# Emerging Gauge Theories in 2D Rydberg Arrays

## Alessio Celi





High-energy physics at ultra-cold temperatures, ECT\* 12/06/2019

# Emerging Gauge Theories in 2D Rydberg Arrays



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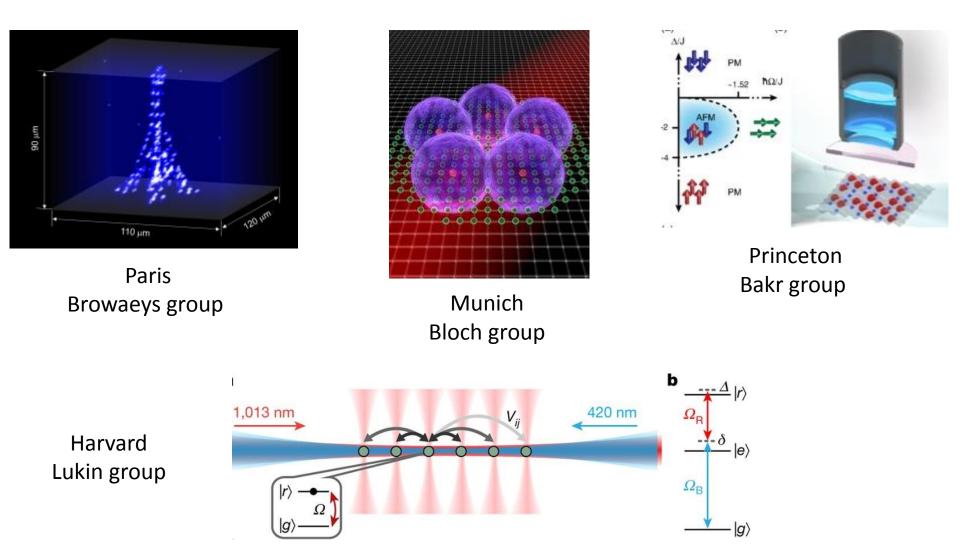


arXiv:1906.xxxx

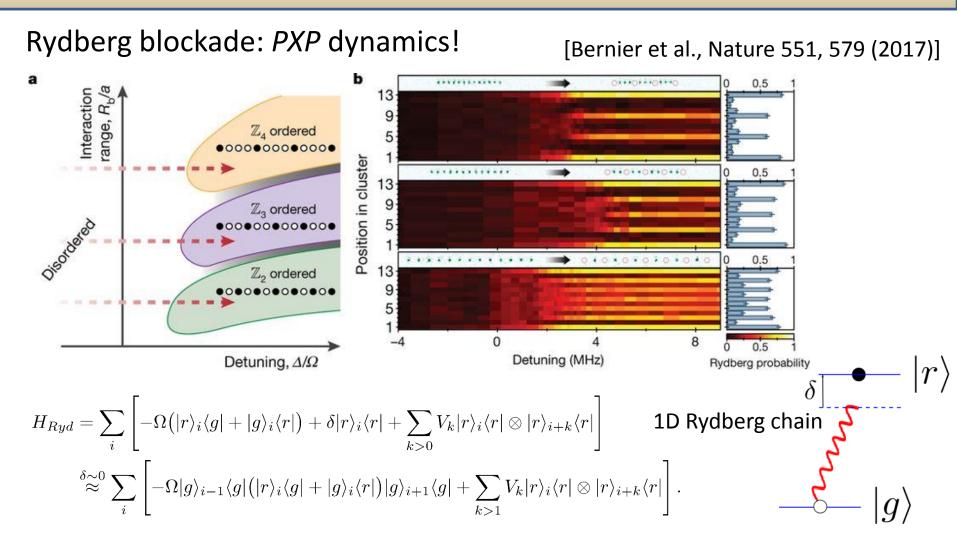
High-energy physics at ultra-cold temperatures, ECT\* 12/06/2019

### Rydberg atoms

#### Arrays of Rydberg atoms: perfect to simulate spin models



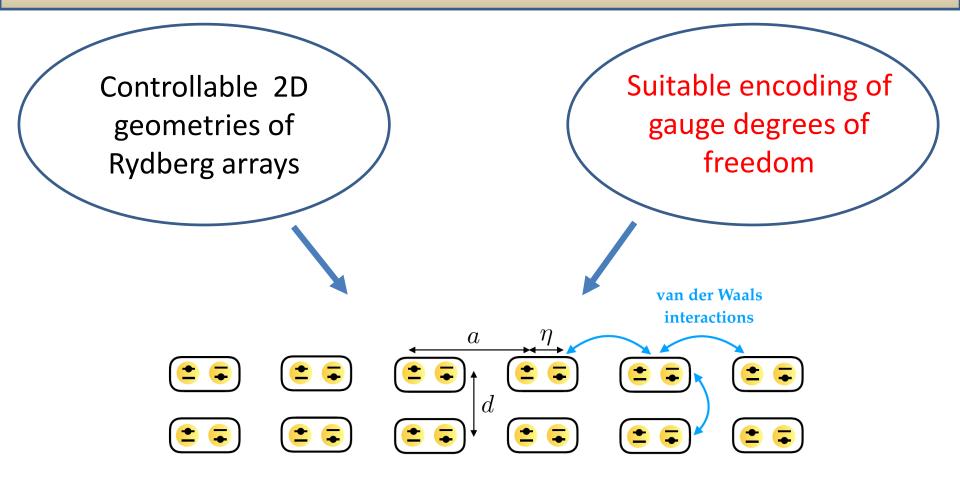
### Rydberg atoms: Blockade



Kibble-Zurek Quantum scars, connection to graph theory

[Keesling et al., arXiv:1809.05540] [Turner et al., Nature Phys. 14, 745 (2018)]

# Take home message



Emerging 2D lattice gauge theory

# Outline

I. Motivations and tutorial

From static to dynamical gauge theories The Rokhsar-Kivelson model Implementation: experimental challenges

- II. Suitable encodingConverting Ring exchange to blockade
- III. Rydberg implementation and examples

# Outline

I. Motivations and tutorial

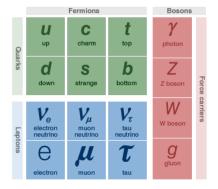
From static to dynamical gauge theories The Rokhsar-Kivelson model Implementation: experimental challenges

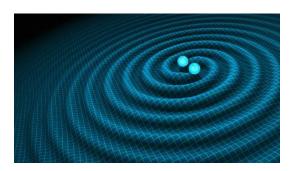
- II. Suitable encoding Converting Ring exchange to blockade
- III. Rydberg implementation and examples

# I.1 Why Gauge Theories?

Gauge theories appear everywhere!

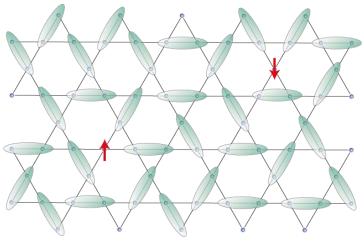
HEP: "All" fundamental interactions are gauge theories (even gravity)





**Condensed Matter:** Emerging local symmetry

Quantum magnetism, high-T<sub>c</sub> superconductivity & topological order



# I.2 Why Lattice Gauge Theories?

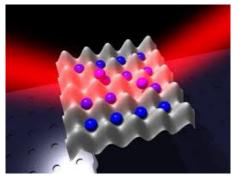
Wilson'74: Lattice formulation to study QCD by Quantum MonteCarlo

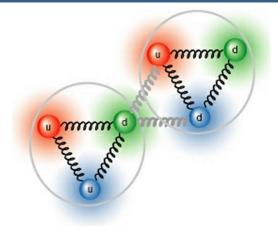
Success: Static properties at low fermion density

Sign problems: Dynamics, phase diagram at high density

Perfect opportunity for tensor networks and quantum simulation!

Hamiltonian approach: engineer an emergent gauge theory





### I.3 Hamiltonian lattice gauge theory [Kogut, Susskind'75]

2D QED: Dynamical gauge field = operator on electro-magnetic quanta

$$[\hat{E}_{s,\mu}, \hat{U}_{s,\mu}] = -\hat{U}_{s,\mu}$$

+ Electro-magnetic energy

+ Gauss law U(1) gauge invariance

$$\hat{E}_{s,x} + \hat{E}_{s,y} - \hat{E}_{s-\hat{x},x} - \hat{E}_{s-\hat{y},y} = Q_s$$

$$H = -\sum_{s,\mu=x,y} g^2 \hat{E}_{s,\mu}^2 - \frac{1}{g^2} (\hat{U}\hat{U}\hat{U}^{\dagger}\hat{U}^{\dagger} + H.c.)$$

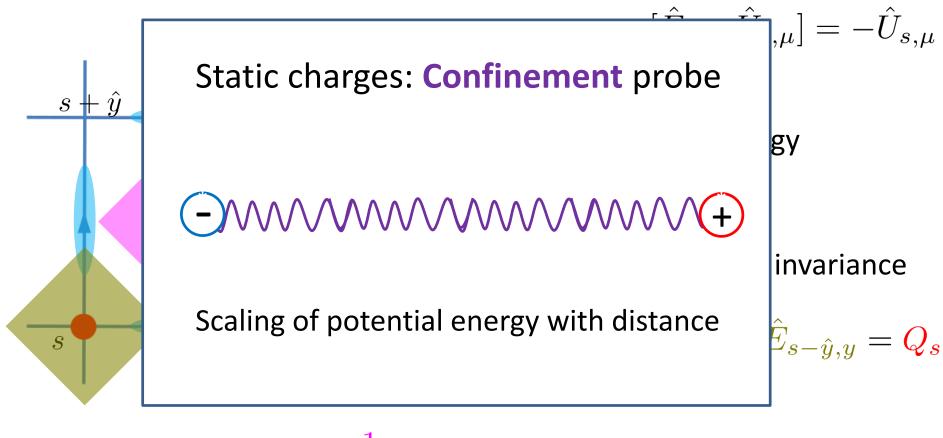
 $s + \hat{x}$ 

S -

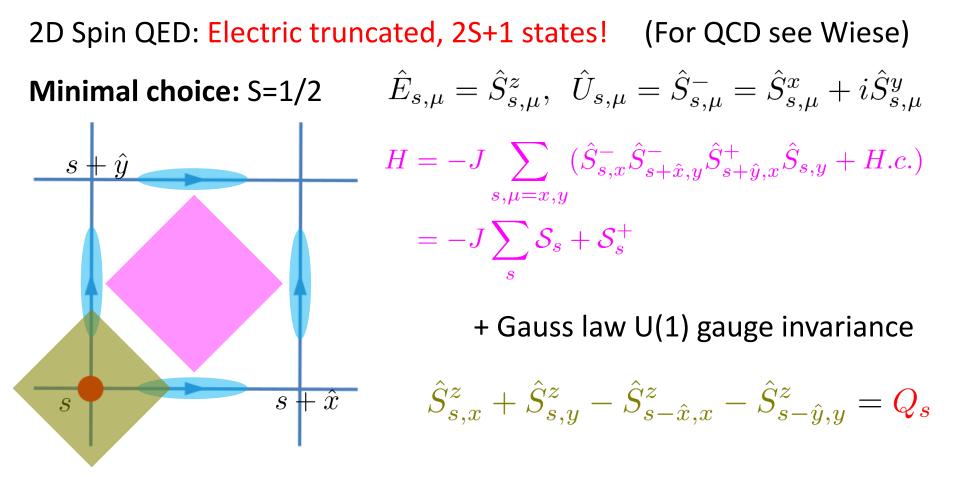
S

### I.3 Hamiltonian lattice gauge theory [Kogut, Susskind'75]

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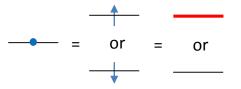


$$H = -\sum_{s,\mu=x,y} g^2 \hat{E}_{s,\mu}^2 - \frac{1}{g^2} (\hat{U}\hat{U}\hat{U}^{\dagger}\hat{U}^{\dagger} + H.c.)$$



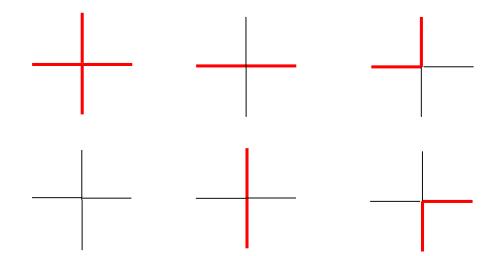
Lattice gauge prototype! Emerging in condensed matter systems!

#### 2D QED S=1/2: Graphical notation

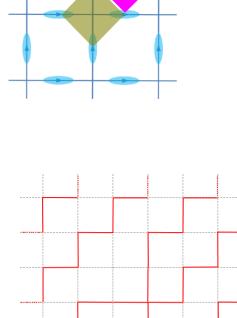


Gauss law no charges: 6 building blocks!

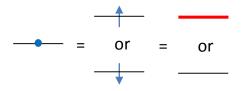
Equivalent to spin ice [Slater'41]



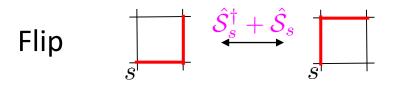
Physical states: strings going up & right

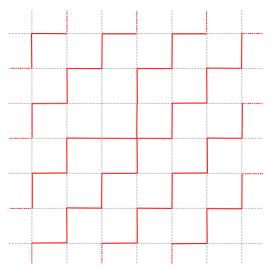




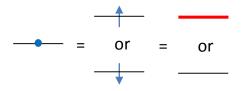


**Plaquette operator:** Kinetic term for strings

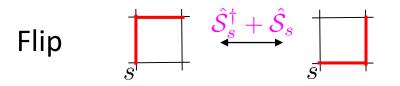


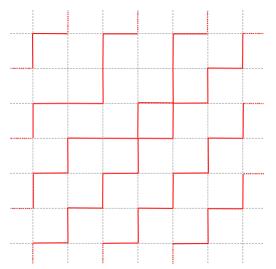




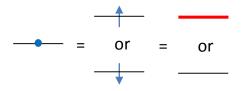


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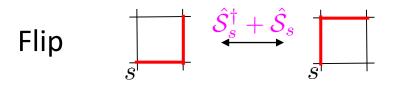


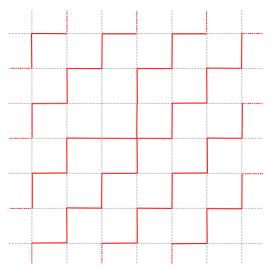




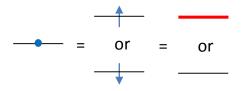


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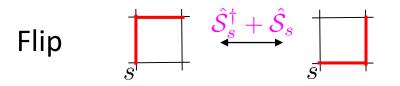


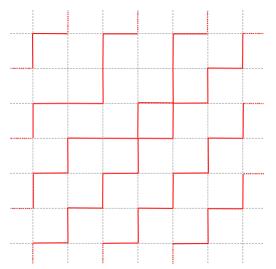




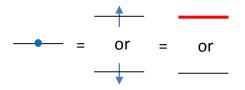


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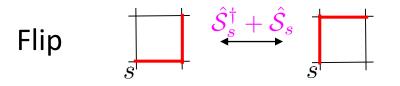


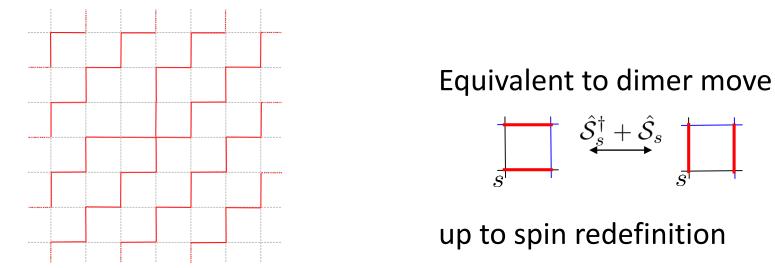






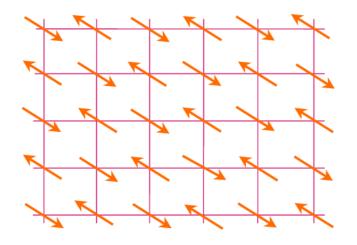
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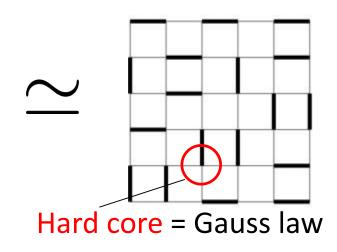


### I.6 Quantum dimer model

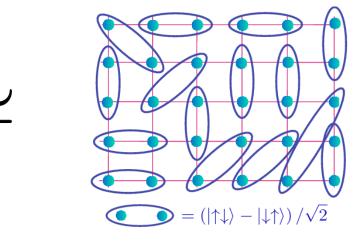
#### Antiferromagnets



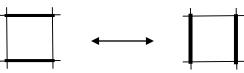
Hard-core dimers



#### Valence bond (VB) covering



Quantum fluctuations



Resonating [And valence bonds [Rok

[Anderson, Baskaran' 87] [Rokhsar, Kivelson' 88]...

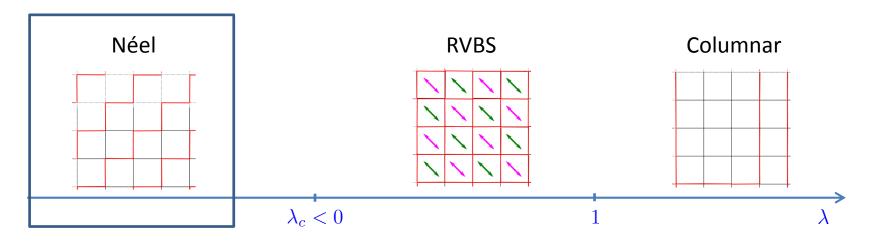
S=1/2 spin gauge theory [Moessner,Sondhi,Fradkin'01]

$$H_{RK} = -J\sum_{s} \left[ \left( \hat{\mathcal{S}}_{s}^{\dagger} + \hat{\mathcal{S}}_{s} \right) - \lambda \left( \hat{\mathcal{S}}_{s}^{\dagger} + \mathcal{S}_{s} \right)^{2} \right] \quad \mathbf{A}_{s} = -J\sum_{s} \left[ \left( \hat{\mathcal{S}}_{s}^{\dagger} + \hat{\mathcal{S}}_{s} \right)^{2} \right] \quad \mathbf{A}_{s} = -J\sum_{s} \left[ \left( \hat{\mathcal{S}}_{s}^{\dagger} + \hat{\mathcal{S}}_{s} \right)^{2} \right] \quad \mathbf{A}_{s} = -J\sum_{s} \left[ \left( \hat{\mathcal{S}}_{s}^{\dagger} + \hat{\mathcal{S}}_{s} \right)^{2} \right] \quad \mathbf{A}_{s} = -J\sum_{s} \left[ \left( \hat{\mathcal{S}}_{s}^{\dagger} + \hat{\mathcal{S}}_{s} \right)^{2} \right] \quad \mathbf{A}_{s} = -J\sum_{s} \left[ \left( \hat{\mathcal{S}}_{s}^{\dagger} + \hat{\mathcal{S}}_{s} \right)^{2} \right] \quad \mathbf{A}_{s} = -J\sum_{s} \left[ \left( \hat{\mathcal{S}}_{s}^{\dagger} + \hat{\mathcal{S}}_{s} \right)^{2} \right] \quad \mathbf{A}_{s} = -J\sum_{s} \left[ \left( \hat{\mathcal{S}}_{s}^{\dagger} + \hat{\mathcal{S}}_{s} \right)^{2} \right] \quad \mathbf{A}_{s} = -J\sum_{s} \left[ \left( \hat{\mathcal{S}}_{s}^{\dagger} + \hat{\mathcal{S}}_{s} \right)^{2} \right] \quad \mathbf{A}_{s} = -J\sum_{s} \left[ \left( \hat{\mathcal{S}}_{s}^{\dagger} + \hat{\mathcal{S}}_{s} \right)^{2} \right] \quad \mathbf{A}_{s} = -J\sum_{s} \left[ \left( \hat{\mathcal{S}}_{s}^{\dagger} + \hat{\mathcal{S}}_{s} \right)^{2} \right] \quad \mathbf{A}_{s} = -J\sum_{s} \left[ \left( \hat{\mathcal{S}}_{s}^{\dagger} + \hat{\mathcal{S}}_{s} \right)^{2} \right] \quad \mathbf{A}_{s} = -J\sum_{s} \left[ \left( \hat{\mathcal{S}}_{s}^{\dagger} + \hat{\mathcal{S}}_{s} \right)^{2} \right] \quad \mathbf{A}_{s} = -J\sum_{s} \left[ \left( \hat{\mathcal{S}}_{s}^{\dagger} + \hat{\mathcal{S}}_{s} \right)^{2} \right] \quad \mathbf{A}_{s} = -J\sum_{s} \left[ \left( \hat{\mathcal{S}}_{s}^{\dagger} + \hat{\mathcal{S}}_{s} \right)^{2} \right] \quad \mathbf{A}_{s} = -J\sum_{s} \left[ \left( \hat{\mathcal{S}}_{s}^{\dagger} + \hat{\mathcal{S}}_{s} \right)^{2} \right] \quad \mathbf{A}_{s} = -J\sum_{s} \left[ \left( \hat{\mathcal{S}}_{s}^{\dagger} + \hat{\mathcal{S}}_{s} \right)^{2} \right] \quad \mathbf{A}_{s} = -J\sum_{s} \left[ \left( \hat{\mathcal{S}}_{s}^{\dagger} + \hat{\mathcal{S}}_{s} \right)^{2} \right] \quad \mathbf{A}_{s} = -J\sum_{s} \left[ \left( \hat{\mathcal{S}}_{s}^{\dagger} + \hat{\mathcal{S}}_{s} \right)^{2} \right] \quad \mathbf{A}_{s} = -J\sum_{s} \left[ \left( \hat{\mathcal{S}}_{s}^{\dagger} + \hat{\mathcal{S}}_{s} \right)^{2} \right] \quad \mathbf{A}_{s} = -J\sum_{s} \left[ \left( \hat{\mathcal{S}}_{s}^{\dagger} + \hat{\mathcal{S}}_{s} \right)^{2} \right] \quad \mathbf{A}_{s} = -J\sum_{s} \left[ \left( \hat{\mathcal{S}}_{s}^{\dagger} + \hat{\mathcal{S}}_{s} \right)^{2} \right] \quad \mathbf{A}_{s} = -J\sum_{s} \left[ \left( \hat{\mathcal{S}}_{s}^{\dagger} + \hat{\mathcal{S}}_{s} \right)^{2} \right] \quad \mathbf{A}_{s} = -J\sum_{s} \left[ \left( \hat{\mathcal{S}}_{s}^{\dagger} + \hat{\mathcal{S}}_{s} \right)^{2} \right] \quad \mathbf{A}_{s} = -J\sum_{s} \left[ \left( \hat{\mathcal{S}}_{s}^{\dagger} + \hat{\mathcal{S}}_{s} \right)^{2} \right] \quad \mathbf{A}_{s} = -J\sum_{s} \left[ \left( \hat{\mathcal{S}}_{s}^{\dagger} + \hat{\mathcal{S}}_{s} \right)^{2} \right] \quad \mathbf{A}_{s} = -J\sum_{s} \left[ \left( \hat{\mathcal{S}}_{s}^{\dagger} + \hat{\mathcal{S}}_{s} \right)^{2} \right] \quad \mathbf{A}_{s} = -J\sum_{s} \left[ \left( \hat{\mathcal{S}}_{s}^{\dagger} + \hat{\mathcal{S}}_{s} \right)^{2} \right] \quad \mathbf{A}_{s} = -J\sum_{s} \left[ \left( \hat{\mathcal{S}}_{s}^{\dagger} + \hat{\mathcal{S}}_{s} \right)^{2} \right] \quad \mathbf{A}_{s} = -J\sum_{s} \left[ \left( \hat{\mathcal{S}}_{s}^{\dagger} + \hat{\mathcal{S}}_{s} \right)^{2} \right] \quad$$

Static phase diagram (no charges)

Chemical potential for flippable plaquettes

ED [Shannon et al.'04] QMC [Banerjee et al.'13] DMRG [Tschirsich et al.'18]



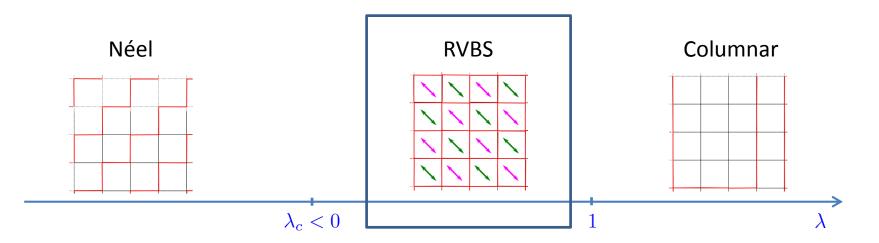
Flippable background Order by disorder

$$H_{RK} = -J\sum_{s} \left[ \left( \hat{\mathcal{S}}_{s}^{\dagger} + \hat{\mathcal{S}}_{s} \right) - \lambda \left( \hat{\mathcal{S}}_{s}^{\dagger} + \mathcal{S}_{s} \right)^{2} \right] \quad \mathbf{A}_{s}$$

Static phase diagram (no charges)

Chemical potential for flippable plaquettes

ED [Shannon et al.'04] QMC [Banerjee et al.'13] DMRG [Tschirsich et al.'18]



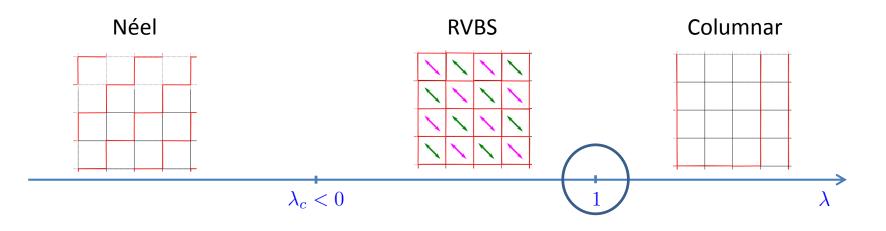
Resonating Valence Bond Solid: Correlations break translational invariance

$$H_{RK} = -J\sum_{s} \left[ \left( \hat{\mathcal{S}}_{s}^{\dagger} + \hat{\mathcal{S}}_{s} \right) - \lambda \left( \hat{\mathcal{S}}_{s}^{\dagger} + \mathcal{S}_{s} \right)^{2} \right] \quad \mathbf{A}$$

Static phase diagram (no charges)

Chemical potential for flippable plaquettes

ED [Shannon et al.'04] QMC [Banerjee et al.'13] DMRG [Tschirsich et al.'18]



 $\lambda = 1$ 

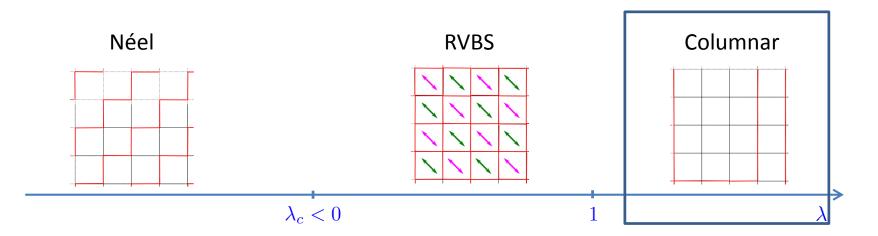
Integrable: superposition all physical states, short-range Resonating Valence bond

$$H_{RK} = -J\sum_{s} \left[ \left( \hat{\mathcal{S}}_{s}^{\dagger} + \hat{\mathcal{S}}_{s} \right) - \lambda \left( \hat{\mathcal{S}}_{s}^{\dagger} + \mathcal{S}_{s} \right)^{2} \right] - \lambda \left( \hat{\mathcal{S}}_{s}^{\dagger} + \mathcal{S}_{s} \right)^{2} \right] - \lambda \left( \hat{\mathcal{S}}_{s}^{\dagger} + \mathcal{S}_{s} \right)^{2} = -\frac{1}{2} \left[ \hat{\mathcal{S}}_{s}^{\dagger} + \hat{\mathcal{S}}_{s} \right]^{2} - \frac{1}{2} \left[ \hat{\mathcal{S}}_{s}^{\dagger} + \hat{\mathcal{S}}_{s} \right]^{2} = -\frac{1}{2} \left[ \hat{\mathcal{S}}_{s}^{\dagger} + \hat{\mathcal{S}}_{s} \right]^{2} - \frac{1}{2} \left[ \hat{\mathcal{S}}_{s}^{\dagger} + \hat{\mathcal{S}}_{s} \right]^{2} = -\frac{1}{2} \left[ \hat{\mathcal{S}}_{s}^{\dagger} + \hat{\mathcal{S}}_{s} \right]^{2} - \frac{1}{2} \left[ \hat{\mathcal{S}}_{s}^{\dagger} + \hat{\mathcal{S}}_{s} \right]^{2} = -\frac{1}{2} \left[ \hat{\mathcal{S}}_{s}^$$

Static phase diagram (no charges)

Chemical potential for flippable plaquettes

ED [Shannon et al.'04] QMC [Banerjee et al.'13] DMRG [Tschirsich et al.'18]



Unflippable background

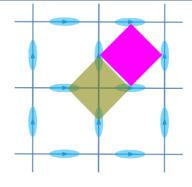
## I.8 Challenge in simulating spin gauge theories

$$H = -J\sum_{s} \left( \hat{S}_{s,x}^{+} \hat{S}_{s+\hat{x},y}^{+} \hat{S}_{s+\hat{y},x}^{-} \hat{S}_{s,y}^{-} + H.c. \right) = -J\sum_{s} \left( \hat{S}_{s}^{\dagger} + \hat{S}_{s} \right)$$

Plaquette interaction hard to implement

Existing proposal to simulate Gauss law and dynamics

- Analogue approach: Gauss law from energy penalty plaquette interactions perturbatively
- Digital simulation with Rydberg gates: [Weimer et al'10] Gauss law by dissipation [Tagliacozzo,AC et al'13] plaquette interactions from sequential operations



[Büchler *et al*'05, Glaetzle *et al*'14]

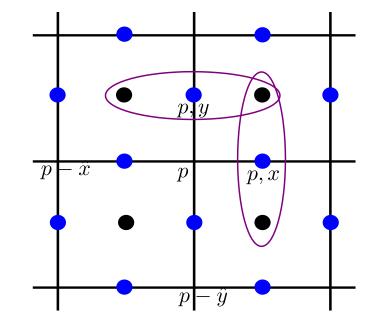
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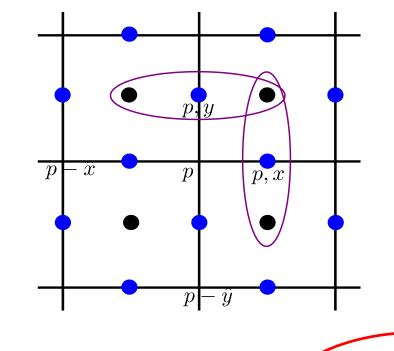
$$H = -J\sum_{p} \left( \hat{S}_{p,x}^{+} \hat{S}_{p+\hat{x},y}^{+} \hat{S}_{p+\hat{y},x}^{-} \hat{S}_{p,y}^{-} + H.c. \right) = -J\sum_{p} \left( \hat{S}_{p}^{\dagger} + \hat{S}_{p} \right)$$



$$\hat{S}_{p,x}^{z} = -2(-1)^{p} \hat{S}_{p}^{z} \hat{S}_{p-\hat{y}}^{z}$$
$$\hat{S}_{p,y}^{z} = 2(-1)^{p} \hat{S}_{p}^{z} \hat{S}_{p-\hat{x}}^{z}$$

$$H_d = -2J\sum_p \left(P_p^{\uparrow\uparrow\uparrow\uparrow} + P_p^{\downarrow\downarrow\downarrow\downarrow}\right) \hat{S}_p^x$$

$$H = -J\sum_{p} \left( \hat{S}_{p,x}^{+} \hat{S}_{p+\hat{x},y}^{+} \hat{S}_{p+\hat{y},x}^{-} \hat{S}_{p,y}^{-} + H.c. \right) = -J\sum_{p} \left( \hat{S}_{p}^{\dagger} + \hat{S}_{p} \right)$$



p

 $H_d = -2J\sum$ 

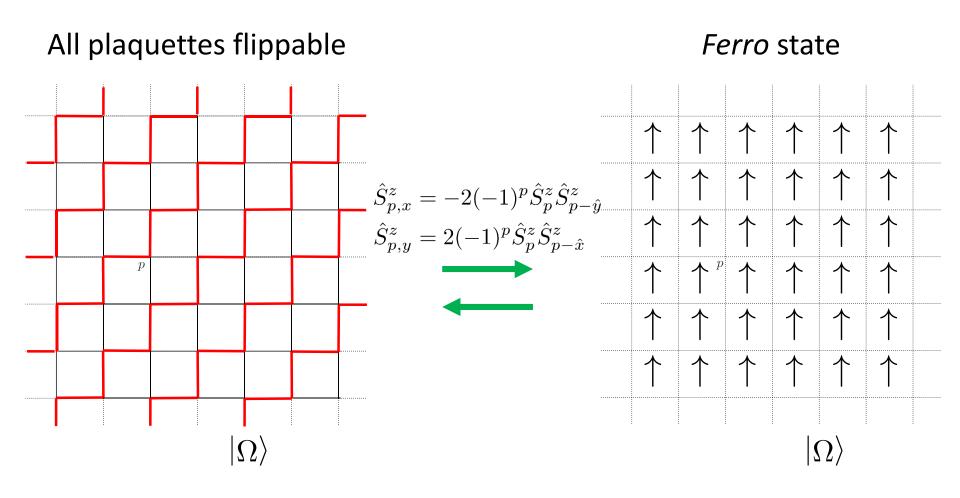
 $(P_p^{\uparrow\uparrow\uparrow\downarrow})$ 

$$\begin{split} \hat{S}^{z}_{p,x} &= -2(-1)^{p} \hat{S}^{z}_{p} \hat{S}^{z}_{p-\hat{y}} \\ \hat{S}^{z}_{p,y} &= 2(-1)^{p} \hat{S}^{z}_{p} \hat{S}^{z}_{p-\hat{x}} \end{split}$$

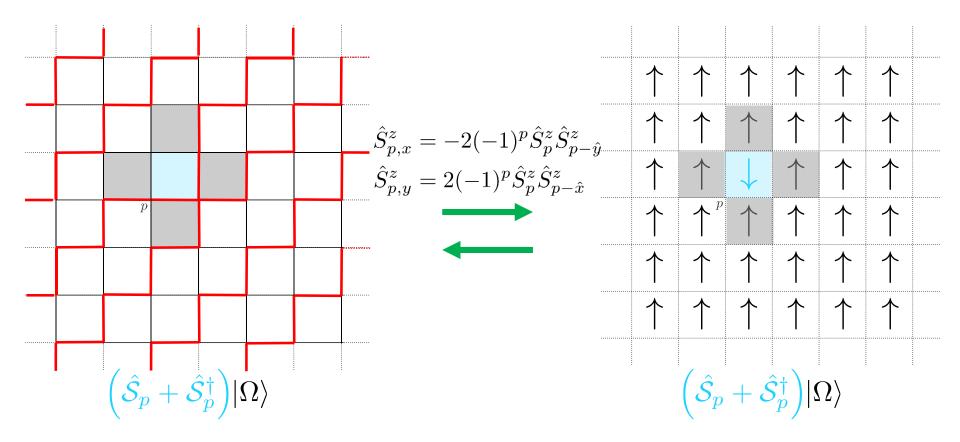
Spin version of height construction

cf. duality in KS U(1), [J. Zhang et al PRL **121**, 223201 (2018)]

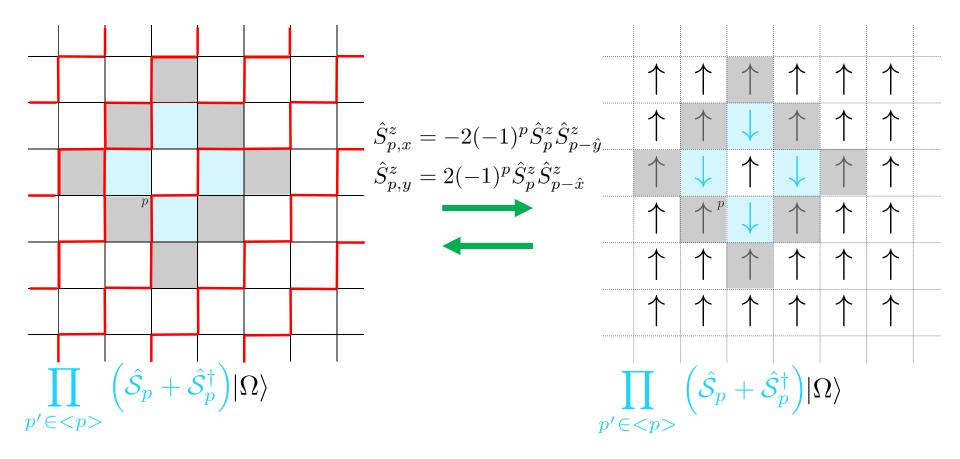
 $\hat{S}_p^x$ 

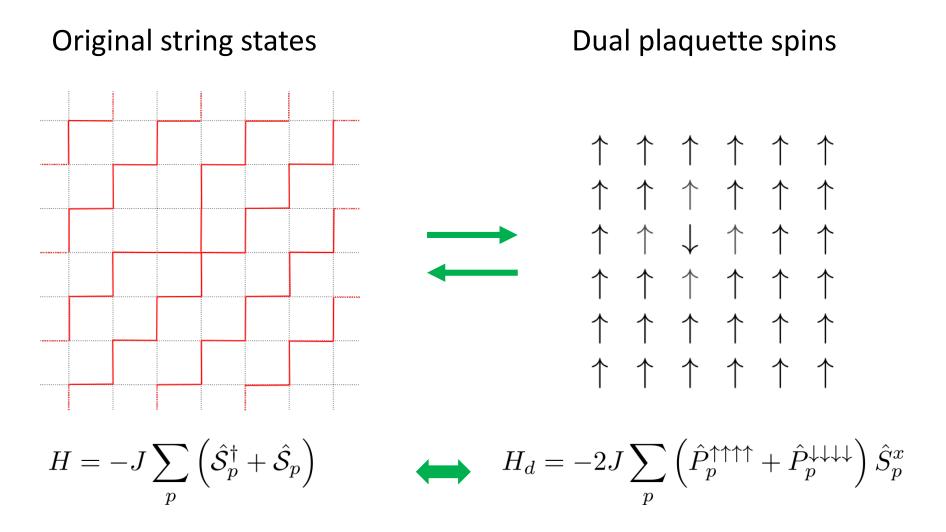






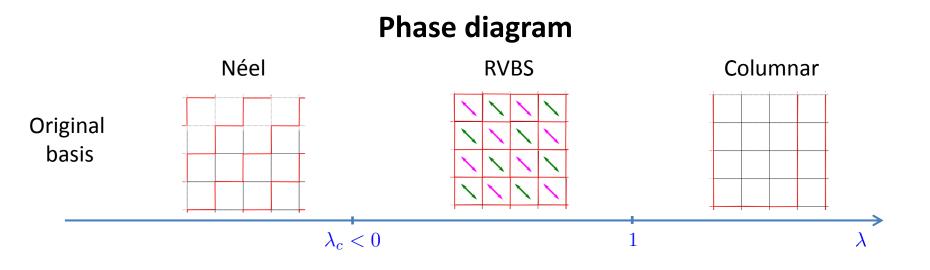
#### Plaquette action: also antiblockade



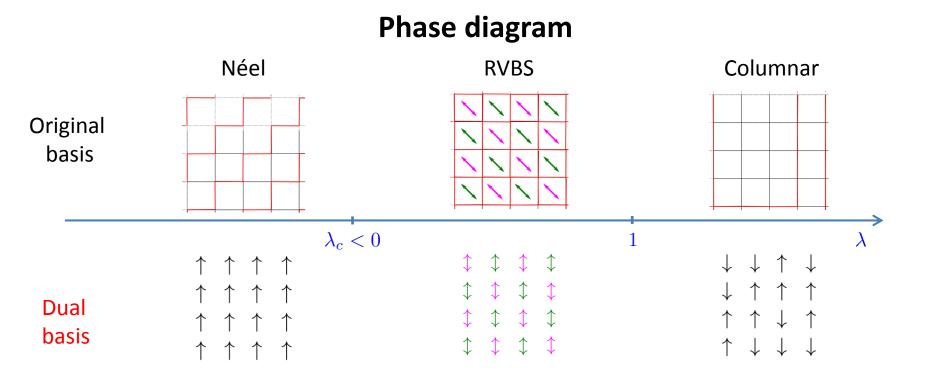


Valid for other lattices and background charges

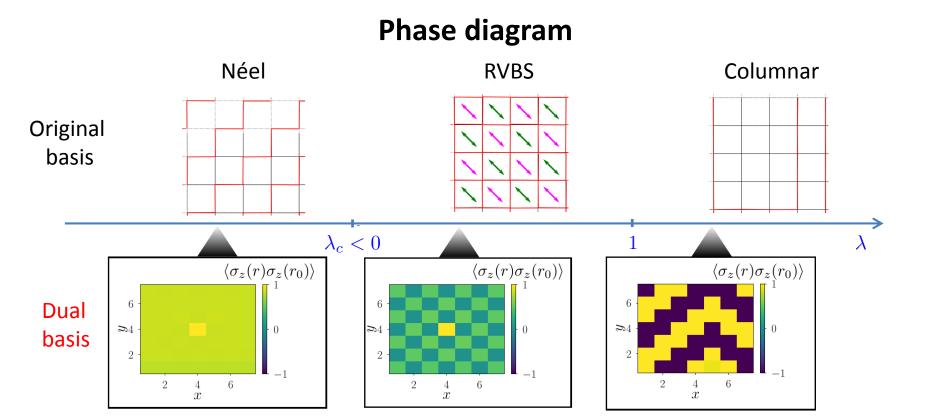
$$H_{RK} = -J\sum_{s} \left( \left( \hat{\mathcal{S}}_{s}^{\dagger} + \hat{\mathcal{S}}_{s} \right) - \lambda \left( \hat{\mathcal{S}}_{s}^{\dagger} + \hat{\mathcal{S}}_{s} \right)^{2} \right) \quad \longleftrightarrow \quad H_{dRK} = -J\sum_{s} \left( P_{s}^{\uparrow\uparrow\uparrow\uparrow} + P_{s}^{\downarrow\downarrow\downarrow\downarrow} \right) \left( 2\hat{S}_{s}^{x} - \lambda \right)$$



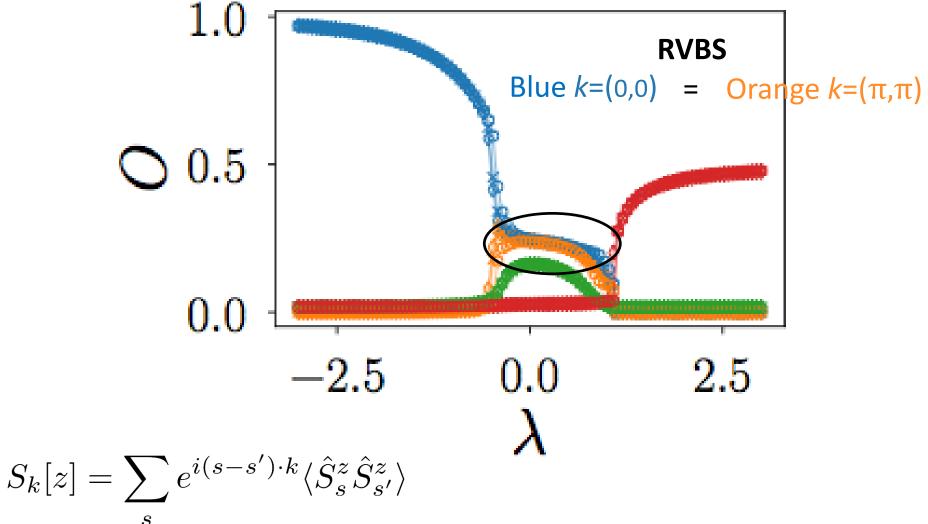
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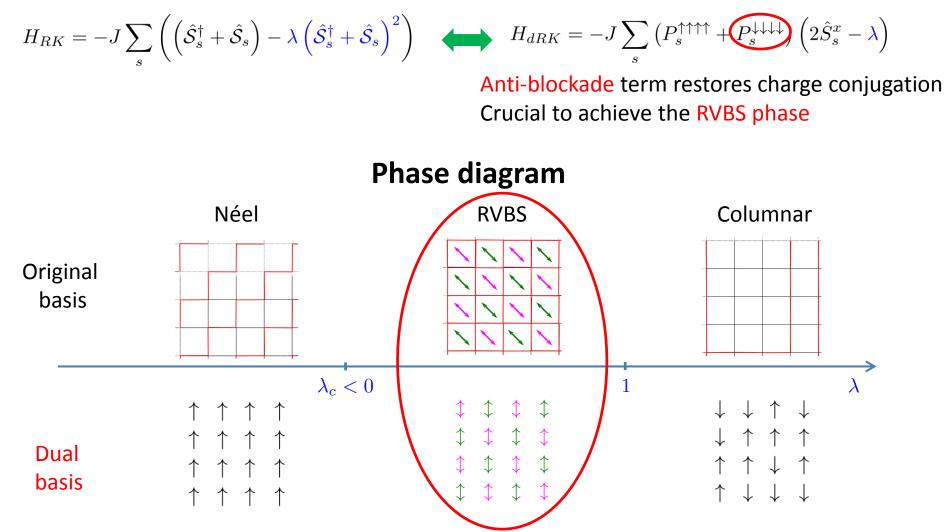


$$H_{RK} = -J\sum_{s} \left( \left( \hat{\mathcal{S}}_{s}^{\dagger} + \hat{\mathcal{S}}_{s} \right) - \lambda \left( \hat{\mathcal{S}}_{s}^{\dagger} + \hat{\mathcal{S}}_{s} \right)^{2} \right) \quad \longleftrightarrow \quad H_{dRK} = -J\sum_{s} \left( P_{s}^{\uparrow\uparrow\uparrow\uparrow} + P_{s}^{\downarrow\downarrow\downarrow\downarrow\downarrow} \right) \left( 2\hat{S}_{s}^{x} - \lambda \right)$$

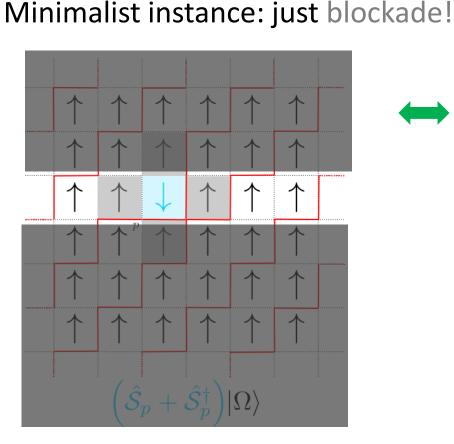


Structure factors and potential energy: good order parameters





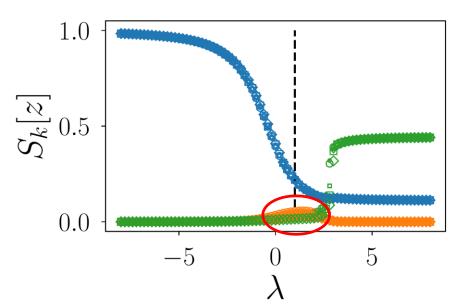
## Special case.1 RK on a ladder [AC et al, to appear]



No free lunch: we need antiblockade! [Moessner,Sondhi'01]  $H_{dRK} \rightarrow -J \sum_{s} P_{s}^{\uparrow\uparrow} \left( 2\hat{S}_{s}^{x} - \lambda \right)$ = PXP + detuning

Realized by a Rydberg chain!

RVBS replaced by a disorder phase



## Special case.2 RK on a periodic ladder [AC et al, to appear]

Periodic ladder: simplest cylinder Bulk plaquettes have all neighbors, 3 independent

$$H_{dRK} \rightarrow -J \sum_{\substack{x,y=[0,1]\\ x,y=[0,1]}} \left(P_{x,y}^{\uparrow\uparrow\uparrow} + P_{x,y}^{\downarrow\downarrow\downarrow}\right) \left(2\hat{S}_{x,y}^{x} - \lambda\right)$$

$$DMRG 64x2$$

$$0.0 -2.5 0.0 2.5$$

$$\lambda$$
Minimal geometry hosting a RVBS phase

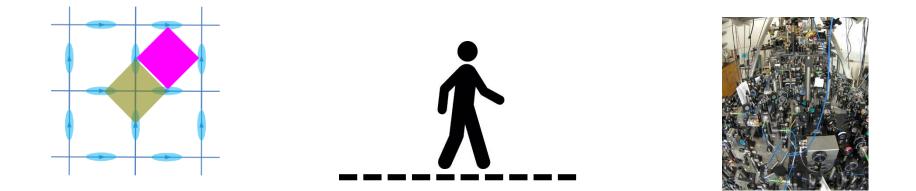
# Outline

I. Motivations and tutorial

From static to dynamical gauge theories The Rokhsar-Kivelson model Implementation: experimental challenges

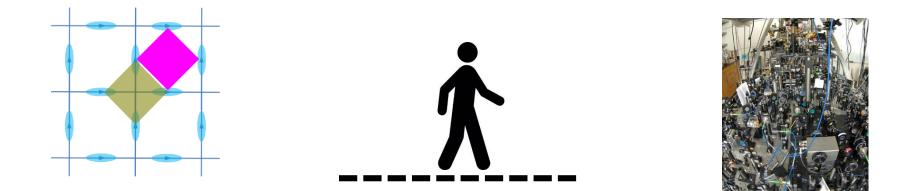
- II. Suitable encoding Converting Ring exchange to blockade
- III. Rydberg implementation and examples

### Engineering the dual Rokhsar-Kivelson model [AC et al, to appear]



$$H_{RK} \to H_{dRK} \to -J\sum_{s} \left( P_s^{\uparrow\uparrow\uparrow\uparrow} + P_s^{\downarrow\downarrow\downarrow\downarrow} \right) \left( 2\hat{S}_s^x - \lambda \right)$$

#### Engineering the dual Rokhsar-Kivelson model [AC et al, to appear]

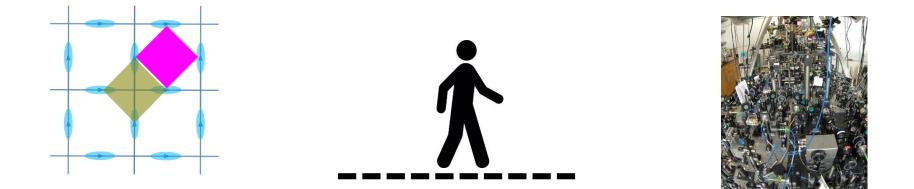


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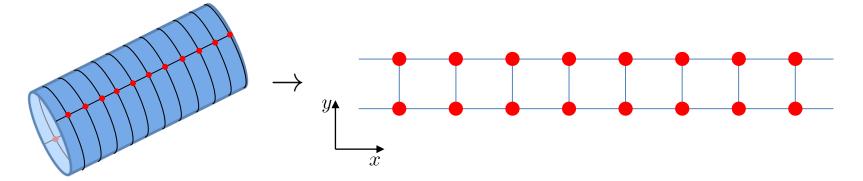
Lesson from blockade: projectors from nearest-neighbor interactions

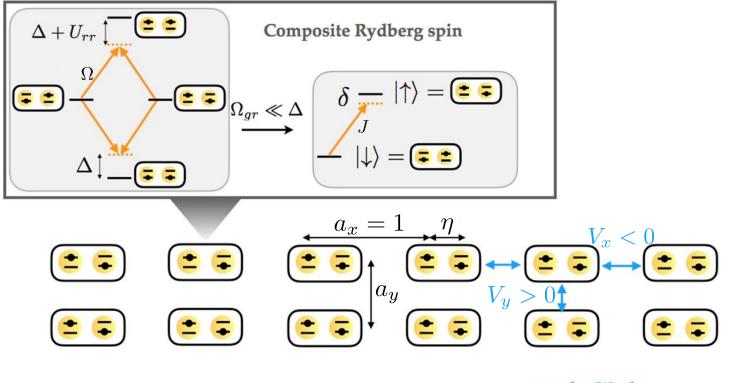
$$\left( P_s^{\uparrow\uparrow\uparrow\uparrow} + P_s^{\downarrow\downarrow\downarrow\downarrow} \right) \hat{S}_s^x \sim \hat{S}_s^x + \sum_{s'=\langle s \rangle} V_{ss'} \hat{S}_s^z \hat{S}_{s'}^z, \quad \text{if } \sum_{s'=\langle s \rangle} V_{ss'} = 0 + \dots$$

### Engineering the dual Rokhsar-Kivelson model [AC et al, to appear]

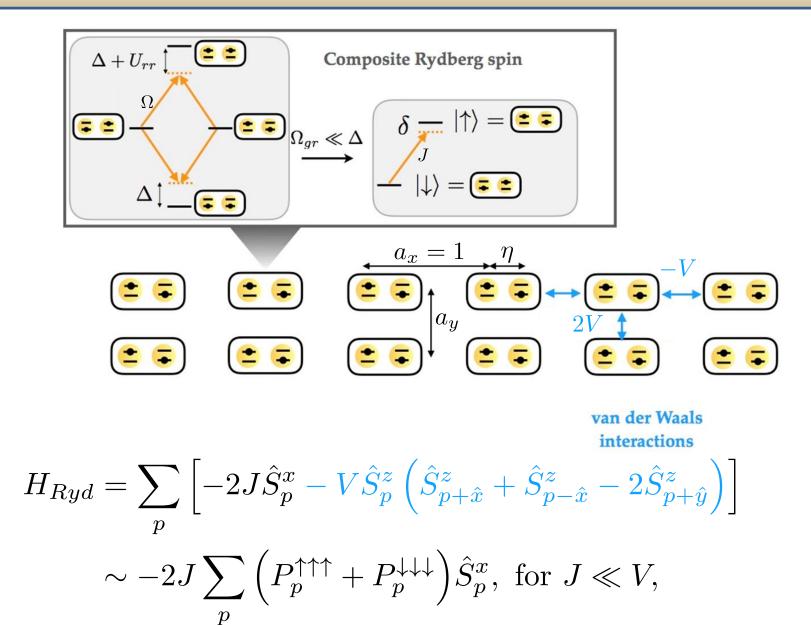


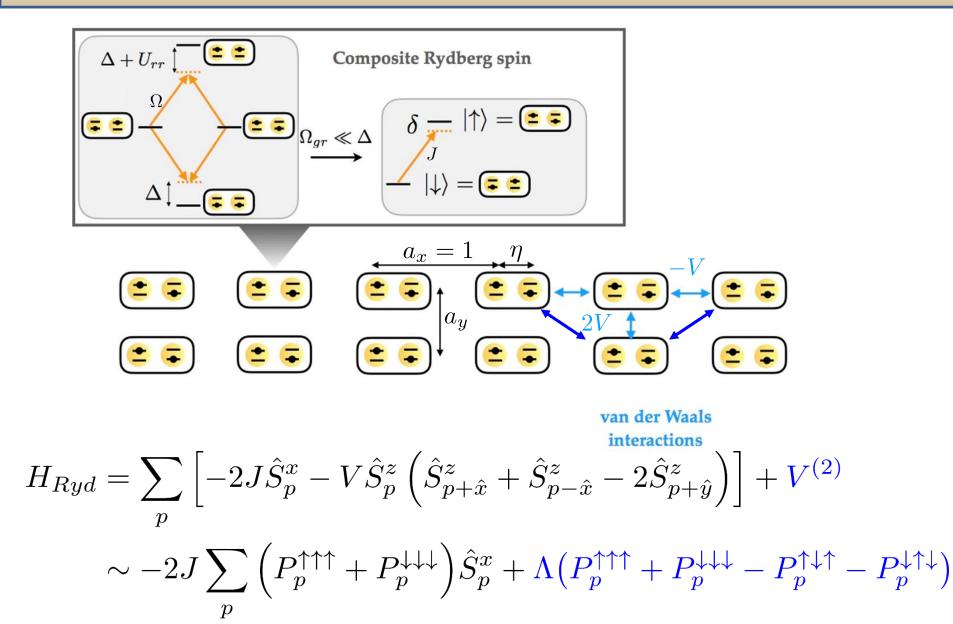
$$H_{RK} \to H_{dRK} \to -J \sum_{x,y=[0,1]} \left( P_{x,y}^{\uparrow\uparrow\uparrow} + P_{x,y}^{\downarrow\downarrow\downarrow} \right) \left( 2\hat{S}_{x,y}^x - \lambda \right)$$



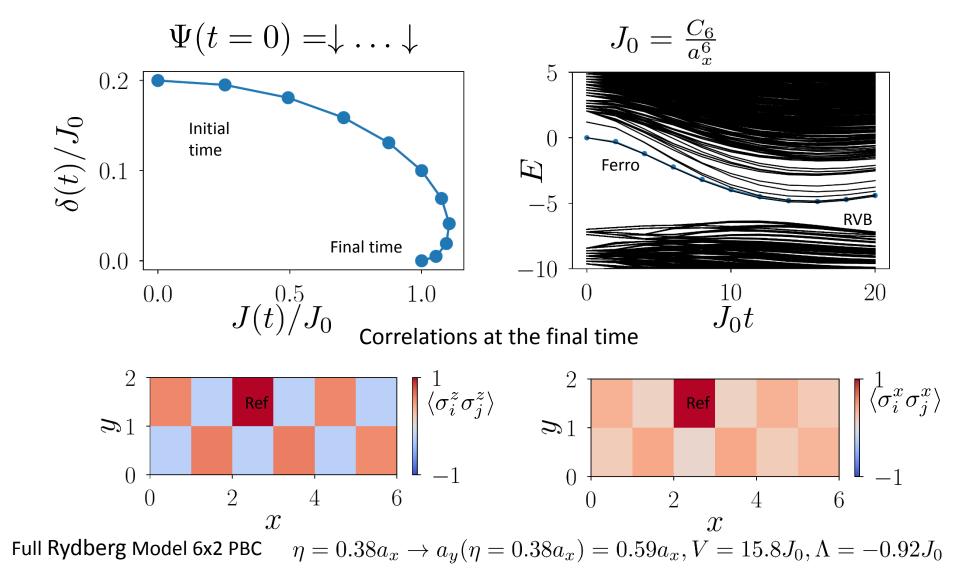


van der Waals interactions



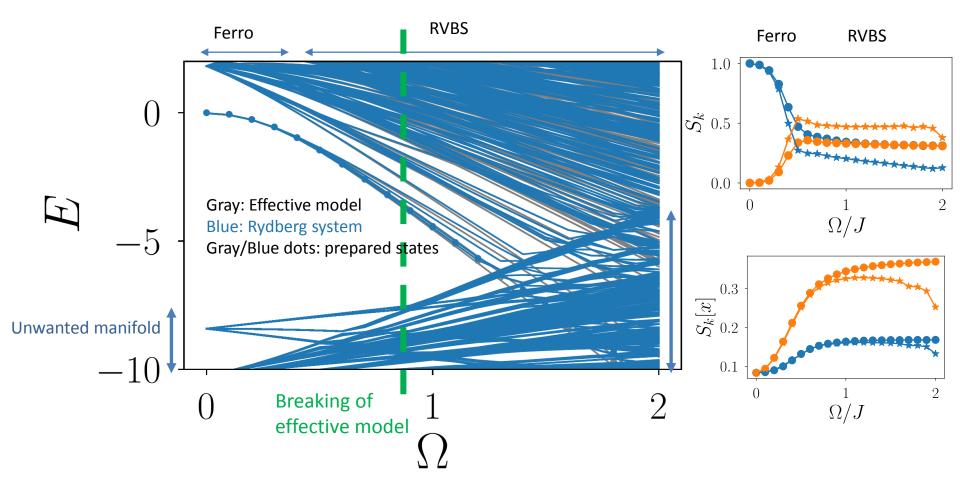


#### Adiabatic preparation of a RVBS state



#### Checking the validity of the effective model

Structure Factor



## Summary

Rydberg atoms "naturally" realize a 2D scalable U(1) lattice gauge theory

We can prepare and detect the RVBS phase in its dual formulation



B. Vermersch



P. Zoller

Probing confinement through static charges

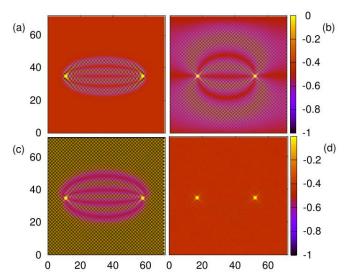
Consider other charge distributions/boundary conditions e.g. dual formulation of quantum dimers

Dynamical charges (bosons) -> Higgs mechanism



M.Lukin

## In progress





B. Vermersch



P. Zoller

Probing confinement through static charges

Consider other charge distributions/boundary conditions e.g. dual formulation of quantum dimers

Dynamical charges (bosons) -> Higgs mechanism



M.Lukin

## Prospects

HEP: First 2D lattice gauge theory soon in the lab!

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Condensed matter: new route to quantum magnetism

- Other geometries
- Charges and excitation (quantum probes)
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AMO physics: new family of models with entangled phases

- Anisotropic tunable coupling
- Larger spin from macro-atoms

#### Rokhsar-Kivelson on the square lattice [AC et al, to appear]

