Relativistic effects in ab initio approaches

Giuseppina Orlandini

Department of Physics
University of Trento

How to extend the reliability of n.r. *ab initio* results for e.w. cross sections to high energy/momentum

Giuseppina Orlandini

Department of Physics
University of Trento

Outline:

- General considerations on the lepton-nucleus hadron tensor and the inclusive response functions $\mathbf{R^{\mu\nu}(q,\omega)}$

Outline:

- General considerations on the lepton-nucleus hadron tensor and the inclusive response functions $\mathbf{R^{\mu\nu}(q,\omega)}$

- *"Ab initio"* $\mathbf{R^{\mu\nu}(q,\omega)}$, i.e. non relativistically *"exact"*, also beyond break up energies: the integral transform approach

Outline:

- General considerations on the lepton-nucleus hadron tensor and the inclusive response functions $\mathbf{R}^{\mu\nu}(\mathbf{q},\omega)$

- *"Ab initio"* $\mathbf{R}^{\mu\nu}(\mathbf{q},\omega)$, i.e. non relativistically *"exact"*, also beyond break up energies: the integral transform approach

- The limits of the non realtivistic approach: the frame dependence

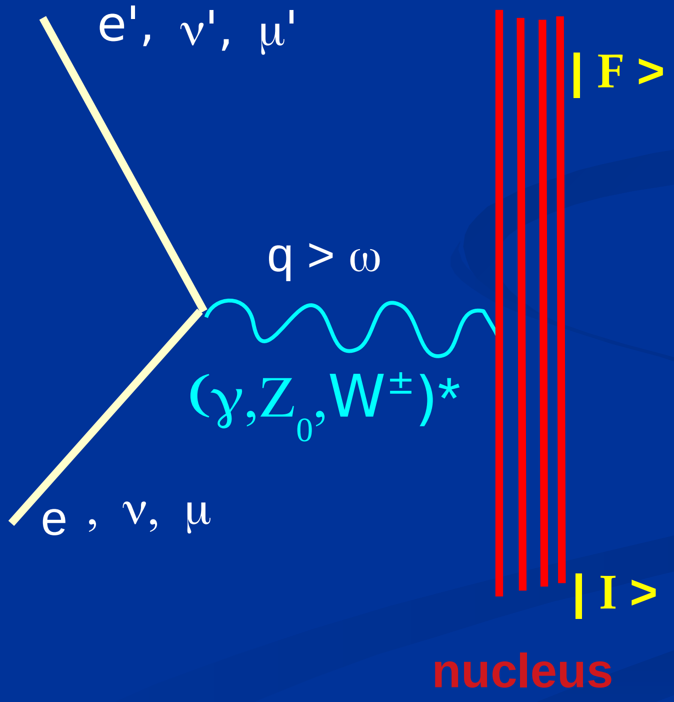
Outline:

- General considerations on the lepton-nucleus hadron tensor and the inclusive response functions $\mathbf{R^{\mu\nu}(q,\omega)}$
- *"Ab initio"* $\mathbf{R^{\mu\nu}(q,\omega)}$, i.e. non relativistically *"exact"*, also beyond break up energies: the integral transform approach
- The limits of the non realtivistic approach: the frame dependence
- From frame dependence to frame independence

Outline:

- General considerations on the lepton-nucleus hadron tensor and the inclusive response functions $\mathbf{R}^{\mu\nu}(\mathbf{q},\omega)$
- *"Ab initio"* $\mathbf{R}^{\mu\nu}(\mathbf{q},\omega)$, i.e. non relativistically *"exact"*, also beyond break up energies: the integral transform approach
- The limits of the non realtivistic approach: the frame dependence
- From frame dependence to frame independence
- Test on the (e,e') scattering

Physics of e.w. Interactions (with nuclei)



at **1st order P.T.** the crucial quantity in the cross section is the Hadron Tensor

$$W^{\mu\nu} = \langle I \mid J^{\mu} \mid F \rangle \langle F \mid J^{\nu} \mid I \rangle \times \delta^4$$

Where:

at **1st order P.T.** the crucial quantity in the cross section is the Hadron Tensor

$$W^{\mu\nu} = \langle I \,|\, J^{\mu} \,|\, F \rangle \; \langle F \,|\, J^{\nu} \,|\, I \rangle \times \delta^4$$

Where:

- **| F > and | I > are nuclear states**

at **1st order P.T.** the crucial quantity in the cross section is the Hadron Tensor

$$W^{\mu\nu} = \langle I \,|\, J^{\mu} \,|\, F \rangle \;\; \langle F \,|\, J^{\nu} \,|\, I \rangle \times \delta^4$$

Where:

- **| F > and | I > are nuclear states**
- **J is the e.w. current operator**

at **1st order P.T.** the crucial quantity in the cross section is the Hadron Tensor

$$W^{\mu\nu} = \langle I | J^{\mu} | F \rangle \langle F | J^{\nu} | I \rangle \times \delta^{4}$$

Where:

- **| F > and | I > are nuclear states**
- **J is the e.w. current operator**
- **| I > is a bound state (g.s. |0>)**

at **1st order P.T.** the crucial quantity in the cross section is the Hadron Tensor

$$W^{\mu\nu} = \langle I | J^{\mu} | F \rangle \langle F | J^{\nu} | I \rangle \times \delta^4$$

Where:

- **| F > and | I > are nuclear states**
- **J is the e.w. current operator**
- **| I > is a bound state (g.s. |0>)**
- **| F > can be a bound or a continuum (scattering) state**

at **1st order P.T.** the crucial quantity in the cross section is the Hadron Tensor

$$W^{\mu\nu} = \langle I \,|\, J^{\mu} \,|\, F \rangle \ \langle F \,|\, J^{\nu} \,|\, I \rangle \times \delta^4$$

Where:

- **| F > and | I > are nuclear states**
- **J is the e.w. current operator**
- **| I > is a bound state (g.s. |0>)**
- **| F > can be a bound or a continuum (scattering) state**
- δ^4 **expresses the energy-momentum conservation**

If $| I >$ is the g.s. $| 0 >$ and $| F >$ is "inclusive"

$$W^{\mu\nu} = < I | J^{\mu} | F > < F | J^{\nu} | I > \times \delta^4$$

$$R^{\mu\nu}(\vec{q}, \omega) = \Sigma_n |<0| J^{\mu}(\vec{q}) |n><n| J^{\nu}(\vec{q}) |0>|^2 \times$$
$$\times \; \delta(\omega - E_n + E_0)$$

Notice!

- The 3-momentum transfer $\vec{q}$ originates from the 3-momentum delta-function δ^3 which now involves the **c.m. of the nucleus**

$W^{\mu\nu}$

$$R^{\mu\nu}(\vec{q}, \omega) = \Sigma_n <0|\, J^{\mu}(\vec{q})\,|n><n|\, J^{\nu}(\vec{q})\,|0>|^2 \ \mathrm{x}$$
$$\mathrm{x} \ \ \delta(\omega - E_n + E_0)$$

Notice!

- The 3-momentum transfer $\vec{q}$ originates from the 3-momentum delta-function δ^3 which now involves the **c.m. of the nucleus**

$W^{\mu\nu}$

- Therefore the non relativistic problem

$$\mathbf{H\,|n\rangle = E_n|n\rangle}$$

has to be solved, referred to the "internal" **(i.e. translation/galileian invariant)** dynamics

$$R^{\mu\nu}(\vec{q},\omega) = \sum_n |\langle 0|\, J^{\mu}(\vec{q})\,|n\rangle\langle n|\, J^{\nu}(\vec{q})\,|0\rangle|^2 \times$$
$$\times\ \delta(\omega - E_n + E_0)$$

The consistent n.r.“ab initio” approach in nuclear physics

- Take as input an Hamiltonian with protons and neutrons as d.o.f. interacting with *realistic* V_{NN} *(i.e. reproducing NN cross sections with* χ*/datum* ~ 1)

The consistent n.r.“ab initio” approach in nuclear physics

- Take as input an Hamiltonian with protons and neutrons as d.o.f. interacting with *realistic* potential *V*
- Take the necessary $J^{[1]}$ *, $J^{[2]}$ .. (at least onsistent with V*)

The consistent n.r.“ab initio” approach in nuclear physics

- Take as input an Hamiltonian with protons and neutrons as d.o.f. interacting with *realistic* potential *V*
- Take the necessary $J^{[1]}, J^{[2]} \cdots$ *at least consistent with V*)
- Calculate

$$R^{\mu\nu}(\vec{q}, \omega) = \sum_n |\langle 0| J^{\mu}(\vec{q}) |n\rangle\langle n| J^{\nu}(\vec{q}) |0\rangle|^2 \times$$
$$\times \ \delta(\omega - E_n + E_0)$$

taking into account the **full many-body dynamics**, respecting **translation/Galileian invariance, controlling the numerical accuracy**

The big problems:

- How to solve the Hamiltonian for |F>, namely the many-body scattering state, when the nucleus breaks into pieces, *(known as "final state interaction" FSI!)*

The big problems:

- How to solve the Hamiltonian for |F>, namely the many-body scattering state, when the nucleus breaks into pieces, *(known as "final state interaction" FSI!)*

*[**Notice:** due to "good" asymptotic boundary conditions the ground stete **|0>** can be calculated with controlled accuracy, at least up to medium heavy systems]*

The big problems:

1. How to solve the Hamiltonian for |F>, namely the many-body scattering state, when the nucleus breaks into pieces, *(known as "final state interaction" FSI!)*

2. Up to which energy/momentum can one push the *ab initio* non relativistic treatment of the dynamics in $R^{\mu\nu}(q, \omega)$??

The big problem:

1. How to solve the Hamiltonian for |F>, namely the many-body scattering state, when the nucleus breaks into pieces, *(known as "final state interaction" FSI!)*

The solution:

The integral transform approach

The big problem:

2. Up to which energy/momentum can one push the *ab initio* non relativistic treatment of the dynamics in $R^{\mu\nu}(q, \omega)$??

The solution:

Analyze the frame dependence, choose the *"right frame"* and the *"proper rel. input kinematics"*

The integral transform approach

KERNEL

$$\Phi(\tau) = \int d\omega \; K(\omega,\tau) \; R(\omega)$$

One is able to calculate $\Phi(\tau)$ but wants $R(\omega)$, which is the quantity of direct physical meaning.

$$\Phi(\tau) = \int d\omega \; K(\omega,\tau) \, R(\omega)$$

One is able to calculate $\Phi(\tau)$ but wants $R(\omega)$, which is the quantity of direct physical meaning.

Warning:

The “inversion” of $\Phi(\tau)$ may is a delicate issue.

It can genereate instabilities

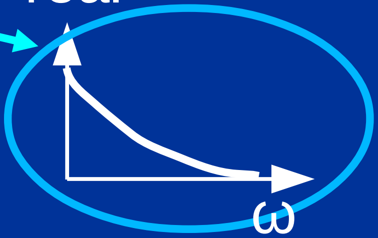
a “good” Kernel has to satisfy two requirements

1) one must be able to calculate the integral transform

2) one must be able to invert the transform controlling the instabilities

Two examples in the literature:

1: **Exponential Kernel:** $K(\omega,\tau) = e^{-\omega\tau}$ τ real



$$\Phi(\tau) = \langle \,|\, \Theta^{\dagger}(\tau)\;\Theta(0)\,|\,\rangle \;\;\text{-->}\;\; \int e^{-\tau\omega}\, R(\omega)\, d\omega$$

$\tau = it$

In Condensed Matter Physics:

Θ = Density Operator

$R(\omega)$ = Dynamical Structure Function

$\Phi(\tau)$ is obtained with Monte Carlo Methods

In Nuclear Physics:

Θ= Charge or current density operator

$R(\omega)$ "Response" Function

(to external perturbative probe)

$\Phi(\tau)$ is obtained with Monte Carlo Methods

In QCD

Θ = quark fields

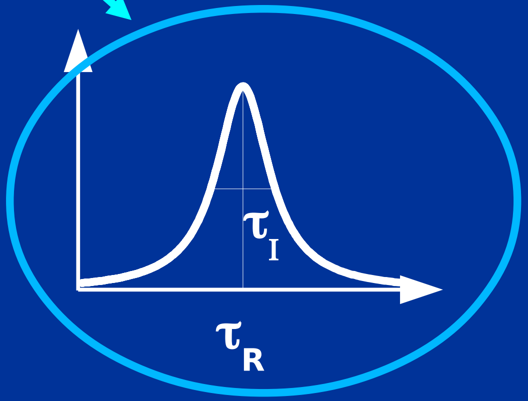
$R(\omega)$ = Hadronic Spectral Function

$\Phi(\tau)$ is obtained by OPE - QCD sum rules or Lattice

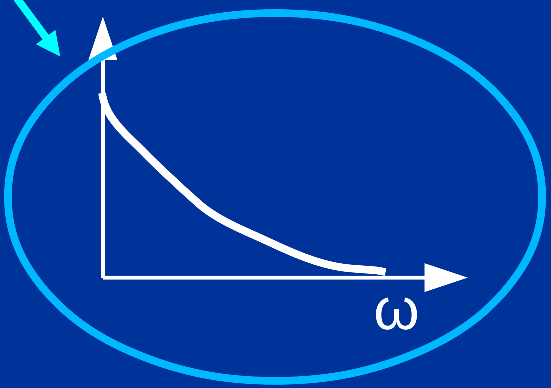
2:

Lorentzian Kernel: $K(\omega,\tau) = [(\omega - \tau)\,(\omega - \tau)^*]^{-1}$

- complex $= \tau_R + \tau_I$
- easy to invert
- $\Phi(\tau)$ calculated via **matrix diagonalization on bound** basis functions

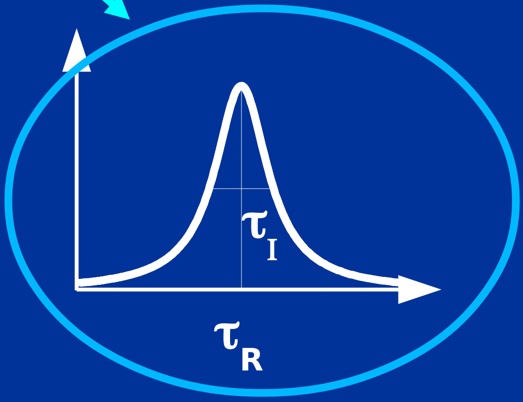


1: **Exponential Kernel:** $K(\omega,\tau) = e^{-\omega\tau}$ τ real



- $\Phi(\tau)$ calculated via **GFMC**
- requires a big effort to be inverted

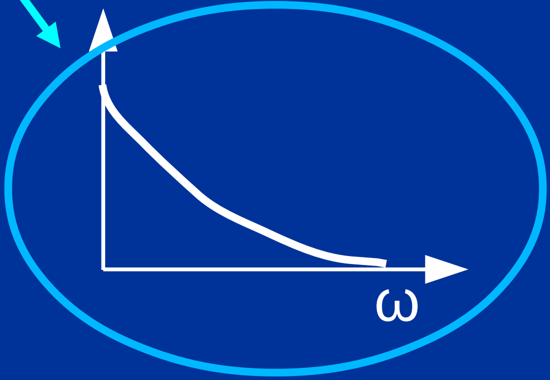
2: **Lorentzian Kernel:** $K(\omega,\tau) = [(\omega - \tau)(\omega - \tau)^*]^{-1}$



- complex = $\tau_R + \tau_I$
- easy to invert
- $\Phi(\tau)$ calculated via **matrix diagonalization on bound** basis functions

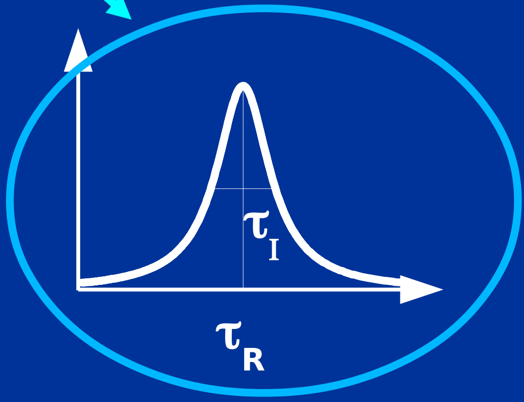
1: **Exponential Kernel:** $K(\omega,\tau) = e^{-\omega\tau}$ τ real

- $\Phi(\tau)$ calculated via **GFMC**
- requires a big effort to be inverted

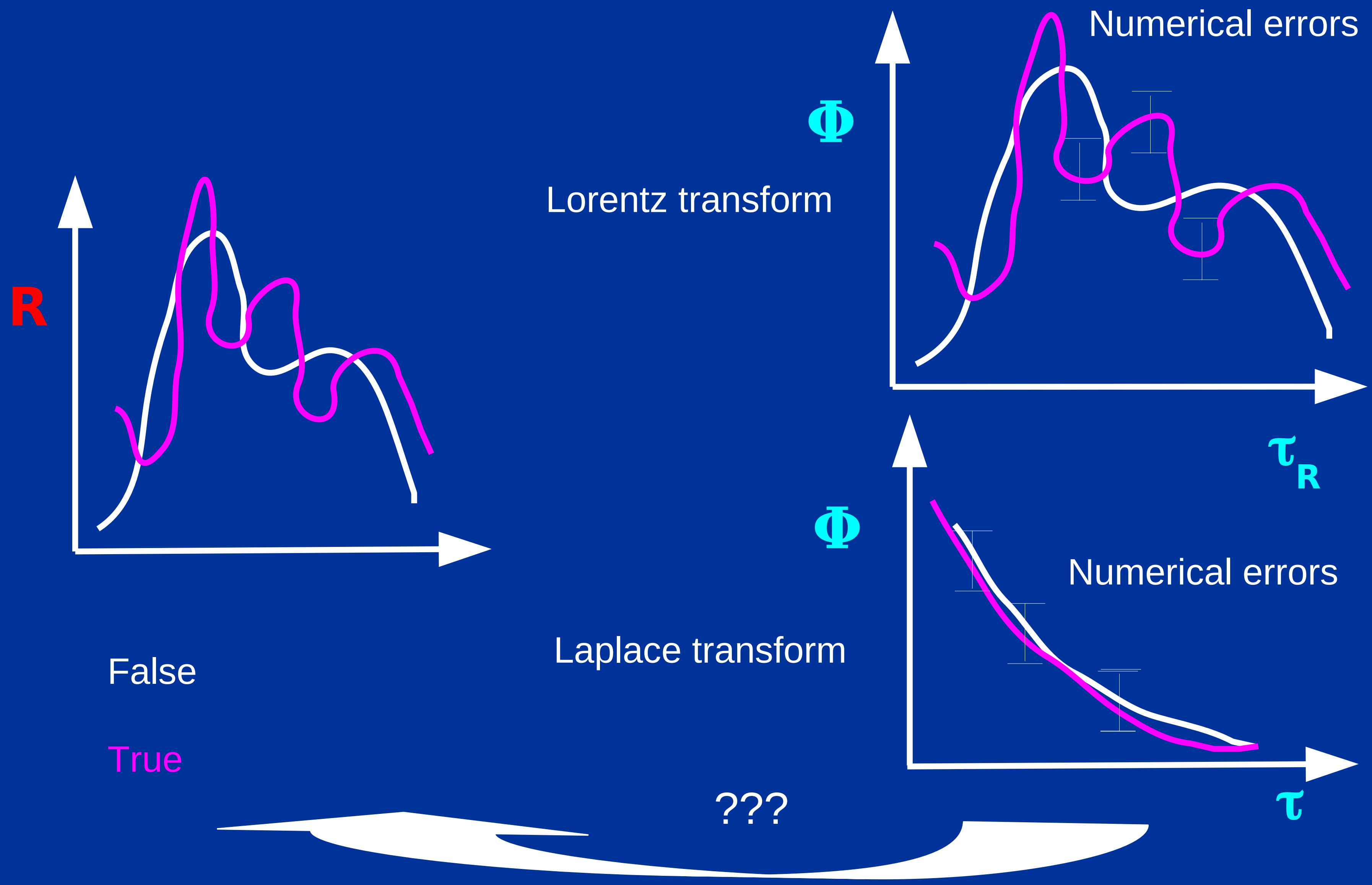


2: **Lorentzian Kernel:** $K(\omega,\tau) = [(\omega - \tau)\,(\omega - \tau)^*]^{-1}$

- complex $= \tau_R + \tau_I$
- easy to invert
- $\Phi(\tau)$ calculated via **matrix diagonalization on bound** basis functions

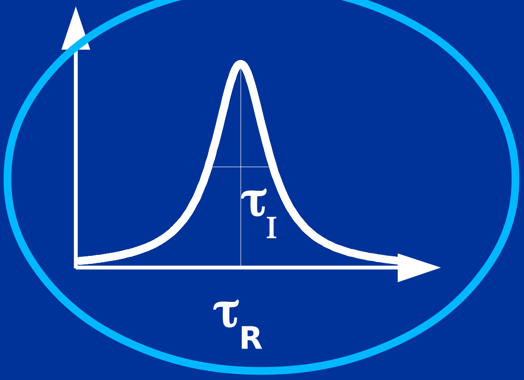


the inversion problem:



How to calculate Φ with Lorentzian kernel?

$$\Phi^{\mu\nu}(q, \tau) = \int d\omega \; K(\omega,\tau) \, R^{\mu\nu}(q, \omega)$$



$$\Phi(\omega_0,\Gamma) = \Gamma/\pi \int [(\omega - \omega_0)^2 + \Gamma^2]^{-1} R(\omega)\, d\omega < \infty$$

$$\Phi(\omega_0,\Gamma) = \Gamma/\pi \int [(\omega - \omega_0)^2 + \Gamma^2]^{-1} R(\omega)\, d\omega < \infty$$

$$= \Gamma/\pi \int d\omega\, [(\omega - \omega_0)^2 + \Gamma^2]^{-1} \Sigma_n |\langle n| \Theta |0\rangle|^2 \delta(\omega - E_n + E_0)$$

$$\Phi(\omega_0,\Gamma) = \Gamma/\pi \int [(\omega - \omega_0)^2 + \Gamma^2]^{-1} R(\omega)\, d\omega < \infty$$

$$= \Gamma/\pi \int d\omega\, [(\omega - \omega_0)^2 + \Gamma^2]^{-1} \Sigma_n |\langle n|\Theta|0\rangle|^2 \delta(\omega - E_n + E_0)$$

$$= \Gamma/\pi\, \Sigma_n \langle 0|\Theta^+ [(H - E_0 - \omega_0)^2 + \Gamma^2]^{-1} |n\rangle\langle n|\Theta|0\rangle$$

$$\Phi(\omega_0,\Gamma) = \Gamma/\pi \int [(\omega - \omega_0)^2 + \Gamma^2]^{-1} R(\omega)\, d\omega < \infty$$

$$= \Gamma/\pi \int d\omega\, [(\omega - \omega_0)^2 + \Gamma^2]^{-1} \Sigma_n |\langle n|\Theta|0\rangle|^2 \delta(\omega - E_n + E_0)$$

$$= \Gamma/\pi \Sigma_n \langle 0|\Theta^+ [(H - E_0 - \omega_0)^2 + \Gamma^2]^{-1} |n\rangle\langle n|\Theta|0\rangle$$

$$= -\Gamma/\pi \; \mathrm{Im}\, [\langle 0|\Theta^+ (H - E_0 - \omega_0 + i\Gamma)^{-1} \Theta |0\rangle]$$

$$\Phi(\omega_0,\Gamma) = \Gamma/\pi \int [(\omega - \omega_0)^2 + \Gamma^2]^{-1} R(\omega)\, d\omega < \infty$$

$$= \Gamma/\pi \int d\omega\, [(\omega - \omega_0)^2 + \Gamma^2]^{-1} \Sigma_n |\langle n| \Theta |0\rangle|^2 \delta(\omega - E_n + E_0)$$

$$= \Gamma/\pi\, \Sigma_n \langle 0 | \Theta^+ [(H - E_0 - \omega_0)^2 + \Gamma^2]^{-1} |n\rangle\langle n| \Theta | 0\rangle$$

$$= - \Gamma/\pi \;\; \mathrm{Im}\, [\langle 0 | \Theta^+ (H - E_0 - \omega_0 + i\Gamma)^{-1} \Theta | 0\rangle]$$

Green O. $[\Pi(\omega)]$ with poles on the complex plane !!

$$\Phi(\omega_0,\Gamma) = \Gamma/\pi \int [(\omega - \omega_0)^2 + \Gamma^2]^{-1} R(\omega)\, d\omega < \infty$$

$$= \Gamma/\pi \int d\omega\, [(\omega - \omega_0)^2 + \Gamma^2]^{-1} \Sigma_n |\langle n| \Theta |0\rangle|^2 \delta(\omega - E_n + E_0)$$

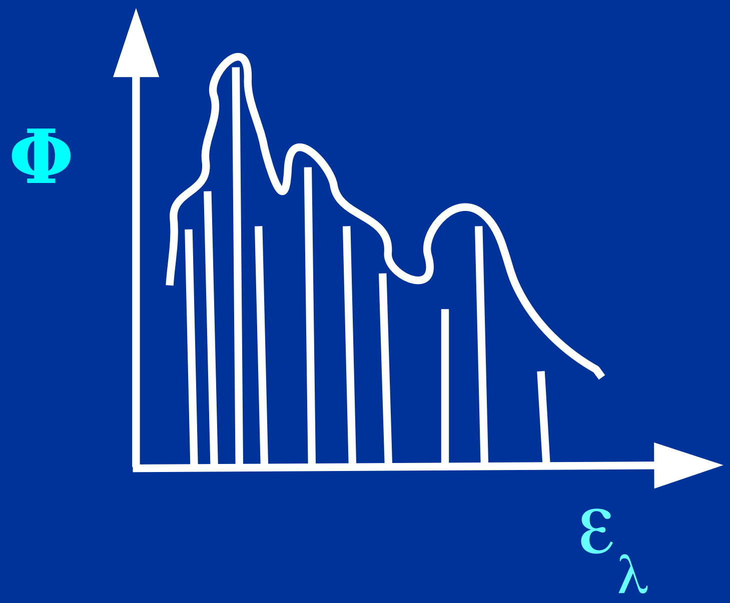
$$= \Gamma/\pi\, \Sigma_n \langle 0 | \Theta^+ [(H - E_0 - \omega_0)^2 + \Gamma^2]^{-1} |n\rangle\langle n| \Theta | 0\rangle$$

$$= - \Gamma/\pi \;\; \mathrm{Im}\, [\langle 0 | \Theta^+ (H - E_0 - \omega_0 + i\Gamma)^{-1} \Theta | 0\rangle]$$

After insertion of complete basis **|n><n|**

... and after diagonalization:

$$\Phi(\tau) = \boxed{\Sigma_\lambda \quad L(\varepsilon_\lambda - E_0, \tau)\ |\langle\lambda|\ \Theta|0\rangle|^2}$$



Namely bars "smeared" with the Lorentzian!

How important are relativistic effects as q increases?

The analysis of frame dependence

One criteria to judge the importance of relativistic effects is the frame dependence of the results

The electron scattering (e,e') response functions in various frames

$$\rho(q)_{fr}$$

$$R_L^{fr}(q^{fr}, \omega^{fr}) = \Sigma_n \langle 0| \, J^0(q)_{fr} |n\rangle\langle n| \, J^0(q)_{fr} |0\rangle|^2 \times$$
$$\times \; \delta(\omega_{fr} - E_n^{fr} + E_0^{fr})$$

$$R_T^{fr}(q^{fr}, \omega^{fr}) = \Sigma_n \langle 0| \, J_\perp(q)_{fr} |n\rangle\langle n| \, J_\perp(q)_{fr} |0\rangle|^2 \times$$
$$\times \; \delta(\omega_{fr} - E_n^{fr} + E_0^{fr})$$

VARIOUS FRAMES:

LAB: **initially nucleons have momenta** $p_i \cong 0$

(in the *quasi elastic* regime the final momentum of the "active nucleon" $p_f \cong q$

VARIOUS FRAMES:

LAB: **initially nucleons have momenta** $p_i \cong 0$

(in the *quasi elastic* regime the final momentum of the "active nucleon" $p_f \cong q$

ANTI-LAB: **initially nucleons have momenta** $p_i \cong -q/A$

(in q.e. the final momentum of the "active nucleon" $p_f \cong q\,(A-1)/A$ **)**

VARIOUS FRAMES:

LAB: initially nucleons have momenta $p_i \cong 0$

(in the *quasi elastic* regime the final momentum of the "active nucleon" $p_f \cong q$

ANTI-LAB: initially nucleons have momenta $p_i \cong -q/A$

(in q.e. the final momentum of the "active nucleon" $p_f \cong q\,(A-1)/A$)

BREIT: initially nucleons have momenta $p_i \cong -q/2A$

(in q.e. the final momentum of the "active nucleon" $p_f \cong q\,(2A-1)/2A$)

VARIOUS FRAMES:

LAB: **initially nucleons have momenta** $p_i \cong 0$

(in the *quasi elastic* regime the final momentum of the "active nucleon" $p_f \cong q$

ANTI-LAB: **initially nucleons have momenta** $p_i \cong -q/A$

(in q.e. the final momentum of the "active nucleon" $p_f \cong q\,(A-1)/A$ **)**

BREIT: **initially nucleons have momenta** $p_i \cong -q/2A$

(in q.e. the final momentum of the "active nucleon" $p_f \cong q\,(2A-1)/2A$ **)**

ANB: **initially nucleons have momenta** $p_i \cong -q/2$

(in q.e. the final momentum of the "active nucleon" $p_f \cong q/2$ **)**

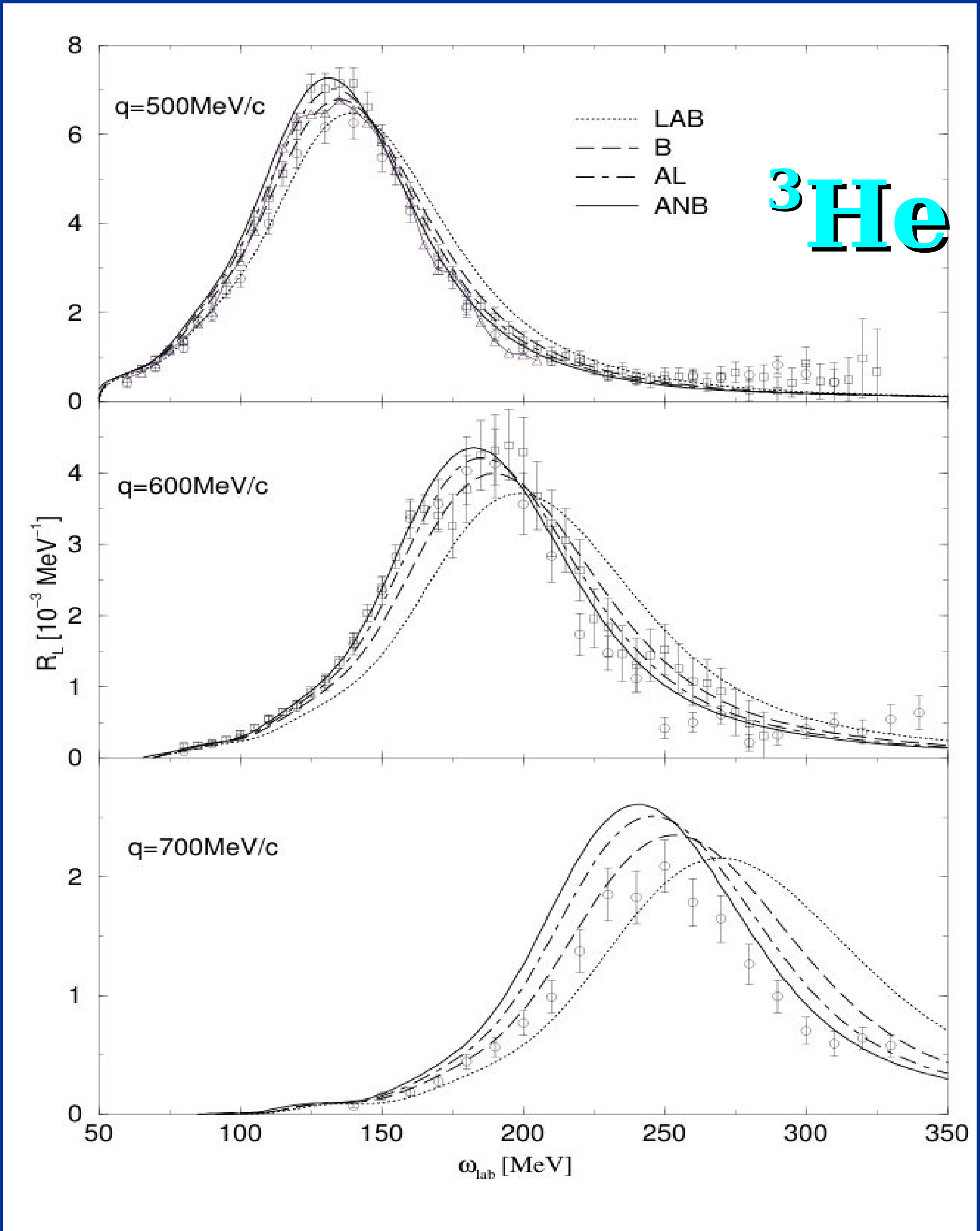
They are connected to the response functions in the LAB frame (where they are measured !)

$$R_L^{\mathbf{LAB}}(q,\omega) = \frac{q^2}{q_{fr}^2}\frac{E_i^{fr}}{M_T} R_L^{fr}(q_{fr},\omega_{fr})$$

$$R_T^{\mathbf{LAB}}(q,\omega) = \frac{E_i^{fr}}{M_T} R_T^{fr}(q_{fr},\omega_{fr})$$

Longitudinal response of ^{3}He

$R_L(q, \omega)$



Large frame dependence!!!

V.Efros, W.Leidemann, G.O., E.L.Tomusiak PRC 72 (2005) 011002

Is there an easy way to cure it?

Is there an easy way to cure it?

use in each frame the kinematical inputs corresponding to the quasi elastic 2-body assumption i.e.

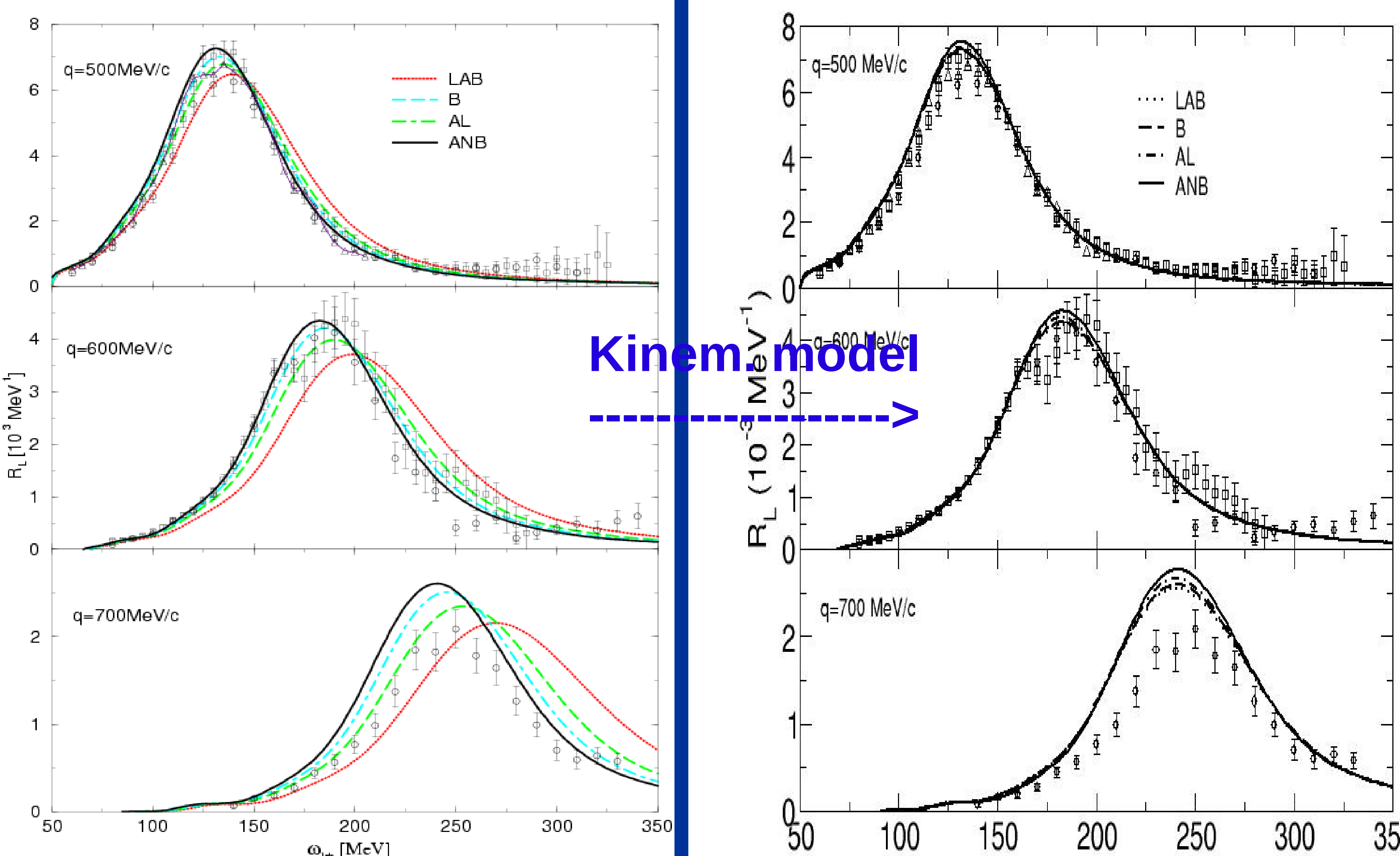
1 + (A-1)-system

The **relative momentum** p_{rel} of the 2 bodies ($1+(A-1)$)

can be calculated in each frame in a **relativistically correct** way.

The **energy** of the final state (the input of a **non relativistic** dynamical calculation) is then taken in its **non relativistic form** $p^2_{rel} / 2\mu$

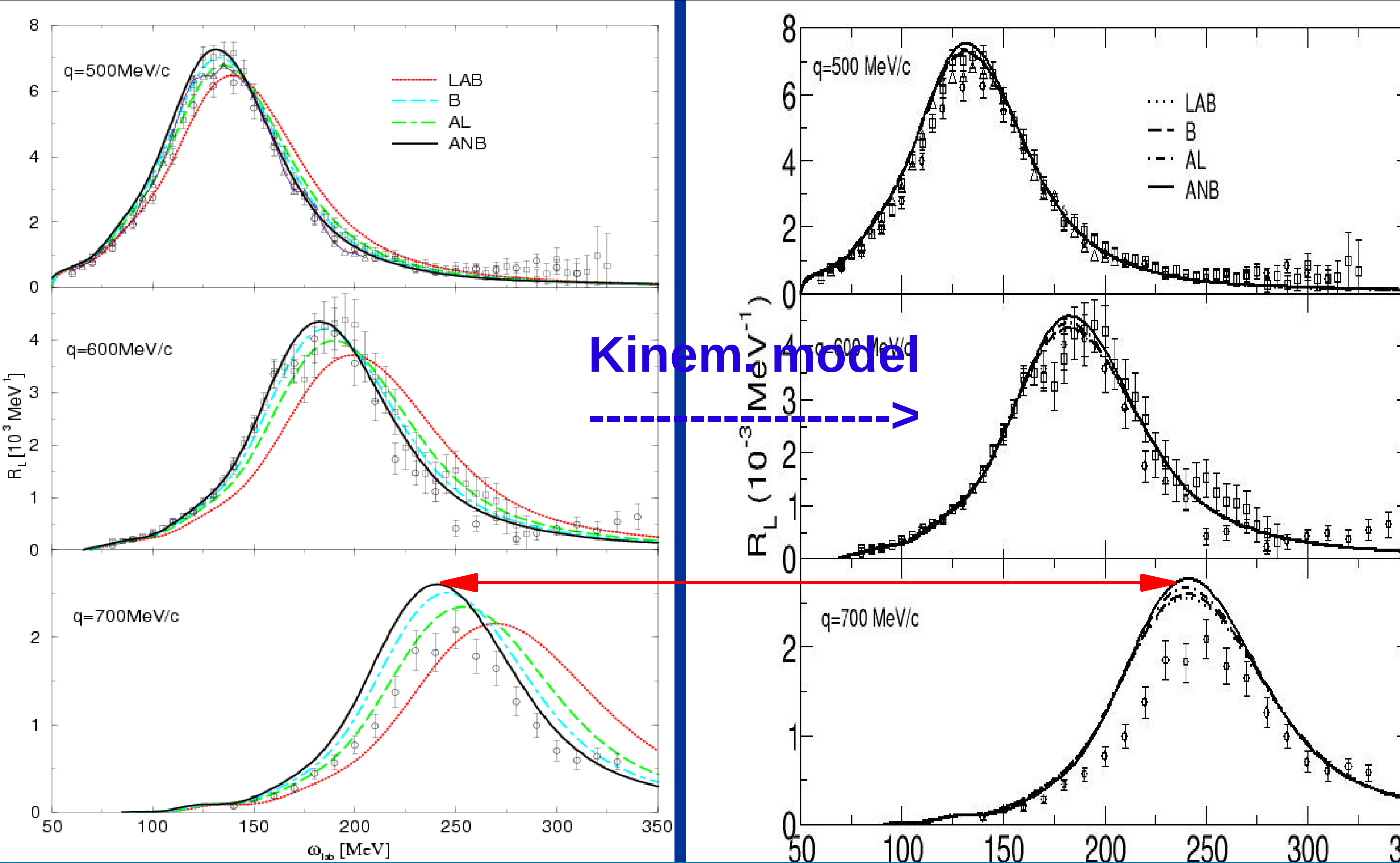
Longitudinal response of ^{3}He $R_L(q, \omega)$



remark:

Of the 4 frames the **ANB** result is the less affected by the relativistically correct kinematical model.

Longitudinal response of ^{3}He $R_L(q, \omega)$



Of the 4 frames the **ANB** result is the less affected by the relativistically correct kinematical model.

The reason is that in the ANB frame the momentum of the active particle is the smallest (about q/2!).

Therefore the error on its kinetic energy is the smallest:

Of the 4 frames the **ANB** result is the less affected by the relativistically correct kinematical model.

The reason is that in the ANB frame the momentum of the active particle is the smallest (about q/2!).

Therefore the error on its kinetic energy is the smallest: in fact, in general:

$$T \cong \frac{p^2}{2\,m} - \frac{p^4}{8\,m^3} + \ldots \qquad\qquad \frac{\Delta T}{T} \cong \frac{p^2}{4\,m^2}$$

Of the 4 frames the **ANB** result is the less affected by the relativistically correct kinematical model.

The reason is that in the ANB frame the momentum of the active particle is the smallest (about q/2!).

Therefore the error on its kinetic energy is the smallest: in fact, in general:

$$T \cong \frac{p^2}{2\,m} - \frac{p^4}{8\,m^3} + \ldots \qquad \frac{\Delta T}{T} \cong \frac{p^2}{4\,m^2}$$

$$\textbf{LAB}: \quad \frac{\Delta T}{T} \cong \frac{q^2}{4\,m^2} \qquad \textbf{ANB}: \quad \frac{\Delta T}{T} \cong \frac{q^2}{16\,m^2} \quad \textbf{!!!}$$

Moreover: the **peak position** in the **ANB** frame is always relativistically correct, in fact in general:

$$\omega_{peak} \cong T(p_f) - T(p_i)$$

Moreover: the **peak position** in the **ANB** frame is always relativistically correct, in fact in general:

$$\omega_{peak} \cong T(p_f) - T(p_i)$$

LAB : $\omega_{peak} \cong T(q) - T(0)$

rel. different from **n.r.** !!!

Moreover: the **peak position** in the **ANB** frame is always relativistically correct, in fact in general:

$$\omega_{peak} \cong T(p_f) - T(p_i)$$

LAB : $\omega_{peak} \cong T(q) - T(0)$

rel. different from **n.r.** !!!

ANB : $\omega_{peak} \cong T(q/2) - T(q/2) = 0$

rel. equal to **n.r.**

always correct **!!!**

Before going to neutrino cross sections test on (e,e') data

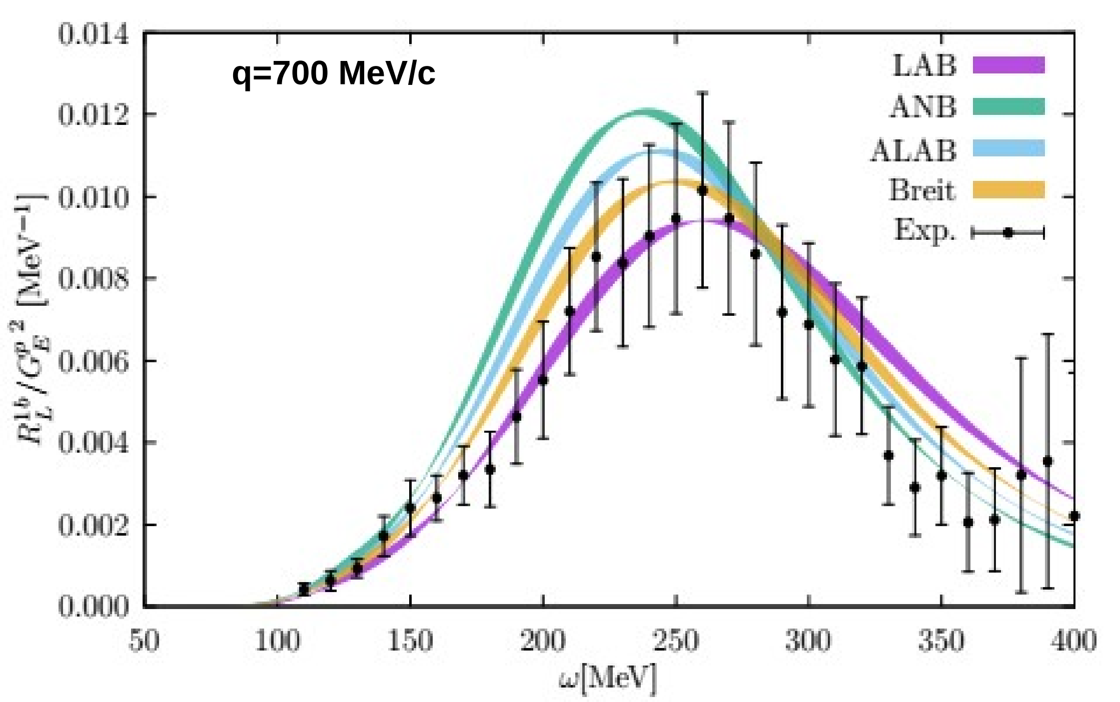
The test on the ^{4}He(e,e') cross section

N.Rocco, W.Leidemann, **A. Lovato**, G.O. Phys. Rev. C 97, 055501 (2018)

$R_L(q, \omega)$ of ^{4}He

Integral transform calculation with

$K(\omega,\tau) = e^{-\omega\tau}$ τ real, GFMC, **full FSI**



Large frame dependence also in ^{4}He!

N.Rocco, W.Leidemann, **A. Lovato**, G.O.
Phys. Rev. C 97, 055501 (2018)

Assuming q.e. kinematics ***[2-body break-up 1-(A-1)]***

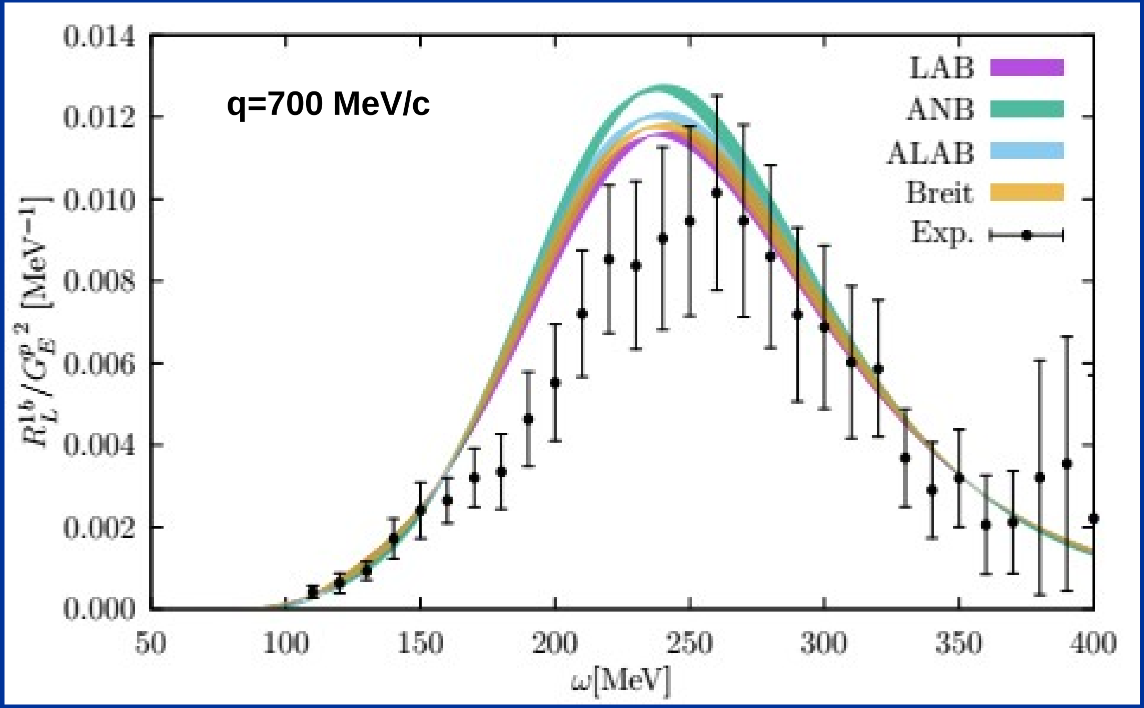
one can treat the relativistic kinematical inputs

correctly!!

$R_L(q, \omega)$ of ^{4}He

Integral transform calculation with

$K(\omega,\tau) = e^{-\omega\tau}$ τ real, GFMC **full FSI**



N.Rocco, W.Leidemann, A. Lovato, G.O.
Phys. Rev. C 97, 055501 (2018)

Assuming q.e. kinematics

[2-body break-up 1-(A-1)]

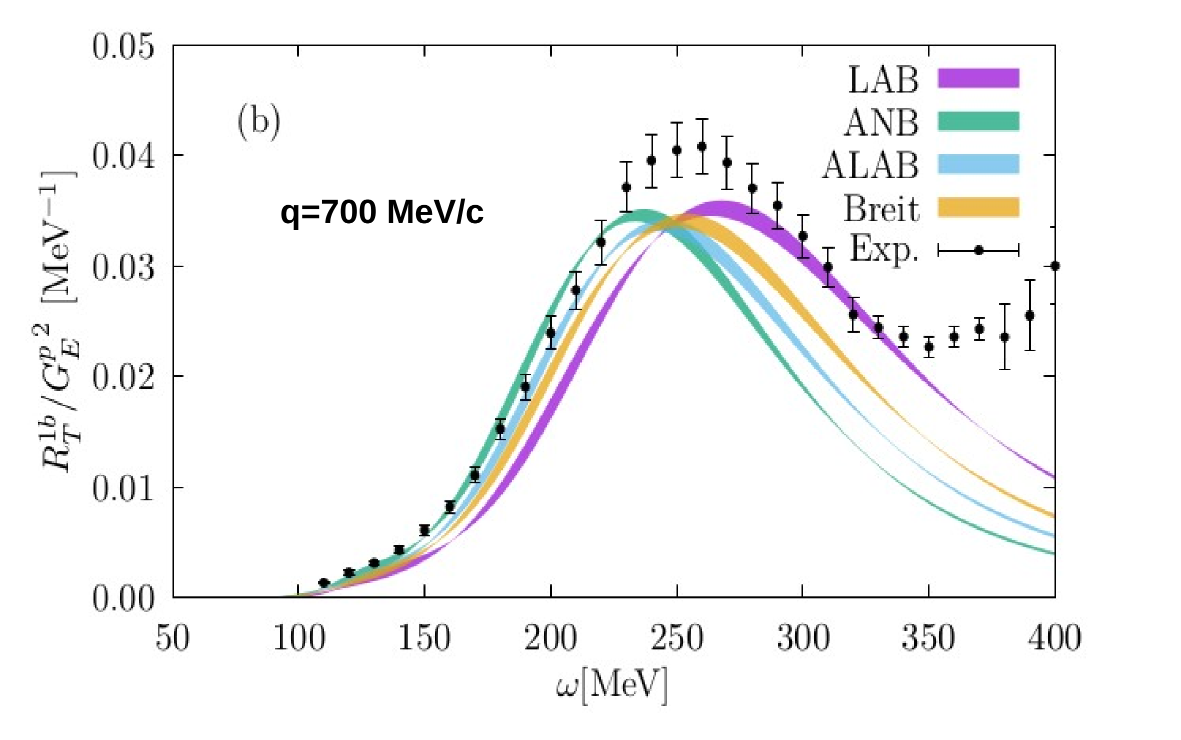
one can treat the relativistic kinematical inputs correctly!!

frame dependence much reduced !!!

$R_T(q, \omega)$ of ^{4}He

Integral transform calculation with

$K(\omega,\tau) = e^{-\omega\tau}$ τ real, GFMC, **full FSI**



Large frame dependence also in ^{4}He!

N.Rocco, W.Leidemann, A. Lovato, G.O.
Phys. Rev. C 97, 055501 (2018)

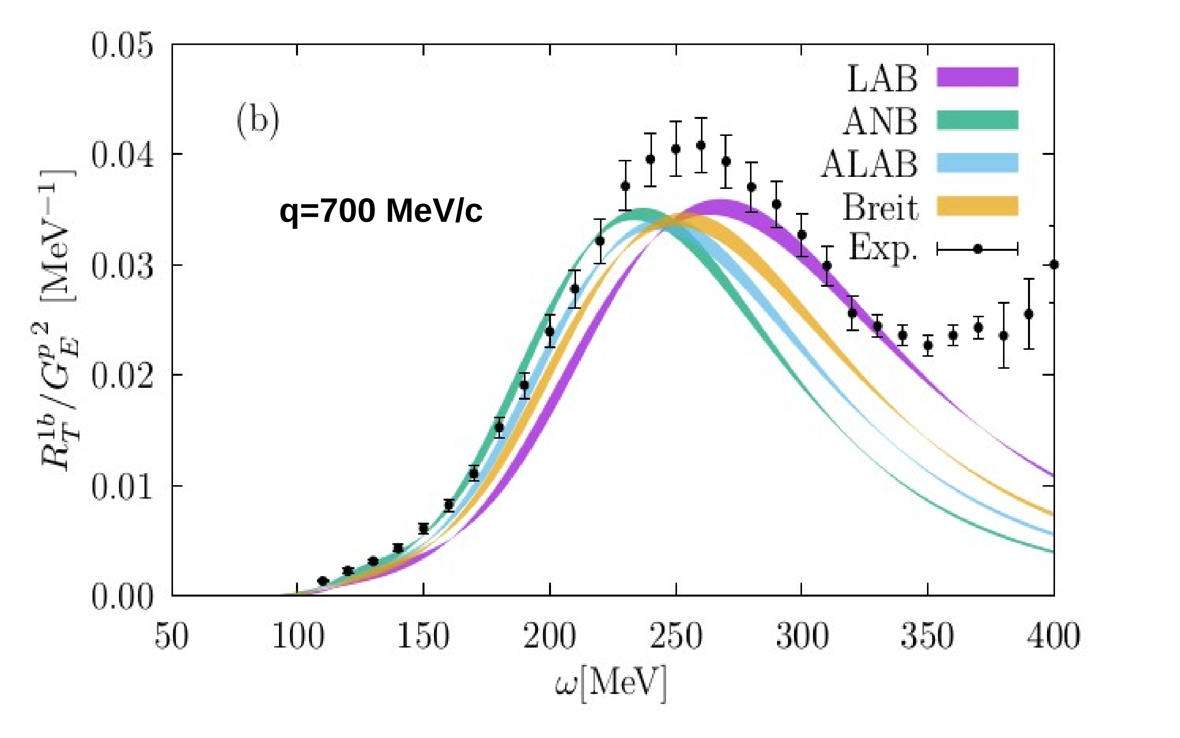
$R_T(q, \omega)$ of ^{4}He

1-body + 2-body currents

No pion production

Integral transform calculation with

$K(\omega,\tau) = e^{-\omega\tau}$ τ real, GFMC, **full FSI**



Large frame dependence also in ^{4}He!

N.Rocco, W.Leidemann, A. Lovato, G.O.
Phys. Rev. C 97, 055501 (2018)

Assuming q.e. kinematics ***[2-body break-up 1-(A-1)]***

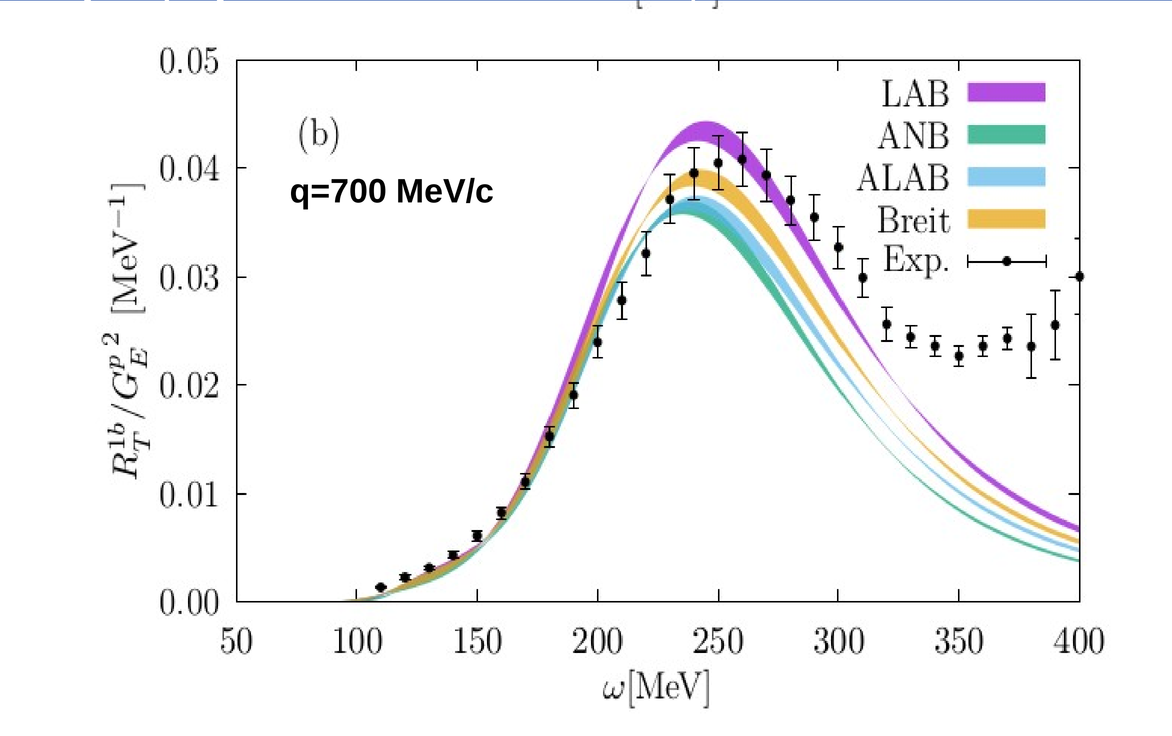
one can treat the relativistic kinematical inputs

correctly!!

$R_T(q, \omega)$ of ^{4}He

Integral transform calculation with

$K(\omega,\tau) = e^{-\omega\tau}$ τ real, GFMC **full FSI**

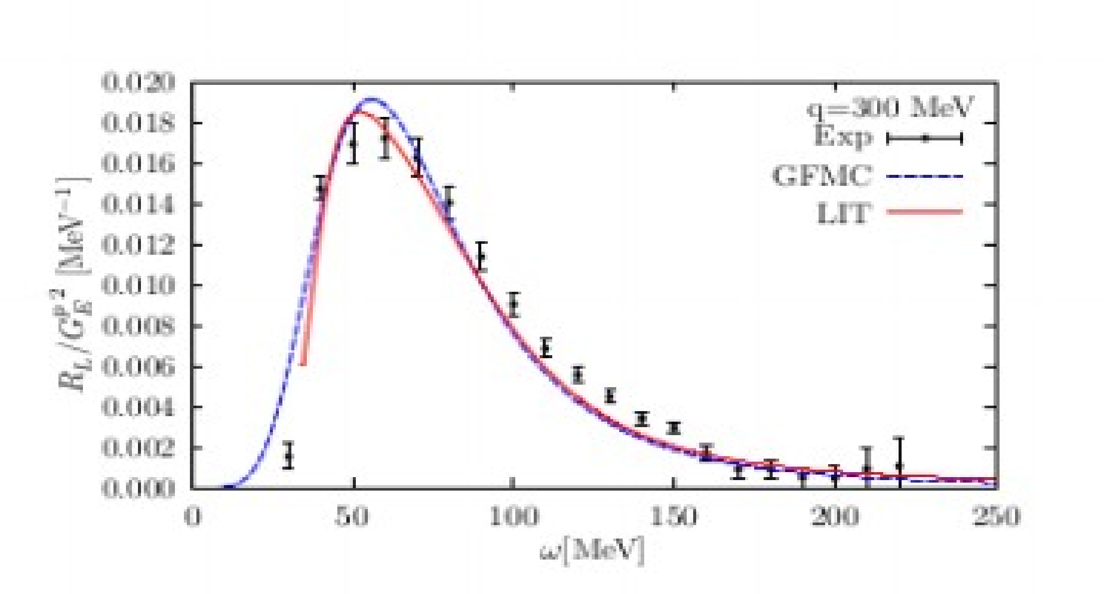


N.Rocco, W.Leidemann, A. Lovato, G.O.
Phys. Rev. C 97, 055501 (2018)

Assuming q.e. kinematics
[2-body break-up 1-(A-1)]
one can treat the relativistic kinematical inputs correctly!!

frame dependence much reduced !!!

estimation of accuracy



Blue line: Exponential Kernel

Red line: Lorentzian Kernel:

LIT: S. Bacca, N. Barnea, W. Leidemann, and G. Orlandini, Phys. Rev. C 80, 064001 (2009).

results on total cross section

Responses are calculated at fixed q as a function of energy transfer

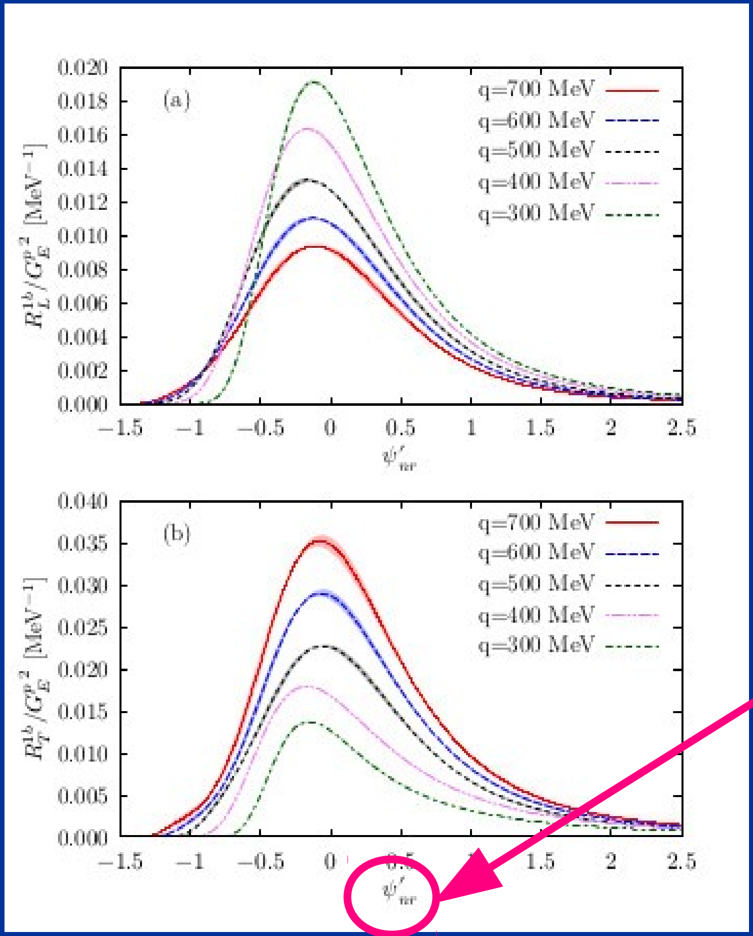
Cross sections are obtained for fixed initial energy and angle and correspond to many values of q!

Computationally very demanding

→ (impossible??)

Regular behaviour if re-plot results as function of scaling variable!

N. Rocco, L. Alvarez-Ruso, A. Lovato, and J. Nieves,
Phys. Rev. C96, 015504 (2017)



$\psi' = k_f /q\ [(\omega-E_s) - q^2/(2m)\ ]$

Scaling variable

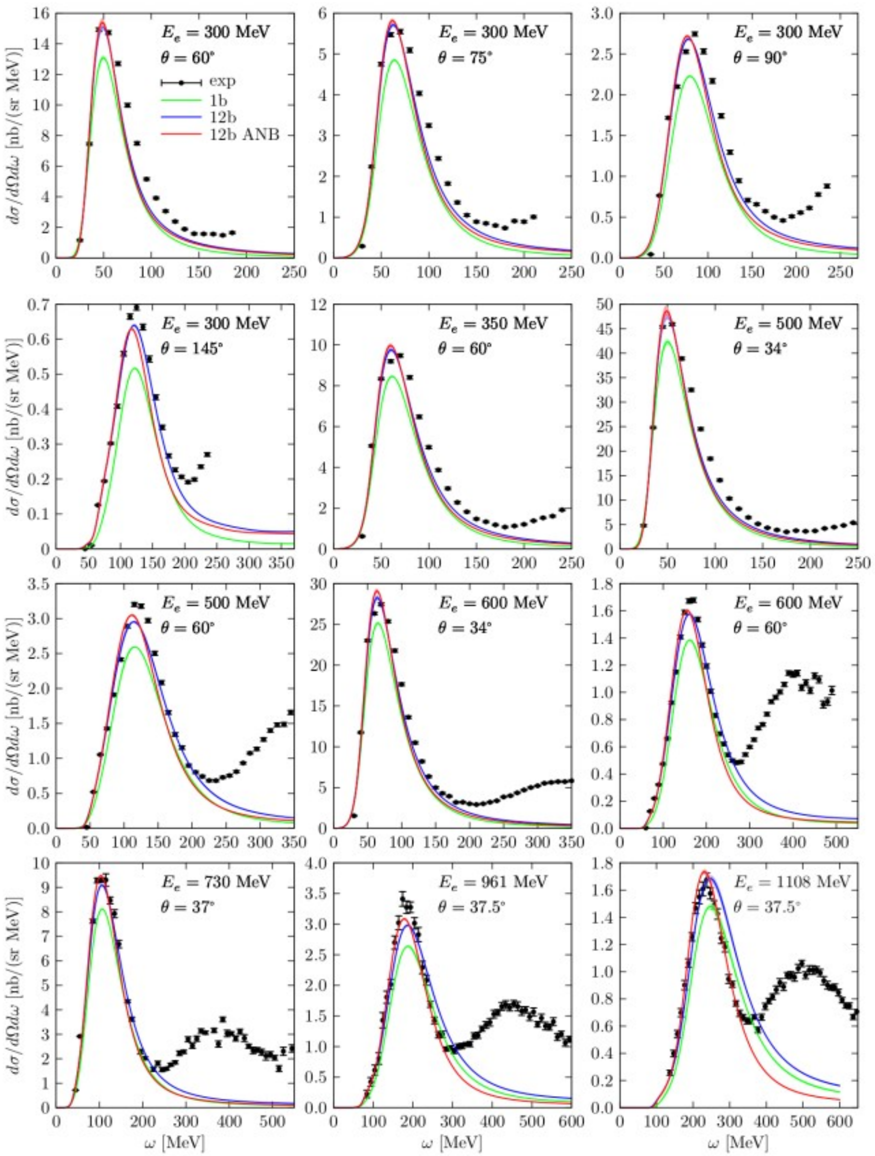


FIG. 7. Double-differential electron-^{4}He cross sections for different values of incident electron energy and scattering angle. The green and blue lines correspond to GFMC calculation were only one- body and one- plus two-body contributions in the electromagnetic currents are accounted for. The red line indicates one plus two-body current results obtained in the ANB frame, employing the two-body fragment model to account for relativistic kinematics. The experimental data are taken from Ref. [?].

N.Rocco, W.Leidemann, A. Lovato, G.O. Phys. Rev. C 97, 055501 (2018)

- Many contributions at different q !
- Very high computational effort demanded
- Smart interpolation via scaling variable performed

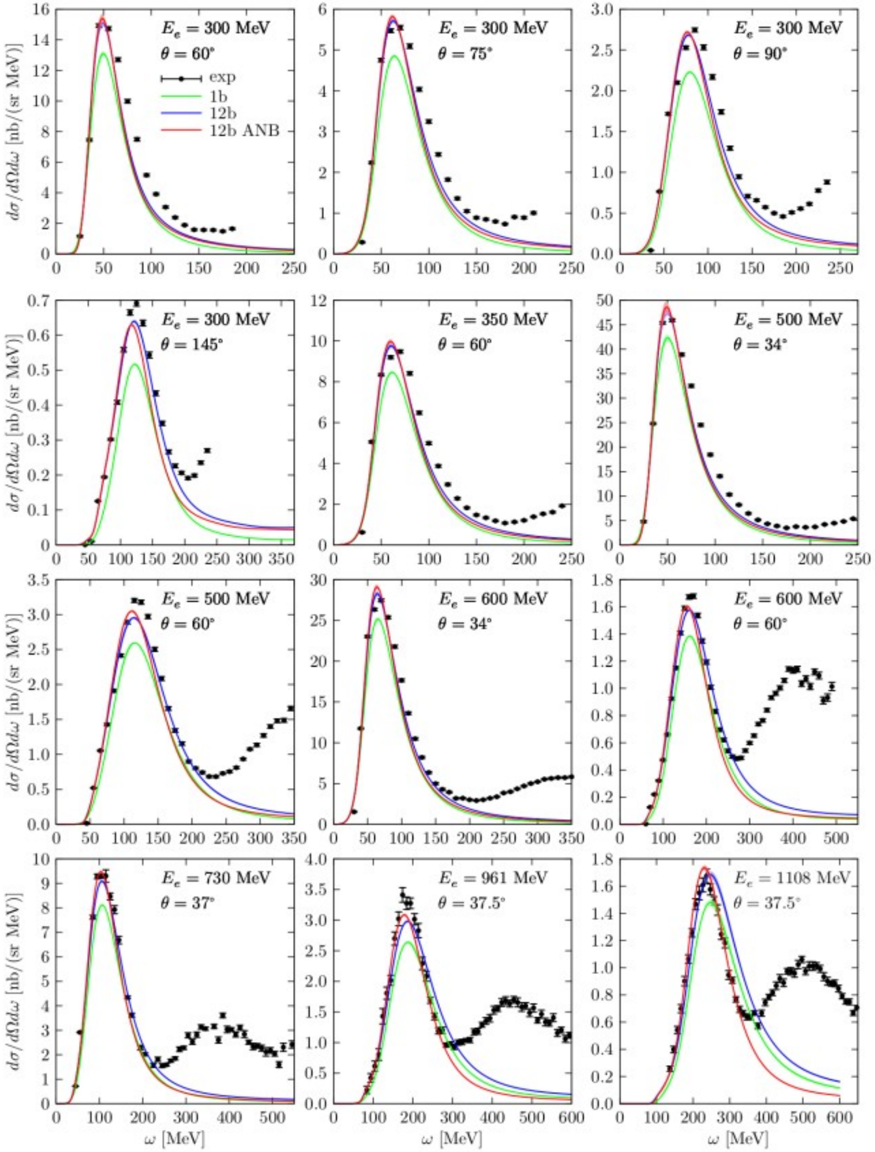


FIG. 7. Double-differential electron-^{4}He cross sections for different values of incident electron energy and scattering angle. The green and blue lines correspond to GFMC calculation were only one- body and one- plus two-body contributions in the electromagnetic currents are accounted for. The red line indicates one plus two-body current results obtained in the ANB frame, employing the two-body fragment model to account for relativistic kinematics. The experimental data are taken from Ref. [?].

N.Rocco, W.Leidemann, A. Lovato, G.O. Phys. Rev. C 97, 055501 (2018)

Test on electron scattering data:

Very good!

One can extend to high q the applicability of an *ab initio* n.r. calculation by choosing the right frame

Summarizing:

- Ab initio **n**on **r**elativistic calculations of the (e,e’) nuclear cross section can be performed considering the **full realistic potential dynamics** both in the initial and in the **final** state

Summarizing:

- Ab initio **n**on **r**elativistic calculations of the (e,e') nuclear cross section can be performed considering the **full realistic potential dynamics** both in the initial and in the **final** state
- **One and two-body currents** (with relativistic corrections up to $(q/m)^2$ are included

Summarizing:

- Ab initio **n**on **r**elativistic calculations of the (e,e') nuclear cross section can be performed considering the **full realistic potential dynamics** both in the initial and in the **final** state
- **One and two-body currents** (with relativistic corrections up to $(q/m)^2$ are included)
- **Frame dependence** is much reduced, using quasi elastic kinematics

Summarizing:

- Ab initio **n**on **r**elativistic calculations of the (e,e') nuclear cross section can be performed considering the **full realistic potential dynamics** both in the initial and in the **final** state
- **One and two-body currents** (with relativistic corrections up to $(q/m)^2$ are included
- **Frame dependence** is much reduced, using quasi elastic relativistic kinematics
- In q.e. regime relativistic effects are minimized in the **ANB frame**

Conclusion and outlook

- The **test** of the described approach on **(e,e')** measured cross section turns out to be **very good**
- Then one can use the same approach for **neutrino** scattering
- Heavier targets than ^{4}He can also be treated
- ^{12}C next

Results obtained with

- Noemi Rocco (Argonne Nat. Lab.)
- Alessandro Lovato (INFN Trento)
- Winfried Leidemann (Univ. Trento)
- Victor Efros (Kurchatov Centre Moscow)
- Ed Tomusiak (Univ. Victoria Canada)