

Neutrino Induced Pion Production Reaction

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- Motivation
- Model of neutrino induced pion production reaction
- Single pion production
- Axial vector current
- Summary

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N. Suzuki, A. Matsuyama, B. Julia-Diaz

JSPS:16K05354,19H05104,25105010

Motivation

- Neutrino Physics:
current and future long base line neutrino experiments, atmospheric neutrino observation

$$\langle \sigma_{\nu A} \rangle = \int dE_{\nu} \sigma_{\nu A}(E_{\nu}) \phi_{\nu}(E_{\nu})$$

T2K	$E_{\nu} \sim$	$0.6 \pm 0.2(\text{GeV})$
Dune		2 ± 2
atmospheric(MH)		a few ~ 10

reaction mechanism in the GeV energy range: QE and **RES**, DIS

- Hadron/nuclear physics: probe for axial vector structure of nucleon/nuclei $\leftrightarrow$ beta-decay, muon/electron capture (in a different W, Q^2 region)

$$\begin{aligned} J_{em}^{\mu} &= V_3^{\mu} + V_{IS}^{\mu} \\ J_{CC}^{\mu} &= V_{1+i2}^{\mu} - A_{1+i3}^{\mu} \end{aligned}$$

Model of neutrino induced pion production reaction

Neutrino induced pion production

- Neutrino induced single pion production (Charged Current: CC)

$$\nu p \rightarrow l^- \pi^+ p : |I = 3/2, I_z = 3/2 >$$

$$\nu n \rightarrow l^- \pi^+ n : \sqrt{\frac{2}{3}} |3/2, 1/2 > + \frac{1}{\sqrt{3}} |3/2, 1/2 >$$

$$\rightarrow l^- \pi^0 p : -\frac{1}{\sqrt{3}} |3/2, 1/2 > + \sqrt{\frac{2}{3}} |3/2, 1/2 >$$

$$\bar{\nu} n \rightarrow l^+ \pi^- n$$

$$\bar{\nu} p \rightarrow l^+ \pi^- p$$

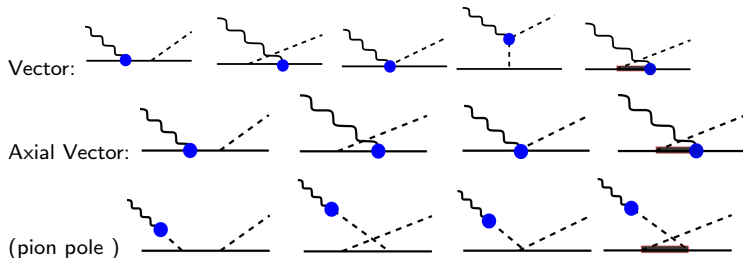
$$\rightarrow l^+ \pi^0 n$$

- Inclusive cross section (massless lepton)

$$\frac{d\sigma}{d\Omega_l dE_l} = \frac{(G_F V_{ud})^2 p_l^2}{2\pi^2} [W_2 \cos^2(\theta/2) + (2W_1 \pm W_3 \frac{E_\nu + E_l}{M}) \sin^2(\theta/2)]$$

Neutrino induced pion production in $\Delta(1232)$ region

- Non-resonant mechanism



- s-channel resonance excitation



Neutrino induced pion production in $\Delta(1232)$ region

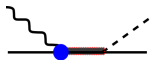
- Contact interaction



$$V^\mu \sim \frac{g_A}{2f_\pi} \gamma^\mu \gamma_5 \boldsymbol{\tau} \times \boldsymbol{\pi} \sim \boldsymbol{\sigma} (\boldsymbol{\tau} \times \boldsymbol{\pi})_{1+i2} \quad E_{0+}$$

$$A^\mu \sim \frac{1}{2f_\pi} \gamma^\mu \boldsymbol{\tau} \times \boldsymbol{\pi} \sim 1 (\boldsymbol{\tau} \times \boldsymbol{\pi})_{1+i2} \delta_{\mu,0} \quad S_{0+}$$

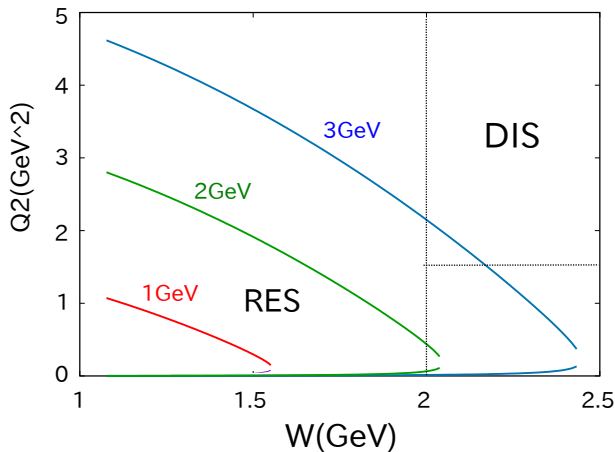
- $\Delta(1232)$ excitation



$$V^\mu \sim \frac{G_M^{N\Delta}}{2M} i \mathbf{S} \times \mathbf{q} T_{1+i2} \quad \text{M1}$$

$$A^\mu \sim G_A^{N\Delta} \mathbf{S} T_{1+i2} \quad \text{GT}$$

GeV neutrino and reaction mechanism

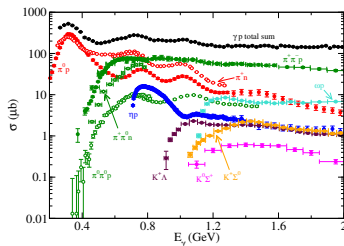
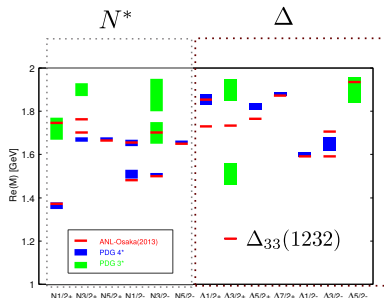


$$W = \sqrt{(p + q)^2}, Q^2 = -q^2 = -(p_\nu - p_l)^2$$

Feature of meson production reactions

$$m_N + m_\pi < W < 2\text{GeV}$$

- meson production through N^* and Δ resonances $M_R < 2\text{GeV}$ and non-resonant mechanism.



- opening of ηN , $\pi\pi N$, $K\Lambda$, $K\Sigma$, , , channels
 $\rightarrow$ multi-channel unitarity including three-body($\pi\pi N$).

Neutrino induced meson production

- Unitarity:

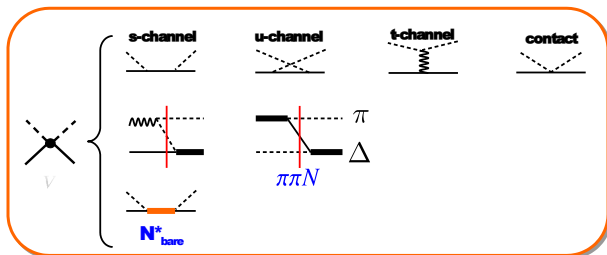
$$\text{Im}(\mathcal{F}_{\pi N, \gamma^* N}) = \sum_{n=\pi N, \pi\pi N, \eta N, \dots} (\mathcal{F}_{n, \pi N})^* \mathcal{F}_{n, \gamma^* N}$$

- HNV : Below 2π , Watson Theorem: $M^\alpha = |M^\alpha| e^{i\delta_{\pi N}^\alpha}$, $\alpha = JLI$
- SL, ANL-Osaka: solve Lippmann-Schwinger equation.
- Q^2 dependence of coupling constants. $G_V^{N, \Delta}(Q^2), G_A^{N, \Delta}(Q^2)$
analysis of data on electron and neutrino induced pion production reactions

Dynamical Coupled Channel model (ANL-Osaka DCC model)

Model developed for N^* physics: spectrum of nucleon excited states, transition form factors (ANL-Osaka Coupled channel model)

- include N^* , Δ resonances
- coupled channel model $\pi N, \eta N, \pi\pi N(\sigma N, \rho N, \pi\Delta), K\Lambda, K\Sigma$
- Satisfy three body ($\pi\pi N$) unitarity



$$\rightarrow T = V + VG_0T$$

- The model is constructed by fitting available data on pion, photon, electron induced meson production reaction

$\pi N \rightarrow MB$:strong interaction dynamics(resonance, non-resonance)

$\gamma^* p, \gamma^* n \rightarrow MB$:iso-vector, iso-scalar vector current

- the model is extended for axial vector current.

vector current vertex $\rightarrow$ axial vector current assuming PCAC and dipole $N - N^* \Delta$ transition form factors.

H. Kamano, S.X. Nakamura, L. TS, PRC88, 035209 (2013),

H. Kamano, S.X. Nakamura, T.-S. H. Lee, TS, PRC94, 015291 (2016)

S. X. Nakamura, H. Kamano, TS, PRD92, 07402 (2015)

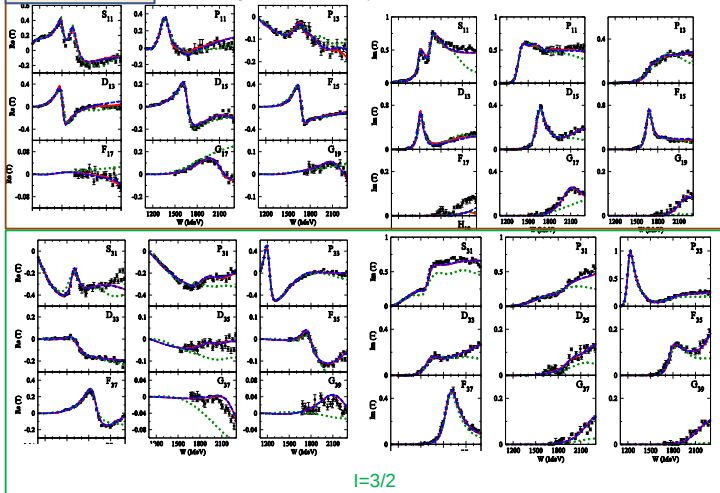
$\rightarrow$ meson production of CC and NC currents

πN amplitude

Re

$I=1/2$

Im



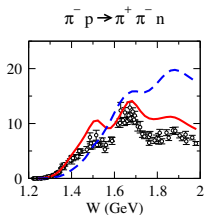
$I=3/2$

$\pi^- N \rightarrow \pi^- \pi^- N$ reaction

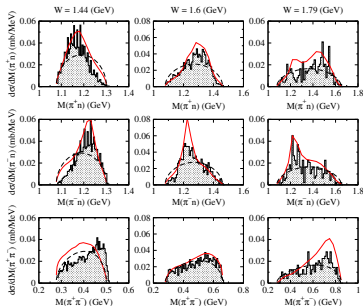
Kamano, Julia-Diaz, Lee, Matsuyama, Sato, PRC79 025206 (2009)

Parameters used in the calculation are from $\pi N \rightarrow \pi N$ analysis.

$$\pi^- p \rightarrow \pi^+ \pi^- n$$



— Full result
- - - C.C. effect off

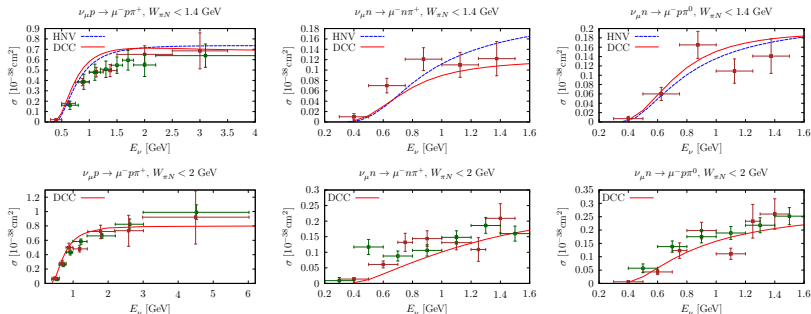


— Full result - - - Phase space
Data handled with the help of R. Arndt



Single pion production

Single pion production in $\Delta(1232)$ region



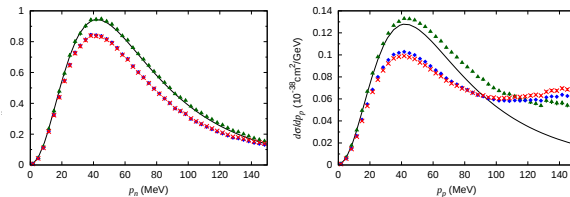
J. Sobczyk, E. Hernandez, S.X. Nakamura, J. Nieves, T. Sato PRD98(2018)073001

- Re-analyzed ANL/BNL data, C. Wilkinson et al. PRD90
- ANL-Osaka DCC, PRD92, Hernandez, Nieves, Valverde PRD76

Caution on $\sigma(\nu N)$ of ANL/BNL data extracted from $\sigma(\nu d)$.

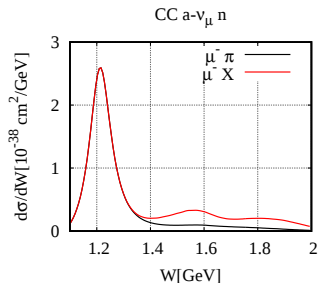
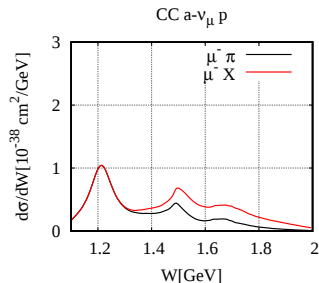
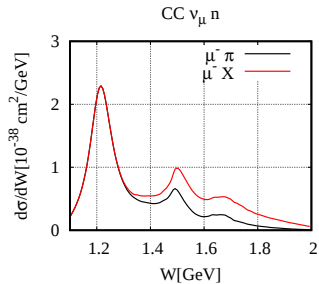
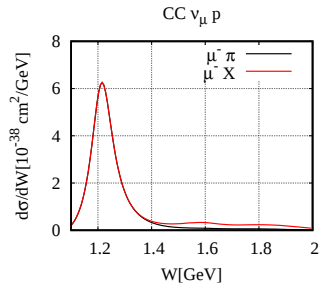
About 10 ~ 30% correction due to FSI effects should be corrected S. Nakamura, H. Kamano, T. Sato PRD99,031301(R)(2019)

Momentum distribution of spectator nucleon ($\nu_\mu d \rightarrow \mu^- \pi^+ pn$)



IA, IA+NN-resc, IA+NN-resc+ πN -resc

$$d\sigma/dW \ E_\nu = 2\text{GeV}$$

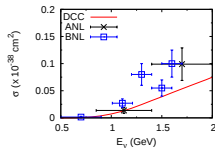
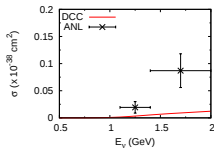
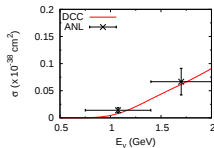


Neutrino induced two pion production

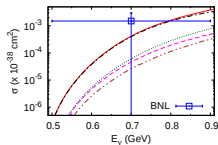
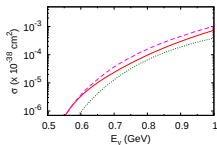
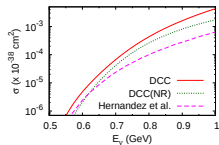
$$\nu_{\mu} p \rightarrow \mu^{-} \pi^{+} \pi^{-} p$$

$$\nu_{\mu} p \rightarrow \mu^{-} \pi^{+} \pi^{+} n$$

$$\nu_{\mu} n \rightarrow \mu^{-} \pi^{+} \pi^{-} n$$



Near threshold (compare with Hernandez et al.(D(1232)+N(1440))



Brief summary: Models of neutrino induced pion production

Summary of models for neutrino reaction in RES

	Res	Non-res	Unit.	1pi	2pi	Tot
RS	Delta, N*	-	X	O		O
LPP	Delta, N*	X	X	O		O
HVM	Delta(1232)	chiral	O	O		
	Delta(1232)+N(1440)	chiral	X	O	O	
Giessen	Delta, N*	phen.	X	O		O
ANL-Osaka	Delta, N*	O	O	O	O	O

RS: D. Rein, L. M. Sehgal AP133(81), LPP: O. Lalakulich, E.A. Paschos, G. Piranishvili, PRD74(2006)

HNW: E. Hernandez, J. Nieves, M. Valverde PRD76(2007) Giessen: T. Leitner, O. Buss, L. Alvarez-Ruso, U. Mosel, PRC79(2009)

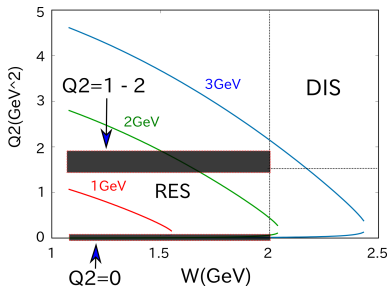
ANL-Osaka DCC: S.X. Nakamura, H. Kamano, TS, PRD92(2015), TS, D. Uno, T.-S.H. Lee PRC67(2003)

R. Gonzales-Jimenes et al. PRD95,113007(2017)+Regge

Axial Vector Current

How good DCC model, Axial vector current?

- $Q^2 = 0$: F_2^{CC} and pi-N cross section.
- $Q^2 \sim 1 - 2 \text{ GeV}^2$: Is DCC model smoothly interpolates to DIS ?



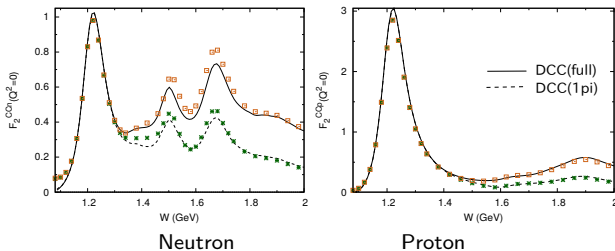
F_2^{CC} and π -N cross section

Axial Vector current F_2^{CC} (total cross section) at $Q^2 = 0$

DCC model : 1π dash, a Total solid

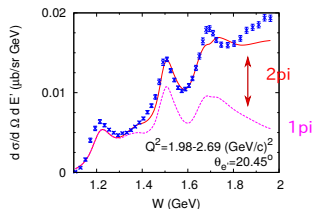
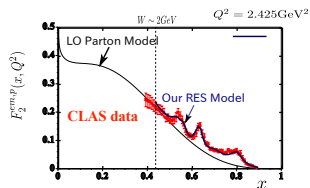
πN cross section data: 1π green, a Total brown

$$F_2^{CC}(Q^2 = 0) = \frac{2f_\pi^2}{\pi} \sigma(\pi + N)$$



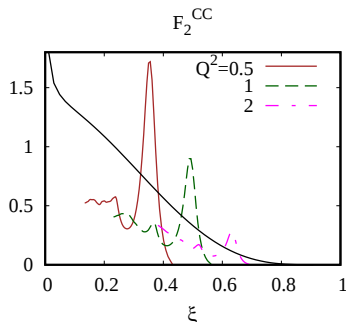
- Description of axial vector current at $Q^2 = 0$ is consistent with pion scattering data.

Is DCC model smoothly interpolates to DIS ?



- EM for proton

Is DCC model smoothly interpolates to DIS ?



$$F_2^\alpha = \frac{F_{2p}^\alpha + F_{2n}^\alpha}{2} \quad \alpha = EM, CC$$

- $m_N + m_\pi < W < 2\text{GeV}$
- around $W \sim 2\text{GeV}$, $Q^2 = 1 \sim 2\text{GeV}^2$, $F_2^{CC} \sim |V|^2$.

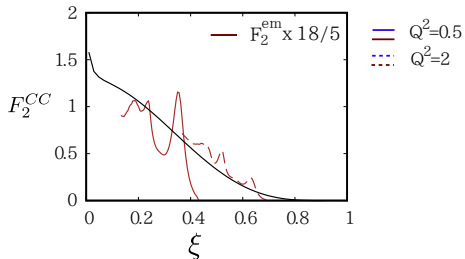
Parton Distribution vs DCC model

- Parton Picture(Isospin-symmetry, neglect s)

$$\begin{aligned}F_2^{EM} &= \frac{x}{2} \left[\left(\frac{2}{3} \right)^2 u_p + \left(\frac{1}{3} \right)^2 d_p \right] + \left[\left(\frac{2}{3} \right)^2 u_n + \left(\frac{1}{3} \right)^2 d_n \right] = x \frac{5}{18} (u + d) \\F_2^{CC} &= \frac{x}{2} (d_p + d_n) (1 + 1) = x(u + d) = \frac{18}{5} F_2^{EM}\end{aligned}$$

- Hadron picture

$$\begin{aligned}F_2^{EM} &\sim \sum_f | \langle f | V_3 + V_{IS} | N \rangle |^2 \\F_2^{CC} &\sim \sum_f [| \langle f | V_{1+i2} | N \rangle |^2 + | \langle f | A_{1+i2} | N \rangle |^2]\end{aligned}$$



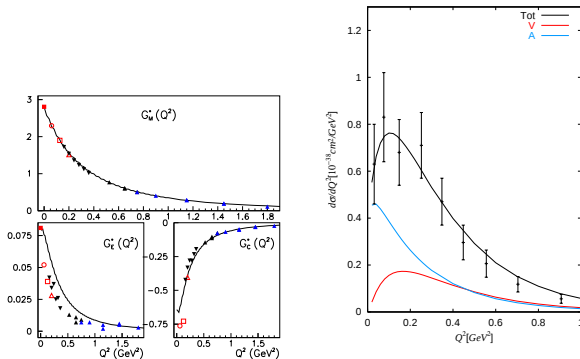
- $F_2^{em} \times 18/5$ of ANL-Osaka Model.

$$\frac{18}{5} F_2^{em}(\text{ANL-OSAKA}) \approx F_2^{CC}(\text{PDF}) = x(u + \bar{u} + d + \bar{d})$$

- contribution of A in DCC-model around large W, Q^2 might be too weak.

Possible solution: transition form factor

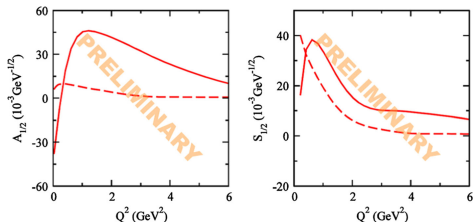
$\Delta(1232)$: Left EM, Right CC



PRC75,015205(2007)(EM), PRC67,65201 (2003)(CC)

Example of transition form factors $N^*(1/2, 1/2^+)$

- Vector(EM): Helicity amplitudes extracted from the residue of partial wave amplitude(DCC-model) at resonance pole (figure from H. Kamano)



- Axial Vector (Quark model)

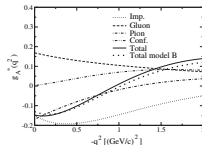
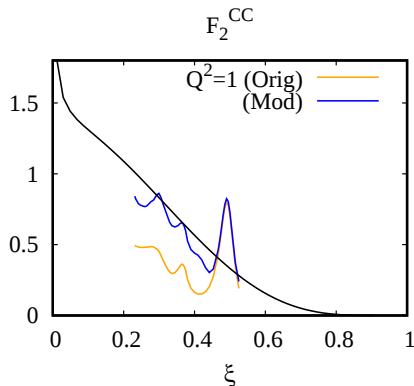


FIG. 7. Axial form factor $g_A^N(q^2)$. Notation as in the upper panel of Fig. 3.

PRC75,065203 (2007) D. Barquilla-Cano, A.J. Buchmann, E. Hernandez

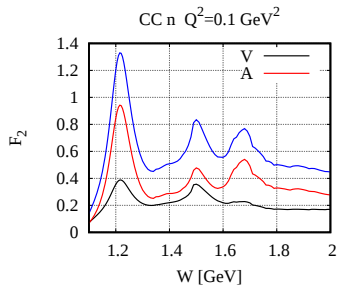
Simple Exercise of DCC model

$$A_A^\lambda(Q^2) = A_A^\lambda(0) \times \frac{A_V^\lambda(Q^2)}{A_V^\lambda(0)} \quad \text{except } P_{33}$$



- ANL-OSAKA DCC model is extended to describe weak meson production reaction up to $W < 2\text{GeV}$.
- Neutrino induced single pion production in N^* , Δ resonance region is studied using ANL-Osaka model.
Comparison with Neutrino event generators(NEUT, GENIE, NuWro,...) and other models will be very useful.
- Model of axial vector current is examined.
At $Q^2 = 0$, DCC model reproduce πN data.
Comparison with PDF at high Q^2 , suggests need for more strength at high W region.
Improving axial transition form factors will be helpful.
PV asymmetry, in principle, gives information of axial vector current.

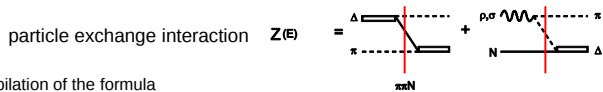
Contribution of A and V at $Q^2 = 0.1$



Scattering amplitude of pion and photon induced meson production amplitudes are given by solving coupled channel LS equation(3-dim reduction) in momentum space

$$T_{\beta,\alpha}(k', k, W) = V_{\beta,\alpha}(k', k) + \sum_{\gamma} \int dq q^2 V_{\beta,\gamma}(k', q) G_{\gamma}^0(q, W) T_{\gamma,\alpha}(q, k, W)$$

Scattering amplitudes satisfy **two-body and three-body unitarity**



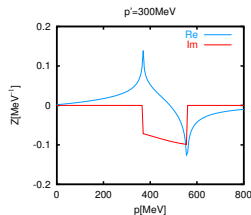
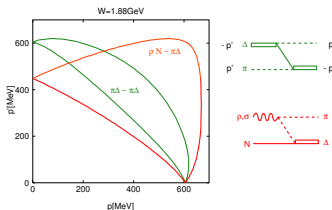
After manipulation of the formula

$$T_{\alpha,\beta}(W) = t_{\alpha,\beta}^{nr}(W) + \sum_{i,j} \bar{\Gamma}_{\alpha,i}(W) \left[\frac{1}{W - m_0 - \Sigma(W)} \right]_{ij} \bar{\Gamma}_{\beta,j}(W)$$

α, β Meson-Baryon channel i, j Resonances

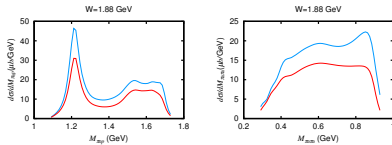
Singularity structure of $Z(E)$

$$\langle \pi\Delta : p' | Z(E) | \pi\Delta : p \rangle \quad l_{\pi\Delta} = 0, j = 3/2$$



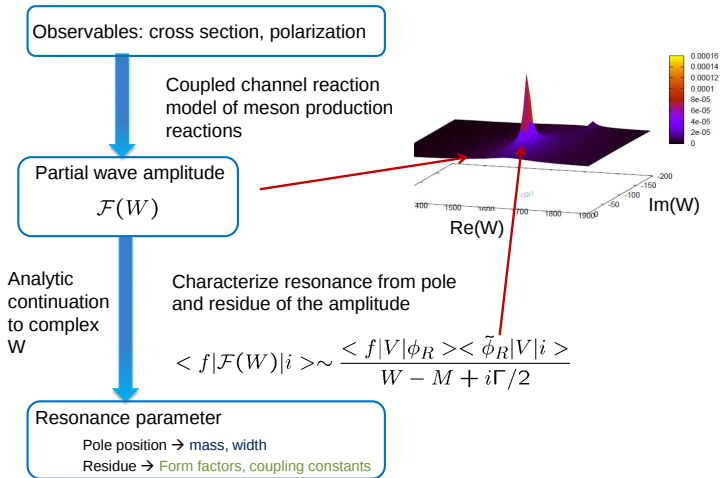
- Rapid momentum dependence of $Z \rightarrow t(\text{non-res})$
- Numerical complication: Contour rotation method is not valid for $\pi\pi N$ production $\rightarrow$ Spline-interpolation method (A. Matsuyama for πNN)

Invariant mass distribution ($\gamma p \rightarrow \pi^0 \pi^0 p$ at $W = 1.88 \text{ GeV}$)



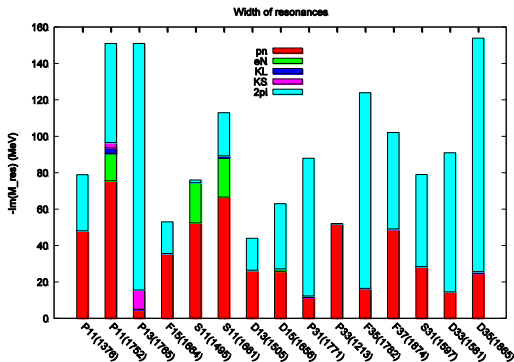
Full $Z(E) = 0$

Extraction of resonance parameters



Half width $\Gamma/2 = -\text{Im}(E_{\text{res}})$

current model



Some freedom exists on the definition of partial width from the residue of the amplitude. The numbers should be taken as a one estimation of the MB-res coupling strength .