From static to dynamical gauge fields with ultracold atoms

Monika Aidelsburger

Ludwig-Maximilians-Universität Munich

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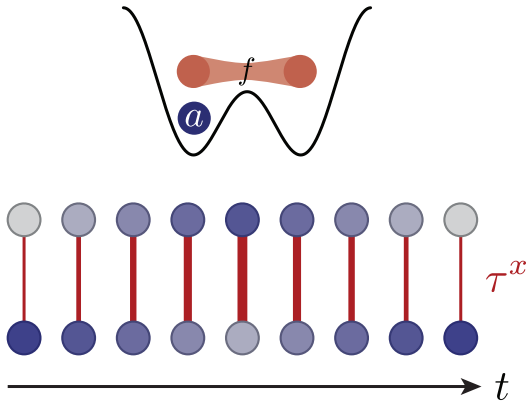
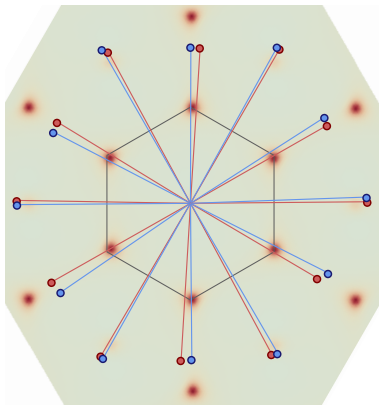
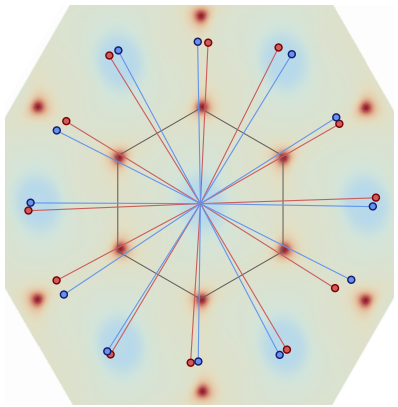






FOR2414

MCQST

UQUAM ultracold quantum matter

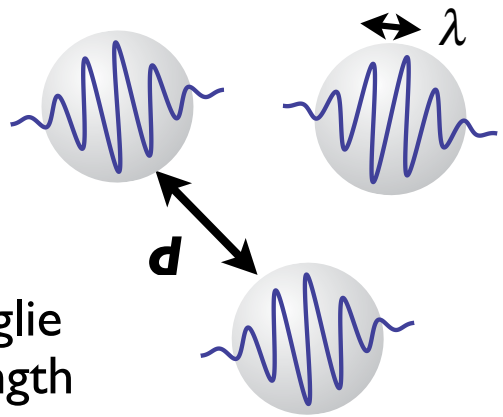
European Research Council

Ultracold Quantum Gases

Laser cooling:

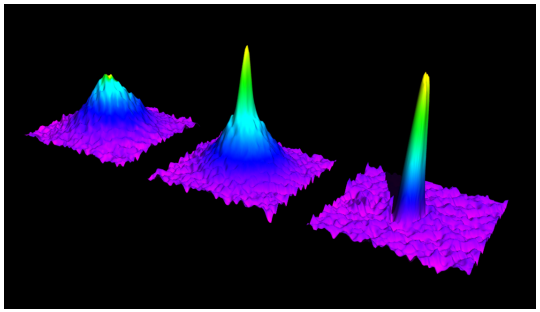
Quantum mechanics:

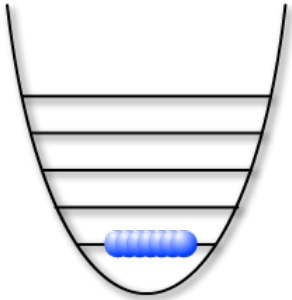
$$\lambda/d \gtrsim 1$$



λ: de-Broglie wavelength

Bose-Einstein Condensate for $T<T_c$:



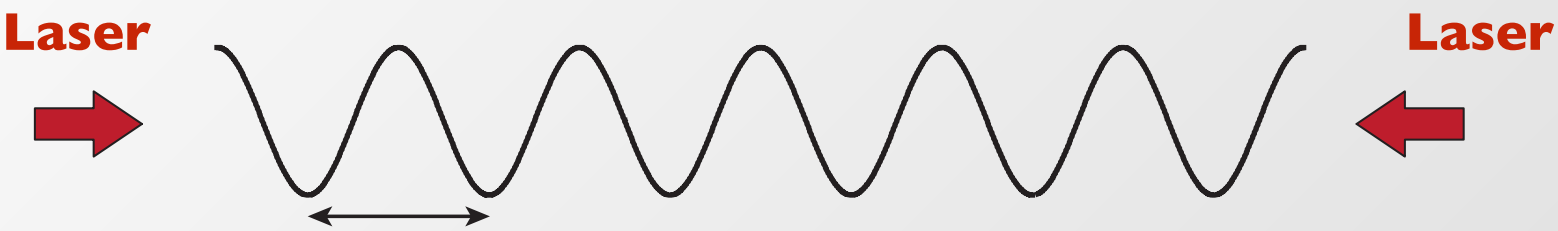


1997: S. Chu, C. Cohen-Tannoudji, B. Phillips

2001: C. Wiemann, W. Ketterle, E. Cornell

Optical lattices

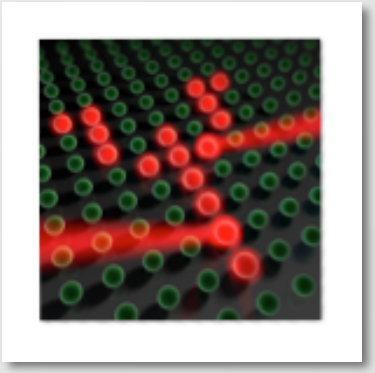
$$V_{\mathrm{dip}}(r) \propto I(r)$$

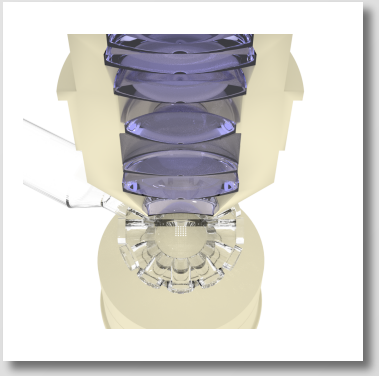


$$a = \lambda/2 \approx 0.3 - 0.8\mu\mathrm{m}$$

Ultracold Quantum Matter

- ***Densities:*** ***$10^{14}/cm^3$***
  (real materials: 10^{24}-$10^{25}/cm^3$)
- ***Temperatures:*** ***few nK***
  (real materials: mK - 300K)

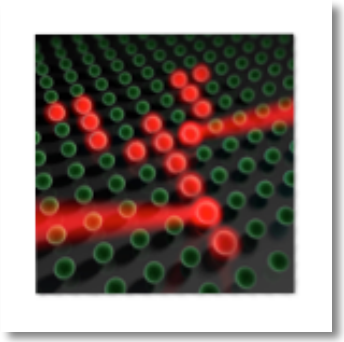




R. Grimm *et al.*, Adv. At. Mol. Opt. Phys. 42, 95-170 (2000)

Quantum simulation

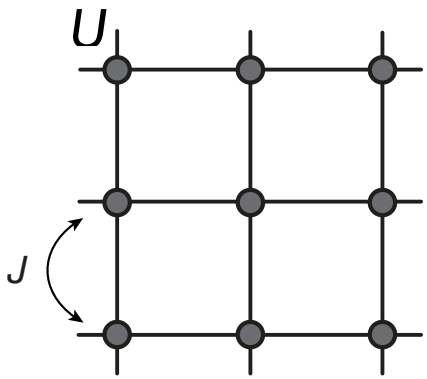
Ultracold atoms in optical lattices:



» Controlled simulation of interacting bosonic and fermionic atoms

$$\hat{H} = -\sum_{\langle i,j \rangle} J_{ij} \left(\hat{a}_i^\dagger \hat{a}_j + \hat{a}_j^\dagger \hat{a}_i \right) + \frac{U}{2} \sum_i \hat{n}_i \left(\hat{n}_i - 1 \right)$$

» Well-isolated from environment



- Synthetic magnetic fields
- Topology
- Gauge theories
- .....

D. Jaksch and P. Zoller, Ann. Phys. 315, 52 (2005); I. Bloch et al. Rev. Mod. Phys. 80, 885 (2008)

Floquet engineering

Floquet engineering as a tool to engineer *non-trivial Hamiltonians*

Basic idea:

» Time-periodic driven Hamiltonian

$$\hat{H}(t) = \hat{H}(t+T)$$

T: driving cycle

» *Stroboscopic* time evolution reproduced by *time-independent Floquet Hamiltonian* $\hat{H}^F$

$$\hat{U}(T,0) = \exp\left(-\frac{i}{\hbar}T\hat{H}^F\right)$$

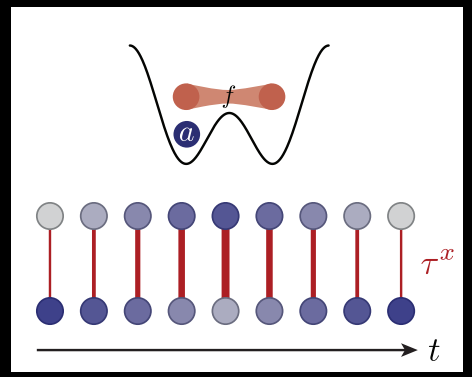
⇒ Possibility to engineer Floquet Hamiltonian with desired properties!

N. Goldman et al. PRX (2014); M. Bukov et al. Adv. in Phys. (2015); A. Eckardt, Rev. Mod. Phys. (2017);

Outline

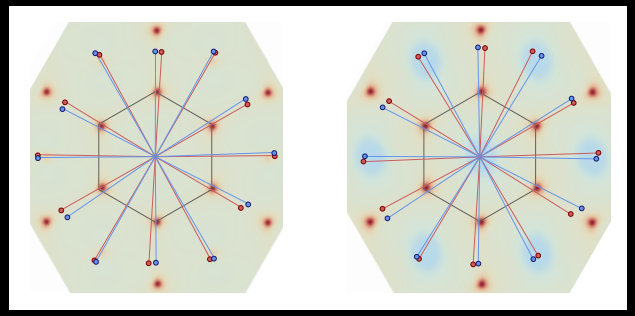
1) Z_2 lattice gauge theories

C. Schweizer *et al.*, arXiv:1901.07103 (2019)



2) Anomalous Floquet phases

K. Wintersperger (in preparation)



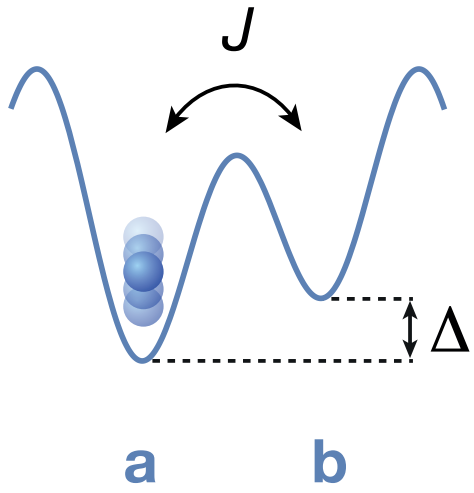
Laser-assisted tunneling

Minimal lattice with two sites:

- Tilted double-well potential

$$\hat{H} = -J\left(\hat{a}^\dagger\hat{b} + \hat{b}^\dagger\hat{a}\right) + \Delta\ \hat{b}^\dagger\hat{b}$$

→ tunneling *inhibited* for $\Delta \gg J$



- *Resonant* modulation at $\hbar\omega = \Delta$ *restores tunneling*

$$\hat{V}(t) = V_0\ \cos(\omega t + \phi)\hat{a}^\dagger\hat{a}$$

- Time-independent *Floquet Hamiltonian* (high-freq. limit $\hbar\omega \gg J$)

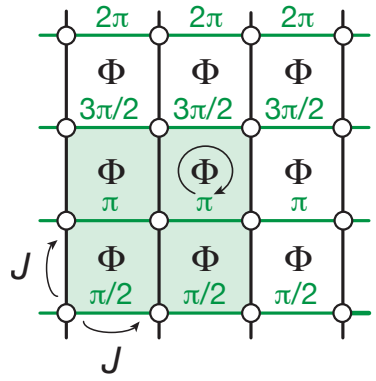
$$\hat{H}^F = -J\mathcal{J}_1\left(\frac{V_0}{\hbar\omega}\right)\mathrm{e}^{i\phi}\hat{a}^\dagger\hat{b} + \mathrm{h.c.}$$

I. Bloch, Munich; W. Ketterle, MIT; M. Greiner, Harvard

Topological lattice models

Hofstadter model

Harper, Proc. Phys. Soc., Sect.A **68**, 874 (1955);
Azbel, Zh. Eksp. Teor. Fiz. **46**, 929 (1964); Hofstadter, PRB **14**, 2239 (1976)

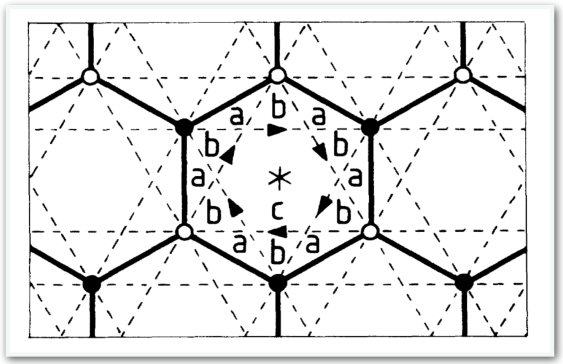


$$\hat{H} = -J \sum_{m,n} \left(\mathrm{e}^{in\Phi} \hat{a}^\dagger_{m+1,n} \hat{a}_{m,n} + \hat{a}^\dagger_{m,n+1} \hat{a}_{m,n} + \mathrm{h.c.} \right)$$

Cold atoms: Bloch Munich, Ketterle MIT, Fallani LENS, Spielman NIST, Greiner Harvard,

Haldane model

Haldane, PRL **61**, 2015 (1988)

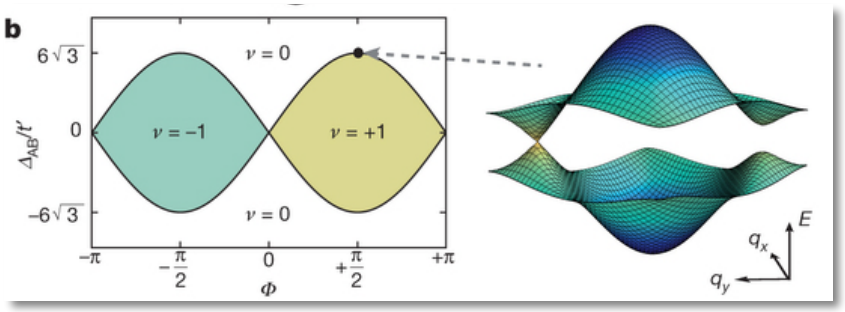


$$\hat{H} = \sum_{\langle ij \rangle} t_{ij} \hat{c}^\dagger_i \hat{c}_j + \sum_{\langle\langle ij \rangle\rangle} \mathrm{e}^{i\Phi_{ij}} t'_{ij} \hat{c}^\dagger_i \hat{c}_j + \Delta_{AB} \sum_{i \in A} \hat{c}^\dagger_i \hat{c}_i$$

Cold atoms: Esslinger ETH, Weitenberg/Sengstock Hamburg

Experimental realizations

Ultracold atoms

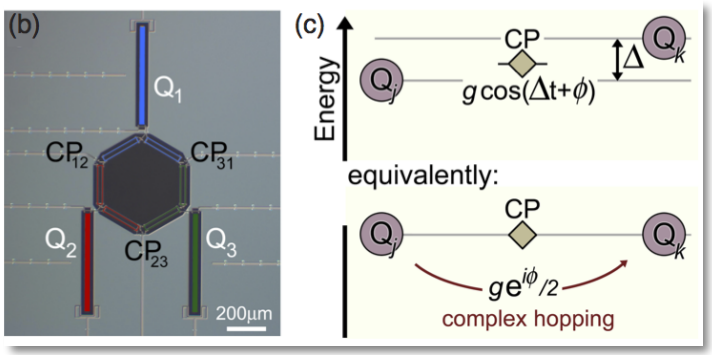


G. Jotzu et al., Nature (2014)

Coupled waveguide arrays

M. C. Rechtsman et al., Nature (2013)

Superconducting circuits



P. Roushan et al., Nat. Phys. (2017)

Review: Aidelsburger, Nascimbène, Goldman, Comptes Rendus Phys. (2018)

Dynamical gauge fields

Synthetic gauge fields:

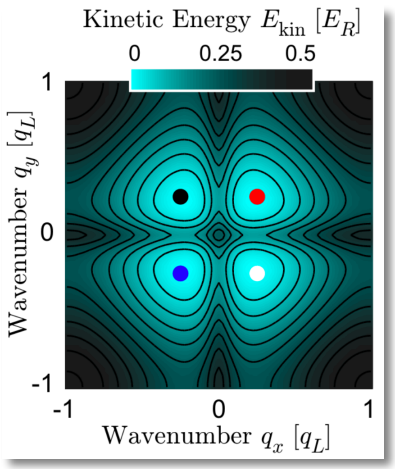
Typically static, i.e. *no backaction* of motion of particles onto fields

Towards lattice gauge theories:

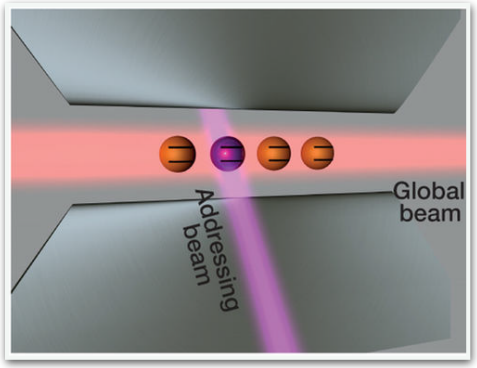
U.-J. Wiese, Nucl. Phys. (2014); E. Zohar Rep. Prog. Phys. (2015);....

- *Density-dependent* gauge fields with atoms

  Clark et al., PRL (2018)
  Görg et al., arXiv (2019)



- *Schwinger model* with ions



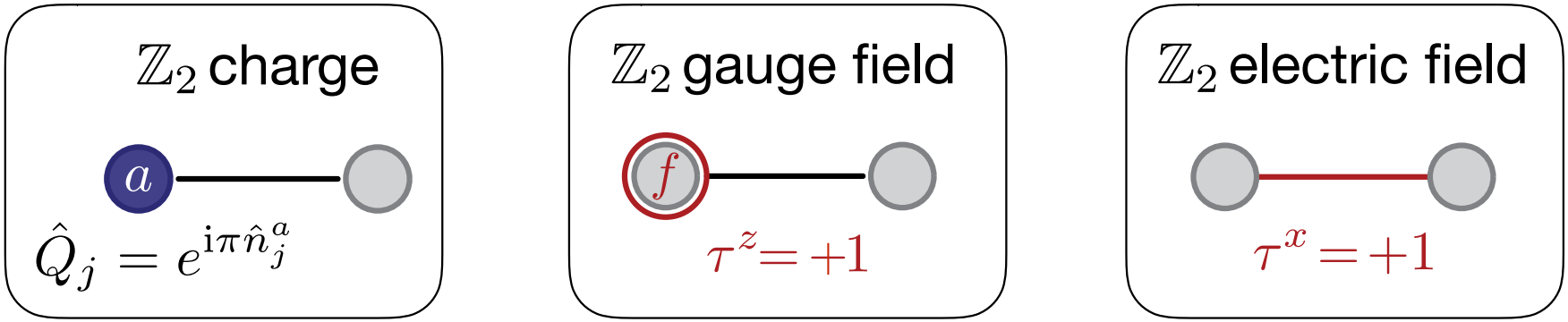
Martinez et al., Nature (2016)

Challenge: Local gauge constraints

Z_2 lattice gauge theory coupled to matter

Elementary ingredients of the model:

$$\hat{H}_{\mathbb{Z}_2} = +\sum_j J_a \left(\hat{\tau}^z_{\langle j,j+1\rangle} \hat{a}^\dagger_j \hat{a}_{j+1} + \text{h.c.} \right) - \sum_j J_f \hat{\tau}^x_{\langle j,j+1\rangle}$$



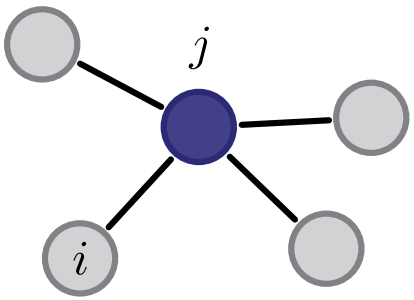
- *matter-gauge field coupling* with strength J_a
- energy of *electric field* J_f

a matter field f gauge field

Symmetries and Gauss's law

Z_2 symmetry:

$$\hat{G}_j = \hat{Q}_j \prod_{i:\langle i,j\rangle} \hat{\tau}^x_{\langle i,j\rangle}, \qquad \left[\hat{H}, \hat{G}_j\right] = 0 \quad \forall j$$



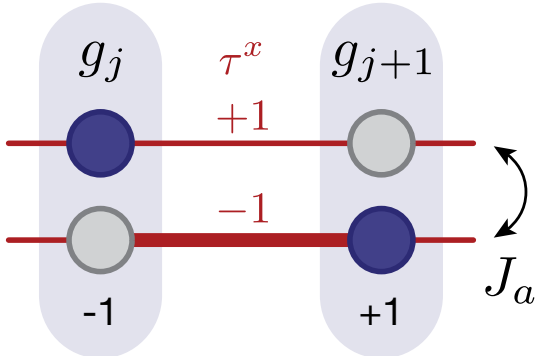
eigenvalues: $g_j = \pm 1$

- g_j=-1 interpreted as *static local background charges*

Z_2 Gauss's law:

$$\boxed{\hat{G}_j|\psi\rangle = g_j|\psi\rangle}$$

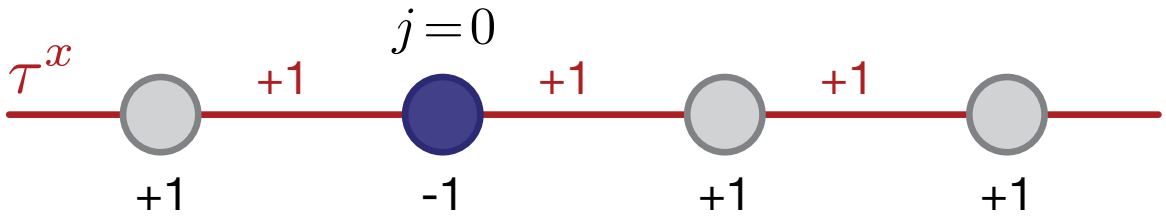
g_j : local conserved quantities



- subsectors characterized by set of conserved quantities $\{g_j\}$

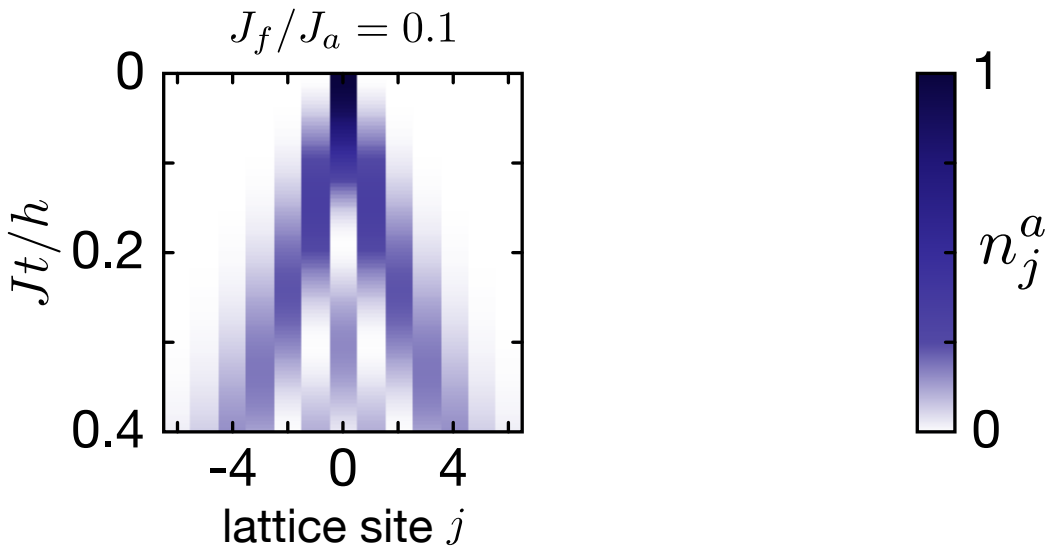
Dynamics 1D model

Initial state:



- Single charge on site *j*=0
- Eigenstate of electric field operator

vanishing electric field $J_f \to 0$



C. Schweizer *et al.*, arXiv:1901.07103 (2019)

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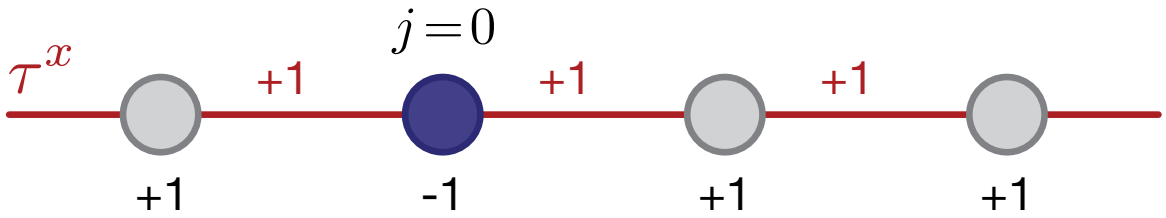

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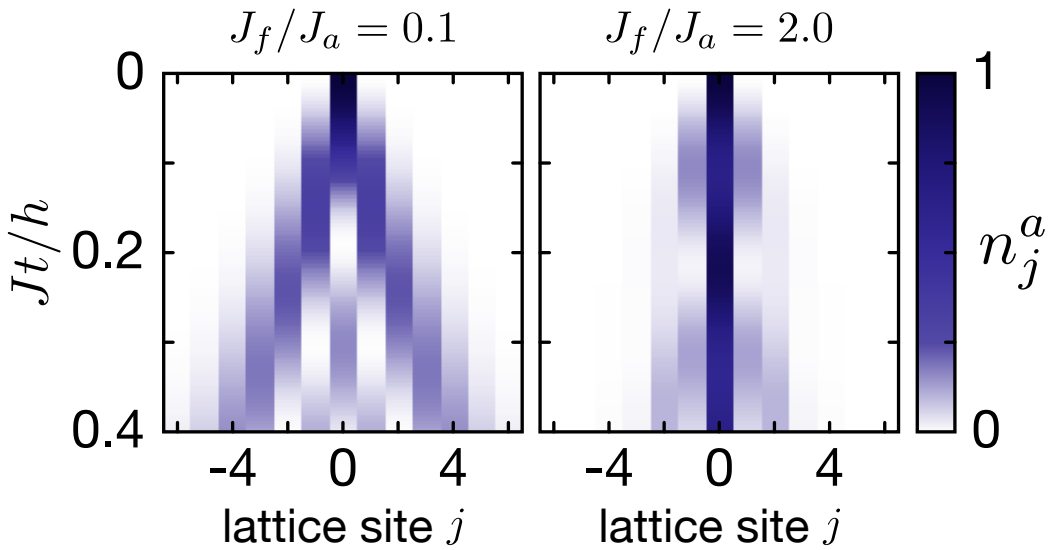


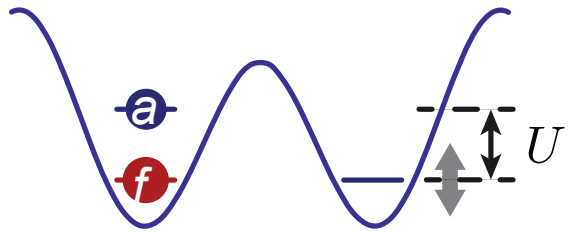
electric field dominated regime $J_f \gg J_a$

C. Schweizer *et al.*, arXiv:1901.07103 (2019)

Floquet engineering

Z_2 lattice gauge theories using Floquet techniques

L. Barbiero et al., arXiv:1810.02777



- Mixture of *two components* (*a* and *f*-particles) to implement matter- and gauge-fields

- *Resonant periodic driving* at the value of the on-site *Hubbard interaction* strength *U*

→ Implement *building block of Z_2 lattice gauge theories* in double well

see also: A. Bermudez & D. Porras, New J. Phys. 17, 103021 (2015); E. Zohar *et al.*, PRL 118, 070501 (2017)

Floquet scheme

Double-well realization:

- Bosonic ^{87}Rb, two states with *opposite magnetic moment*

a matter field $F = 1, m_F = -1$

f gauge field $F = 1, m_F = +1$

- *State-dependent* offset $\Delta_f = U \rightarrow$ *break symmetry between a and f-particles!*
- *State-independent* modulation $\hbar\omega = U$

$$\hat{H}(t) = -J\left(\hat{a}_2^\dagger \hat{a}_1 + \hat{f}_2^\dagger \hat{f}_1 + \text{h.c.}\right) + U\sum_j \hat{n}_j^a \hat{n}_j^f$$
$$+\Delta_f \hat{n}_1^f + A\cos(\omega t + \phi)\left(\hat{n}_1^a + \hat{n}_1^f\right)$$

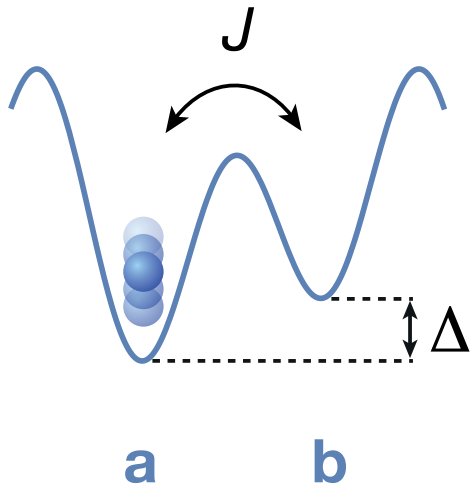
see also: A. Bermudez & D. Porras, New J. Phys. 17, 103021 (2015)

Laser-assisted tunneling

Multi-photon processes:

- Tilted double-well potential

$$\hat{H} = -J\left(\hat{a}^\dagger\hat{b} + \hat{b}^\dagger\hat{a}\right) + \Delta\ \hat{b}^\dagger\hat{b}$$



- *Resonant* modulation at $\Delta_\nu = \nu\hbar\omega$

$$\hat{V}(t) = V_0\ \cos(\omega t + \phi)\hat{a}^\dagger\hat{a}$$

- Restored tunneling $\boxed{J_{\text{eff}} = J\mathcal{J}_\nu(\chi)\mathrm{e}^{i\nu\phi}}$ $\chi = \dfrac{V_0}{\hbar\omega}$

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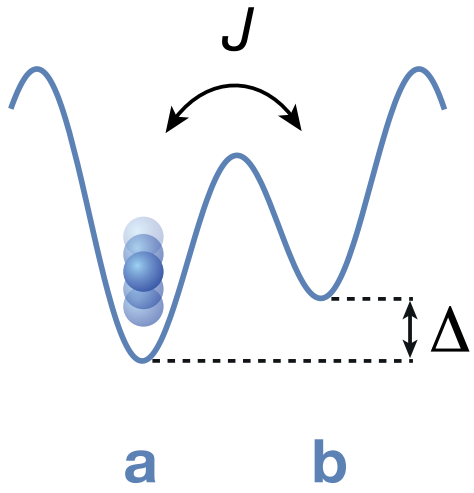

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- Restored tunneling $\boxed{J_{\text{eff}} = J\mathcal{J}_\nu(\chi)\mathrm{e}^{i\nu\phi}}$ $\chi = \dfrac{V_0}{\hbar\omega}$

- Reflection properties of Bessel function: $\mathcal{J}_{-\nu}(\chi) = (-1)^\nu\mathcal{J}_\nu(\chi)$

Effective Floquet model

Resonant driving $\hbar\omega = U$ & *high-frequency limit* $\hbar\omega \gg J$

$$\phi = \{0, \pi\}$$

a matter field *f* gauge field

Effective Floquet model

Resonant driving $\hbar\omega = U$ & *high-frequency limit* $\hbar\omega \gg J$

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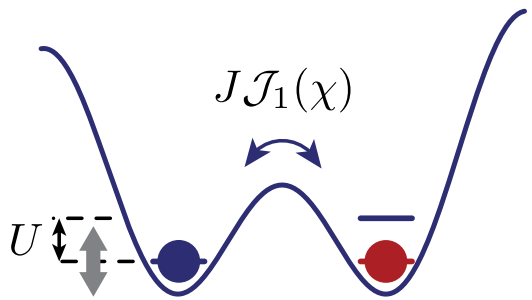
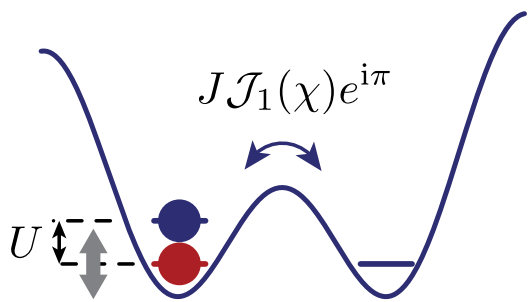


Tunneling of matter particle:



Reflection properties of Bessel function:

$$\mathcal{J}_{-\nu}(\chi) = (-1)^{\nu}\mathcal{J}_{\nu}(\chi)$$

a matter field *f* gauge field

Effective Floquet model

Resonant driving $\hbar\omega = U$ & *high-frequency limit* $\hbar\omega \gg J$

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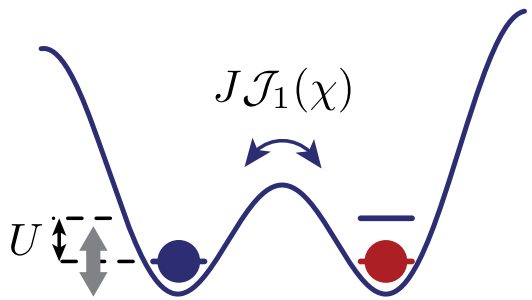
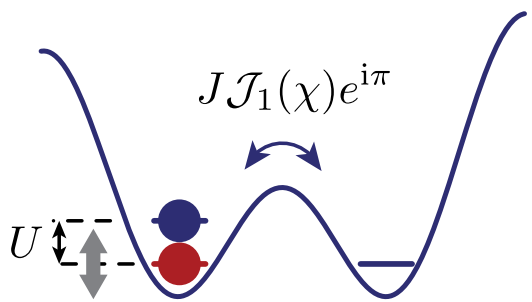


Tunneling of matter particle:

Link variable: $\hat{\tau}^z = \hat{n}_1^f - \hat{n}_2^f$



Reflection properties of Bessel function:

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a matter field *f* gauge field

Effective Floquet model

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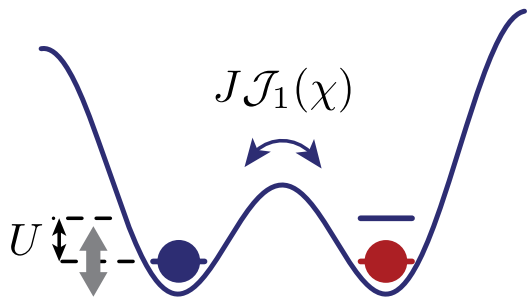

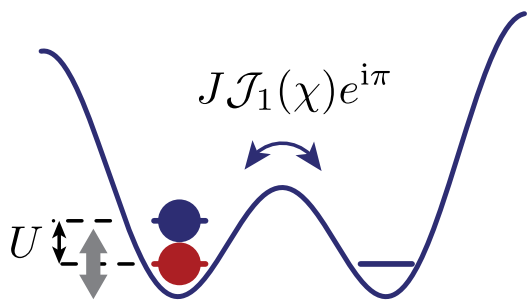

Tunneling of matter particle:

Link variable: $\hat{\tau}^z = \hat{n}_1^f - \hat{n}_2^f$

$$\hat{H}_{\text{eff}} = J_a\, \hat{\tau}^z \left(\hat{a}_2^\dagger \hat{a}_1 + \hat{a}_1^\dagger \hat{a}_2\right) \qquad J_a = J\mathcal{J}_1(\chi)$$

Reflection properties of Bessel function:

$$\mathcal{J}_{-\nu}(\chi) = (-1)^\nu \mathcal{J}_\nu(\chi)$$

a matter field *f* gauge field

Effective Floquet model

Resonant driving $\hbar\omega = U$ & *high-frequency limit* $\hbar\omega \gg J$

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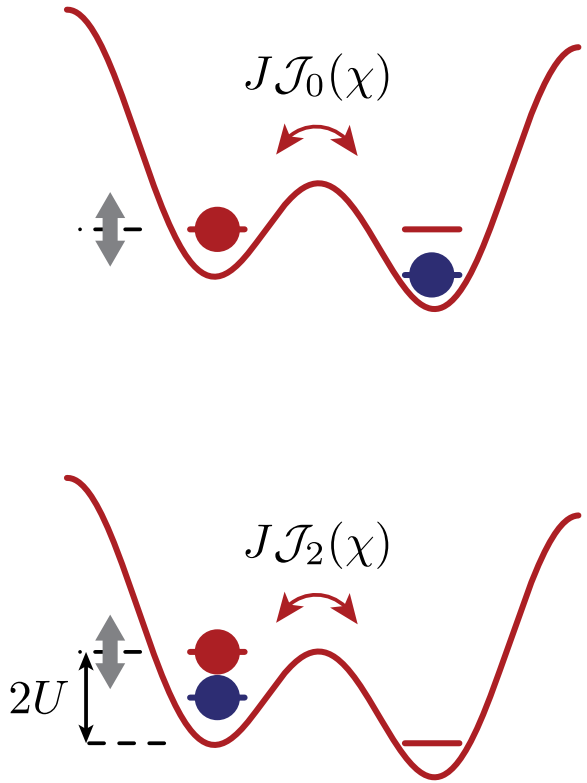


Tunneling of gauge-field particle:

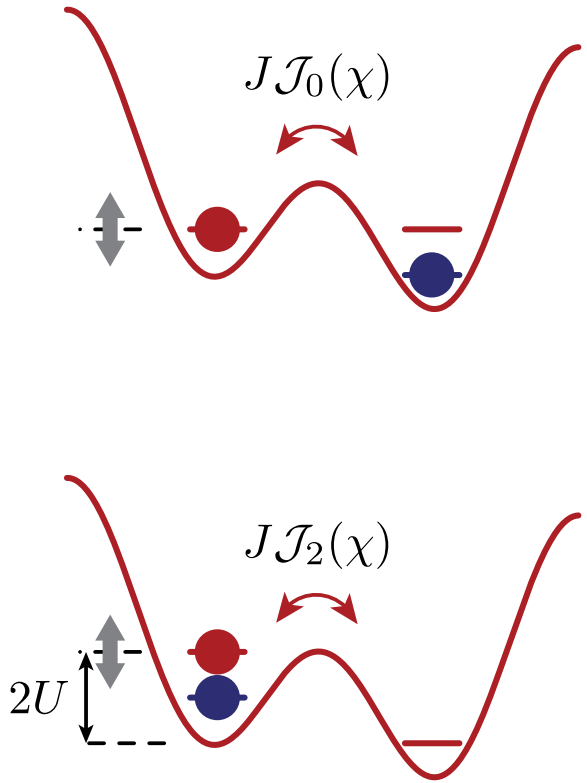
Needs to be real: $\hat{\tau}^x = \hat{f}_1^\dagger \hat{f}_2 + \hat{f}_2^\dagger \hat{f}_1$

a matter field *f* gauge field

Effective Floquet model

Resonant driving $\hbar\omega = U$ & *high-frequency limit* $\hbar\omega \gg J$

$$\phi = \{0, \pi\}$$



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$$\hat{J}_f = J\mathcal{J}_0(\chi)\,\hat{n}_1^a + J\mathcal{J}_2(\chi)\,\hat{n}_2^a$$

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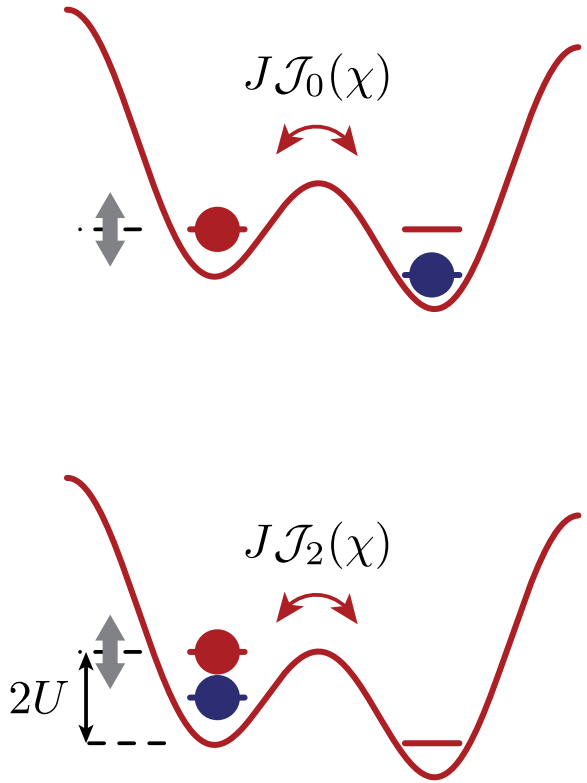

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Can be avoided for:

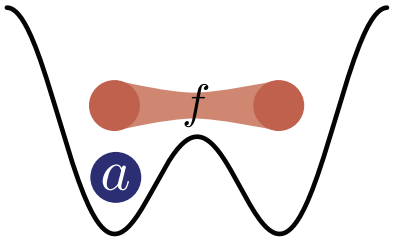
$$\chi = 1.84: \quad \mathcal{J}_0(\chi) = \mathcal{J}_2(\chi)$$

a matter field *f* gauge field

Dynamics double-well

Initial state I:

eigenstate of electric-field operator $\hat{\tau}^x$



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C. Schweizer *et al.*, arXiv:1901.07103 (2019)

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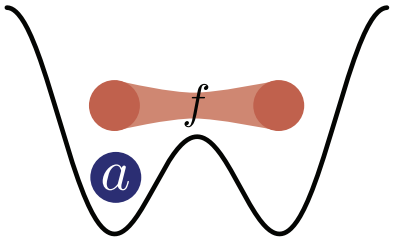

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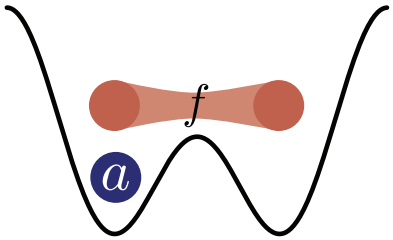

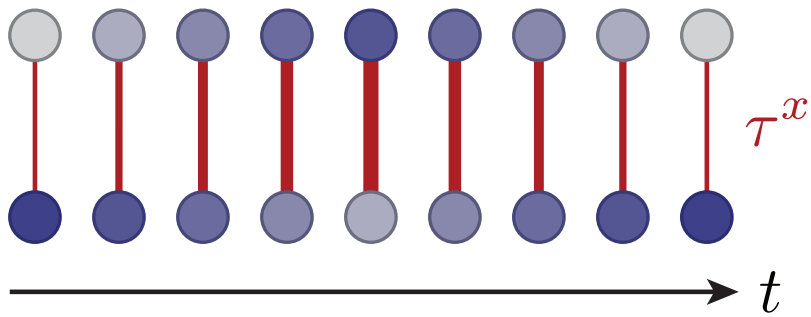

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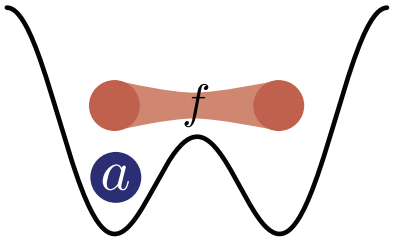

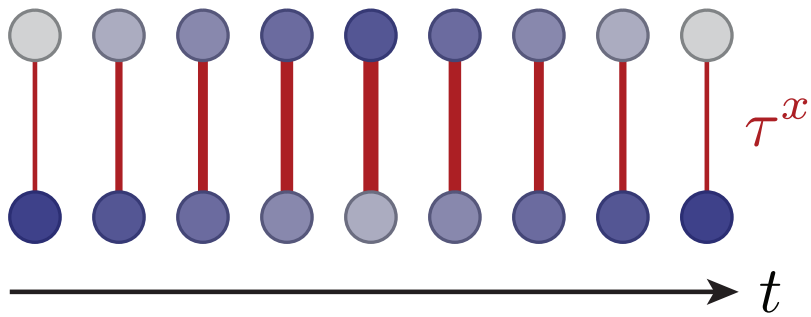

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⇒ oscillation amplitude / frequency depends on ratio J_f / J_a !

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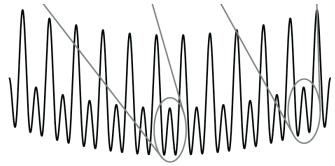

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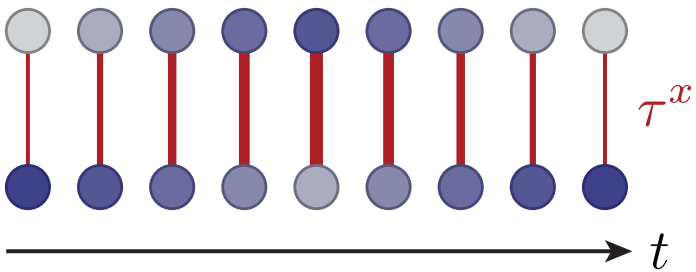

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$\Rightarrow$ *Z_2 charge* + *Z_2 gauge field*

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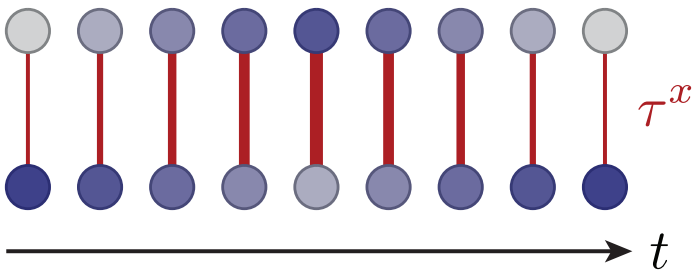

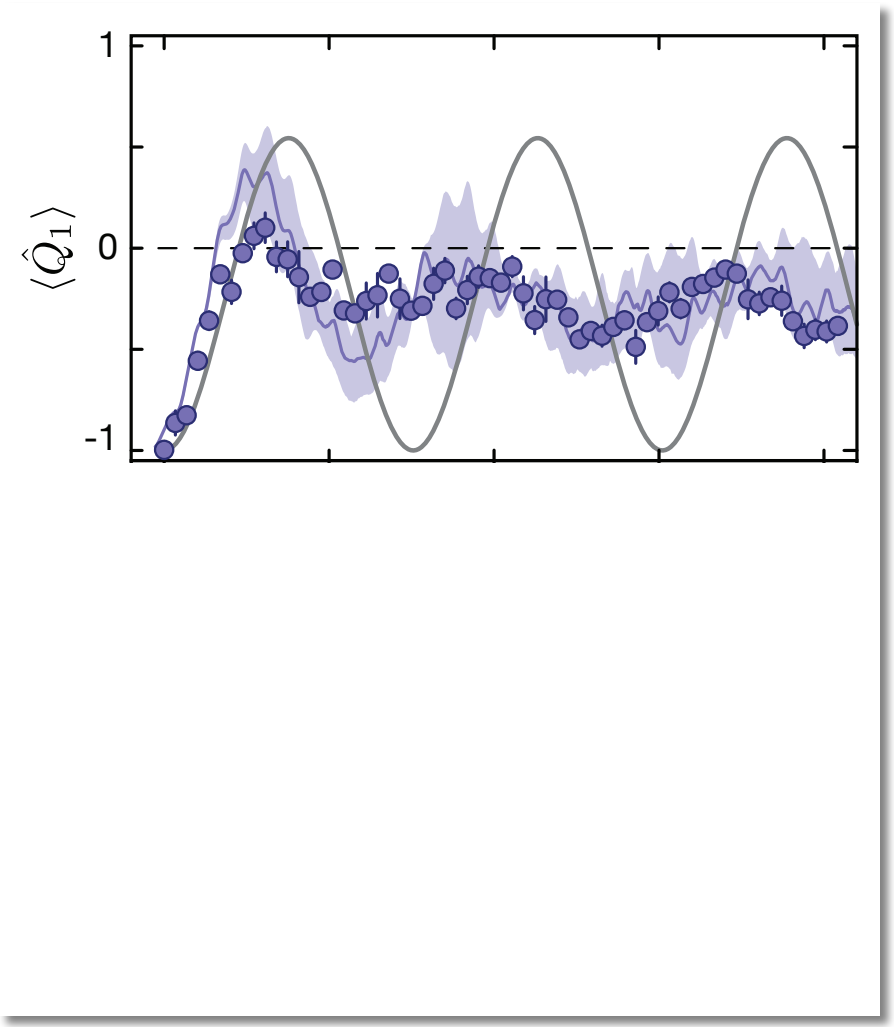

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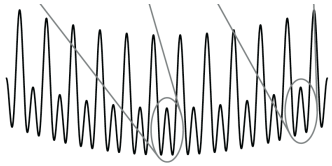

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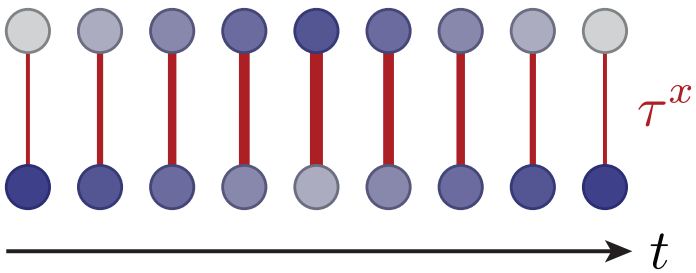

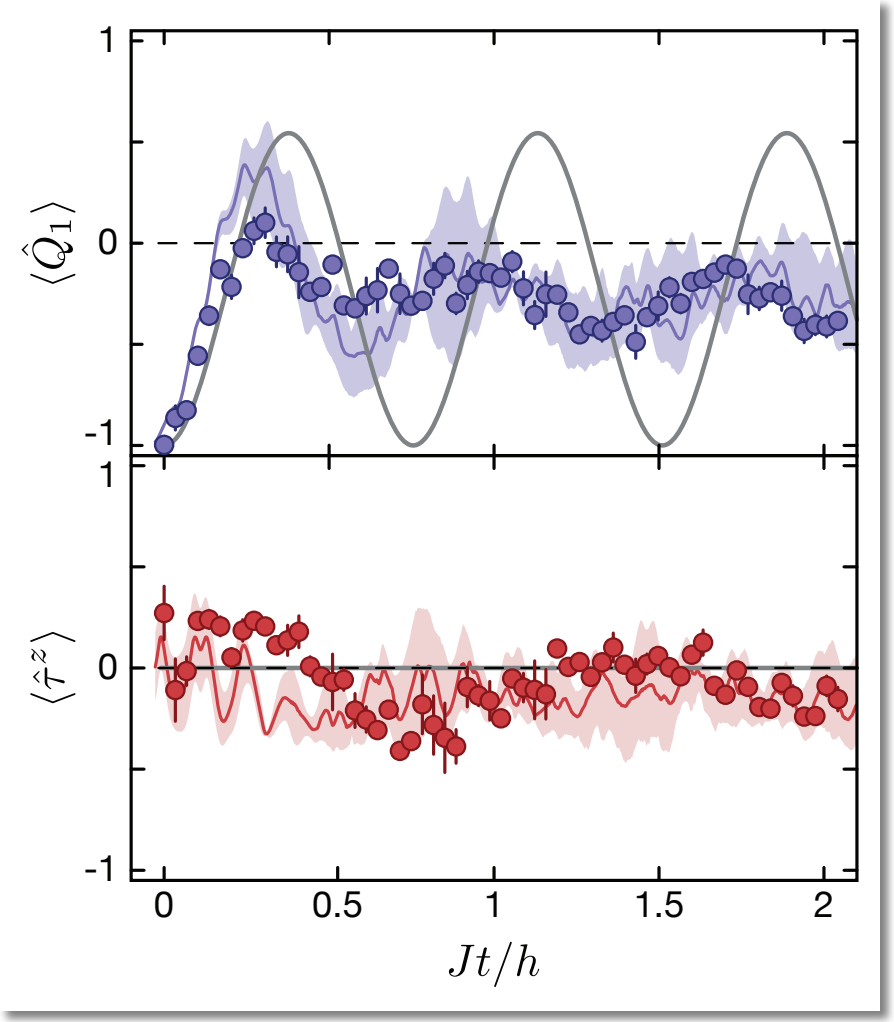

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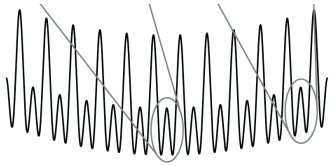

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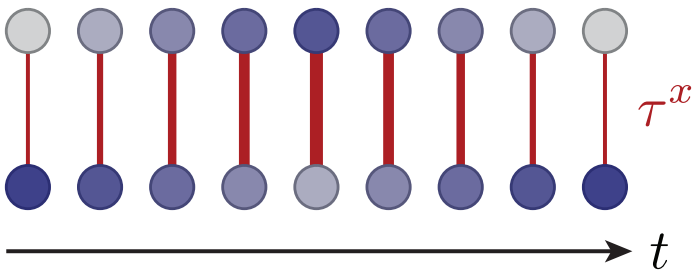

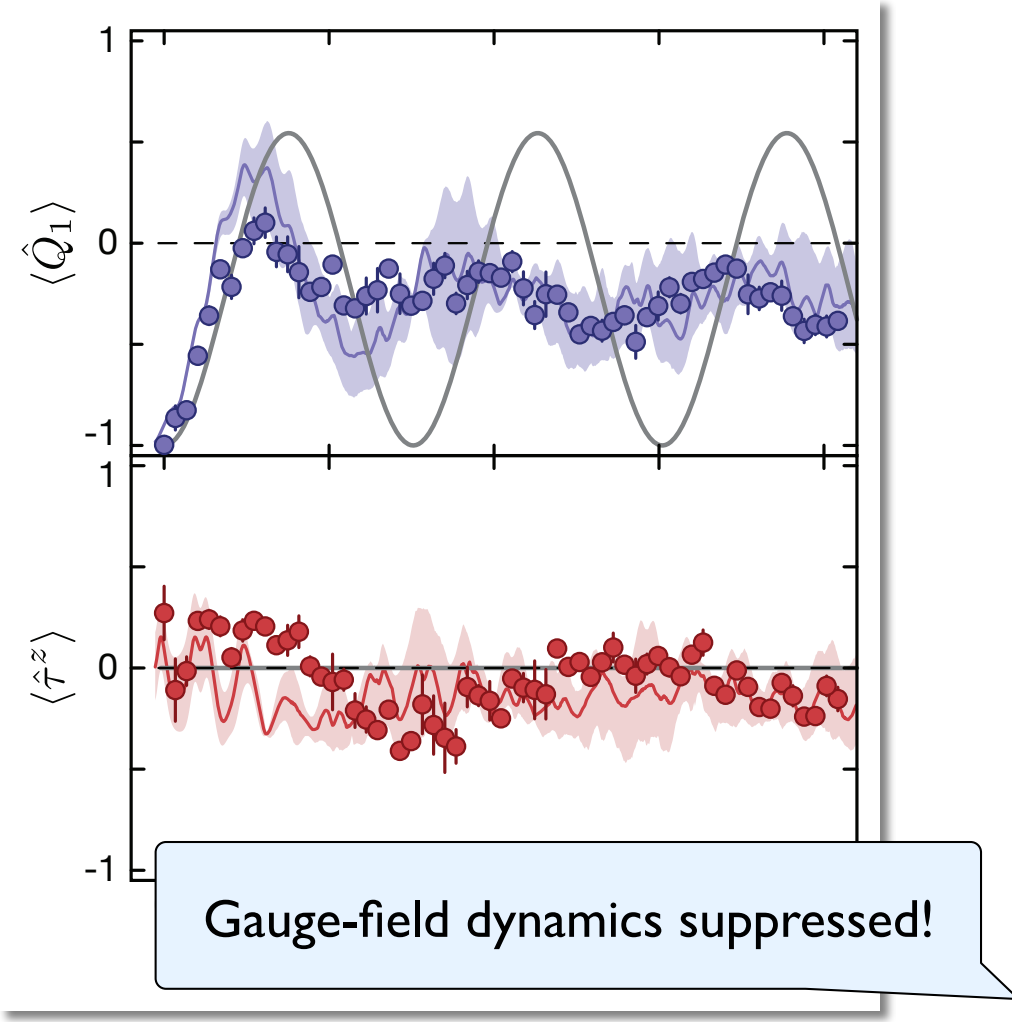

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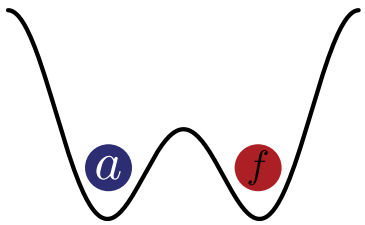

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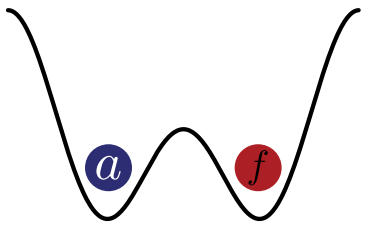

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coherent superposition of two subsectors
with $g_1 = -g_2 = \pm 1$

$$\Rightarrow \quad \langle \hat{G}_1 \rangle = \langle \hat{G}_2 \rangle = 0$$

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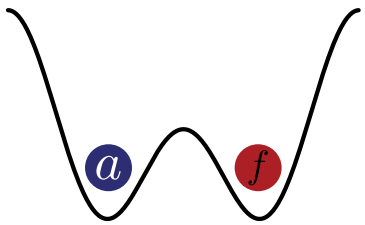

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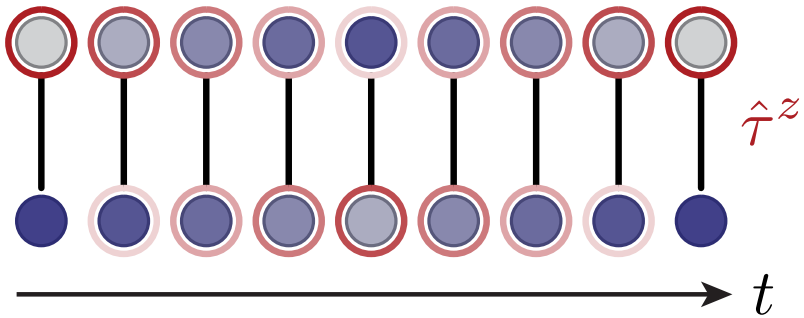

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- Note, subsectors are *not coupled*
- Dynamics of charge unchanged

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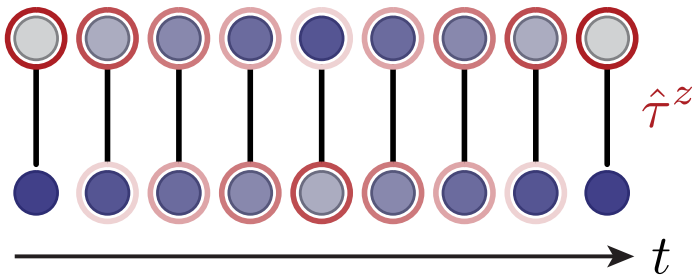

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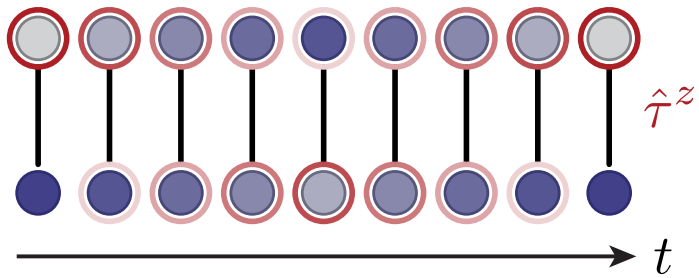

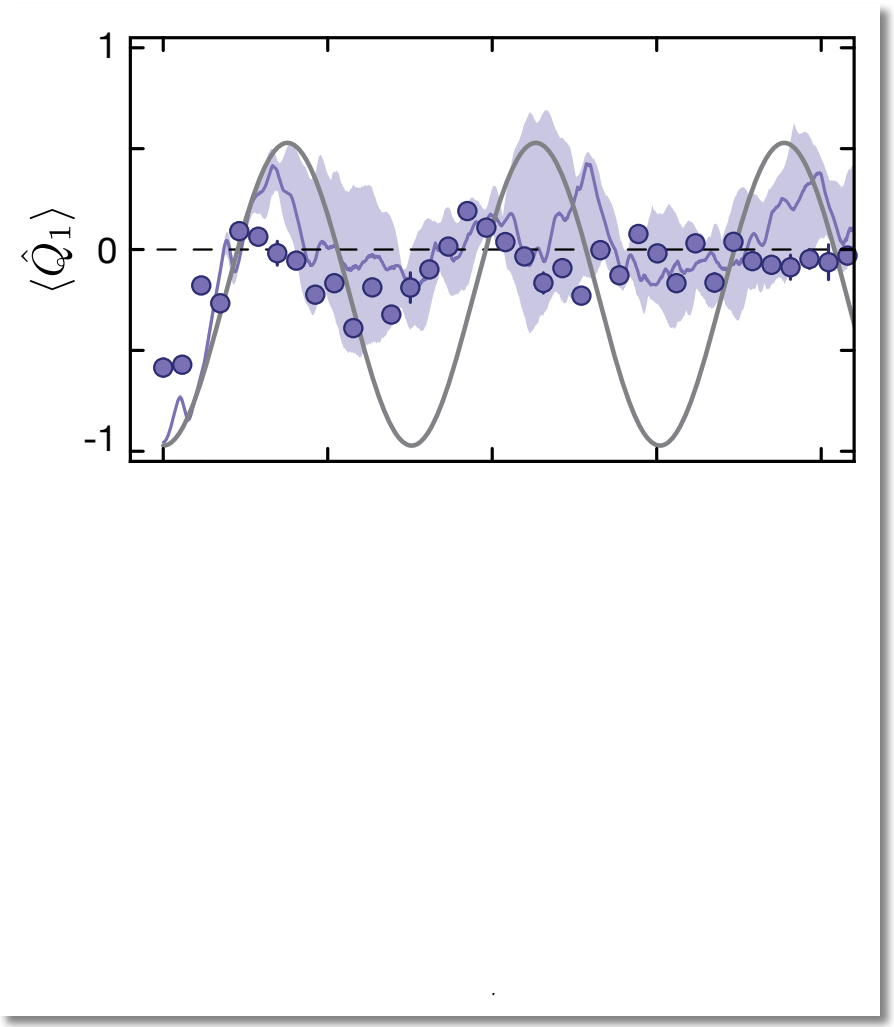

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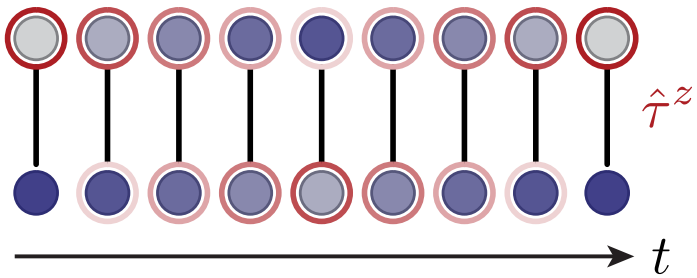

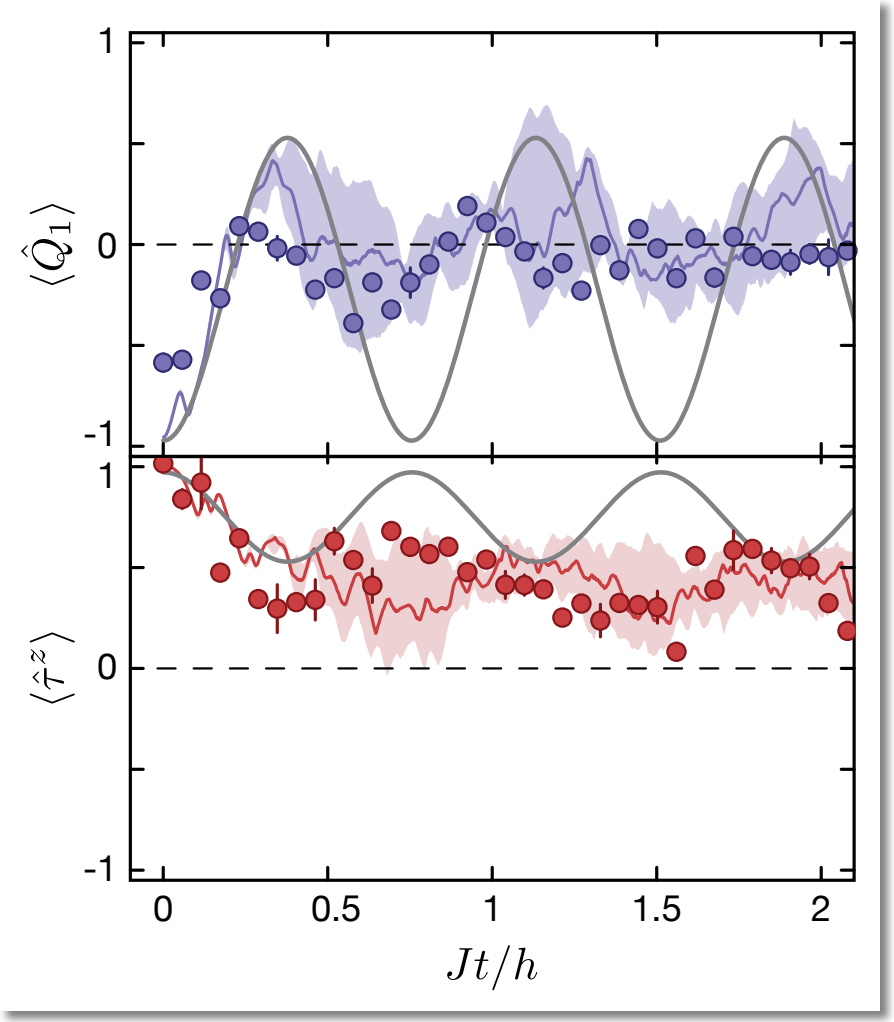

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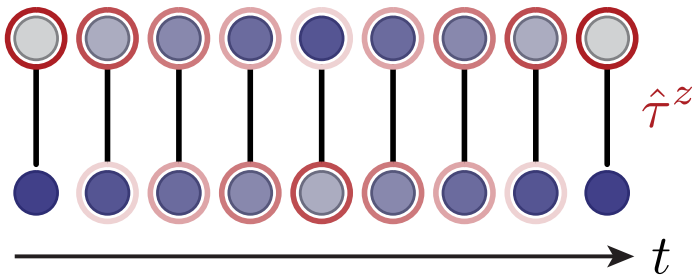

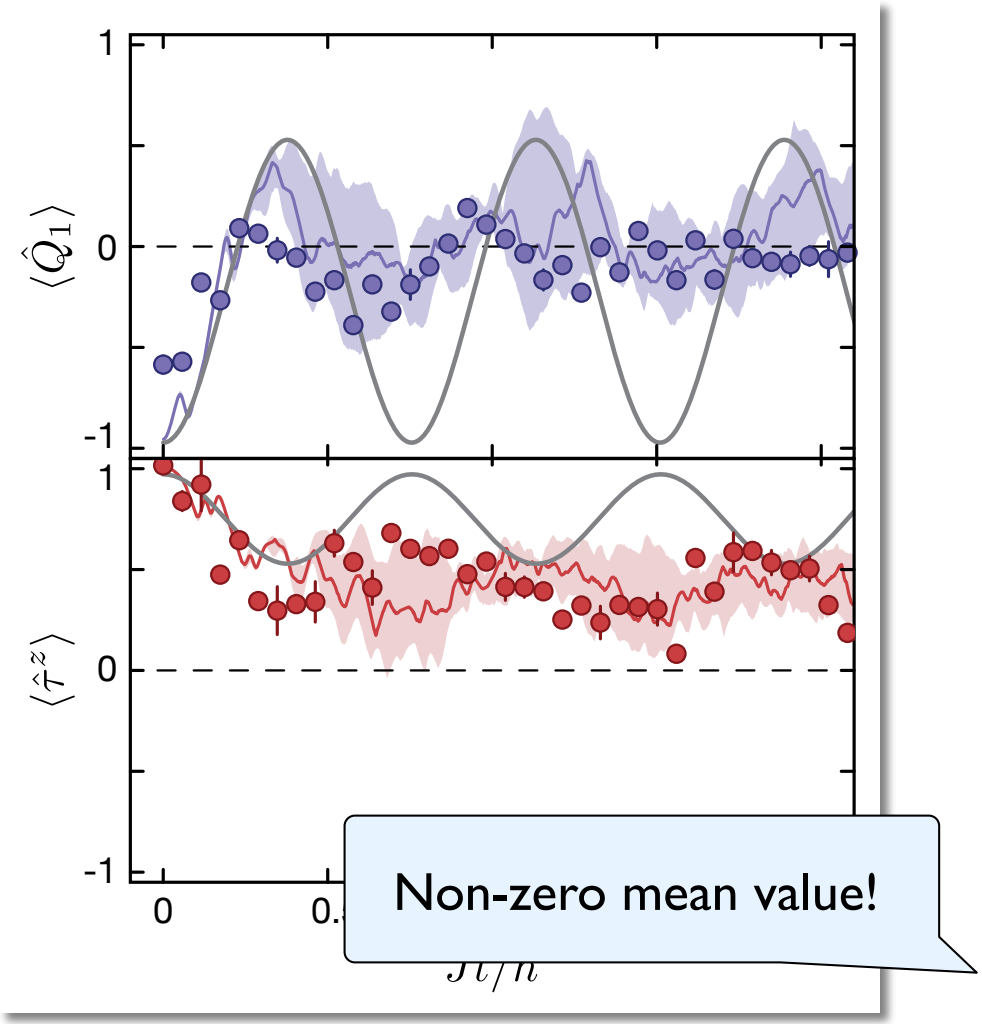

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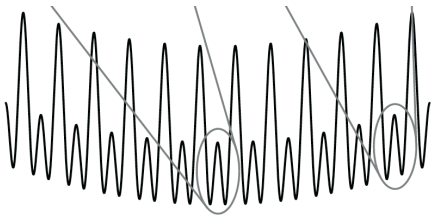
Corrections

Symmetry-breaking terms



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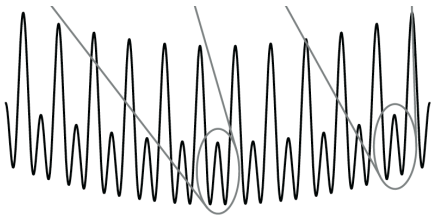

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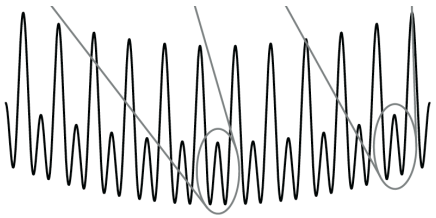

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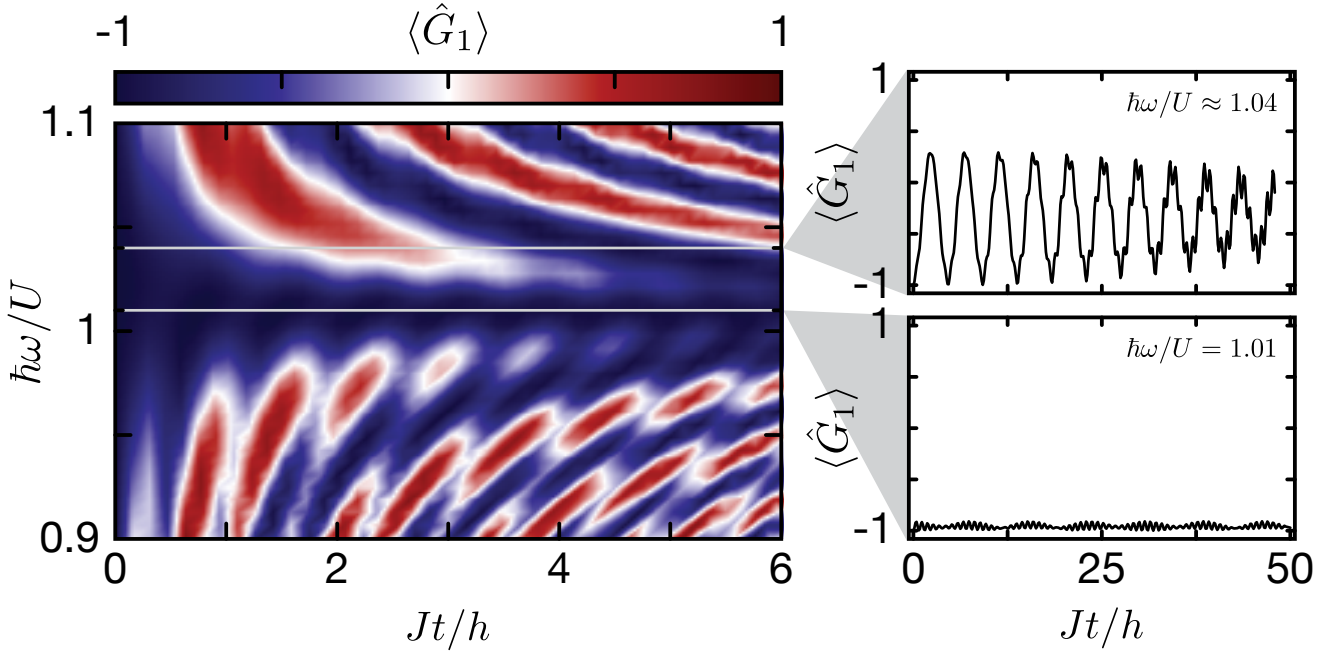

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Summary & Outlook

Double-well realization

- Good understanding of the *full time-dependent dynamics*
- Observed *non-trivial dynamics* due to gauge constraints
- Detailed analysis of *symmetry breaking terms* + routes to suppress them

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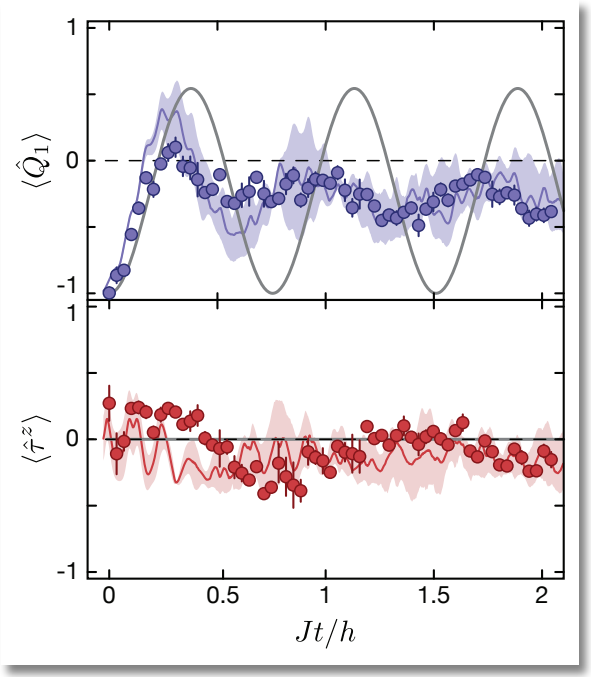

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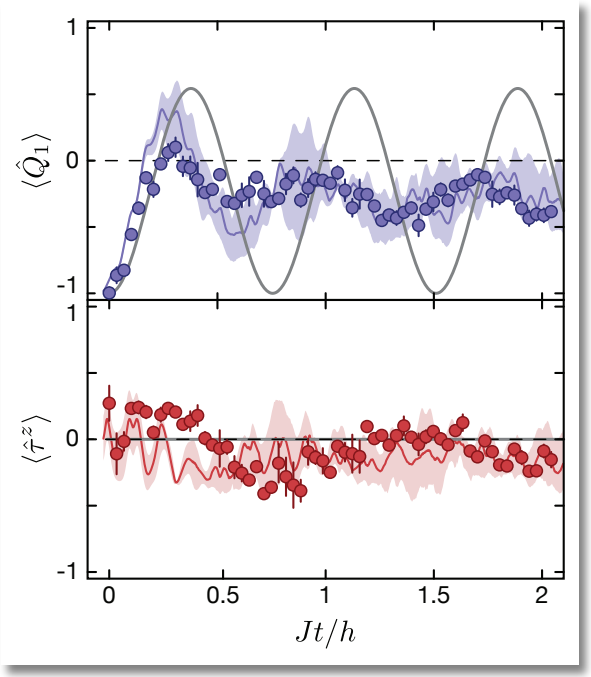
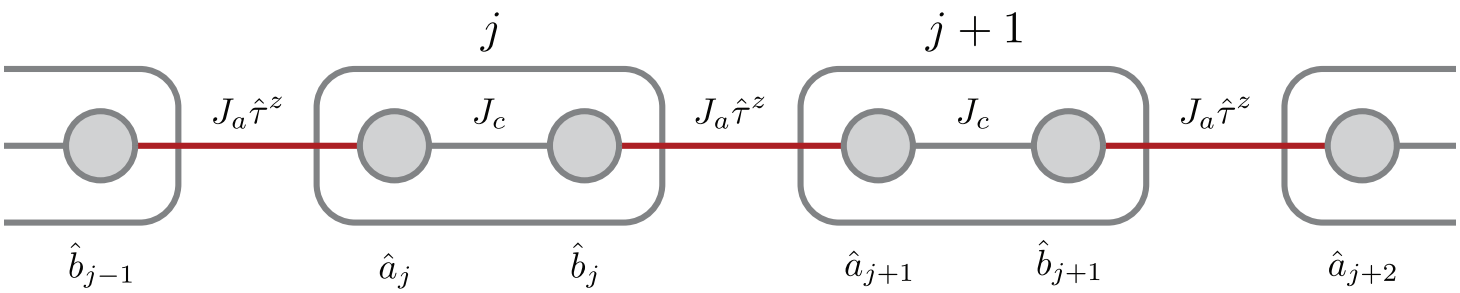


Floquet-scheme / phase diagram of extended 1D models



The team

Experiment:

Christian Schweizer

Moritz Berngruber

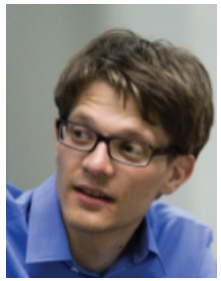
Karen Wintersperger

Christoph Braun

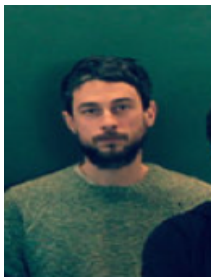
Immanuel Bloch

MA

Theory:



Fabian Grusdt
(TU Munich)



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Eugene Demler
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Marco Di Liberto
(ULB Bruxelles)

Nathan Goldman
(ULB Bruxelles)