

Engineering Z_2 Lattice Gauge Theories with a strongly interacting atomic mixture

Luca Barbiero

Center for Nonlinear Phenomena and Complex Systems, Université Libre de Bruxelles, Belgium

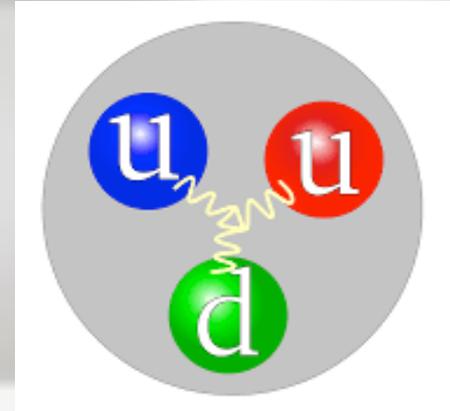
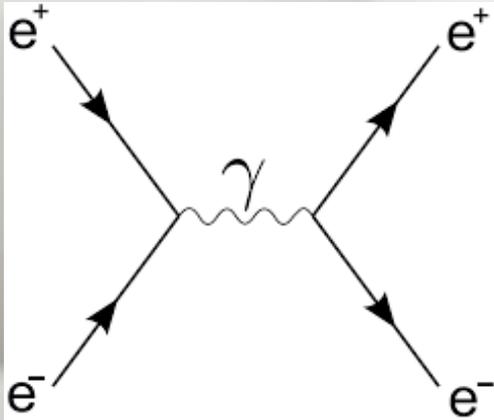
LB, C. Schweizer, M. Aidelsburger, E. Demler, N. Goldman, F. Grusdt arXiv:1810.02777

C. Schweizer, F. Grusdt, M. Berngruber, LB, E. Demler, N. Goldman, I. Bloch, M. Aidelsburger
arXiv:1901.07103

Gauge Theories in High Energy

U(1) GT: QED interaction between matter and light (gauge bosons)

SU(3) GT: QCD interaction between quarks and gluons (gauge bosons)

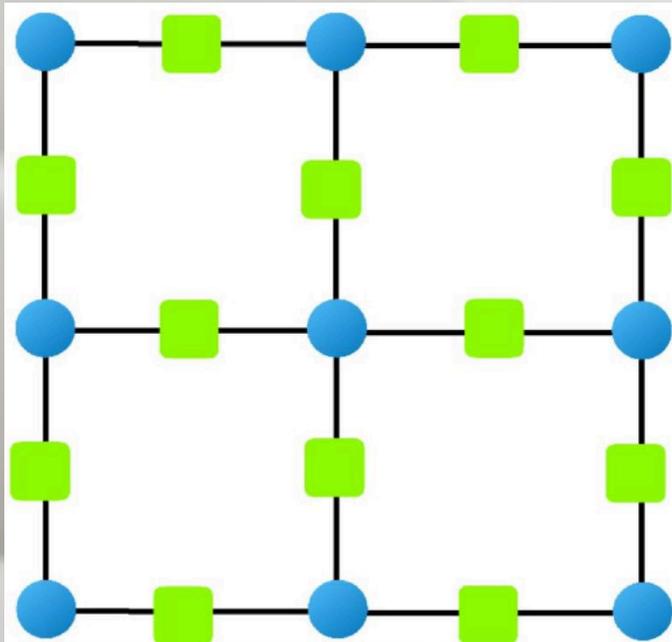


standard model of particles physics

mass = 2.3 MeV/c ² charge = 2/3 spin = 1/2	mass = 1.276 GeV/c ² charge = 2/3 spin = 1/2	mass = 173.21 GeV/c ² charge = 2/3 spin = 1/2	mass = 0 charge = 0 spin = 1	mass = 126 GeV/c ² charge = 0 spin = 0
u up	c charm	t top	g gluon	H Higgs boson
mass = 4.8 MeV/c ² charge = -1/3 spin = 1/2	mass = 95 MeV/c ² charge = -1/3 spin = 1/2	mass = 4.18 GeV/c ² charge = -1/3 spin = 1/2	mass = 0 charge = 0 spin = 1	
d down	s strange	b bottom	γ photon	
mass = 0.511 MeV/c ² charge = -1 spin = 1/2	mass = 105.7 MeV/c ² charge = -1 spin = 1/2	mass = 1.777 GeV/c ² charge = -1 spin = 1/2	mass = 91.2 GeV/c ² charge = 0 spin = 1	
e electron	μ muon	τ tau	Z Z boson	
mass = 0 charge = 0 spin = 1/2	mass = 0.17 MeV/c ² charge = 0 spin = 1/2	mass = 1.777 MeV/c ² charge = 0 spin = 1/2	mass = 80.4 GeV/c ² charge = 1 spin = 1	
ν _e electron neutrino	ν _μ muon neutrino	ν _τ tau neutrino	W W boson	
LEPTONS			GAUGE BOSONS	

GFs are fundamental degrees of freedom + discretization is needed to avoid UV divergence → LGT

Gauge Theories in Condensed Matter



particles

gauge field

LGT High- T_c Superconductors, topological state of matter,
frustrated magnets, quantum computation,.....
(Kitaev, Sachdev, Fisher, Vishwanath, Balents,...)

Gauge fields arise as a consequence of rewriting the Hamiltonian in terms of new collective degrees of freedom (slave-particle decomposition), distinct from those of the original many-particle system but which dominate the low-energy physics in the region of parameter space of interest.

In most cases the gauge theory is as hard (or harder) to solve than the original many-body problem without gauge fields :(

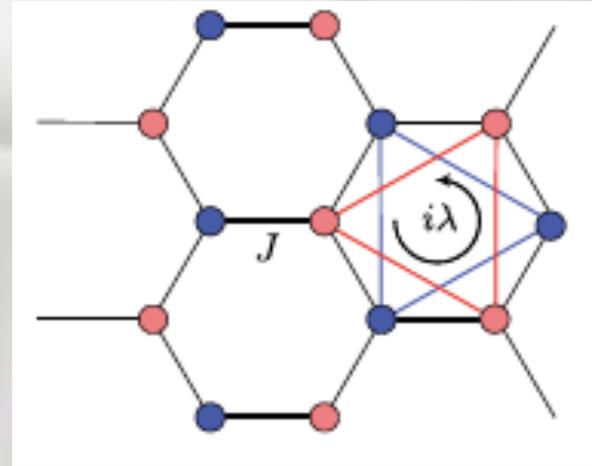
We need a quantum simulator!!!

see proposals to simulate LGT in AMO platforms IQOQI, ICFO, MPQ,.....

Topology with Cold Atoms

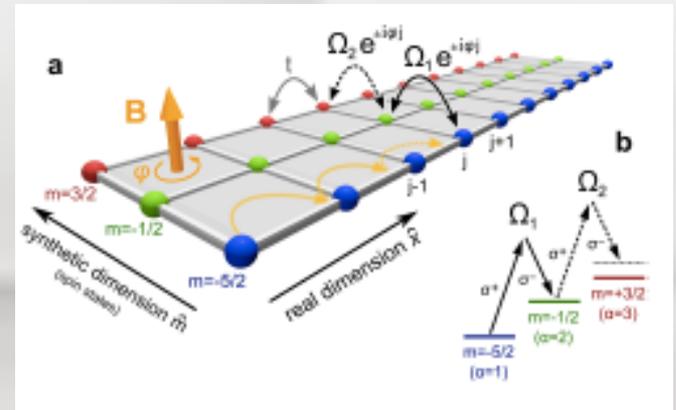
Haldane model (ETH)

$$H = -J \sum_{\langle j,k \rangle} a_j^\dagger a_k + \lambda \sum_{\langle\langle j,k \rangle\rangle} i^\odot a_j^\dagger a_k$$

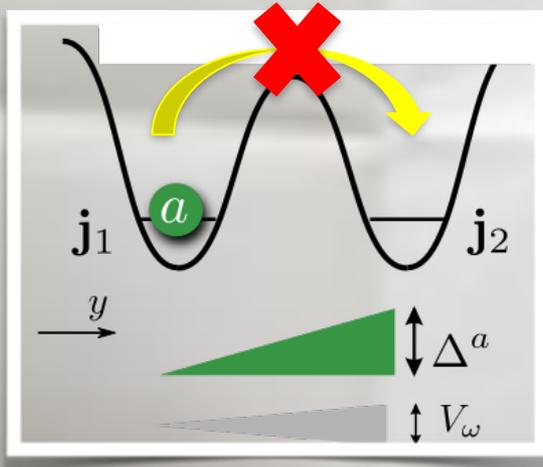


Fermi Ladder (LENS)

$$H = \sum_{j,\alpha} [-t(c_{j,\alpha}^\dagger c_{j+1,\alpha} + h.c.) + \mu_j n_{j,\alpha}] + \sum_{j,\alpha} \left[\frac{\Omega_\alpha}{2} (e^{i\varphi_j} c_{j,\alpha}^\dagger c_{j,\alpha+1} + h.c.) + \xi_\alpha n_{j,\alpha} \right]$$



Building Block #1: single particle in a double well



Theory by Zoller, Jaksch NJP 2003

$$H = -t(|j_1\rangle\langle j_2| + |j_2\rangle\langle j_1|) + \Delta^a |j_2\rangle\langle j_2|$$

$$|\Delta^a| \gg t$$

+

$$\Delta^\omega |j_2\rangle\langle j_2| = A \cos(\omega t + \phi_{2,1}) |j_2\rangle\langle j_2|$$

$$\Delta^a = n\omega$$

$$H_{eff} = -\tilde{t}(|j_2\rangle\langle j_1| e^{in\phi_{2,1}} + |j_1\rangle\langle j_2| e^{-in\phi_{2,1}})$$

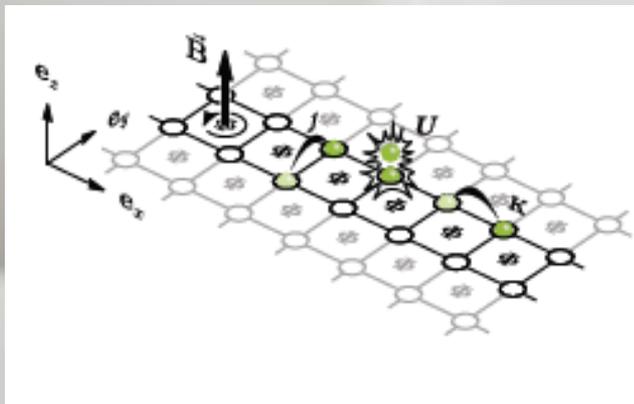
$$\Delta^a = -n\omega$$

$$H_{eff} = -\tilde{t}(|j_2\rangle\langle j_1| e^{in(\pi - \phi_{2,1})} + |j_1\rangle\langle j_2| e^{-in(\pi - \phi_{2,1})})$$

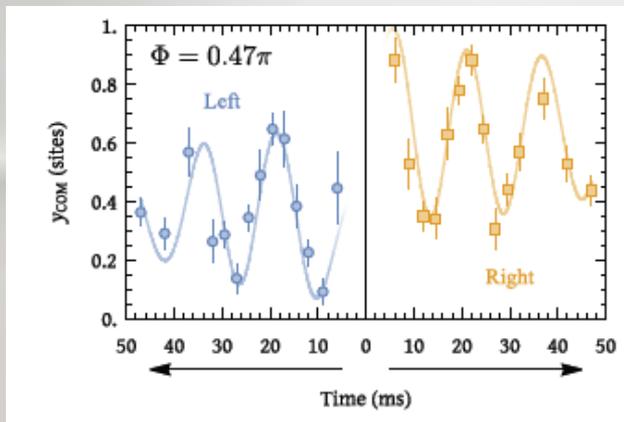
the field remains insensitive to the spatial motion of the atoms.....classical!!!

Microscopy of the interacting Harper–Hofstadter model in the two-body limit

M. Eric Tai¹, Alexander Lukin¹, Matthew Rispoli¹, Robert Schittko¹, Tim Menke¹, Dan Borgnia¹, Philipp M. Preiss^{1,†}, Fabian Grusdt¹, Adam M. Kaufman¹ & Markus Greiner¹



gradient
+
shaking



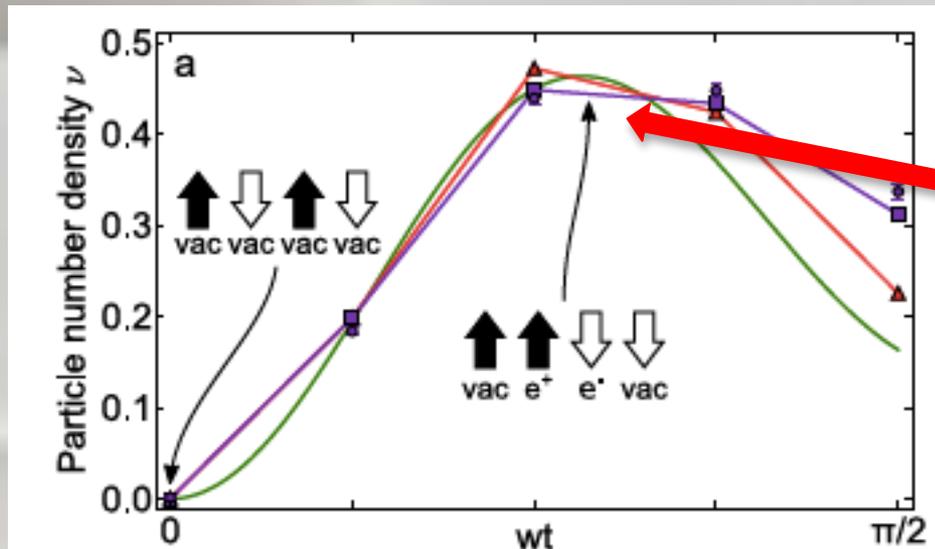
chiral dynamics

See experiments also at LMU, MIT, NIST, PARIS, CHICAGO, LENS....

Real-time dynamics of lattice gauge theories with a few-qubit quantum computer

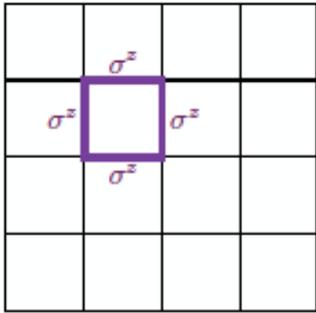
Esteban A. Martinez^{1*}, Christine A. Muschik^{2,3*}, Philipp Schindler¹, Daniel Nigg¹, Alexander Erhard¹, Markus Heyl^{2,4}, Philipp Hauke^{2,3}, Marcello Dalmonte^{2,3}, Thomas Monz¹, Peter Zoller^{2,3} & Rainer Blatt^{1,2}

Kogut-Susskind Hamiltonian with trapped ions



creation of electron-positron

Z_2 Lattice Gauge Theories (gapped matter dof)



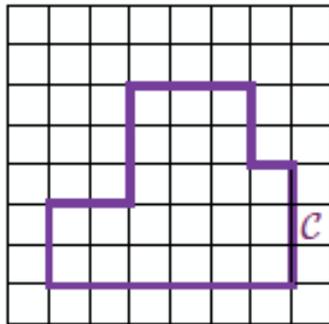
(a)

$$G_i = \begin{array}{c|c} \sigma^x & \sigma^x \\ \hline \sigma^x & \sigma^x \end{array} \quad (b)$$

$$H_{Z_2} = -K \sum_{\square} \prod_{l \in \square} \sigma_l^z - g \sum_l \sigma_l^x$$

$$G_i = \prod_{l \in +} \sigma_l^x \quad [H_{Z_2}, G_i] = 0$$

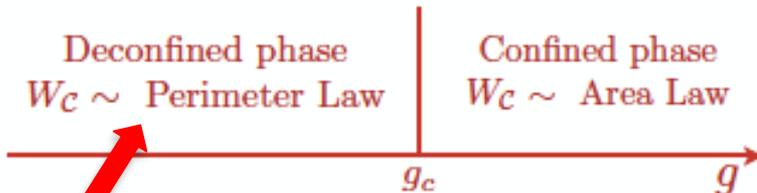
confinement-deconfinement phase transition



$$W_C = \prod_C \sigma^z$$

Deconfined phase
 $W_C \sim$ Perimeter Law

Confined phase
 $W_C \sim$ Area Law



$g \gg K$ strong fluctuations of the Z_2 flux \rightarrow
fast decay

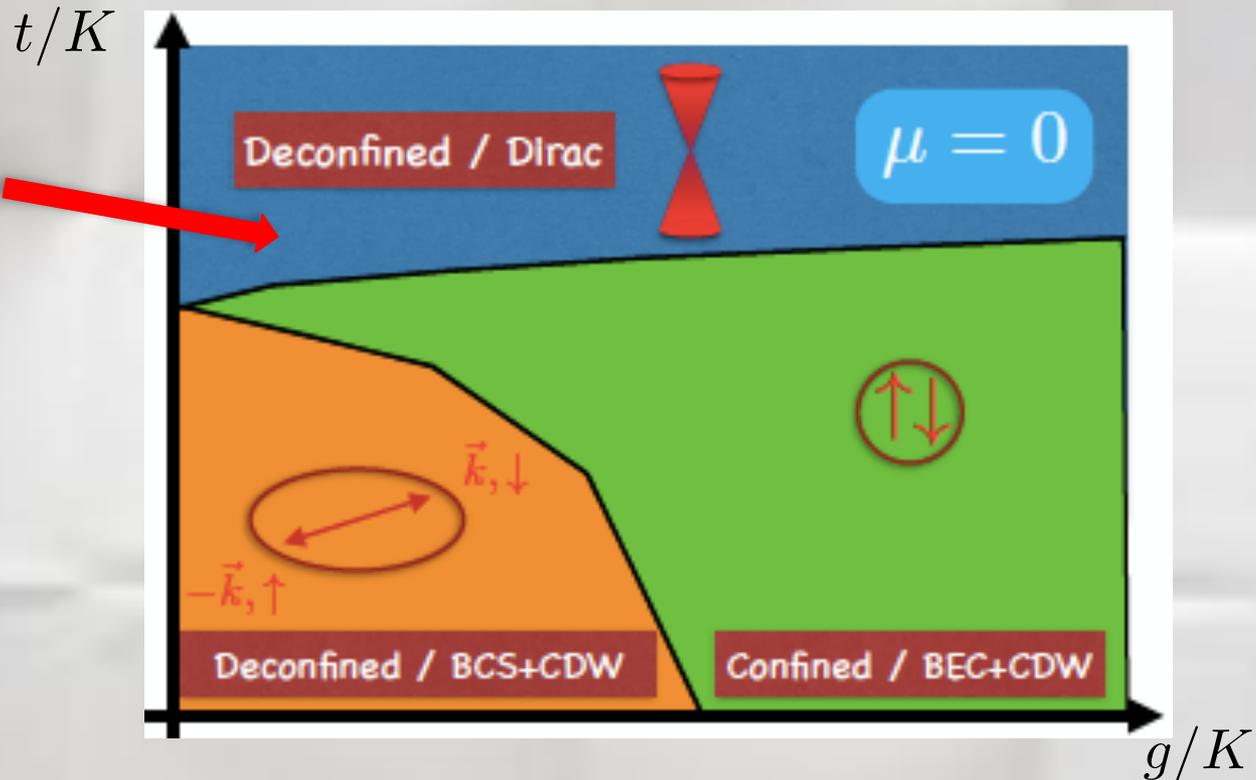
$g \ll K$ small fluctuations of the Z_2 flux \rightarrow
slow decay

topological

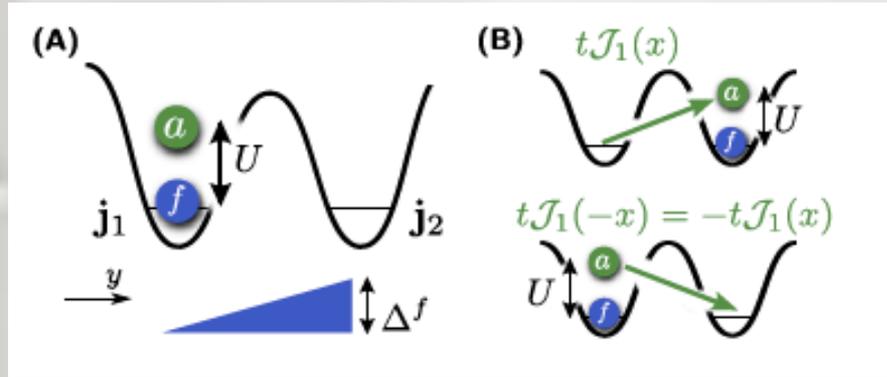
Z_2 Lattice Gauge Theories (gapless matter dof)

$$H_{Z_2} = -K \sum_{\square} \prod_{l \in \square} \sigma_l^z - g \sum_l \sigma_l^x + \boxed{-t \sum_{l,\sigma} \sigma_l^z c_{l,\sigma}^\dagger c_{l+r,\sigma} + h.c.}$$

spontaneous formation of a π -flux through each plaquette



Two particles with strong onsite interaction



$$H = -t(a_{j_1}^\dagger a_{j_2} + f_{j_1}^\dagger f_{j_2} + h.c.) + U \sum_{j=j_1, j_2} n_j^a n_j^f + V_j(j) n_j^f + \sum_j V_\omega(j, t)(n_j^a + n_j^f)$$

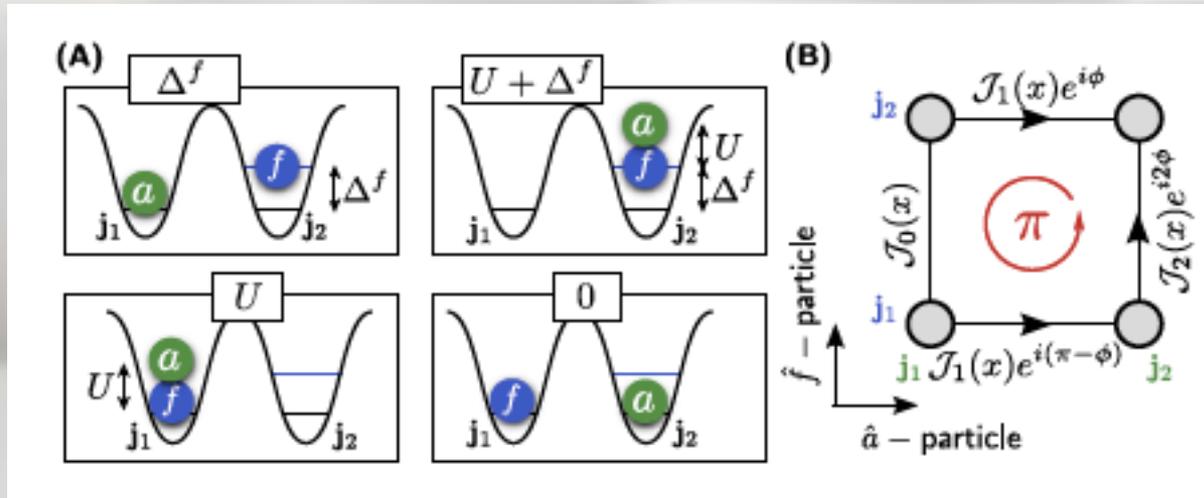
$$V_f(j_2) = \Delta^f + V_f(j_1) \quad \Delta^f = U = \omega \quad \phi_{j_1, j_2} = 0$$

at the 0th order in the Floquet expansion

$$H_{eff}^{2well} = -t_y^a \lambda e^{i\varphi} a_{j_2}^\dagger a_{j_1} - t_y^f \Lambda e^{i\theta} f_{j_2}^\dagger f_{j_1} + h.c.$$

Effective dynamical gauge field

$$H_{eff}^{2well} = -t_y^a \lambda e^{i\varphi} a_{j_2}^\dagger a_{j_1} - t_y^f \Lambda e^{i\theta} f_{j_2}^\dagger f_{j_1} + h.c.$$

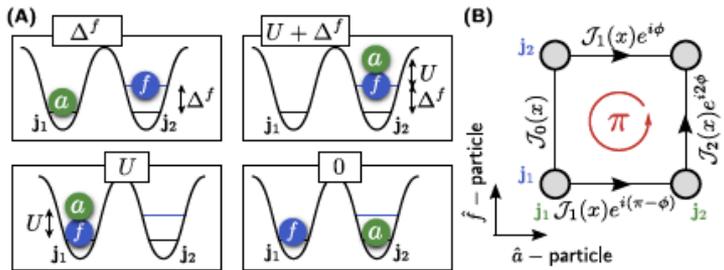


$$\varphi = \frac{\pi}{2} [1 - (n_{j_1}^f - n_{j_2}^f)] \longrightarrow e^{i\varphi} = n_{j_2}^f - n_{j_1}^f = \tau_{\langle j_2, j_1 \rangle}^z$$

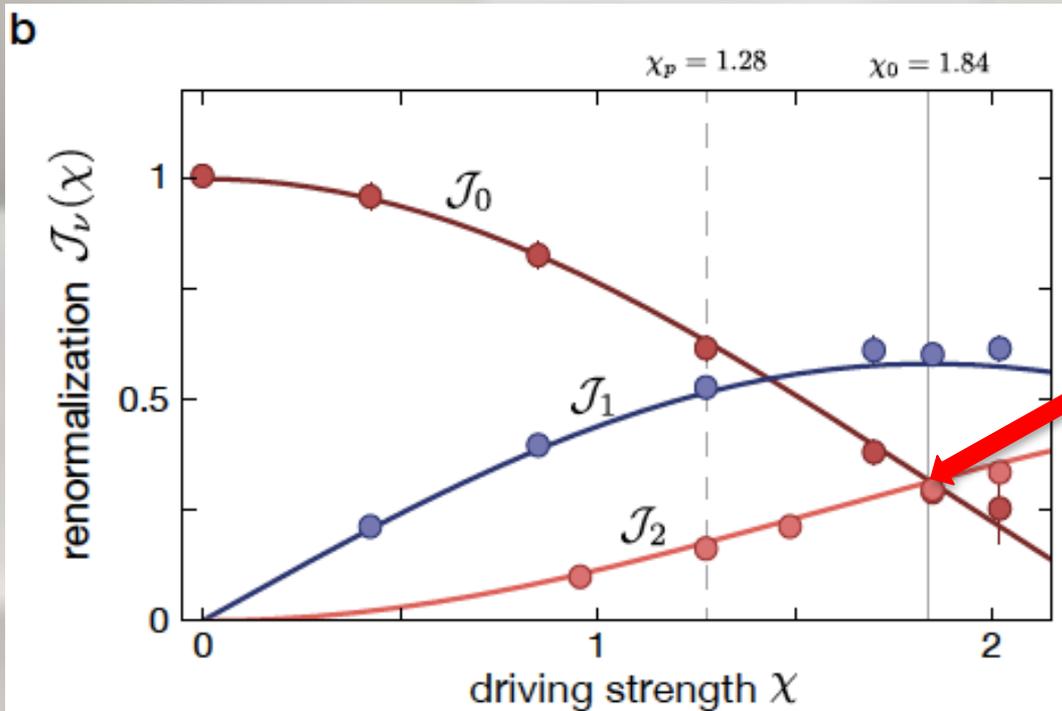
$$\theta = 0 \longrightarrow e^{i\theta} = 1$$

The \$a\$-particle tunneling acquires a phase which depends by the occupation number of the \$f\$-particles.

Shaking trick



$$\tilde{t}^f = t\mathcal{J}_0(\chi)n_1^a + t\mathcal{J}_2(\chi)n_2^a$$



For this specific driving frequency J_0 and J_2 have the same value!

$$\hat{G}_i = \hat{Q}_i \prod_{i:\langle j,i \rangle} \hat{\tau}_{\langle j,i \rangle}^x$$

$$\hat{G}_i |\psi\rangle = g_i |\psi\rangle$$

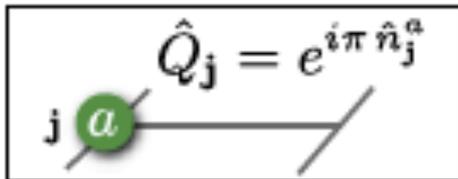
$$g_i = +1, -1$$

Gauss Law

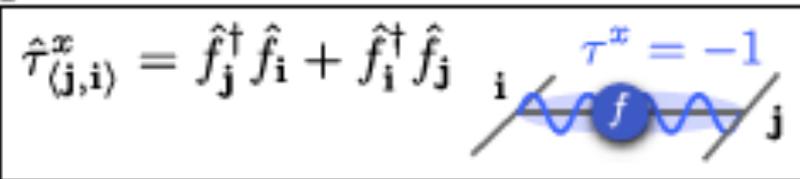
Effective Z_2 LGT in a double-well

$$H_{LGT}^{2well} = -\tilde{t}^a \tau_{\langle j_2, j_1 \rangle}^z (a_{j_2}^\dagger a_{j_1} + h.c.) - \tilde{t}^f \tau_{\langle j_2, j_1 \rangle}^x$$

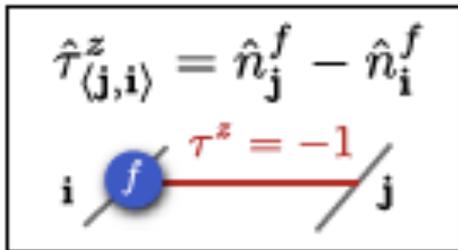
Z_2 matter field



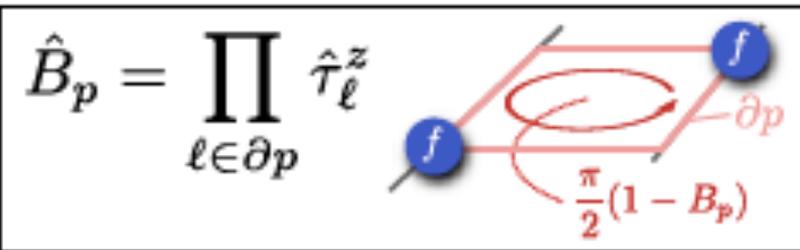
Z_2 electric field



Z_2 gauge field

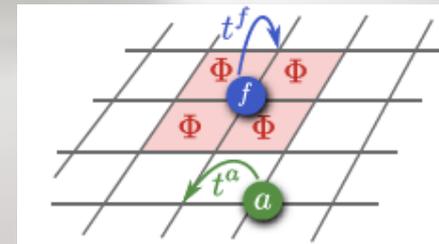


Z_2 magnetic field

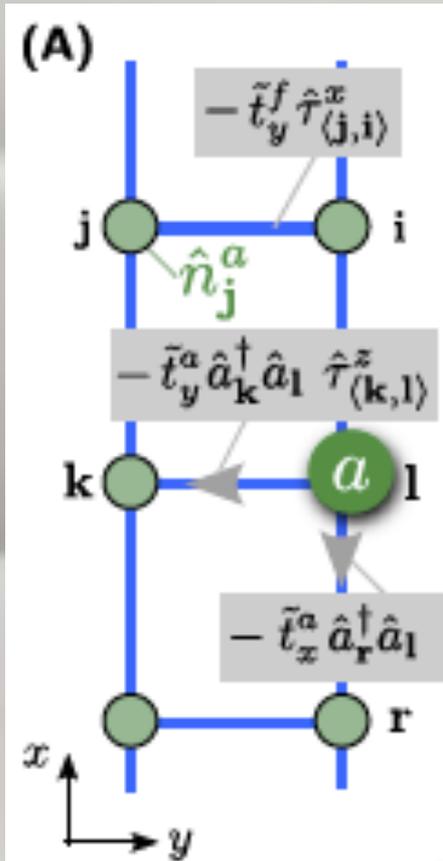


$$\tau_{\langle j_2, j_1 \rangle}^z \simeq e^{i\pi A} \quad \text{with } A=0,1 \text{ thus quantized!!!}$$

The f-particle becomes source of magnetic flux for the a particle, flux attachment!!!



Matter-Gauge field coupling in a ladder



$$H_{2leg}^{LGT} = - \sum_{\langle i,j \rangle_x} \tilde{t}_x^a (a_j^\dagger a_i + h.c.) -$$

$$- \sum_{\langle i,j \rangle_y} [\tilde{t}_y^a (a_j^\dagger a_i \tau_{\langle i,j \rangle}^z + h.c.) + \tilde{t}_y^f \tau_{\langle i,j \rangle}^x]$$

with exactly one f-particle per rung and an arbitrary number of a-particles. Global $U(1) \times Z_2$ symmetry

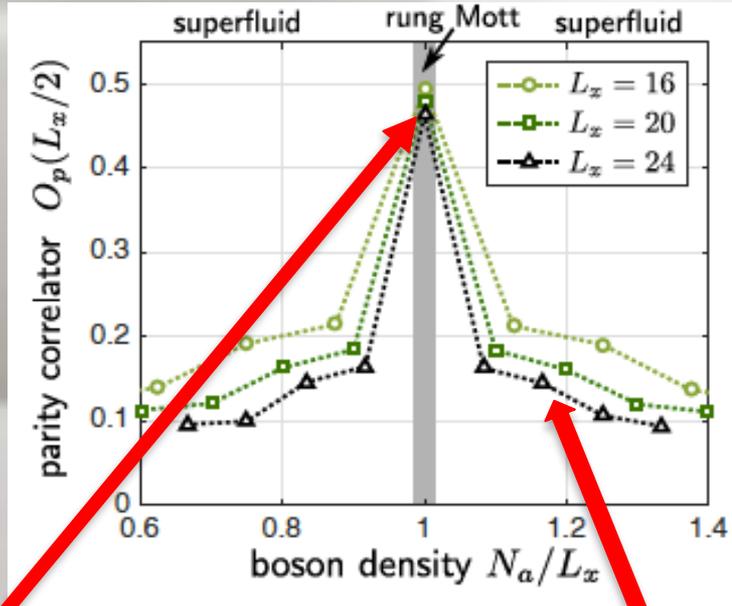
$$\hat{V}_i = \prod_{j=1}^{L_x} \hat{g}_1(je_x), \quad i = 1, 2 \quad \hat{g}_i(je_x) = (-1)^{\hat{Q}_{je_x + (i-1)e_y}} \hat{\tau}_{\langle je_x + e_y, je_x \rangle_y}^x$$

What about the phase diagram???

see related models in the Daniel's seminar

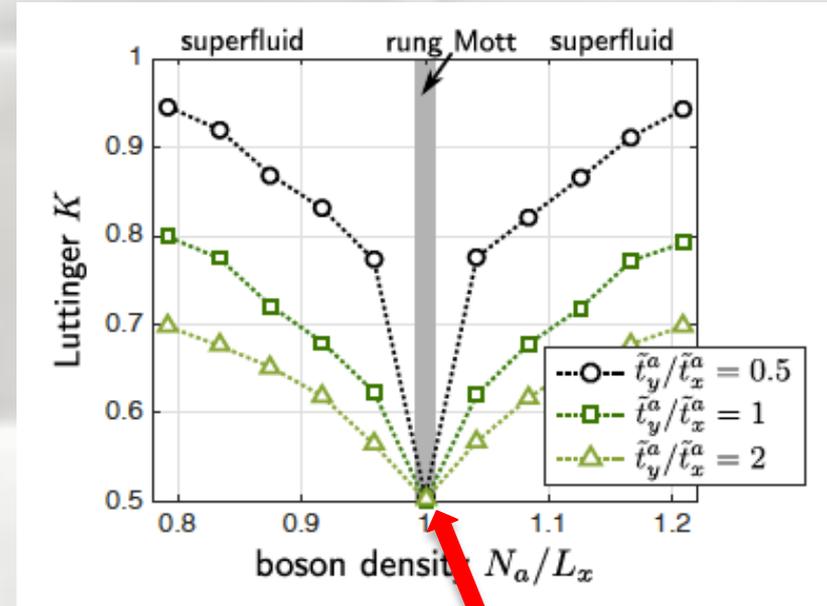
Phase transition in the matter sector

$$O_p(l) = \left\langle \exp \left[i\pi \sum_{j < l} (n_{je_x}^a + n_{je_x+e_y}^a - N_a/L_x) \right] \right\rangle$$



big value and
weak size
dependence
rung-MOTT

small value and
strong size
dependence
Superfluid

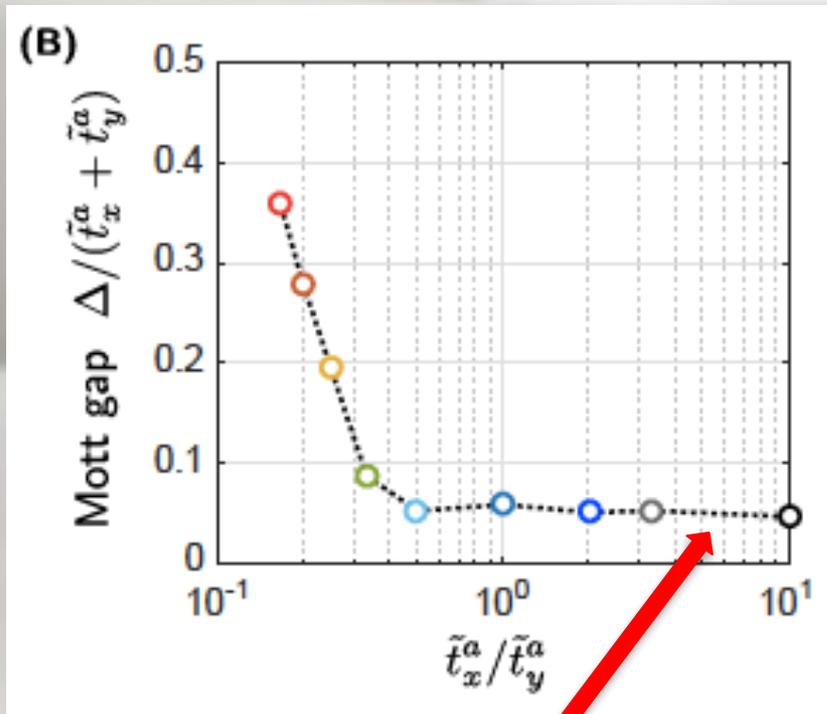


no phase transition at fixed density?

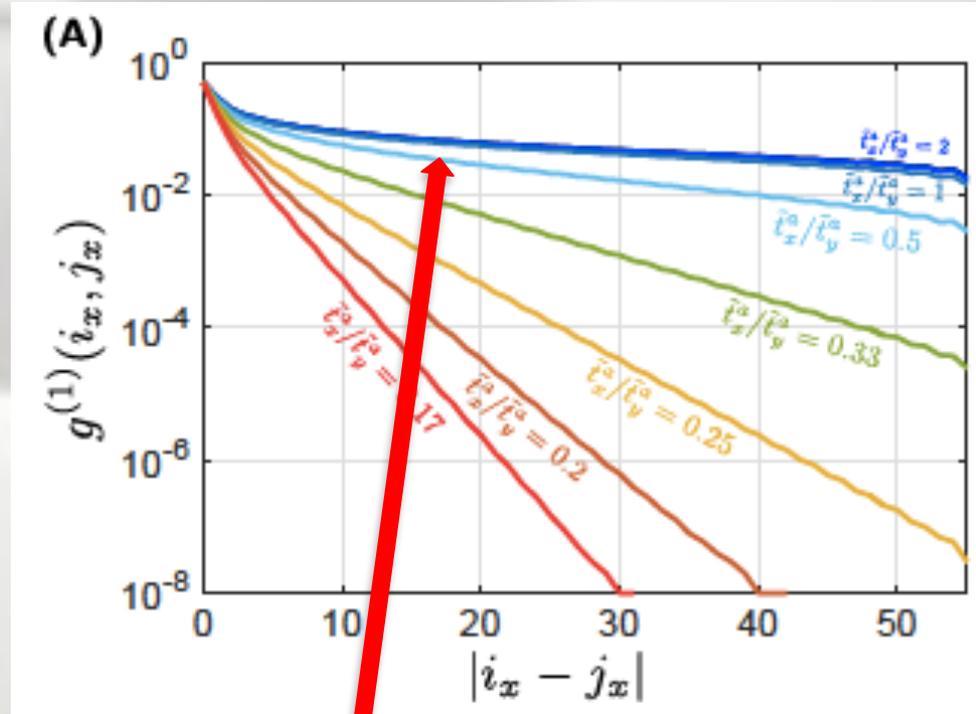
Commensurate-incommensurate Phase Transition

Phase transition in the matter sector

$$\Delta = E_0(N_a + 1) + E_0(N_a - 1) - 2E_0(N_a) \quad g^{(1)}(i_x, j_x) = \langle a_{i_x e_x}^\dagger \left(\prod_{l_x=j_x}^{i_x-1} \tau_{\langle (l_x+1)e_x, l_x e_x \rangle}^z \right) a_{j_x e_x} \rangle$$



0 or very small?

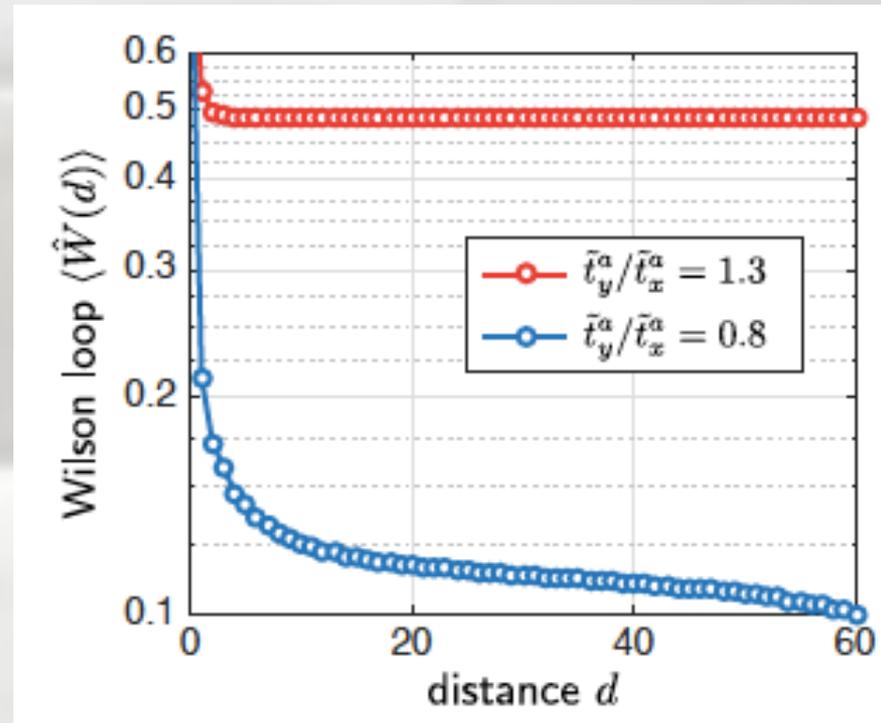
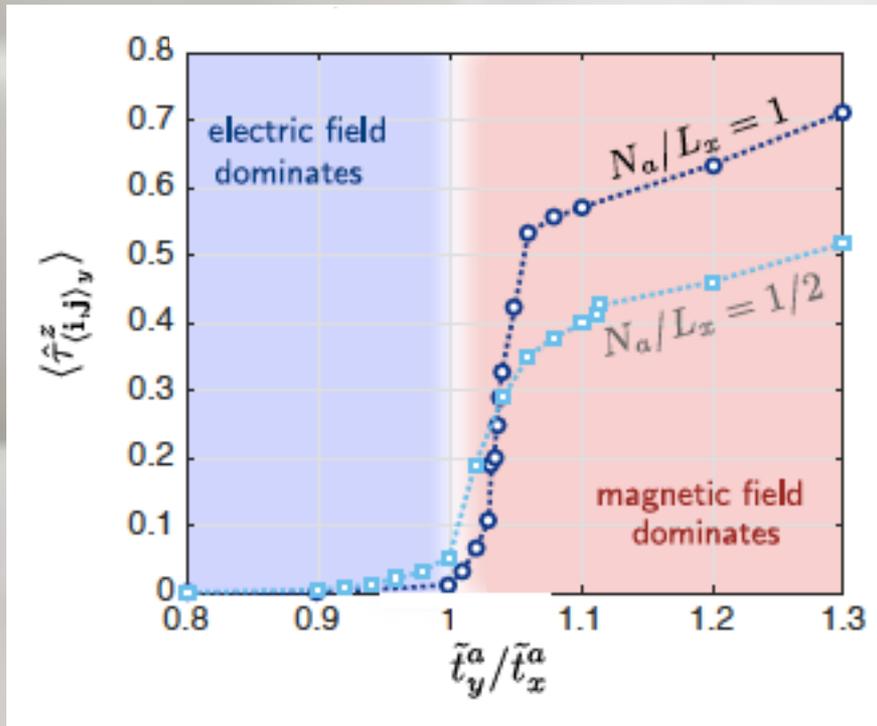


true power-law decay or finite size effects?

Mott-SF transition at fixed density? maybe....

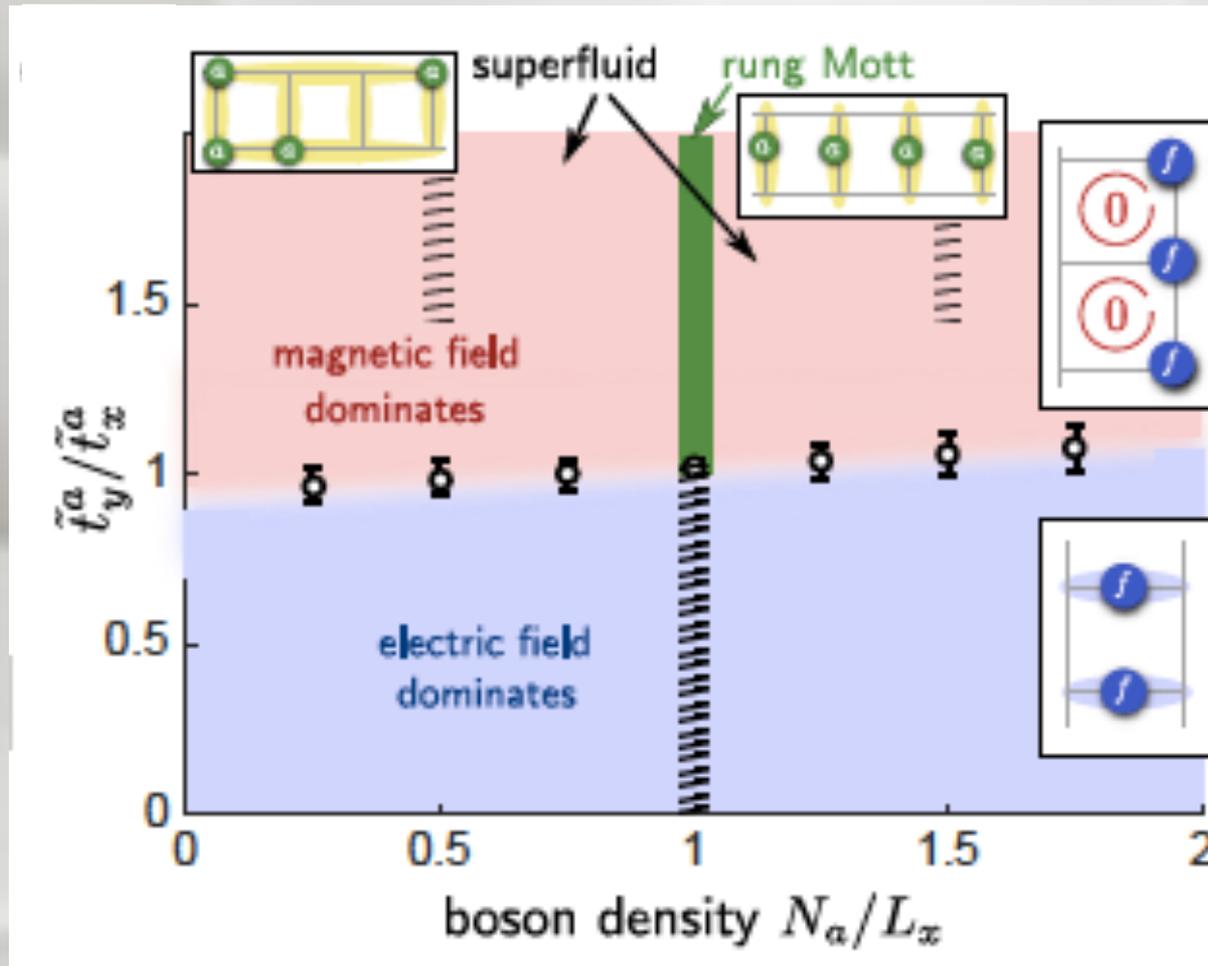
Phase transition in the gauge sector

$$W(d) = \langle \tau_{\langle i,j \rangle}^z \tau_{\langle i+de_x, j+de_x \rangle}^z \rangle$$

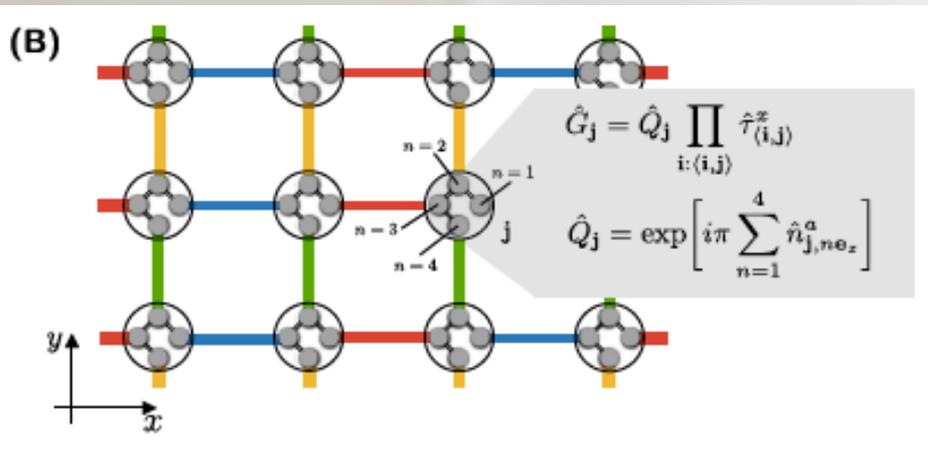


Reminiscent of a confined-deconfined transition, but describing a global symmetry breaking

Phase Diagram



Effective Z_2 LGT in 2D



$$\begin{aligned} \hat{\mathcal{H}}_{2\text{DLGT}}^{\text{simp}} = & \varepsilon_a \sum_j \hat{a}_j^\dagger \hat{a}_j - t_{xy}^f \sum_{\langle i,j \rangle} \hat{\Lambda}_{\langle i,j \rangle} \hat{\tau}_{\langle i,j \rangle}^x \\ & - t_{xy}^a \lambda_{02} |\phi_1|^2 \sum_{\langle i,j \rangle \in E} \left(\hat{\tau}_{\langle i,j \rangle}^z \hat{a}_i^\dagger \hat{a}_j + \text{h.c.} \right) \\ & - t_{xy}^a \lambda_{02} |\phi_2|^2 \sum_{\langle i,j \rangle \in B} \left(\hat{\tau}_{\langle i,j \rangle}^z \hat{a}_i^\dagger \hat{a}_j + \text{h.c.} \right) \end{aligned}$$

Local Z_2 Symmetry

$$\hat{G}_j = \hat{Q}_j \prod_{i:(j,i)} \hat{\tau}_{(j,i)}^x$$

Confinement-Deconfinement PT, Toric Code,
Anyons, FQHE....

EXP



C. Schweizer
LMU-MPQ Munich



I. Bloch
LMU-MPQ Munich



M. Aidelsburger
LMU-MPQ Munich

THANK YOU

THEORY



E. Demler
Harvard



N. Goldman
Bruxelles



F. Grusdt
Harvard-TUM Munich

LB, C. Schweizer, M. Aidelsburger, E. Demler, N. Goldman, F. Grusdt arXiv:1810.02777
C. Schweizer, F. Grusdt, M. Berngruber, LB, E. Demler, N. Goldman, I. Bloch, M. Aidelsburger arXiv:1901.07103