Anyons in cold atoms:
How to detect them?

Leonardo Mazza

High-energy physics at ultracold temperatures
ECT*, Villazzano, 13 June 2019

LPTMS, Université Paris-Sud, CNRS

arXiv:1903.03011

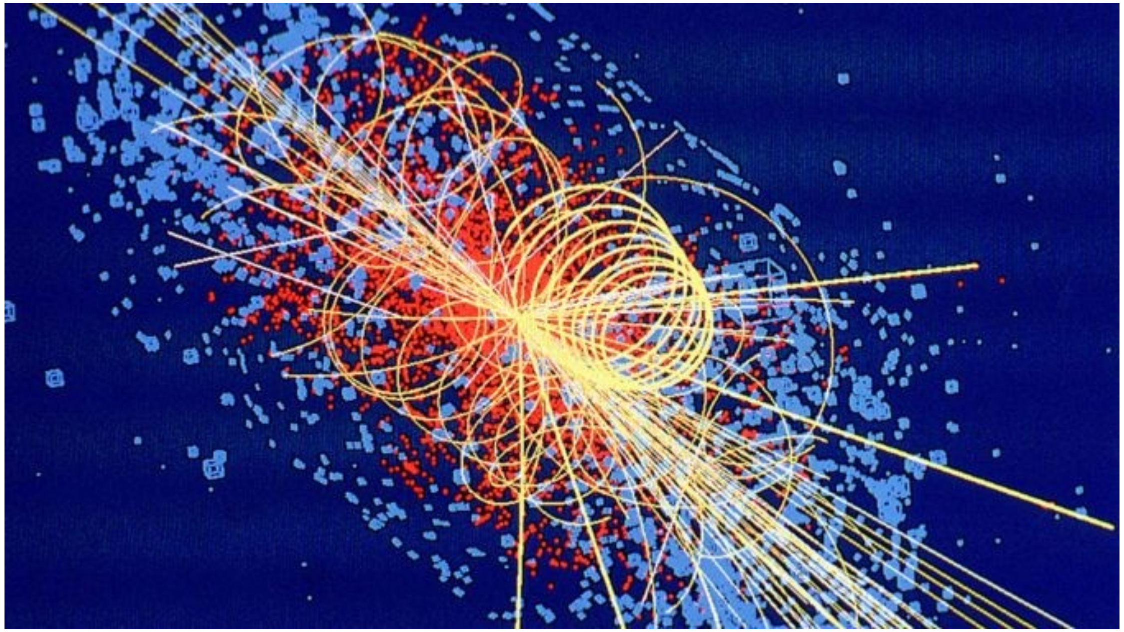
Acknowledgements

Elia Macaluso
Tommaso Comparin and
Iacopo Carusotto,

Trento, BEC center

arXiv:1903.03011

Quantum statistics

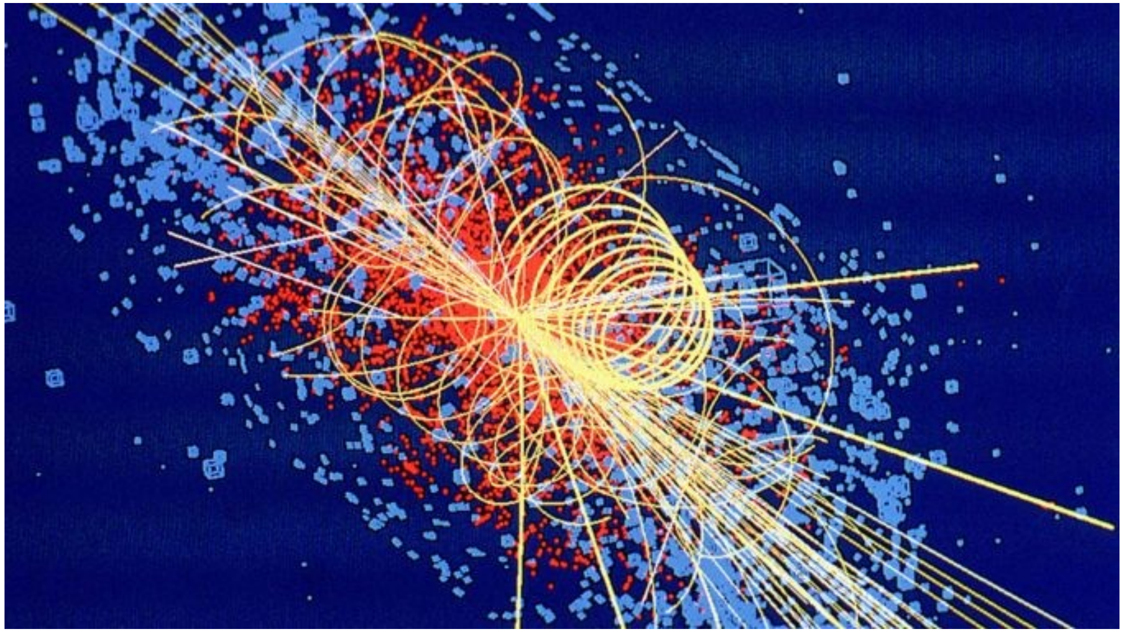


Fundamental particles of the
4-dimensional space-time are

bosons or **fermions**

- Quantum statistics
- Symmetrization postulate
- Spin

Quantum statistics



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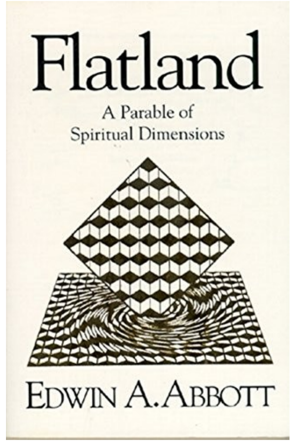
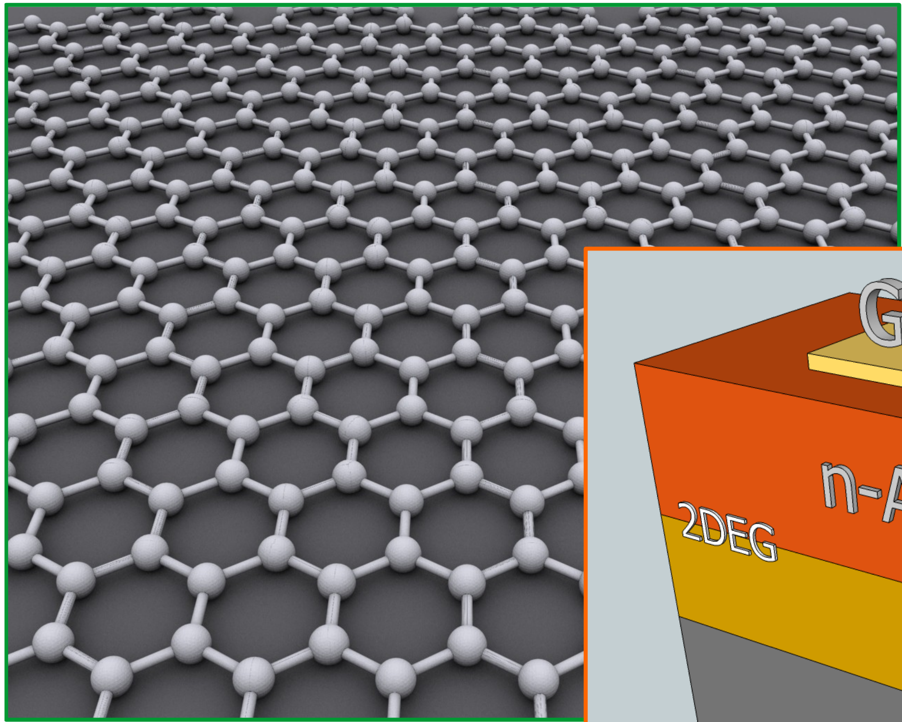
Anything else is an **anyon**

- Two spatial dimensions
- Quasi-particle excitation of a many-body system
- No unambiguous observation so far
- A field called “topological quantum computing” is based on anyons

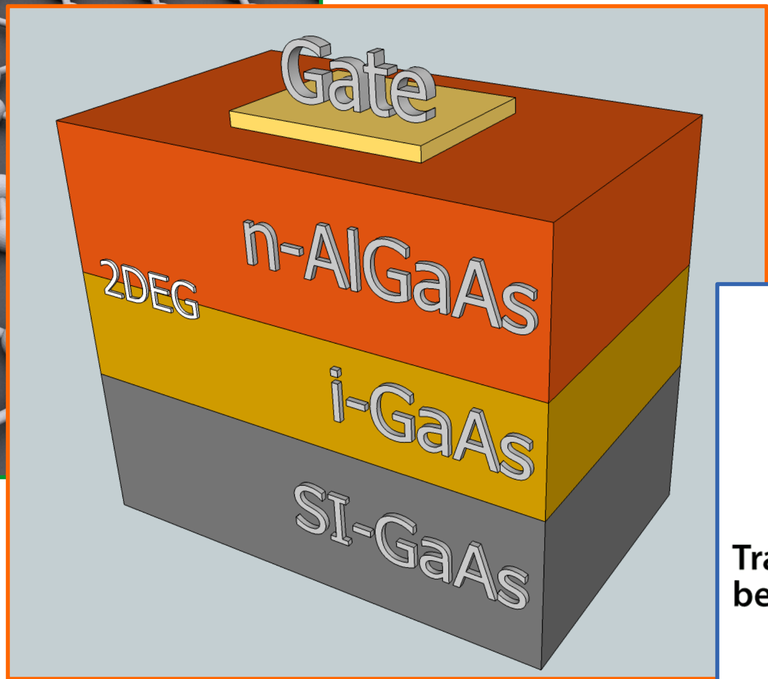
Preskill, lecture notes on quantum computation (2004);
A. Khare, Fractional Statistics and Quantum Theory (2005) and many more…

Quantum flatland



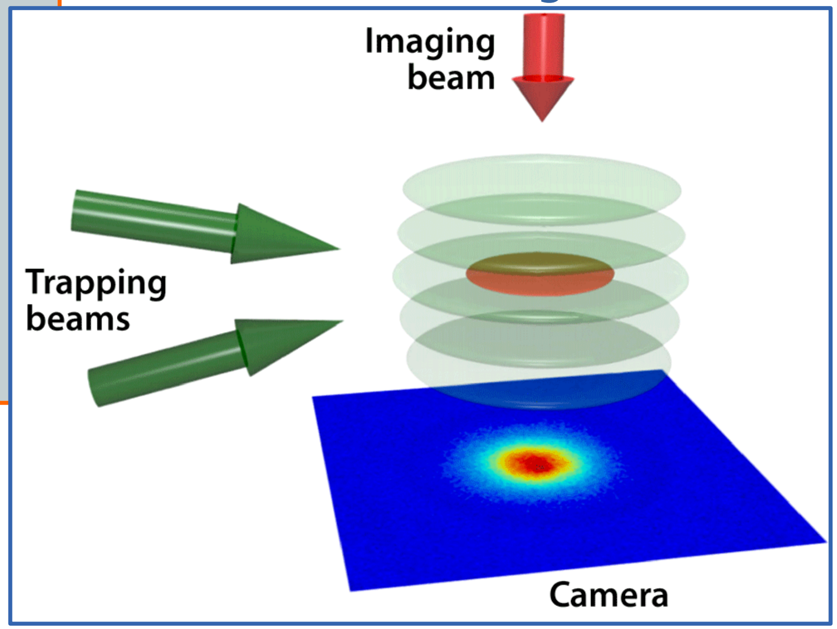


Graphene



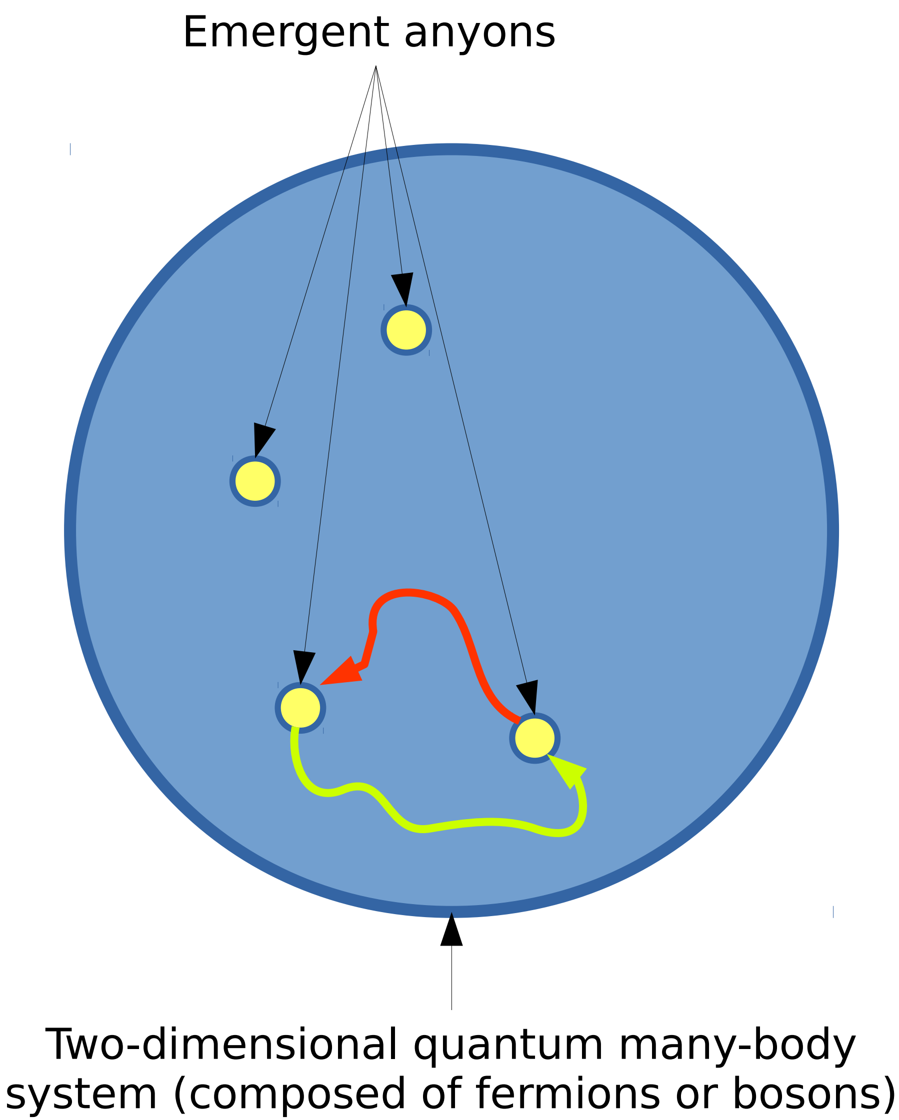
Two-dimensional electron gases

Two-dimensional atomic gases



- In these systems, we will (hopefully) find anyons

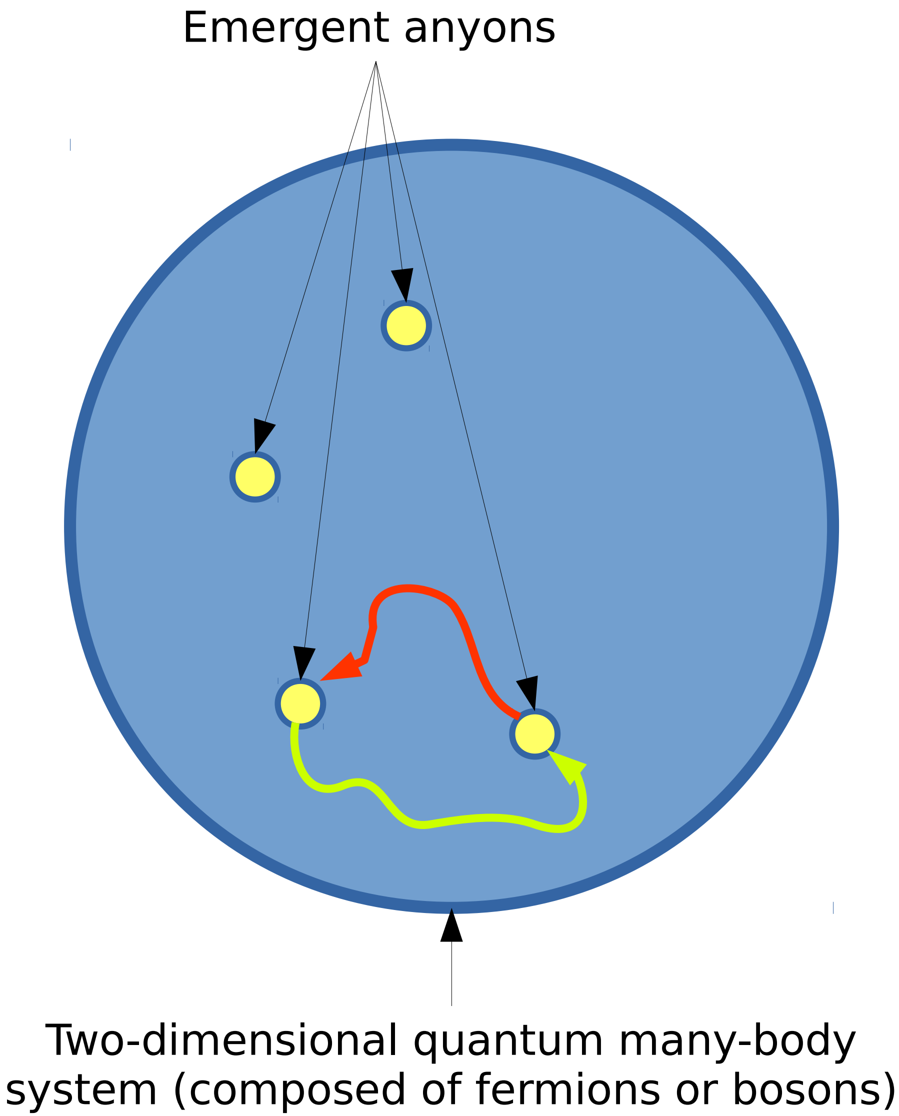
Anyonic statistics



Quantum statistics for identical particles
*from the notion of **exchange** of particles*

(it works only in 2D)

Anyonic statistics



Quantum statistics for identical particles
from the notion of ***exchange*** *of particles*

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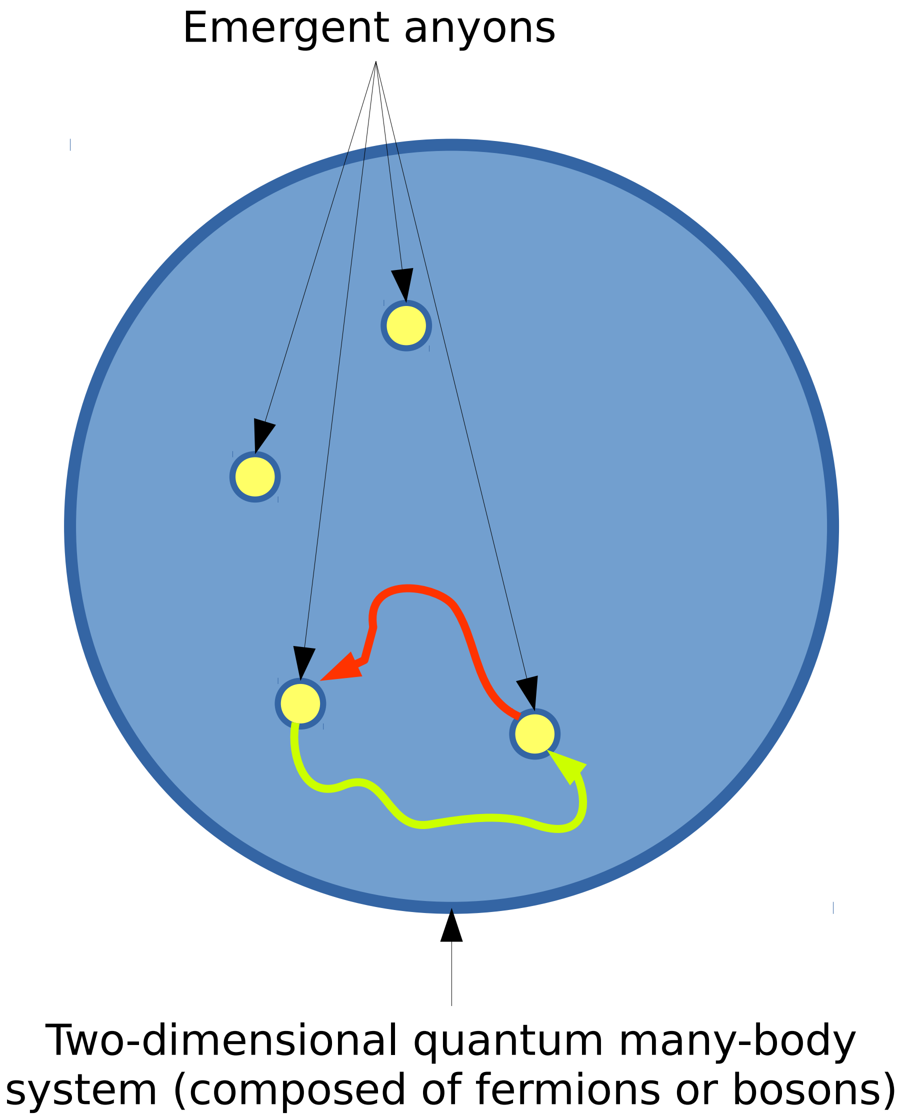
- Bosons

$$\Psi(\vec{r}_1, \vec{r}_2) \to +\Psi(\vec{r}_2, \vec{r}_1)$$

Anyonic statistics



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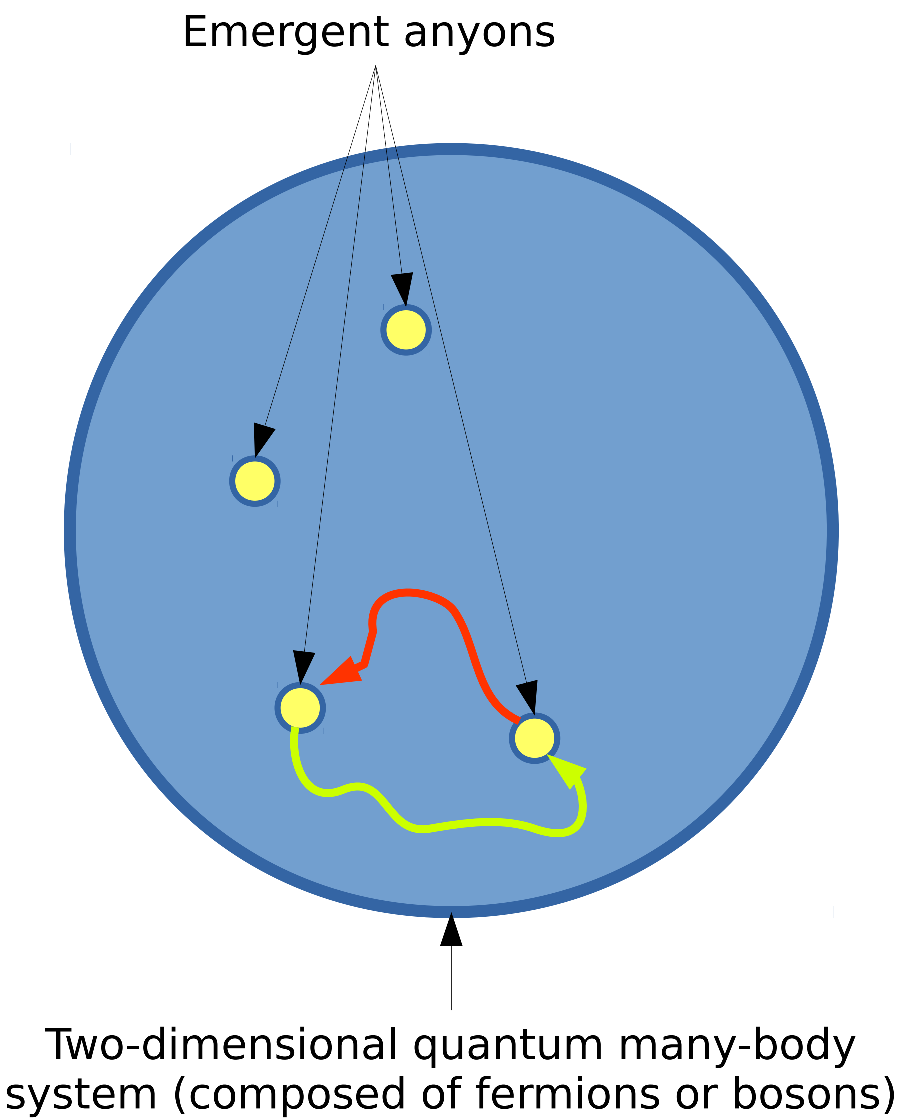
Anyonic statistics



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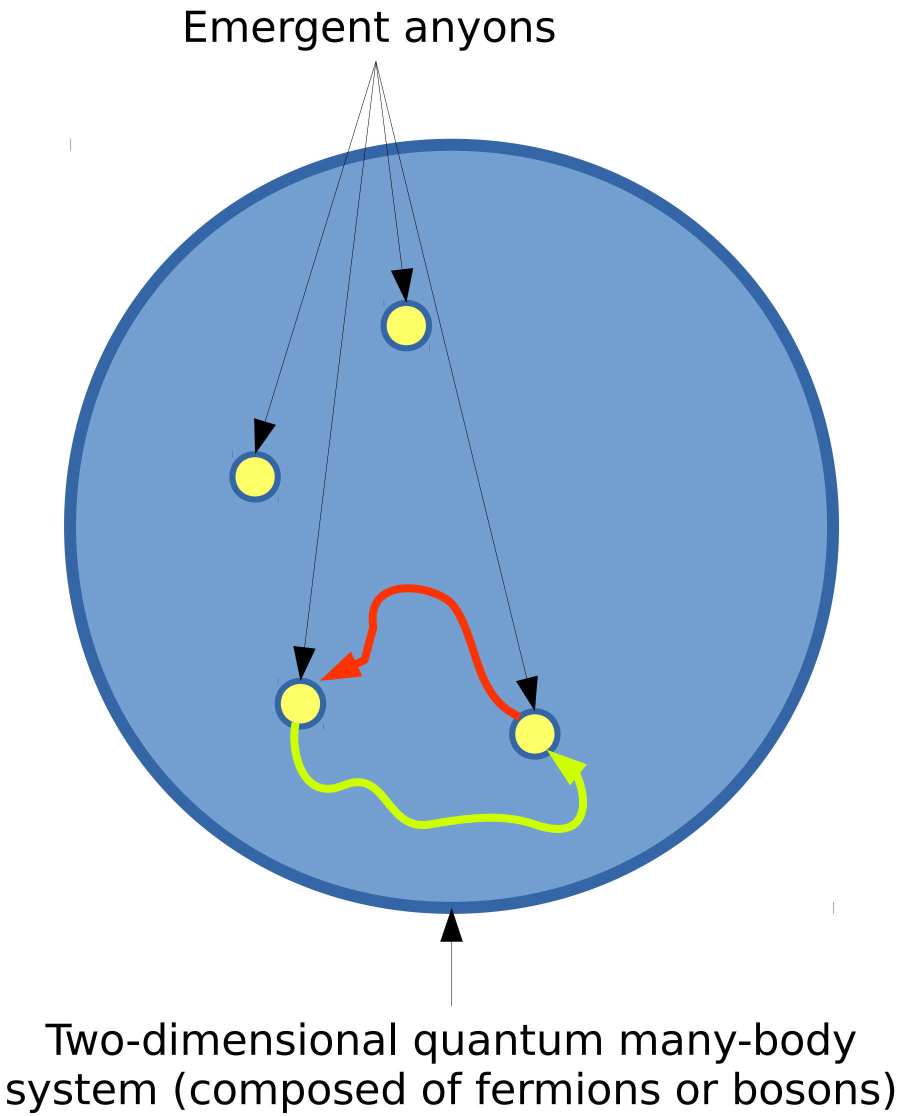
- Abelian anyons
$$\Psi(\vec{r}_1, \vec{r}_2) \to e^{i\varphi}\Psi(\vec{r}_2, \vec{r}_1)$$

Anyonic statistics



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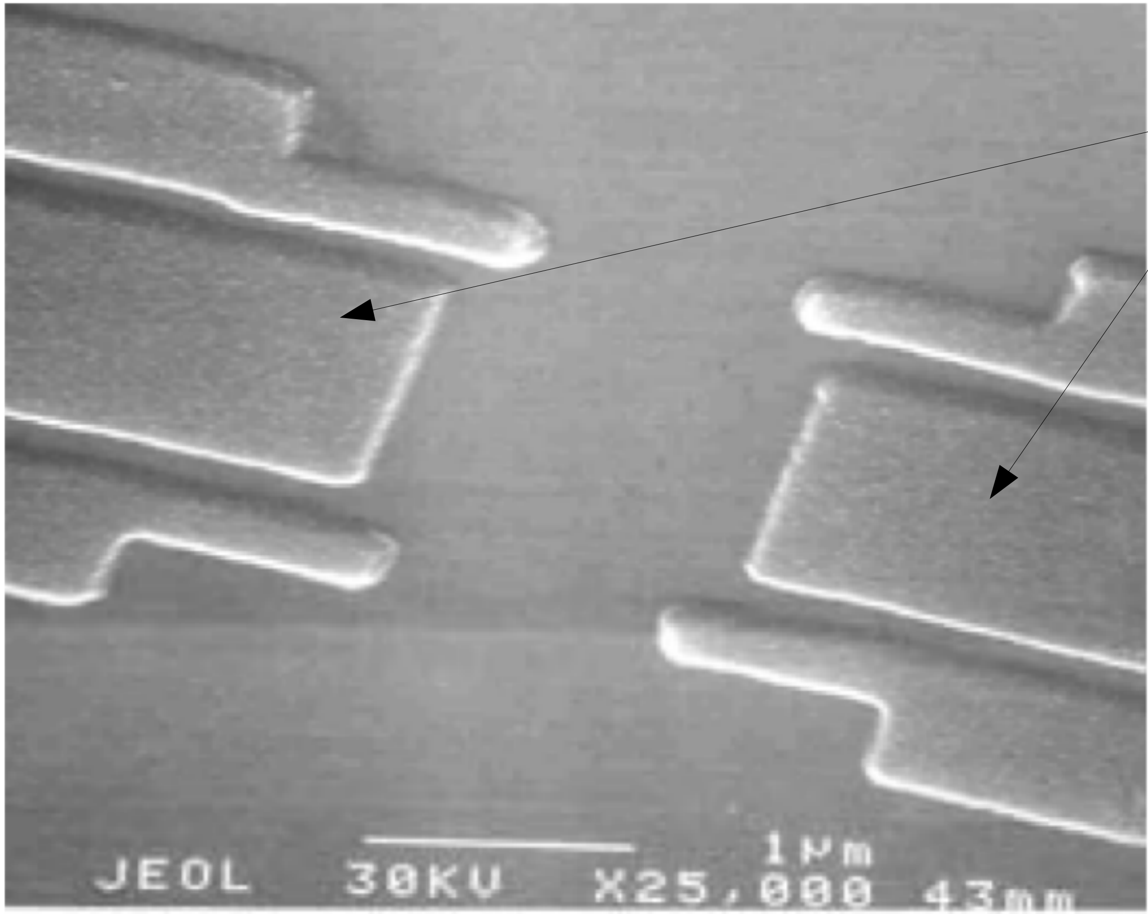
$$\Psi(\vec{r}_1, \vec{r}_2) \to e^{i\varphi}\Psi(\vec{r}_2, \vec{r}_1)$$
- Non-Abelian anyons
$$\vec{\Psi}(\vec{r}_1, \vec{r}_2) \to \mathcal{U}\vec{\Psi}(\vec{r}_2, \vec{r}_1)$$

Quantum Hall bars



Two Hall bars

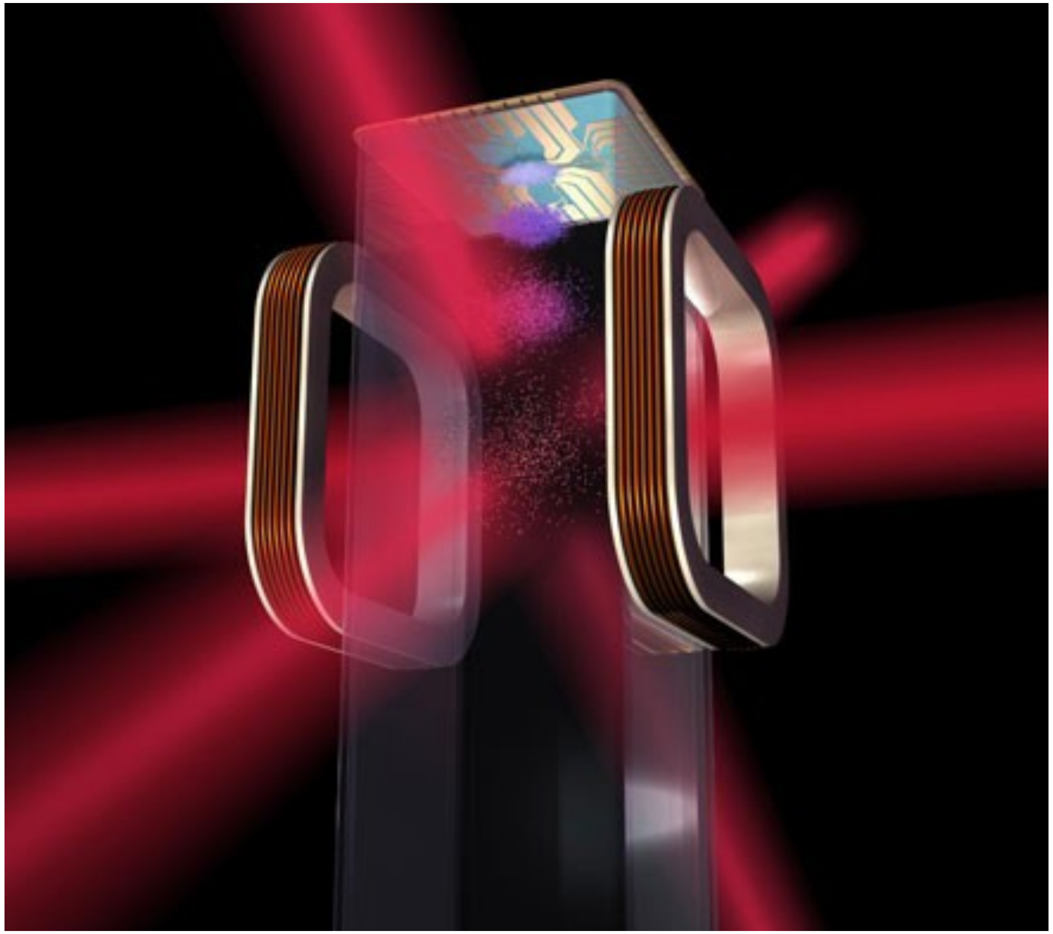
- 2D electron gas with perpendicular magnetic field
- should host anyons
- AlGaAs – GaAs heterostructures
- 25-100 mK
- 10 Tesla

So far no unambiguous proof that these systems host anyons

(but a lot of great physics!)

Willett, Pfeiffer, West, PNAS, 106, 8853 (2009)

Ultra-cold atoms



Anyons should exist in every many-body quantum system:

Can we look for them in cold atomic gases?

- Bose-Einstein condensates
- Degenerate Fermi gases
- 10-100 nK
- Isolated (~ no environment)
- Highly-idealized conditions

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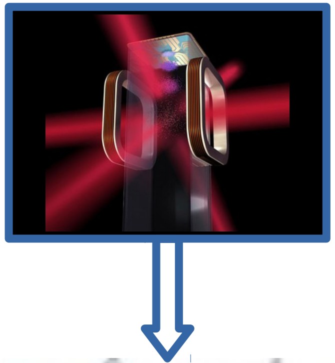
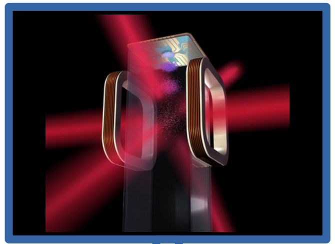
A theorist view...



A lot of knobs for doing a lot of things!

A theorist view...



- Tune confining potential (harmonic, box,...)
- Add a lattice
- Tune strength and range of atom-atom interaction
- Add disorder

Measurements

- Density profile
- Momentum distribution

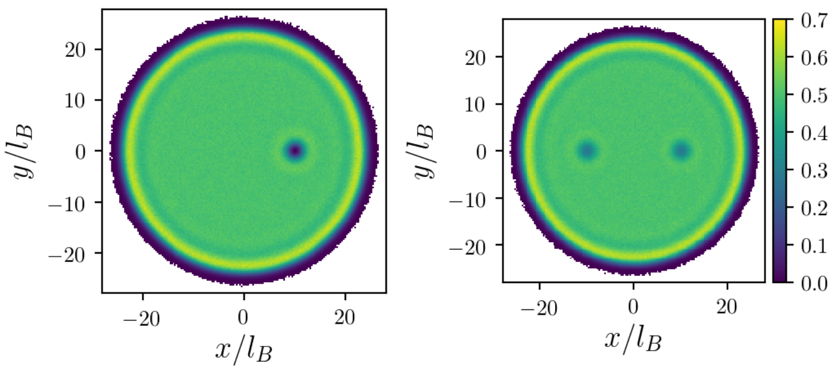
A lot of knobs for doing a lot of things!

This talk

Let us assume that it is possible to realize cold atomic gases supporting anyons:

- how can we prove that they are there?

This talk: anyonic statistics can be revealed by the density profile of a system hosting anyons



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Outline

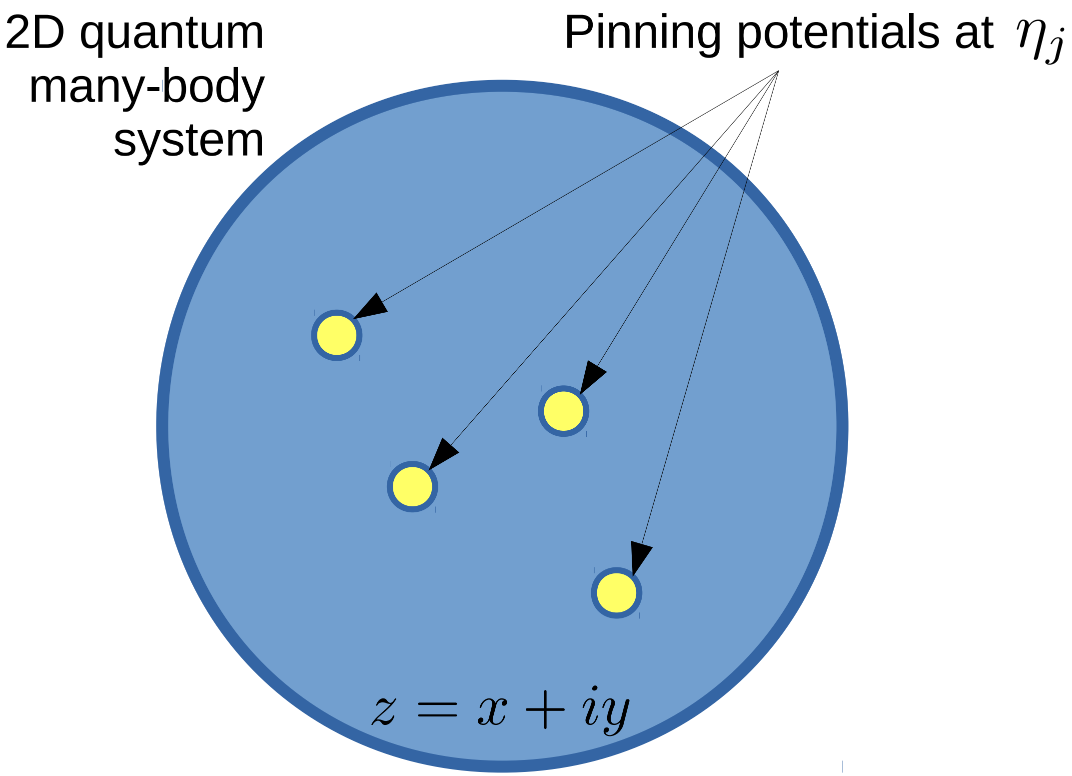
- Introduction: Anyons

- Part #1: Detecting anyons with local measurements

- Part #2: The case of non-Abelian anyons

- Conclusions

Anyons in quantum many-body systems



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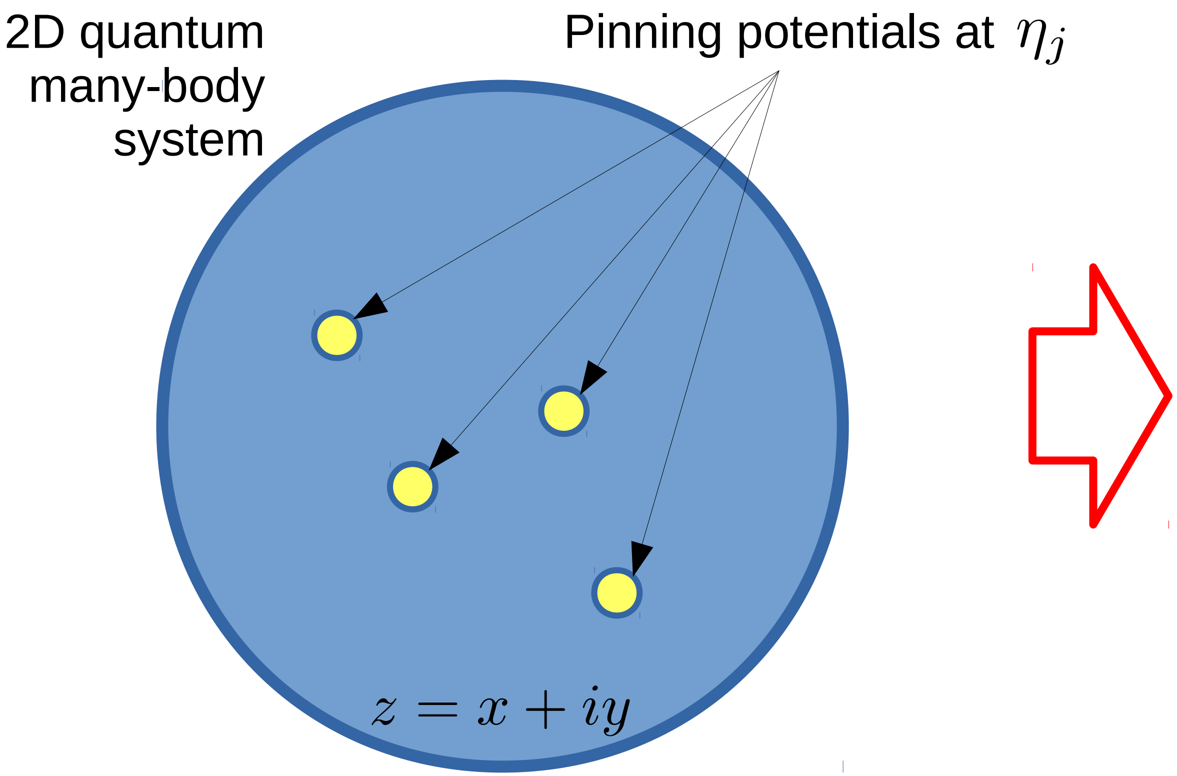

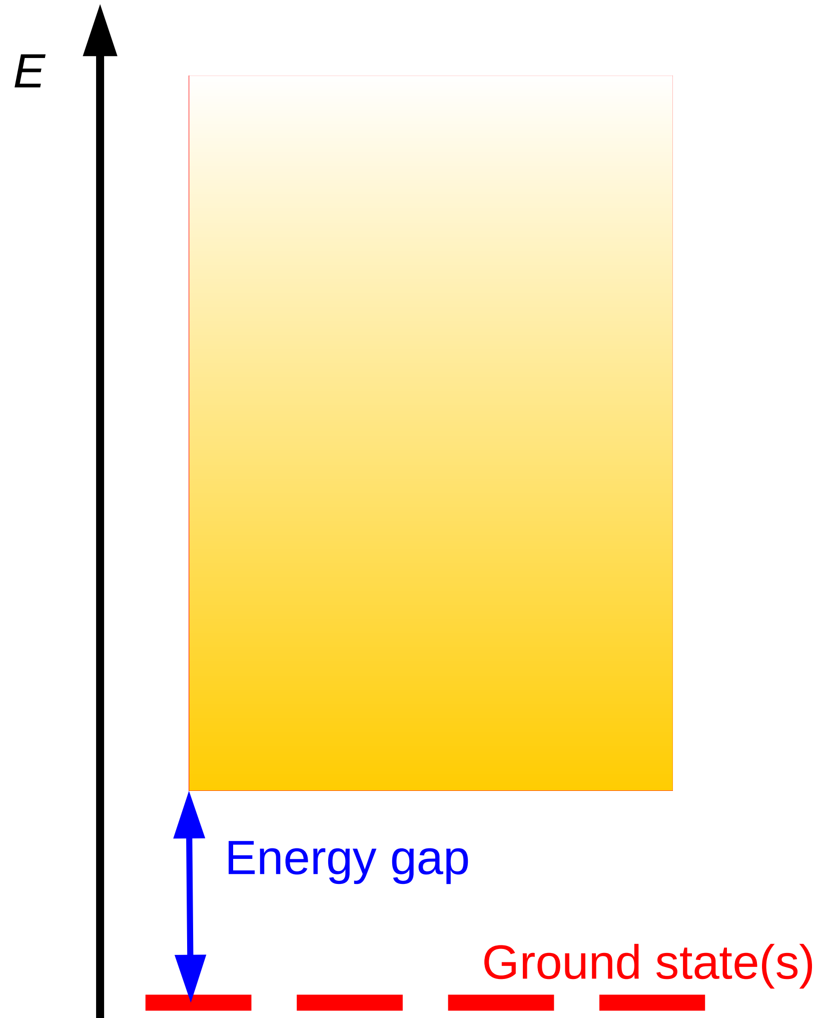

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$$\psi_\alpha(\{z_i\}, \{\eta_j\})$$

Old things, nicely reviewed here:
Bonderson, Gurarie and Nayak, PRB **83**, 075303 (2011)

Exotic quantum statistics

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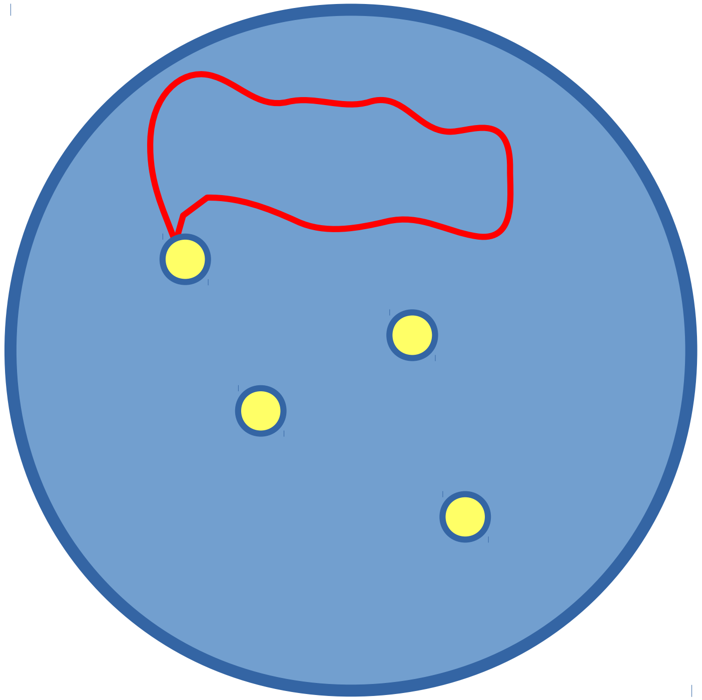

Quantum statistics for identical particles
from the notion of adiabatic exchange of particles

Adiabaticity:

- ensured by the many-body energy gap

$$T \gg h/E_{\text{gap}}$$

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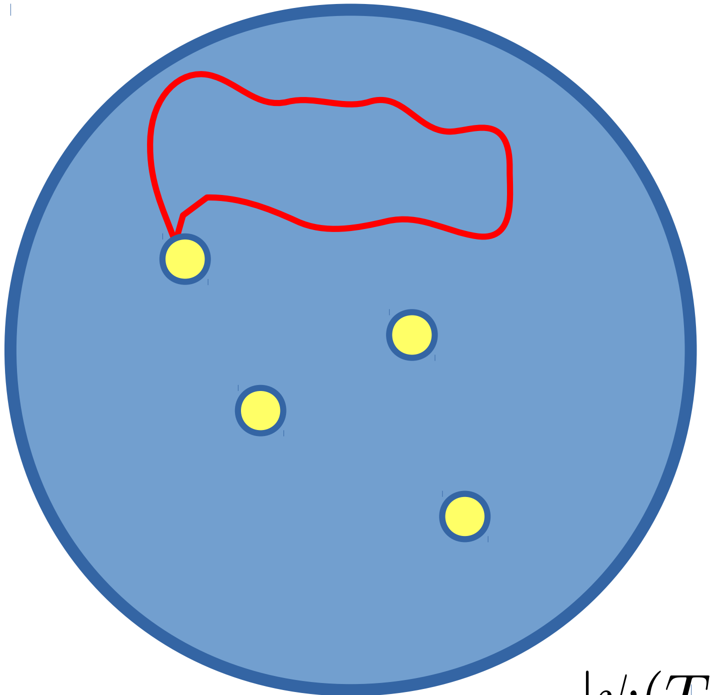

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- geometric contribution to the time evolution (Berry phase)

$|\mathrm{GS}(\eta_j)\rangle$ *Ground state for each pinning configuration*

$$|\psi(T)\rangle = \underbrace{e^{-\frac{i}{\hbar}E_{GS}T}}_{\text{Dynamical phase}} \underbrace{\exp\left[i\int_0^T \mathcal{A}(t')\mathrm{d}t'\right]}_{\text{Geometric contribution}} |\mathrm{GS}(\eta_j(T))\rangle$$

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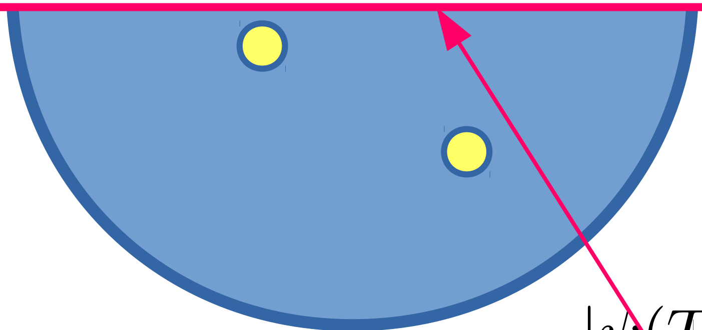
Geometric contribution: Berry connection

$$\mathcal{A}(t) = i\langle\ \mathrm{GS}(\eta_j(t))\ |\frac{\mathrm{d}}{\mathrm{d}t}|\ \mathrm{GS}(\eta_j(t))\ \rangle$$



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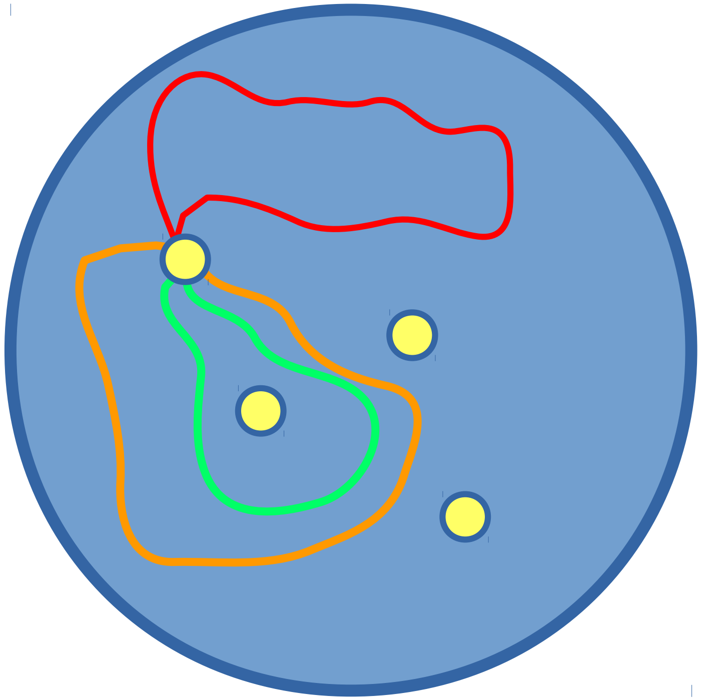

Quantum statistics for identical particles
from the notion of adiabatic exchange of particles

The geometric contribution:

- **Non-topological part**
  - Different in the three cases
- **Topological part**
  - Absent for the **red** path
  - Present and equal for the **orange** and **green** paths

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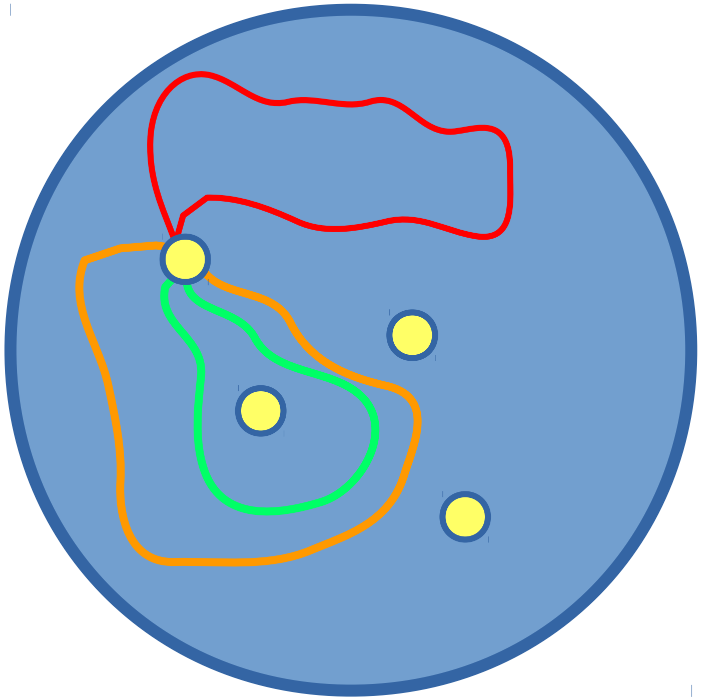

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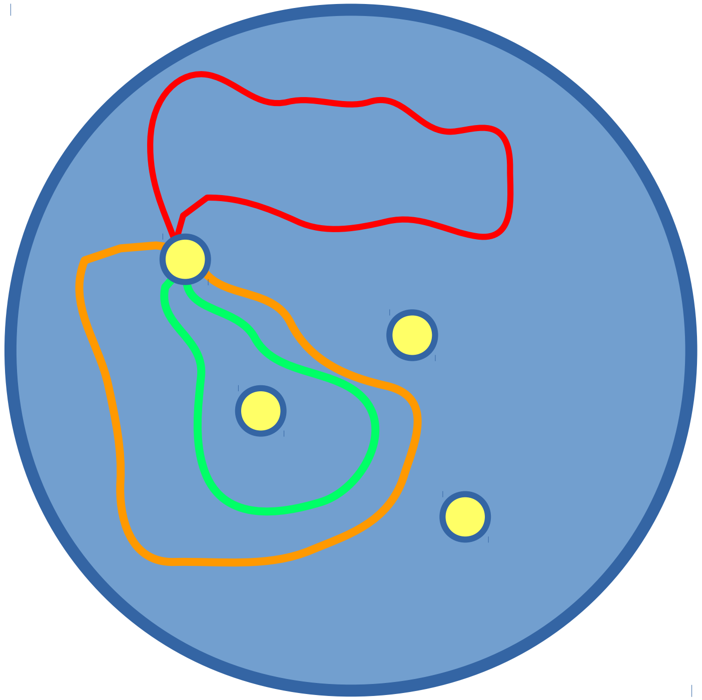

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This encodes the quantum statistics
(and works only in 2D)

- Bosons, fermions and Abelian anyons
  - ➔ The ground state is non degenerate
  - ➔ Topological contribution: $e^{i\varphi}$
- Non-Abelian anyons
  - ➔ The ground state is degenerate
  - ➔ Topological contribution: $\mathcal{U}$

Anyons in the Laughlin wavefunction

The Laughlin wavefunction (M=3 for fermions)**:**

$$\Psi_{2qh}(\{z_j\},\eta_1,\eta_2) \sim \prod_j (z_j-\eta_1)(z_j-\eta_2)\prod_{i<j}(z_i-z_j)^M e^{-\sum_j |z_j|^2/(4\ell_B^2)}$$

Explicit analytical computation of the topological contribution to the geometric term possible.

$$e^{i\varphi} = e^{i2\pi/M}$$

- Analytical method
- Numerical method

Geometric contribution: Berry connection

$$\mathcal{A}(t) = i\langle\, \mathrm{GS}(\eta_j(t))\,|\frac{\mathrm{d}}{\mathrm{d}t}|\,\mathrm{GS}(\eta_j(t))\,\rangle$$

David Tong, The Quantum Hall Effect, Lecture notes.

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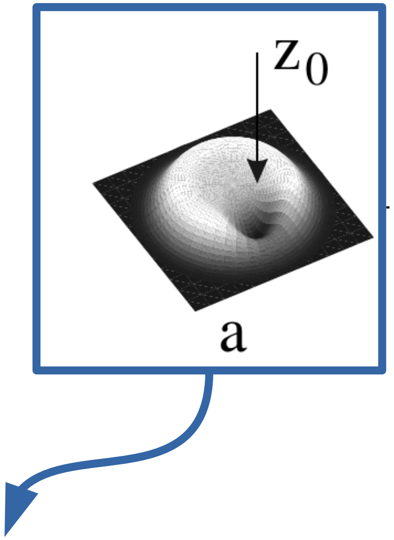
Experimental approach

VOLUME 87, NUMBER 1 PHYSICAL REVIEW LETTERS 2 JULY 2001

$\frac{1}{2}$-Anyons in Small Atomic Bose-Einstein Condensates

B. Paredes,* P. Fedichev, J. I. Cirac, and P. Zoller
Institute for Theoretical Physics, University of Innsbruck, Innsbruck, Austria
(Received 1 March 2001; published 15 June 2001)

Assumption:
we prepare a bosonic
Laughlin state



Ψ_{1qh}

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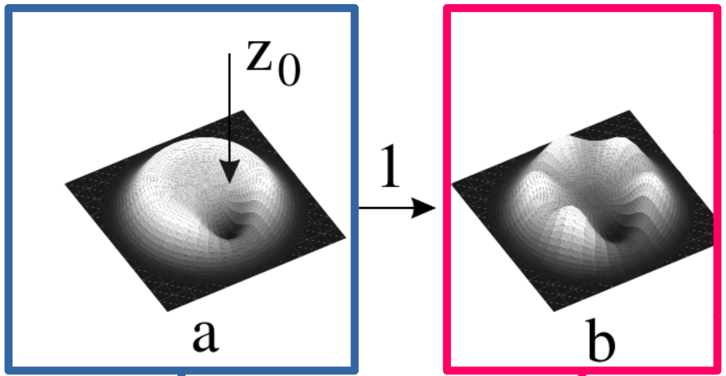

$$\Psi_{1qh}$$

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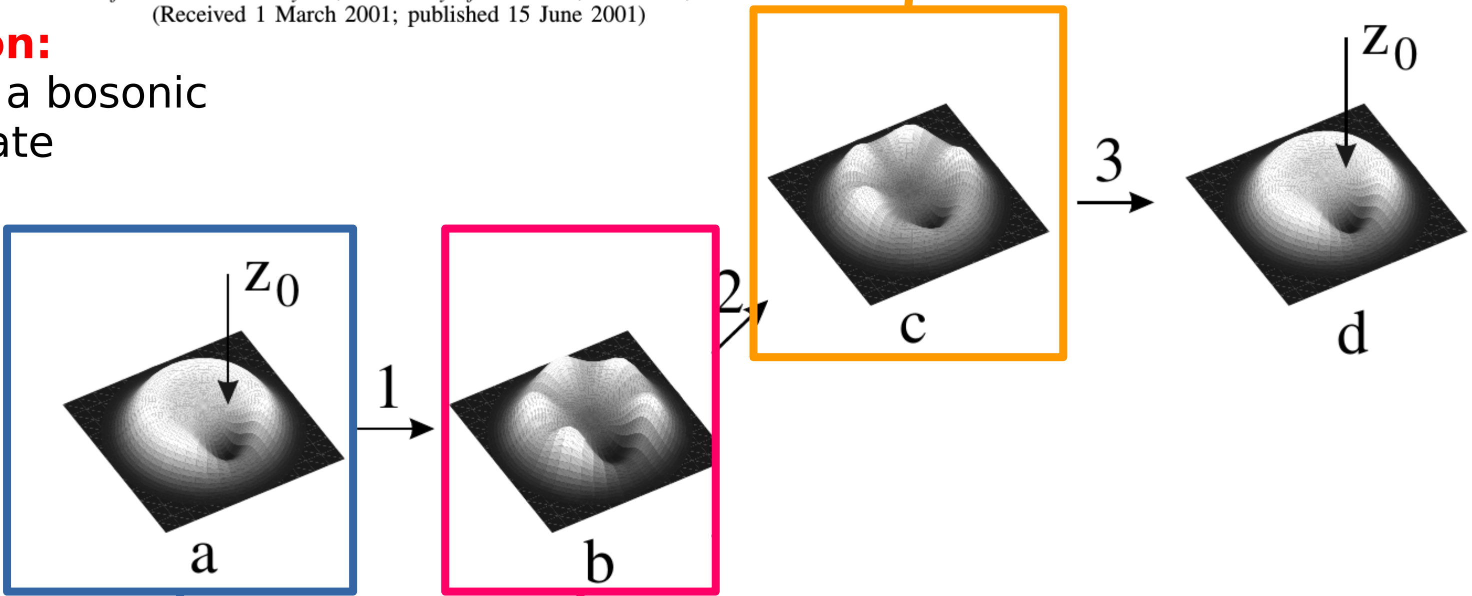
1 adiabatic rotation

$$\Psi_{1qh} - \Psi_{2qh}$$



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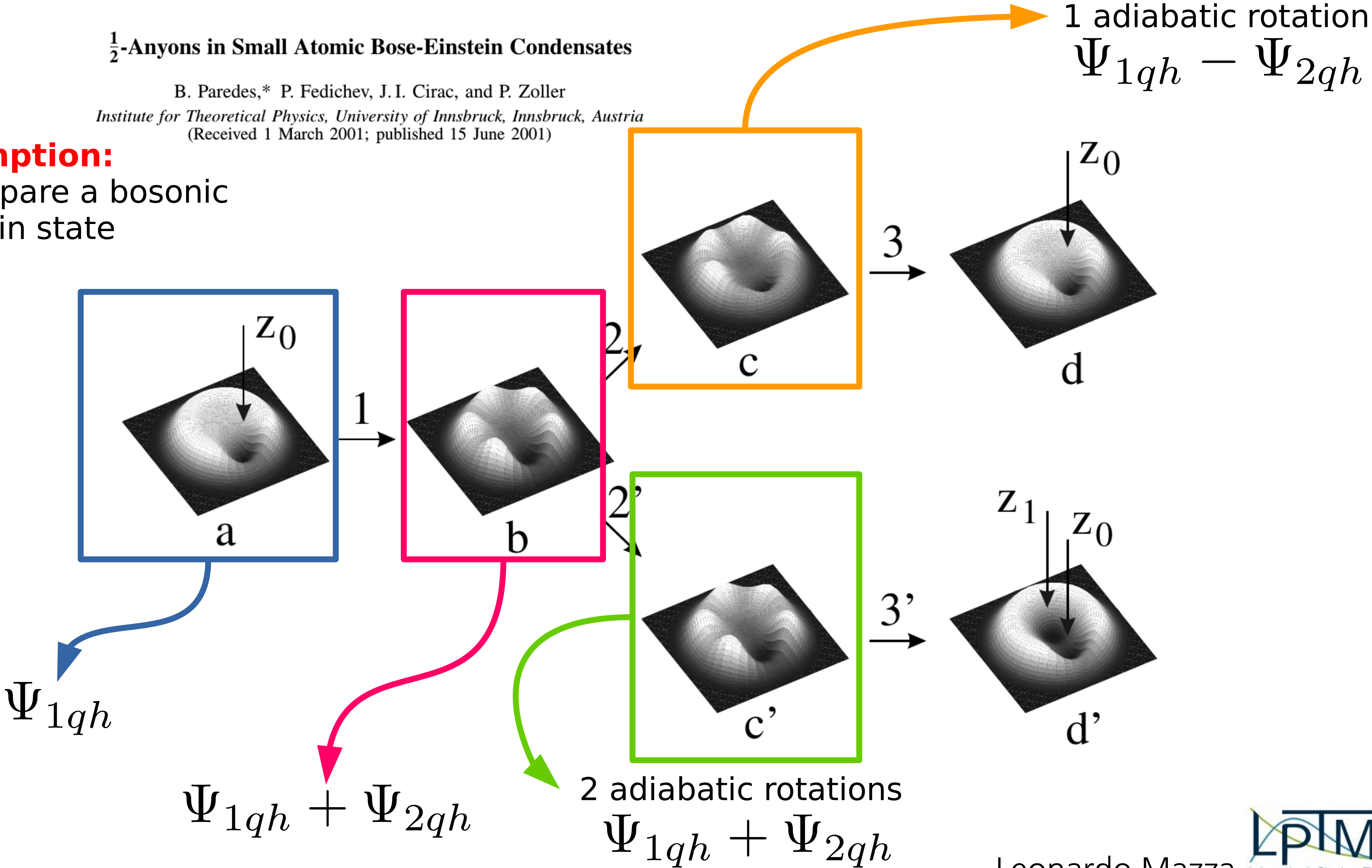

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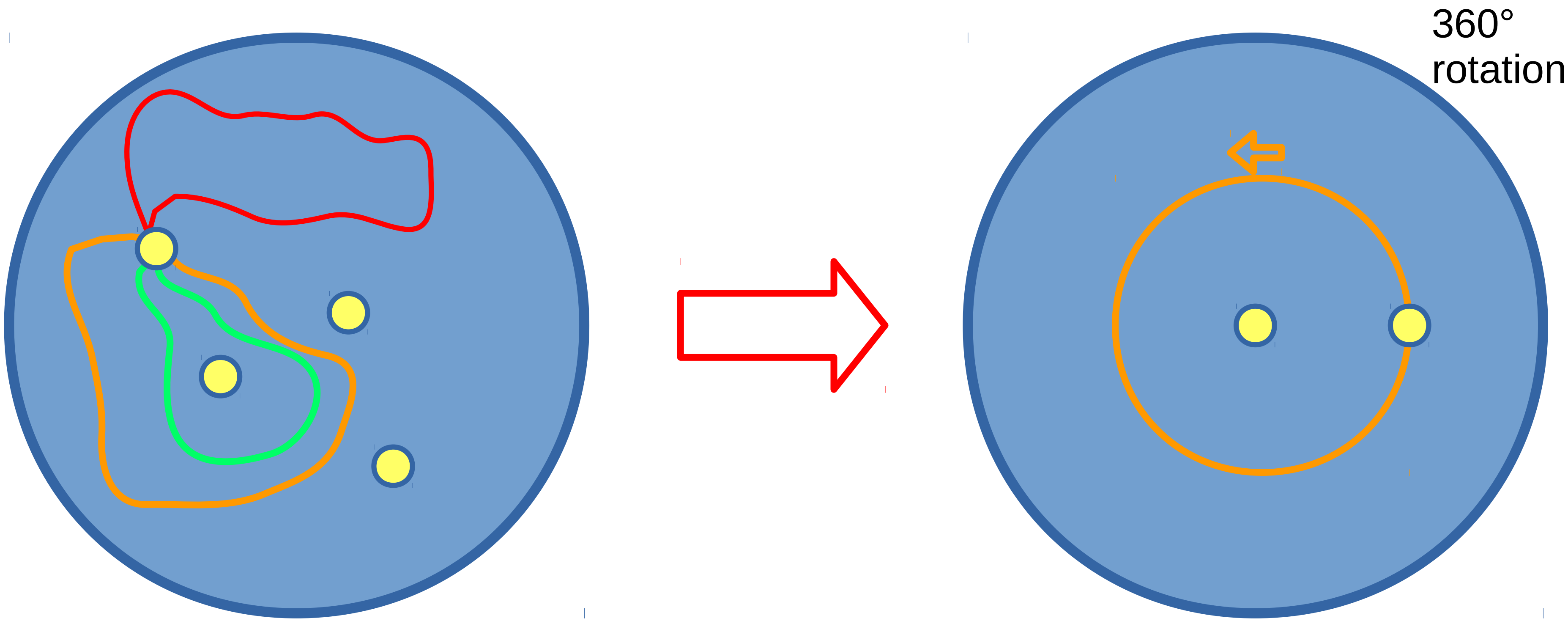
Highly-symmetric paths

Use a simple and symmetric configuration to compute the statistical properties of anyons



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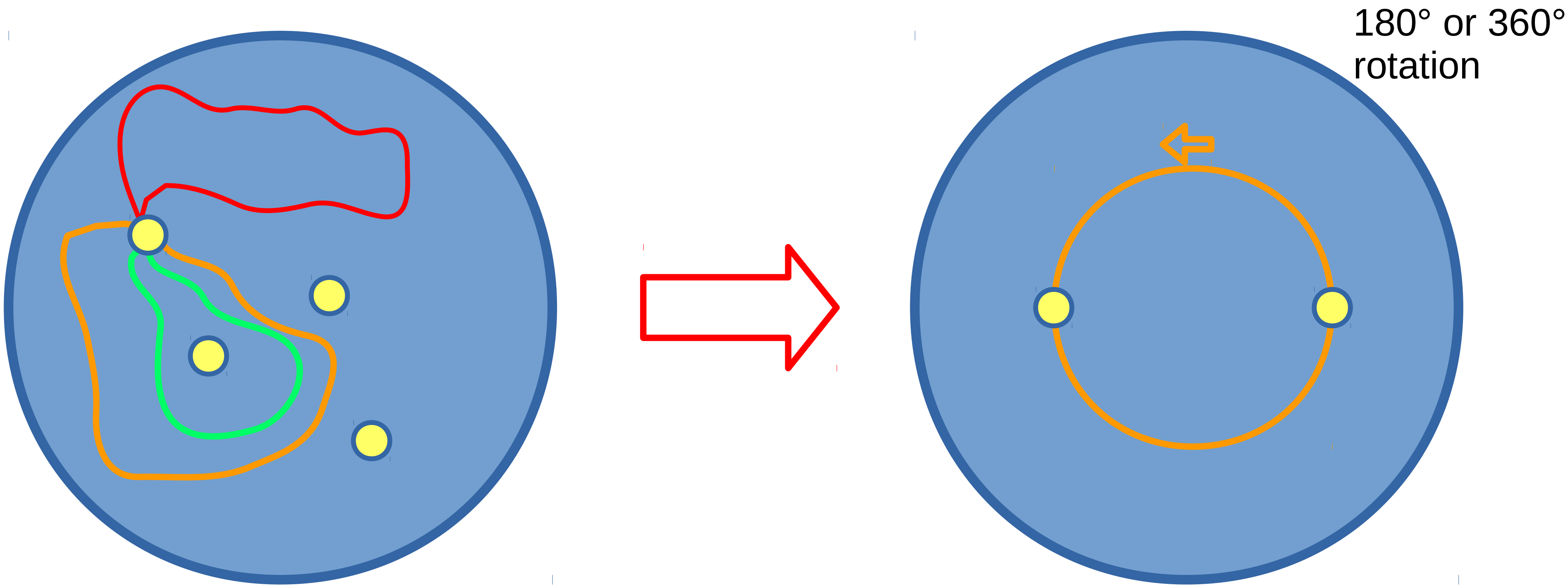
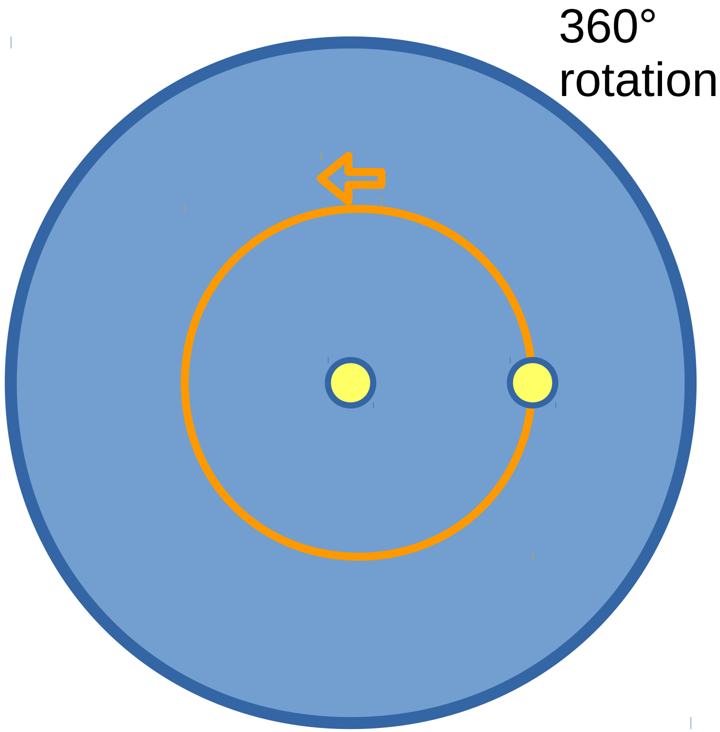
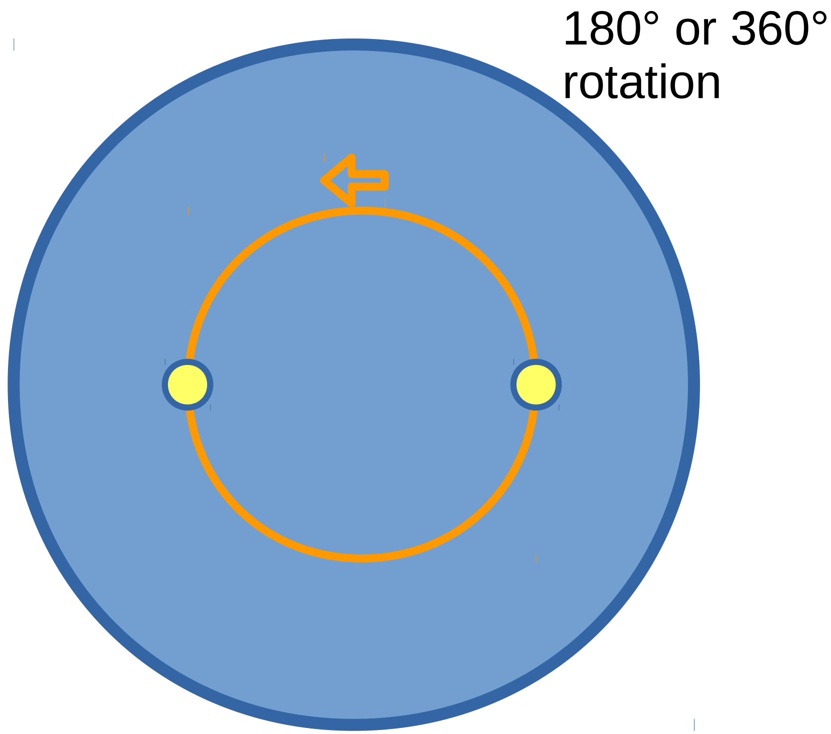


Rigid anyonic rotations

Use a simple and symmetric configuration to compute the statistical properties of anyons



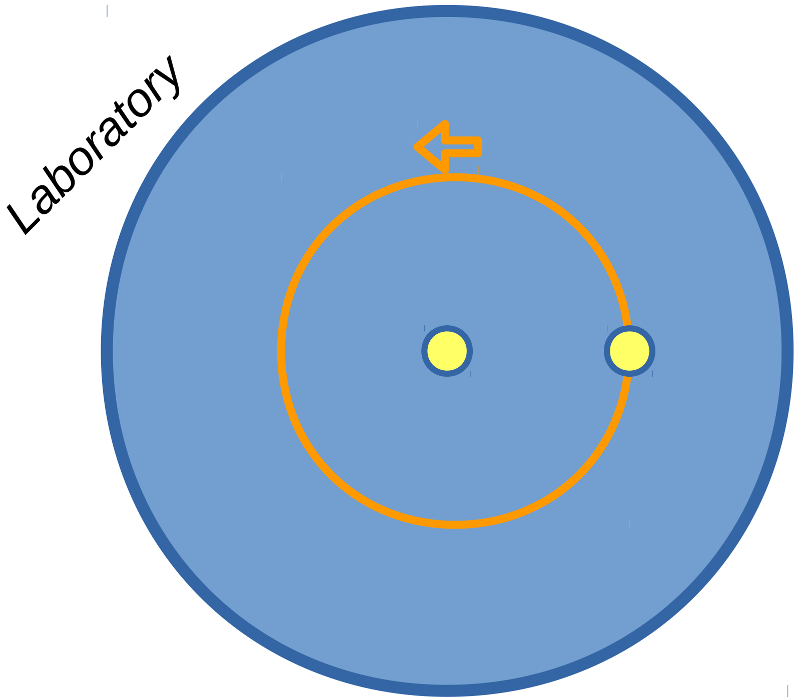


$$\eta_j(t) = \eta_j(0) e^{i \frac{\theta_f}{T} t} \quad \forall j$$

Co-rotating reference frame

Rigid rotations imply the existence of a reference frame where anyons are at rest

Assumption: rotational invariance of $\hat{H}_0$



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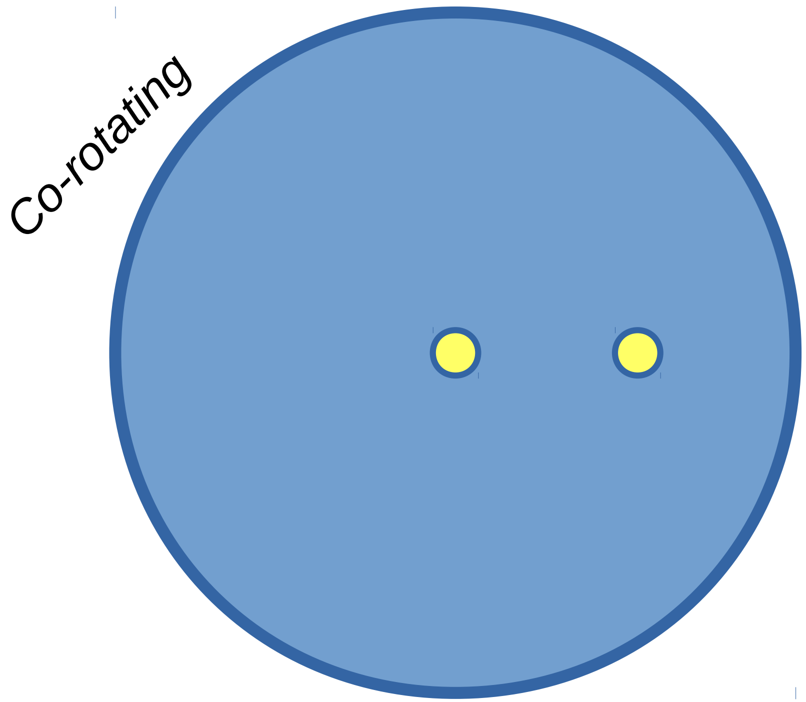
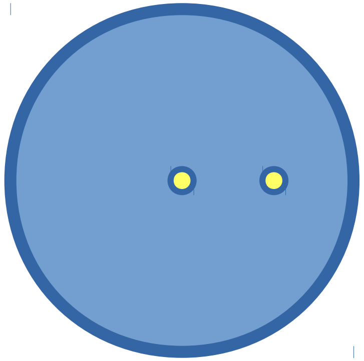


$$\hat{H} = \hat{H}_0 + \hat{H}_{\text{pinning}}(\eta_j(0)) - \frac{\theta_f}{T}\hat{L}_z$$

Co-moving Schroedinger equation

$$\hat{H} = \underbrace{\hat{H}_0 + \hat{H}_{\text{pinning}}(\eta_j(0))}_{\text{Gapped many-body Hamiltonian}} \underbrace{- \frac{\theta_f}{T}\hat{L}_z}_{\substack{\text{Small perturbation} \\ t \in [0,T]}}$$



$|GS\rangle$

Key: use of time-dependent perturbation theory

Result: in the infinite T limit, the system does not leave the ground space because of the perturbation (adiabatic theorem)

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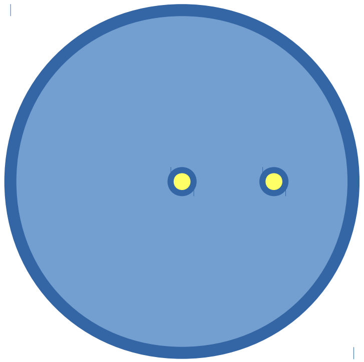

$|GS\rangle$

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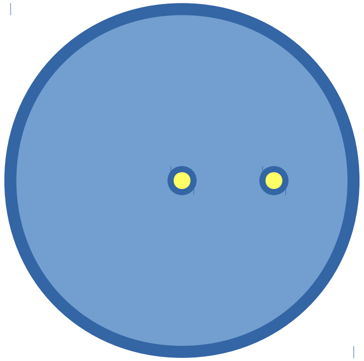
Result: in the infinite T limit, the system does not leave the ground space because of the perturbation (adiabatic theorem)

$$|\psi_2(t)\rangle = e^{-\frac{i}{\hbar}E_0 t}\gamma(t)|GS\rangle$$

$$i\hbar\frac{\mathrm{d}}{\mathrm{d}t}\gamma(t) = -\frac{\theta_f}{T}\langle GS|\hat{L}_z|GS\rangle\gamma(t)$$

Co-moving Schroedinger equation

$$\hat{H} = \underbrace{\hat{H}_0 + \hat{H}_{\mathrm{pinning}}(\eta_j(0))}_{\text{Gapped many-body Hamiltonian}} \underbrace{- \frac{\theta_f}{T}\hat{L}_z}_{\substack{\text{Small perturbation} \\ t \in [0,T]}}$$



$|GS\rangle$

Key: use of time-dependent perturbation theory

Result: in the infinite T limit, the system does not leave the ground space because of the perturbation (adiabatic theorem)

$$|\psi_2(t)\rangle = e^{-\frac{i}{\hbar}E_0 t}\gamma(t)|GS\rangle$$

$$i\hbar\frac{\mathrm{d}}{\mathrm{d}t}\gamma(t) = -\frac{\theta_f}{T}\langle GS|\hat{L}_z|GS\rangle\gamma(t)$$

Non-degenerate

$$|\psi_2(T)\rangle = e^{-\frac{i}{\hbar}E_0 T} e^{\frac{i}{\hbar}\theta_f \langle \hat{L}_z \rangle}|GS\rangle$$

In the lab reference frame

$$|\psi(T)\rangle = e^{-\frac{i}{\hbar}\hat{L}_z\theta_f}|\psi_2(T)\rangle$$

Non-degenerate

$$|\psi(T)\rangle = e^{-\frac{i}{\hbar}E_0 T}\; e^{-\frac{i}{\hbar}\hat{L}_z\theta_f}\; e^{\frac{i}{\hbar}\theta_f\langle\hat{L}_z\rangle}\;|GS\rangle$$

Experimental consideration:

$$\theta_f = 2\pi \quad \Rightarrow \quad e^{-\frac{i}{\hbar}\hat{L}_z 2\pi} = 1$$

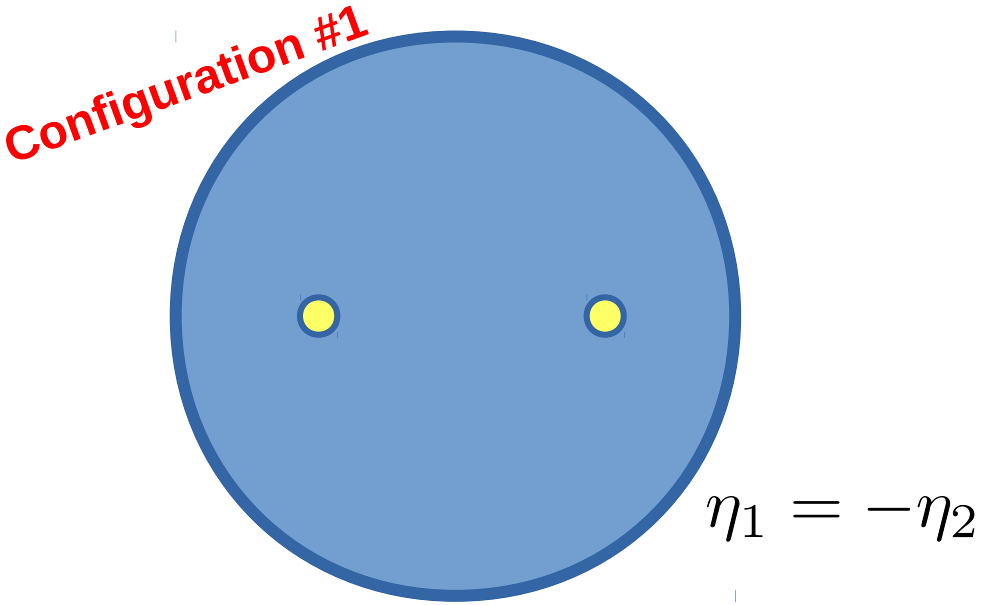
In this case the geometric information is all contained in the expectation of the angular momentum over the ground space: MEASURABLE

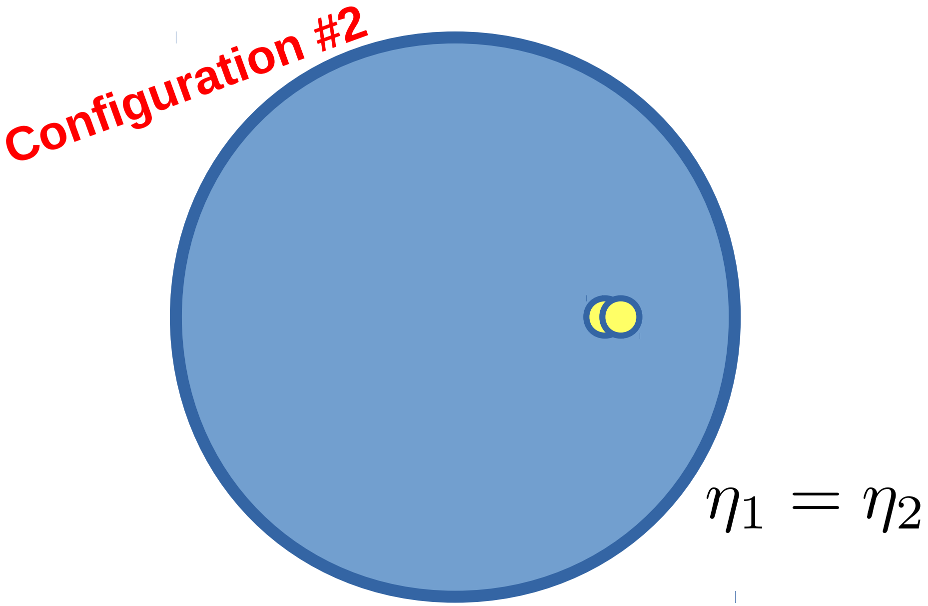
Theoretical consideration:

If you have a good method to handle your many-body wavefunction, you can compute the geometric contribution without time-evolution

Tserkovnyak and Simon, PRL **90** 016802 (2003)
Wu, Estienne, Regnault and Bernevig, PRL **113** 116801 (2014)

Topological contribution





360° rotation in the configuration #1
- Topological contribution
- Non-topological contribution

360° rotation in the configuration #2
- Non-topological contribution

$$\boxed{\varphi_{br} = \frac{2\pi}{\hbar}\left[\langle \hat{L}_z \rangle_{\eta_1=-\eta_2} - \langle \hat{L}_z \rangle_{\eta_1=\eta_2}\right]}$$

(Non-degenerate case)

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$$\varphi_{br} = \frac{2\pi}{\hbar} \left[\langle \hat{L}_z \rangle_{\eta_1 = -\eta_2} - \langle \hat{L}_z \rangle_{\eta_1 = \eta_2} \right]$$

Can we use this relation to unambiguously reveal non-Abelian anyons?

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Can we use this relation to unambiguously reveal non-Abelian anyons?

In the lowest Landau level:

$$\frac{1}{\hbar}\langle \hat{L}_z\rangle = \frac{N}{2\ell_B^2}\langle r^2\rangle - N$$

Results on the Abelian anyons of the Laughlin state:
Umucalilar, Macaluso, Comparin and Carusotto, PRL **120** 230403 (2018)

Outline

- Introduction: Anyons
- Part #1: Detecting anyons with local measurements
- Part #2: The case of non-Abelian anyons
- Conclusions

Moore-Read wavefunction

$$\Psi_{2qh}(\{z_j\}, \eta_1, \eta_2) = \mathrm{Pf}\left(\frac{(\eta_1 - z_i)(\eta_2 - z_j) + i \leftrightarrow j}{z_i - z_j}\right) \prod_{i<j} (z_i - z_j)^M e^{-\sum_j |z_j|^2/(4\ell_B^2)}$$

- Lowest Landau level wavefunction
- Defined for bosons (M=1) or fermions (M=2)
- Filling factor ν = 1/M
- Expected to explain the plateau at ν = 5/2

G. Moore and N. Read, Nuclear Physics B 360, 362 (1991)
Bonderson, Gurarie and Nayak, PRB **83**, 075303 (2011)

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- Lowest Landau level wavefunction
- Defined for bosons (M=1) or fermions (M=2)
- Filling factor ν = 1/M
- Expected to explain the plateau at ν = 5/2

- For two quasiholes, the wavefunction is not degenerate
- The quasiholes are not Abelian

$$\sigma \times \sigma = 1 + \psi$$

- The fusion channel depends on the parity of the number of particles

G. Moore and N. Read, Nuclear Physics B 360, 362 (1991)
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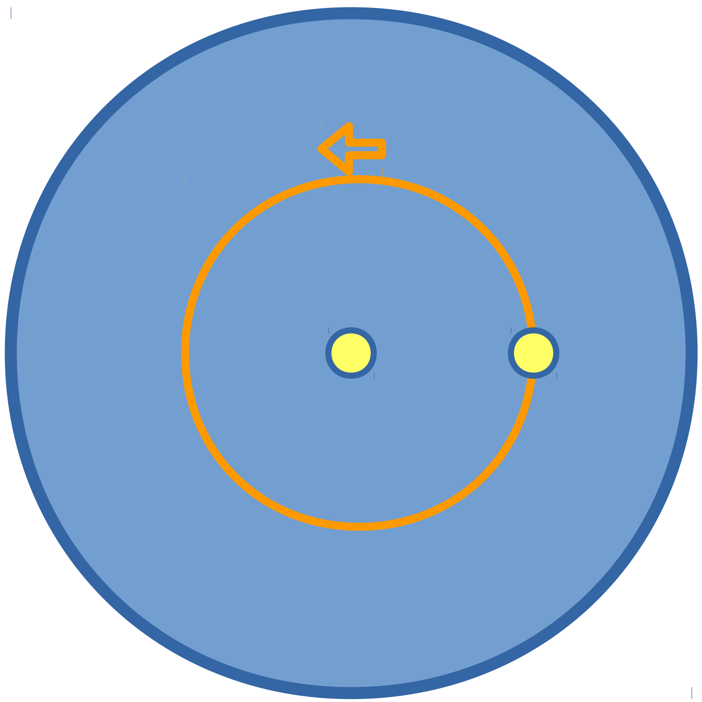
In practice...



When you have two non-Abelian anyons, the braiding phase is not uniquely determined.

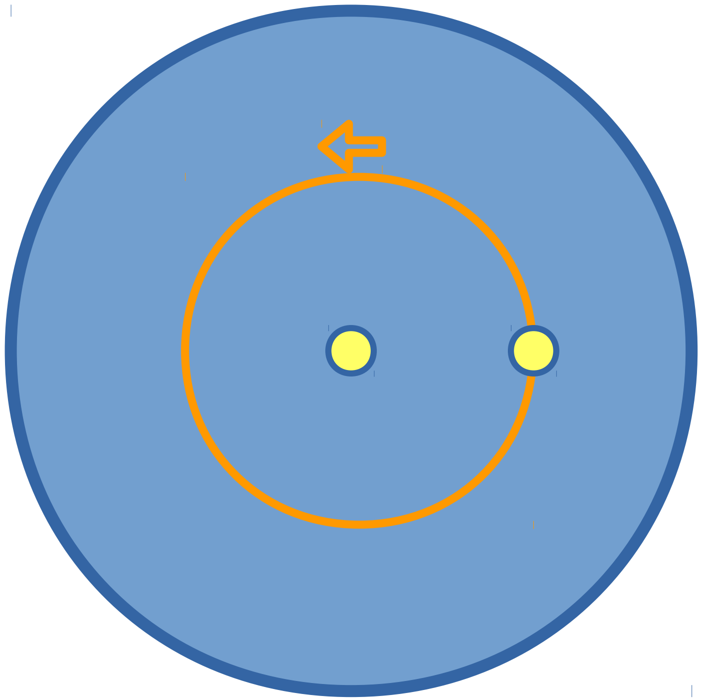
- It depends on the fusion channel

$$\sigma \times \sigma = \underset{\downarrow\,\varphi_{br,1}}{1} + \underset{\downarrow\,\varphi_{br,\psi}}{\psi}$$

- The fusion channel depends on the parity of the number of particles

$$\varphi_{br} = 2\pi \left[\frac{1}{4M} - \frac{1}{8} + \frac{P_N}{2} \right]$$

In practice...



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In practice...

- Measuring two different braiding phases is an unambiguous signature of non-Abelian statistics
- There is a research field called “topological quantum computation” that is based on non-Abelian anyons
- No unambiguous observation so far

When you have two non-Abelian anyons, the braiding phase is not uniquely determined.

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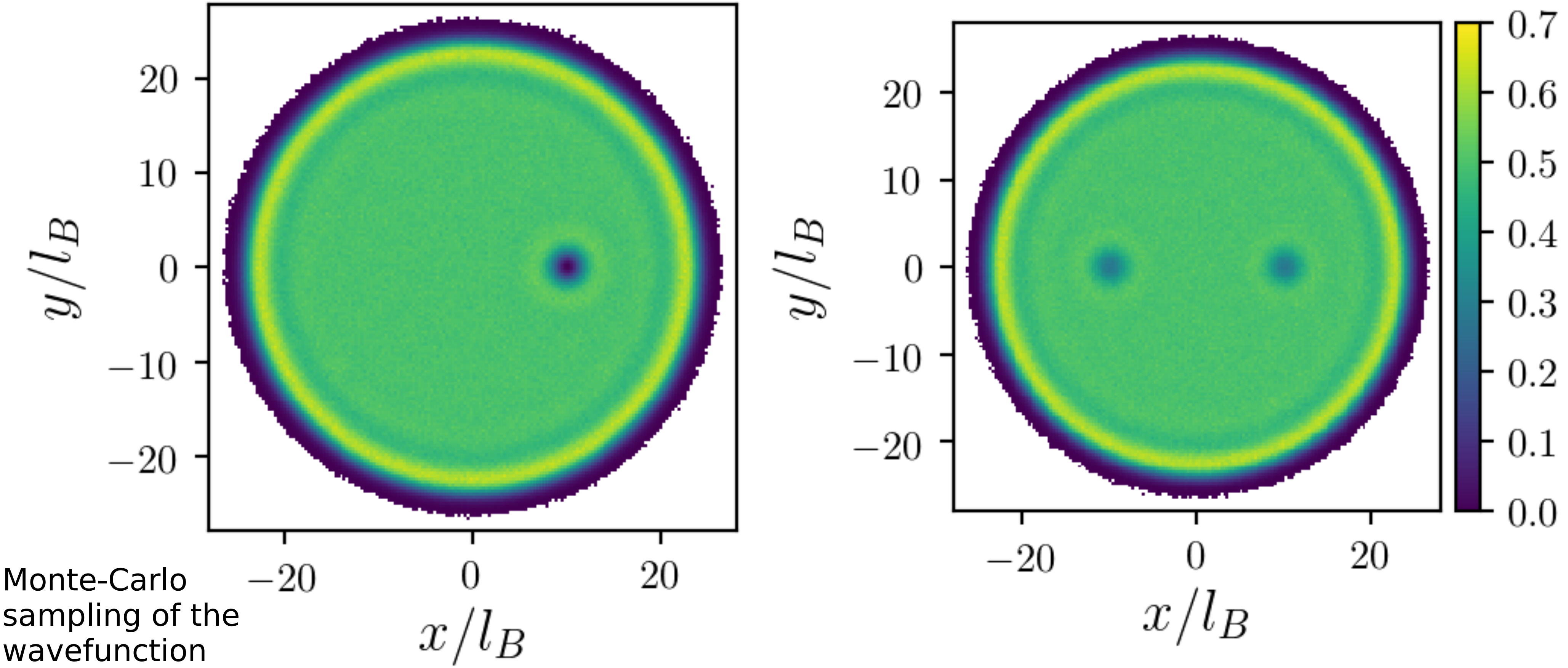
Braiding phase from density profile

In the lowest Landau level:

$$\frac{1}{\hbar}\langle \hat{L}_z \rangle = \frac{N}{2\ell_B^2}\langle r^2 \rangle - N$$

$$\varphi_{\mathrm{br}} = 2\pi \frac{N}{2\ell_B^2}\left[\langle r^2 \rangle_{\eta_1=-\eta_2} - \langle r^2 \rangle_{\eta_1=\eta_2}\right]$$



Monte-Carlo sampling of the wavefunction N=150; M=2

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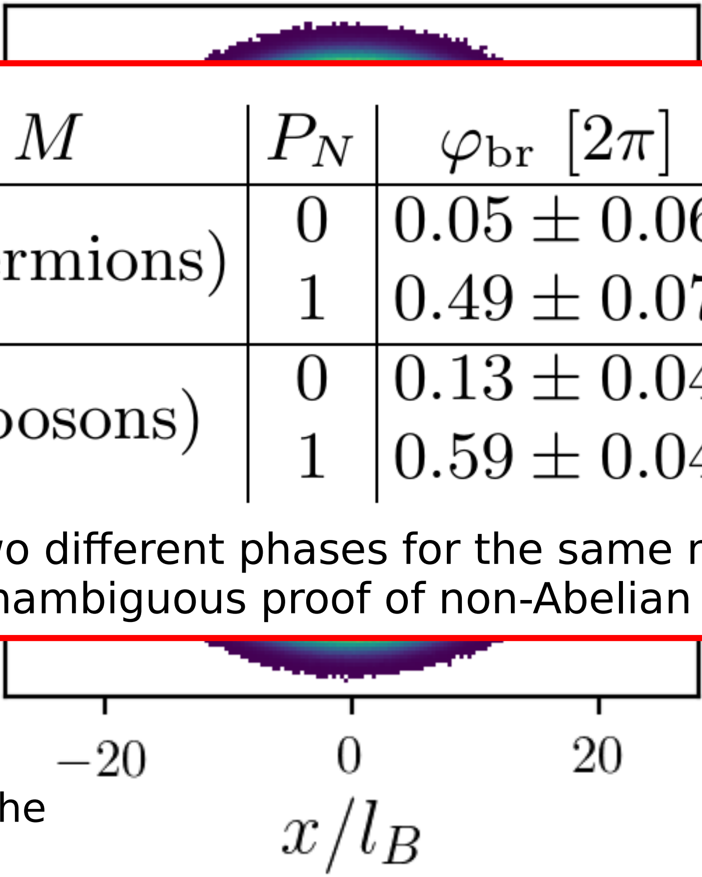
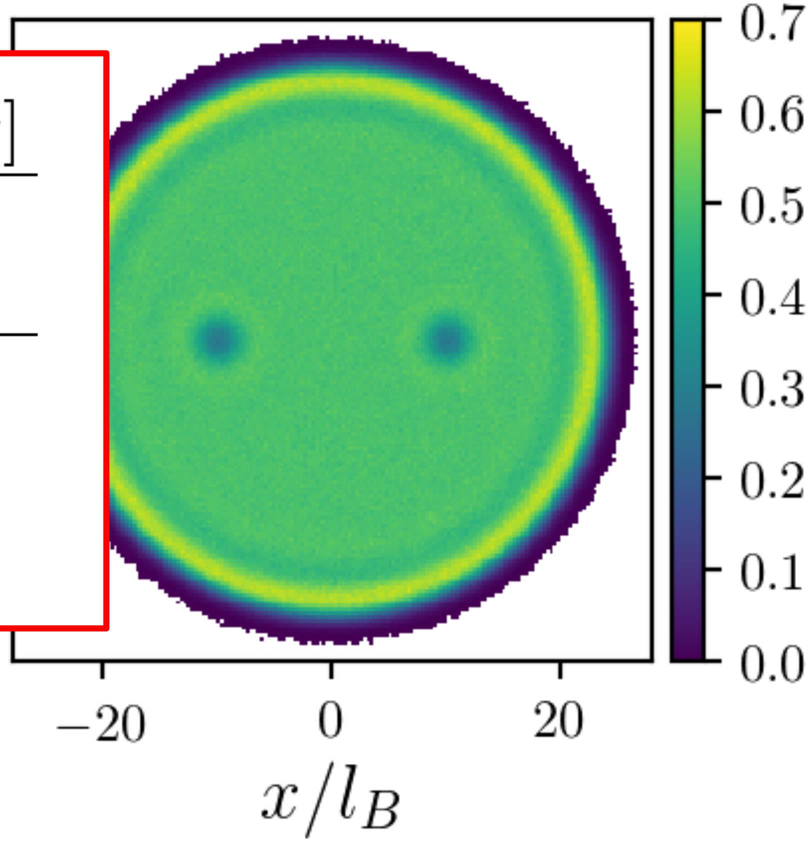


M	P_N	φ_{br} $[2\pi]$	φ_{br} $[2\pi]$
2 (fermions)	0	0.05 ± 0.06	0
	1	0.49 ± 0.07	0.5
1 (bosons)	0	0.13 ± 0.04	0.125
	1	0.59 ± 0.04	0.625

Two different phases for the same model are unambiguous proof of non-Abelian statistics



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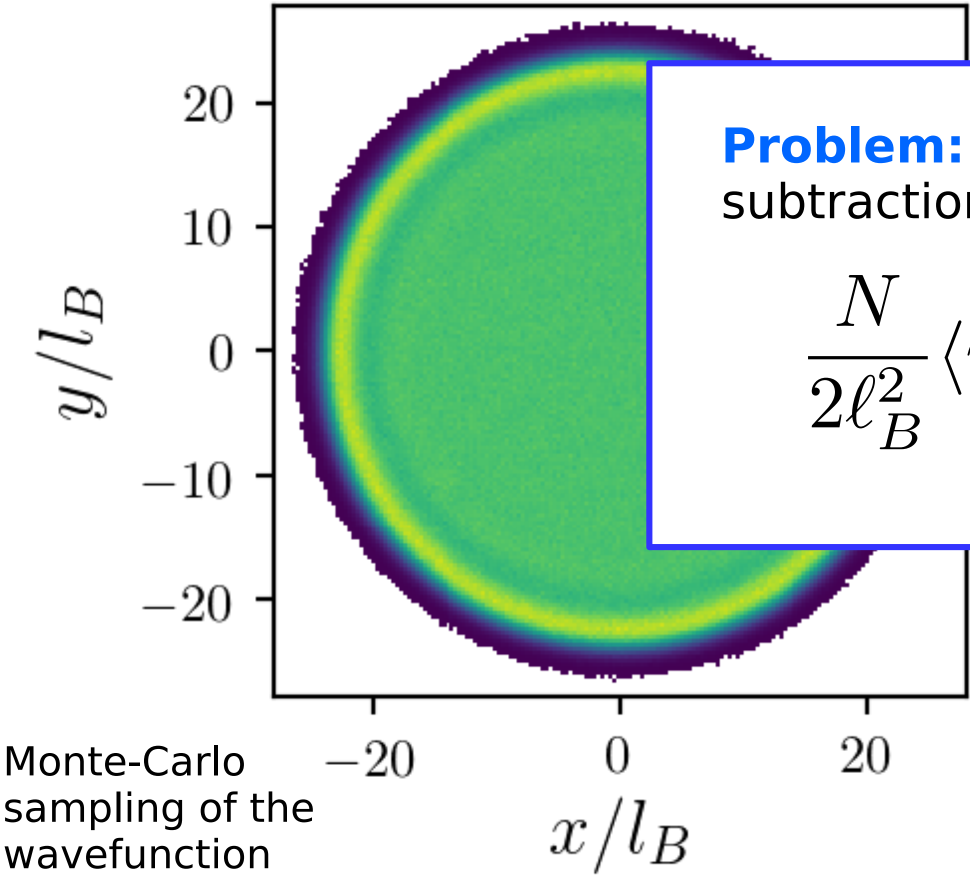

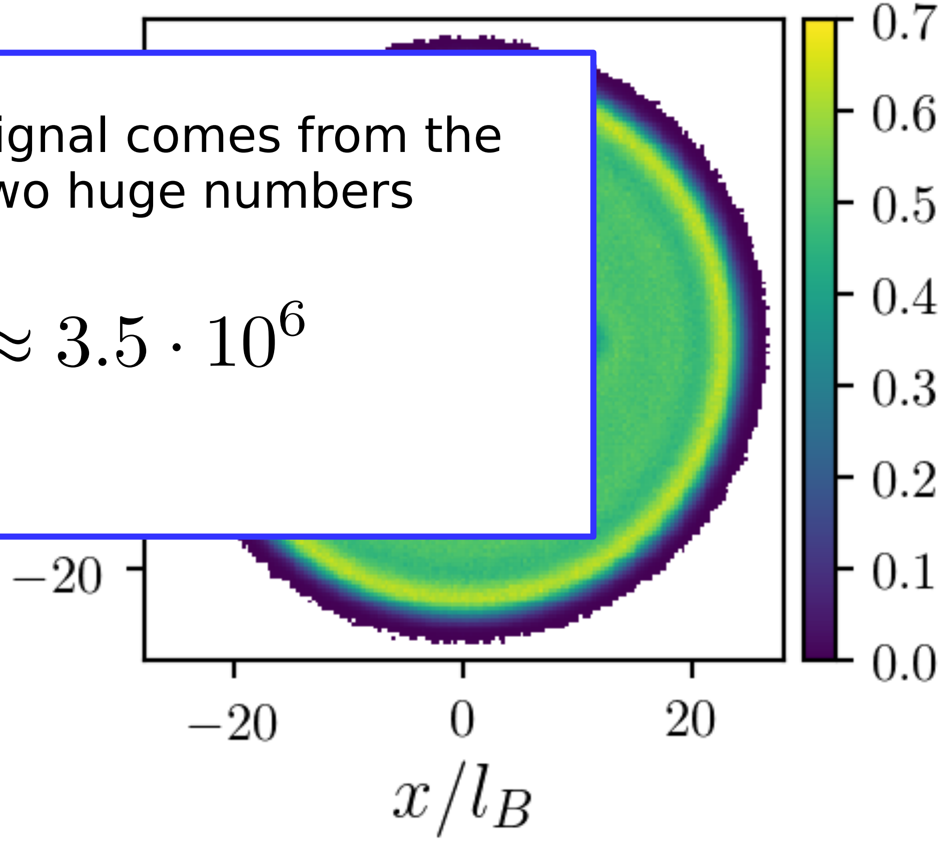

Problem: the signal comes from the subtraction of two huge numbers

$$\frac{N}{2\ell_B^2}\langle r^2 \rangle \approx 3.5 \cdot 10^6$$

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Braiding phase from density profile

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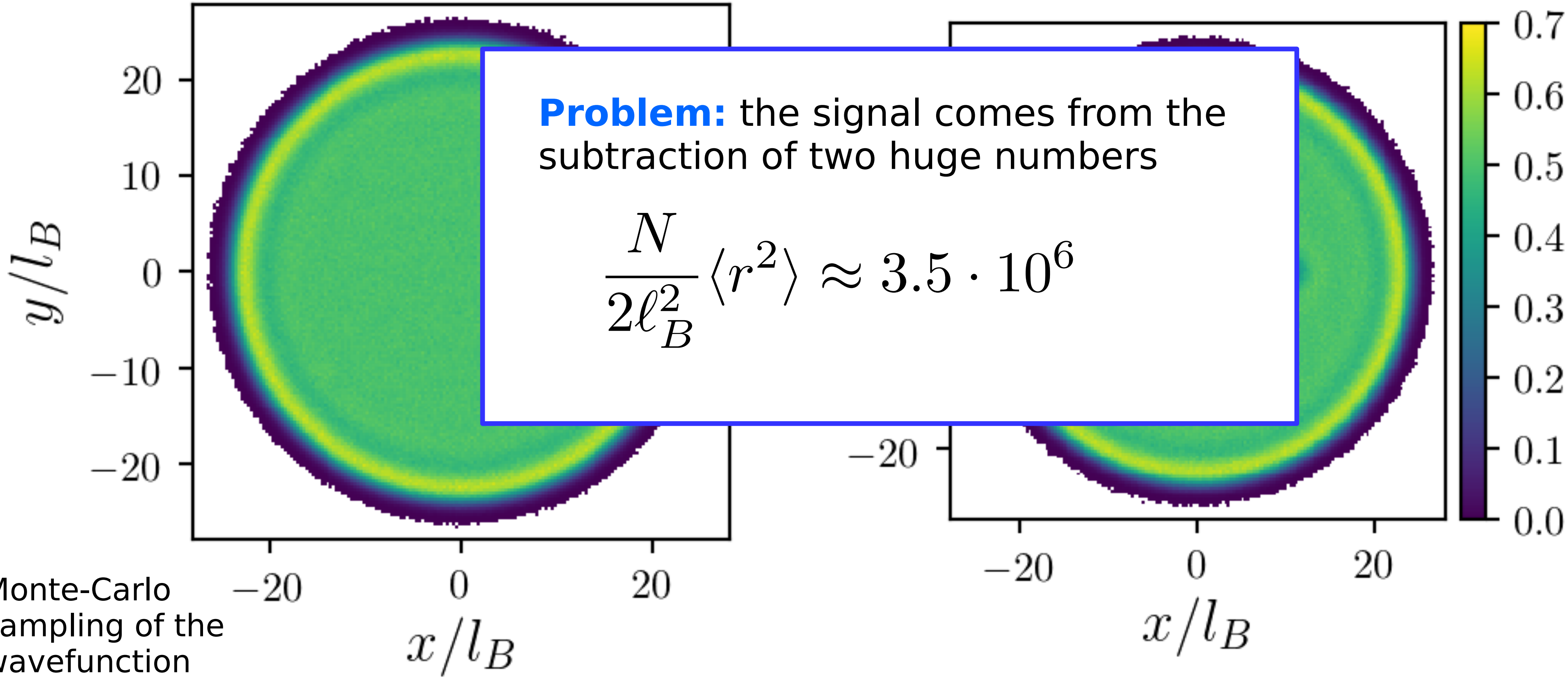
Where is this difference significant?

$$\varphi_{\mathrm{br}} = 2\pi \frac{N}{2\ell_B^2}\boxed{\left[\langle r^2 \rangle_{\eta_1=-\eta_2} - \langle r^2 \rangle_{\eta_1=\eta_2}\right]}$$



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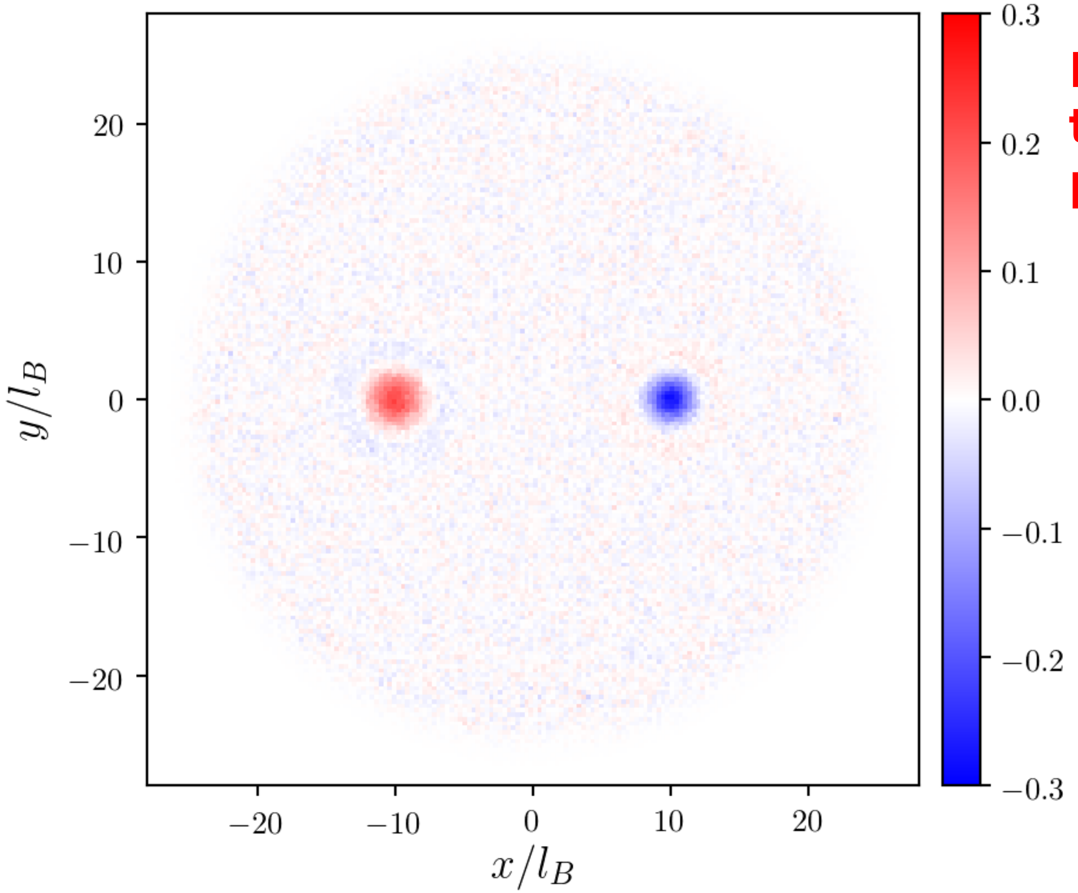

Difference of the two density profiles

This suggests that the braiding phase might be locally encoded around the quasiholes

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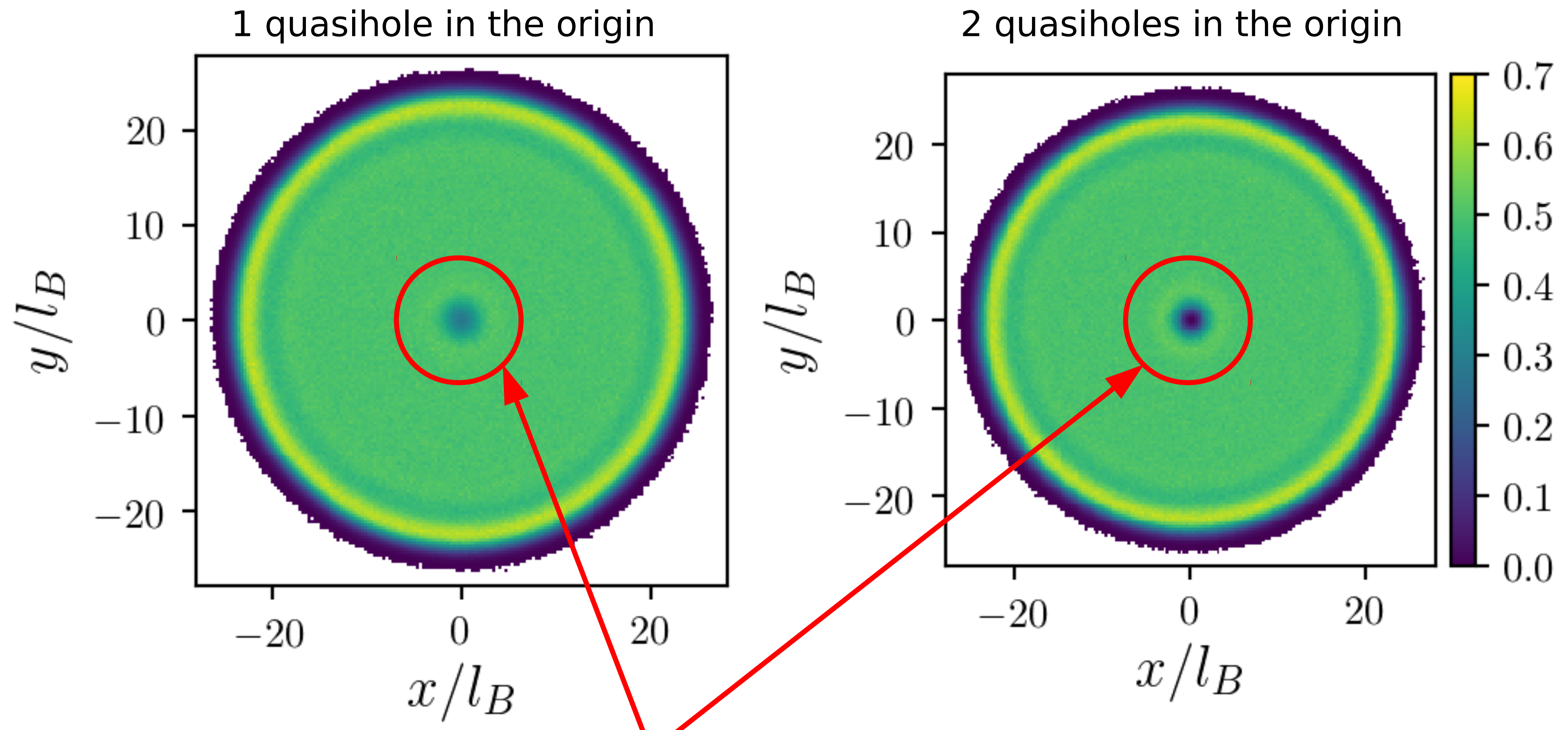
Braiding phase and depletion density

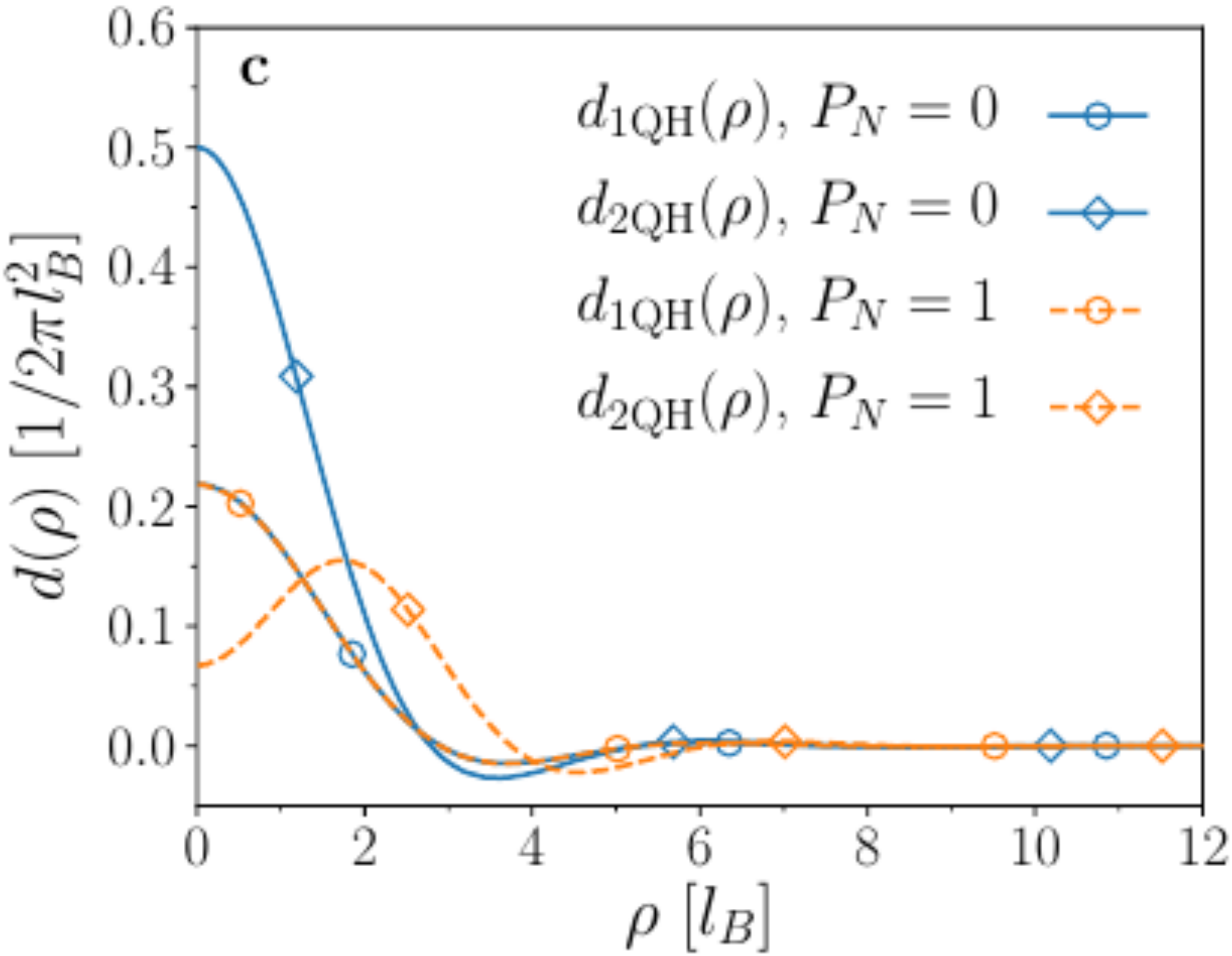
Depletion density: $d(r) = \bar{n} - n(r)$



$$\varphi_{\mathrm{br}} = 2\pi \frac{2\pi}{2\ell_B^2} \int_0^{\boxed{R}} r^2 \big(d_{2qh}(r) - 2 d_{1qh}(r) \big) r \mathrm{d}r$$

Results

Bosons, M=1



We obtained similar results for fermions!

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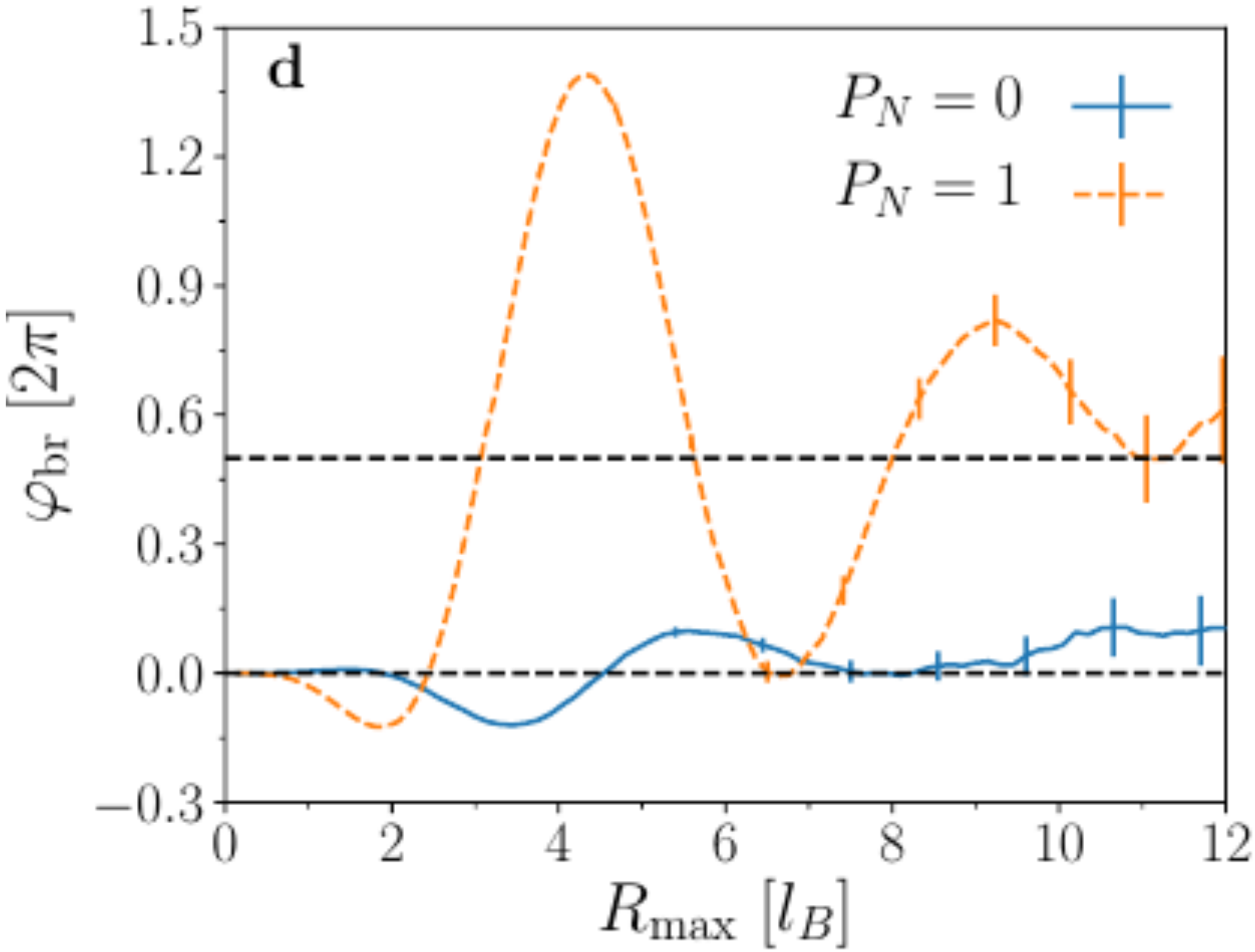


We obtained similar results for fermions!

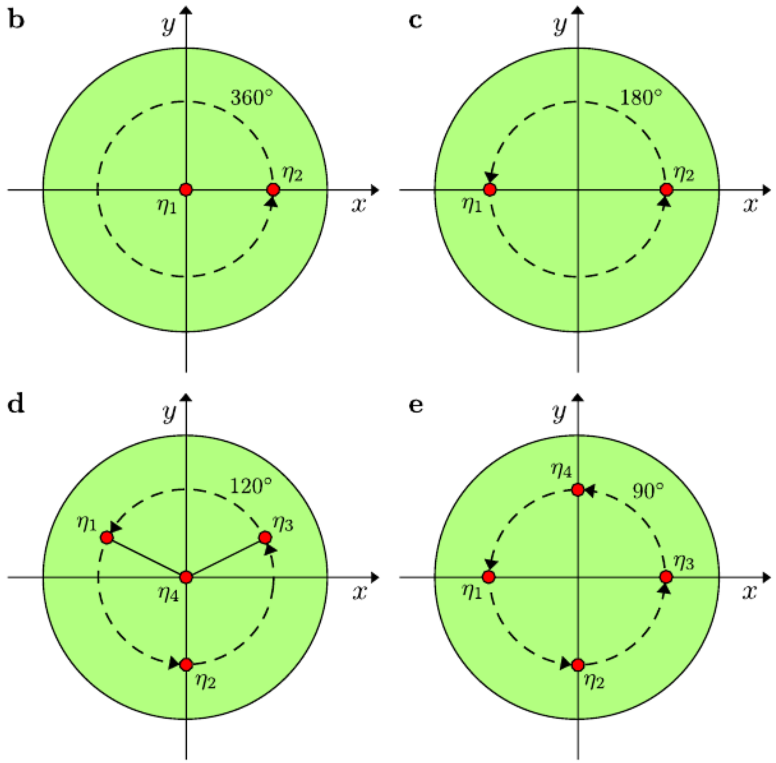
How should the experiment work?

- **First:** create two separated quasiholes
  - Reconstruct their density profile
- **Second:** create two separated quasiholes
  - Bring them close by and reconstruct the density profile of a “double” quasi hole
  - Both fusion channels are possible
- **Third:** compute the statistical phase

Summary

Rigid anyonic rotations vs arbitrary paths in parameter space

- An effective method to extract anyonic properties
- **Question:** Can we in principle extract all the information on the anyons of a wavefunction?



Macaluso, Comparin, LM, Carusotto, arXiv:1903.03011 (2019)

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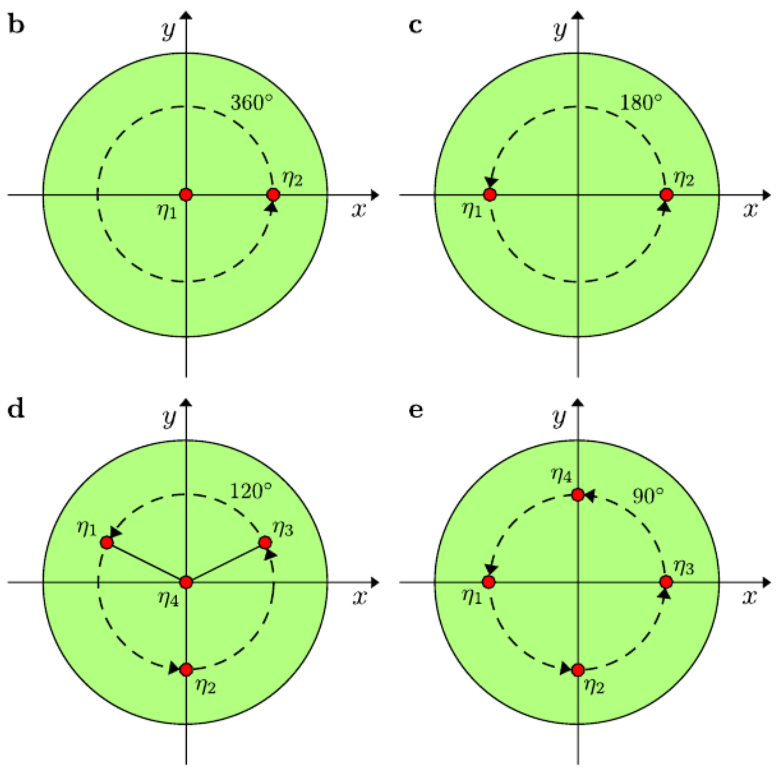
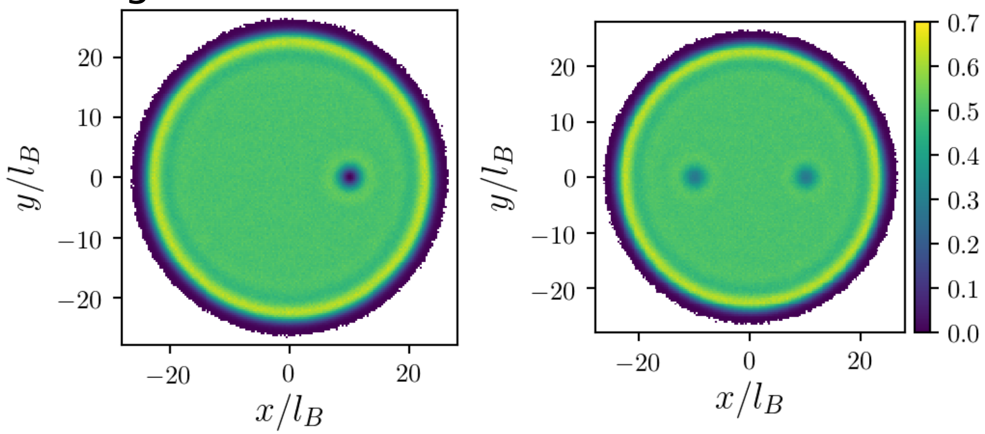


Anyonic properties are **experimentally accessible**

- Density profile measurements, no time evolution
- Importance of the density depletion around quasiholes!!
- Unambiguous evidence of non-Abelian statistics



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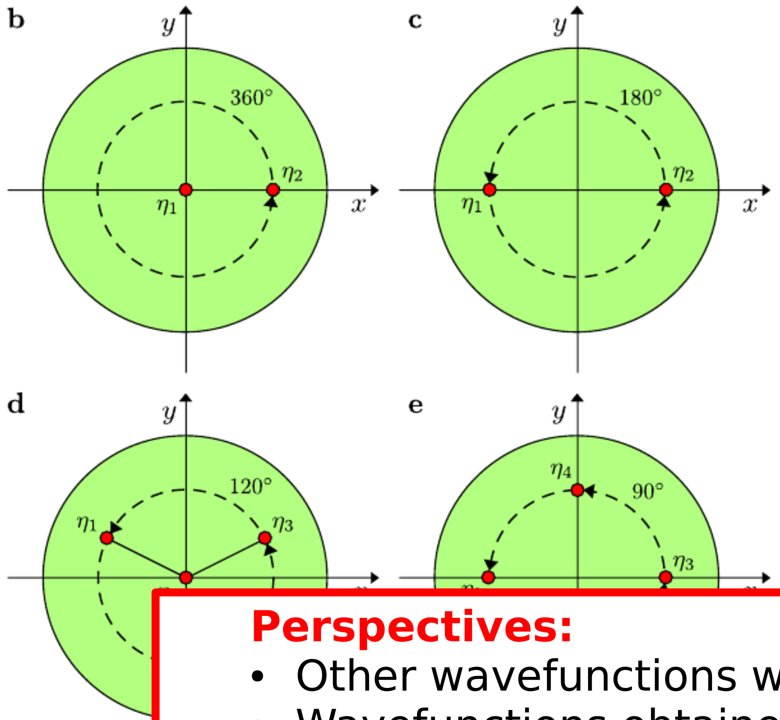

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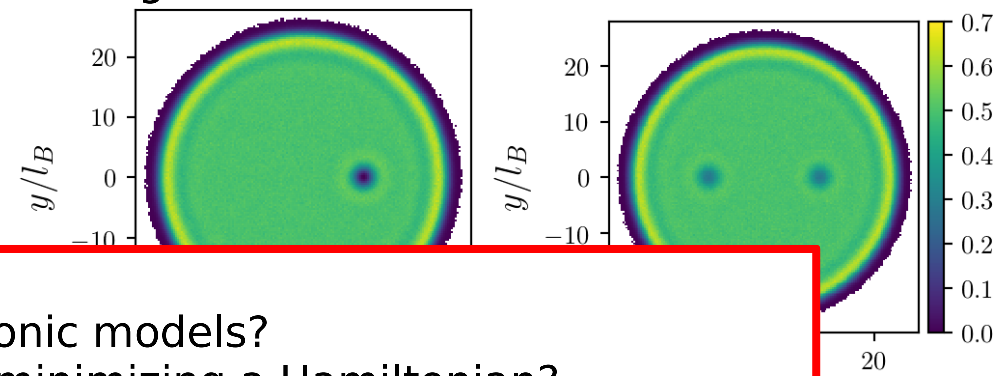
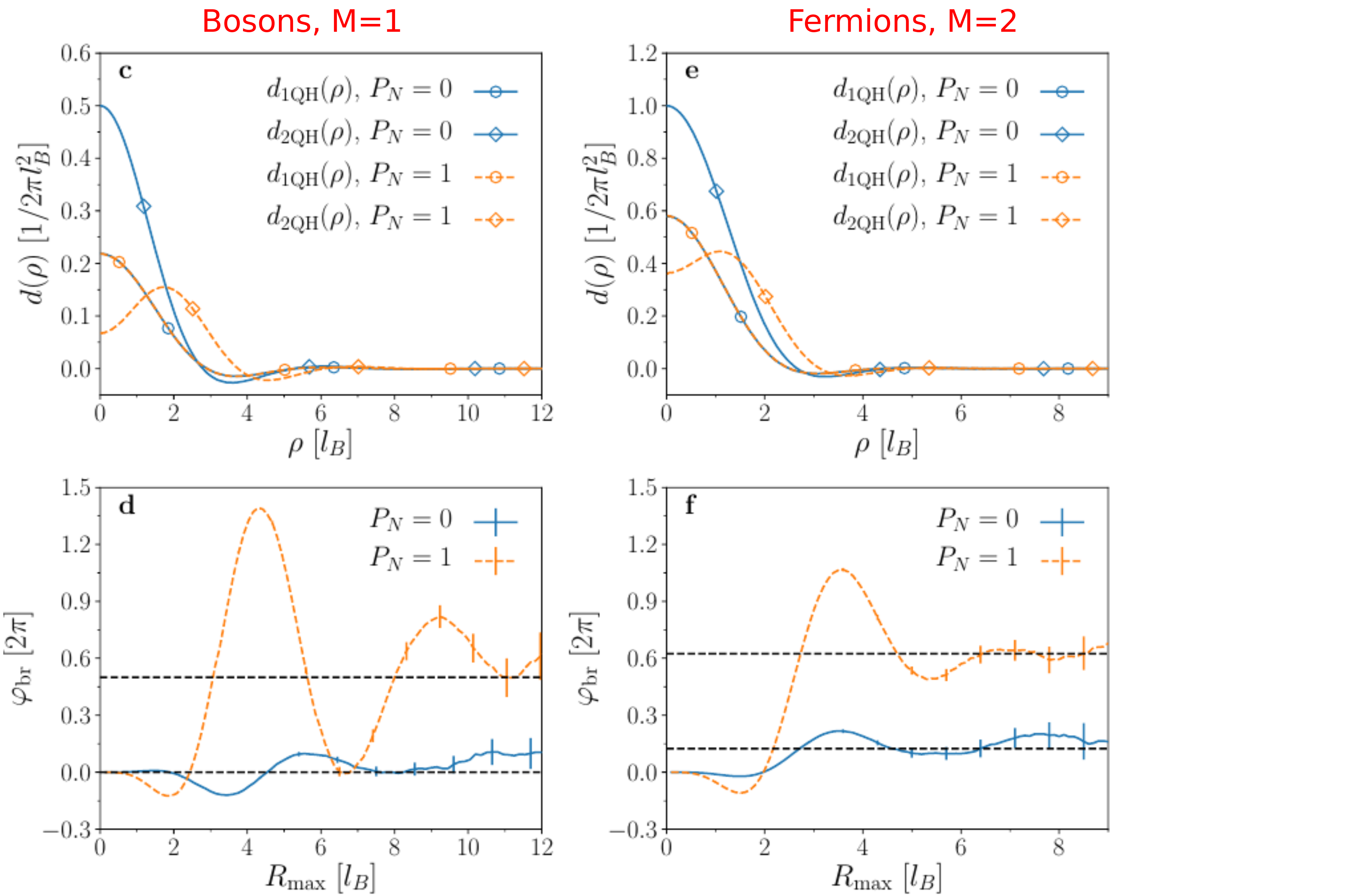


Perspectives:
- Other wavefunctions with anyonic models?
- Wavefunctions obtained from minimizing a Hamiltonian?
- Lattice models?
- Improve experimental protocols?

Macaluso, Comparin, LM, Carusotto, arXiv:1903.03011 (2019)

Thank you

Results



Bosons, M=1
Fermions, M=2
c
$d_{1QH}(\rho)$, $P_N = 0$
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ρ $[l_B]$
e
d
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$P_N = 1$
φ_{br} $[2\pi]$
R_{max} $[l_B]$
f