



UNIWERSYTET JAGIELŁOŃSKI
W KRAKOWIE



Pair-production and real-time confinement in 1+1 discretized scalar QED

Work in progress

In collaboration with: Luca Tagliacozzo, Maciej Lewenstein, Jakub Zakrzewski

High-energy physics at ultra-cold temperatures, ECT*, Trento

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Uniwersytet Jagielloński, Kraków, Poland



NARODOWE CENTRUM NAUKI

Introduction

Quantum Field Theories (QFT) on lattice

QFT emerges as a unification of quantum mechanics and special relativity



Description of light-matter interaction
Quantum electrodynamics (QED)
U(1) gauge theory

“Second quantization”
Description of many-body systems in low-energy theories in non-relativistic regimes

In 1954 non-Abelian gauge theory (Yang-Mills) was introduced to explain strong interaction...

And by early 1970's the “Standard model” of particle physics was complete...

Perturbative approaches of Feynman diagrams worked greatly in high-energies and/or short distances

Standard Model of Elementary Particles			
three generations of matter (fermions)			interactions / force carriers (bosons)
I	II	III	
mass = 2.2 MeV/c ² charge 2/3 spin 1/2 u up	mass = 1.28 GeV/c ² charge 2/3 spin 1/2 c charm	mass = 173.1 GeV/c ² charge 2/3 spin 1/2 t top	mass = 0 charge 0 spin 1 g gluon
mass = 4.7 MeV/c ² charge -1/3 spin 1/2 d down	mass = 96 MeV/c ² charge -1/3 spin 1/2 s strange	mass = 4.18 GeV/c ² charge -1/3 spin 1/2 b bottom	mass = 0 charge 0 spin 1 γ photon
mass = 0.511 MeV/c ² charge -1 spin 1/2 e electron	mass = 105.66 MeV/c ² charge -1 spin 1/2 μ muon	mass = 177.68 GeV/c ² charge -1 spin 1/2 τ tau	mass = 91.19 GeV/c ² charge 0 spin 1 Z Z boson
mass < 2.2 eV/c ² charge 0 spin 1/2 ν _e electron neutrino	mass < 0.17 MeV/c ² charge 0 spin 1/2 ν _μ muon neutrino	mass < 18.2 MeV/c ² charge 0 spin 1/2 ν _τ tau neutrino	mass = 80.39 GeV/c ² charge ±1 spin 1 W W boson
LEPTONS			
QUARKS			
SCALAR BOSONS			
GAUGE BOSONS / VECTOR BOSONS			

However, such approaches failed to explain “confinement of quarks” to form composite hadrons, which works at the non-perturbative limit of low energies and/or large distances

Introduction

Quantum Field Theories (QFT) on lattice

Lattice gauge theory (LGT) on Euclidean space-time

Opened up new possibilities to approach
non-perturbative limits...

Since then Monte-Carlo simulations have been
used to study various facets of high energy physics
on lattice...

Within a few months, we got the Hamiltonian
Formulation of LGT...

Natural language of ultra-cold atomic systems,
for **analogue quantum simulation of LGT**...

PHYSICAL REVIEW D

VOLUME 10, NUMBER 8

15 OCTOBER 1974

Confinement of quarks*

Kenneth G. Wilson

Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14850

(Received 12 June 1974)

A mechanism for total confinement of quarks, similar to that of Schwinger, is defined which requires the existence of Abelian or non-Abelian gauge fields. It is shown how to quantize a gauge field theory on a discrete lattice in Euclidean space-time, preserving exact gauge invariance and treating the gauge fields as angular variables (which makes a gauge-fixing term unnecessary). The lattice gauge theory has a computable strong-coupling limit; in this limit the binding mechanism applies and there are no free quarks. There is unfortunately no Lorentz (or Euclidean) invariance in the strong-coupling limit. The strong-coupling expansion involves sums over all quark paths and sums over all surfaces (on the lattice) joining quark paths. This structure is reminiscent of relativistic string models of hadrons.

PHYSICAL REVIEW D

VOLUME 11, NUMBER 2

15 JANUARY 1975

Hamiltonian formulation of Wilson's lattice gauge theories

John Kogut*

Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14853

Leonard Susskind†

*Belfer Graduate School of Science, Yeshiva University, New York, New York
and Tel Aviv University, Ramat Aviv, Israel*

and Laboratory of Nuclear Studies, Cornell University, Ithaca, New York

(Received 9 July 1974)

Wilson's lattice gauge model is presented as a canonical Hamiltonian theory. The structure of the model is reduced to the interactions of an infinite collection of coupled rigid rotators. The gauge-invariant configuration space consists of a collection of strings with quarks at their ends. The strings are lines of non-Abelian electric flux. In the strong-coupling limit the dynamics is best described in terms of these strings. Quark confinement is a result of the inability to break a string without producing a pair.

Introduction

High energy physics at ultra-cold temperatures

Analogue quantum simulation

As a classical computer will take exponentially large time to simulate a quantum system of many particles

“An analogue quantum simulator is a bespoke device used to simulate aspects of the dynamics of another physical system using continuous parameters. An important example of an analogue quantum simulator is provided by a ‘source system’ comprised of ultra-cold atoms confined to an optical lattice. In a certain regime this system has been found to realize Hubbard Hamiltonians that describe ‘target systems’ that exhibit strongly correlated many body physics.”

- *Analogue Quantum Simulation: A Philosophical Prospectus*, by D. Hangleiter, J. Carolan, K. Thébault



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Analogue quantum simulation

As a classical computer will take exponentially large time to simulate a quantum system of many particles

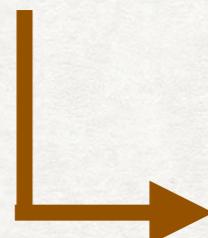
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- *Analogue Quantum Simulation: A Philosophical Prospectus*, by D. Hangleiter, J. Carolan, K. Thébault

Analogue quantum simulation of LGT

Ultracold quantum gases and lattice systems: quantum simulation of lattice gauge theories, U.-J. Wiese, Ann. Phys. (Berlin) **525**, 777 (2013).

Quantum simulations of lattice gauge theories using ultracold atoms in optical lattices, E. Zohar, J. I. Cirac, and B. Reznik, Rep. Prog. Phys. **79**, 014401 (2016).



“Quantum simulation of high energy physics, involving gauge field(s), may be both theoretically and experimentally harder than simulating other physical phenomena, in condensed matter physics, for example. In fact, three **basic requirements** must be met in order to obtain a quantum simulator of a gauge theory: *the simulation must include both fermionic and bosonic degrees of freedom (matter and gauge fields/particles). It must have a Relativistic, Lorentz invariance (a causal structure); and it must involve local gauge invariance, in order to obtain conservation of charges, and of course - the required interactions.*”

Introduction

High energy physics at ultra-cold temperatures

Our work

- We study the “simplest” high-energy model on lattice → discretized 1+1 dimensional scalar QED
- Matter particles are also bosonic →
 1. many works have been done with fermionic matter (eg. Schwinger model), but not so many with bosons
 2. not so simple from the low-energy perspective (both theory and experiment)
- Can be simulated using ultra-cold atomic systems

Requires large local Hilbert space dimension for numerical simulations



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Requires large local Hilbert space dimension for numerical simulations

- We use tensor network (TN) simulation in 1+1 dimension, i.e., we use matrix product state (MPS) ansatz.
- Two-site density matrix renormalization group (DMRG) and one-site variational optimization to find the ground state.
- Hybrid time-dependent variational principle (TDVP) to obtain time-evolution. Hybrid → first two-site scheme upto a certain bond-dimension, then one-site scheme.

Why TN:

1. Does not suffer from sign problem
2. Heavily successful in simulating strongly correlated systems in 1+1 dimension efficiently and accurately.
3. Can be extended to higher dimensions (e.g., PEPS).

Various talks in this workshop in this direction

Introduction

High energy physics at ultra-cold temperatures

Our goal

- Signatures of Schwinger pair-production in the dynamics



PHYSICAL REVIEW VOLUME 82, NUMBER 5 JUNE 1, 1951

On Gauge Invariance and Vacuum Polarization

JULIAN SCHWINGER
Harvard University, Cambridge, Massachusetts
(Received December 22, 1950)

This paper is based on the elementary remark that the extraction of gauge invariant results from a formally gauge invariant theory is ensured if one employs methods of solution that involve only gauge covariant quantities. We illustrate this statement in connection with the problem of vacuum polarization by a prescribed electromagnetic field. The vacuum current of a charged Dirac field, which can be expressed in terms of the Green's function of that field, implies an addition to the action integral of the electromagnetic field. Now these quantities can be related to the dynamical properties of a "particle" with space-time coordinates that depend upon a proper-time parameter. The proper-time equations of motion involve only electromagnetic field strengths, and provide a suitable gauge invariant basis for treating problems. Rigorous solutions of the equations of motion can be obtained for a constant field, and for a plane wave field. A renormalization of field strength and charge, applied to the modified lagrange function for constant fields, yields a finite, gauge invariant result which implies nonlinear properties for the electromagnetic field in the vacuum. The contribution of a zero spin charged field is also stated. After the same field strength renormalization, the modified physical quantities describing a plane wave in the vacuum reduce to just those of the Maxwell field; there are no nonlinear phenomena for a single plane wave, of arbitrary strength and spectral composition. The results obtained for constant (that is, slowly varying fields), are then applied to treat the two-photon disintegration of

a spin zero neutral meson arising from the polarization of the proton vacuum. We obtain approximate, gauge invariant expressions for the effective interaction between the meson and the electromagnetic field, in which the nuclear coupling may be scalar, pseudoscalar, or pseudovector in nature. The direct verification of equivalence between the pseudoscalar and pseudovector interactions only requires a proper statement of the limiting processes involved. For arbitrarily varying fields, perturbation methods can be applied to the equations of motion, as discussed in Appendix A, or one can employ an expansion in powers of the potential vector. The latter automatically yields gauge invariant results, provided only that the proper-time integration is reserved to the last. This indicates that the significant aspect of the proper-time method is its isolation of divergences in integrals with respect to the proper-time parameter, which is independent of the coordinate system and of the gauge. The connection between the proper-time method and the technique of "invariant regularization" is discussed. Incidentally, the probability of actual pair creation is obtained from the imaginary part of the electromagnetic field action integral. Finally, as an application of the Green's function for a constant field, we construct the mass operator of an electron in a weak, homogeneous external field, and derive the additional spin magnetic moment of $\alpha/2\pi$ magnetons by means of a perturbation calculation in which proper-mass plays the customary role of energy.

Pairs of particle and anti-particle are created by a strong electric field

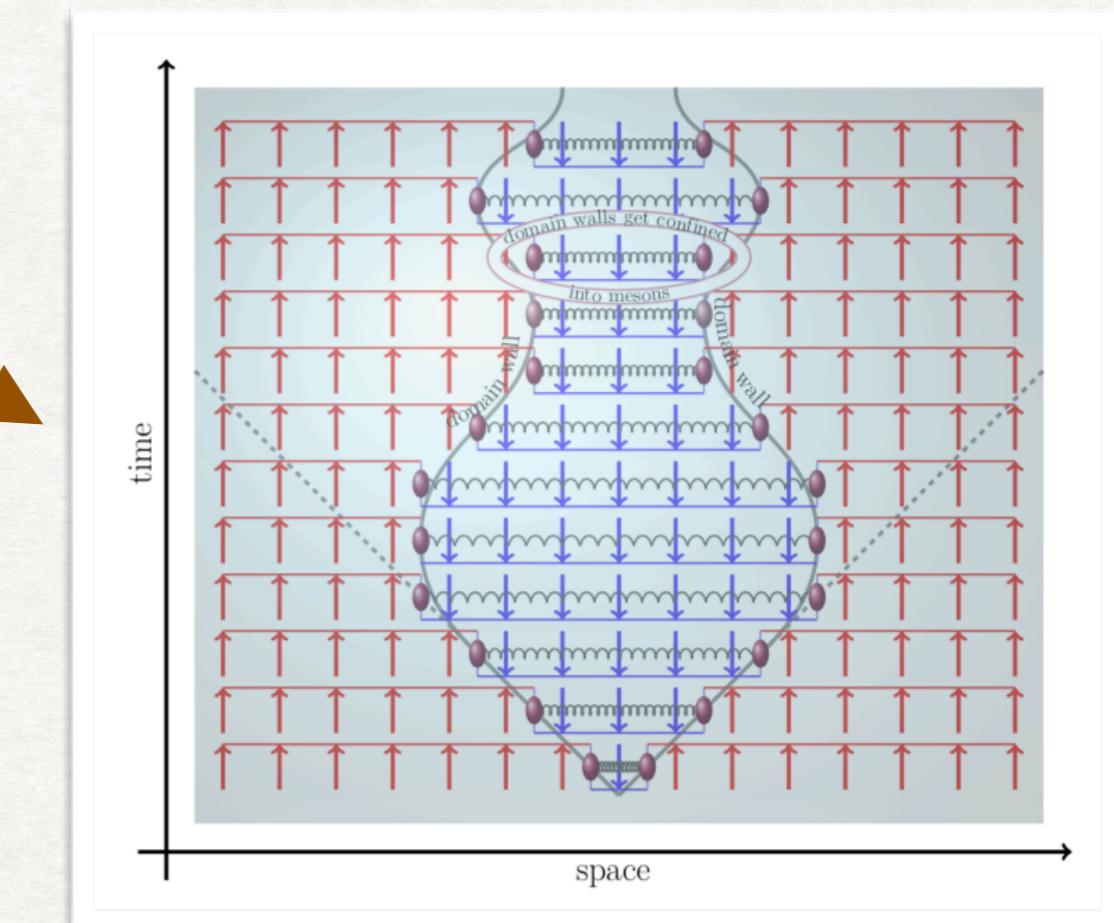
Introduction

High energy physics at ultra-cold temperatures

Our goal

- Signatures of Schwinger pair-production in the dynamics
- Real-time confinement

Attraction between particles and anti-particles increases with distance as they move away from each other due to the presence of confining potential. This strongly hinders the spreading of information in the dynamics, and results in bending of light-cone.



M. Kormos et. al., Nature Physics **13**, 246–249 (2017)

Introduction

High energy physics at ultra-cold temperatures

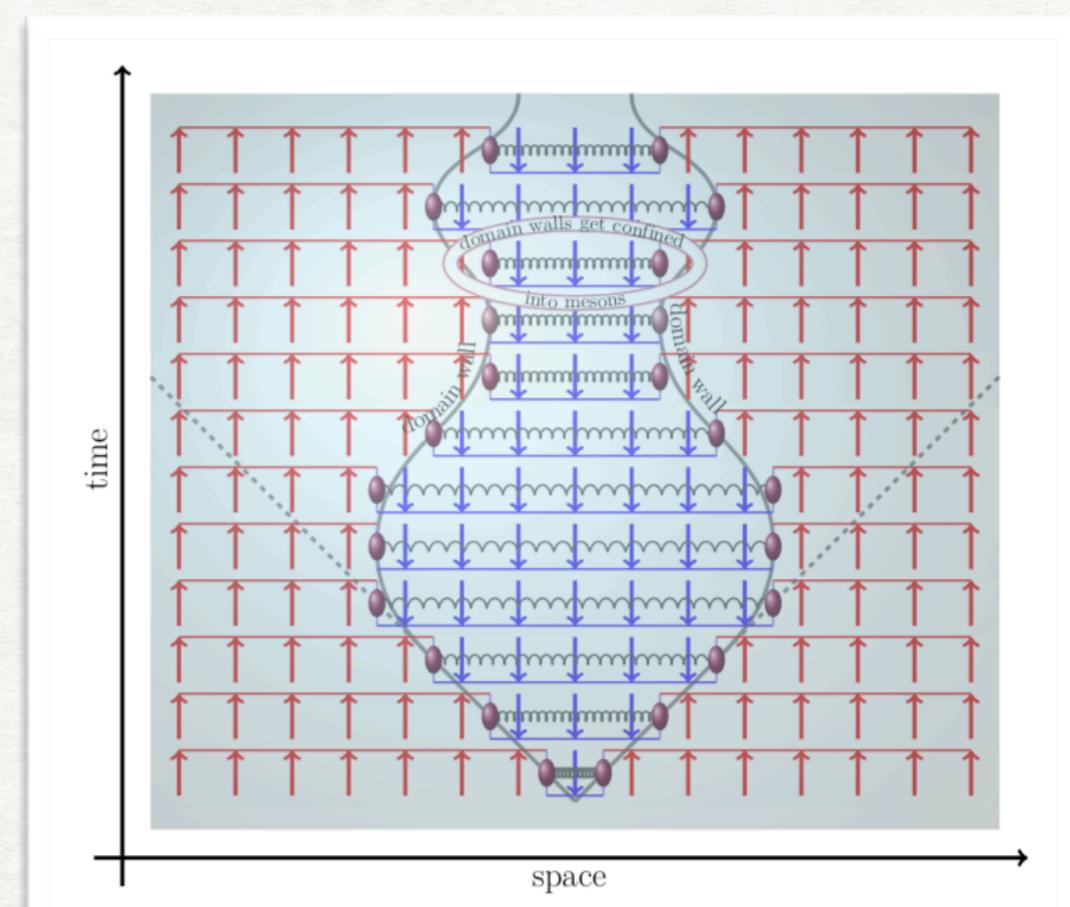
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Alternative to study confinement-deconfinement in equilibrium, which needs:

1. calculation of different Wilson loops
2. experimentally, a large system at very low temperature.



M. Kormos et. al., Nature Physics **13**, 246–249 (2017)

Introduction

High energy physics at ultra-cold temperatures

Our goal

- Signatures of Schwinger pair-production in the dynamics
- Real-time confinement
- Showcasing the use of TN to solve and simulate bosonic lattice gauge theories of high energy origins



For Schwinger model...



1. J. High Energy Phys. **11**, 158 (2013)
2. Phys. Rev. A **90**, 042305 (2014)
3. Phys. Rev. Lett. **113**, 091601 (2014)
4. Phys. Rev. D **92**, 034519 (2015)
5. Phys. Rev. D **94**, 085018 (2016)
6. Phys. Rev. D **96**, 114501 (2017)

Also Dr. Bañuls's talk...

The model

1+1 dimensional scalar QED on lattice

$$\mathcal{L} = - [D_\mu \phi]^* D^\mu \phi - m^2 |\phi|^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$
$$D_\mu = (\partial_\mu + iqA_\mu)$$

Metric convention $\rightarrow (-1, 1, 1, 1)$ or $(-1, 1)$

Note: with added $|\phi|^4$ interaction and imaginary mass,
this model exhibits Abelian Higgs mechanism

For quantum simulation: [D. Gonzalez-Cuadra et. al., New J. Phys. 19, 063038 \(2017\)](#)

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In 1+1 dimension... fixing temporal gauge... $A_t(x, t) = 0$

$$\mathcal{L} = |\partial_t \phi|^2 - (\partial_x - iqA_x)\phi^*(\partial_x + iqA_x)\phi - m^2 |\phi|^2 + \frac{1}{2}(\partial_t A_x)^2$$

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Hamiltonian after quantization...

$$\hat{H} = \int dx \left[\frac{1}{2} \hat{E}_x^2 + \hat{\Pi}^\dagger \hat{\Pi} + (\partial_x - iq\hat{A}_x)\hat{\phi}^*(\partial_x + iq\hat{A}_x)\hat{\phi} + m^2 \hat{\phi}^\dagger \hat{\phi} \right]$$

$$E_x = \partial_t A_x; \quad \Pi = \partial_t \phi^*; \quad \Pi^* = \partial_t \phi$$

$$[\hat{A}_x(x_1), \hat{E}_x(x_2)] = i\delta(x_1 - x_2); \quad [\hat{\phi}(x_1), \hat{\Pi}(x_2)] = [\hat{\phi}^\dagger(x_1), \hat{\Pi}^\dagger(x_2)] = i\delta(x_1 - x_2)$$

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Hamiltonian after discretization...

$$\hat{H} = \frac{a}{2} \sum_j \hat{E}_j^2 + \frac{1}{a} \sum_j \hat{\Pi}_j^\dagger \hat{\Pi}_j - \left[\frac{1}{a} \sum_j \hat{\phi}_{j+1}^\dagger \exp(-iq\hat{A}_j) \hat{\phi}_j + \text{h.c.} \right] + (am^2 + \frac{2}{a}) \sum_j \hat{\phi}_j^\dagger \hat{\phi}_j$$

Lattice spacing $\rightarrow a$

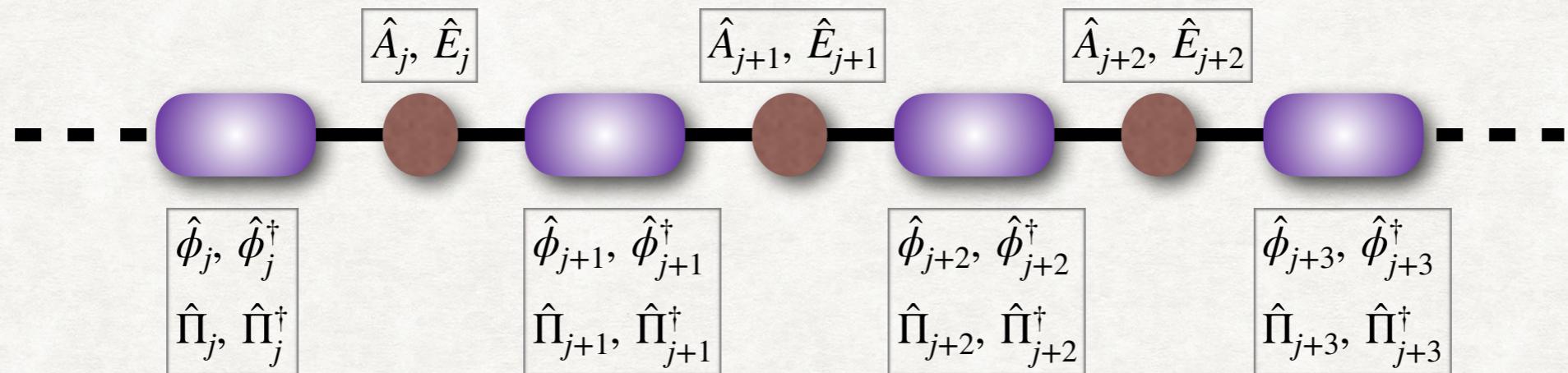
$$[\hat{A}_j, \hat{E}_k] = i\delta_{jk}; \quad [\hat{\phi}_j, \hat{\Pi}_k] = [\hat{\phi}_j^\dagger, \hat{\Pi}_k^\dagger] = i\delta_{jk}$$

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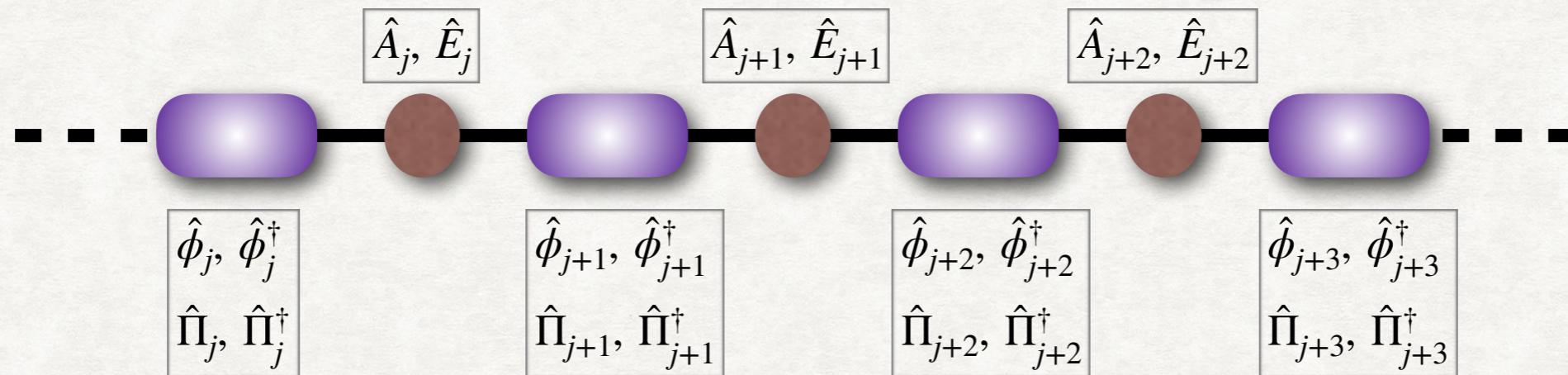


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Introduce bosonic creation and annihilation operators

$$\hat{\phi}_j = \frac{1}{\sqrt{2}}(\hat{a}_j + \hat{b}_j^\dagger); \quad \hat{\Pi}_j = \frac{i}{\sqrt{2}}(\hat{a}_j^\dagger - \hat{b}_j)$$

$$\hat{\phi}_j^\dagger = \frac{1}{\sqrt{2}}(\hat{a}_j^\dagger + \hat{b}_j); \quad \hat{\Pi}_j^\dagger = \frac{i}{\sqrt{2}}(\hat{b}_j^\dagger - \hat{a}_j)$$

Rescale gauge fields and introduce “ladder” operators

$$\hat{L}_j = \hat{E}_j/q; \quad \hat{\theta}_j = q\hat{A}_j; \quad \hat{U}_j = \exp(-i\hat{\theta}_j)$$

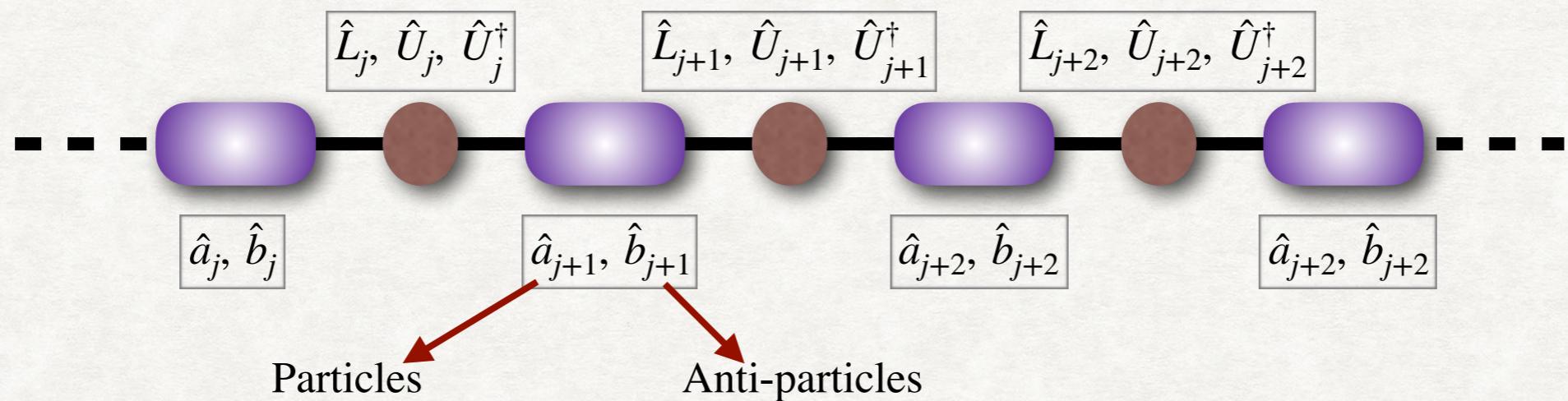
$$[\hat{L}_j, \hat{U}_j] = -\hat{U}_j; \quad [\hat{L}_j, \hat{U}_j^\dagger] = \hat{U}_j^\dagger$$

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Hamiltonian after discretization...

$$\hat{H} = \frac{aq^2}{2} \sum_j \hat{L}_j^2 + \left(\frac{am^2}{2} + \frac{3}{2a} \right) \sum_j (\hat{a}_j^\dagger \hat{a}_j + \hat{b}_j^\dagger \hat{b}_j^\dagger) + \left(\frac{am^2}{2} + \frac{1}{2a} \right) \sum_j (\hat{a}_j^\dagger \hat{b}_j^\dagger + \hat{a}_j^\dagger \hat{b}_j) - \frac{1}{2a} \sum_j \left[(\hat{a}_{j+1}^\dagger + \hat{b}_{j+1}) \hat{U}_j (\hat{a}_j + \hat{b}_j^\dagger) + \text{h.c.} \right]$$

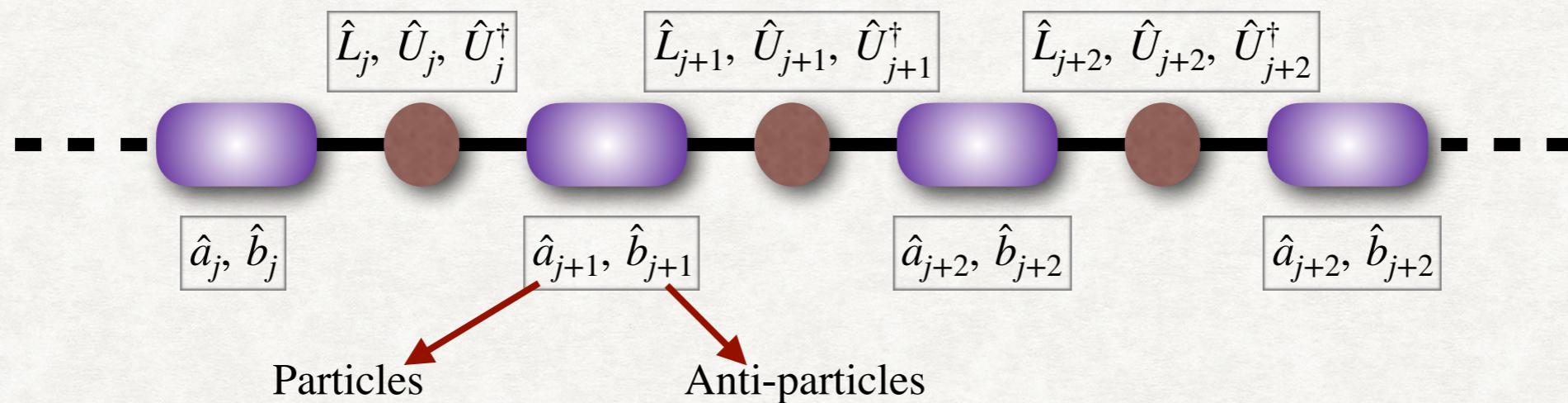


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Local U(1) invariance...

$$\begin{aligned}\hat{a}_j &\rightarrow e^{i\alpha_j} \hat{a}_j \\ \hat{b}_j &\rightarrow e^{-i\alpha_j} \hat{b}_j \\ \hat{U}_j &\rightarrow e^{-i\alpha_j} \hat{U}_j e^{i\alpha_{j+1}}\end{aligned}$$

Corresponding Gauss law generators...

$$\hat{G}_j = \hat{L}_j - \hat{L}_{j-1} - \left(\hat{a}_j^\dagger \hat{a}_j - \hat{b}_j^\dagger \hat{b}_j \right)$$

Dynamical charge: Particle—anti-particle number difference

We restrict ourself to $\hat{G}_j |\psi\rangle = 0$ sector for $\forall j$

The model

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Dispersion relation without gauge fields...

$$\omega(k) = \frac{1}{a} \sqrt{2 + a^2 m^2 - 2 \cos(ka)}$$
$$\lim_{a \rightarrow 0} \omega(k) = \sqrt{k^2 + m^2}$$

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We work in the dimensionless units...

$$\frac{2\hat{H}}{aq^2} \rightarrow \hat{H} = \sum_j \hat{L}_j^2 + ((m/q)^2 + 3x) \sum_j (\hat{a}_j^\dagger \hat{a}_j + \hat{b}_j^\dagger \hat{b}_j) + ((m/q)^2 + x) \sum_j (\hat{a}_j^\dagger \hat{b}_j^\dagger + \hat{a}_j \hat{b}_j) - x \sum_j \left[(\hat{a}_{j+1}^\dagger + \hat{b}_{j+1}) \hat{U}_j (\hat{a}_j + \hat{b}_j^\dagger) + \text{h.c.} \right]$$
$$x = 1/a^2 q^2$$

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$x = 1/a^2 q^2$

Using Gauss law, we can also integrate-out the gauge fields for an open chain...

$$\hat{L}_j = \sum_{l \leq j} (\hat{n}_l^a - \hat{n}_l^b)$$

leads to a Hamiltonian with long-range interaction

intra-species repulsion — inter-species attraction

Results

- We consider an open chain of $N = 60$ sites ($N - 1 = 59$ bonds).
- We truncate bosonic dimension to 5 on each sites for both the types.
- We use two-site DMRG upto maximum bond dimension, $D_{max} = 200$, and then one-site variational optimization (“one-site DMRG”) to converge to the lowest energy state ($|\Omega\rangle$) in the variational manifold.
- For time-evolution, we employ first two-site TDVP upto maximum bond dimension, $D_{max} = 640$, then we switch to one-site version for better accuracy.

Results

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Quenches performed...

	Local	Global
Type-I	Initially inject energy to the ground state	
Type-II	Evolve the system under a background field	

Results

Local quench: Type-I

$$|\psi(t=0)\rangle = (\hat{a}_{N/2}^\dagger + \hat{b}_{N/2}) \hat{U}_{N/2}^\dagger (\hat{a}_{N/2+1} + \hat{b}_{N/2+1}^\dagger) |\Omega\rangle$$

Hamiltonian...

$$\hat{H} = \sum_j \hat{L}_j^2 + ((m/q)^2 + 3x) \sum_j (\hat{a}_j^\dagger \hat{a}_j + \hat{b}_j \hat{b}_j^\dagger) + ((m/q)^2 + x) \sum_j (\hat{a}_j^\dagger \hat{b}_j^\dagger + \hat{a}_j \hat{b}_j) - x \sum_j \left[(\hat{a}_{j+1}^\dagger + \hat{b}_{j+1}) \hat{U}_j (\hat{a}_j + \hat{b}_j^\dagger) + \text{h.c.} \right]$$

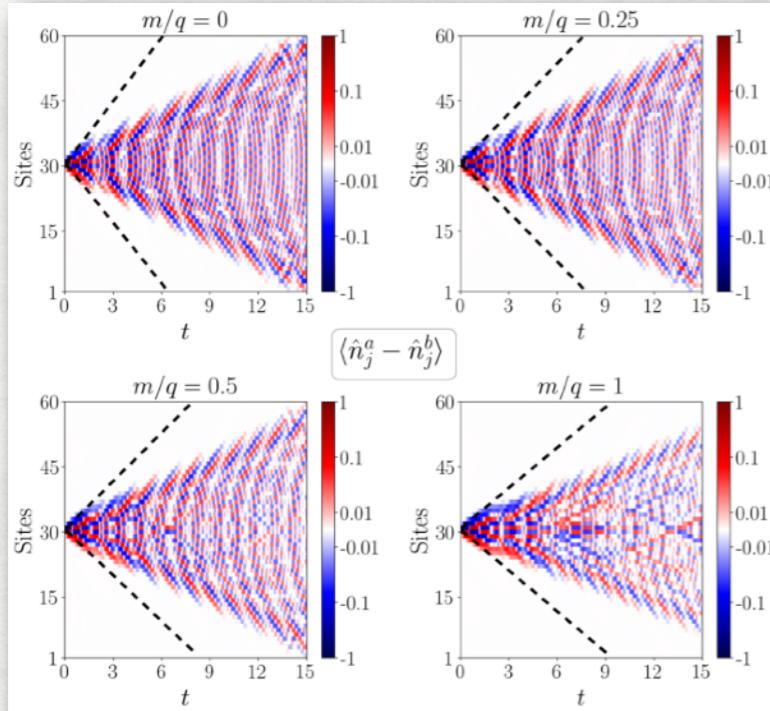
$x = 1/a^2 q^2$

Results

Local quench: Type-I

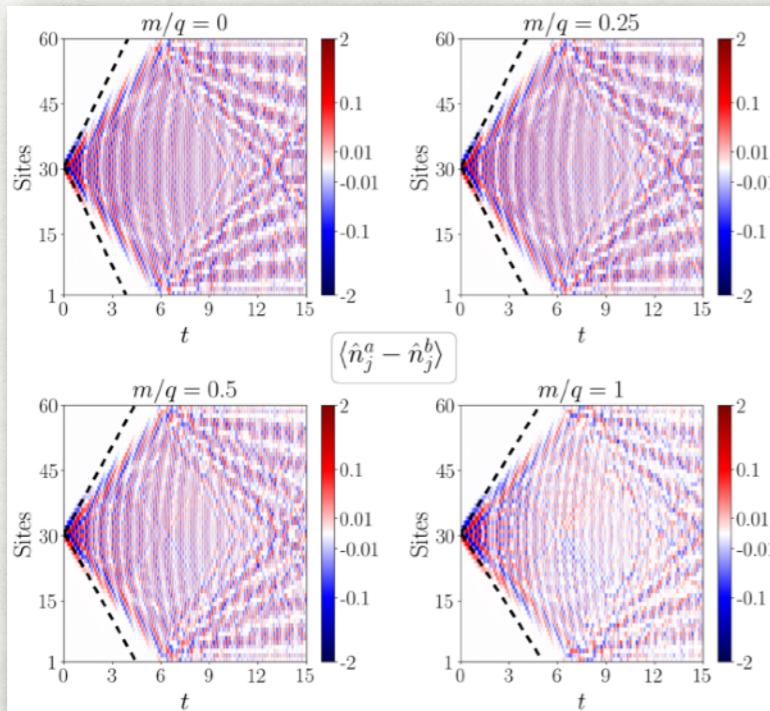
$$|\psi(t=0)\rangle = (\hat{a}_{N/2}^\dagger + \hat{b}_{N/2}) \hat{U}_{N/2}^\dagger (\hat{a}_{N/2+1}^\dagger + \hat{b}_{N/2+1}^\dagger) |\Omega\rangle$$

$x = 2$



Particles and anti-particles fly away at opposite directions
But velocity get reduced due to confinement...

$x = 4$

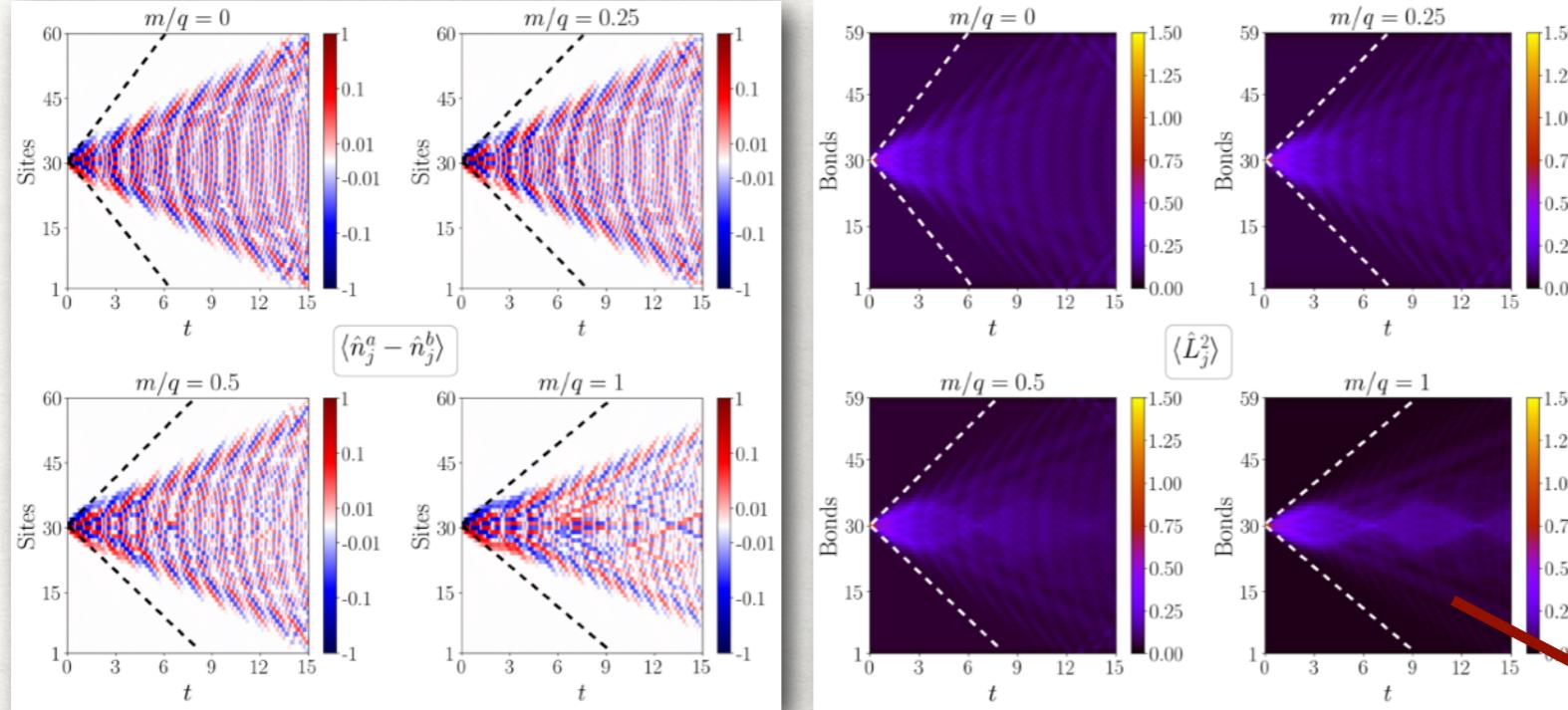


Results

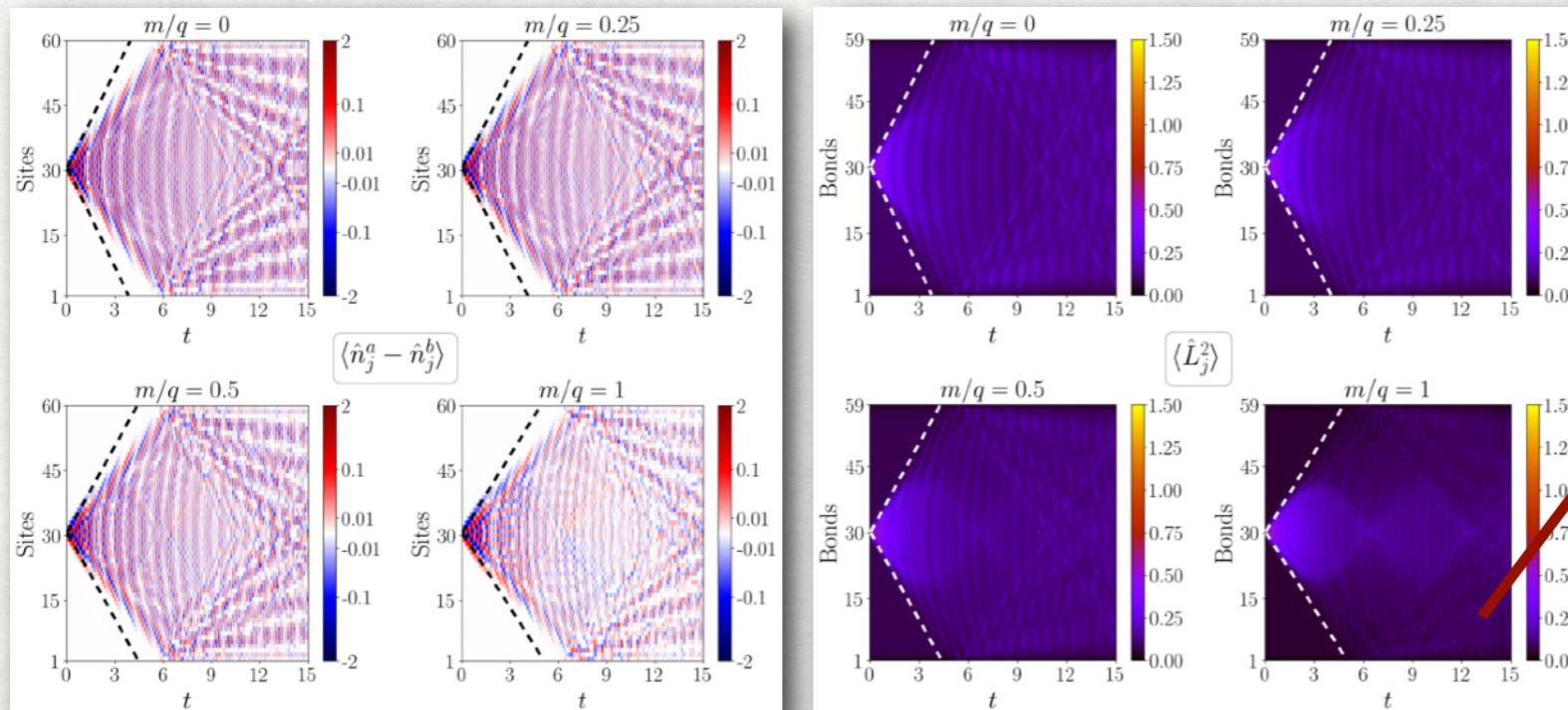
Local quench: Type-I

$$|\psi(t=0)\rangle = (\hat{a}_{N/2}^\dagger + \hat{b}_{N/2}) \hat{U}_{N/2}^\dagger (\hat{a}_{N/2+1}^\dagger + \hat{b}_{N/2+1}^\dagger) |\Omega\rangle$$

$x = 2$



$x = 4$



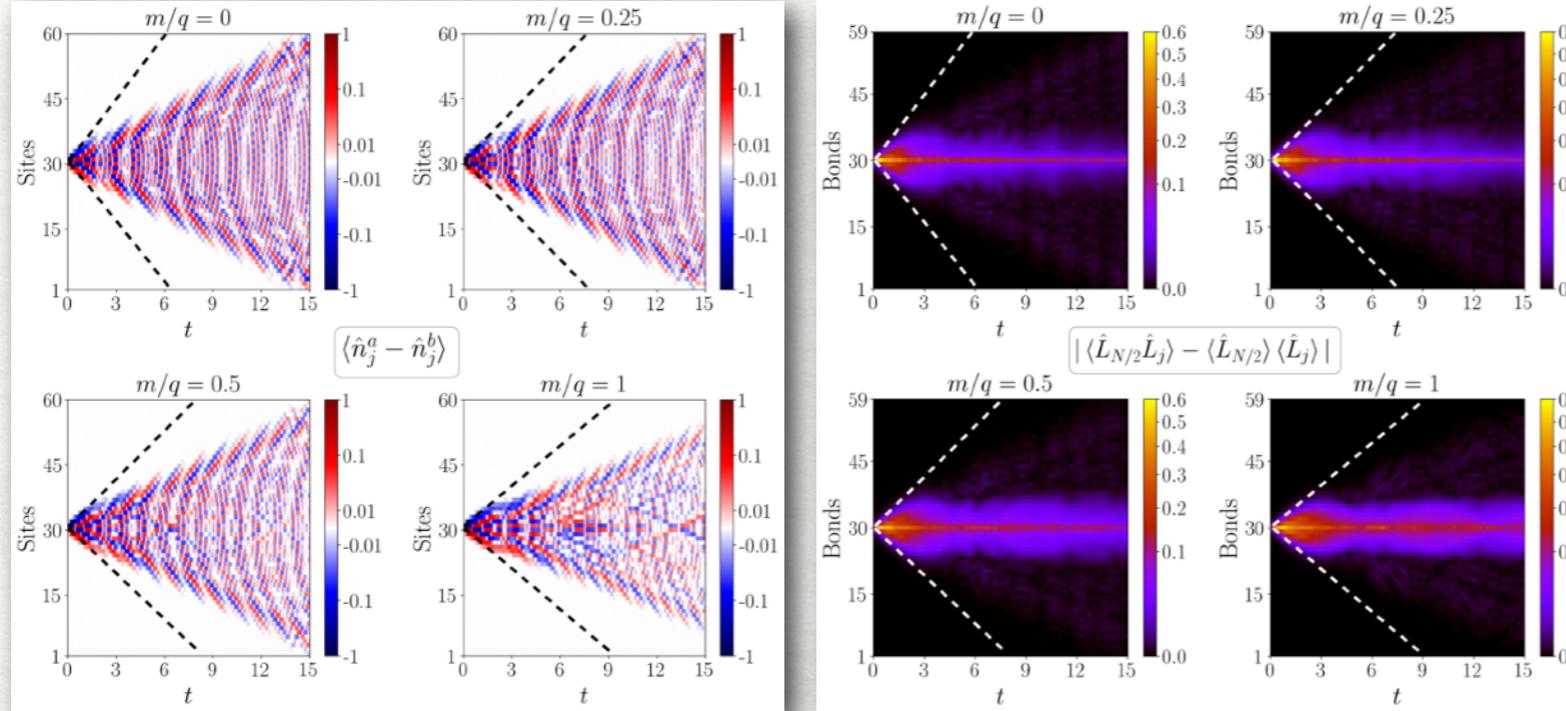
Clear signatures of
domain-wall confinement

Results

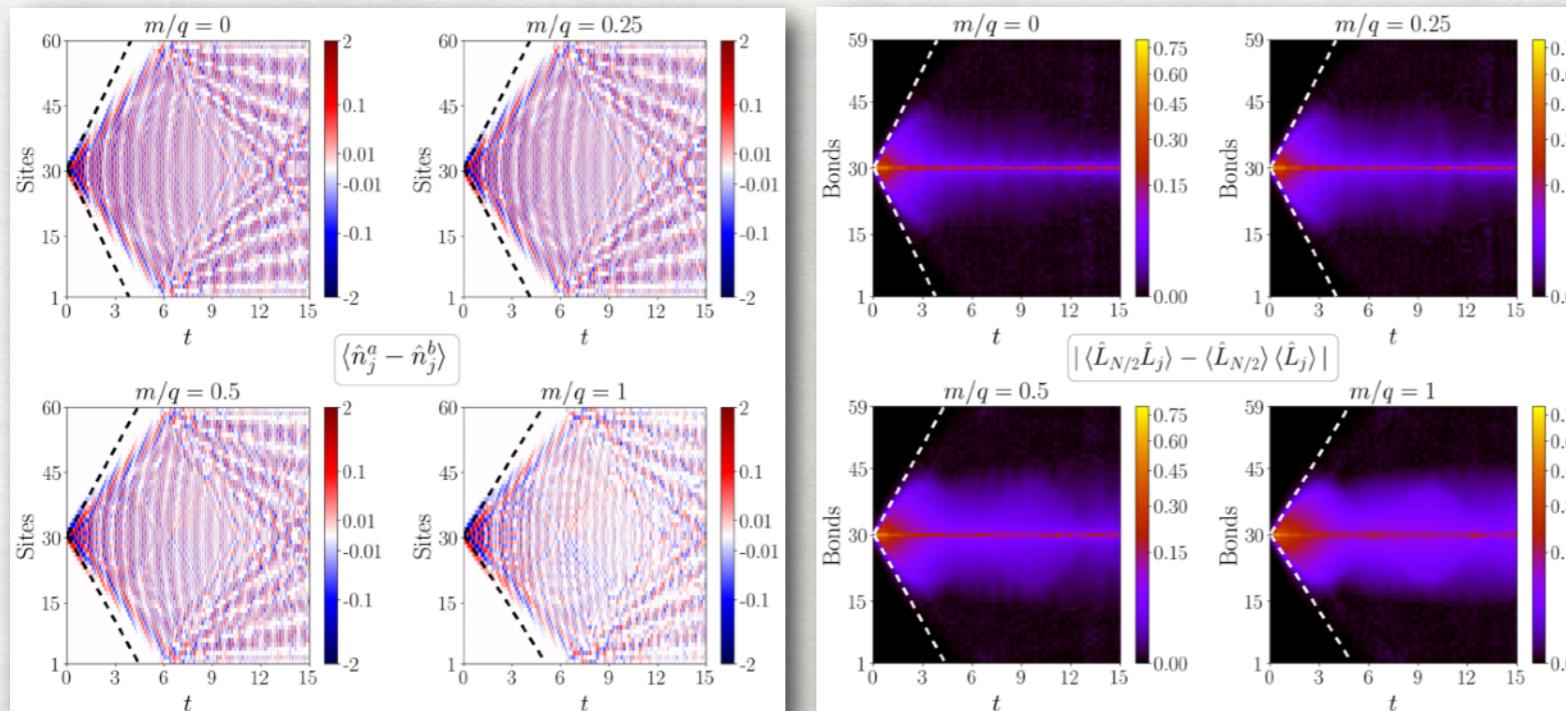
Local quench: Type-I

$$|\psi(t=0)\rangle = (\hat{a}_{N/2}^\dagger + \hat{b}_{N/2}) \hat{U}_{N/2}^\dagger (\hat{a}_{N/2+1}^\dagger + \hat{b}_{N/2+1}^\dagger) |\Omega\rangle$$

$x = 2$



$x = 4$



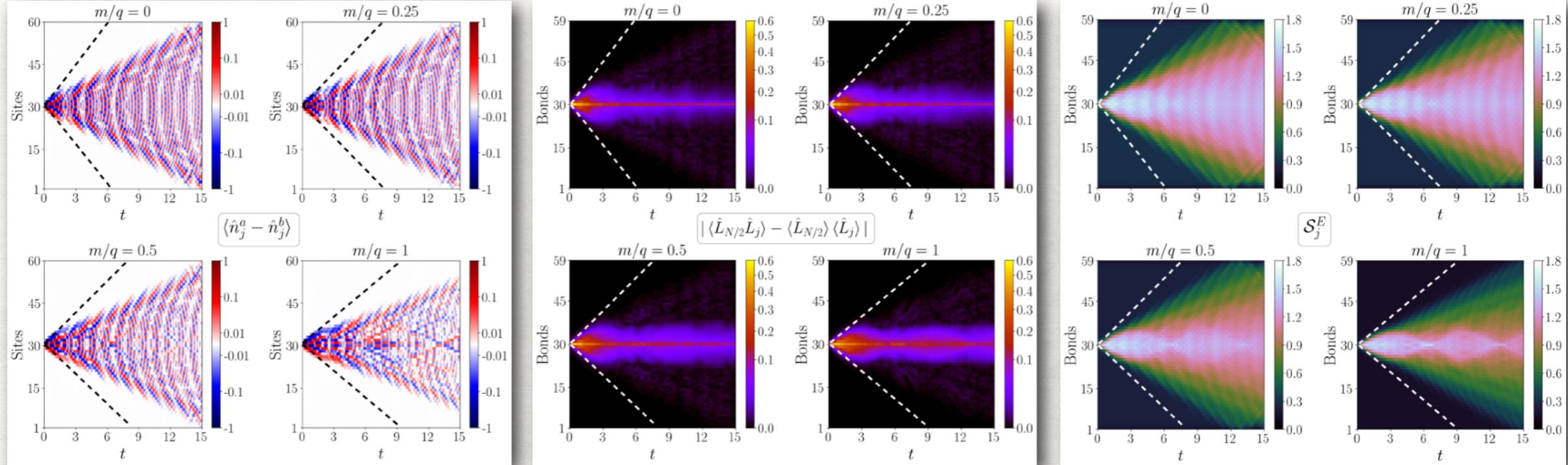
Clear signatures of domain-wall confinement

Results

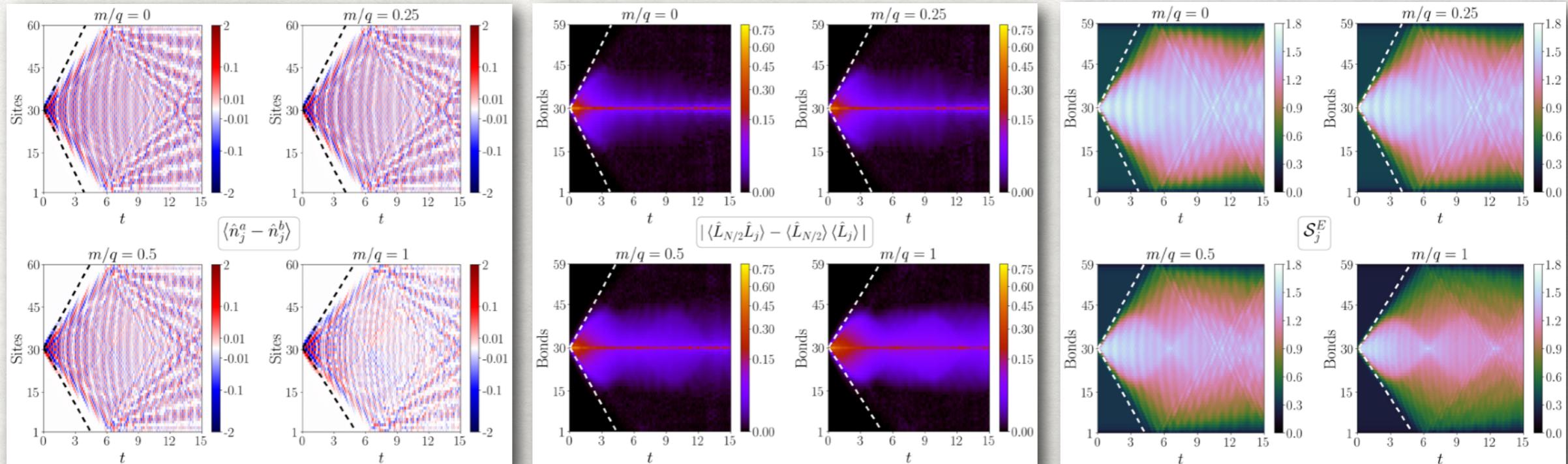
Local quench: Type-I

$$|\psi(t=0)\rangle = (\hat{a}_{N/2}^\dagger + \hat{b}_{N/2}) \hat{U}_{N/2}^\dagger (\hat{a}_{N/2+1}^\dagger + \hat{b}_{N/2+1}^\dagger) |\Omega\rangle$$

$x = 2$



$x = 4$



Results

Local quench: Type-II

Evolution with... $\hat{L}_{N/2} \rightarrow \hat{L}_{N/2} + \alpha$

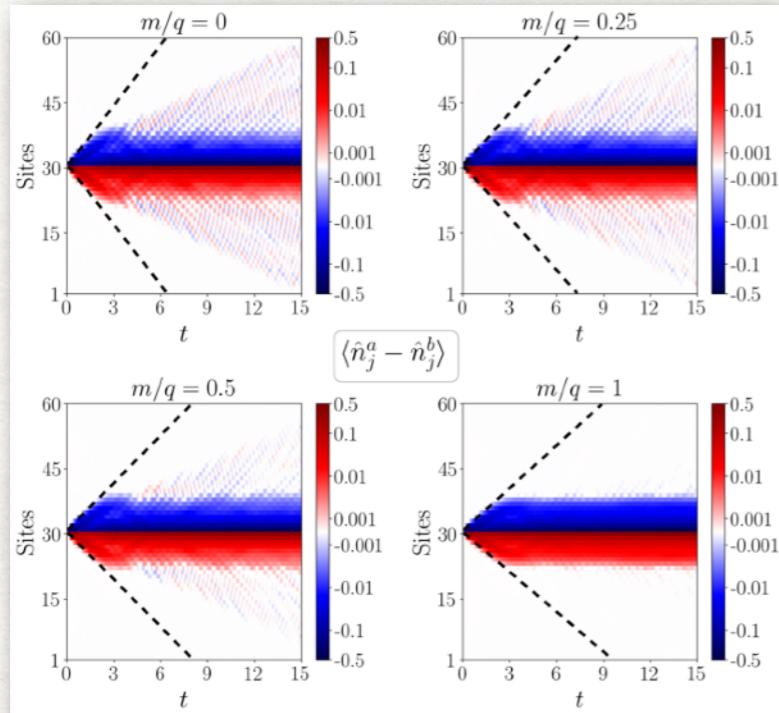
Results

Local quench: Type-II

Evolution with... $\hat{L}_{N/2} \rightarrow \hat{L}_{N/2} + \alpha$

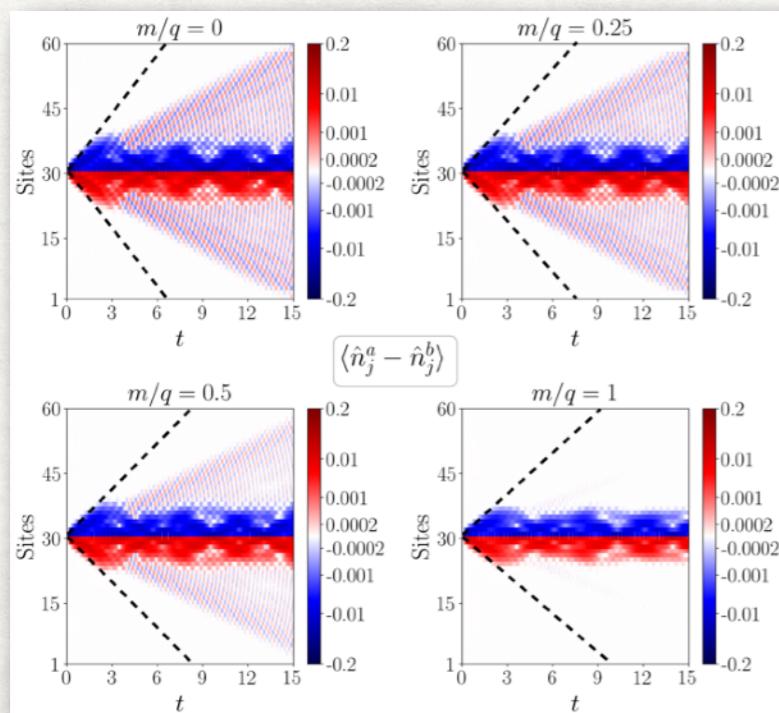
$\alpha = -10$

$x = 2$



$\alpha = -20$

$x = 2$



**Pair-production, confinement
Light-cone bends as expected**

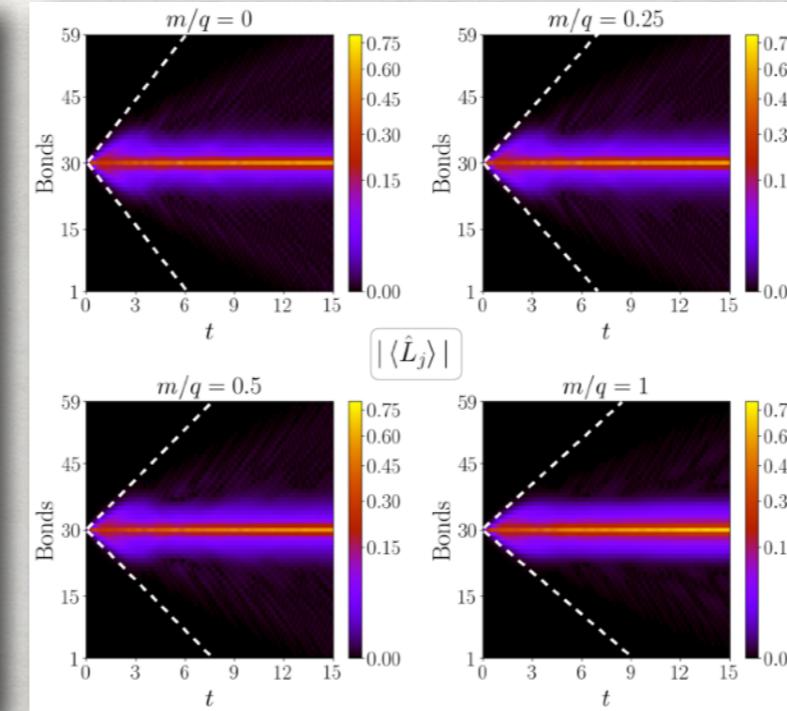
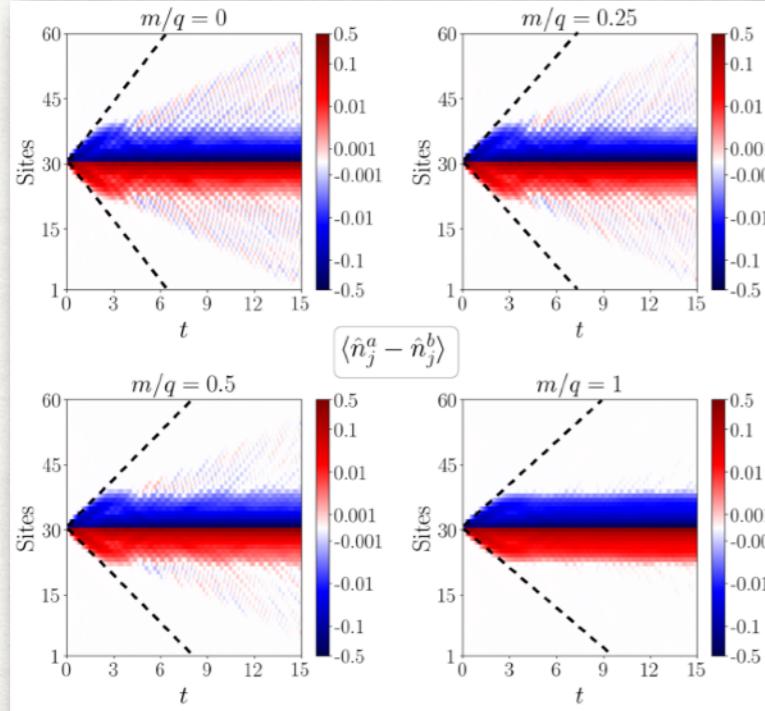
Results

Local quench: Type-II

Evolution with... $\hat{L}_{N/2} \rightarrow \hat{L}_{N/2} + \alpha$

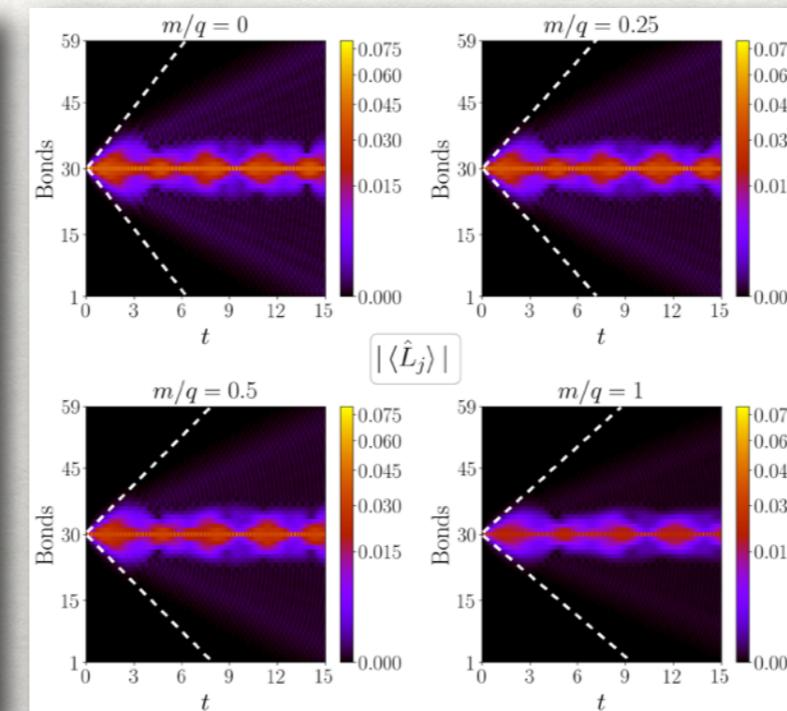
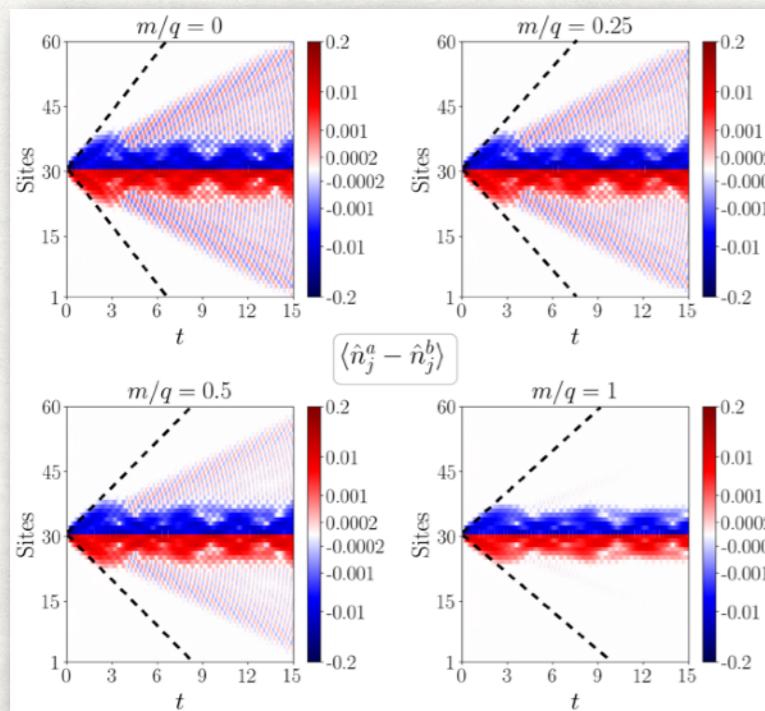
$\alpha = -10$

$x = 2$



$\alpha = -20$

$x = 2$



**Stronger background field
Stronger confinement**

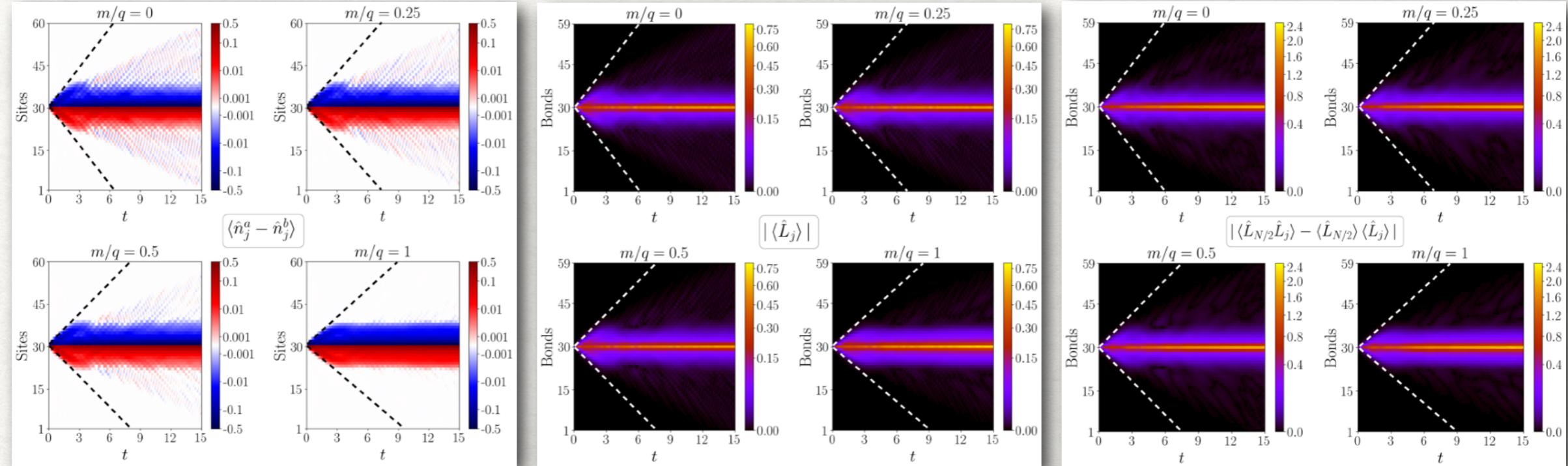
Results

Local quench: Type-II

Evolution with... $\hat{L}_{N/2} \rightarrow \hat{L}_{N/2} + \alpha$

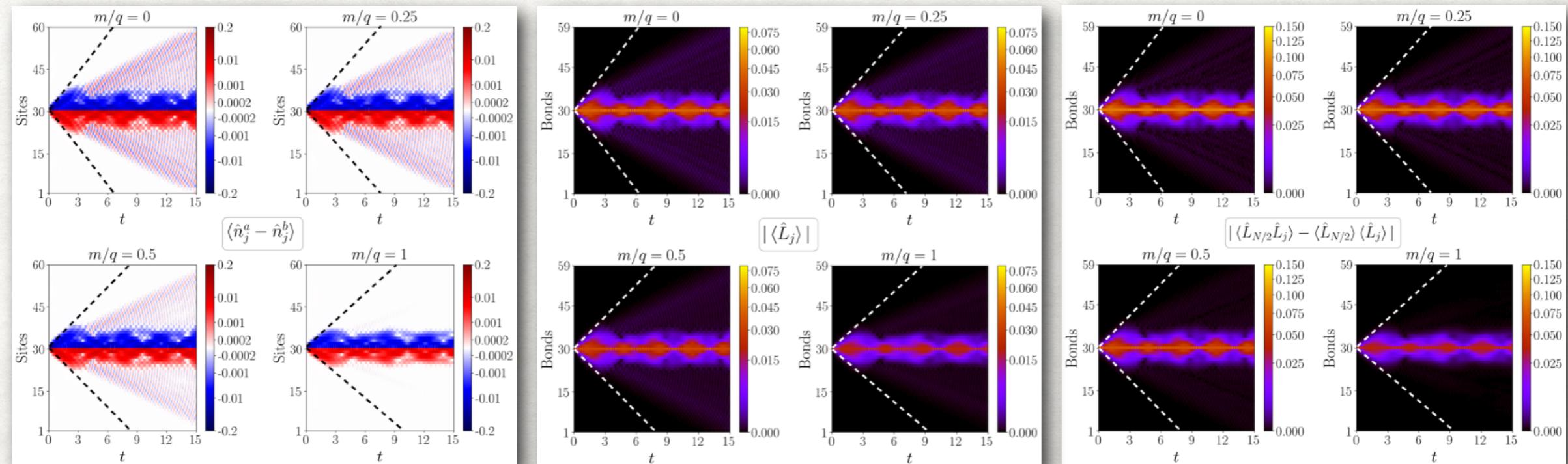
$\alpha = -10$

$x = 2$



$\alpha = -20$

$x = 2$



Results

Global quench: Type-I

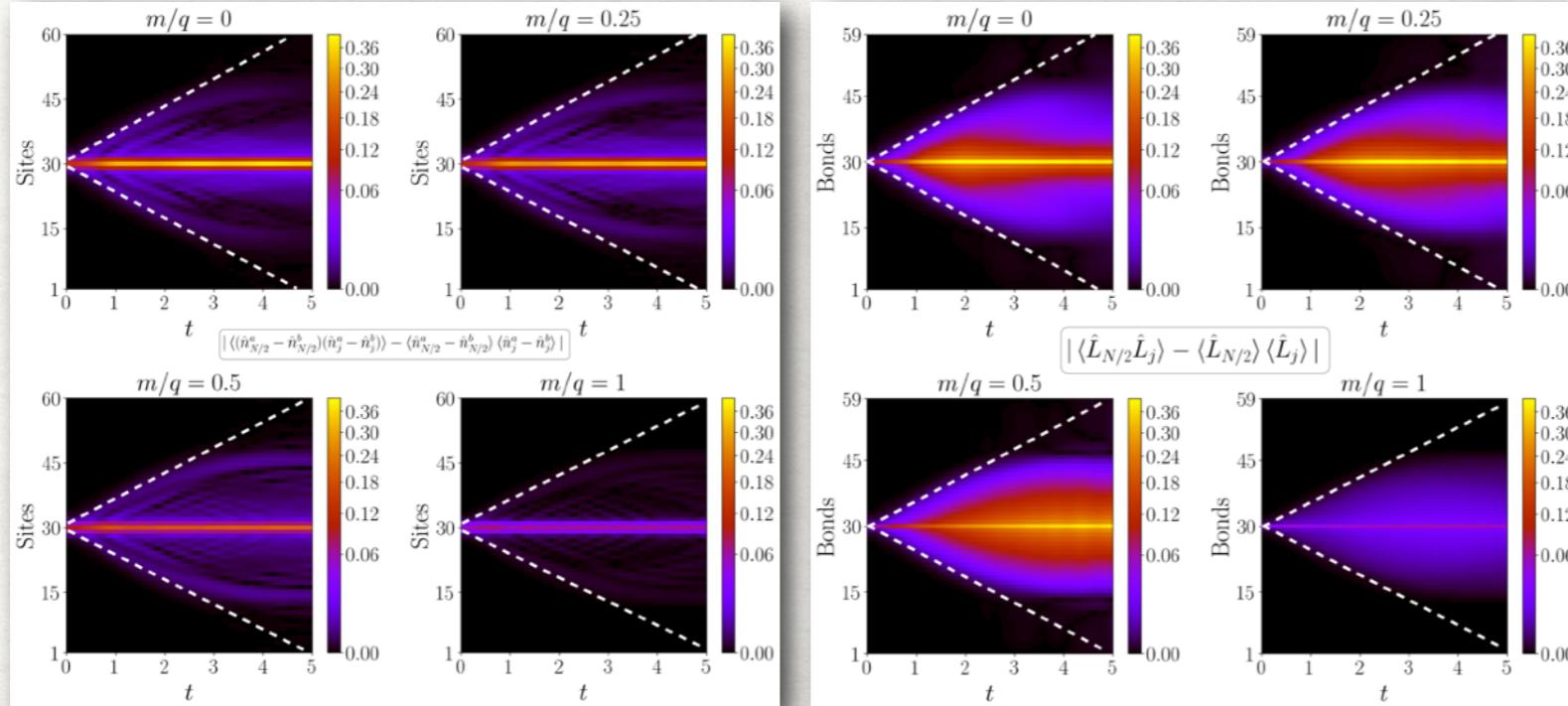
$$|\psi(t=0)\rangle = (\hat{a}_1^\dagger + \hat{b}_1) \left[\prod_{j=1}^{N-1} \hat{U}_j^\dagger \right] (\hat{a}_N + \hat{b}_N^\dagger) |\Omega\rangle$$

Results

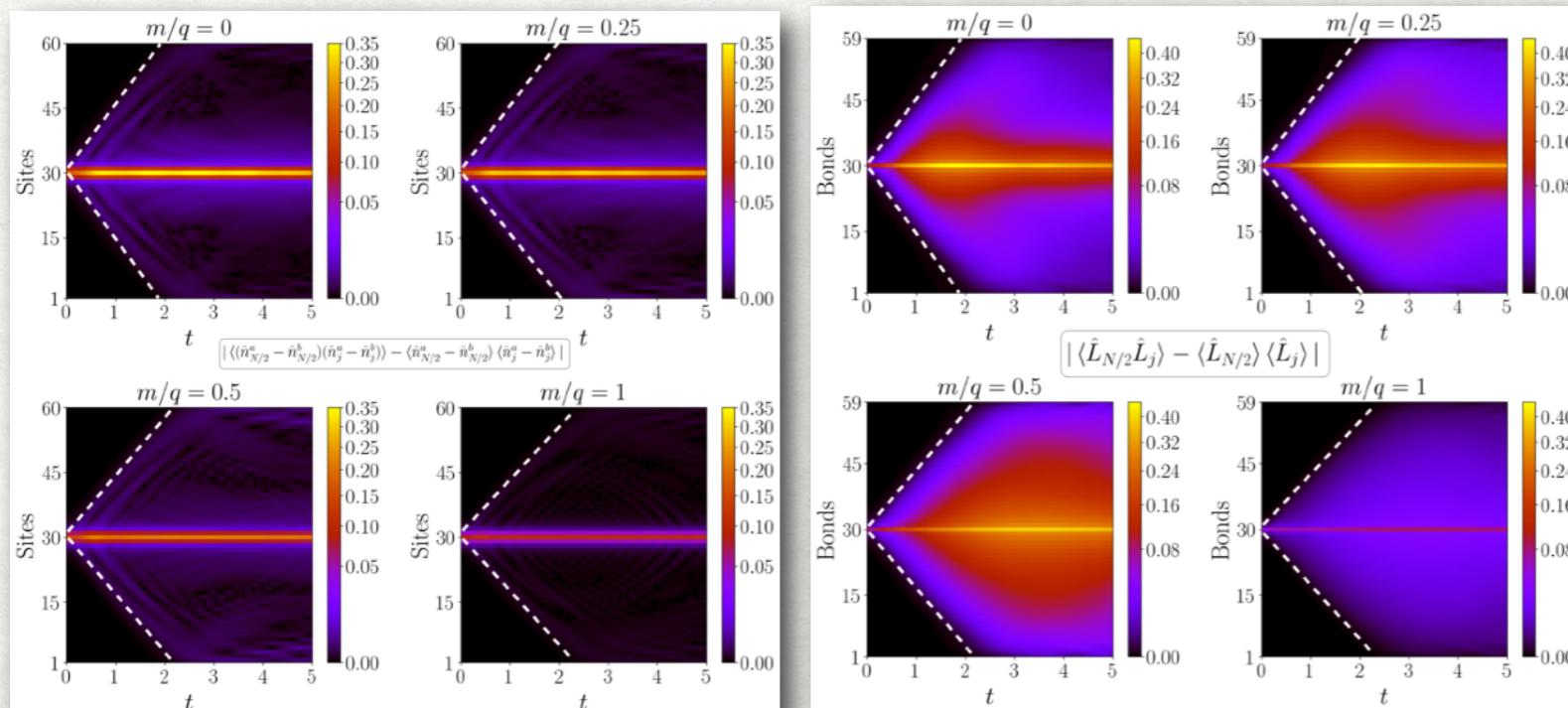
Global quench: Type-I

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$x = 2$



$x = 4$

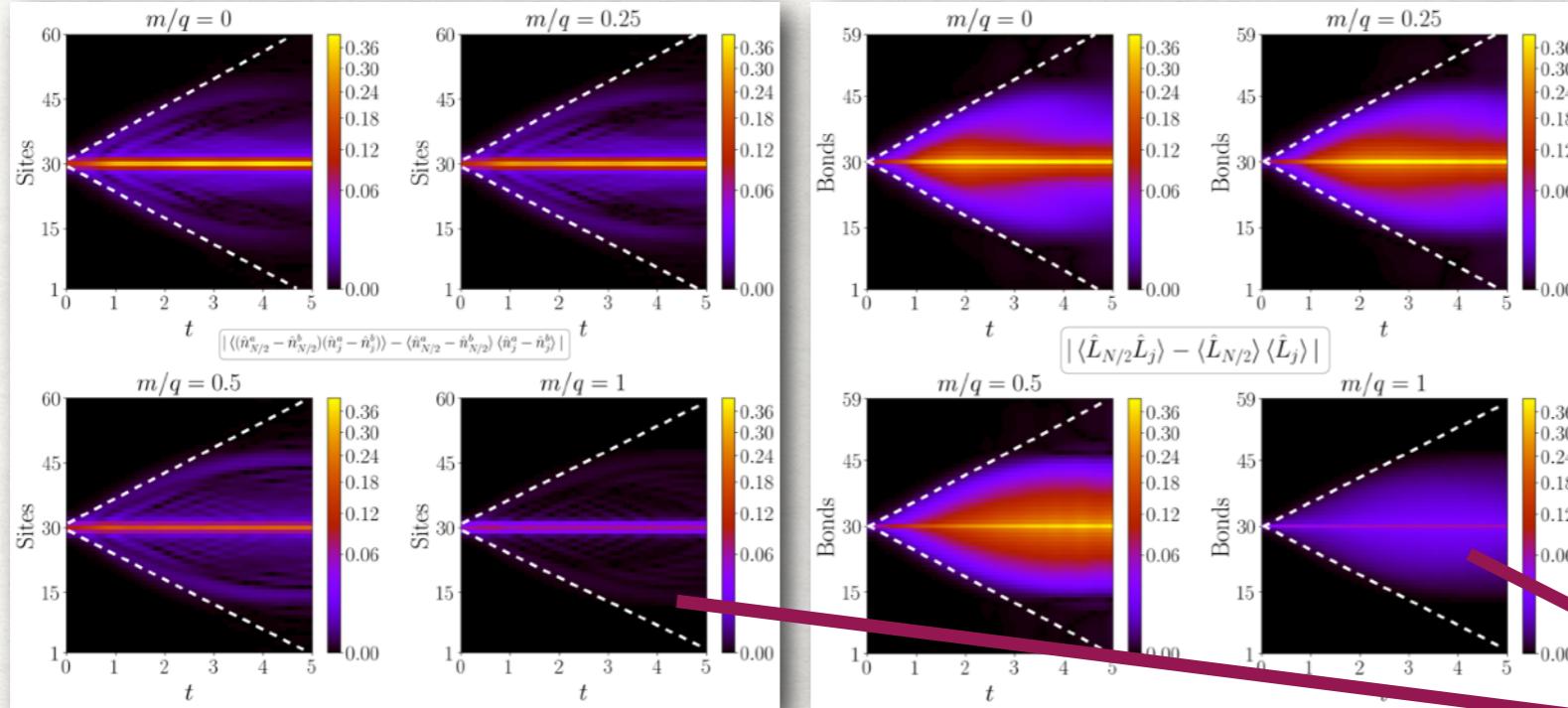


Results

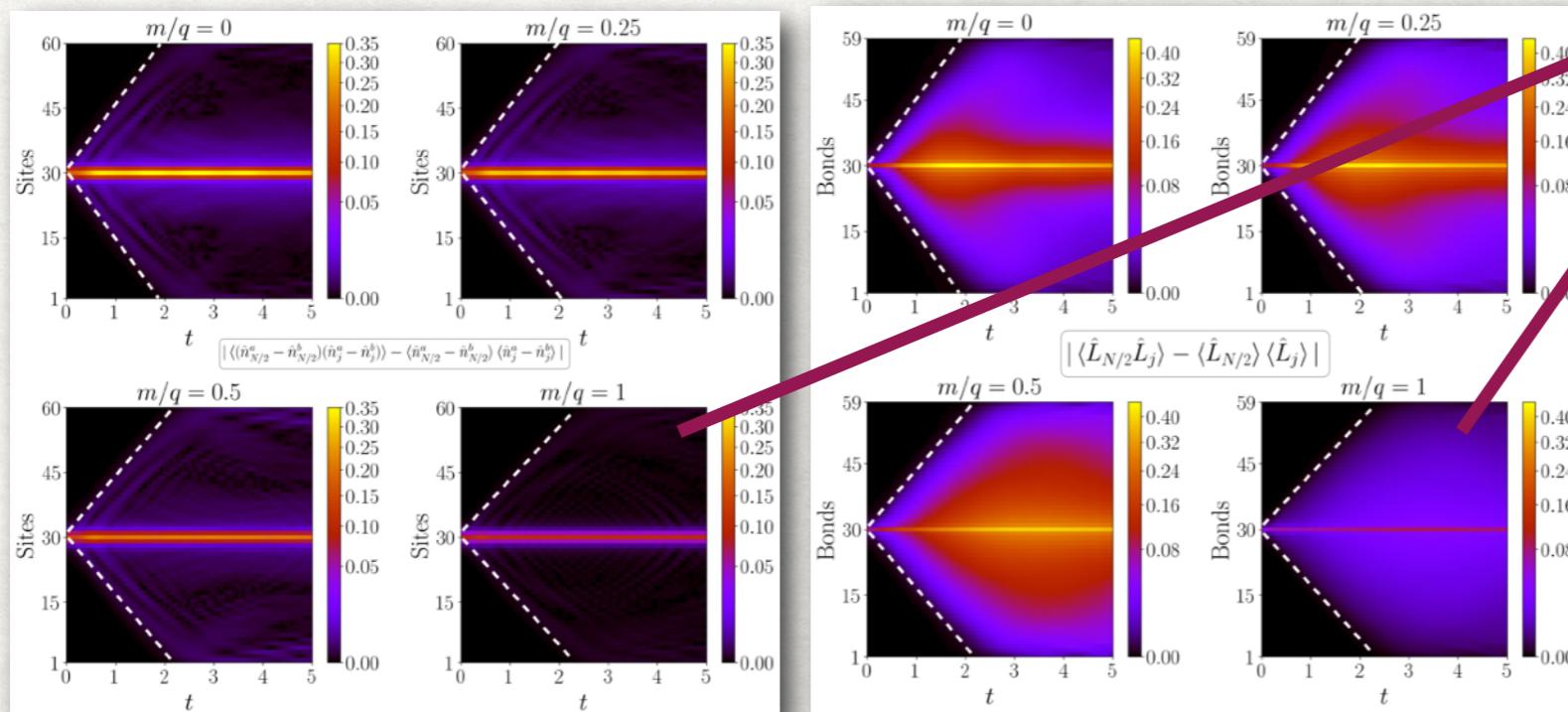
Global quench: Type-I

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$x = 2$



$x = 4$



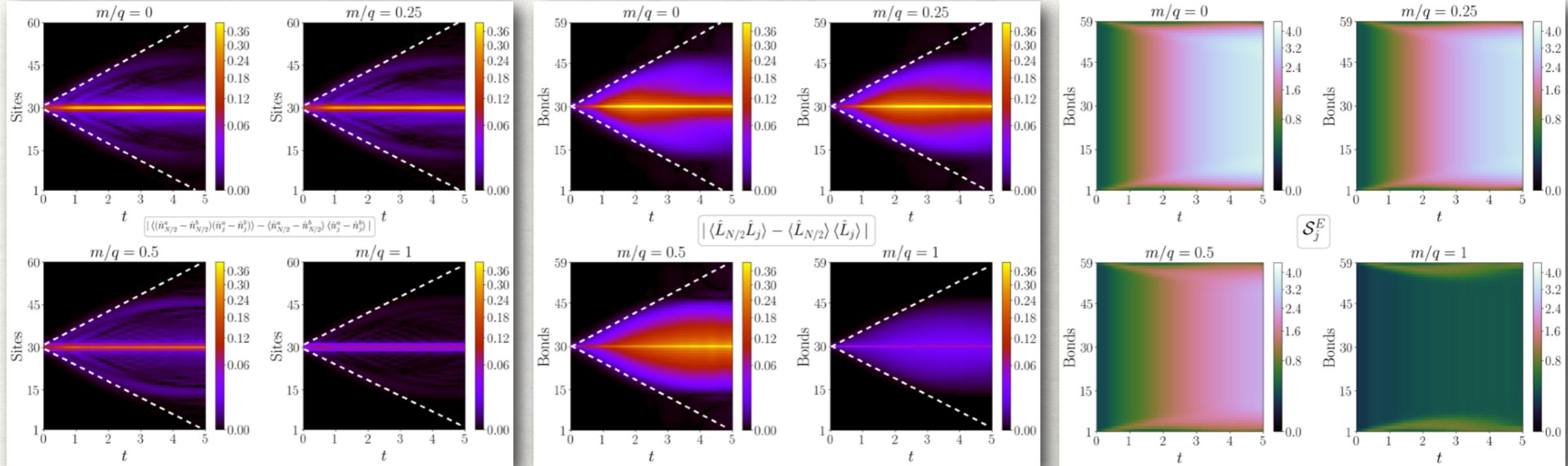
Insufficient energy in the initial state to produce and move pairs of larger mass

Results

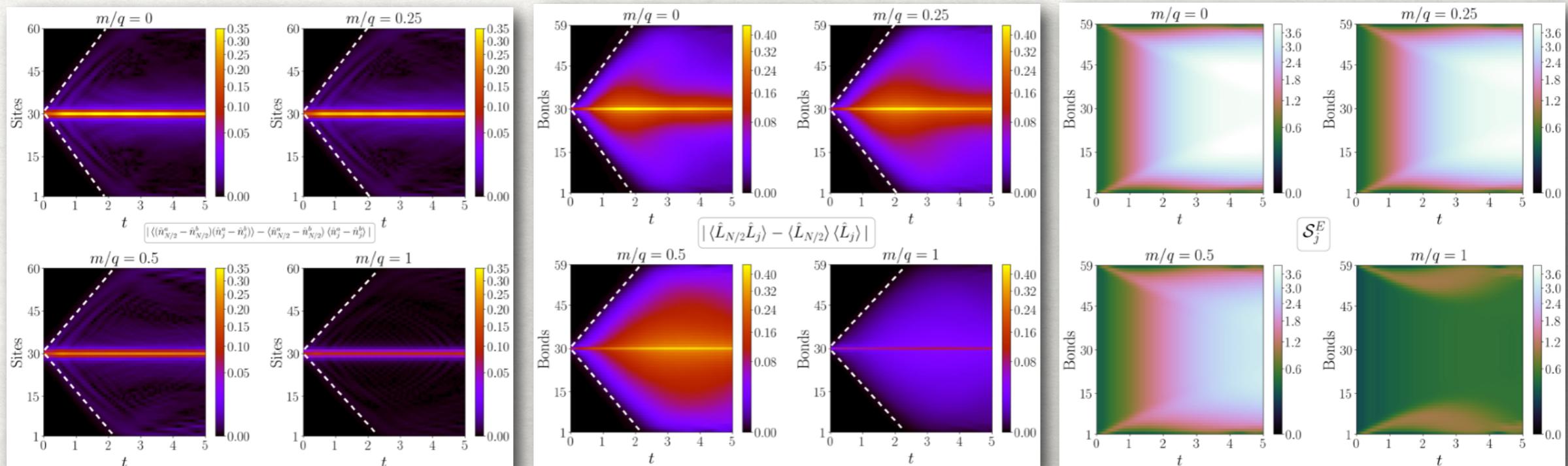
Global quench: Type-I

$$|\psi(t=0)\rangle = (\hat{a}_1^\dagger + \hat{b}_1) \left[\prod_{j=1}^{N-1} \hat{U}_j^\dagger \right] (\hat{a}_N + \hat{b}_N^\dagger) |\Omega\rangle$$

$x = 2$



$x = 4$



Results

Global quench: Type-II

Evolution with... $\hat{L}_j \rightarrow \hat{L}_j + \alpha, \forall j$

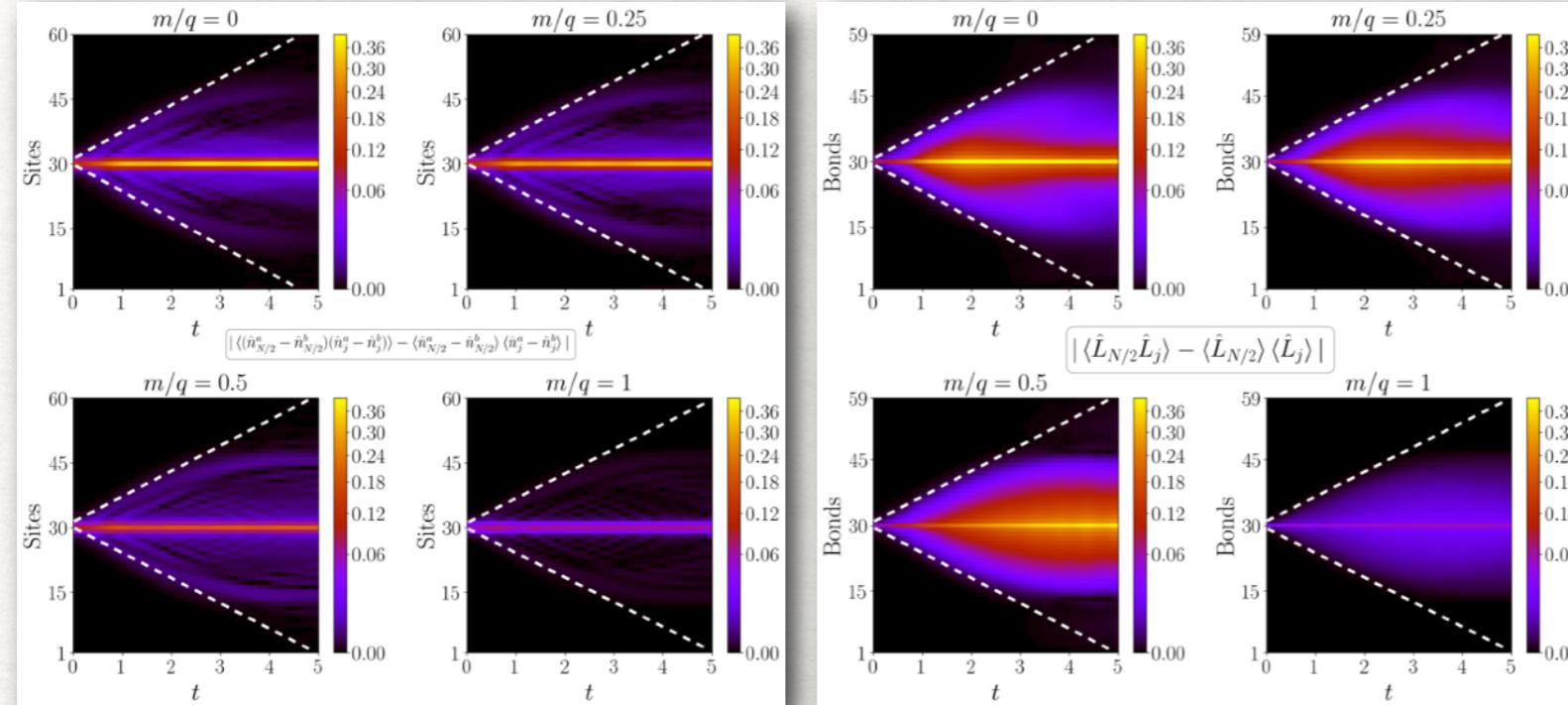
Results

Global quench: Type-II

Evolution with... $\hat{L}_j \rightarrow \hat{L}_j + \alpha, \forall j$

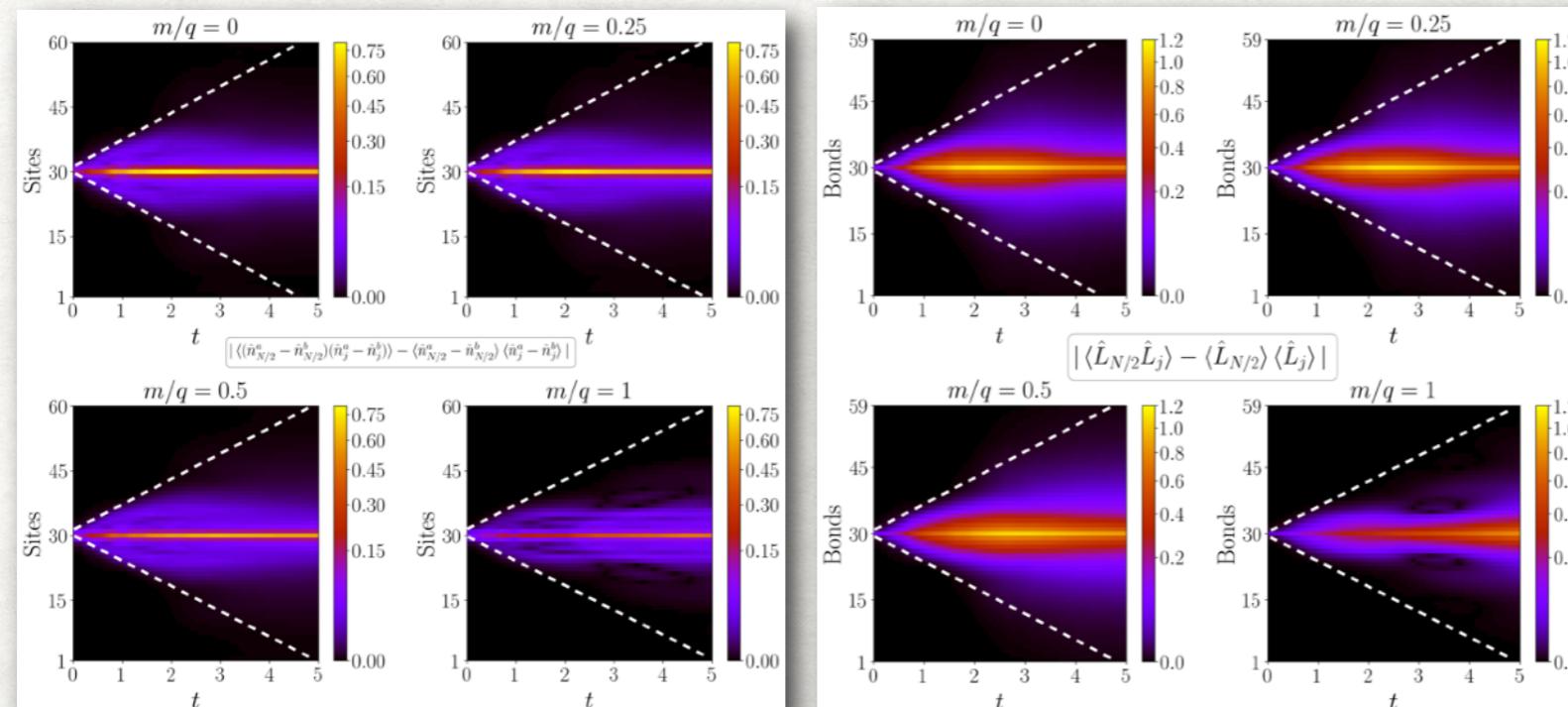
$\alpha = -1$

$x = 2$



$\alpha = -2$

$x = 2$



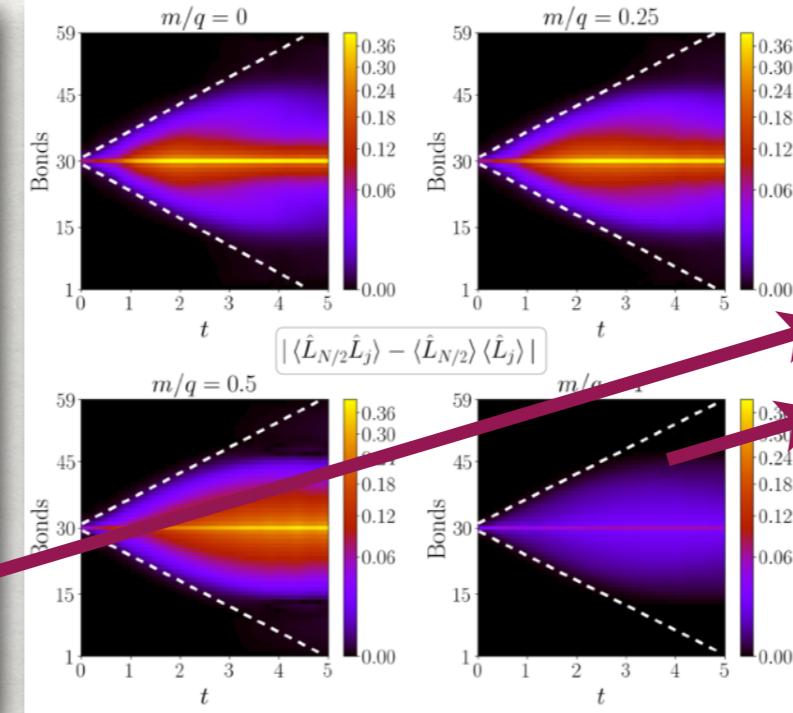
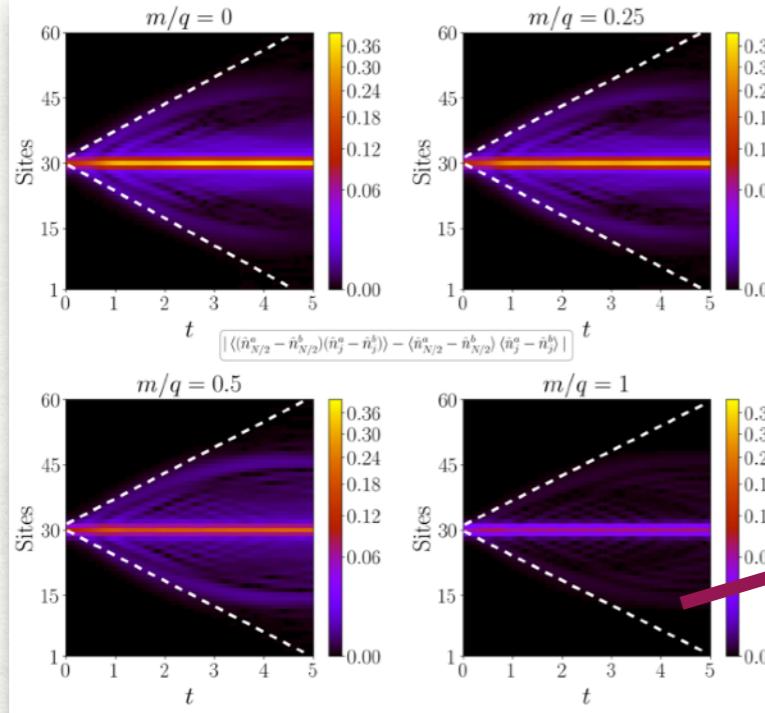
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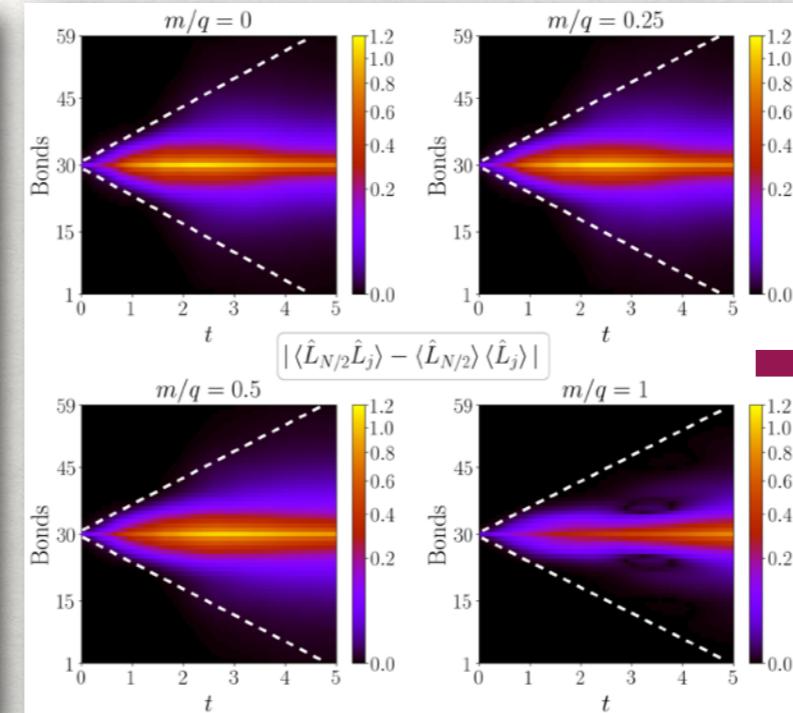
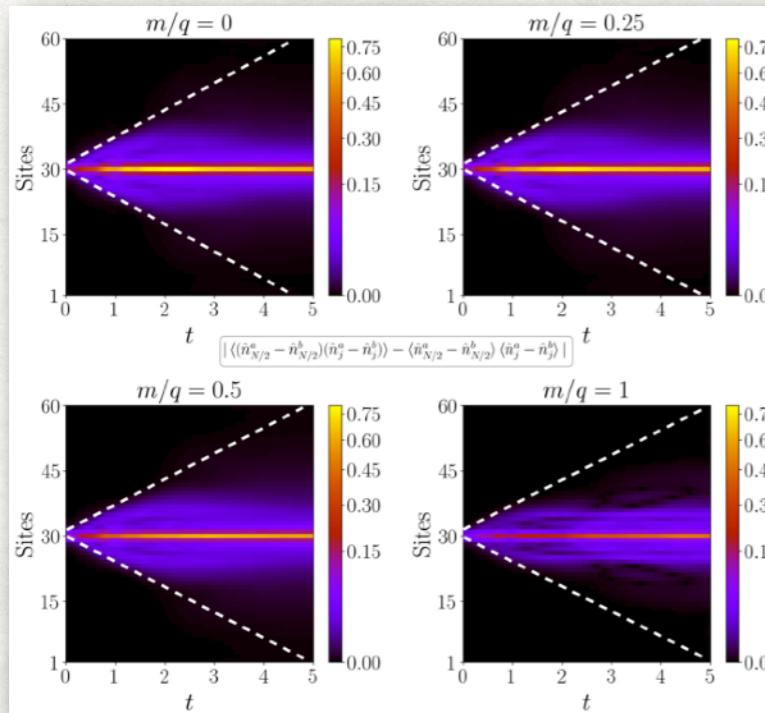
$\alpha = -1$

$x = 2$



$\alpha = -2$

$x = 2$



Insufficient energy in the background field to produce pairs of larger mass

Ease of pair-production
More confined, larger bending of light-cone

Results

Global quench: Type-II

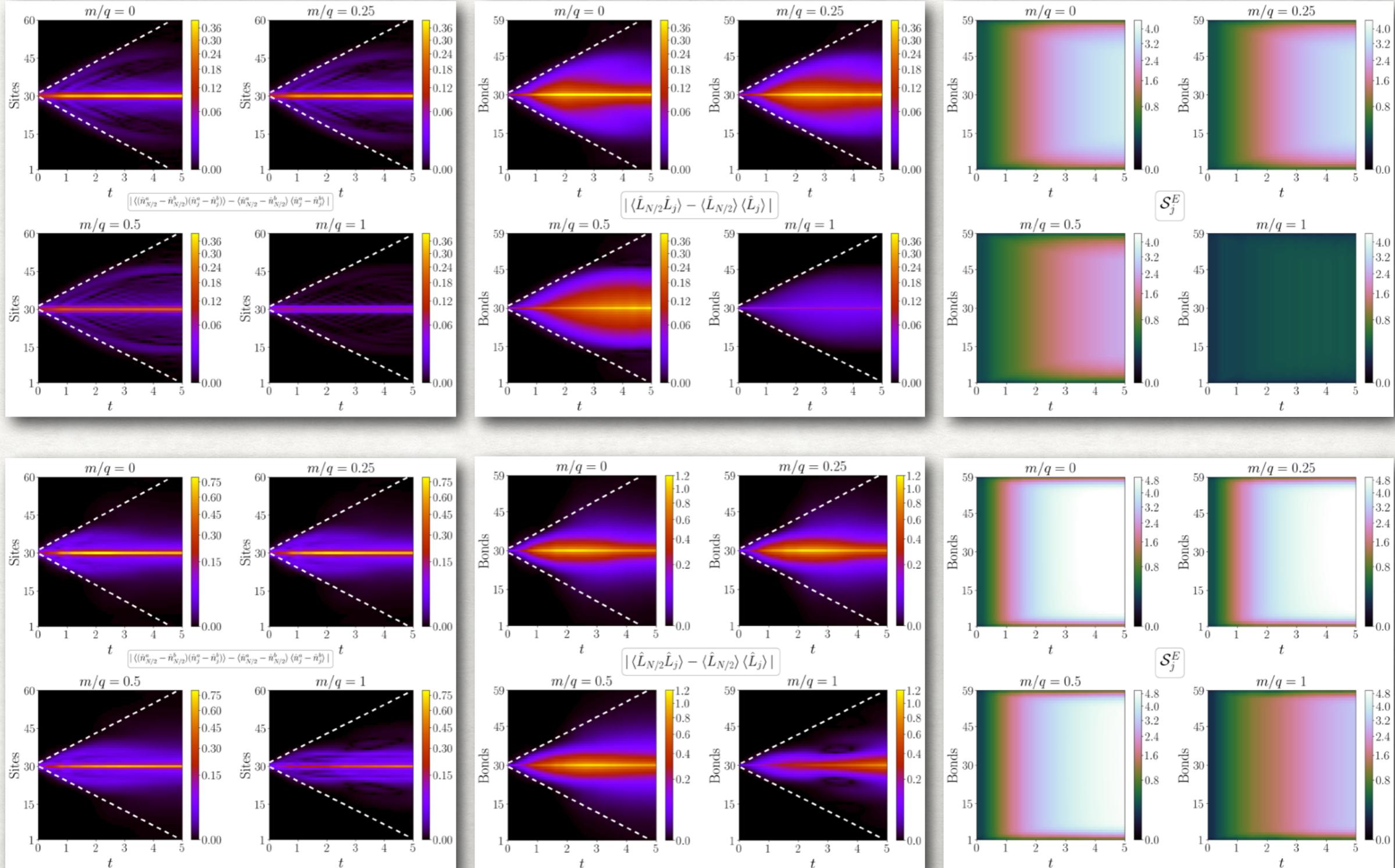
Evolution with... $\hat{L}_j \rightarrow \hat{L}_j + \alpha, \forall j$

$\alpha = -1$

$x = 2$

$\alpha = -2$

$x = 2$



Results

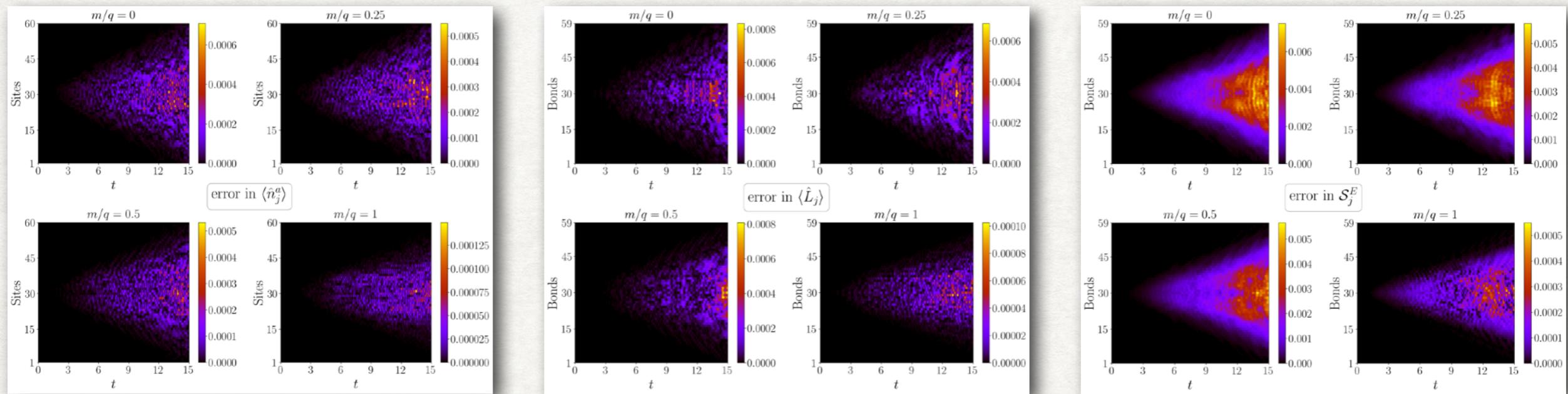
Errors in local quench: Type-I

Difference in values for $D_{max} = 320$ and $D_{max} = 640$

Results

Errors in local quench: Type-I

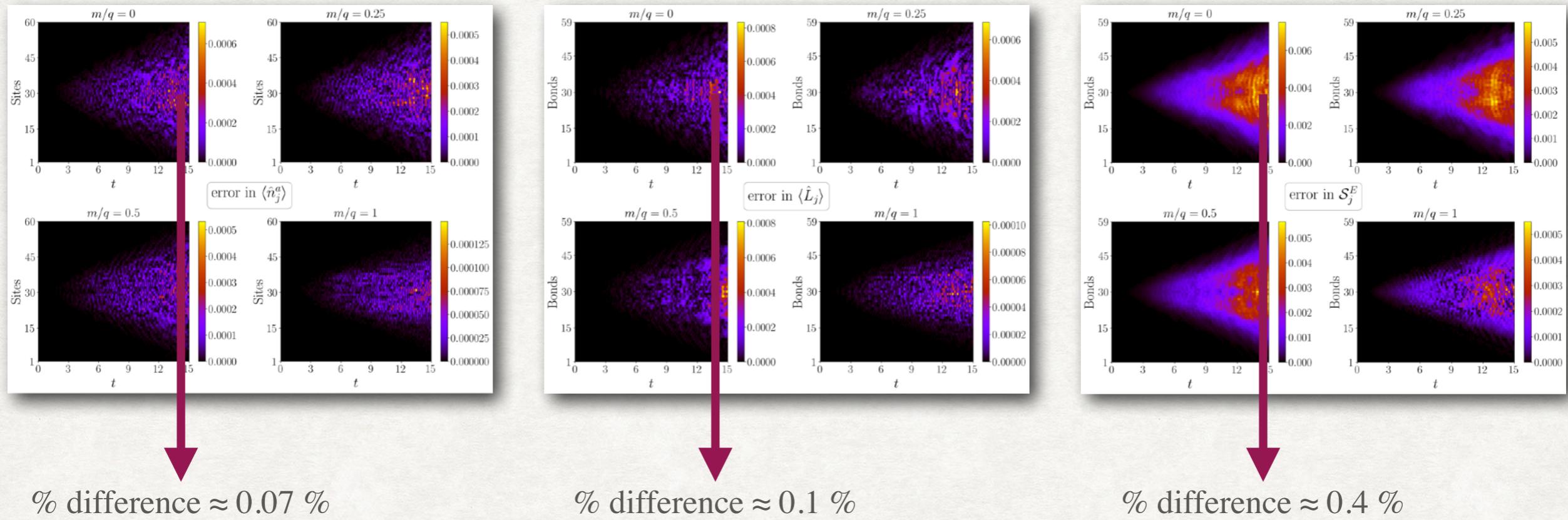
Difference in values for $D_{max} = 320$ and $D_{max} = 640$



Results

Errors in local quench: Type-I

Difference in values for $D_{max} = 320$ and $D_{max} = 640$



Similar for Type-II quench

Bottom-line: Success of TNS to **faithfully** simulate bosonic LGT models of high energy origins

Results

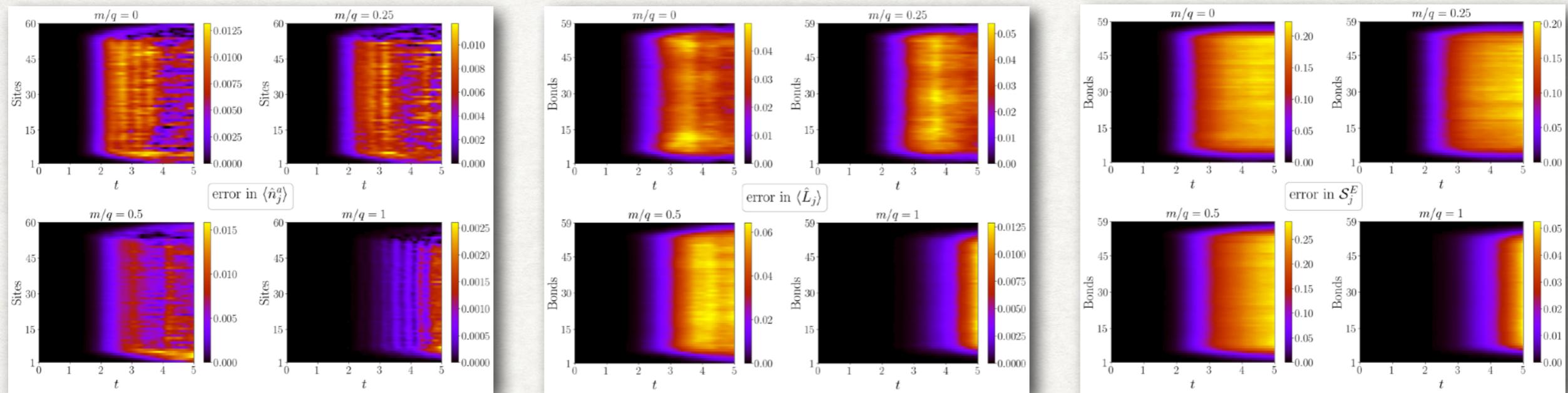
Errors in global quench: Type-II

Difference in values for $D_{max} = 320$ and $D_{max} = 640$

Results

Errors in global quench: Type-II

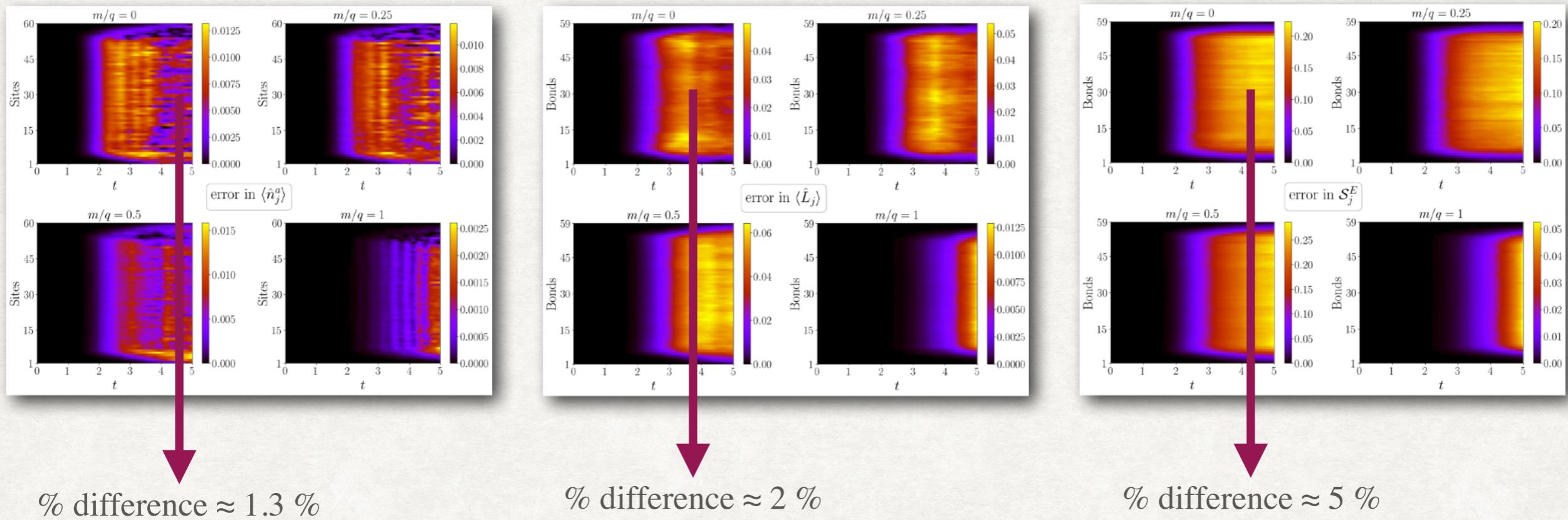
Difference in values for $D_{max} = 320$ and $D_{max} = 640$



Results

Errors in global quench: Type-II

Difference in values for $D_{max} = 320$ and $D_{max} = 640$



Faithful upto $t \approx 2$ for all masses. But can capture qualitative features for later time.
We are pushing our limits on D_{max} to get better results.

Better for Type-I quench

Conclusions

Things to do...

Conclusions

Things to do...

- Thorough analysis of the observations, better analysis of errors
- Proposition of possible experimental realization using cold atomic systems

$$\hat{H} = \sum_j \hat{L}_j^2 + ((m/q)^2 + 3x) \sum_j (\hat{a}_j^\dagger \hat{a}_j + \hat{b}_j \hat{b}_j^\dagger) + ((m/q)^2 + x) \sum_j (\hat{a}_j^\dagger \hat{b}_j^\dagger + \hat{a}_j \hat{b}_j) - x \sum_j \left[(\hat{a}_{j+1}^\dagger + \hat{b}_{j+1}) \hat{U}_j (\hat{a}_j + \hat{b}_j^\dagger) + \text{h.c.} \right]$$

Conclusions

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→

$$\begin{aligned} \hat{H}_{CA} = & \sum_j \hat{L}_j^2 + \mu_1 \sum_j (\hat{a}_j^\dagger \hat{a}_j + \hat{b}_j \hat{b}_j^\dagger) + \mu_2 \sum_j (\hat{a}_j^\dagger \hat{b}_j^\dagger + \hat{a}_j \hat{b}_j) - x \sum_j [(\hat{a}_{j+1}^\dagger + \hat{b}_{j+1}) \hat{U}_j (\hat{a}_j + \hat{b}_j^\dagger) + \text{h.c.}] \\ & + V_1 \sum_j ((\hat{a}_j^\dagger \hat{a}_j)^2 + (\hat{b}_j^\dagger \hat{b}_j)^2) + V_2 \sum_j \hat{a}_j^\dagger \hat{a}_j \hat{b}_j^\dagger \hat{b}_j \end{aligned}$$

Dispersion relation without gauge fields and interactions...

$$\omega(k) = \sqrt{(\mu_1 - \mu_2)(\mu_1 + \mu_2 - 4x \cos(ka))}$$



stable iff...

$$\mu_1 \geq \mu_2$$

$$\mu_1 + \mu_2 \geq 4x$$

Or...

$$\mu_1 \leq \mu_2$$

$$\mu_1 + \mu_2 \leq -4x$$

Conclusions

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- Proposition of possible experimental realization using cold atomic systems

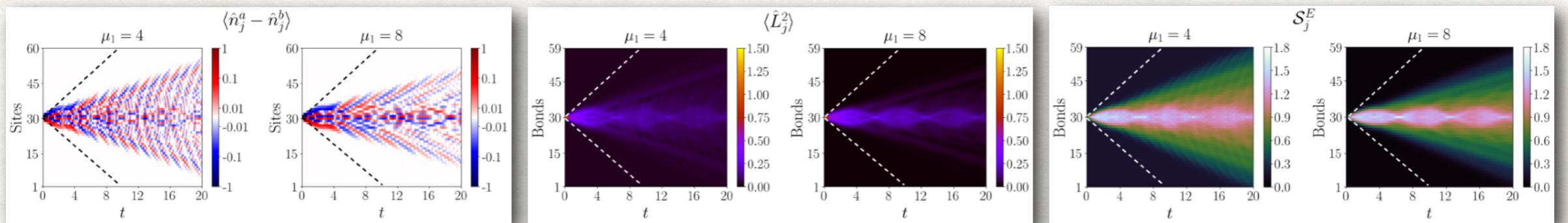
$$\hat{H} = \sum_j \hat{L}_j^2 + ((m/q)^2 + 3x) \sum_j (\hat{a}_j^\dagger \hat{a}_j + \hat{b}_j^\dagger \hat{b}_j^\dagger) + ((m/q)^2 + x) \sum_j (\hat{a}_j^\dagger \hat{b}_j^\dagger + \hat{a}_j \hat{b}_j) - x \sum_j [(\hat{a}_{j+1}^\dagger + \hat{b}_{j+1}) \hat{U}_j (\hat{a}_j + \hat{b}_j^\dagger) + \text{h.c.}]$$

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$$x = 1; \quad \mu_2 = 0; \quad V_1 = 1/2; \quad V_2 = 0$$

Local quench: Type-I



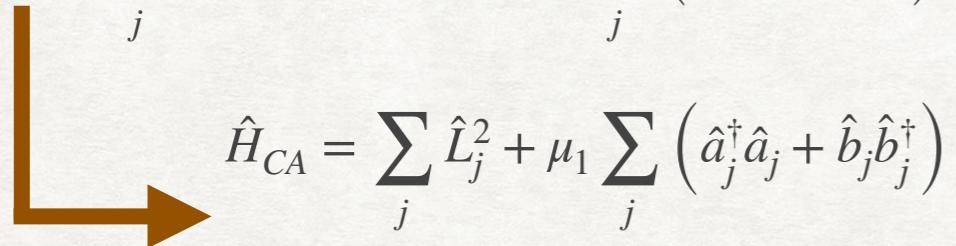
Similar behavior as the scalar QED Hamiltonian

Conclusions

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- Proposition of possible experimental realization using cold atomic systems

$$\hat{H} = \sum_j \hat{L}_j^2 + ((m/q)^2 + 3x) \sum_j (\hat{a}_j^\dagger \hat{a}_j + \hat{b}_j \hat{b}_j^\dagger) + ((m/q)^2 + x) \sum_j (\hat{a}_j^\dagger \hat{b}_j^\dagger + \hat{a}_j \hat{b}_j) - x \sum_j [(\hat{a}_{j+1}^\dagger + \hat{b}_{j+1}) \hat{U}_j (\hat{a}_j + \hat{b}_j^\dagger) + \text{h.c.}]$$


$$\hat{H}_{CA} = \sum_j \hat{L}_j^2 + \mu_1 \sum_j (\hat{a}_j^\dagger \hat{a}_j + \hat{b}_j \hat{b}_j^\dagger) + \mu_2 \sum_j (\hat{a}_j^\dagger \hat{b}_j^\dagger + \hat{a}_j \hat{b}_j) - x \sum_j [(\hat{a}_{j+1}^\dagger + \hat{b}_{j+1}) \hat{U}_j (\hat{a}_j + \hat{b}_j^\dagger) + \text{h.c.}]$$
$$+ V_1 \sum_j ((\hat{a}_j^\dagger \hat{a}_j)^2 + (\hat{b}_j^\dagger \hat{b}_j)^2) + V_2 \sum_j \hat{a}_j^\dagger \hat{a}_j \hat{b}_j^\dagger \hat{b}_j$$

- Confined-deconfined transition (!!!)

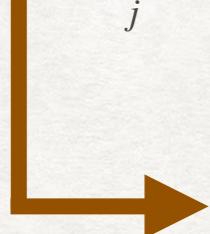


Conclusions

Things to do...

- Thorough analysis of the observations, better analysis of errors
- Proposition of possible experimental realization using cold atomic systems

$$\hat{H} = \sum_j \hat{L}_j^2 + ((m/q)^2 + 3x) \sum_j (\hat{a}_j^\dagger \hat{a}_j + \hat{b}_j \hat{b}_j^\dagger) + ((m/q)^2 + x) \sum_j (\hat{a}_j^\dagger \hat{b}_j^\dagger + \hat{a}_j \hat{b}_j) - x \sum_j [(\hat{a}_{j+1}^\dagger + \hat{b}_{j+1}) \hat{U}_j (\hat{a}_j + \hat{b}_j^\dagger) + \text{h.c.}]$$



$$\begin{aligned} \hat{H}_{CA} = & \sum_j \hat{L}_j^2 + \mu_1 \sum_j (\hat{a}_j^\dagger \hat{a}_j + \hat{b}_j \hat{b}_j^\dagger) + \mu_2 \sum_j (\hat{a}_j^\dagger \hat{b}_j^\dagger + \hat{a}_j \hat{b}_j) - x \sum_j [(\hat{a}_{j+1}^\dagger + \hat{b}_{j+1}) \hat{U}_j (\hat{a}_j + \hat{b}_j^\dagger) + \text{h.c.}] \\ & + V_1 \sum_j ((\hat{a}_j^\dagger \hat{a}_j)^2 + (\hat{b}_j^\dagger \hat{b}_j)^2) + V_2 \sum_j \hat{a}_j^\dagger \hat{a}_j \hat{b}_j^\dagger \hat{b}_j \end{aligned}$$

- Confined-deconfined transition (!!!)



What we achieved so far...

- Study of a bosonic system of high-energy origin in 1+1 dimension using TN, which can be simulated using cold atoms
- Signatures of pair-production and real-time confinement via different types of quenches
- Showcased faithful simulation of such systems using TN

Thank you!!!

Collaborators...



Luca Tagliacozzo



Maciej Lewenstein



Jakub Zakrzewski

