University of Strathclyde Glasgow

ICCUB | Institut de Ciències del Cosmos UNIVERSITAT DE BARCELONA

UNIVERSITAT DE BARCELONA

Robust phenomena in and out of equilibirium and gauge systems

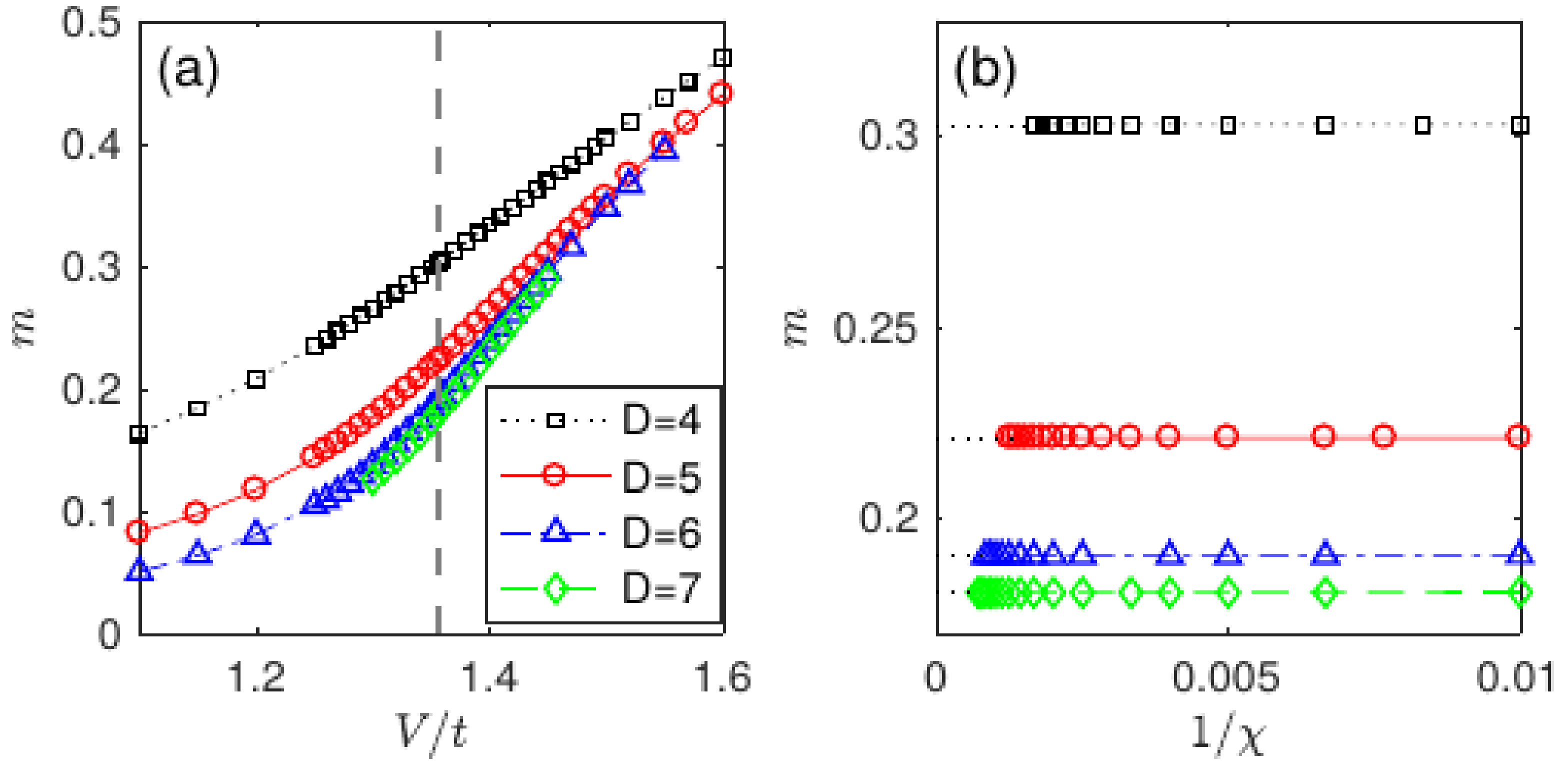
P. Corboz, P. Czarnik, G. Kapteijns, L. Tagliacozzo

arXiv:1803.08445 (PRX 2018)

....

See also Radar and Lauchli arXiv:1803.08566

Part 1 iPEPS



PHYSICAL REVIEW X

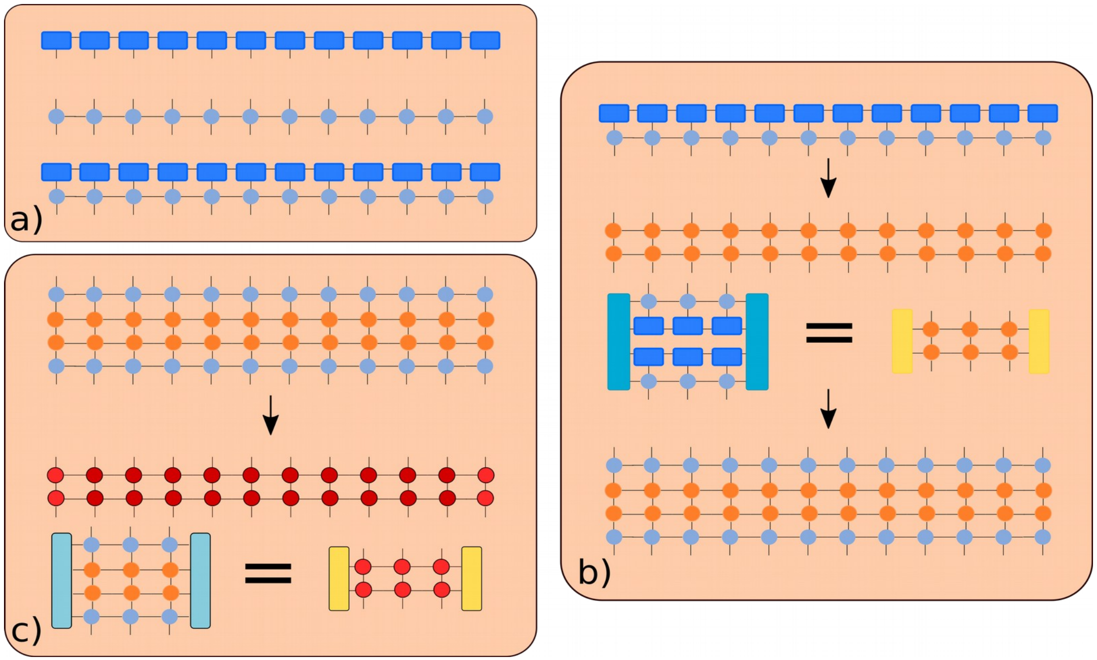
Highlights Recent Subjects Accepted Collections Authors Referees Search Press About Staff

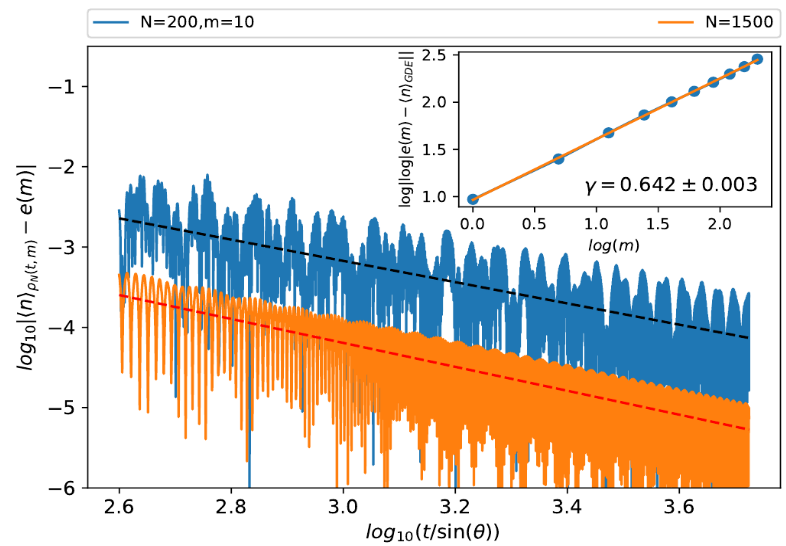
Open Access

Finite Correlation Length Scaling with Infinite Projected Entangled-Pair States

Philippe Corboz, Piotr Czarnik, Geert Kapteijns, and Luca Tagliacozzo
Phys. Rev. X **8**, 031031 – Published 30 July 2018

Part 2 out of equilibrium long-time predictions





PHYSICAL REVIEW B
covering condensed matter and materials physics

Highlights Recent Accepted Authors Referees Search Press About

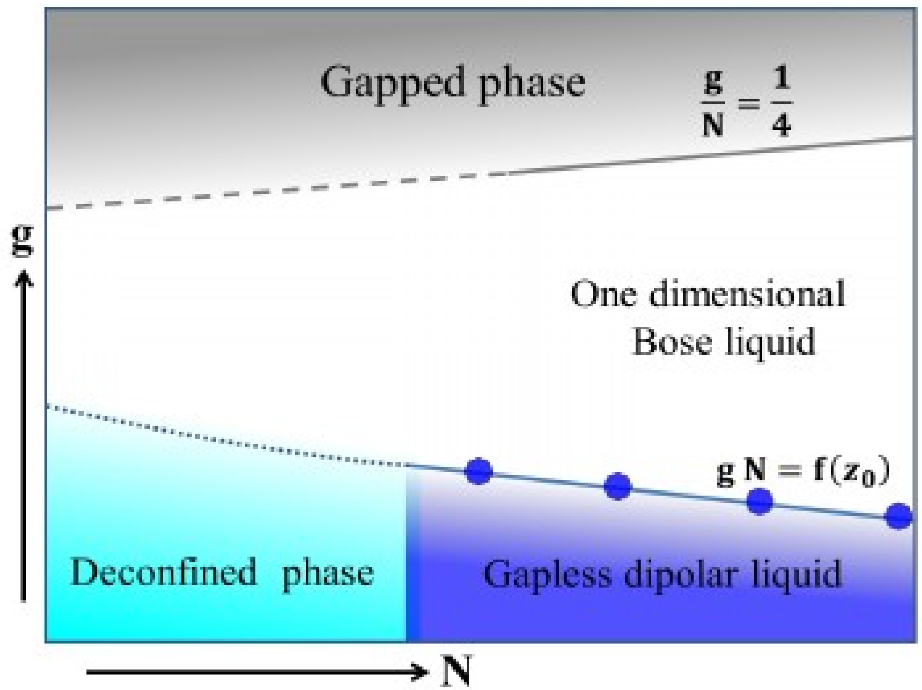
Simulating the out-of-equilibrium dynamics of local observables by trading entanglement for mixture

J. Surace, M. Piani, and L. Tagliacozzo

Phys. Rev. B **99**, 235115 – Published 7 June 2019

More

Part 3
Emerging gauge theories



PHYSICAL REVIEW A
covering atomic, molecular, and optical physics and quantum information

Toolbox for Abelian lattice gauge theories with synthetic matter

Omjyoti Dutta, Luca Tagliacozzo, Maciej Lewenstein, and Jakub Zakrzewski

Phys. Rev. A **95**, 053608 – Published 4 May 2017

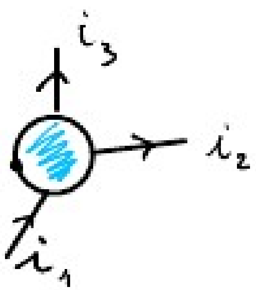
PART 1, Finitely correlated IPEPS

Outline

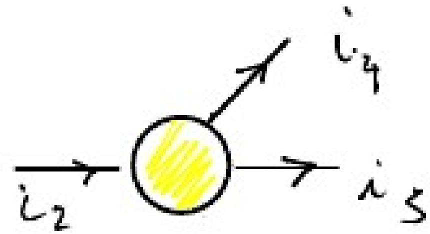
- **Background**
  - Tensor networks
  - Critical ground states
  - MPS for critical systems (D scaling)
  - PEPS for critical systems
- **Results**
  - Finitely correlated PEPS,
  - Applications, finite correlation length scaling and extrapolations
  - TN beyond the entanglement

Notation

Tensors:

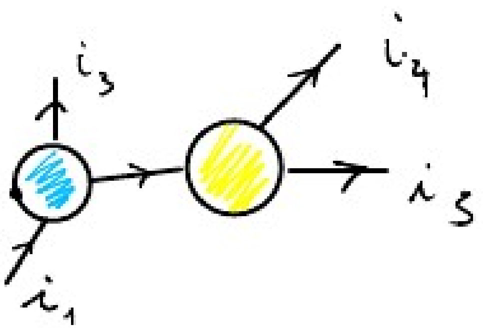


$$T_{i_1}^{\ \ i_2 i_3}$$



$$S_{i_2}^{\ \ i_5 i_4}$$

Operations among tensors:



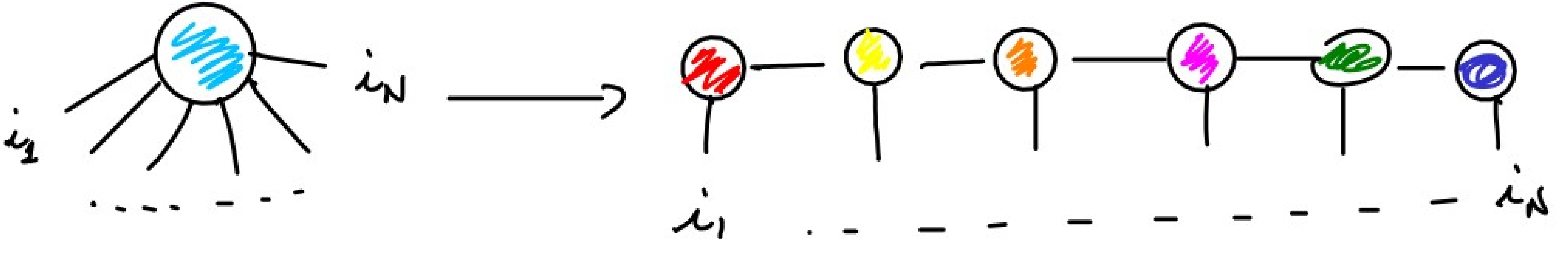
$$(TS)_{i_1}^{\ \ i_3 i_4 i_5} = \sum_{i_2} T_{i_1}^{\ \ i_3 i_2} \; S_{i_2}^{\ \ i_4 i_5}$$

Tensor Network states

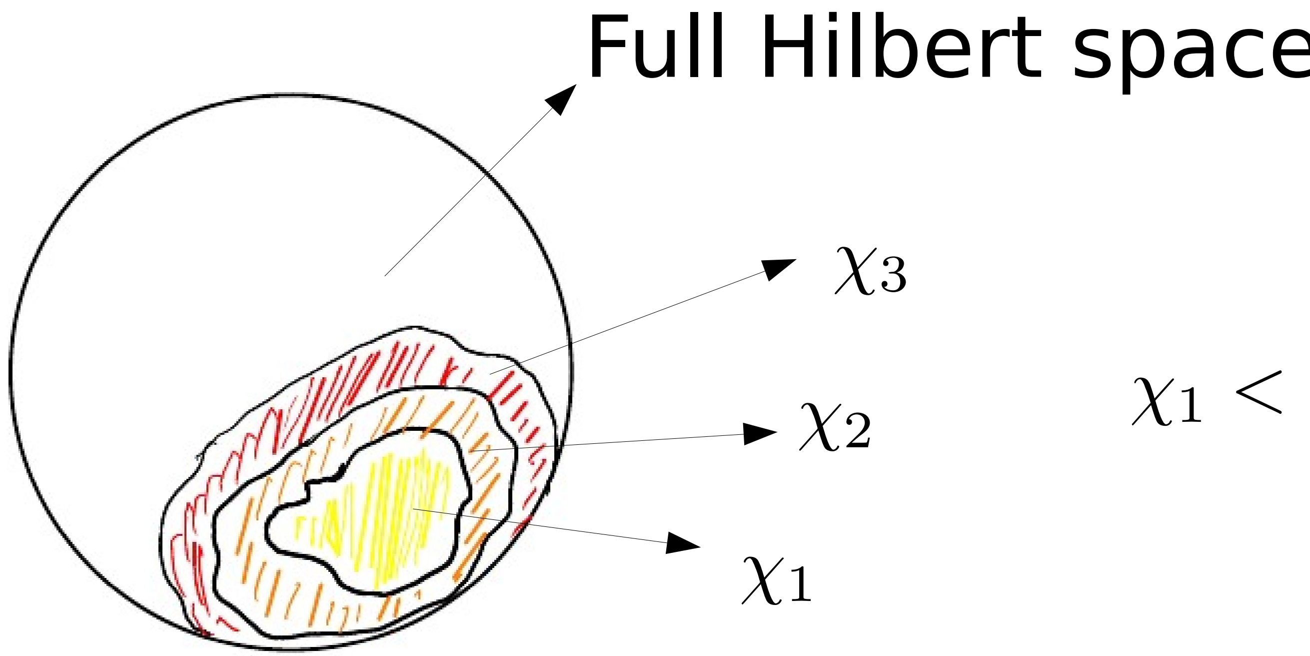
- Express exponentially large tensors

$$c^{i_1 \cdots i_N}$$

- As contraction of small elementary tensors, example matrix product state



Low entangled states

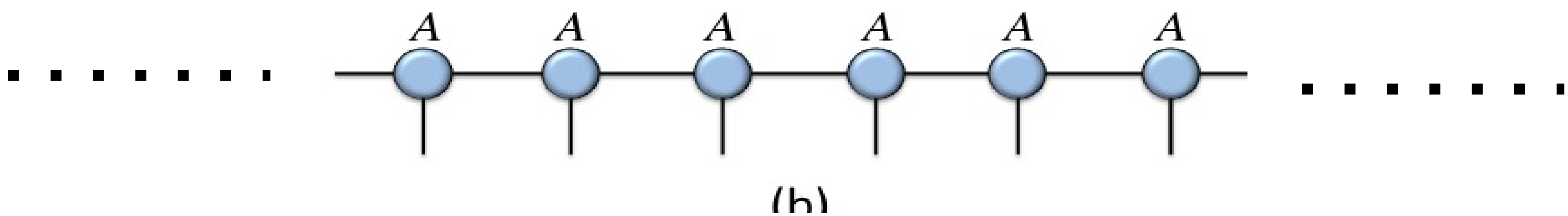


$$\chi_1 < \chi_2 < \chi_3$$

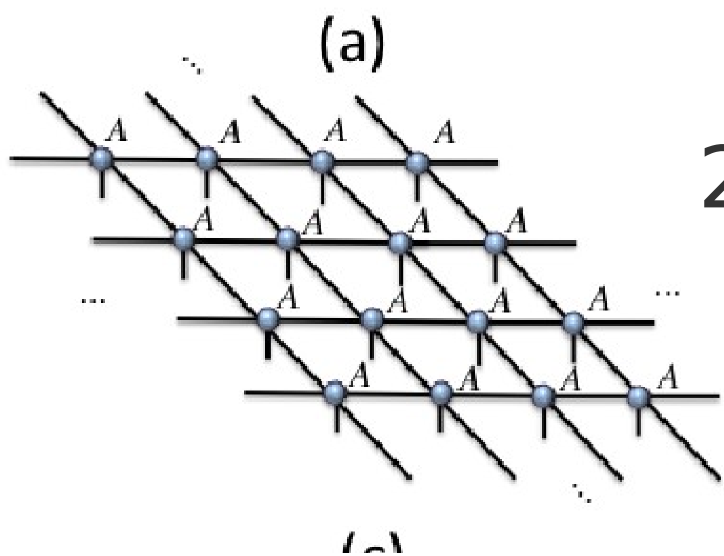
Theorems for
gapped, local Hamiltonians (1D)
this is the relevant corner
for low energy states

Physics out of a single tensor

- State of an infinite 1D chain

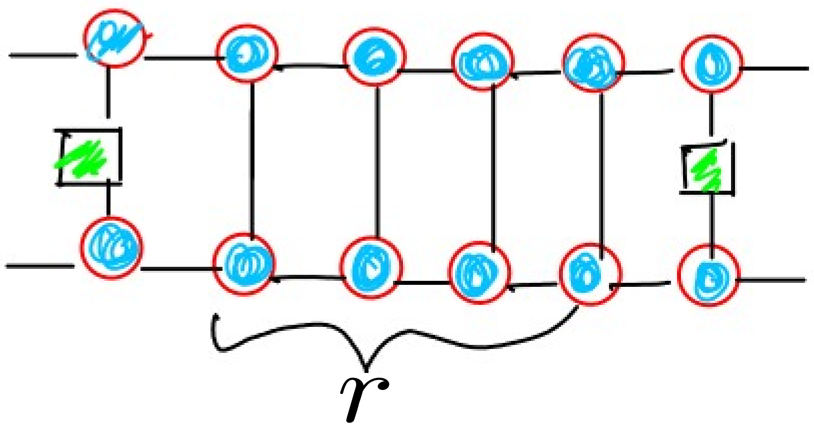


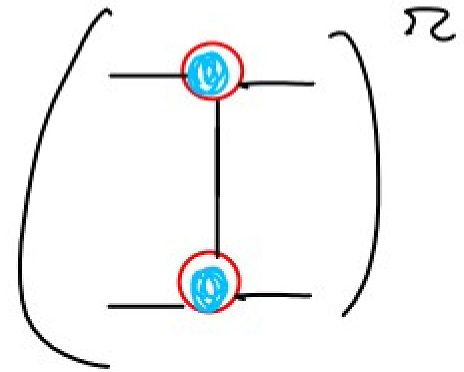
or infinite 2D lattice



Properties of MPS

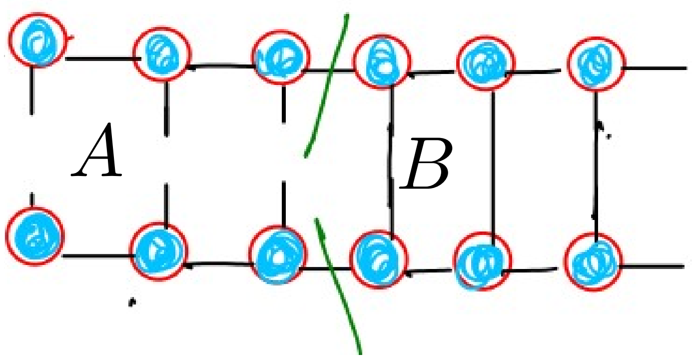
- The correlations decay exponentially





$$E = \sum_i e_i |e_i\rangle\langle e_i| \qquad E^r \simeq (e_2/e_1)^r$$

- There is an area law for entanglement



$$\mathrm{rank}(\rho_A) \propto D$$

Outline

- **Background**
  - Tensor networks
  - Critical ground states
  - MPS for critical systems (D scaling)
  - PEPS for critical systems
- **Results**
  - Finitely correlated PEPS,
  - Applications, finite correlation length scaling and extrapolations
  - TN beyond entanglement

Critical ground states

Algebraically decaying correlation functions

$$\langle O(0)O(r)\rangle \propto r^{-\eta}$$

Logarithmic increase of entanglement entropy

$$S \propto \log L$$

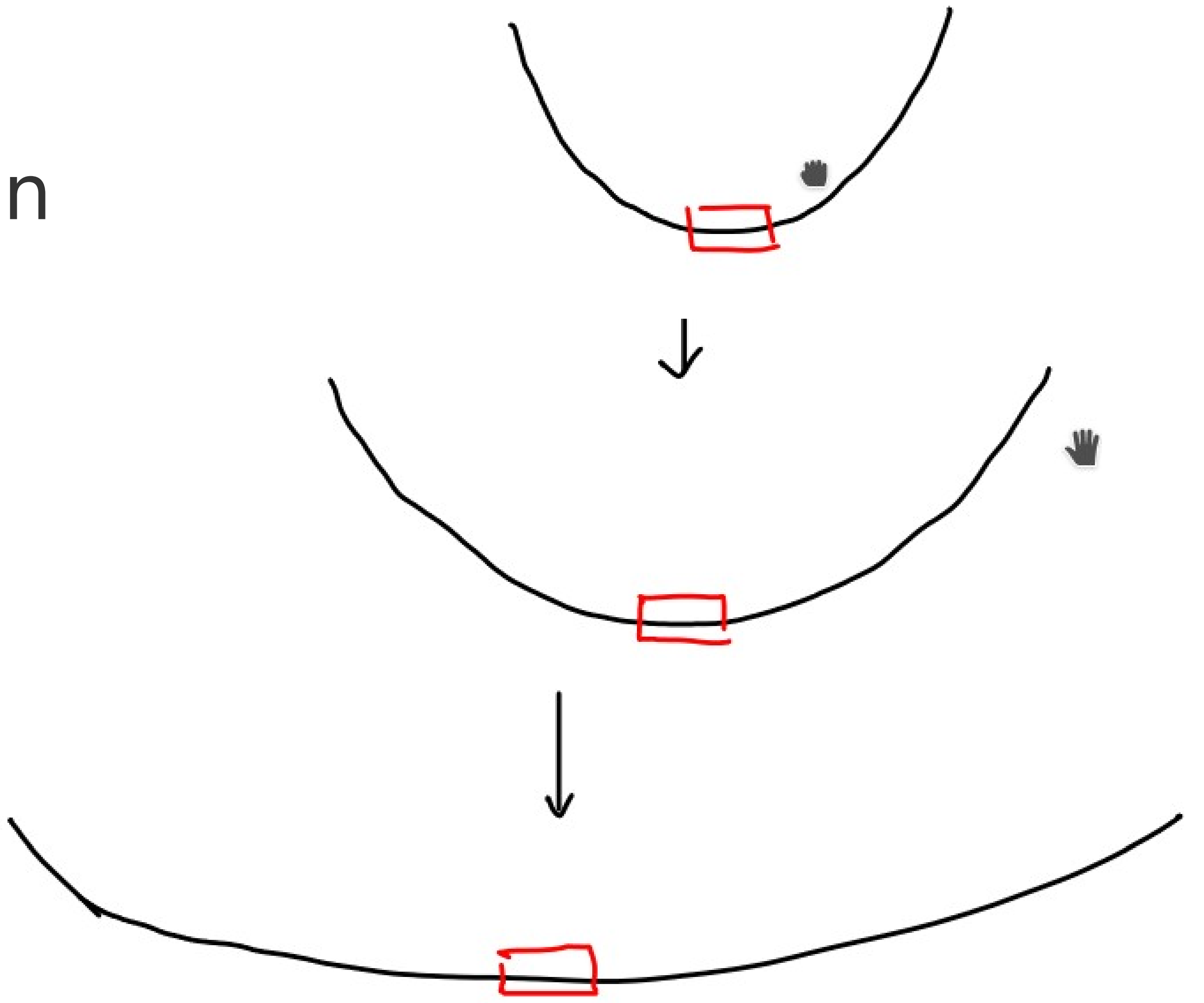
Thermodynamic limit

Same region

On larger

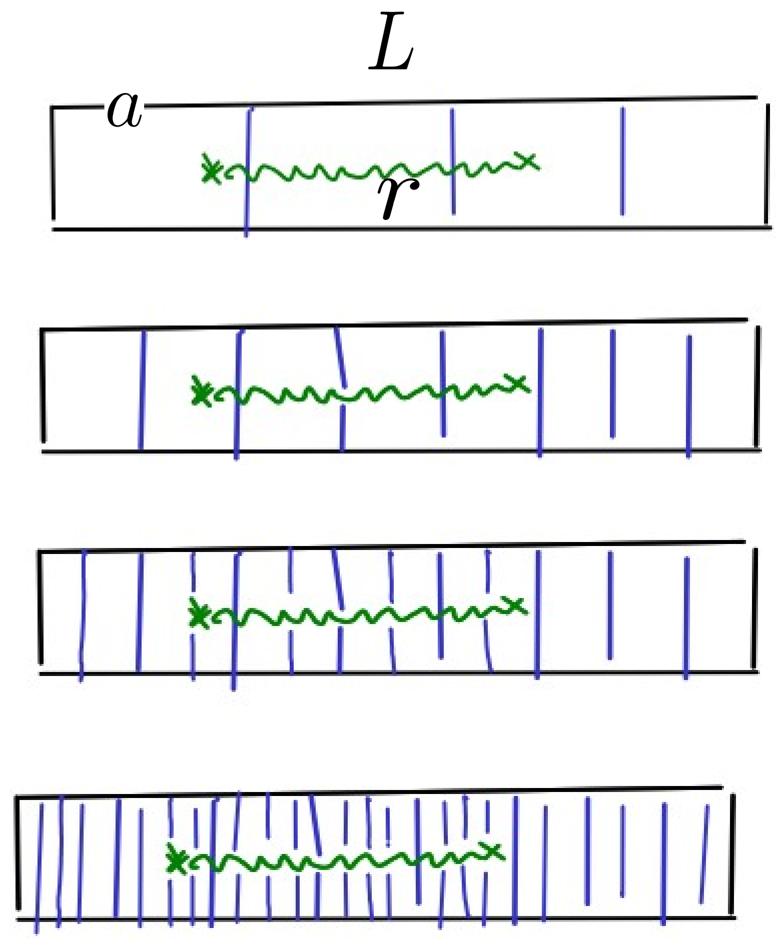
and larger

Systems



Constructing field theories with TN

- Continuous field theory, divergences, regularized on the lattice



$$a = L/4 \quad r = 2a$$

$$a = L/8 \quad r = 4a$$

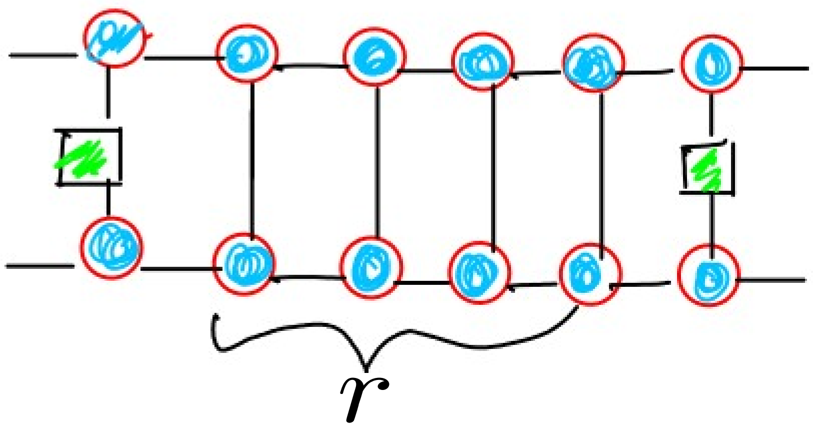
$$a = L/16 \quad r = 8a$$

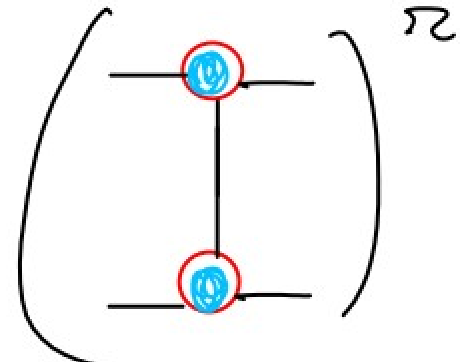
- Take the continuum limit

$$a = L/32 \quad r = 16a$$

MPS for critical systems?

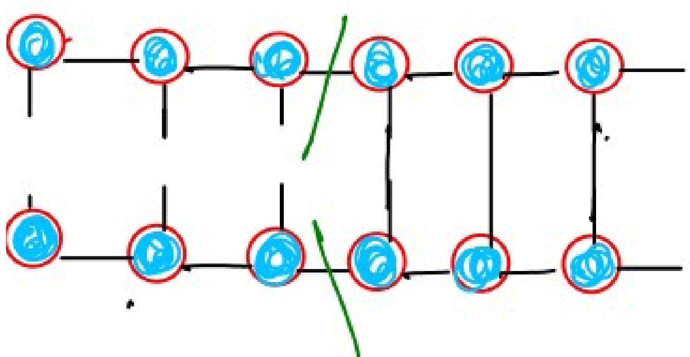
- The correlations should decay as power





$$E = \sum_i e_i |e_i\rangle\langle e_i| \qquad E^r \simeq (e_2/e_1)^r$$

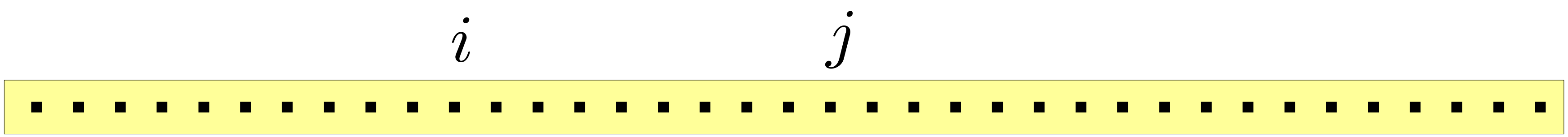
- Rank ρA should increase with size of A



$$\mathrm{rank}(\rho_A) \propto D$$

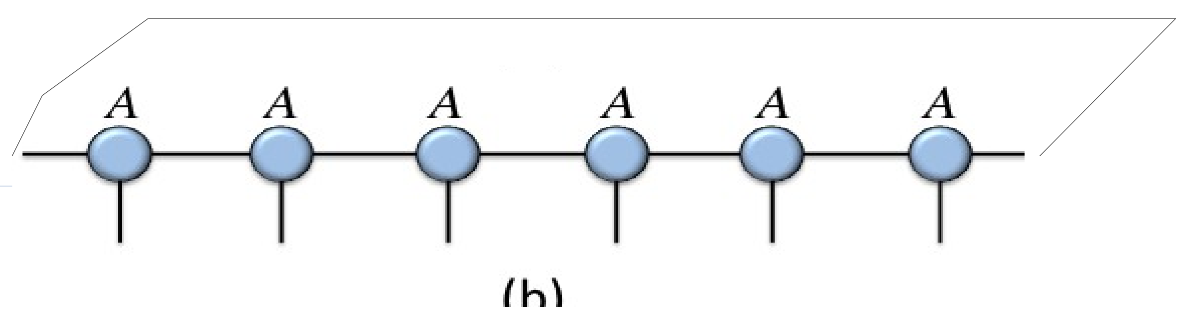
Numerical results 1D critical MPS

- Ising model in transverse field



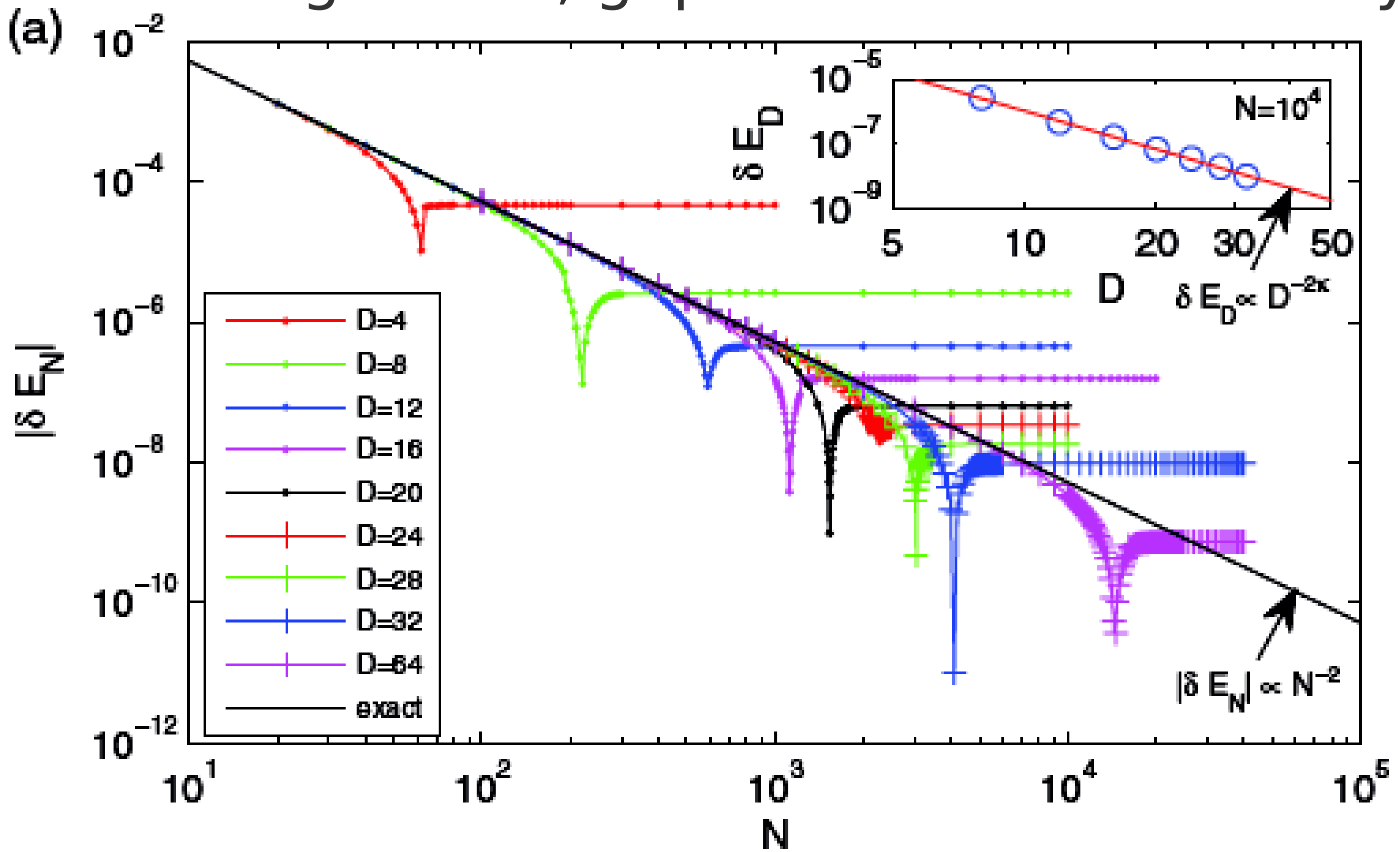
$$H = \sum_i \sigma_x^i \sigma_x^{i+1} + \sigma_z^i$$

- We optimize the MPS matrices over increasingly large rings **with fix D**



Energy results

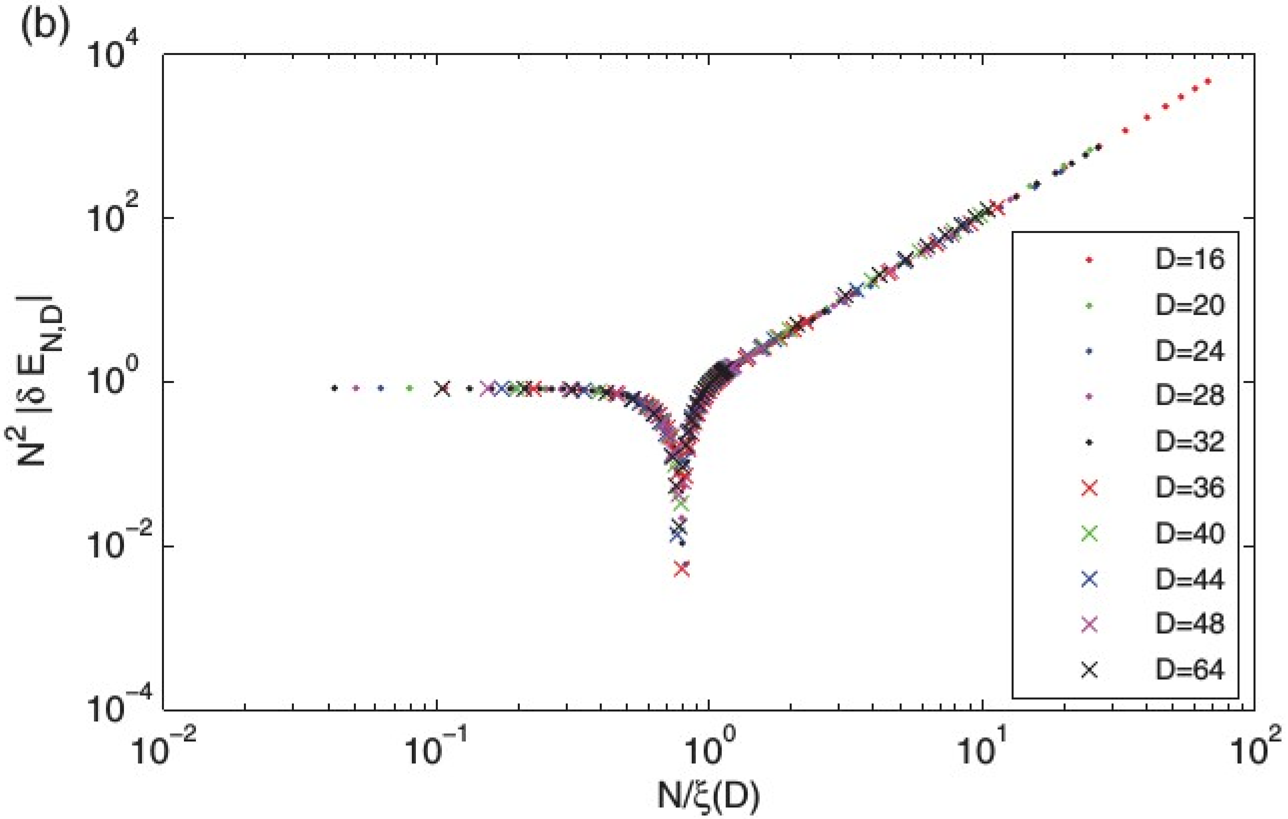
- Critical Ising model, gap decreases with the system size



At fixed bond dimension the results deviate from the exact line,the deviation is systematic, we can extrapolate

LT et al. PRB 08, Pirvu et. al. (LT) 2012
see also Nishino et al. 96, Pollman et al. 09 ...

Finite correlation (D) scaling



$$\Delta E_D = \xi_D^{-1} \propto D^{-\kappa}$$

Outline

- **Background**
  - Tensor networks
  - Critical ground states
  - MPS for critical systems (D scaling)
  - PEPS for critical systems
- **Results**
  - Finitely correlated PEPS,
  - Applications, finite correlation length scaling and extrapolations
  - TN beyond entanglement

Properties of conformal critical points in 2D (isolated)

- Correlations decay algebraically

$$\langle O(0)O(r)\rangle \propto r^{-\eta}$$

- Area law for entanglement entropy

$$S \propto L + \cdots$$

Properties of PEPS

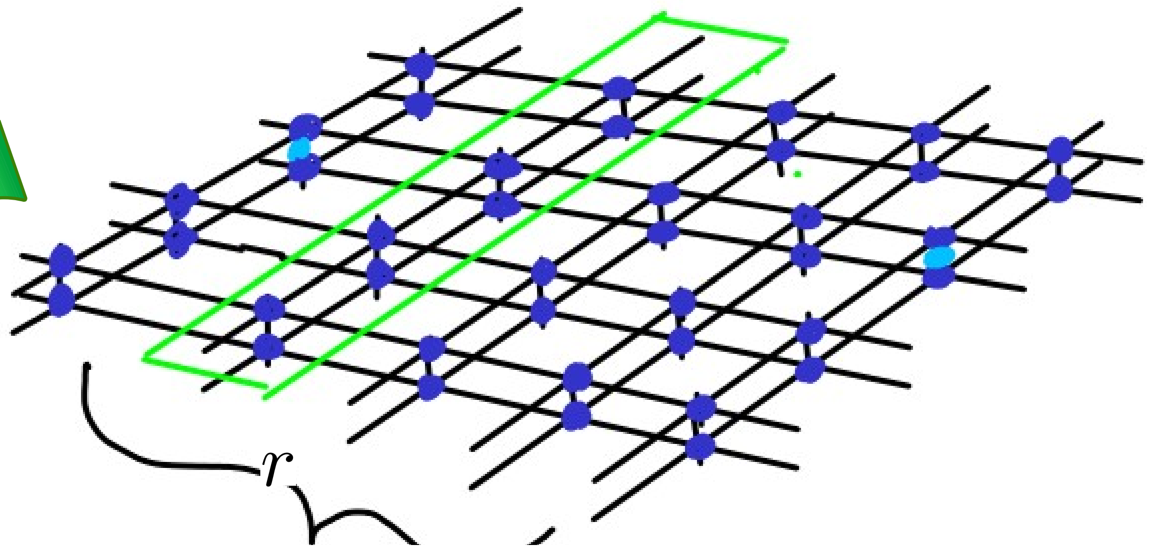
- The correlation function are infinite 2D TN and can decay:

  - exponentially

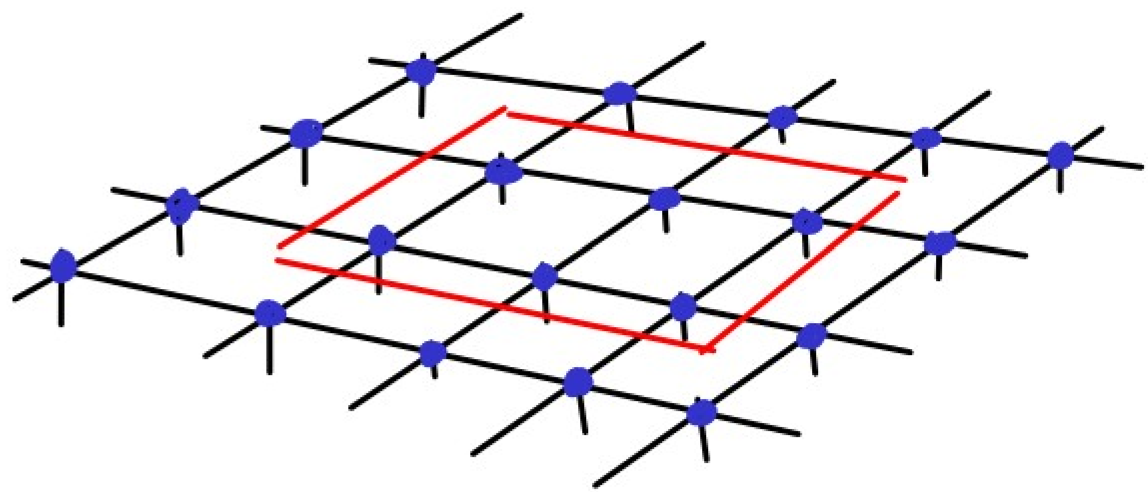
    $$exp(-r/\xi)$$

  -but also algebraically)

    $$r^{-\eta}$$



- Area law
  for entanglement



Cirac Verstraete 2004 ...

Outline

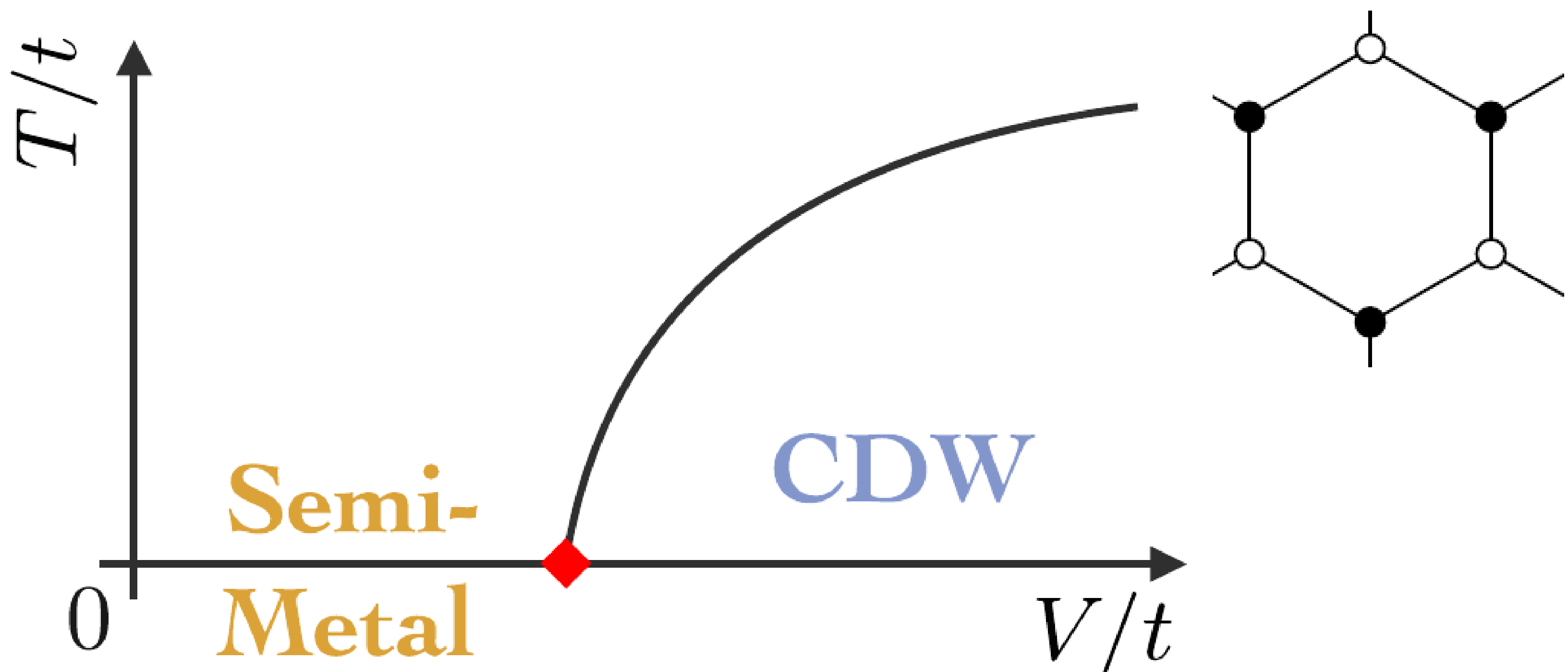
- **Background**
  - Tensor networks
  - Critical ground states
  - MPS for critical systems (D scaling)
  - PEPS for critical systems
- **Results**
  - Finitely correlated iPEPS,
  - Applications, finite correlation length scaling and extrapolations
  - TN beyond entanglement

Numerical results 2D

Interacting fermions on the Honey Comb lattice with iPEPS

$$\hat{H} = -t \sum_{\langle \mathbf{i},\mathbf{j} \rangle} \left[\hat{c}^{\dagger}_{\mathbf{i}} \hat{c}_{\mathbf{j}} + h.c. \right] + V \sum_{\langle \mathbf{i},\mathbf{j} \rangle} \hat{n}_{\mathbf{i}} \hat{n}_{\mathbf{j}},$$

$$m = \left| n_A - n_B \right|$$

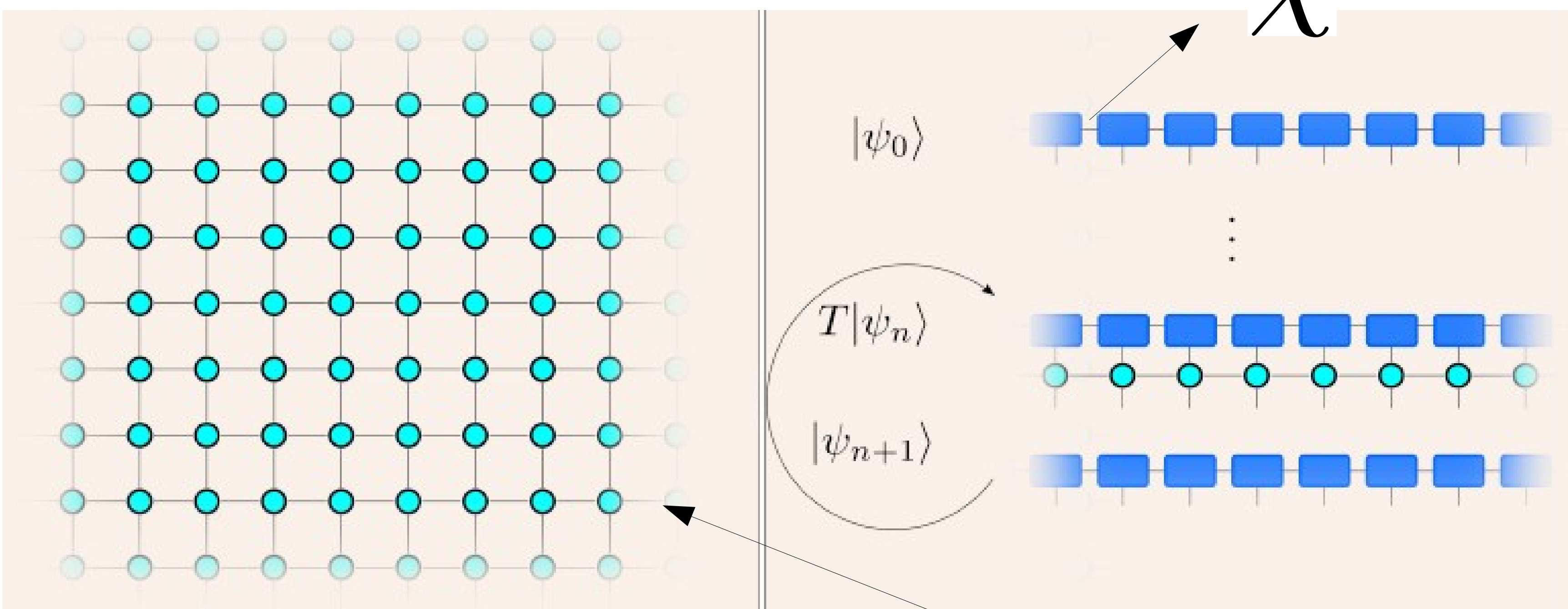


Wang et al. 2014

Slight technical complication,

Contracting the 2D TN introduces an extra parameter

χ



- Morally we have 2 parameters D, χ

Our relevant parameters

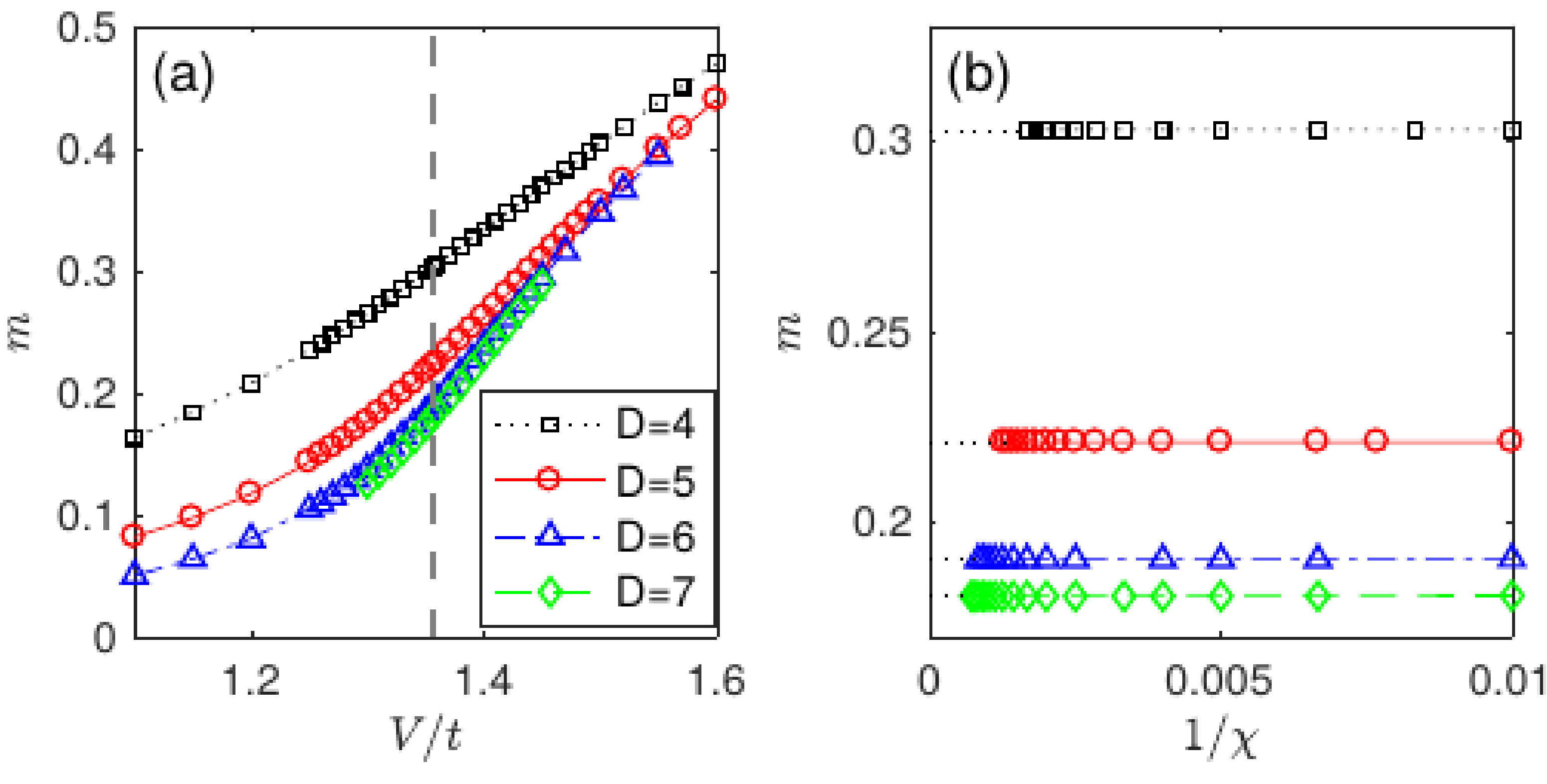
- The distance from the critical point

$$g = (V - V_c)/V_c$$

- The PEPS bond dimension D
- The boundary bond dimension χ
- We get rid of the last by taking it sufficiently large

The order parameter

- The location of the critical point is known from Monte-Carlo (dashed line) Wang et al 2014
- There m is expected to vanish as $\mathrm{m} \propto g^{\beta}$

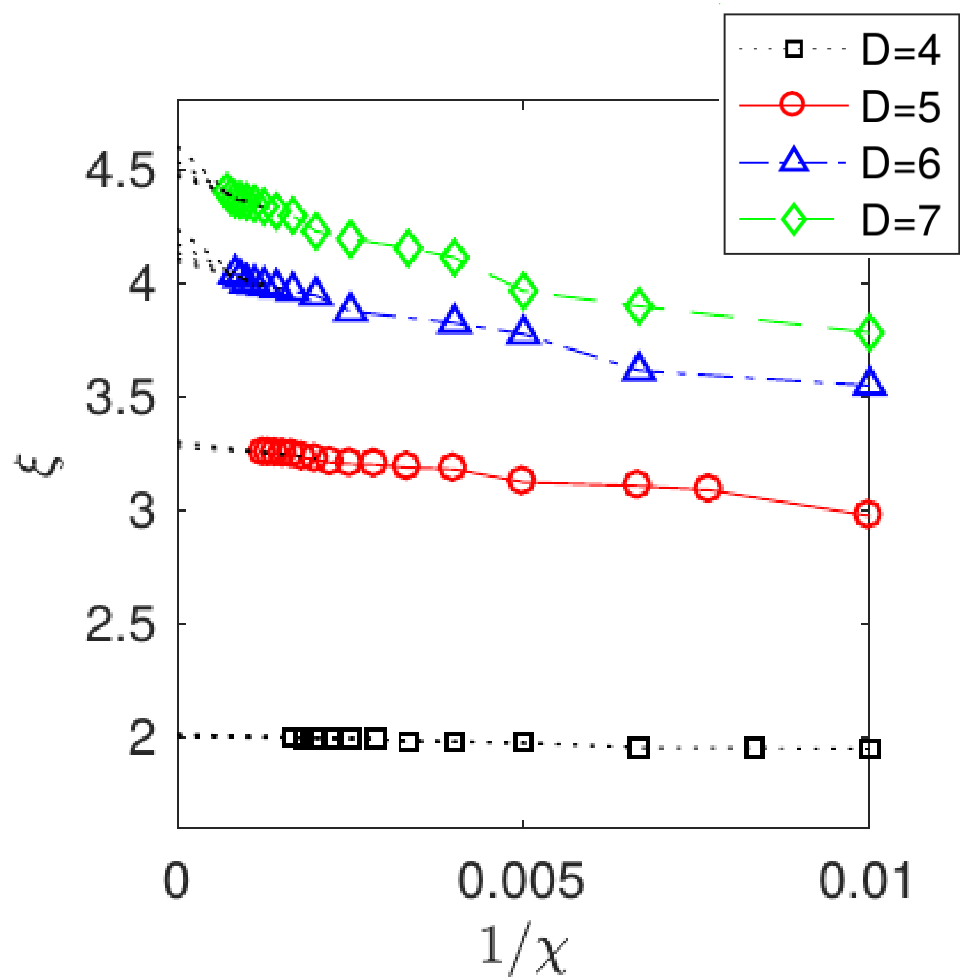


Corboz...(LT) 2018

Finite correlation length

- Strong dependence of the correlation length on χ

- We need to extrapolate it

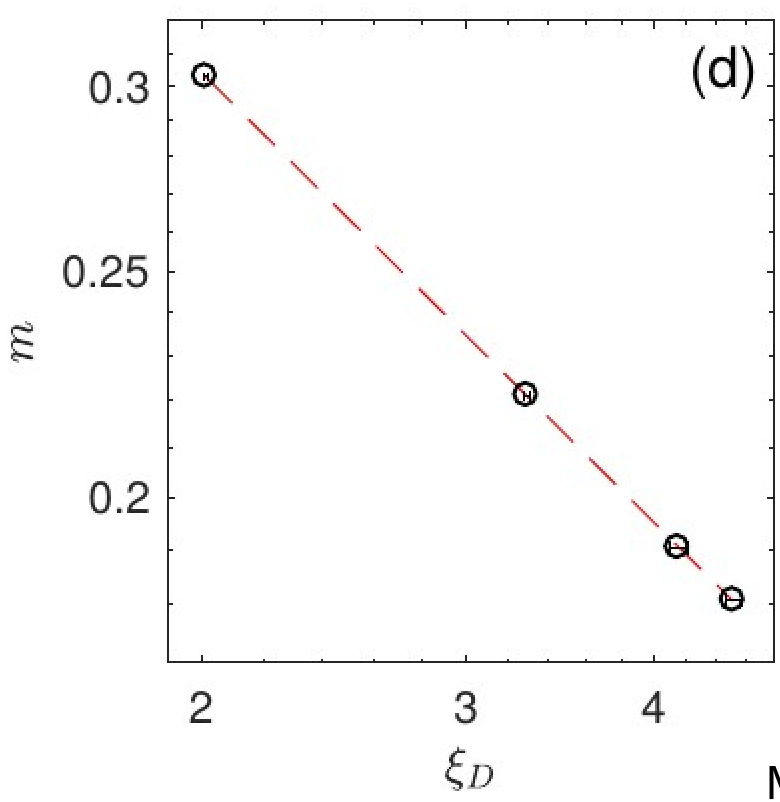


Outline

- **Background**
  - Tensor networks
  - Critical ground states
  - MPS for critical systems (D scaling)
  - PEPS for critical systems
- **Results**
  - Finitely correlated PEPS,
  - Applications, finite correlation length scaling and extrapolations
  - TN beyond entanglement

Order parameter vs correlation length

- Using the scaling hypothesis, we expect that m scales as



$$m \propto \xi_D^{-\frac{\beta}{\nu}}$$

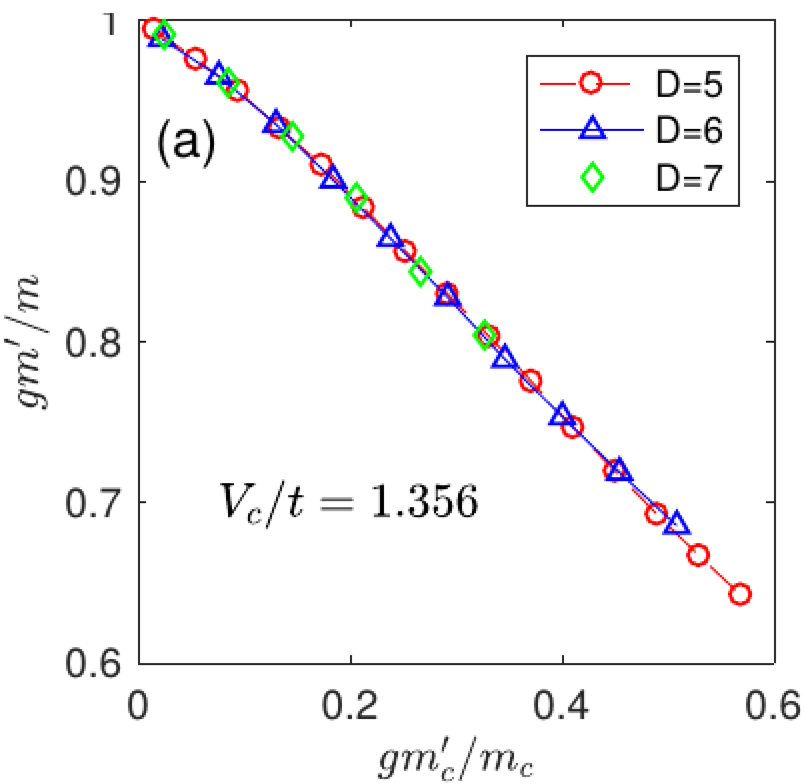
$$\beta/\nu = 0.64(2)$$

MonteCarlo prediction 0.65(4)
Wang et al 2014

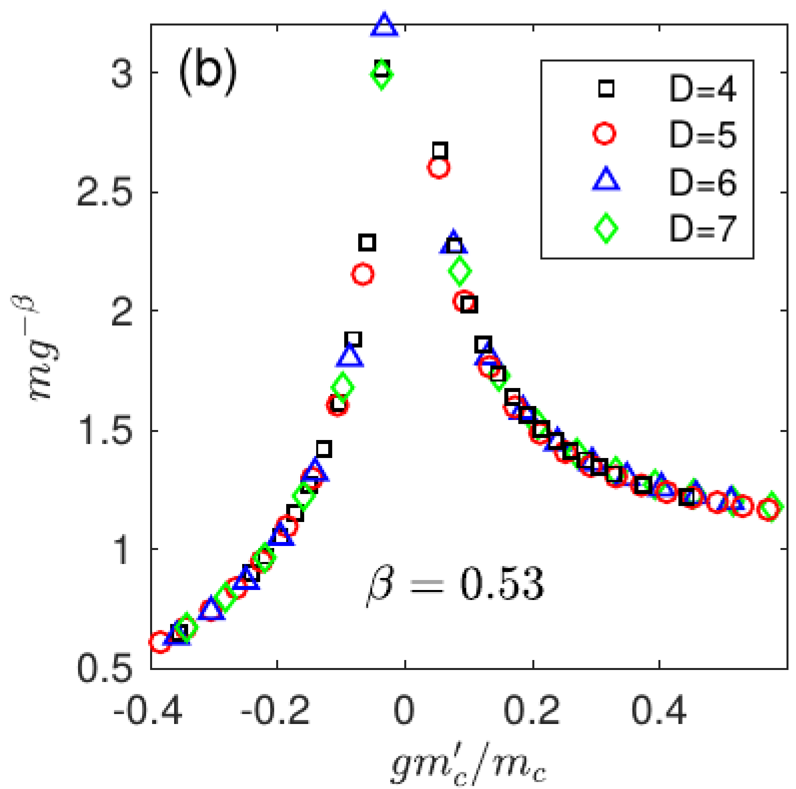
Locating the critical point, extracting critical exponents

$$\frac{m_c'(D)}{m_c(D)} := \frac{m'(g=0,D)}{m(g=0,D)} \sim \xi_D^{1/\nu}.$$

$$g\frac{m'(g,D)}{m(g,D)} = \mathcal{P}\left(g\frac{m_c'(D)}{m_c(D)}\right). \qquad m(g,D)\,g^{-\beta} = \tilde{\mathcal{P}}\left(g\frac{m_c'(D)}{m_c(D)}\right),$$



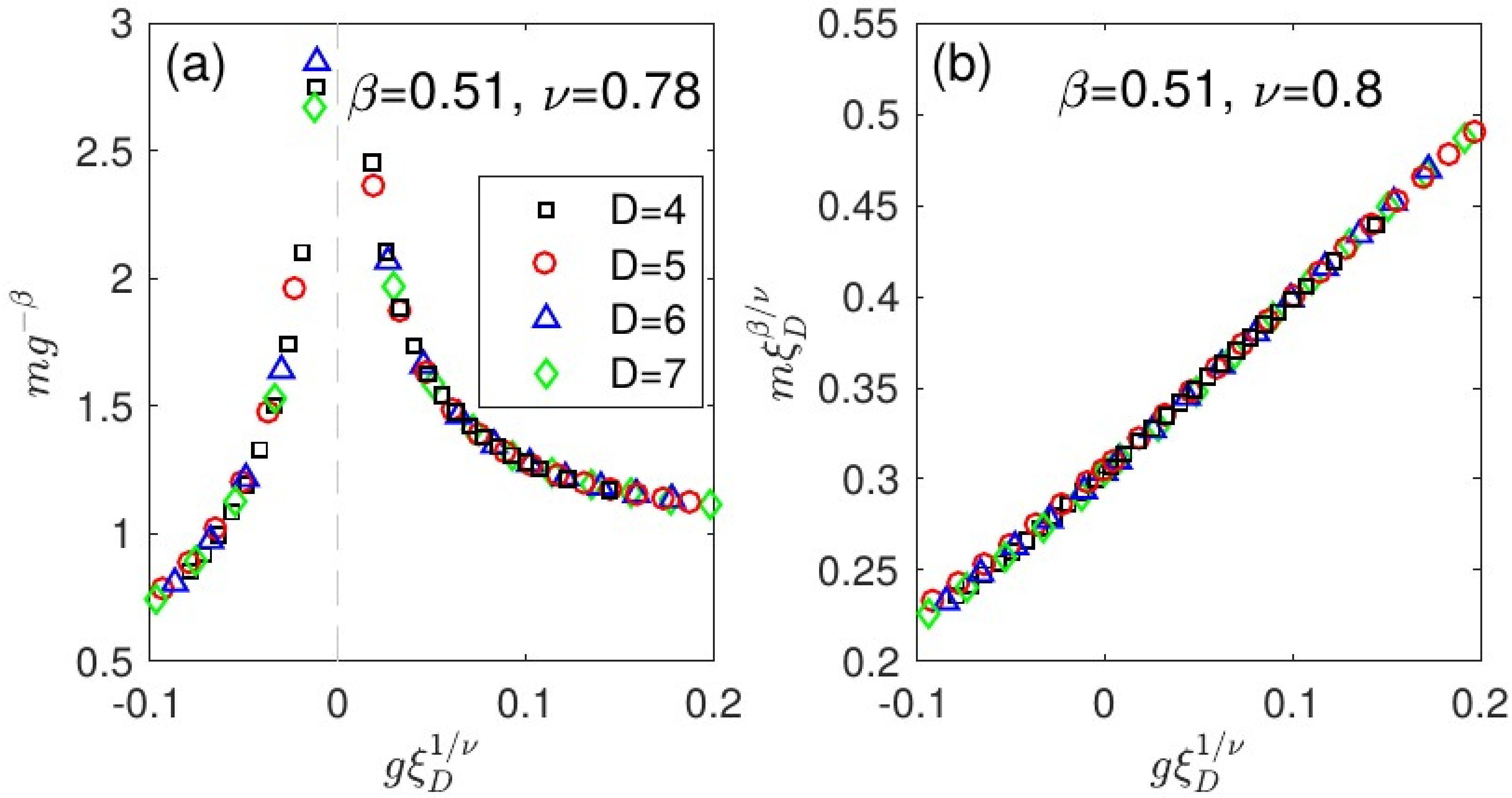
1.356(1) Wang et al (2014)



0.52(3) Wang et al (2014)

The critical exponents

$$m(g,D)\,\xi_D^{\beta/\nu} = \mathcal{M}(g\xi_D^{1/\nu}), \qquad m(g,D)\,g^{-\beta} = \tilde{\mathcal{M}}(g\xi_D^{1/\nu}),$$



0.52(3) 0.80(3) Wang et al. (2014)

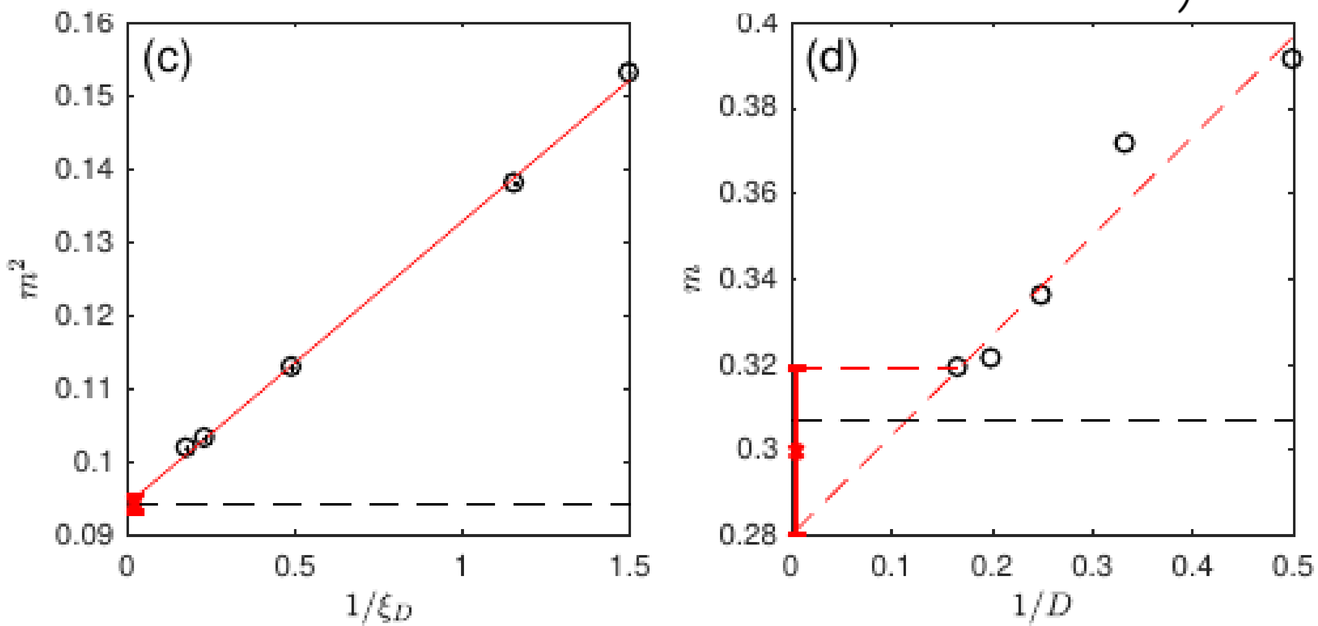
Applications, extrapolating finite D results

Heisenberg model on the square lattice

$$H = \sum_{<i,j>} \vec{\sigma}_i \cdot \vec{\sigma}_j$$

We plot the square magnetization as a function of the correlation length

$$m = 0.307 \pm 0.002,$$



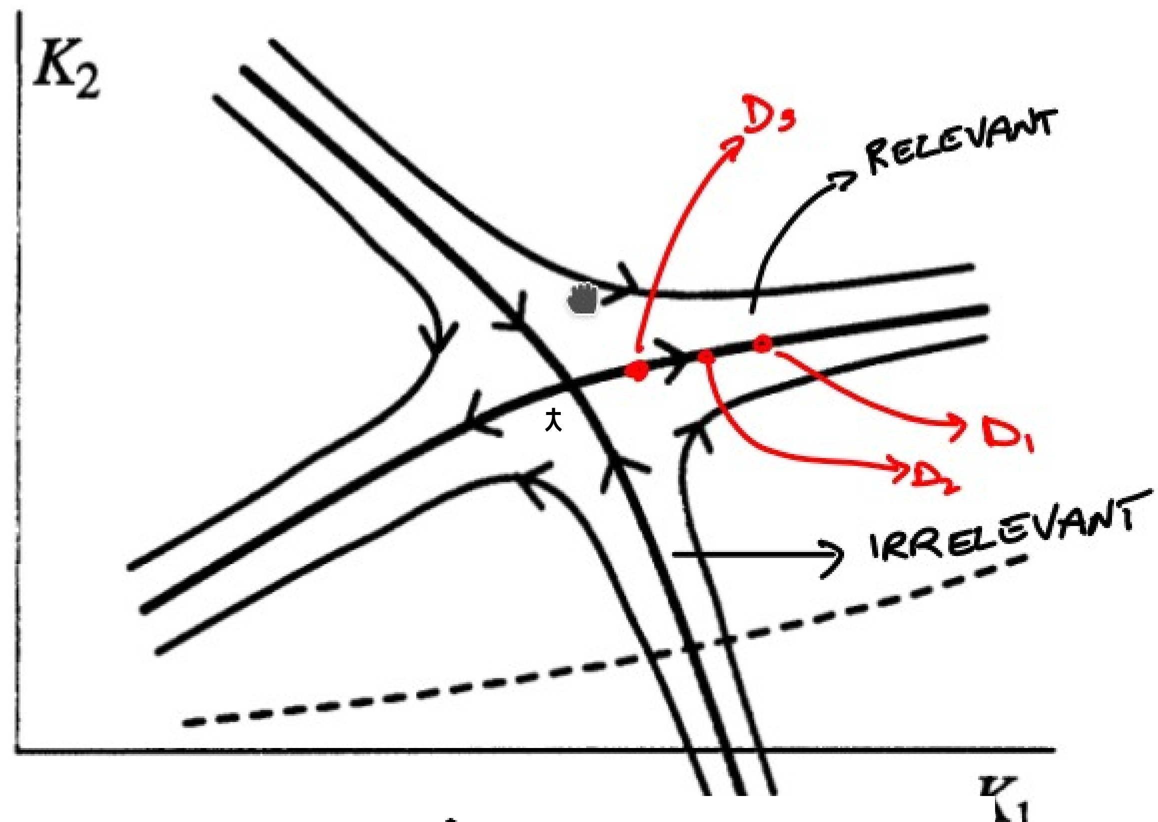
The state of the art is $m = 0.30743(1)$ (Sandvik 2010)

Outline

- **Background**
  - Tensor networks
  - Critical ground states
  - MPS for critical systems (D scaling)
  - PEPS for critical systems
- **Results**
  - Finitely correlated PEPS,
  - Applications, finite correlation length scaling and extrapolations
  - TN beyond entanglement

Beyond entanglement universality,

Robustness to approximations,
RG framework



$$f_s(u_t, u_h) = b^{-d} f_s(b^{y_t} u_t, b^{y_h} u_h)$$

Conclusions part 1

Finitely correlated TN states

- From our numerical results it seems that both 1D and 2D TN states with finite D are finitely correlated.
- This in 1D matches our expectations based on entanglement scaling, but in 2D it does not.
- Once we identify the presence of a finite correlation length and understand how to tune it (change D), we can use it to unveil universal information about the critical points.
- There is life beyond entanglement, robust phenomena

Part 2 out of equilibrium

Long time, equilibration

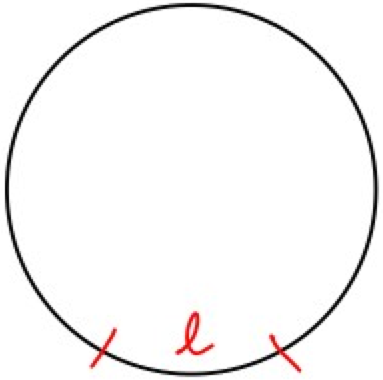
- Surace Piani Tagliacozzo arXiv:1810.01231
  see also Levlatan et al arXiv:1702.08894
  C. D. White et al arXiv:1707.01506
  ....
  C. B. Mendl, arXiv:1812.11876

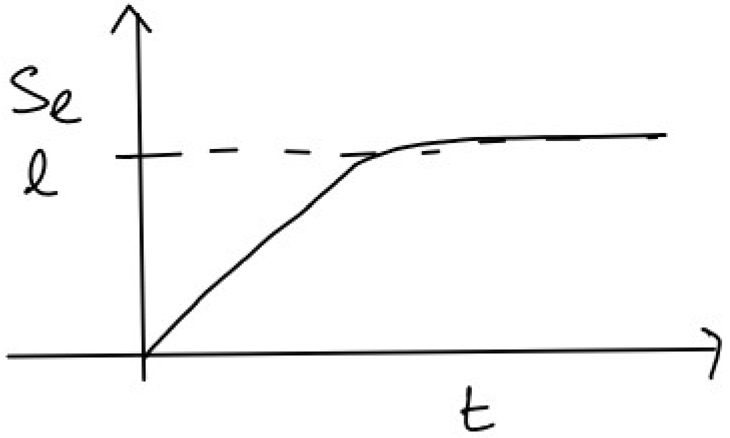
Outline

- **Background**
  - The computational complexity
  - Going beyond entanglement
- **Results**
  - A new algorithm for the out-of-equilibrium evolution
  - Benchmark on free fermions

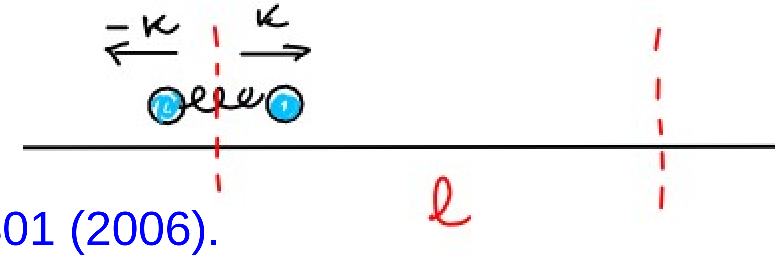
Translational invariant picture Calabrese Cardy

- After a quench, the excess energy produces a radiation of correlations





- Can be explained in terms of radiation of entangled pseudo-particles



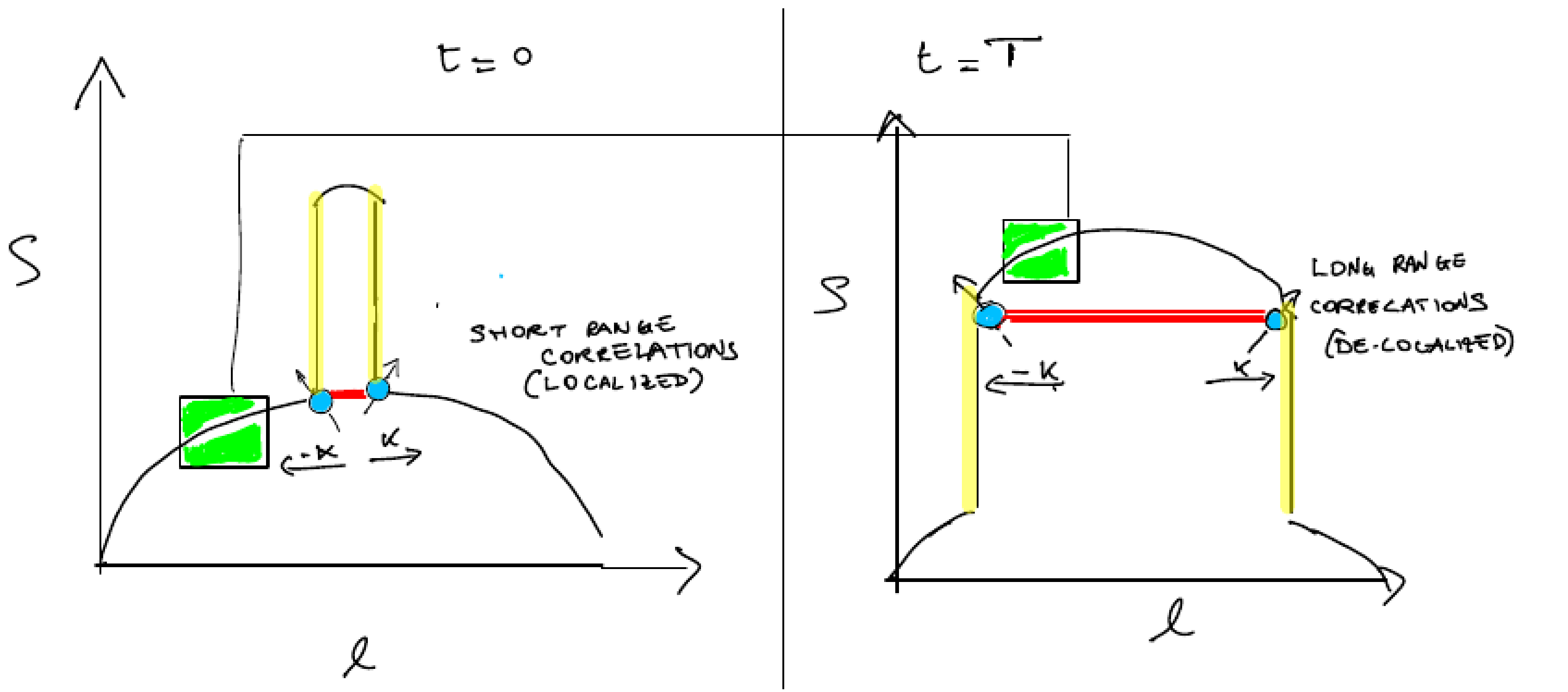
Calabrese Cardy, Phys. Rev. Lett. 96, 136801 (2006).

Outline

- **Background**
  - The computational complexity
  - Going beyond entanglement
- **Results**
  - A new algorithm for the out-of-equilibrium evolution
  - Benchmark on free fermions

Equilibration

- A closed system undergoes unitary dynamics and thus does not equilibrate, but keeps rotating
- Locally however the observables relax
- The evolution creates very complex states
- These states locally they look pretty simple



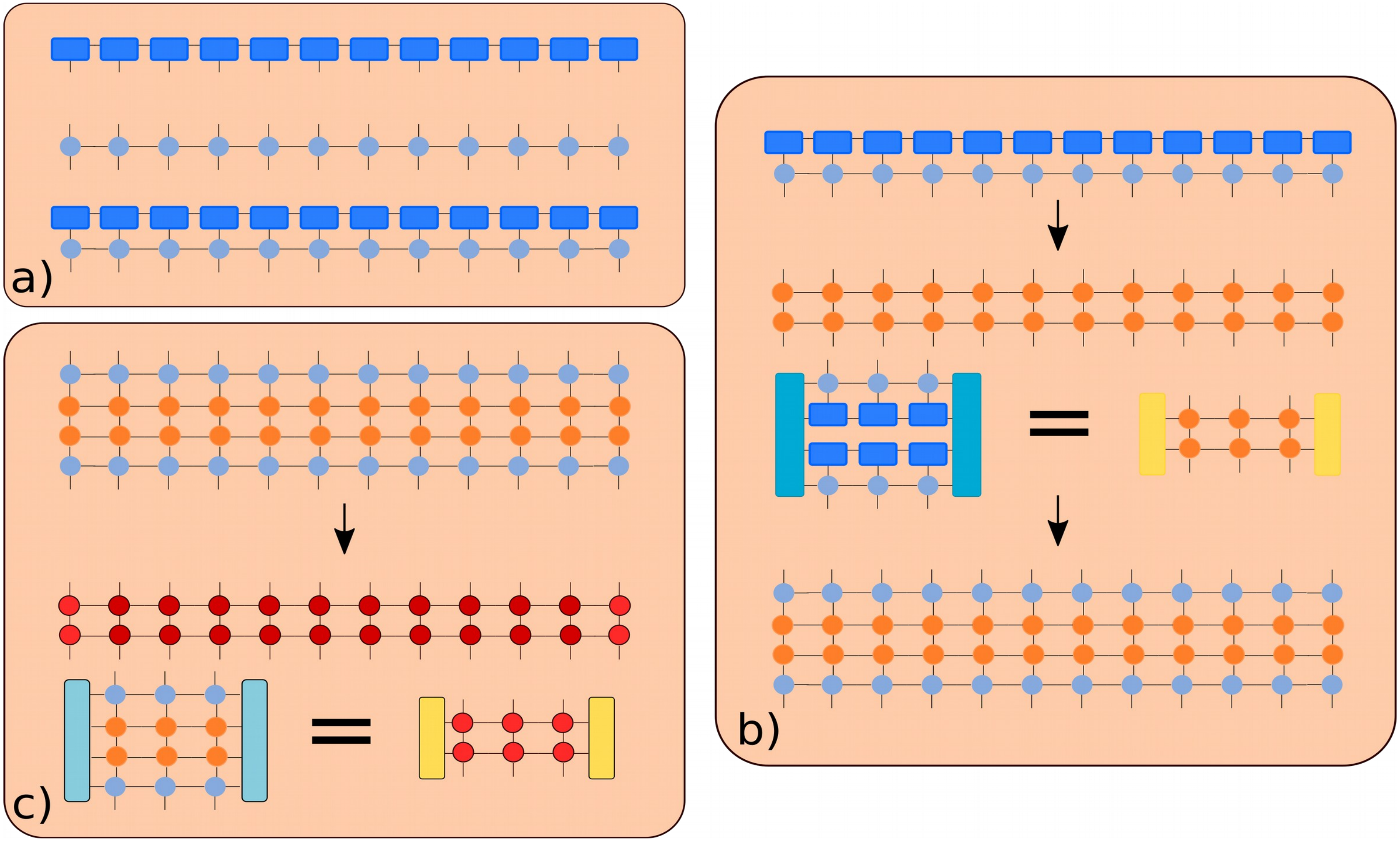
Robustness obtained by protecting local correlations

- If we perform an approximate dynamics that protects local correlations we should locally equilibrate to the correct state,
- The local dynamic is thus robust against changing the long-distance properties of the states,
- We can choose the simplest state that has the correct local properties, and project the dynamics on that state, **from pure to mixed**

Outline

- **Background**
  - The computational complexity
  - Going beyond entanglement
- **Results**
  - A new algorithm for the out-of-equilibrium evolution
  - Benchmark on free fermions

TN algorithm that protects local correlations



Outline

- **Background**
  - The computational complexity
  - Going beyond entanglement
- **Results**
  - A new algorithm for the out-of-equilibrium evolution
  - Benchmark on free fermions

Test it on fermionic Gaussian states

- Ground states of quadratic Hamiltonian of fermionic/bosonic operators are gaussian

$$H = \sum_{<ij>} a_i^\dagger a_j + \gamma a_i a_j + \mathrm{h.c.} + \mu \sum_i a_i^\dagger a_j$$

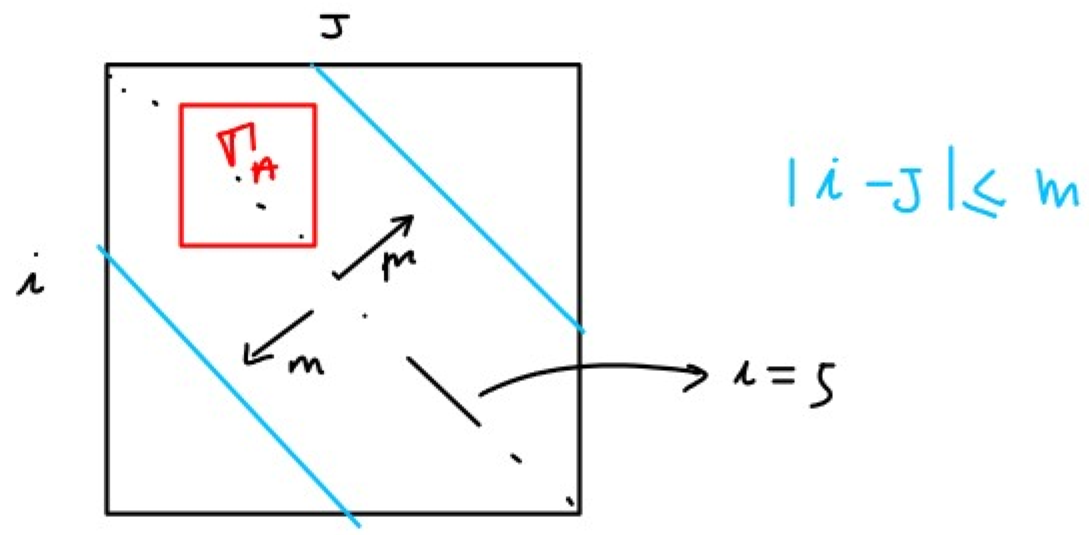
- Ground states are completely described by their correlation matrices

$$\Gamma_{ij} = \langle a_i^\dagger a_j \rangle$$

See also White Fishman (14)

Practically on the correlation matrix

- The relevant correlations are inside a band of size m in the correlation matrix

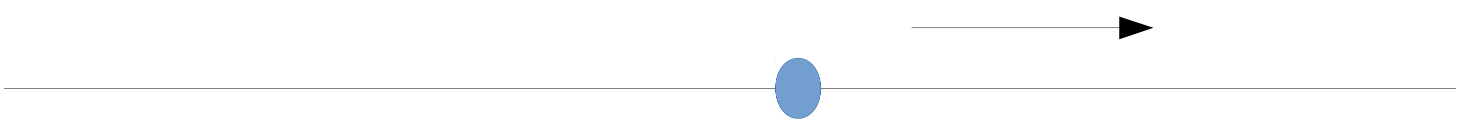


- At the truncation stage all correlation that are outside the band are zeroed

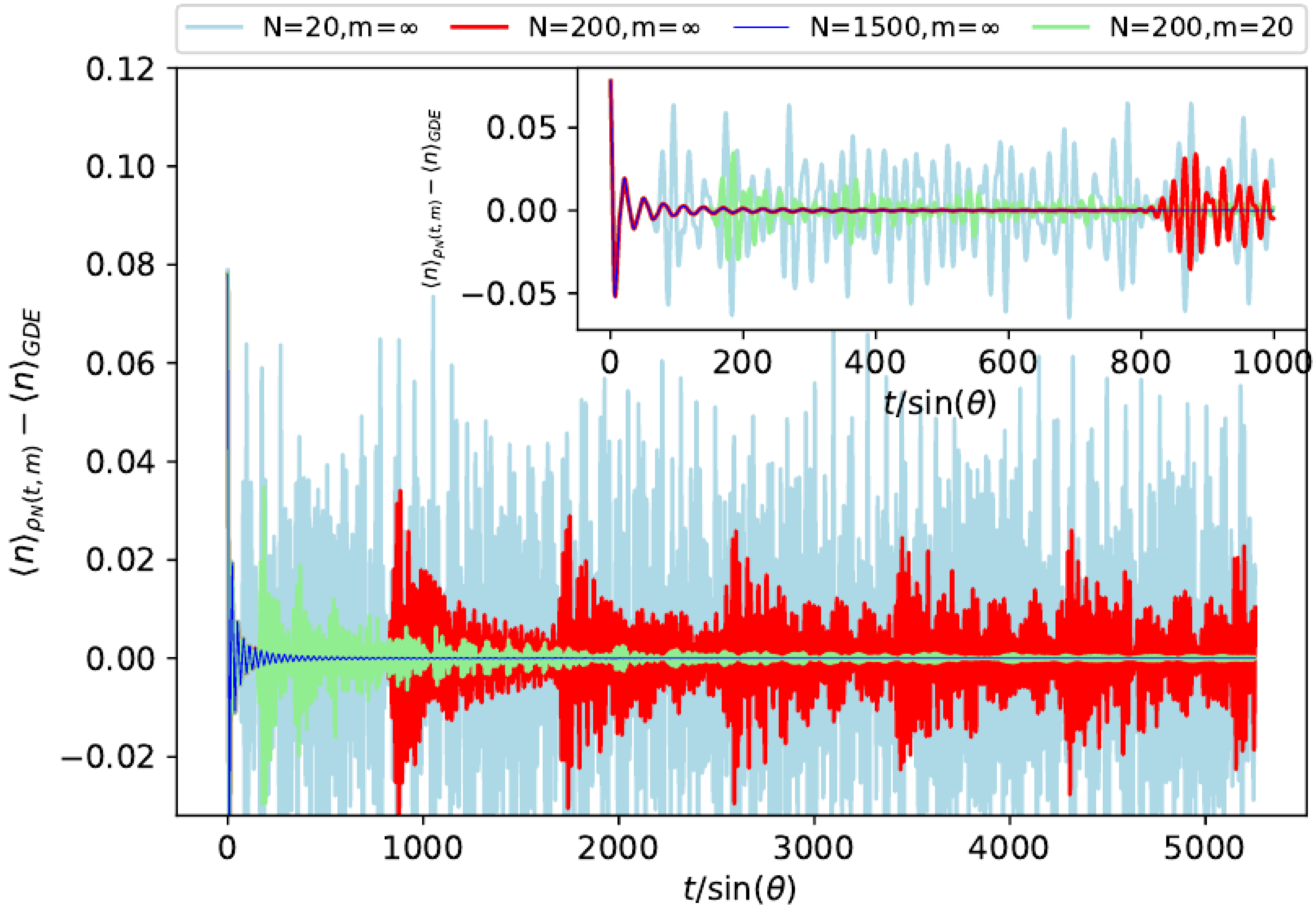
The quench protocol

$$H(\theta) = -\sin(\theta)\sum_{i=0}^{N-1}\sigma_i^x\sigma_{i+1}^x - \cos(\theta)\sum_{i=0}^{N-1}\sigma_i^z$$

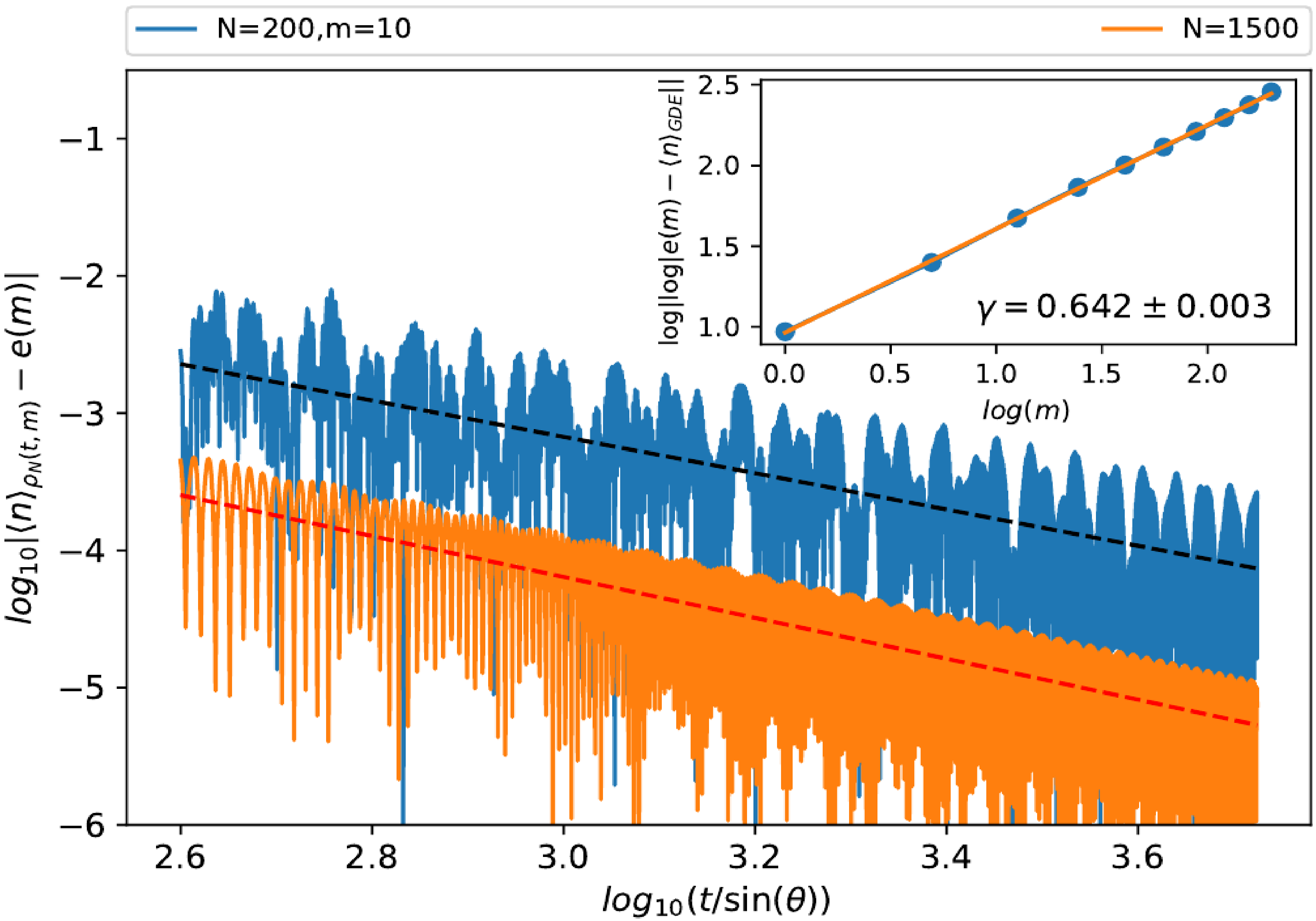
$$\theta : \frac{\pi}{4} + 0.1 \rightarrow \frac{\pi}{4} + 0.3$$



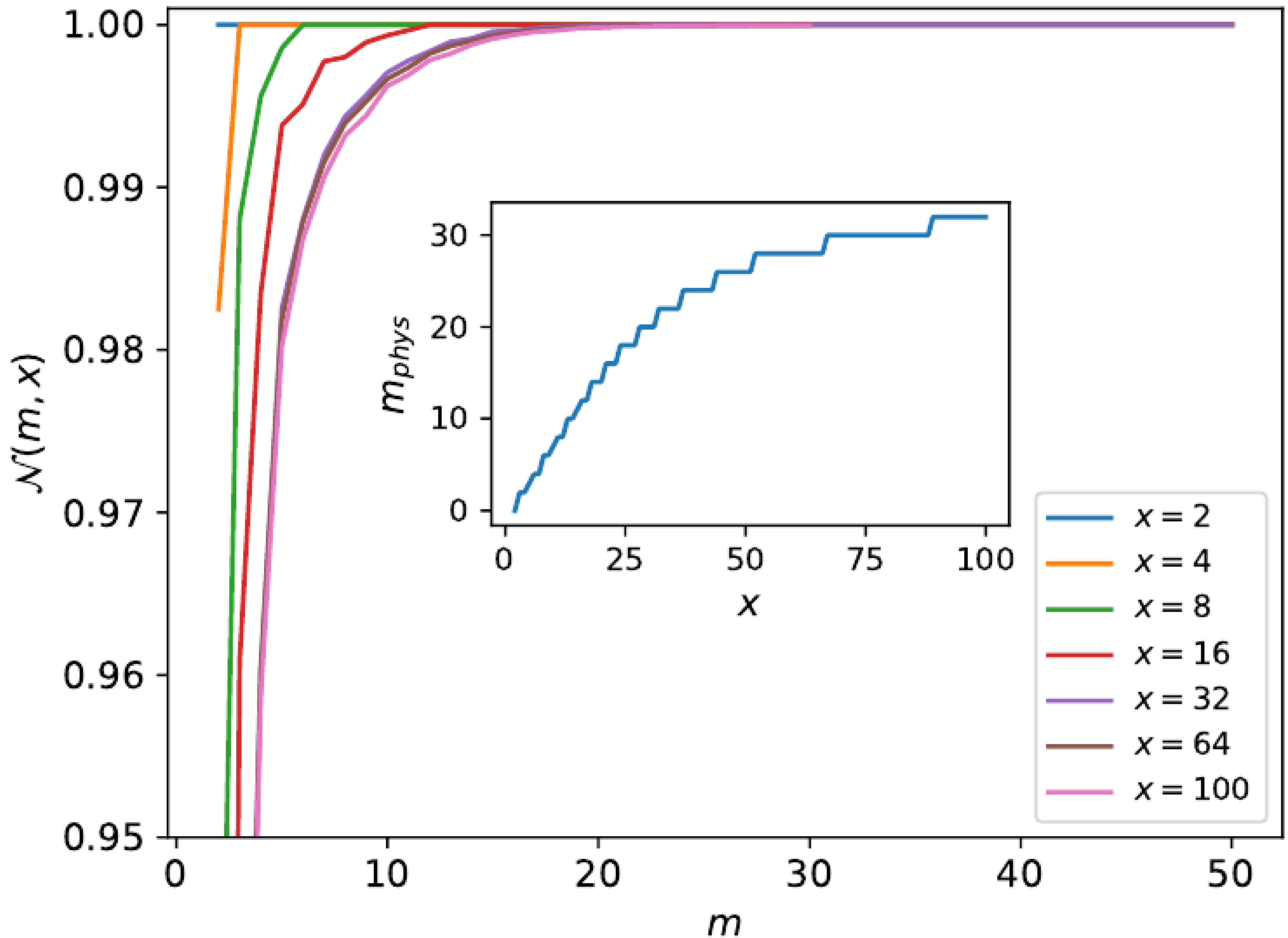
Quenches in Ising, using only local correlations



Precision of the long time predictions



Physicality of the state



Conclusions

Conclusions

- There are robust aspects in physics that can be predicted with limited computational resources
- Criticality at equilibrium and its universality.
- At short times, the presence and shape of the light cones are robust even to the addition of long range interactions
- At long times, the local equilibration process is robust once we protect from errors and imperfections certain local correlations
- We can exploit this robustness in order to design approximate algorithms that allow to predict those robust aspects of the OED.

Part 3 emerging gauge symmetry

Second solution, emerging gauge theories

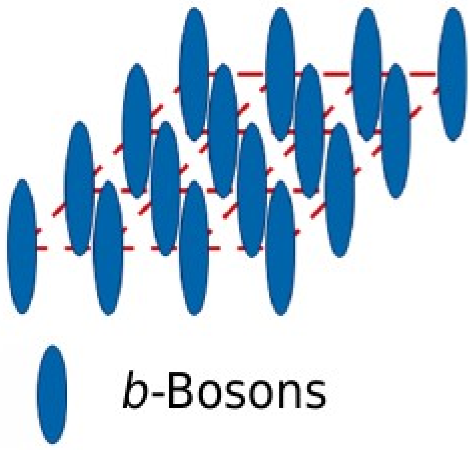
- Rather than working at the microscopic level, we can look for systems where gauge theories are known to emerge at low energies (e.g spin liquids)
- Natural candidates are fermions on frustrated lattice, but this is still challenging experimentally
- Here we focus on an exotic bosonic system that should be easier to realize [LT4] see also seminal works by Pachos, Fisher...

Exotic bosonic system

- Two species a bosons, and b bosons

$$H = -\sum_{\mathbf{j},\hat{\delta}} \left(J_a(\mathbf{j},\hat{\delta}) \hat{a}^\dagger_{\mathbf{j}} \exp\left[i\alpha_{\hat{\delta}} \hat{n}_{(\mathbf{j},\hat{\delta})} \right] \hat{a}_{\mathbf{j}+\hat{\delta}} + h.c. \right) - J_b \sum_{\mathbf{j},\hat{\delta}} \left(\hat{b}^\dagger_{\mathbf{j}} \hat{b}_{\mathbf{j}+\hat{\delta}} + h.c. \right) + \frac{U}{2} \sum_{\mathbf{j}} (\hat{a}^\dagger_{\mathbf{j}})^2 \hat{a}^2_{\mathbf{j}},$$

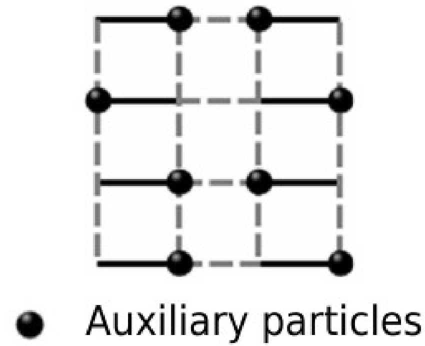
- The b bosons don’t interact and form tubes



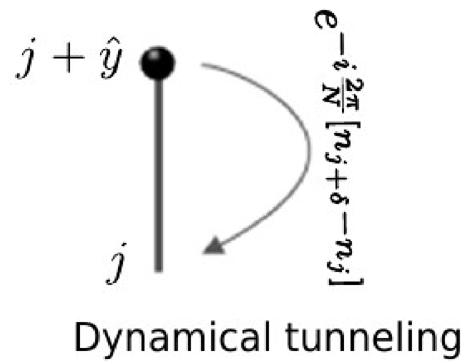
Dutta LT et al .Phys. Rev. A 95, 053608 (2017)

Bosonic system 2

- The hopping of the a boson is modulated
  - In amplitude



  - In phase



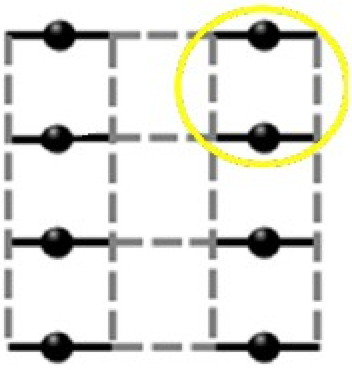
- The a bosons are hard-core

Low energy effective theory

The unperturbed Hamiltonian (a hard-core)

$$H_0 = -J_{1x} \sum_{j_x odd, j_y} \left(\hat{a}^\dagger_{\mathbf{j}} \hat{a}_{\mathbf{j}+\hat{x}} + h.c. \right)$$

At half filling for a bosons



$$\mathrm{J}_{2x} \ll J_b \ll J_y$$

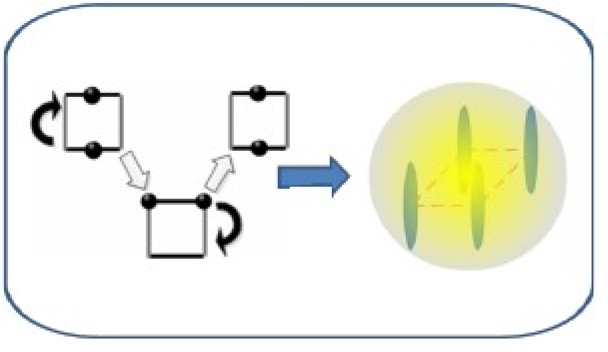
Dimer state

The b boson states are all degenerate,
no interactions, no hopping.

Low energy effective theory 2

- We add the y hopping of the a bosons as a perturbation

$$-J_y \sum_{\mathbf{j}} \left(\hat{a}^\dagger_{\mathbf{j}} \exp\left[i\frac{2\pi}{N} \hat{n}_{(\mathbf{j},\hat{y})} \right] \hat{a}_{\mathbf{j}+\hat{y}} + h.c. \right)$$



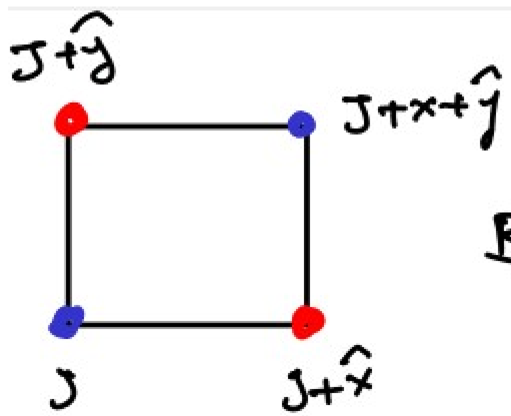
Effective plaquette interaction

- Second order processes produce an effective potential for the b bosons,

Four body effective interaction

- The four body term, depends on the difference of occupations inside a plaquette

$$H_{\text{pot}} = -2K \sum_p \cos\left[\hat{\mathcal{B}}_p\right]$$



$$B_p = \frac{2\pi}{N}\left(n_j + n_{j+x+y} - n_{j+x} - n_{j+y}\right)$$

$$\mathrm{K}{=}2\mathrm{J}_y^2/J_{1x}$$

The plaquette algebra

- The plaquette variable is compact, Z_N

$$\mathrm{B}_p \in (2\pi/N)\,[-(N-1)/2, ..-1, 0, 1, ...(N-1)/2].$$

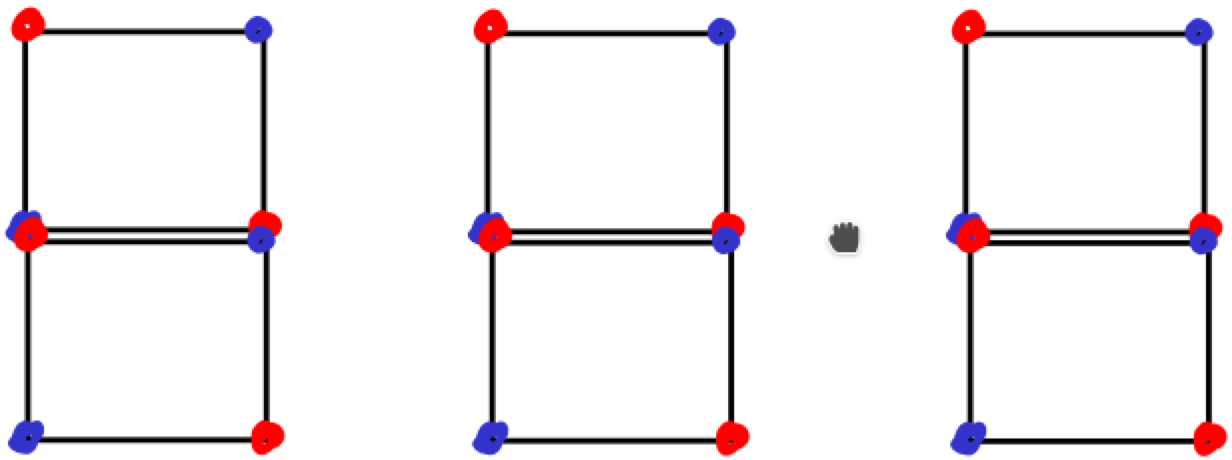
- We can introduce the conjugate operator that increases, or decreases B

$$\left[\hat{\mathcal{L}}_p, e^{\mp i\hat{\mathcal{B}}_p}\right] = \pm e^{\mp i\hat{\mathcal{B}}_p},$$

$$\left[\hat{\mathcal{B}}_p, e^{\pm i\frac{2\pi}{N}\hat{\mathcal{L}}_p}\right] = \pm\frac{2\pi}{N} e^{\pm i\frac{2\pi}{N}\hat{\mathcal{L}}_p}.$$

Stripes of non interacting plaquettes

- Each column is independent (as a consequence of dimerized a bosons)

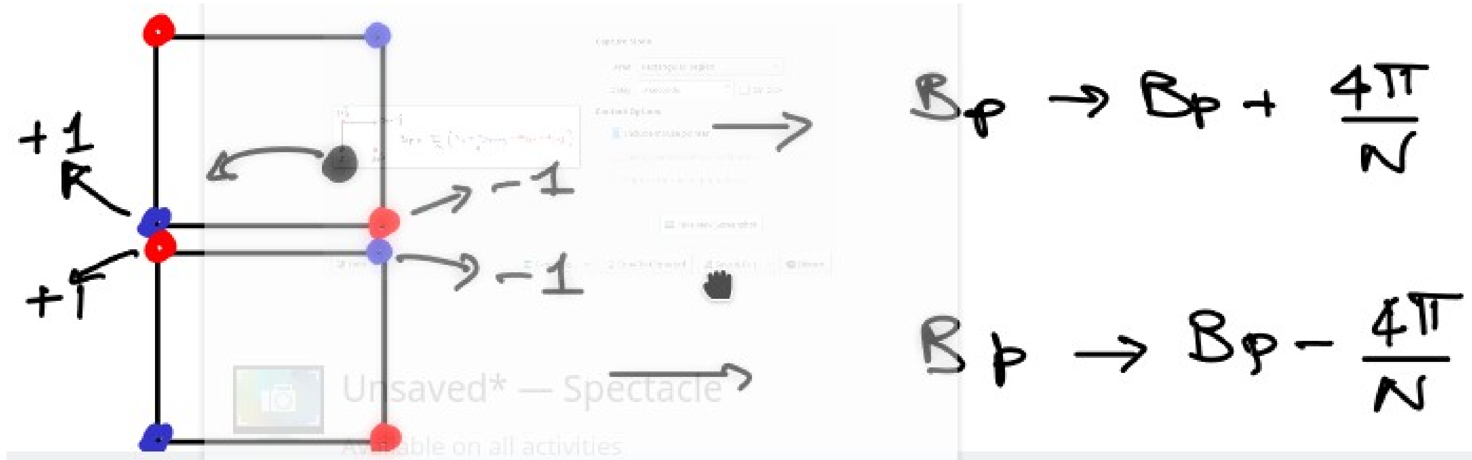


- We can now add the hopping of b bosons

$$J_b \sum_{\mathbf{j},\hat{\delta}} \left(\hat{b}^{\dagger}_{\mathbf{j}} \hat{b}_{\mathbf{j}+\hat{\delta}} + h.c. \right)$$

Inside the column

- Horizontal



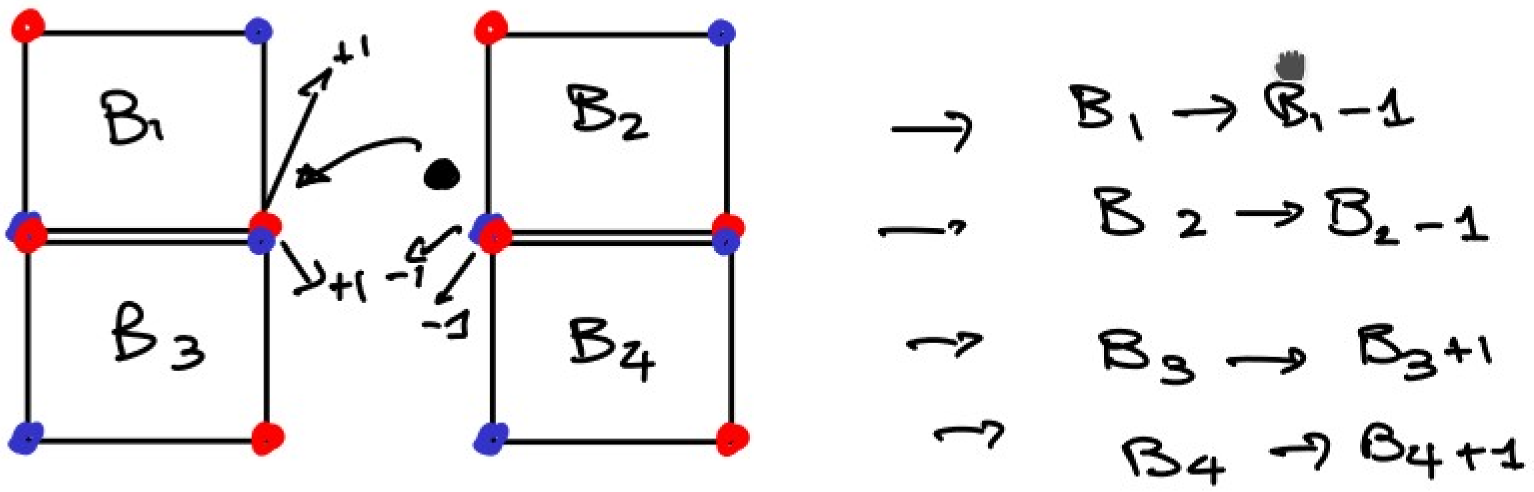
$$\hat{b}^{\dagger}_{\mathbf{j}}\hat{b}_{\mathbf{j}+\hat{x}}\,|\mathcal{B}_p,\mathcal{B}_{p-\hat{y}}\rangle\,;\quad j_x \in odd$$

$$=\sqrt{(n_{\mathbf{j}}+1)n_{\mathbf{j}+\hat{x}}}\left|\mathcal{B}_p+\frac{4\pi}{N},\mathcal{B}_{p-\hat{y}}-\frac{4\pi}{N}\right\rangle$$

- Vertical ….

Among the columns

- The hopping of b bosons changes four plaquettes ,



That up to a local transformation is equivalent to

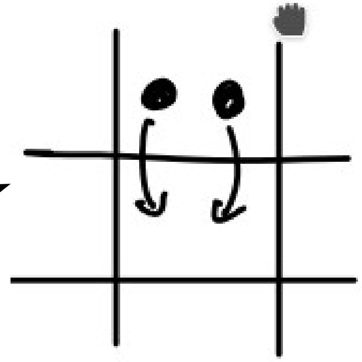
$$\bar{n} e^{i \frac{2\pi}{N} \left[\hat{\mathcal{L}}_p + \hat{\mathcal{L}}_{p+\hat{x}} - \hat{\mathcal{L}}_{p-\hat{y}} - \hat{\mathcal{L}}_{p+\hat{x}-\hat{y}}\right]} |\mathcal{B}_p, \mathcal{B}_{p-\hat{y}}, \mathcal{B}_{p+\hat{x}}, \mathcal{B}_{p+\hat{x}-\hat{y}}\rangle$$

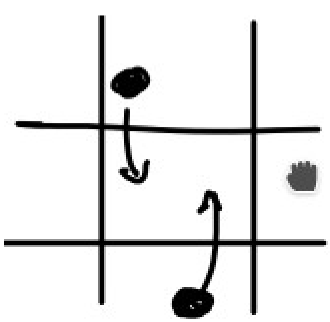
Summarizing, low energy bosonic model (compact)

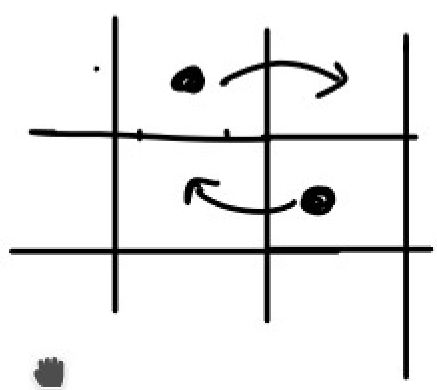
- The effective H is $H_{\text{pot}}/K + g^2 H_{\text{kin}}$

$$H_{\text{pot}} = -2K \sum_p \cos\left[\hat{\mathcal{B}}_p\right]$$

$$\begin{aligned} H_{\text{kin}} = 2\sum_p \Big[&\cos\left(\frac{4\pi}{N}\left[\hat{\mathcal{L}}_p - \hat{\mathcal{L}}_{p-\hat{y}}\right]\right) \\ &+ \cos\left(\frac{2\pi}{N}\left[\hat{\mathcal{L}}_p - \hat{\mathcal{L}}_{p+\hat{x}} - \hat{\mathcal{L}}_{p+\hat{y}} + \hat{\mathcal{L}}_{p+\hat{x}+\hat{y}}\right]\right) \\ &+ 2\cos\left(\frac{2\pi}{N}\left[2\hat{\mathcal{L}}_p - \hat{\mathcal{L}}_{p+\hat{y}} - \hat{\mathcal{L}}_{p-\hat{y}}\right]\right)\Big], \end{aligned}$$

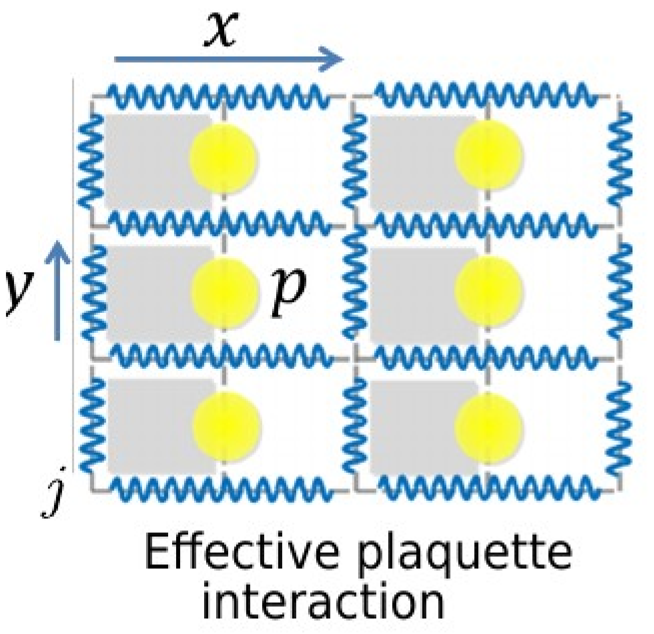






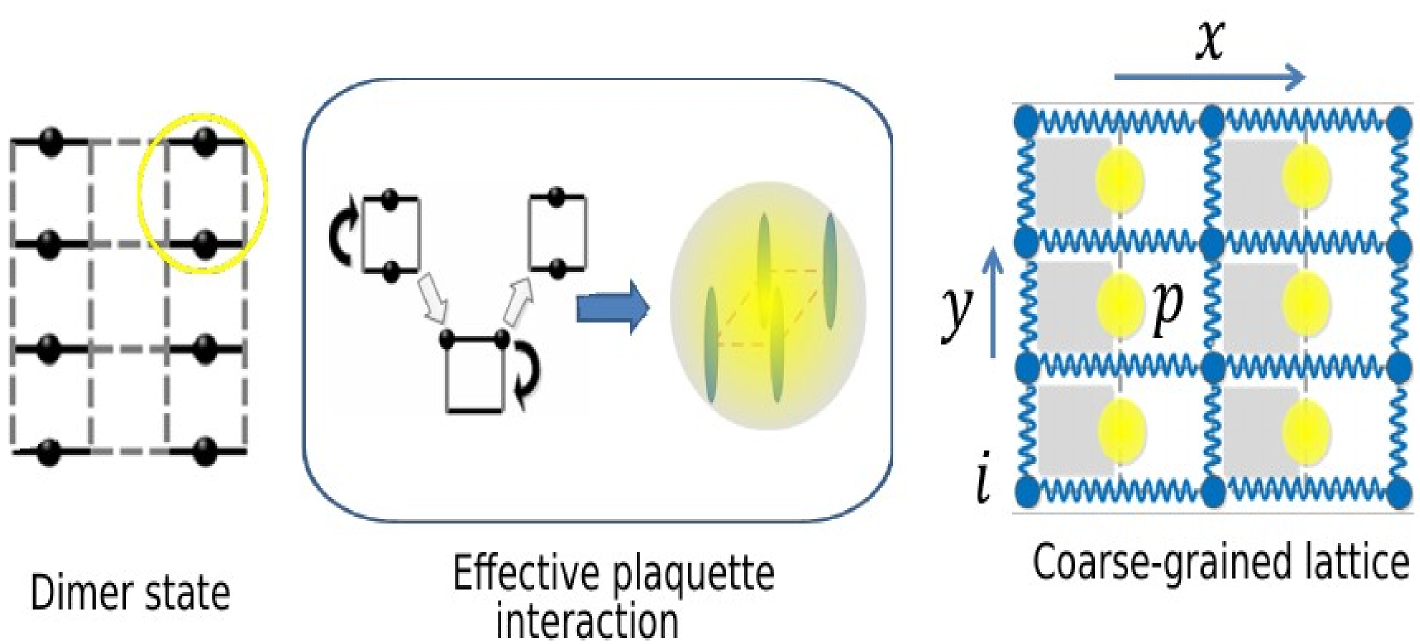
Why is it interesting?

- We have a spin/bosonic model (of plaquettes) that we can reinterpret as a gauge theory



$$\hat{\mathcal{E}}_{(\mathbf{j},\hat{x})} = \hat{\mathcal{L}}_p - \hat{\mathcal{L}}_{p-\hat{y}}$$

Until now,
Integrating out a bosons



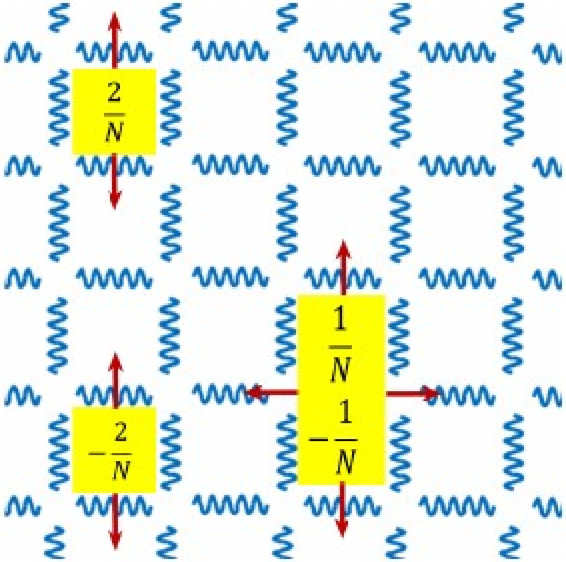
From staggered configuration of a bosons,
we obtain a gauge theory for b bosons

Exotic gauge theory

$$H_{\text{gauge}} = -2\sum_i \cos\hat{\mathcal{B}}_p - 2g^2\left[\sum_i\left[\cos\left(\frac{4\pi}{N}\mathcal{E}_{(i,\hat{x})}\right)\right.\right.$$
$$\left.\left.+2\cos\frac{2\pi}{N}\left[\mathcal{E}_{(i,\hat{x})}-\mathcal{E}_{(i+\hat{y},\hat{x})}\right]+\cos\frac{2\pi}{N}\left[\mathcal{E}_{(i,\hat{y})}-\mathcal{E}_{(i+\hat{y},\hat{y})}\right]\right]\right]$$

$$g \ll 1$$

Ground state is the vacuum of the plaquettes



Both monopoles (standard) and dipoles elementary excitations (exotic)

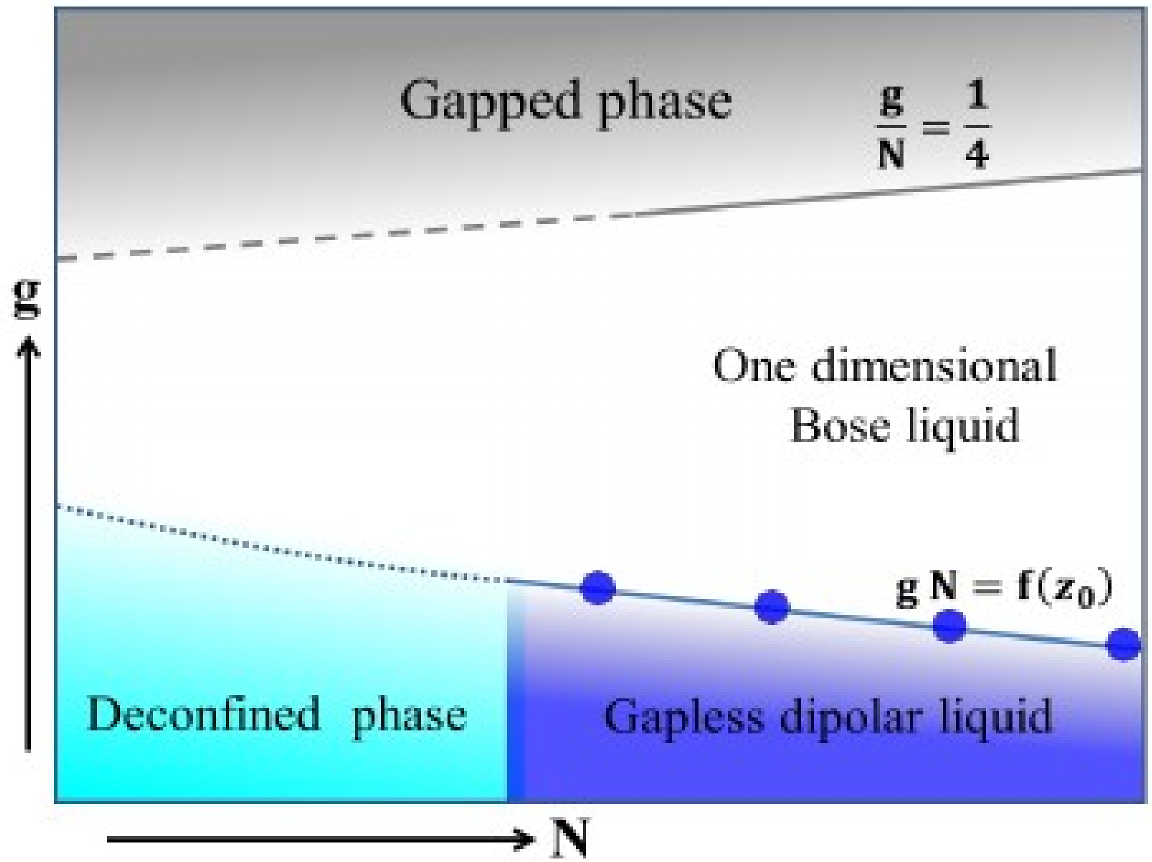
Gas of monopoles vs gas of dipoles

In 2D a Coulomb gas is screened (monopoles), thus monopole condense, and theory is confined Polyakov (70s).

Dipoles on the other hand do not screen, so they condense but still lead to long-range interactions (see Frolich 80s).

We can expect to have a phase with a masless photon in 2D.

Sketch of the phase diagram



Based on mapping to an effective action
(Sine Gordon) for dipoles

Deconfined QED3

Thank you !!!