm{pf}\, \mathcal{D}}{\int [d\Phi] \;\; e^{-S_B} \;\; \mathrm{pf}\, \mathcal{D}} \quad \longrightarrow \quad \frac{\int [d\Phi] \;\; \mathcal{O} e^{i\alpha} \;\; e^{-S_B} \;\; |\mathrm{pf}\, \mathcal{D}|}{\int [d\Phi] \;\; e^{i\alpha} \;\; e^{-S_B} \;\; |\mathrm{pf}\, \mathcal{D}|} = \frac{\left\langle \mathcal{O} e^{i\alpha} \right\rangle_{pq}}{\left\langle e^{i\alpha} \right\rangle_{pq}}$$

"Witten index" $W = \mathrm{Tr}\left[(-1)^F e^{-\beta H}\right] \propto \left\langle e^{i\alpha} \right\rangle_{pq}$ (at zero temperature)

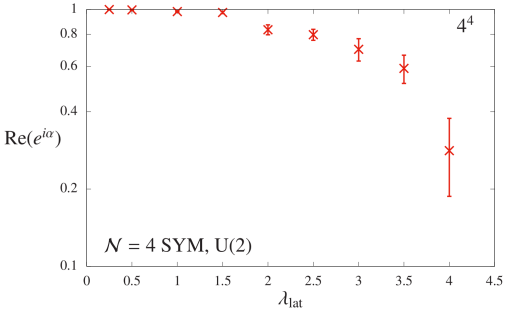
$\Longrightarrow W = 0$ for dynamical supersymmetry breaking $\longrightarrow$ severe sign problem

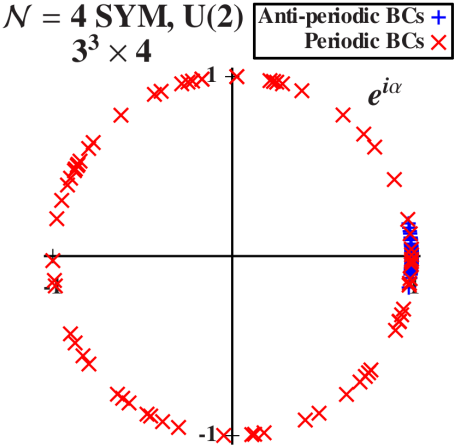
Quantum opportunities: Sign problems

Also sign problems without susy breaking

Phase fluctuations increase with coupling

[figs for (3+1)d $\mathcal{N} = 4$ SYM]





Quantum opportunities: Real-time dynamics

$\exp[-S] \longrightarrow \exp[iS]$

Already several talks about quantum simulation for real-time dynamics

Holography connects maximal SYM to non-perturbative quantum gravity

$\longrightarrow$ entropy production, quantum chaos, Hawking evaporation, ...

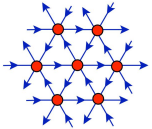
In near term, simpler supersymmetric systems more realistic targets

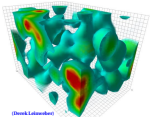
[e.g., (1+1)-dimensional Wess–Zumino model]

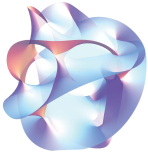
Recap: An exciting time for lattice supersymmetry

Significant progress currently being made
in classical lattice studies of supersymmetric QFTs

Opportunities for near-future quantum simulations

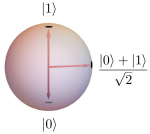






Lower-dimensional studies test holography

- Super-Yang–Mills (SYM) quantum mechanics
- Plane-wave matrix model
- (1+1)-dimensional SYM



Quantum opportunities to address sign problems
$\longrightarrow$ supersymmetry breaking, real-time dynamics, ...

Thank you!

Collaborators

Simon Catterall, Raghav Jha, Anosh Joseph, Toby Wiseman

also Georg Bergner, Poul Damgaard, Joel Giedt

Funding and computing resources

UK Research and Innovation

USQCD

Backup: Breakdown of Leibniz rule on the lattice

$\left\{Q_\alpha, \overline{Q}_{\dot\alpha}\right\} = 2\sigma^\mu_{\alpha\dot\alpha}P_\mu = 2i\sigma^\mu_{\alpha\dot\alpha}\partial_\mu$ is problematic

$\Longrightarrow$ try finite difference $\partial\phi(x) \longrightarrow \Delta\phi(x) = \frac{1}{a}\left[\phi(x+a) - \phi(x)\right]$

Crucial difference between ∂ and Δ

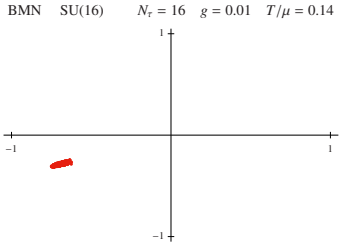
$$\Delta\left[\phi\eta\right] = a^{-1}\left[\phi(x+a)\eta(x+a) - \phi(x)\eta(x)\right]$$
$$= \left[\Delta\phi\right]\eta + \phi\Delta\eta + a\left[\Delta\phi\right]\Delta\eta$$

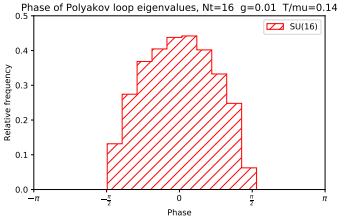
Full supersymmetry requires Leibniz rule $\partial\left[\phi\eta\right] = \left[\partial\phi\right]\eta + \phi\partial\eta$

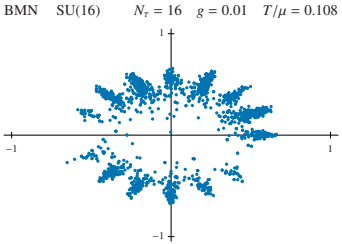
only recoverd in $a \to 0$ continuum limit for any local finite difference

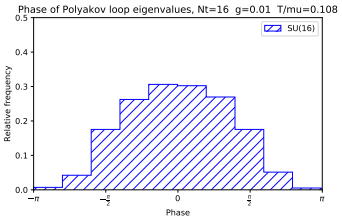
Backup: Another plane-wave matrix model confinement observable

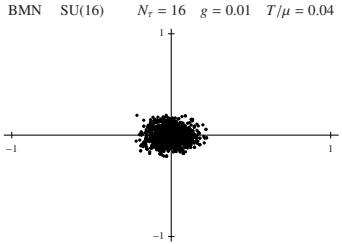
Polyakov loop eigenvalue distribution evolves from localized to uniform

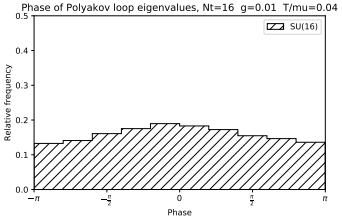






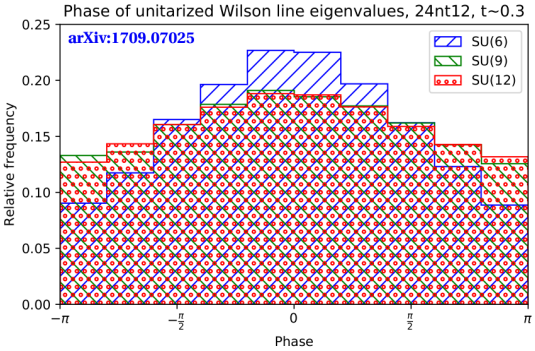


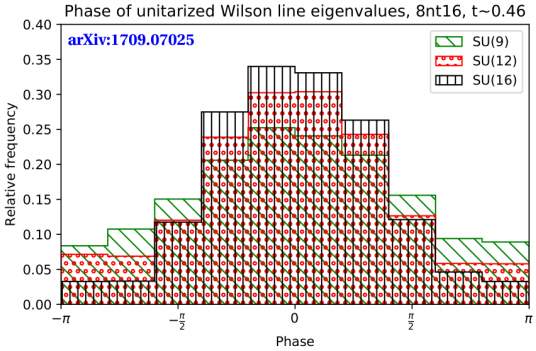




Backup: (1+1)-dimensional SYM Wilson line eigenvalues

Check ‘spatial deconfinement’ through Wilson line eigenvalue phases





Left: $\alpha = 2$ distributions more extended as N increases $\longrightarrow$ D1 black string

Right: $\alpha = 1/2$ distributions more compact as N increases $\longrightarrow$ D0 black hole

Backup: Dimensional reduction to (1+1)-dimensional SYM

Naive for now: 4d $\mathcal{N} = 4$ SYM code with $N_x = N_y = 1$

$A_4^* \longrightarrow A_2^*$ (triangular) lattice

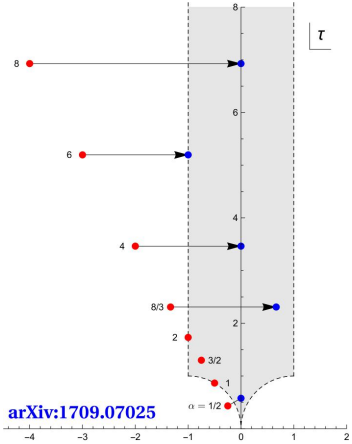
Torus **skewed** depending on $\alpha = N_t/L$

Modular trans. into fund. domain

$\longrightarrow$ some skewed tori actually rectangular

Also need to stabilize compactified links

to ensure broken center symmetries



arXiv:1709.07025

Backup: More on dynamical susy breaking

Spontaneous susy breaking means $\langle 0\,|H|\,0\rangle > 0$ or equivalently $\langle \mathcal{Q}\mathcal{O}\rangle \neq 0$

Twisted superQCD auxiliary field e.o.m. $\longleftrightarrow$ Fayet–Iliopoulos D-term potential

$$d = \overline{\mathcal{D}}_a \mathcal{U}_a + \sum_{i=1}^{F} \phi_i \overline{\phi}_i - r\mathbb{I}_N \qquad \longleftrightarrow \qquad \mathrm{Tr}\left[\left(\sum\nolimits_i \phi_i \overline{\phi}_i - r\mathbb{I}_N\right)^2\right] \in H$$

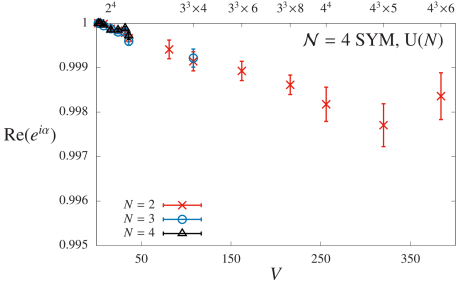
Have F scalar vevs to zero out N diagonal elements

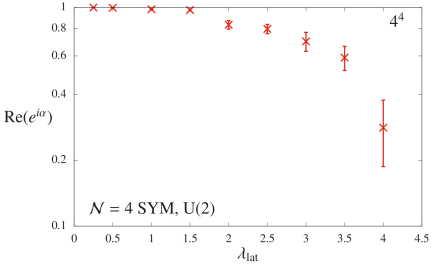
$\longrightarrow N > F$ suggests susy breaking, $\langle 0\,|H|\,0\rangle > 0 \longleftrightarrow \langle \mathcal{Q}\eta\rangle = \langle d\rangle \neq 0$

Backup: More on $\mathcal{N} = 4$ SYM sign problem in 3+1 dimensions

Fix $\lambda_{\text{lat}} = g_{\text{lat}}^2 N = 0.5$

Pfaffian nearly real positive for all accessible volumes





Fix 4^4 volume

Fluctuations increase with coupling

Signal-to-noise becomes obstruction for $\lambda_{\text{lat}} \gtrsim 4$