FACULTY OF SCIENCES

DEPARTMENT OF PHYSICS AND ASTRONOMY

QE NEUTRINO-NUCLEUS SCATTERING AND EXPERIMENTS

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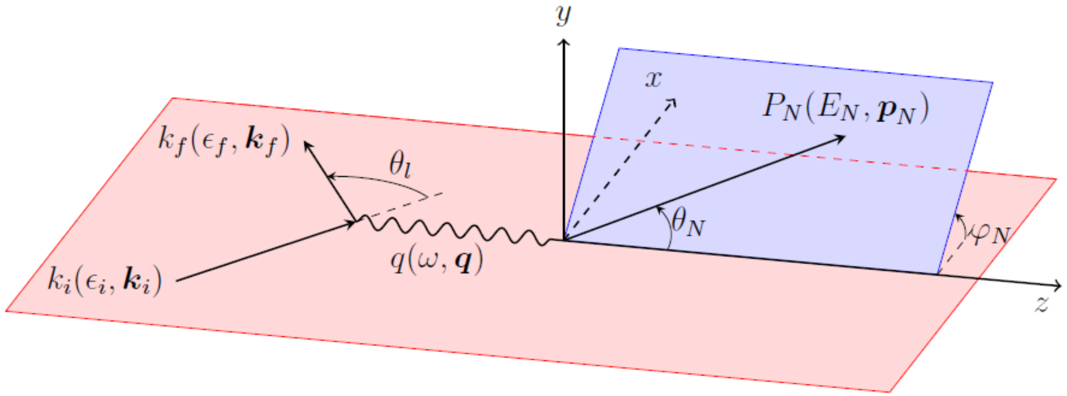
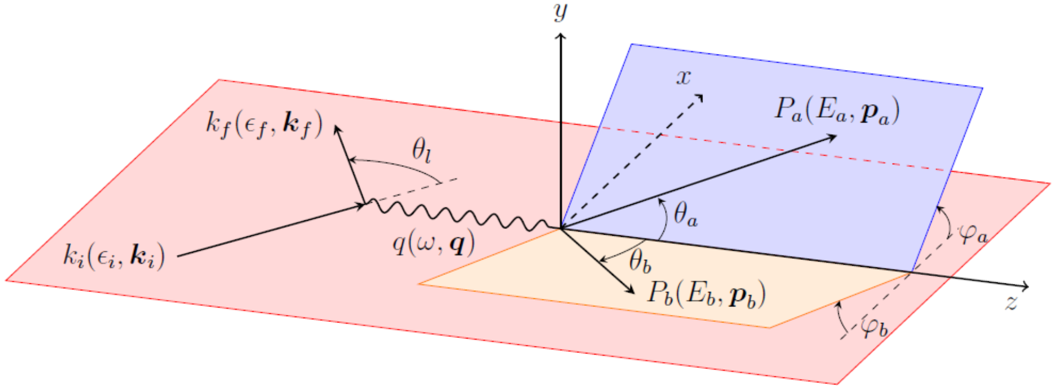
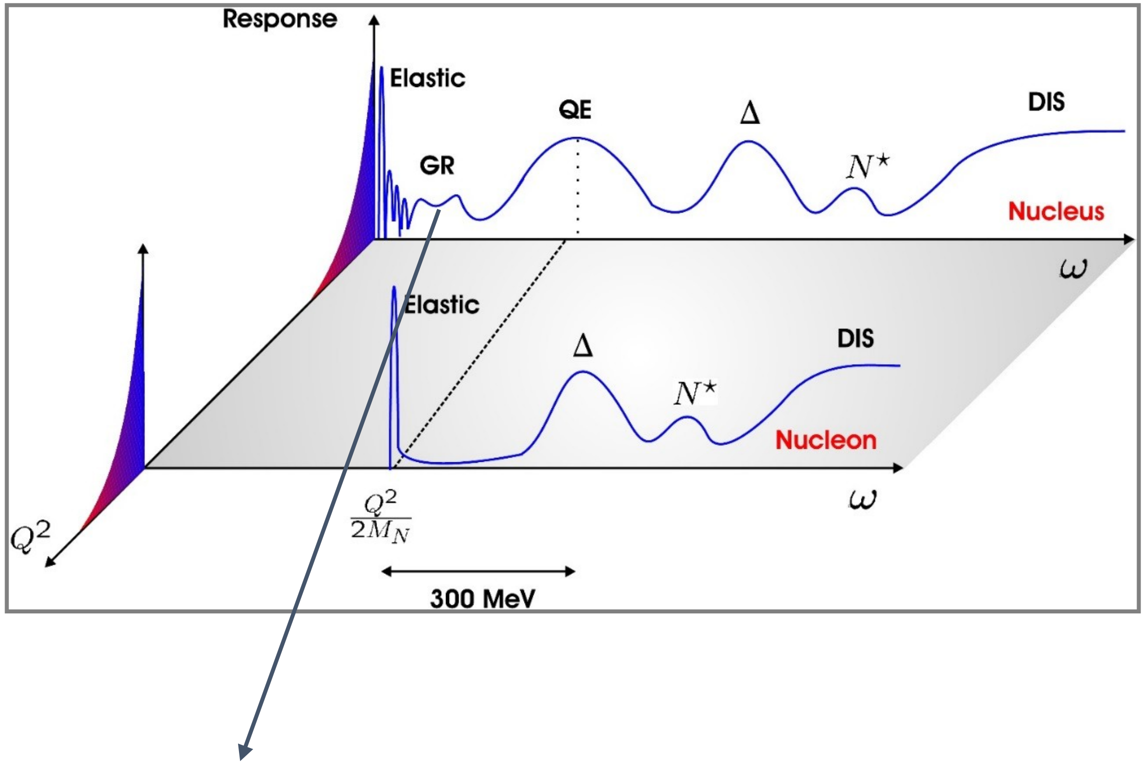
Outline

- Cross sections calculations for QE scattering
- Influence of long-range correlations
- Mean field and RPA effects in neutrino experiments

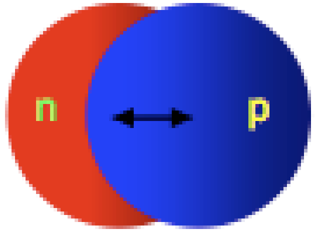
Neutrino-hadron scattering

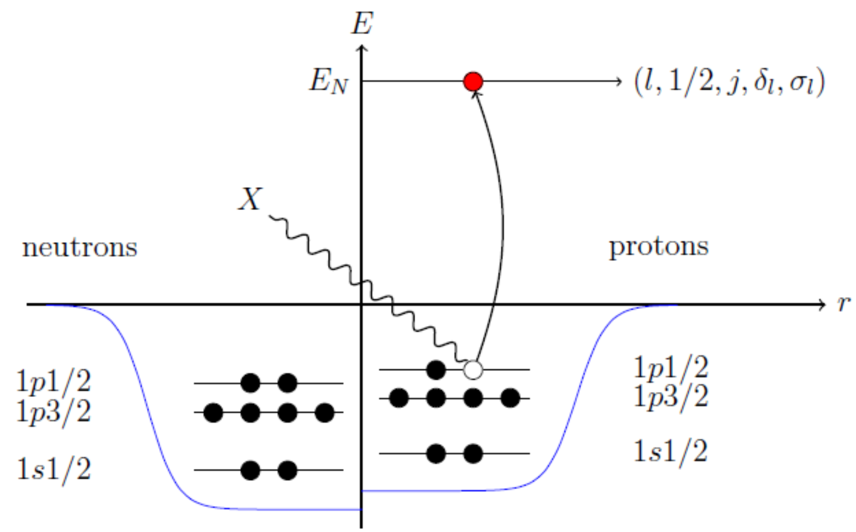


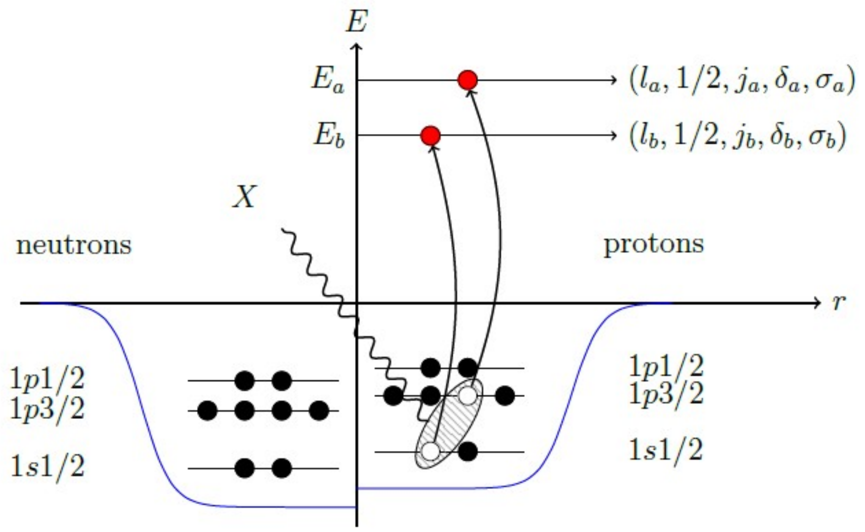




e.g.







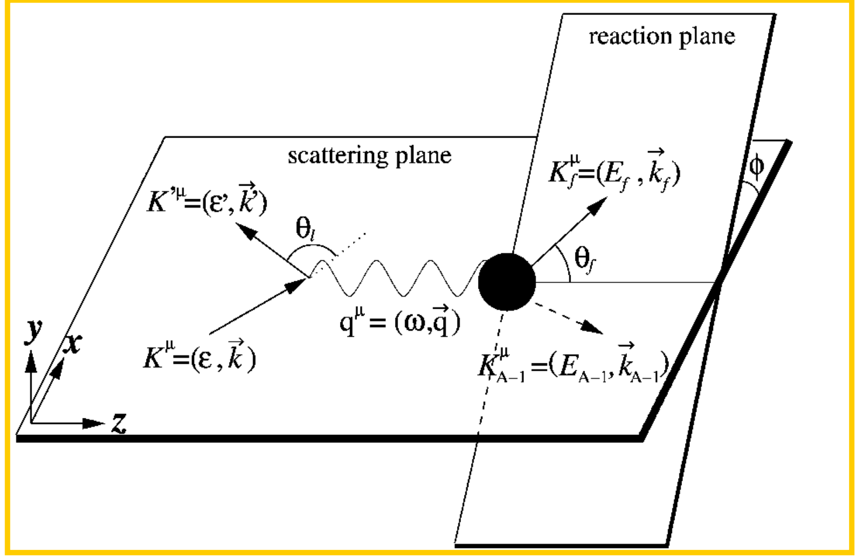
Cross section calculations

- Starting point : mean-field nucleus with Hartree-Fock single-particle wave functions
- Skyrme SkE2 force used to build the potential
- BindingPauli-blocking (orthogonality)
- Distortion of outgoing nucleon in real potential

2 pqrticle excitations : not discussed here

Neutrino-nucleus interactions

$$\hat{H}_W = \frac{G}{\sqrt{2}} \int d\vec{x}\, \hat{j}_{\mu,lepton}(\vec{x})\, \hat{J}^{\mu,hadron}(\vec{x})$$



Hadron current

$$J^\mu = F_1(Q^2)\gamma^\mu + i\frac{\kappa}{2M_N}F_2(Q^2)\sigma^{\mu\nu}q_\nu + G_A(Q^2)\gamma^\mu\gamma_5 + \frac{1}{2M_N}G_P(Q^2)q^\mu\gamma_5$$

Lepton tensor

$$l_{\alpha\beta} \equiv \overline{\sum_{s,s'}}[\overline{u}_l\gamma_\alpha(1-\gamma_5)u_l]^\dagger[\overline{u}_\nu\gamma_\beta(1-\gamma_5)u_\nu]$$

Non-relativistic reduction of the currents

$$\vec{J}_V^{\alpha}(\vec{x}) = \vec{J}^{\alpha}_{convection}(\vec{x}) + \vec{J}^{\alpha}_{magnetization}(\vec{x})$$

with $\vec{J}_c^{\alpha}(\vec{x}) = \frac{1}{2Mi}\sum_{i=1}^{A} G_E^{i,\alpha}\left[\delta(\vec{x}-\vec{x}_i)\overrightarrow{\nabla}_i - \overleftarrow{\nabla}_i\,\delta(\vec{x}-\vec{x}_i)\right],$

$$\vec{J}_m^{\alpha}(\vec{x}) = \frac{1}{2M}\sum_{i=1}^{A} G_M^{i,\alpha}\,\overrightarrow{\nabla}\times\vec{\sigma}_i\,\delta(\vec{x}-\vec{x}_i),$$

$$\vec{J}_A^{\alpha}(\vec{x}) = \sum_{i=1}^{A} G_A^{i,\alpha}\,\vec{\sigma}_i\,\delta(\vec{x}-\vec{x}_i),$$

$$J_V^{0,\alpha}(\vec{x}) = \rho_V^{\alpha}(\vec{x}) = \sum_{i=1}^{A} G_E^{i,\alpha}\,\delta(\vec{x}-\vec{x}_i),$$

$$J_A^{0,\alpha}(\vec{x}) = \rho_A^{\alpha}(\vec{x}) = \frac{1}{2Mi}\sum_{i=1}^{A} G_A^{i,\alpha}\,\vec{\sigma}_i\cdot\left[\delta(\vec{x}-\vec{x}_i)\overrightarrow{\nabla}_i - \overleftarrow{\nabla}_i\,\delta(\vec{x}-\vec{x}_i)\right]$$

$$J_P^{0,\alpha}(\vec{x}) = \rho_P^{\alpha}(\vec{x}) = \frac{m_\mu}{2M}\sum_{i=1}^{A} G_P^{i,\alpha}\,\vec{\nabla}\cdot\vec{\sigma}_i\,\delta(\vec{x}-\vec{x}_i)$$

for NC reactions

$$G_E^{V,o} = \left(\frac{1}{2}-\sin^2\theta_W\right)\tau_3 - \sin^2\theta_W,$$

$$G_M^{V,o} = \left(\frac{1}{2}-\sin^2\theta_W\right)(\mu_p-\mu_n)\,\tau_3 - \sin^2\theta_W\,(\mu_p+\mu_n)$$

$$G^{A,0} = g_a\,\frac{\tau_3}{2} = -\frac{1.262}{2}\,\tau_3$$

for CC reactions

$$G_E^{V,\pm} = \tau_\pm$$

$$G_M^{V,\pm} = (\mu_p-\mu_n)\,\tau_\pm$$

$$G^{A,\pm} = g_a\,\tau_\pm = -1.262\,\tau_\pm$$

$$G = (1+Q^2/M^2)^{-2}$$

Q^2 dependence : dipole parametrization
Or BBBA05

Inclusive 1-nucleon knockout cross sections

$$\frac{d^2\sigma}{d\Omega\, d\omega} = (2\pi)^4\; k_f \varepsilon_f \sum_{s_f, s_i} \frac{1}{2J_i + 1} \sum_{M_f, M_i} \left| \left\langle f \left| \widehat{H}_W \right| i \right\rangle \right|^2$$

Multipole expansion:

$$\left(\frac{d^2\sigma_{i\to f}}{d\Omega d\omega} \right)_{\frac{\nu}{\bar{\nu}}} = \frac{G^2 \varepsilon_f^2}{\pi} \frac{2\cos^2\left(\frac{\theta}{2}\right)}{2J_i + 1} \left[\sum_{J=0}^{\infty} \sigma_{CL}^J + \sum_{J=1}^{\infty} \sigma_T^J \right]$$

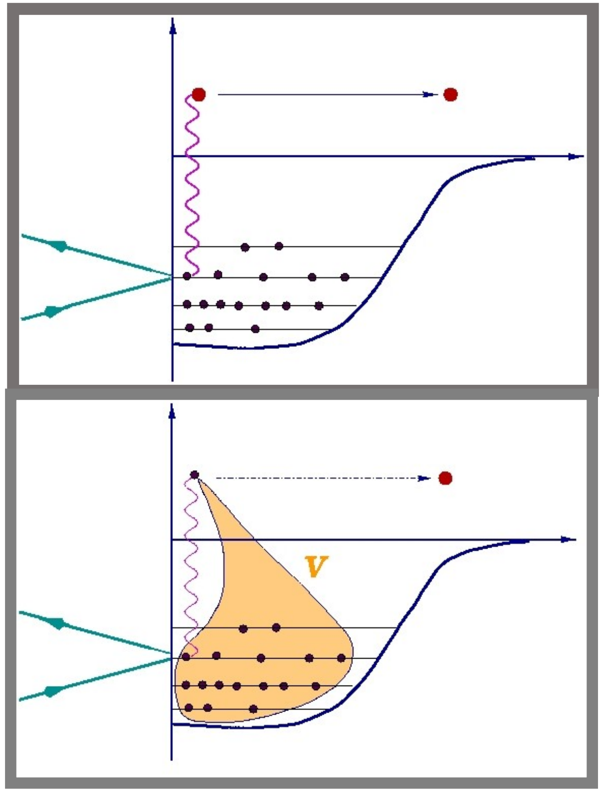
$$\sigma_{CL}^J = \left| \left\langle J_f \left|\left| \widehat{\mathcal{M}}_J(\kappa) + \frac{\omega}{|\vec{q}|} \widehat{\mathcal{L}}_J(\kappa) \right|\right| J_i \right\rangle \right|^2$$

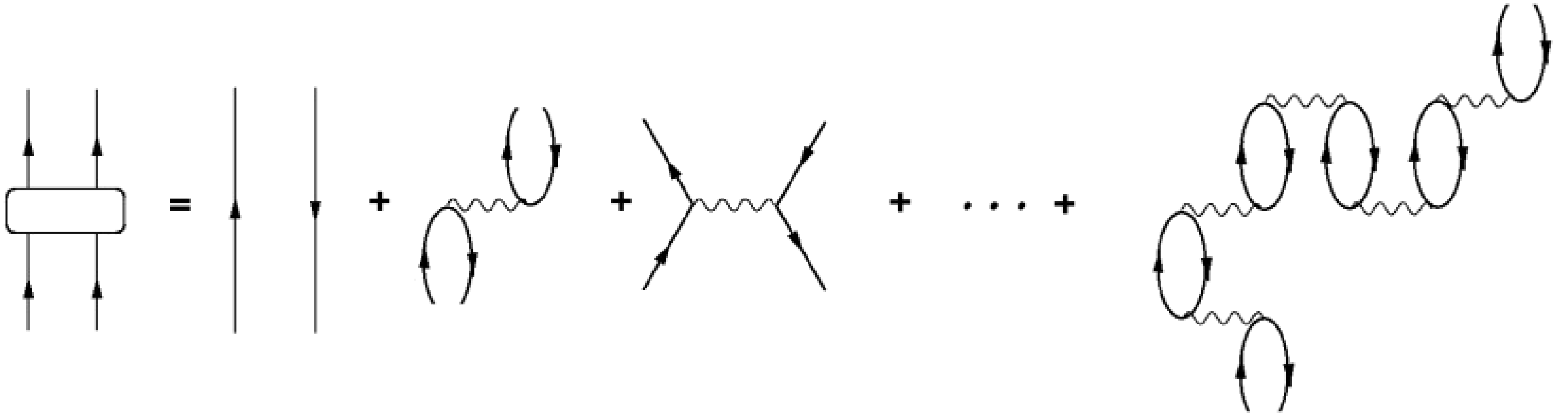
$$\sigma_T^J = \left(-\frac{q_\mu^2}{2|\vec{q}|^2} + \tan^2\left(\frac{\theta}{2}\right) \right) \left[\left| \left\langle J_f \left|\left| \widehat{\mathcal{J}}_J^{mag}(\kappa) \right|\right| J_i \right\rangle \right|^2 + \left| \left\langle J_f \left|\left| \widehat{\mathcal{J}}_J^{el}(\kappa) \right|\right| J_i \right\rangle \right|^2 \right]$$

$$\mp \tan\left(\frac{\theta}{2}\right) \sqrt{-\frac{q_\mu^2}{|\vec{q}|^2} + \tan^2\left(\frac{\theta}{2}\right)} \left[2\Re\left(\left\langle J_f \left|\left| \widehat{\mathcal{J}}_J^{mag}(\kappa) \right|\right| J_i \right\rangle \left\langle J_f \left|\left| \widehat{\mathcal{J}}_J^{el}(\kappa) \right|\right| J_i \right\rangle^* \right) \right]$$

Long-range correlations : Continuum RPA

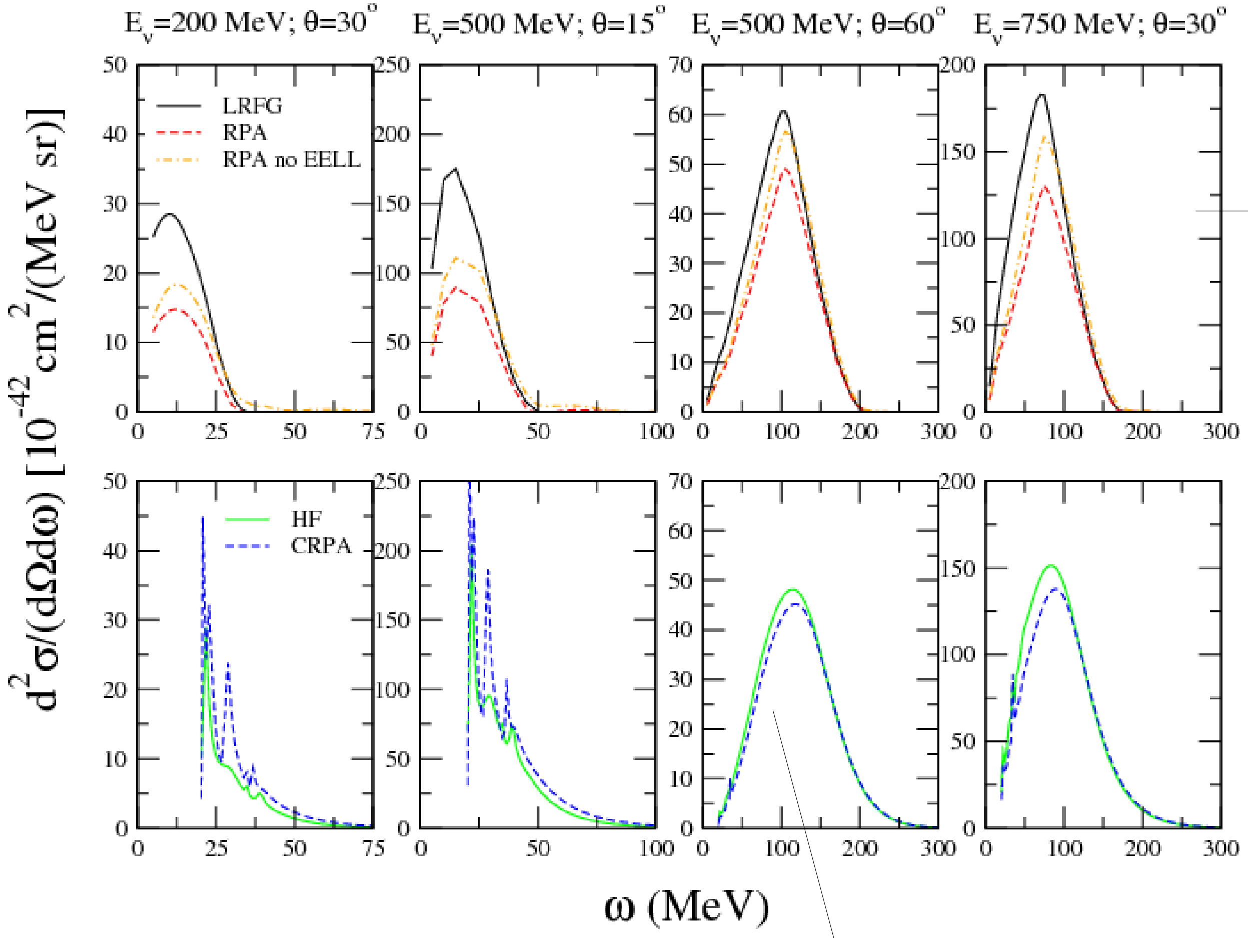
- Green’s function approach
- Skyrme SkE2 residual interaction
- self-consistent calculations





$$|\Psi_{RPA}\rangle = \sum_{c} \left\{ X_{(\Psi,C)} \left|ph^{-1}\right\rangle - Y_{(\Psi,C)} \left|hp^{-1}\right\rangle \right\}$$

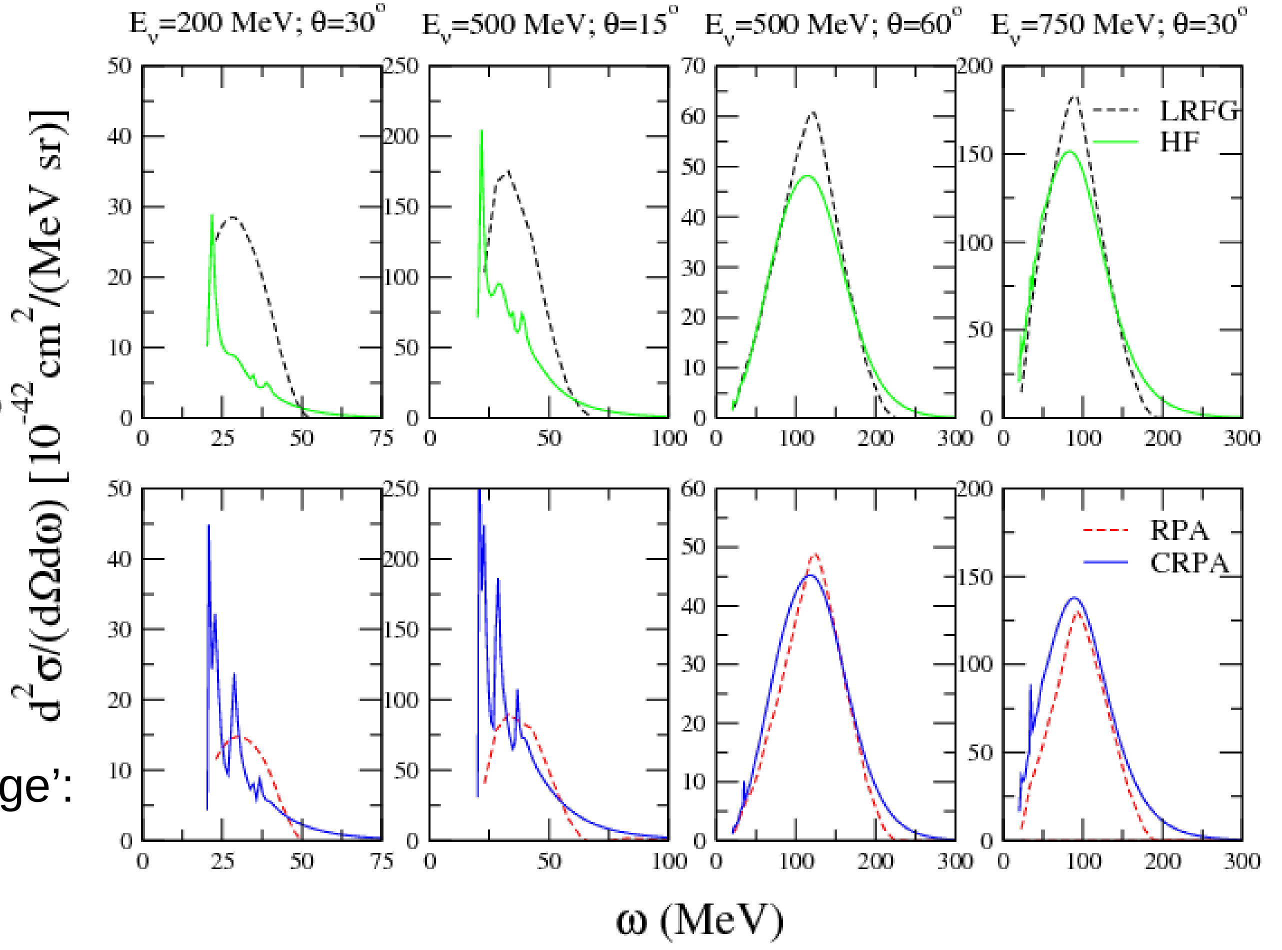
$$\Pi^{(RPA)}(x_1, x_2; \omega) = \Pi^{(0)}(x_1, x_2; \omega) + \frac{1}{\hbar} \int dx \int dx' \; \Pi^{(0)}(x_1, x; \omega) \, \widetilde{V}(x, x') \, \Pi^{(RPA)}(x', x_2; \omega)$$



- Start from (basically) free initial and final states
- ➔ Large effect of RPA is needed to introduce interactions

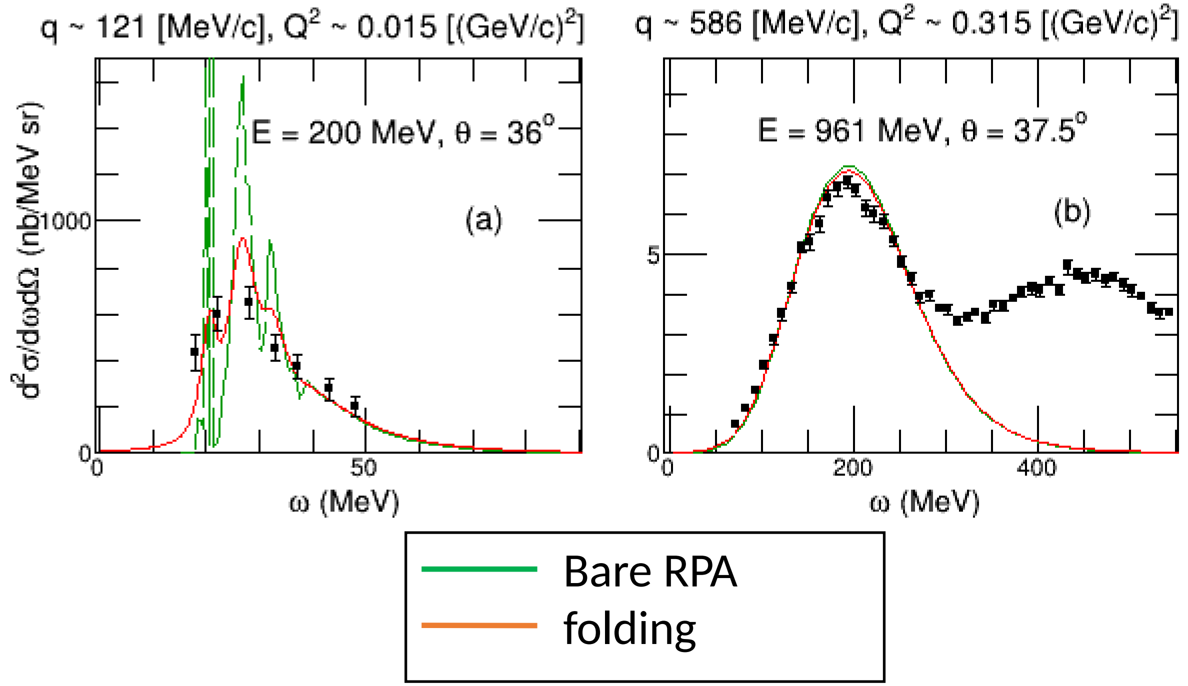
- MF initial and final states
- ➔ Effect of RPA is smaller

Eventually the models 'converge':



Final state interactions

-Calculations of the wave function of the outgoing nucleon in the same (real) nuclear potential used for the initial state
-influence of the spreading width of the particle states is implemented through a folding procedure



$$R'(q,\omega') = \int_{-\infty}^{\infty} d\omega \; R(q,\omega) \; L(\omega,\omega'),$$

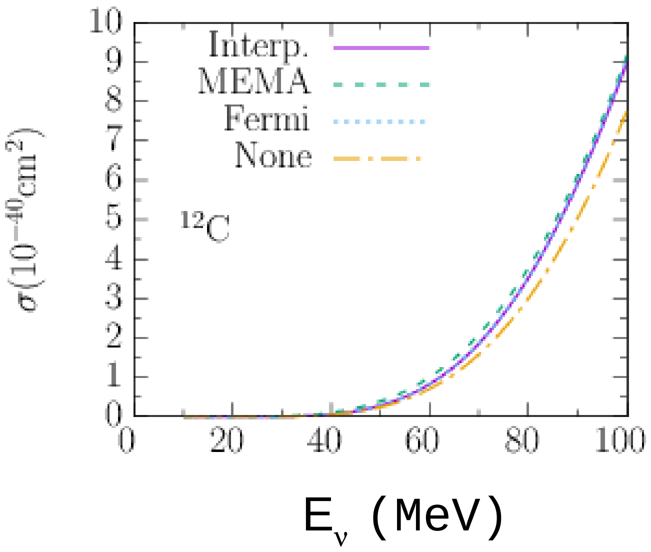
$$L(\omega,\omega') = \frac{1}{2\pi}\left[\frac{\Gamma}{(\omega-\omega')^2 + (\Gamma/2)^2}\right].$$

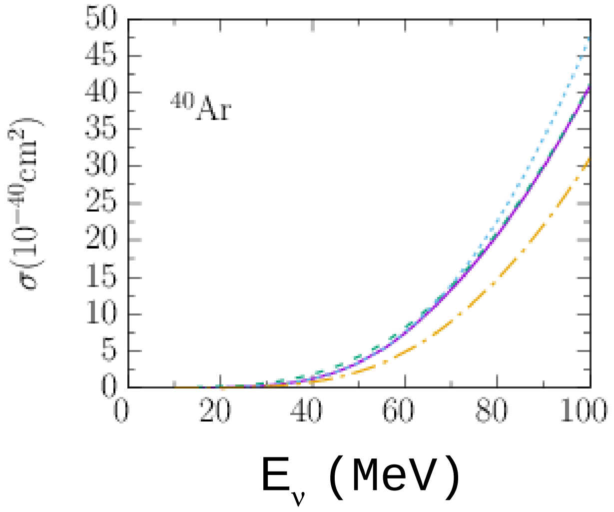
Coulomb corrections

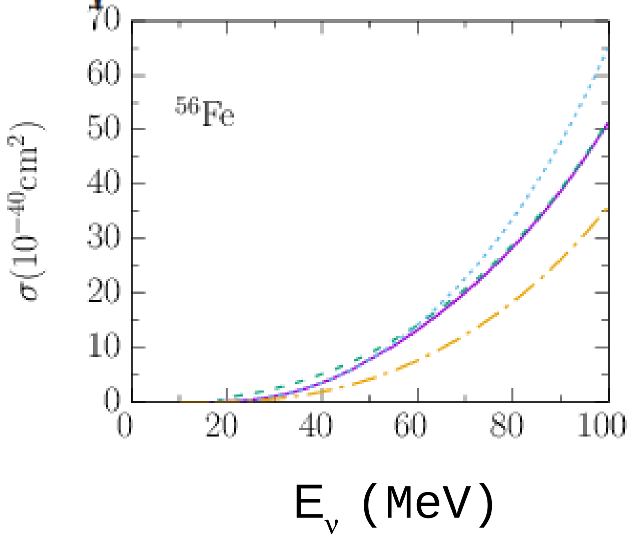
- ✓ Low energies : Fermi function (s-wave correction factor) $F(Z',E) = \dfrac{2\pi\eta}{1-e^{-2\pi\eta}}$ $\quad \eta \sim \mp Z'\alpha$
- ✓ High energies : modified effective momentum approximation (J. Engel, PRC57,2004 (1998))

$$q_{eff} = q + 1.5\left(\frac{Z'\alpha\hbar c}{R}\right), \qquad \Psi_l^{eff} = \zeta(Z',E,q)\,\Psi_l\,,$$

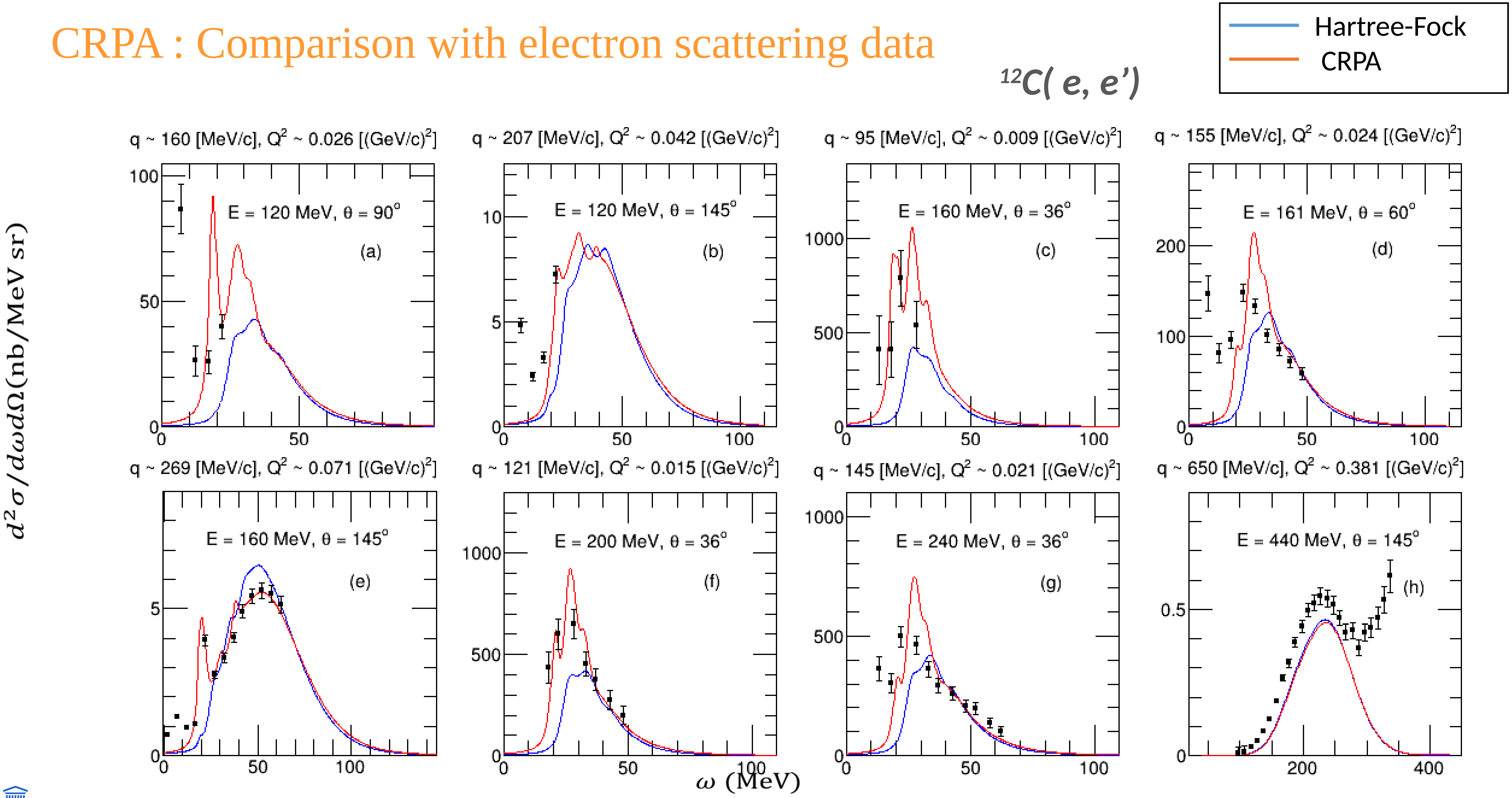
$$\zeta(Z',E,q) = \sqrt{\frac{q_{eff}E_{eff}}{qE}}$$

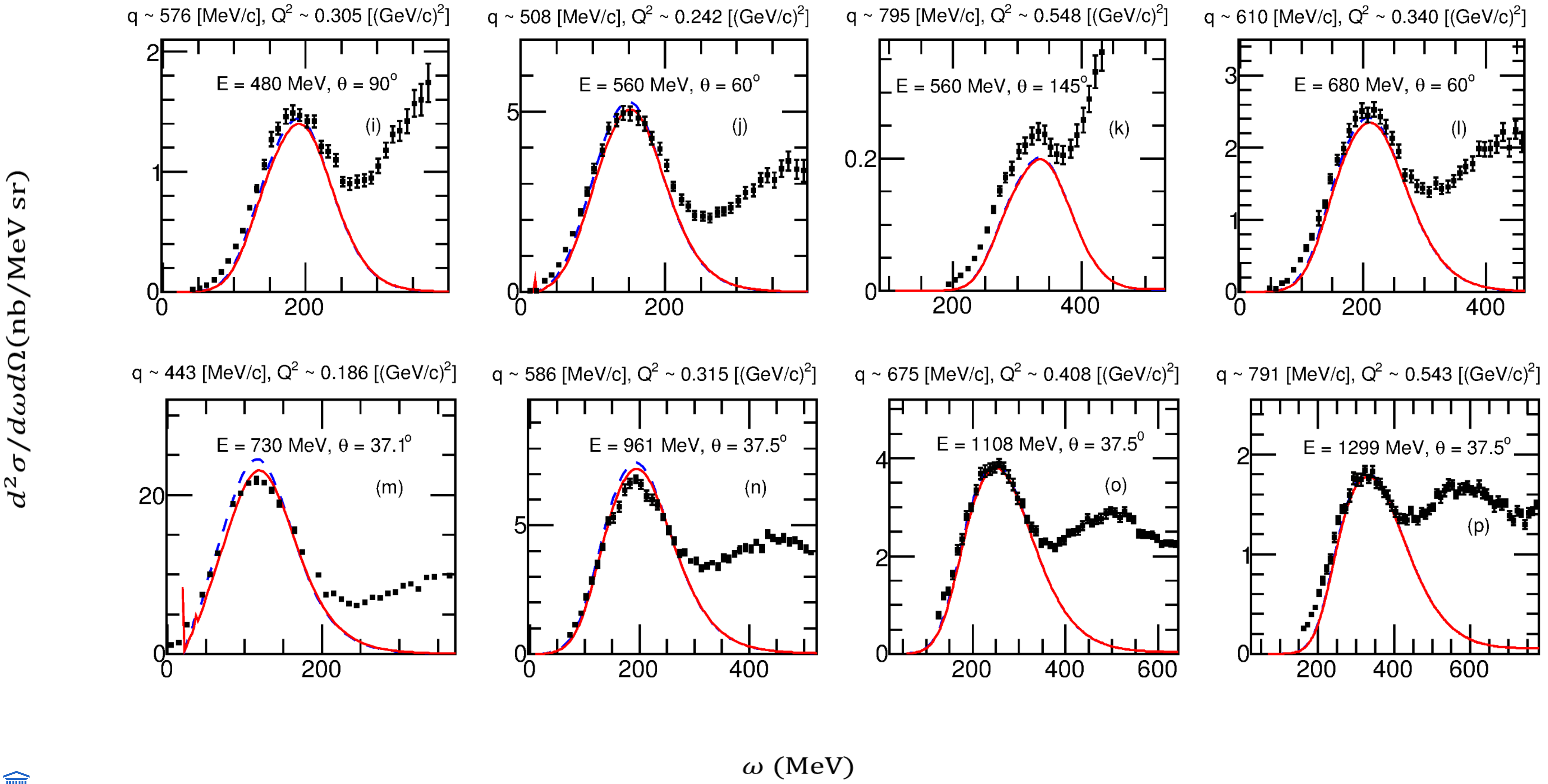




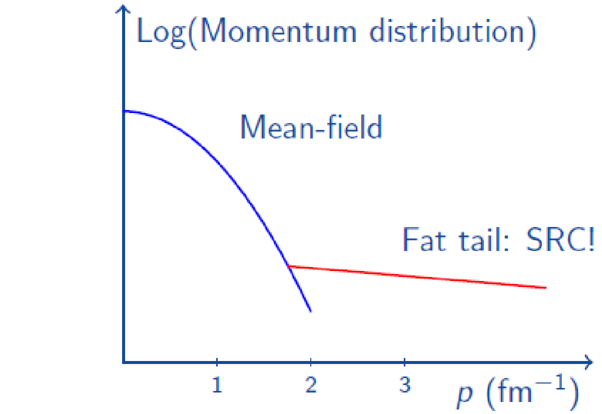


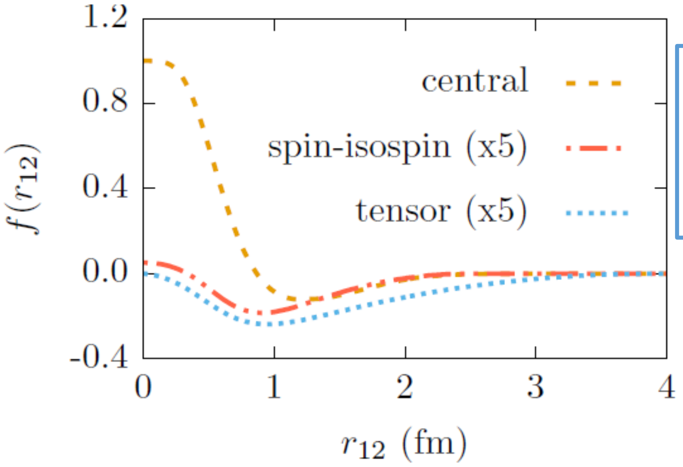
CRPA : Comparison with electron scattering data

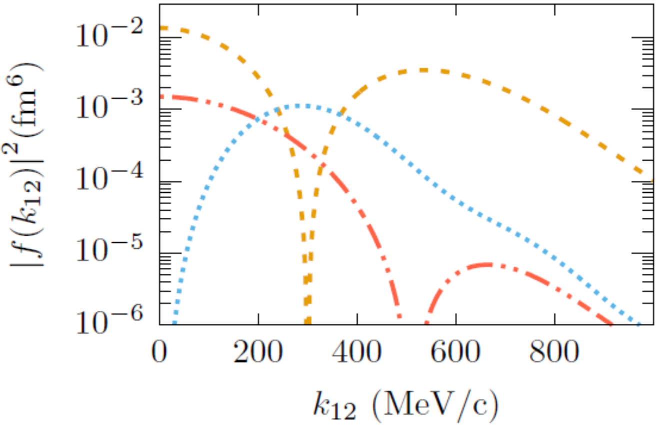



q ~ 576 [MeV/c], Q^2 ~ 0.305 [(GeV/c)^2]
E = 480 MeV, θ = 90°
(i)
q ~ 508 [MeV/c], Q^2 ~ 0.242 [(GeV/c)^2]
E = 560 MeV, θ = 60°
(j)
q ~ 795 [MeV/c], Q^2 ~ 0.548 [(GeV/c)^2]
E = 560 MeV, θ = 145°
(k)
q ~ 610 [MeV/c], Q^2 ~ 0.340 [(GeV/c)^2]
E = 680 MeV, θ = 60°
(l)
q ~ 443 [MeV/c], Q^2 ~ 0.186 [(GeV/c)^2]
E = 730 MeV, θ = 37.1°
(m)
q ~ 586 [MeV/c], Q^2 ~ 0.315 [(GeV/c)^2]
E = 961 MeV, θ = 37.5°
(n)
q ~ 675 [MeV/c], Q^2 ~ 0.408 [(GeV/c)^2]
E = 1108 MeV, θ = 37.5°
(o)
q ~ 791 [MeV/c], Q^2 ~ 0.543 [(GeV/c)^2]
E = 1299 MeV, θ = 37.5°
(p)
d²σ/dωdΩ(nb/MeV sr)
ω (MeV)

Short-range correlations







Gearheart (1994) Pieper (1992)

$$|\Psi\rangle = \frac{1}{\sqrt{\mathcal{N}}}\widehat{\mathcal{G}}|\Phi\rangle$$

with

$$\widehat{\mathcal{G}} \approx \widehat{\mathcal{S}}\left(\prod_{i<j}^{A}\left[1+\widehat{l}(i,j)\right]\right)$$

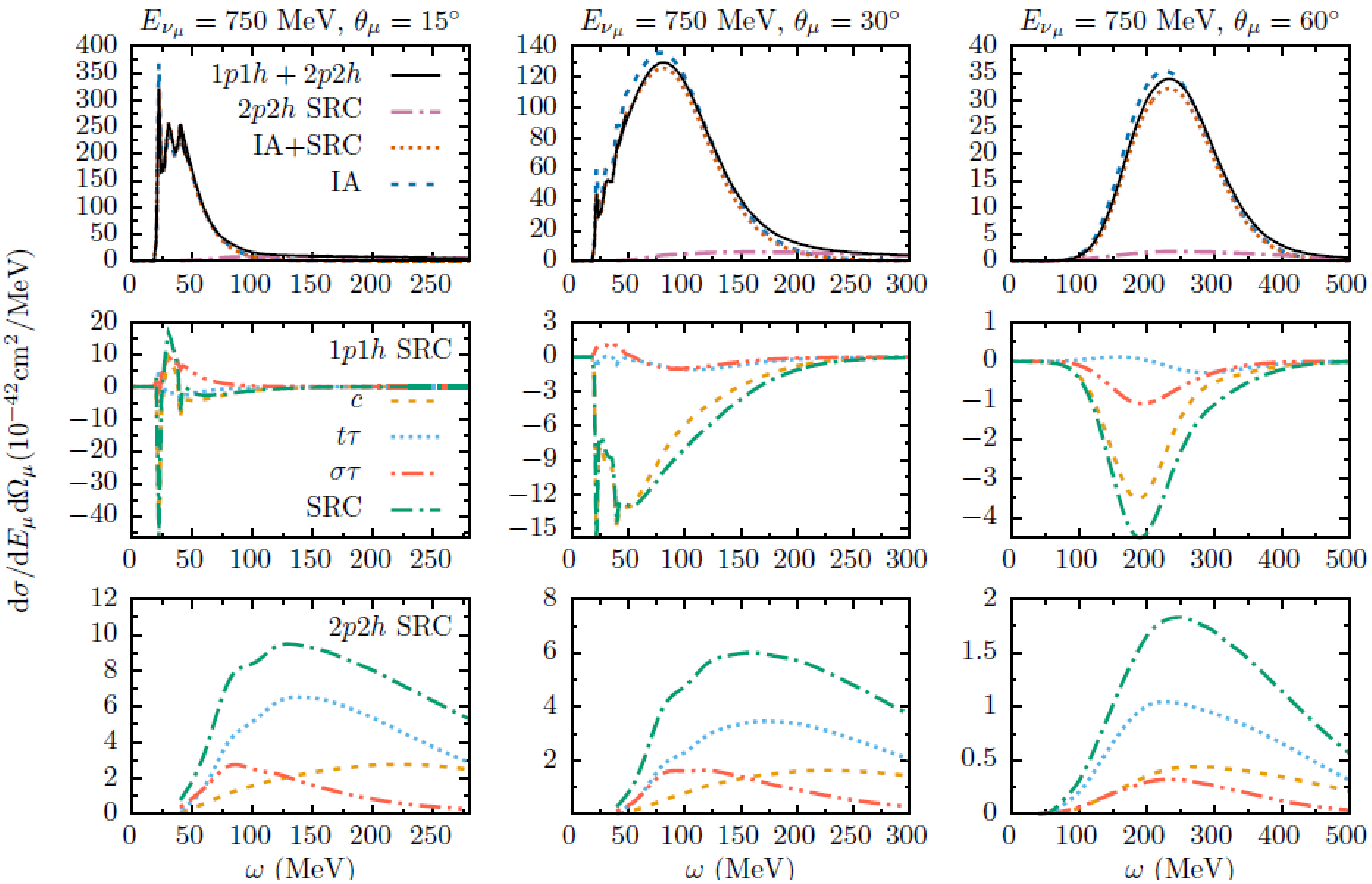
$$\widehat{l}(i,j) = -\,g_c(r_{ij}) + f_{\sigma\tau}(r_{ij})\,(\vec{\sigma}_i\cdot\vec{\sigma}_j)\,(\vec{\tau}_i\cdot\vec{\tau}_j) + f_{t\tau}(r_{ij})\widehat{S}_{ij}\,(\vec{\tau}_i\cdot\vec{\tau}_j)\,,$$

Shifting the complexity induced by correlations from the wave functions to the operators

$$\langle\Psi_{\mathrm{f}}|\widehat{J}_{\mu}^{\mathrm{nucl}}|\Psi_{\mathrm{i}}\rangle = \frac{1}{\sqrt{\mathcal{N}_{\mathrm{i}}\mathcal{N}_{\mathrm{f}}}}\langle\Phi_{\mathrm{f}}|\widehat{J}_{\mu}^{\mathrm{eff}}|\Phi_{\mathrm{i}}\rangle$$

$$\widehat{J}_{\mu}^{\mathrm{eff}} \approx \sum_{i=1}^{A}\widehat{J}_{\mu}^{[1]}(i \;+ \sum_{i<j}^{A}\widehat{J}_{\mu}^{[1],\mathrm{in}}(i,j) + \left[\sum_{i<j}^{A}\widehat{J}_{\mu}^{[1],\mathrm{in}}(i,j)\right]^{\dagger}$$

$$\widehat{J}_{\mu}^{[1],\mathrm{in}}(i,j) = \left[\widehat{J}_{\mu}^{[1]}(i) + \widehat{J}_{\mu}^{[1]}(j)\right]\widehat{l}(i,j)$$



SRC
neutrinos
1p1h+2p2h

Start from :

- Non-relativistic wave functions
- Non-relativistic description of the hadron dynamics

The nucleus is a relativistic system …
but when do we notice?

$$J^\mu = F_1(Q^2)\gamma^\mu + i\frac{\kappa}{2M_N}F_2(Q^2)\sigma^{\mu\nu}q_\nu + G_A(Q^2)\gamma^\mu\gamma_5 + \frac{1}{2M_N}G_P(Q^2)q^\mu\gamma_5$$

$\langle\varphi_n|\hat{J}^\mu|\varphi_n\rangle$

Retain only lowest order contributions in $\frac{E}{M}$

$$\vec{J}_V^\alpha(\vec{x}) = \vec{J}^\alpha_{convection}(\vec{x}) + \vec{J}^\alpha_{magnetization}(\vec{x})$$

with

$$\vec{J}_c^\alpha(\vec{x}) = \frac{1}{2Mi}\sum_{i=1}^{A} G_E^{i,\alpha}\left[\delta(\vec{x}-\vec{x}_i)\overrightarrow{\nabla}_i - \overleftarrow{\nabla}_i\delta(\vec{x}-\vec{x}_i)\right],$$

$$\vec{J}_m^\alpha(\vec{x}) = \frac{1}{2M}\sum_{i=1}^{A} G_M^{i,\alpha}\overrightarrow{\nabla}\times\vec{\sigma}_i\,\delta(\vec{x}-\vec{x}_i),$$

$$\vec{J}_A^\alpha(\vec{x}) = \sum_{i=1}^{A} G_A^{i,\alpha}\vec{\sigma}_i\,\delta(\vec{x}-\vec{x}_i),$$

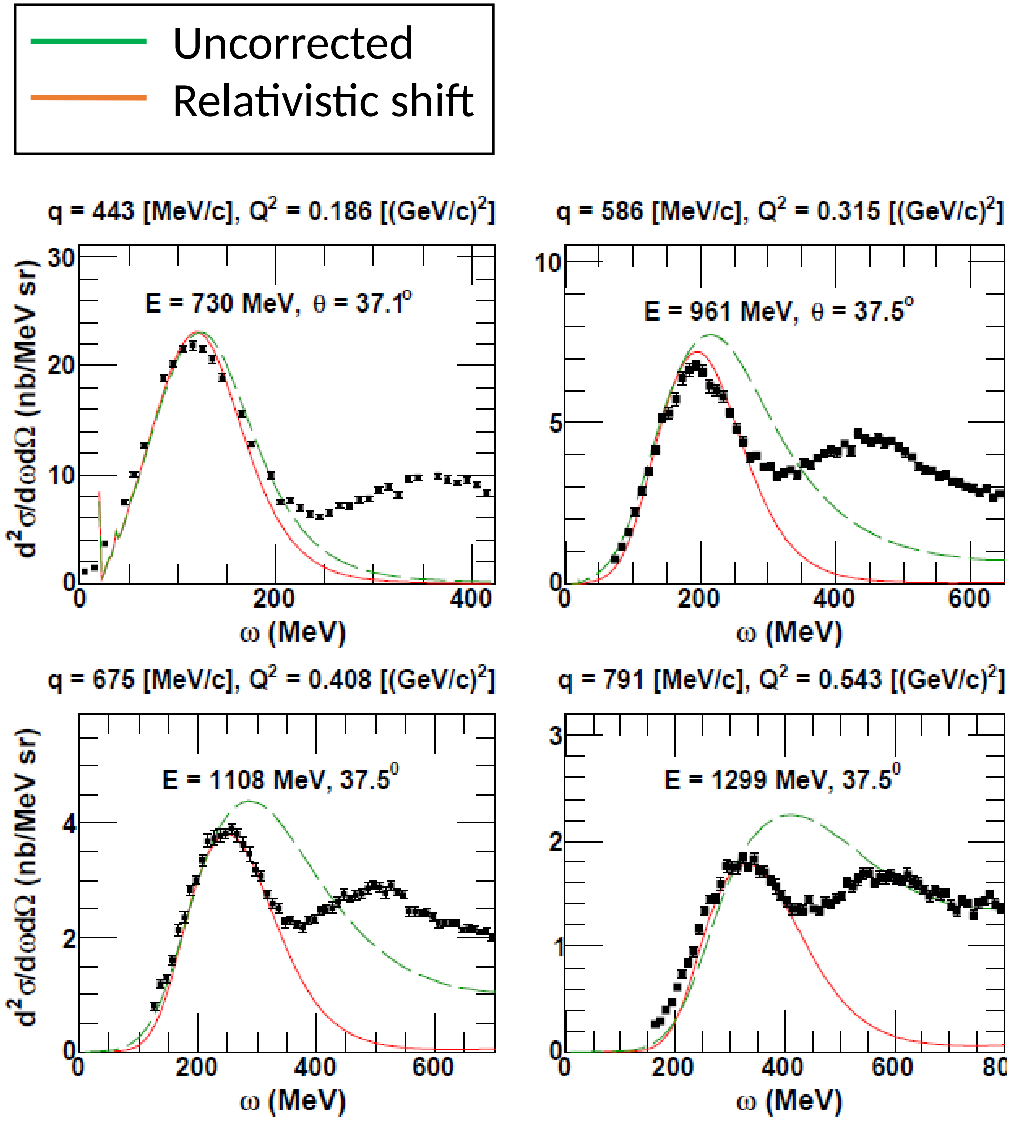
$$J_V^{0,\alpha}(\vec{x}) = \rho_V^\alpha(\vec{x}) = \sum_{i=1}^{A} G_E^{i,\alpha}\delta(\vec{x}-\vec{x}_i),$$

$$J_A^{0,\alpha}(\vec{x}) = \rho_A^\alpha(\vec{x}) = \frac{1}{2Mi}\sum_{i=1}^{A} G_A^{i,\alpha}\vec{\sigma}_i\cdot\left[\delta(\vec{x}-\vec{x}_i)\overrightarrow{\nabla}_i - \overleftarrow{\nabla}_i\delta(\vec{x}-\vec{x}_i)\right]$$

$$J_P^{0,\alpha}(\vec{x}) = \rho_P^\alpha(\vec{x}) = \frac{m_\mu}{2M}\sum_{i=1}^{A} G_P^{i,\alpha}\overrightarrow{\nabla}\cdot\vec{\sigma}_i\,\delta(\vec{x}-\vec{x}_i)$$

Relativistic corrections

(J. Jeschonnek and T. Donnelly, PRC57, 2438 (1998))

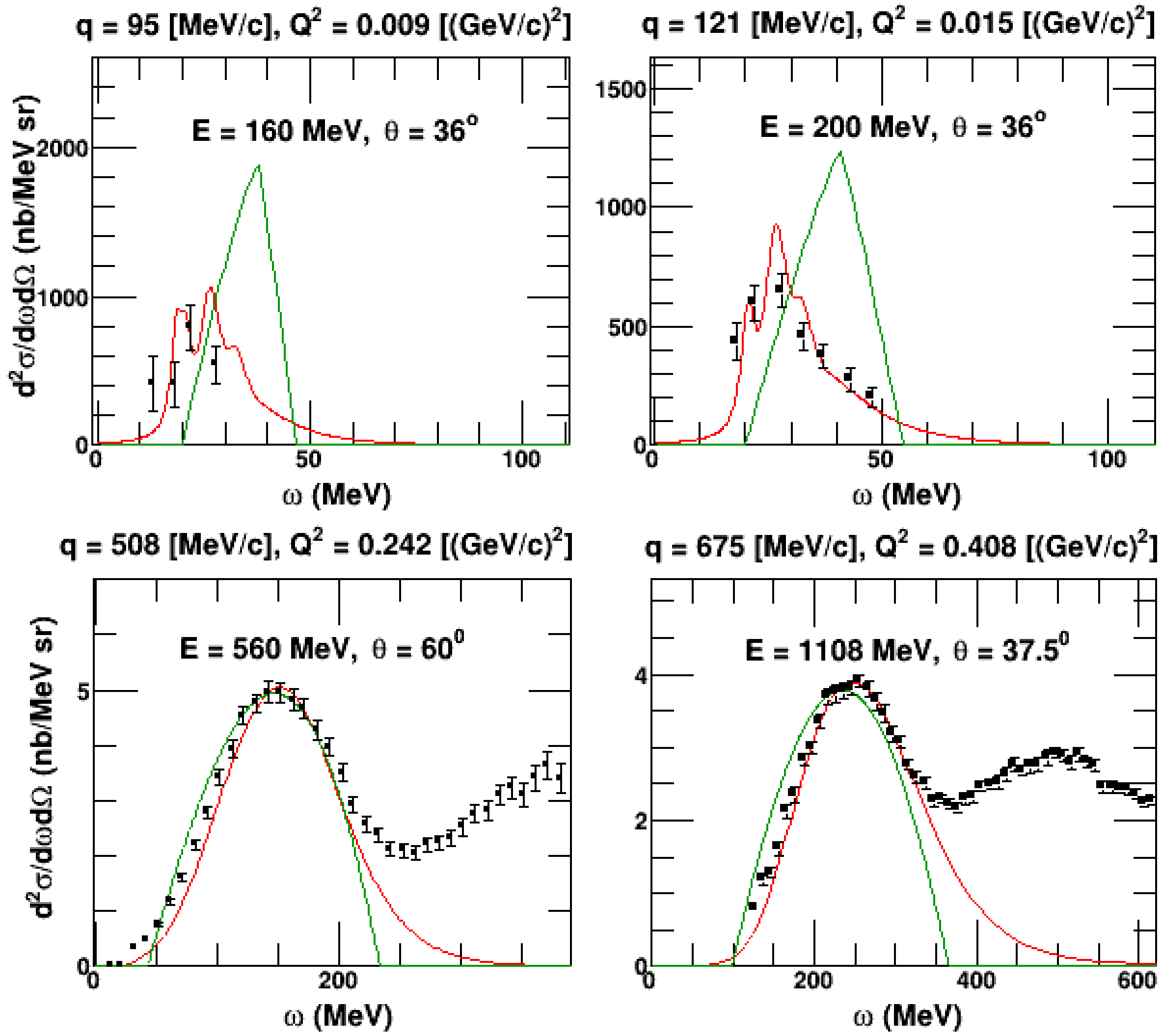


Shift :

$$\lambda \rightarrow \lambda(\lambda + 1) \qquad \lambda = \omega / 2M_N$$

- The outgoing nucleon obtains the correct relativistic momentum
$$p = \sqrt{T^2 + 2MT}$$
- Shifts the QE peak to the right relativistic position

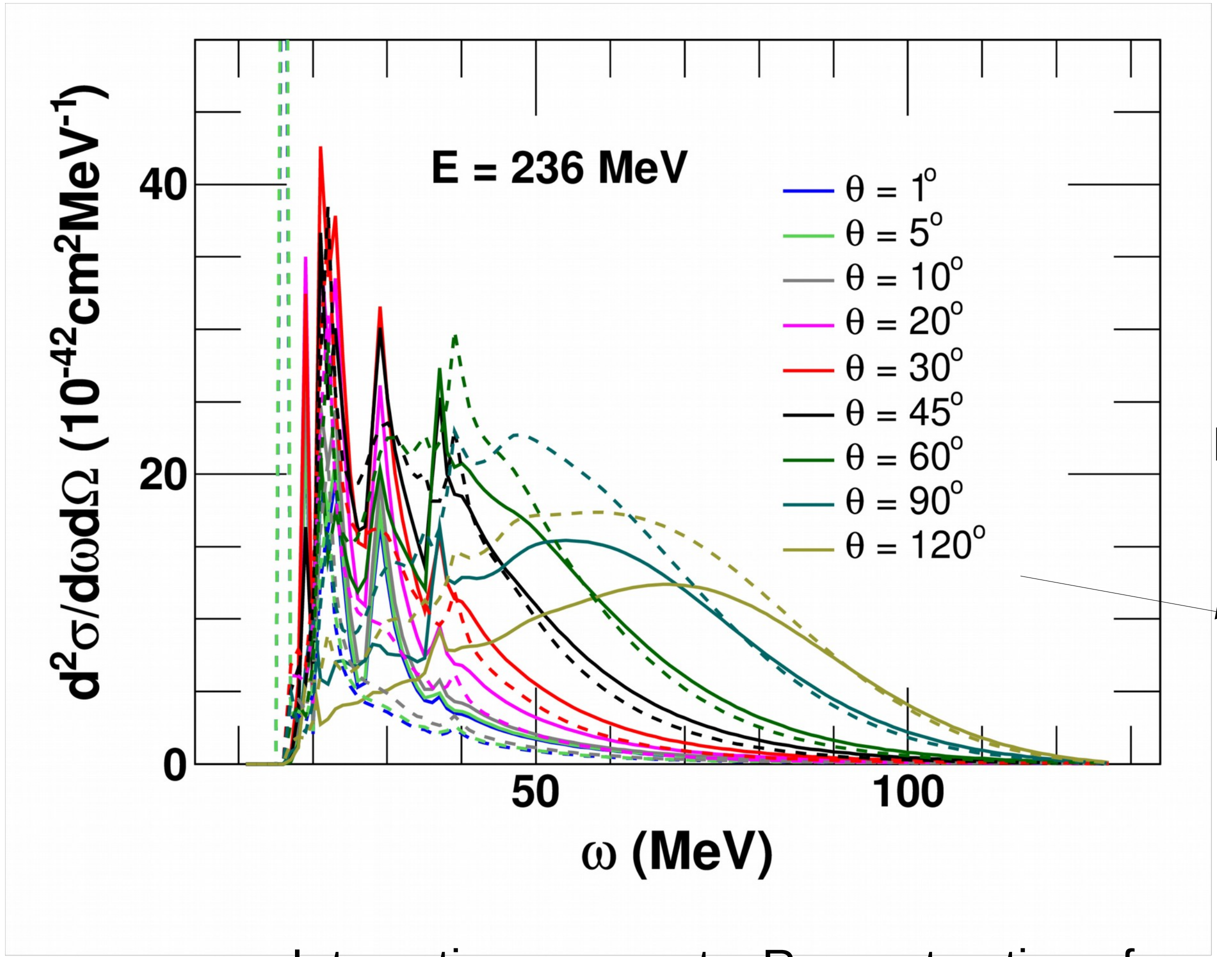
Oscillation analysis is often done with RFG

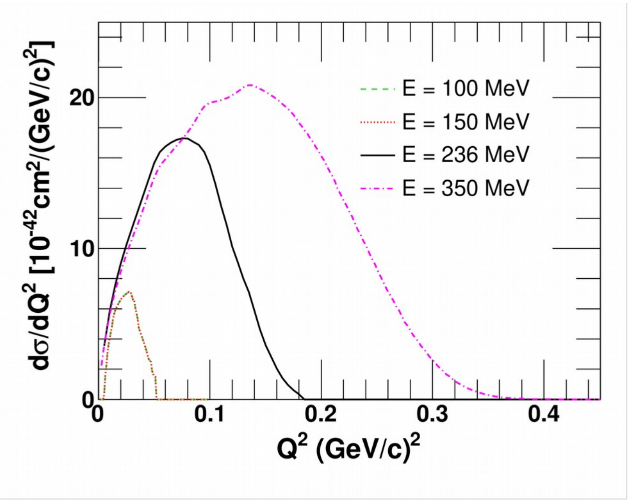


Fermi gas
CRPA

Low energy neutrino experiments

- Kaon Decay At Rest (KDAR) neutrinos, monoenergetic E = 236 MeV





Kinematic region between genuine low excitation Energies and QE scattering: ‘Transition’

First measurement (MiniBooNE): only Tmu dependence

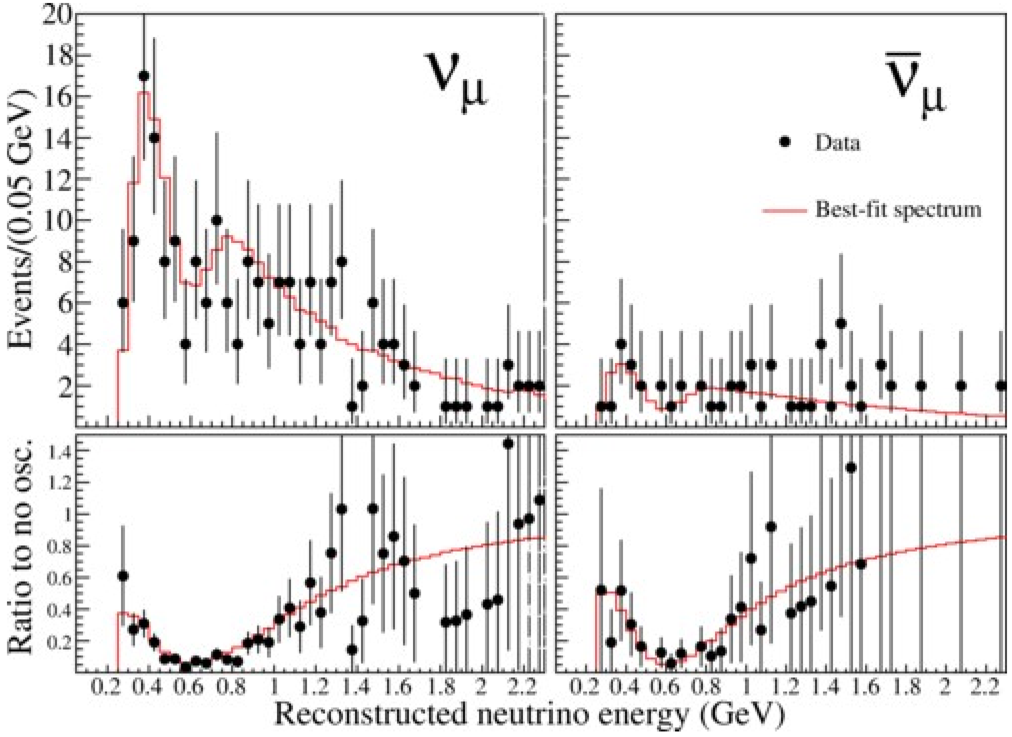
Angular information of the muon, discern clear differences HF and CRPA

Interesting prospects: Reconstruction of w and q + measurements of hadronic final state!

Reconstructed energies

Experimental kinematic variable

$$\overline{E}_\nu = \frac{2M'_n E_l - ({M'_n}^2 + m_l^2 - M_p^2)}{2(M'_n - E_l + P_l \cos\theta)},$$



What they **want**

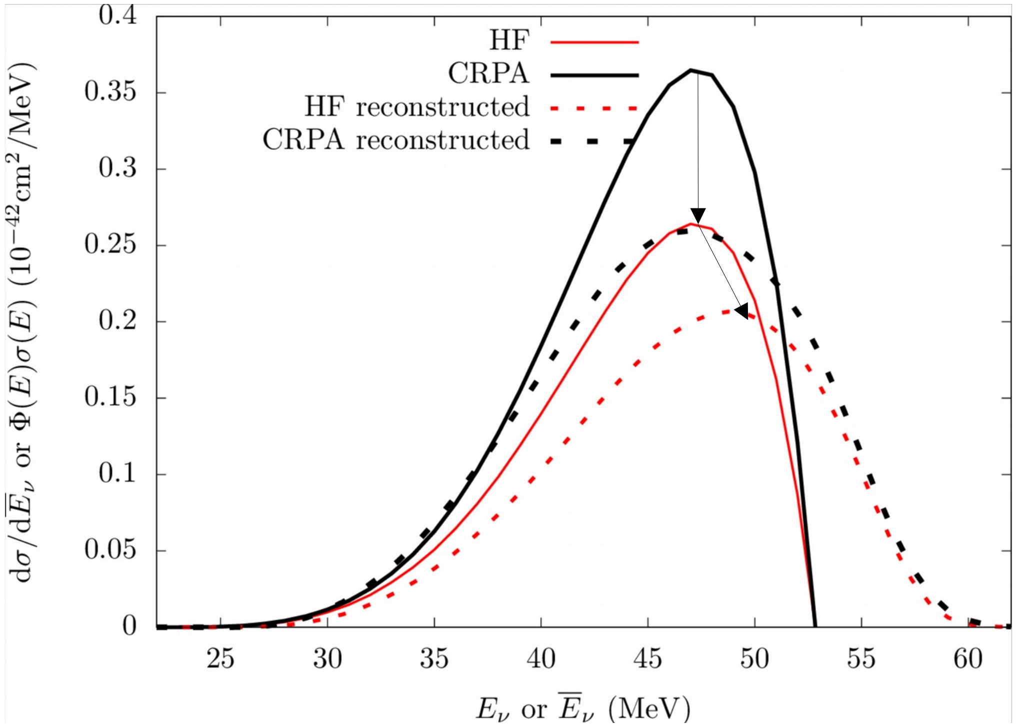
$$N(\overline{E}_\nu) = \int dE_\nu \Phi(E_\nu) d(E_\nu, \overline{E}_\nu). \qquad (3)$$

What they **have**

What theory provides

Low energy neutrino experiments

- Pion Decay At Rest (piDAR) neutrinos, monoenergetic muon neutrinos at E = 30 MeV
- Michel spectrum for electron neutrinos

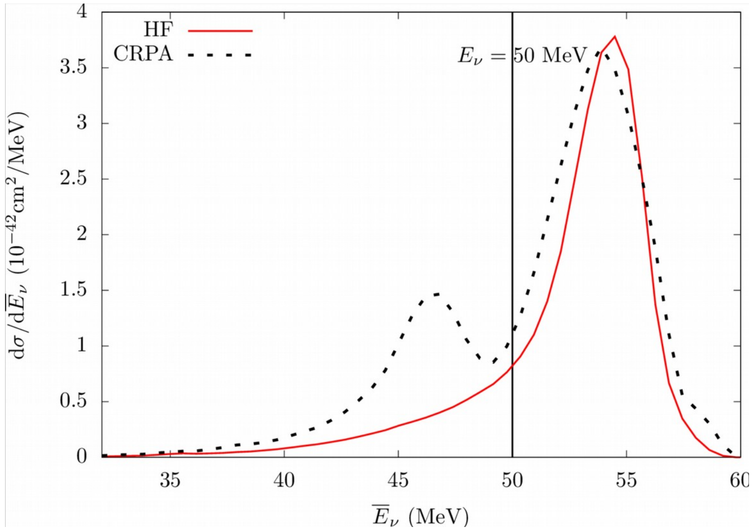


Very low energies, correlations add significant strength

Continuum is not enough, need discrete excitations

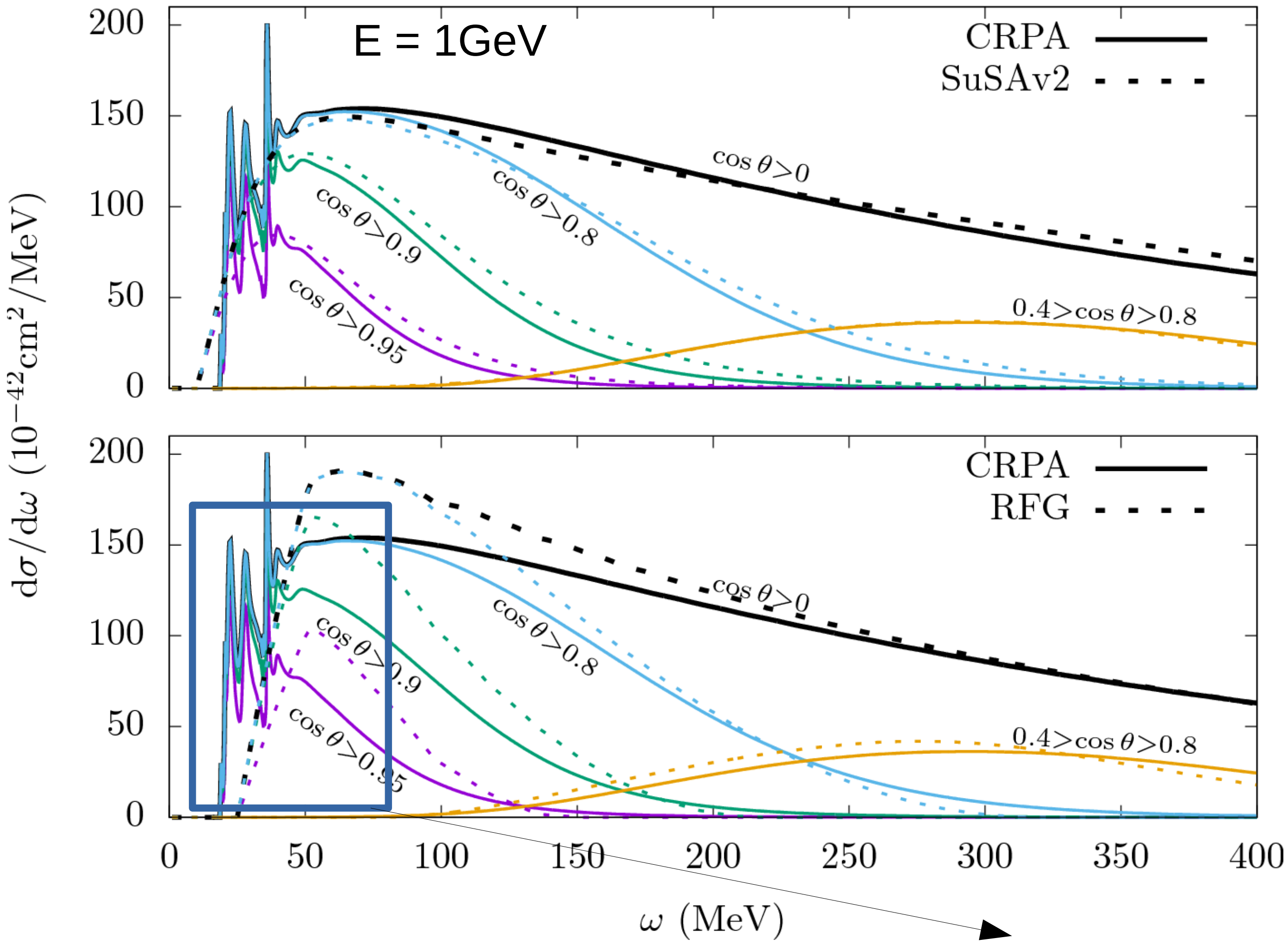
Combining lepton and hadronic information in LArTPC
To measure w-q dependence of weak response

Still an example to understand kinematic reconstruction of energies



Intermediate energy neutrino experiments

- MiniBooNE, T2K mostly dominated by QE Response, but low-ω excitations are still present

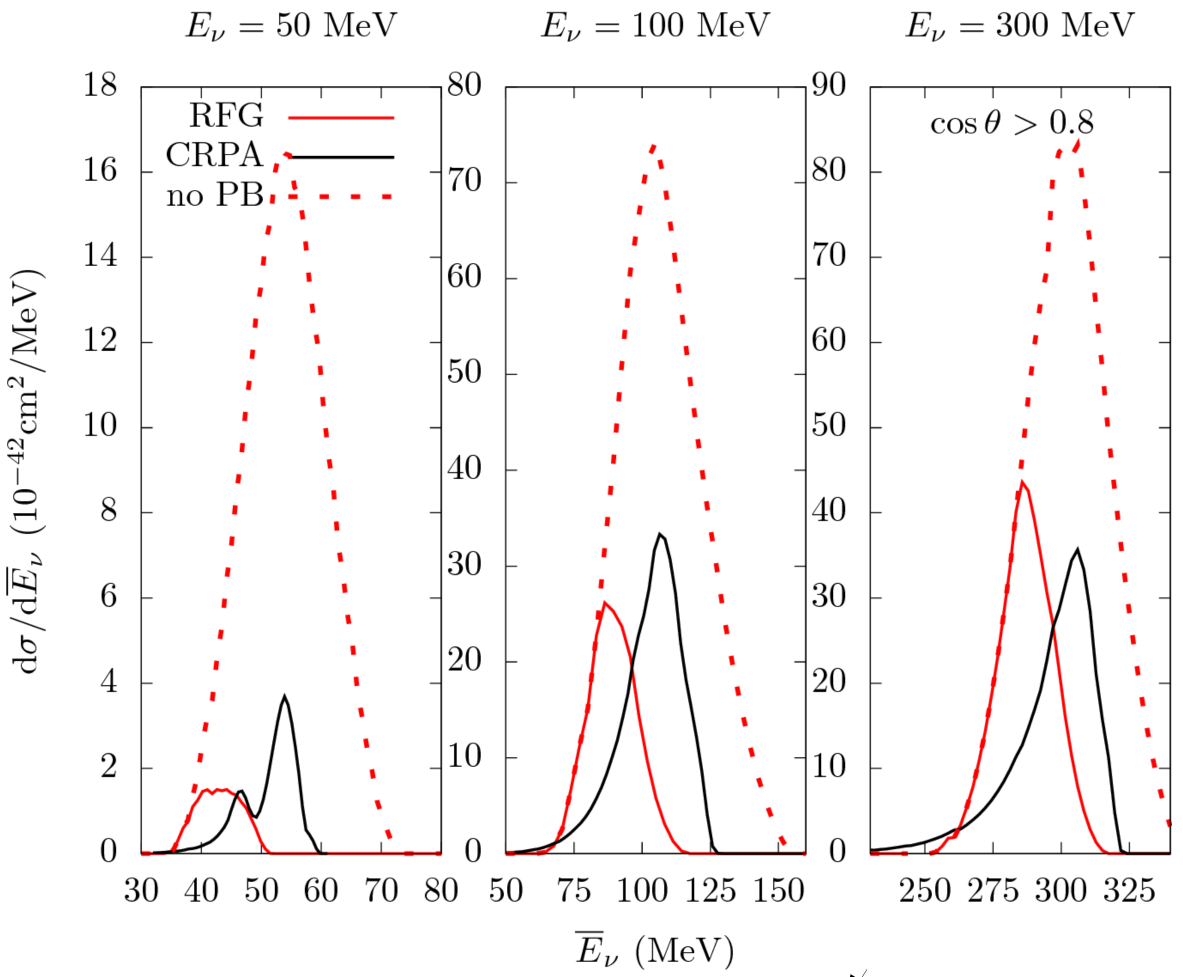


Goal: Taking into account in a satisfactory way the nuclear response for QE starting from low to intermediate ω and q.

Unsatisfactory description of low ω region

Intermediate energy neutrino experiments

- MiniBooNE, T2K mostly dominated by QE Response, but low-w excitations are still present

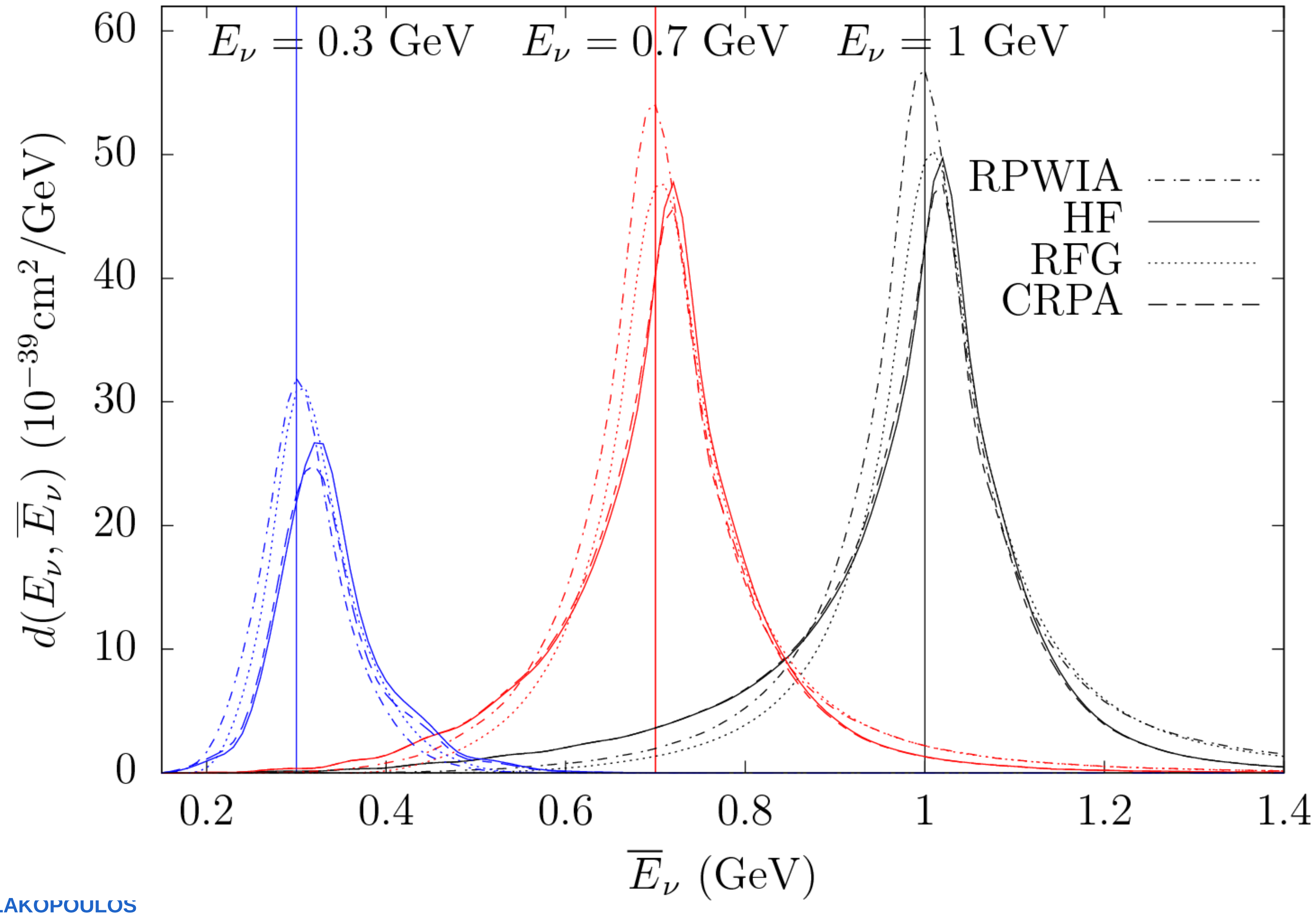


Bias of kinematic energy reconstruction at low energy and scattering angles ~E_b

Unsatisfactory description of low ω region

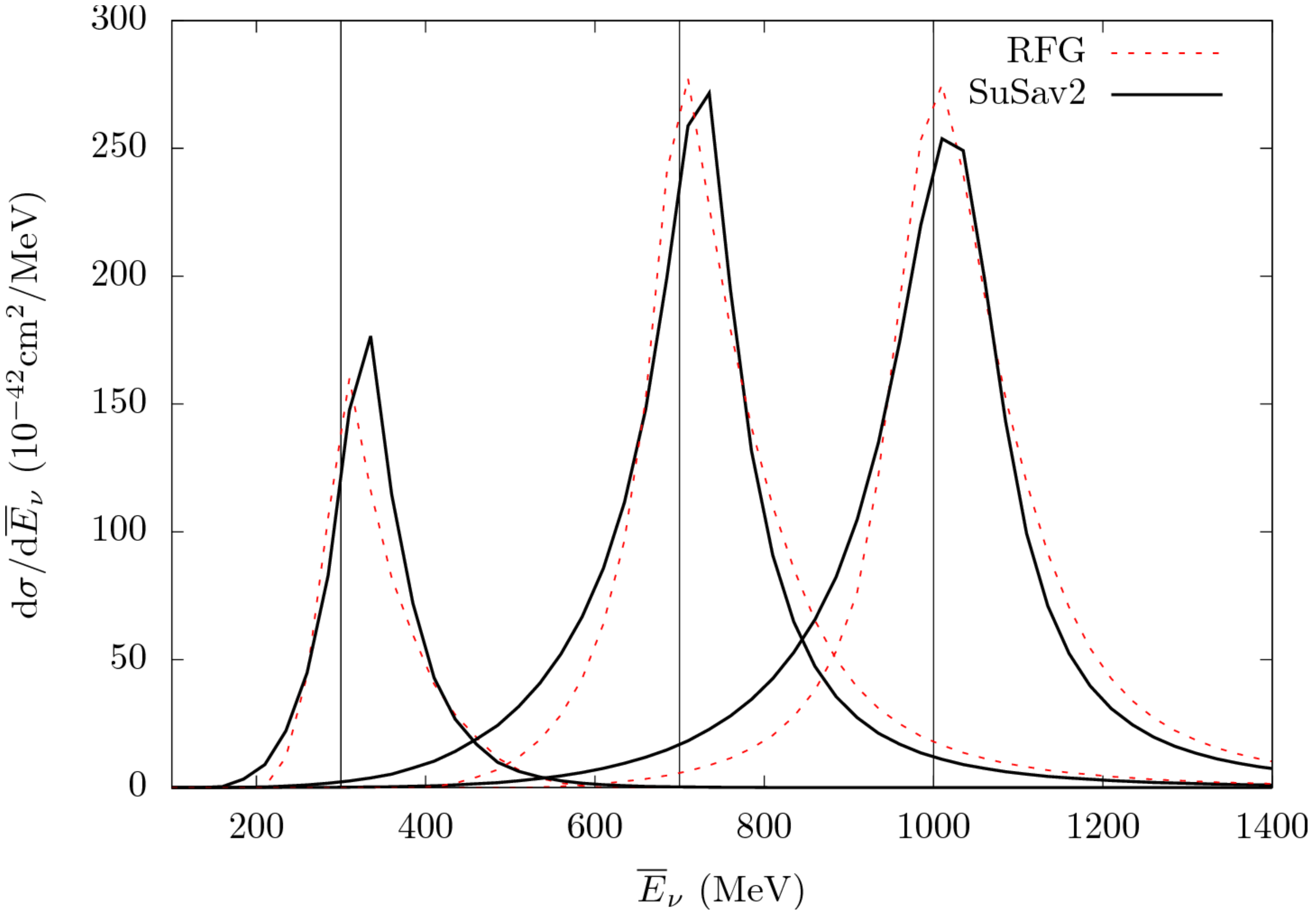
Intermediate energy neutrino experiments

Mean-field treatment gives different shapes of reconstructed energy distributions



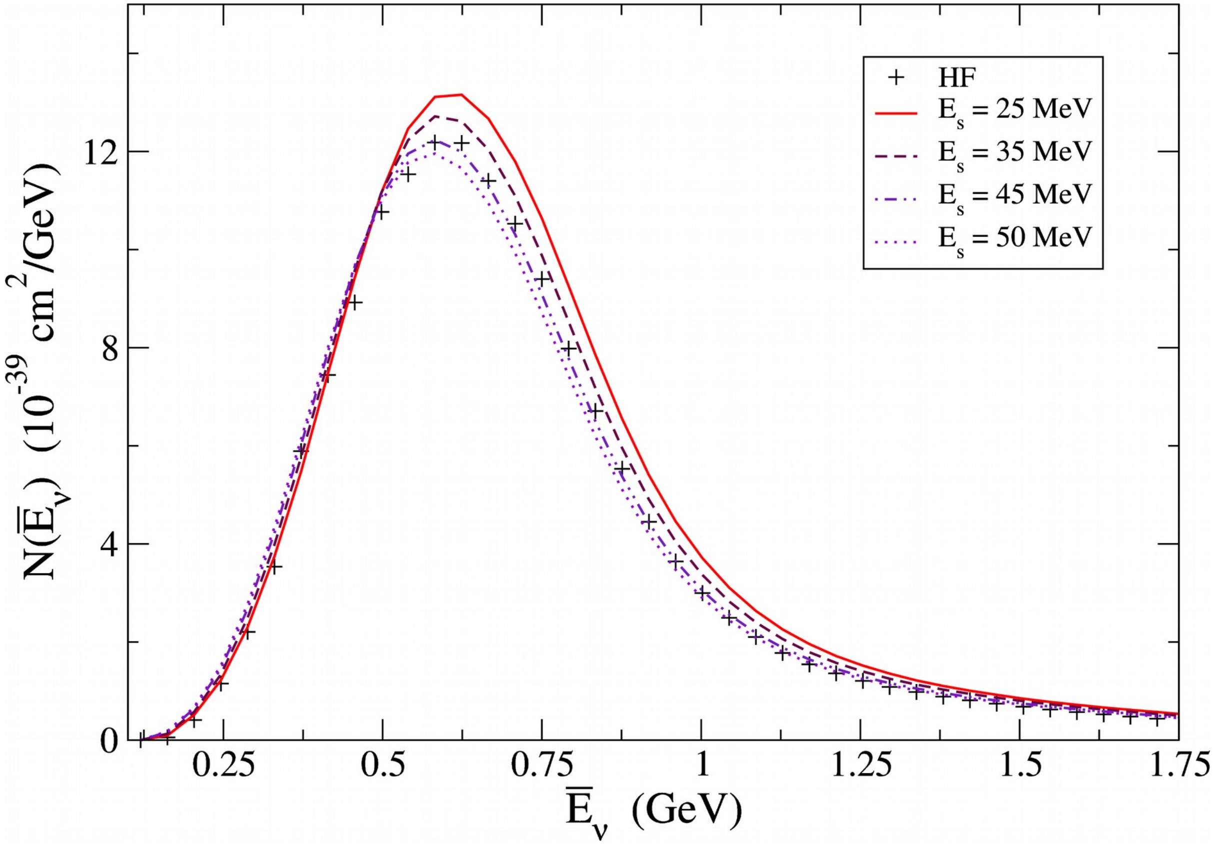
Intermediate energy neutrino experiments

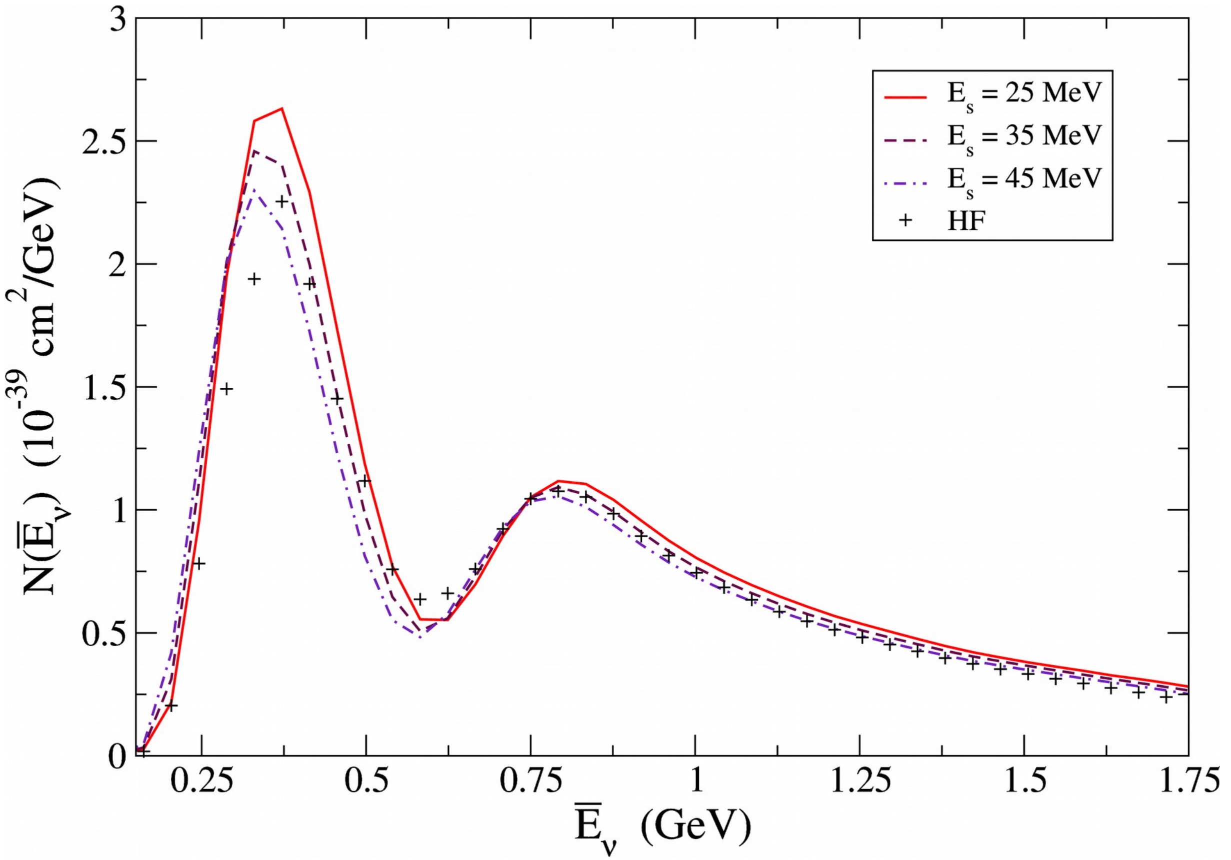
Mean-field treatment gives different shapes of reconstructed energy distributions



Intermediate energy neutrino experiments

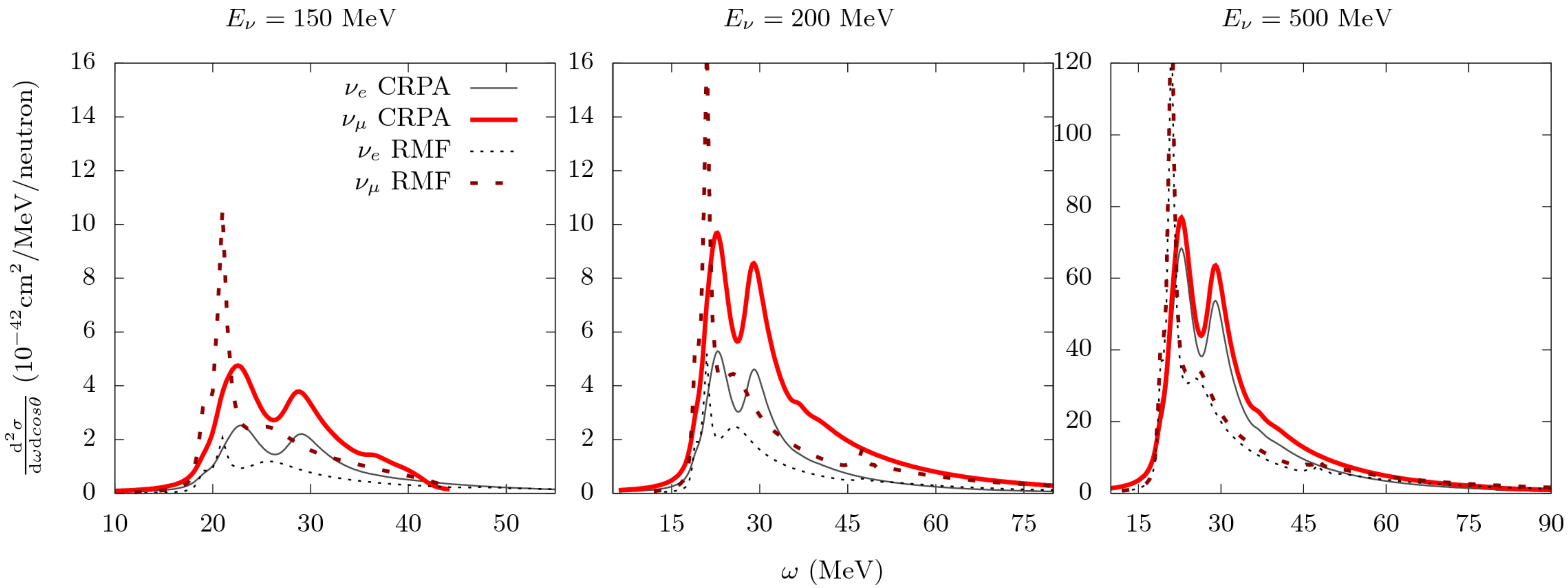
'Fit' HF result by varying the binding in RFG → RFG does not reproduce HF for different flux





Cross section differences between ν_μ and ν_e

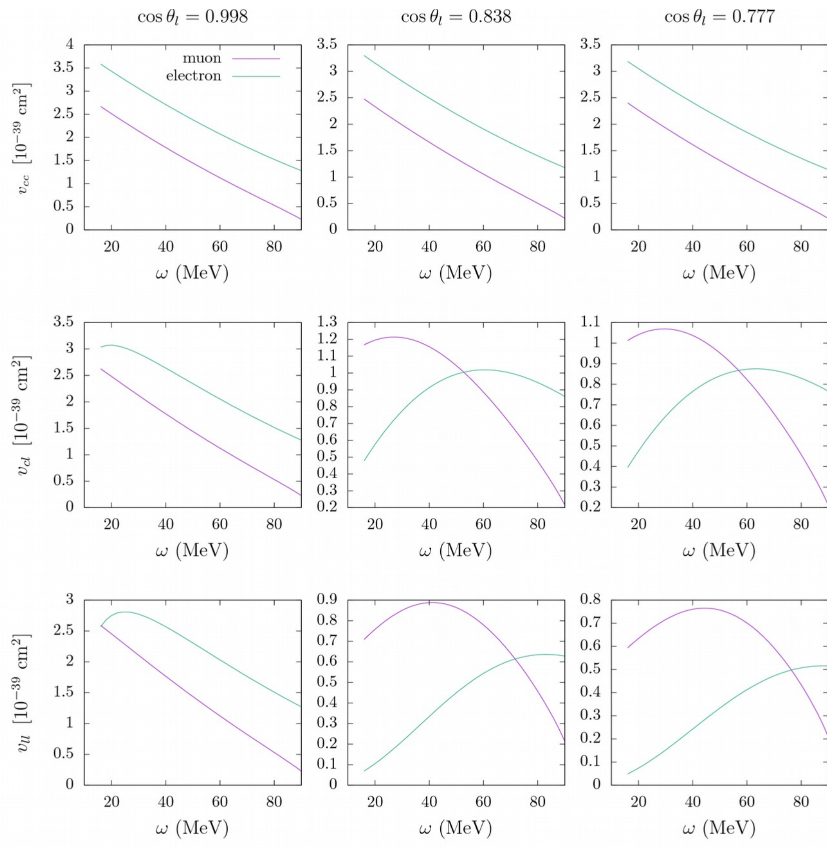
- Mean field models give larger ν_μ than ν_e cross sections for low ω and q

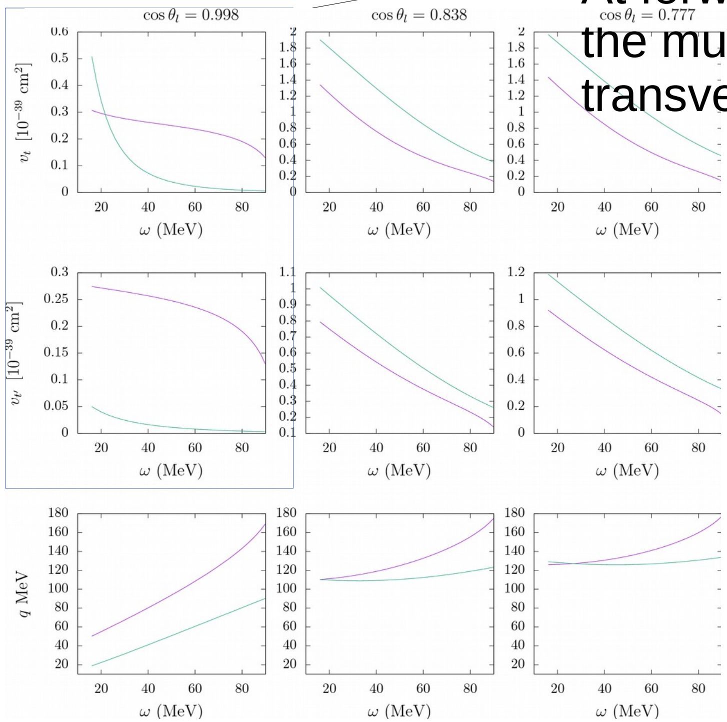


But why ? (Spoiler: orthogonality of initial and final state)

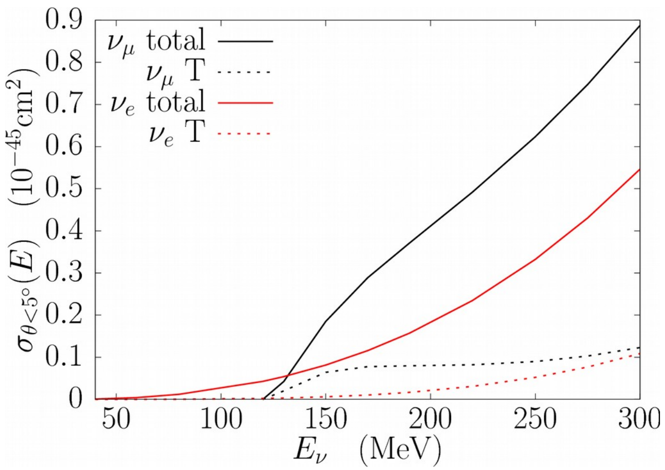
- Mean field models give larger ν_μ than ν_e cross sections for low ω and q

Leptonic prefactors: generally favor larger cross section for electron neutrinos





At forward angles (close to threshold) the muon mass terms gives transverse contributions

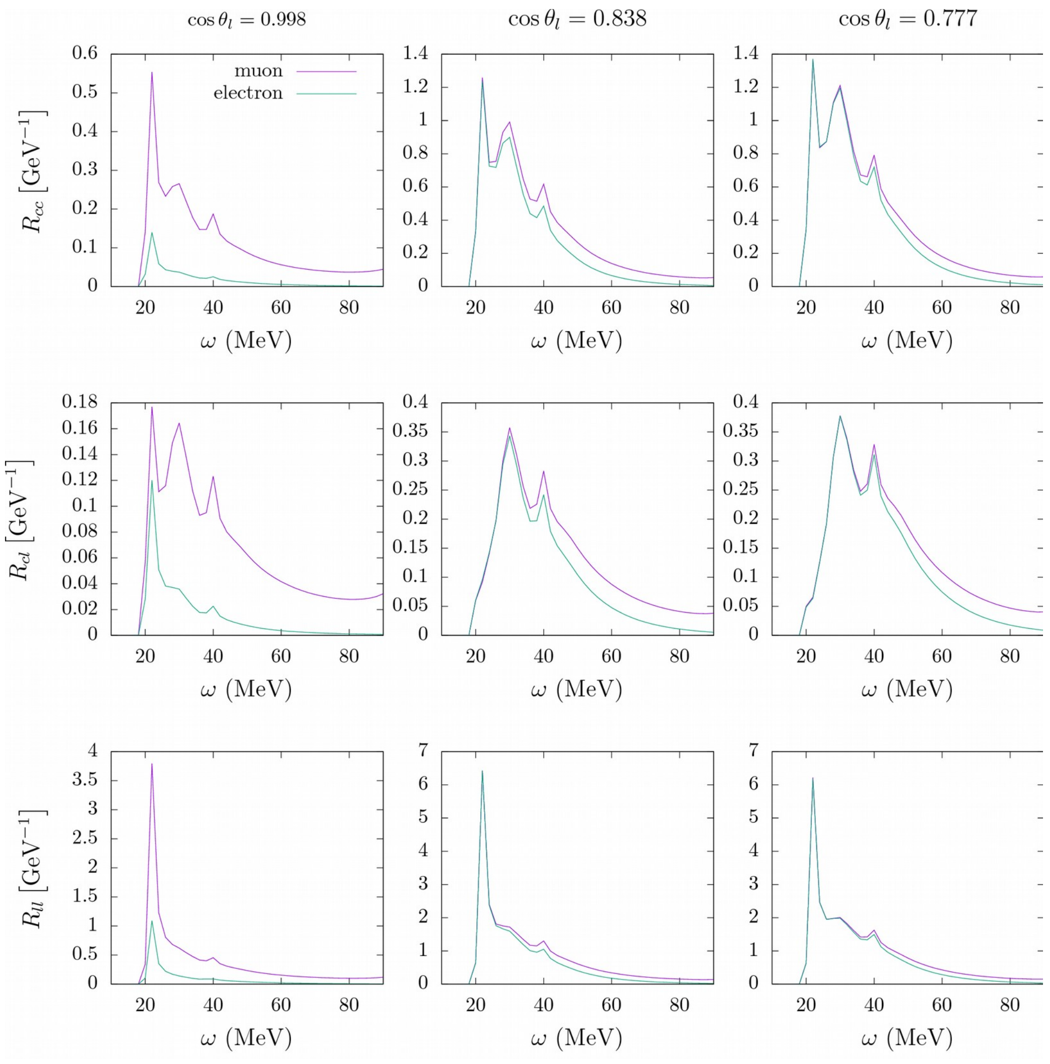


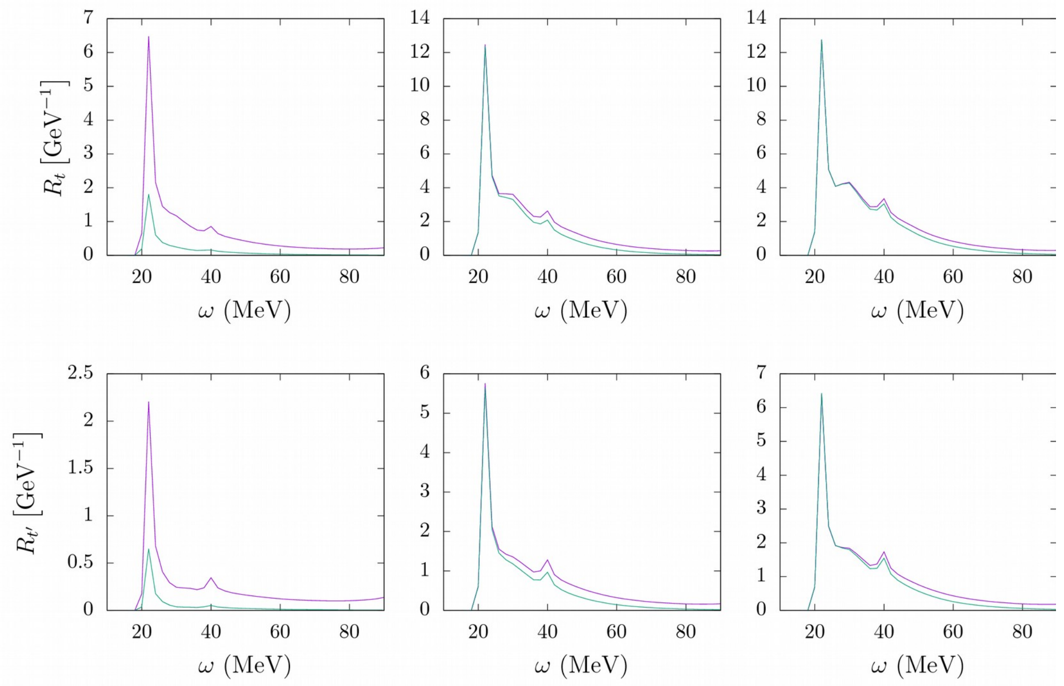
$$\omega_T W_T = \left\{ \frac{\epsilon_i k_f \sin^2\theta}{2|\vec{k}\,|^2}\cos(2\phi_F)\left(|J_{||}|^2 - |J_\perp|^2\right) + \left[\frac{\epsilon_i k_f \sin^2\theta}{2|\vec{k}\,|^2} - \frac{1}{2}\left(\frac{-\epsilon_f}{k_f}+\cos\theta\right)\right]\left(|J_{||}|^2+|J_\perp|^2\right)\right\} \qquad \text{(A31)}$$

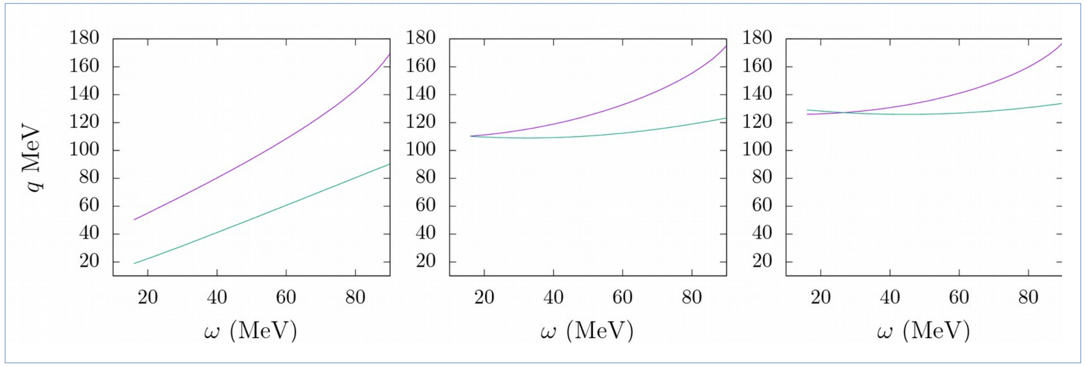
$$\omega_{TT'} W_{TT'} = -\frac{1}{|\vec{k}\,|}\left(\frac{\epsilon_i\epsilon_f}{k_f} + k_f - (\epsilon_i+\epsilon_f)\cos\theta\right) Im\left(J_{||}J_\perp^*\right) \qquad \text{(A35)}$$

- Mean field models give larger ν_μ than ν_e cross sections for low ω and q

<u>Responses:</u> favor larger cross section for muon neutrinos





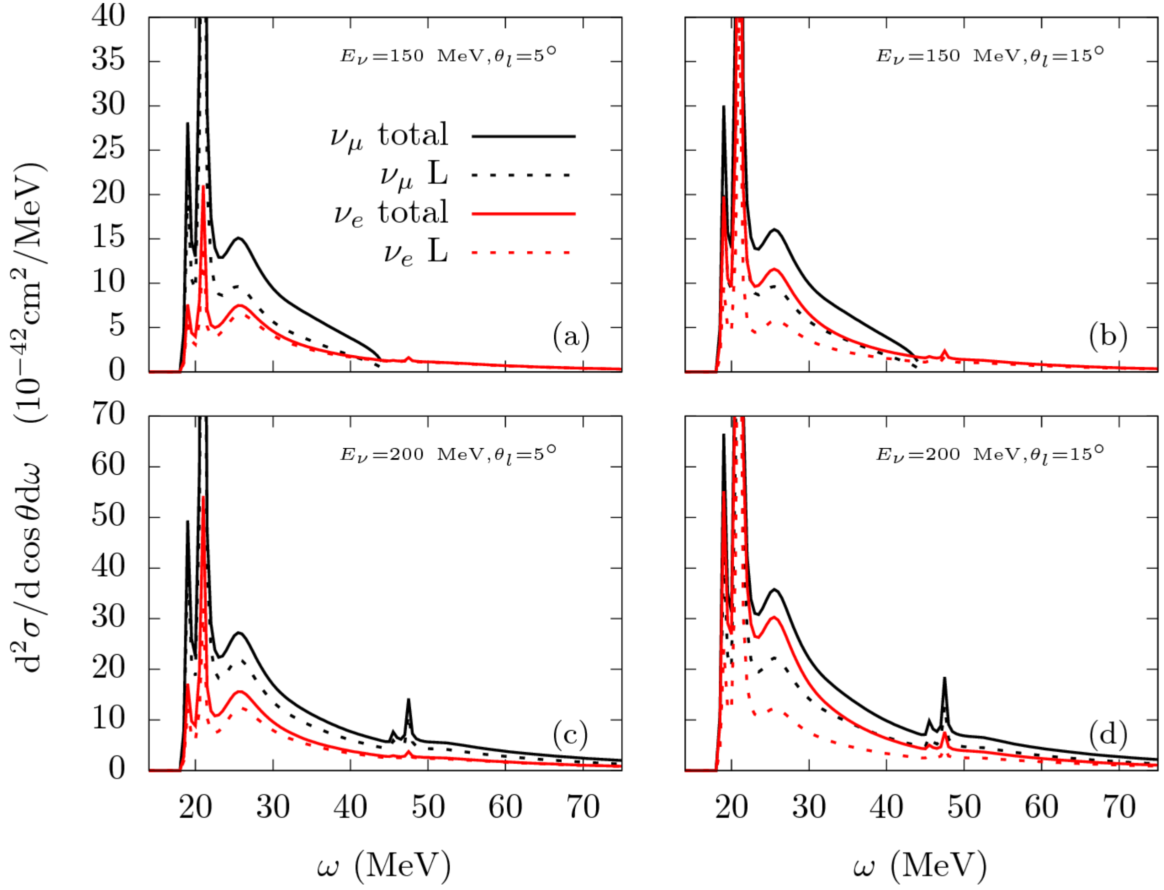


Difference in momentum transfers shifts response to higher values

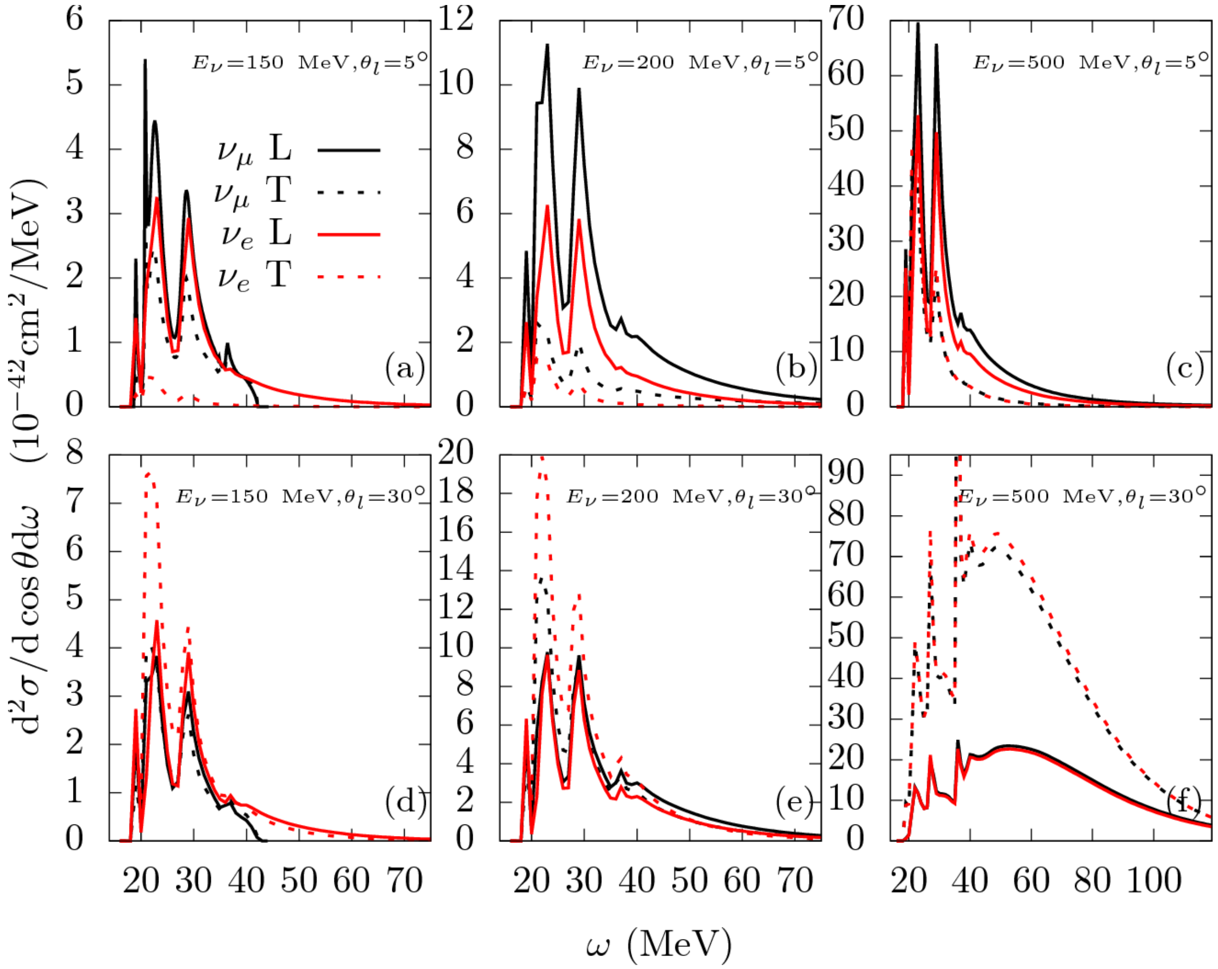
- Mean field models give larger ν_μ than ν_e cross sections for low ω and q

Responses*Lepton factors: favor larger cross section for muon neutrinos

RMF

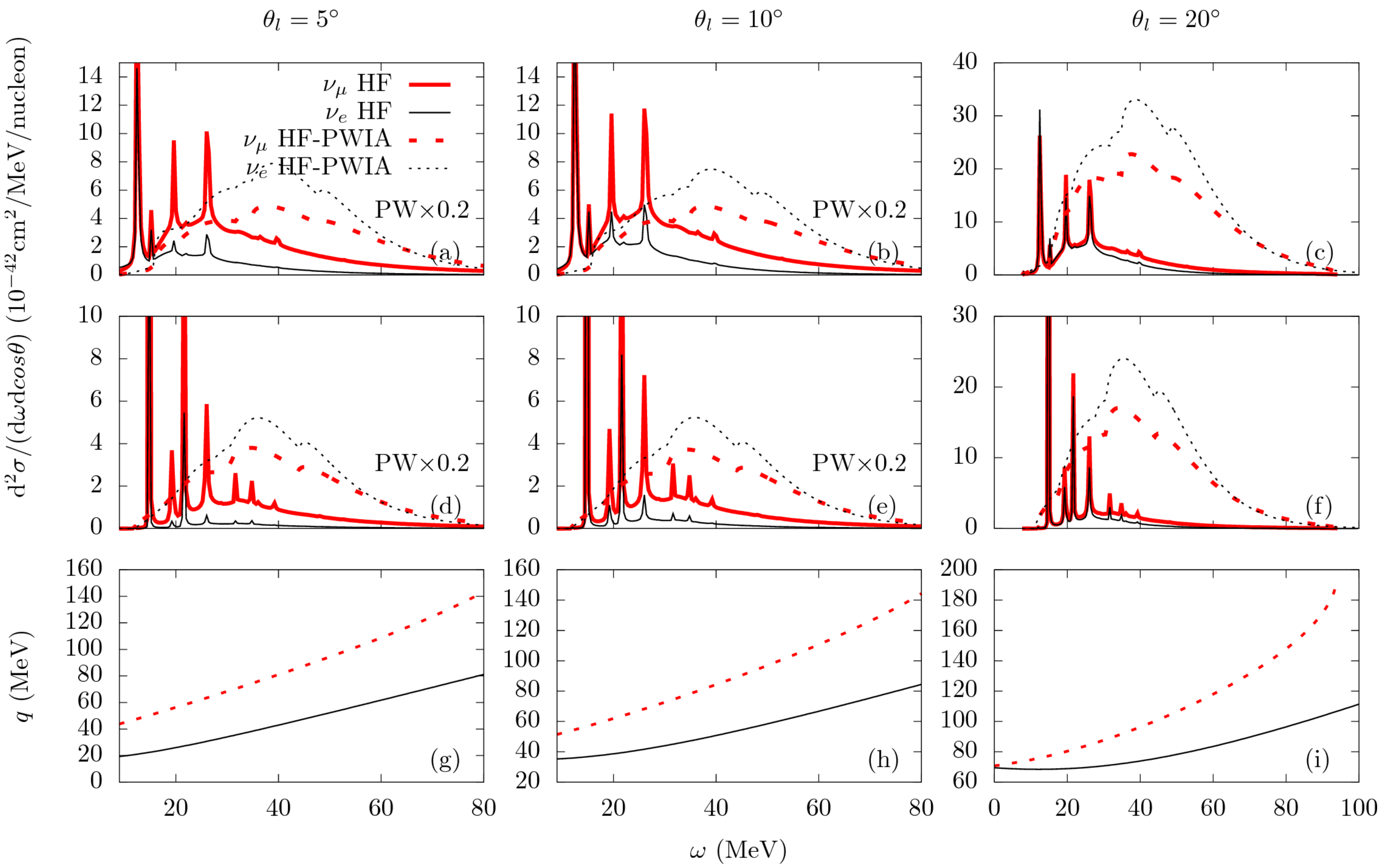


CRPA



- Mean field models give larger ν_μ than ν_e cross sections for low ω and q

Responses*Lepton factors: favor larger cross section for muon neutrinos
But not in the (R)PWIA



Large reduction at low ω, due to orthogonality, Pauli-blocking

Effects of orthogonality

Responses*Lepton factors: favor larger cross section for muon neutrinos
But not in the (R)PWIA

Orthogonality → **Pauli blocked RPWIA (PB-RPWIA)** (arXiv:1904.10696, R. Gonzalez-Jimenez)

$$|\Psi^{s_N}(\mathbf{p}_N)\rangle = |\psi^{s_N}_{pw}(\mathbf{p}_N)\rangle - \sum_{\kappa,m_j} [C^{m_j,s_N}_\kappa(\mathbf{p}_N)]^\dagger \, |\psi^{m_j}_\kappa\rangle$$

$$C^{m_j,s_N}_\kappa(\mathbf{p}_N) \equiv \langle\psi^{s_N}_{pw}(\mathbf{p}_N)|\psi^{m_j}_\kappa\rangle \, .$$

$$C^{m_j,s_N}_\kappa(\mathbf{p}_N) = (2\pi)^{3/2}\sqrt{\frac{M}{VE_N}} \times u(\mathbf{p}_N, s_N)^\dagger \psi^{m_j}_\kappa(\mathbf{p}_N)\, .$$

→

$$C^{m_j,s_N}_\kappa(\mathbf{p}_N) = \frac{1}{\sqrt{V}}\eta_\kappa(p_N)\left[\chi^\dagger_{s_N} \;\varphi^{m_j}_\kappa(\Omega_{\mathbf{p}_N})\right]$$

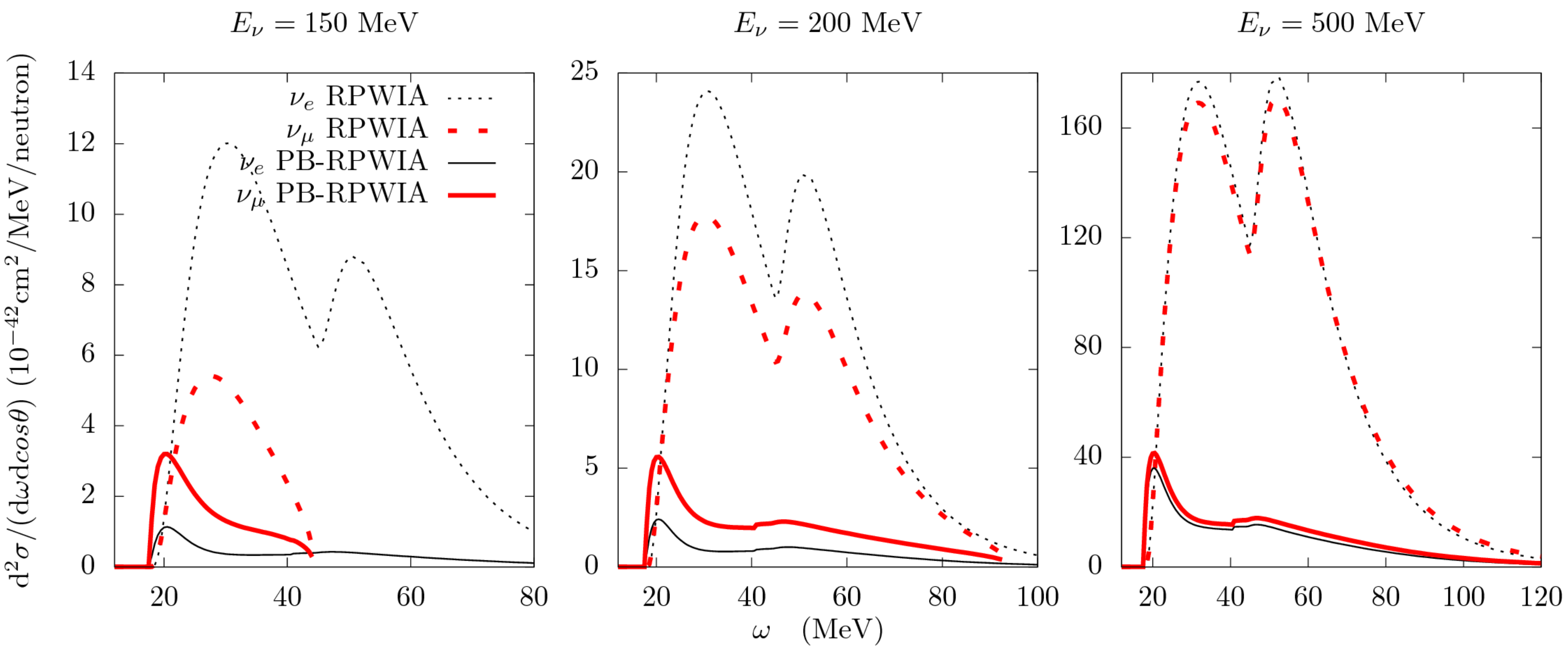
With:

$$\eta_\kappa(p_N) = (2\pi)^{3/2}\sqrt{\frac{M}{E_N}}(-i)^\ell \times \left(g_\kappa(p_N) + S_\kappa f_\kappa(p_N)\frac{p_N}{E_N+M}\right) .$$

Effects of orthogonality

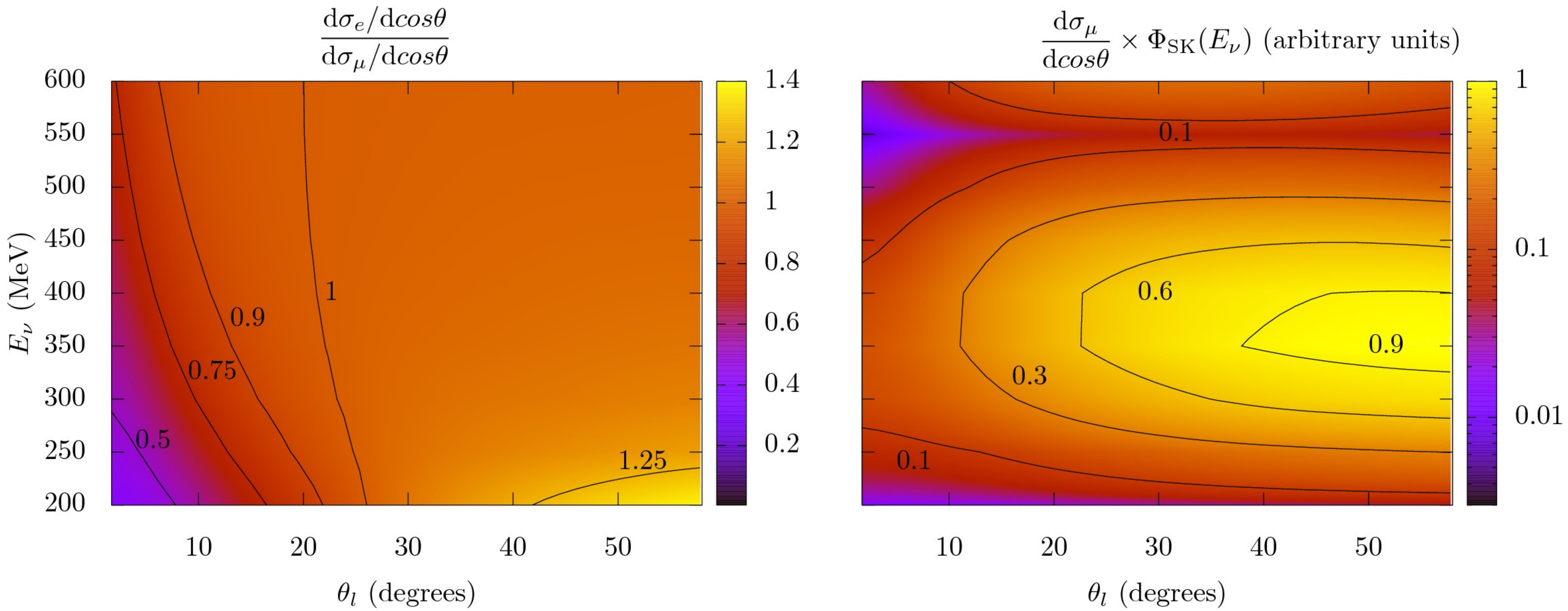
<u>Responses*Lepton factors:</u> favor larger cross section for muon neutrinos
But not in the (R)PWIA

Orthogonality ↔ Pauli-blocking → **Pauli blocked RPWIA (PB-RPWIA)**



Including only the effect of orthogonality gives the same cross section ratio, and good shape/magnitude compared to the full calculation

Effects of orthogonality



Summary

- Classical mean field with FSI provides an adequate description of the QE region from low to intermediate energy and momentum transfer
- Mean field distortion leads to shape differences in reconstructed energy distributions
- The ratio of electron to muon neutrino cross sections at low energy and momentum transfer is reproduced by models that have orthogonality of initial and final state