Washington University in St. Louis

FRIB Theory Alliance

From light-nuclei to neutron stars within chiral dynamics

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May 28, 2019

The *basic model* of nuclear theory

The *basic model* of nuclear theory: description of the static and dynamic properties of nuclear systems.

- Nucleon-nucleon (NN) scattering data: "thousands" of experimental data available
- The spectra, properties, and transition of nuclei: BE, radii, magnetic moments, beta decays rates, weak/radiative captures, electroweak form factors, etc.
- The nucleonic matter equation of state: for ex. EOS neutron matter

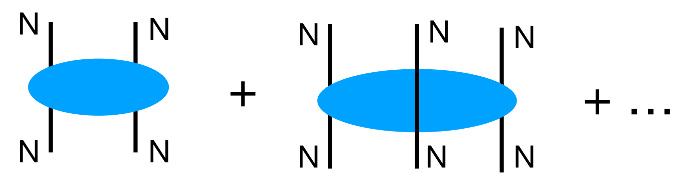
………

Inputs for the *basic model*:

Many-body interactions between the constituents

$$H = \sum_{i=1}^{A} \frac{\mathbf{p}_i^2}{2m_i} + \sum_{i<j=1}^{A} \overbrace{v_{ij}}^{\text{th+exp}} + \sum_{i<j<k=1}^{A} \overbrace{V_{ijk}}^{\text{th+exp}} + \ldots..$$

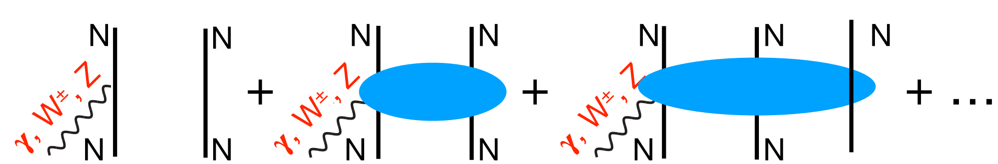
One-body Two-body (NN) Three-body (3N)



Electroweak current operators:

$$j^{\rm EW} = \sum_{i=1}^{A} j_i + \sum_{i<j=1}^{A} j_{ij} + \sum_{i<j<k=1}^{A} j_{ijk} + \ldots.$$

One-body Two-body Many-body



Chiral EFT: from QCD to nuclear systems

S. Weinberg, Phys. Lett. **B251**, 288 (1990); Nucl. Phys. **B363**, 3 (1991); Phys. Lett **B295**, 114 (1992)

QCD

Symmetries in particular the approximate chiral symmetry between hadronic d.o.f (π, N, Δ)

Approximate chiral symmetry requires the pion to couple to other pions and to baryons by powers of its momentum

Effective chiral Lagrangian $\mathcal{L}_{eff}(\pi, N, \Delta)$

$$\mathcal{L}_{eff} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \dots$$

Given a power counting scheme

$$\mathcal{L}^{(n)} \sim \left(\frac{Q}{\Lambda_\chi}\right)^n$$

~ 100 MeV soft scale

~ 1 GeV hard scale

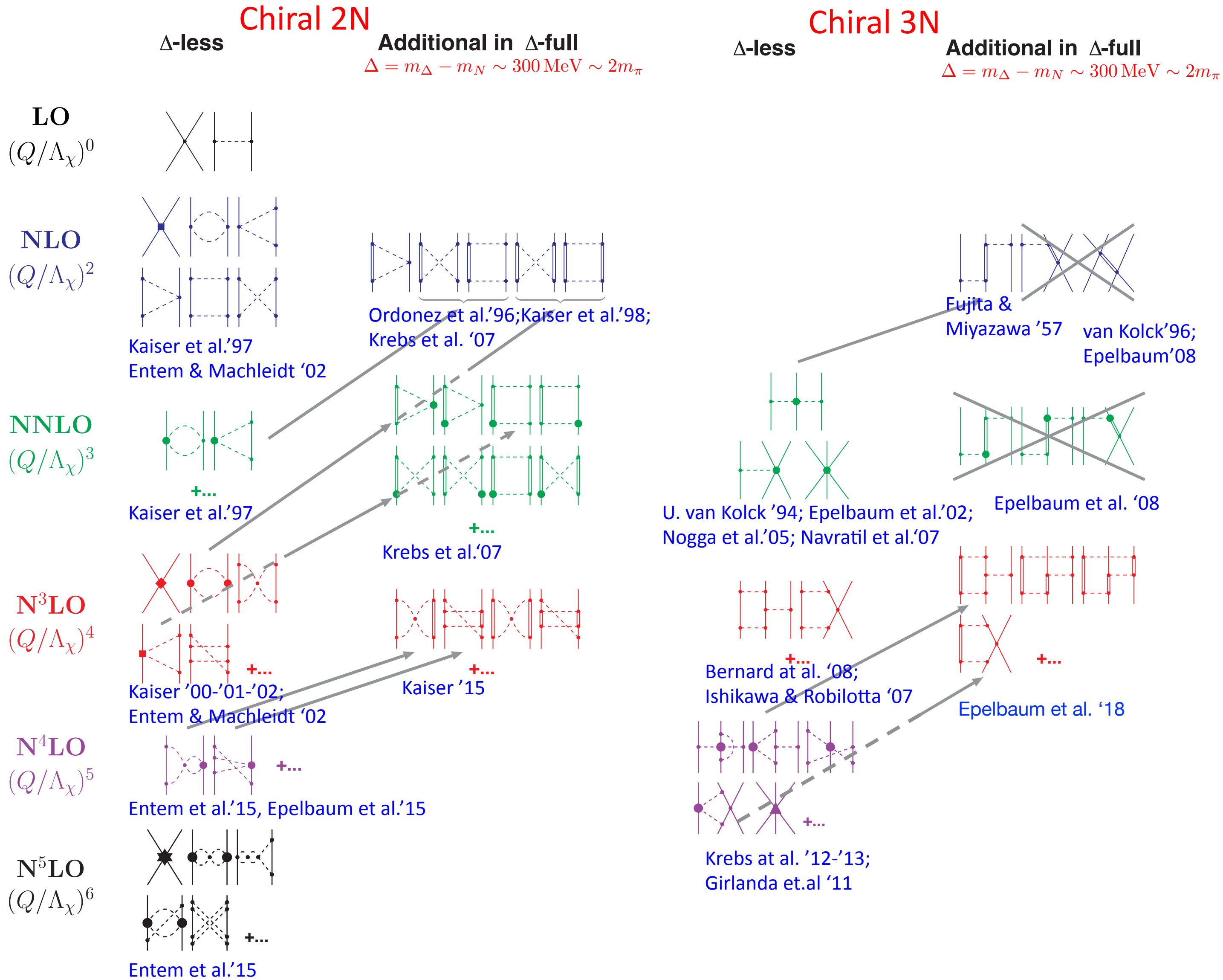
Calculate amplitudes+prescription to obtain potentials + regularization (of high momentum components)

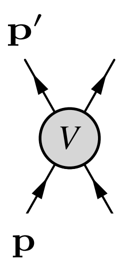
Nuclear forces and currents

Few- and many-body methods: QMC, NCSM, CC, etc

Nuclear structure and dynamics

Nuclear Hamiltonian: Chiral EFT formulation of the *basic model*





Note: Many of the available versions of chiral potentials are formulated in momentum-space and are strongly nonlocal: ➡ $\mathbf{p} \to -i\boldsymbol{\nabla}$ hard to use in QMC methods

Gezerlis et al. PRL 111, 032501 2013; PRC 90, 054323 2014; Lynn et al. PRL 113, 192501 2014

Piarulli et al. PRC 91, 024003 2015; PRC 94, 054007 2016

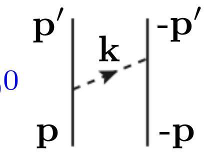
Local chiral NN potential with Δ's

Piarulli et al. PRC **91**, 024003 2015; PRC **94**, 054007 2016

$$v_{12} = v_{12}^{\rm EM} + v_{12}^{\rm L} + v_{12}^{\rm S}$$

$v_{12}^{\rm EM}$: EM component including corrections up to α^2

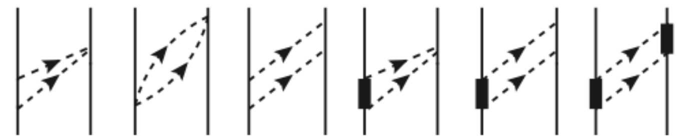
$\rm LO :\ Q^0$



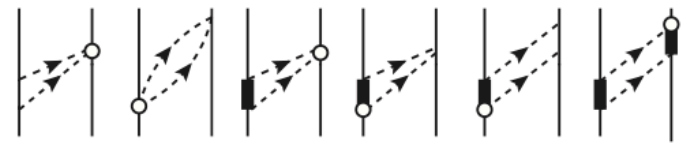
$v_{12}^{\rm L}$: chiral OPE and TPE component with Δ's

- dependence only on the momentum transfer **k=p′-p**

$\rm NLO :\ Q^2$



$\rm N2LO :\ Q^3$



$v_{12}^{\rm S}$: short-range contact component up to order N3LO (Q⁴) parametrized by (2+7+11) CI and (2+4) IB LECs

- the functional form taken as $C_{R_S}(r) \propto e^{-(r/R_S)^2}$ with $R_S = 0.8\ (0.7)$ fm a (b) models

In coordinate-space it reads as:

$$v_{12} = \sum_{l=1}^{16} v^l(r) O_{12}^l$$

$$O_{12}^{l=1,\dots,6} = [\mathbf{1}\,,\, \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\,,\, S_{12}] \otimes [\mathbf{1}\,,\, \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2]$$

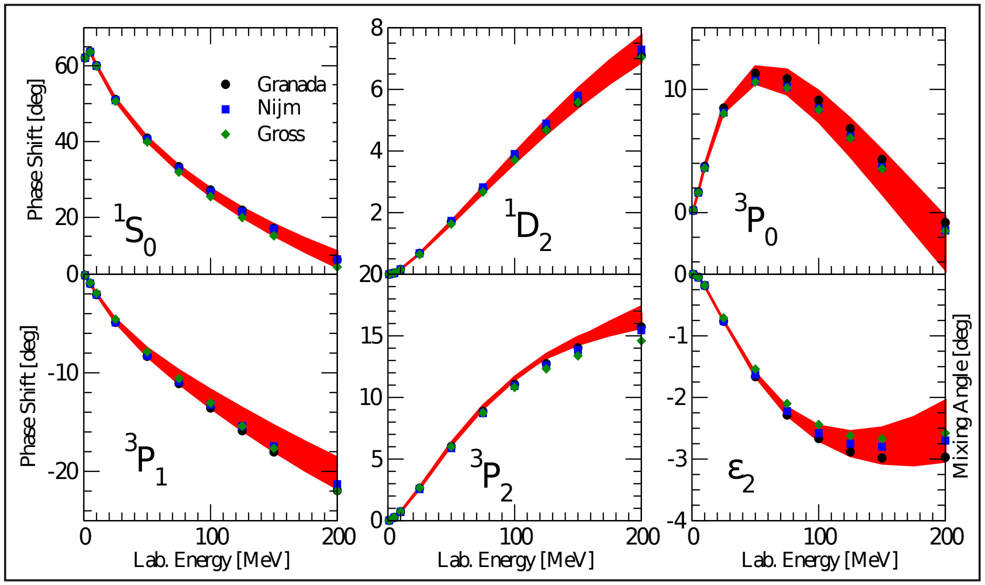
$$O_{12}^{l=7,\dots,11} = \mathbf{L}\cdot\mathbf{S}\,,\, \mathbf{L}\cdot\mathbf{S}\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\,,\, (\mathbf{L}\cdot\mathbf{S})^2\,,\, \mathbf{L}^2\,,\, \mathbf{L}^2\,\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$$

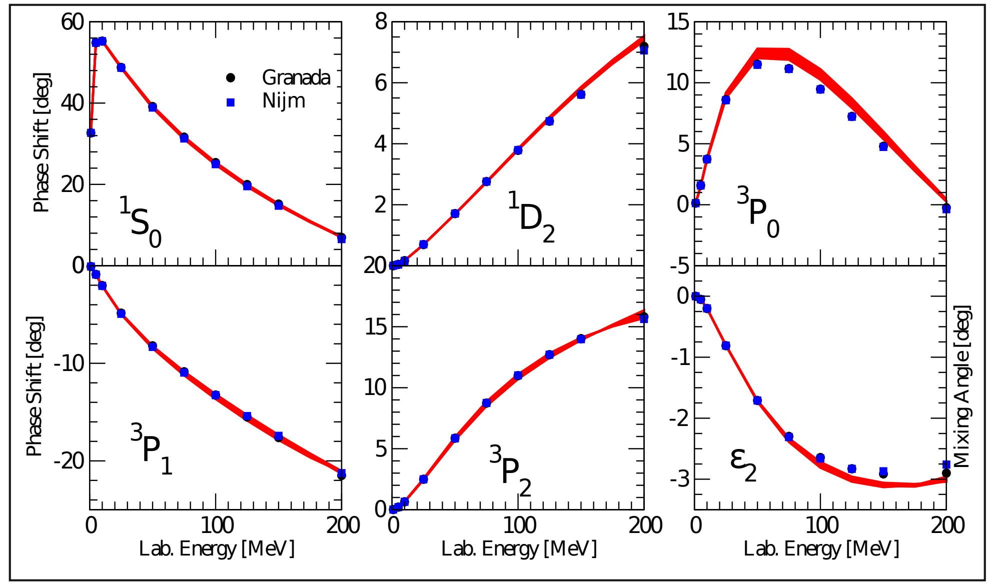
$$O_{12}^{l=12,\dots,16} = T_{12}\,,\, (\tau_1^z + \tau_2^z)\,,\, \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\, T_{12}\,,\, S_{12}\, T_{12}\,,\, \mathbf{L}\cdot\mathbf{S}\, T_{12}$$

Fitting procedure: NN PWA and database

The 26 LECs are fixed by fitting the pp and np Granada database up to two ranges of E_{lab} = 125 MeV and 200 MeV, the deuteron BE and the nn scattering length

To minimizing the χ^2 we have used the Practical Optimization Using No Derivatives (for Squares), POUNDers (M. Kortelainen, PRC **82**, 024313 2010)



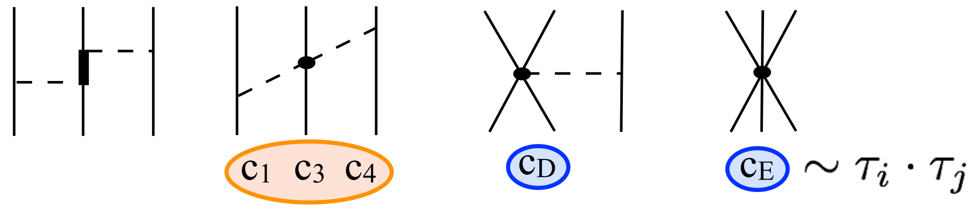


model	order	$E_{\rm Lab}$ (MeV)	N_{pp+np}	χ^2/datum
Ia	N3LO	0–125	2668	1.05
Ib	N3LO	0–125	2665	1.07
IIa	N3LO	0–200	3698	1.37
IIb	N3LO	0–200	3695	1.37

Models a (b) cutoff~500 MeV (600 MeV) in momentum-space

Local chiral 3N potential with Δ's

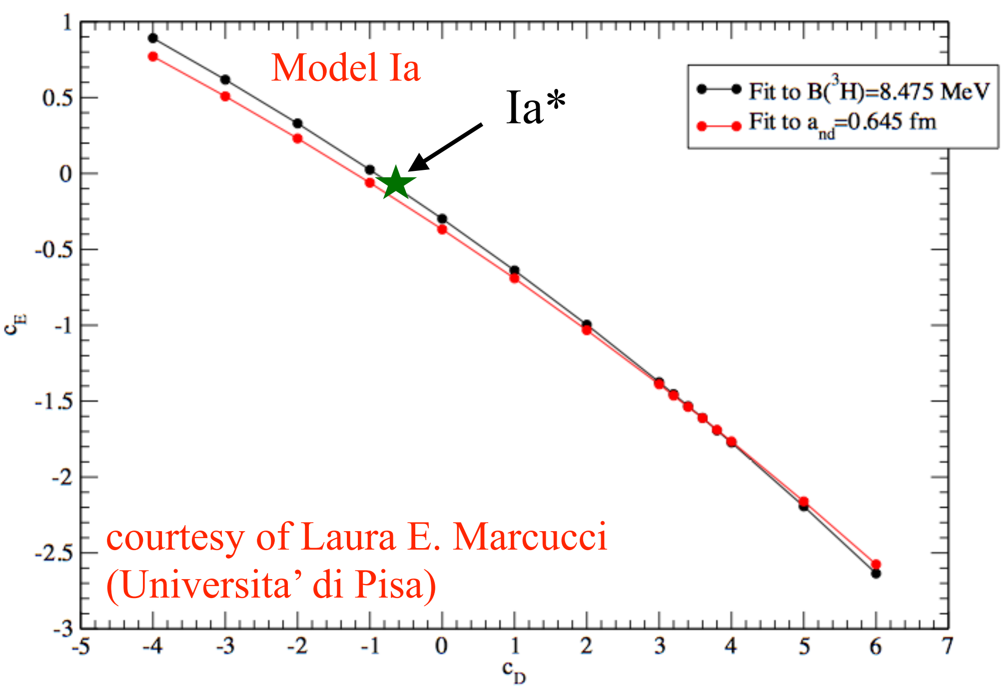
Inclusion of 3N forces at N2LO:



1) Fit to:

- $E_0(^3\mathrm{H}) = -8.482$ MeV
- $^2a_{nd} = (0.645 \pm 0.010)$ fm

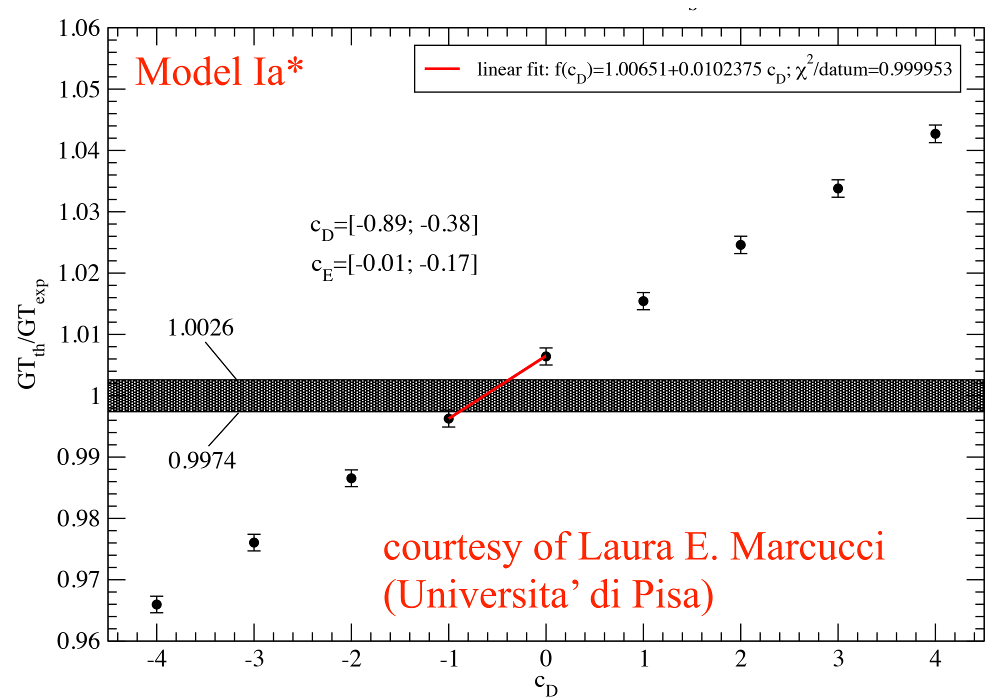
Model	c_D	c_E
Ia	3.666	–1.638
Ib	–2.061	–0.982
IIa	1.278	–1.029
IIb	–4.480	–0.412



2) Fit to:

- $E_0(^3\mathrm{H}) = -8.482$ MeV
- GT m.e. in ^{3}H β-decay

Model	c_D	c_E
Ia*	–0.635(255)	–0.09(8)
Ib*	–4.705(285)	0.550(150)
IIa*	–0.610(280)	–0.350(100)
IIb*	–5.250(310)	0.05(180)



Ab initio Methods: HH and QMC

Hyperspherical Harmonics (HH) expansion for A=3 and 4 bound and continuum states

$$|\Psi\rangle = \sum_\mu c_\mu \underbrace{|\Phi_\mu\rangle}_{\text{HH basis}} \qquad c_\mu \quad \text{from} \quad E = \frac{\langle\Psi|H|\Psi\rangle}{\langle\Psi|\Psi\rangle}$$

Kievsky *et al.*, JPG: NPP **35,** 063101 (2008)

Quantum Monte Carlo (QMC) methods encompass a large family of computational methods whose common aim is the study of complex quantum systems

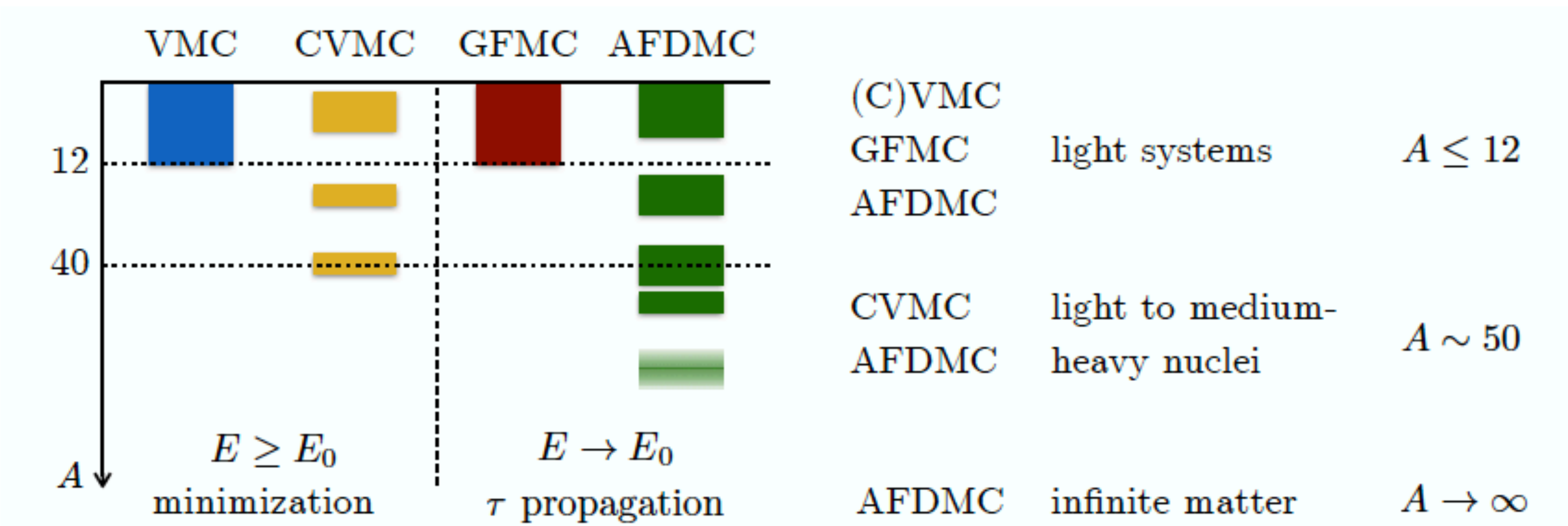


Figure by Diego Lonardoni, LANL

QMC: Variational Monte Carlo (VMC)

R.B. Wiringa, PRC **43**, 1585 (1991)

Minimize the expectation value of H:

$$E_T = \frac{\langle \Psi_T | H | \Psi_T \rangle}{\langle \Psi_T | \Psi_T \rangle} \geq E_0$$

Trial wave function (involves variational parameters):

$$|\Psi_T\rangle = \left[1 + \sum_{i<j<k} U_{ijk}\right] \left[S \prod_{i<j} (1 + U_{ij})\right] |\Psi_J\rangle$$

$|\Psi_J\rangle = \left[\prod_{i<j} f_c(r_{ij})\right] |\Phi(JMTT_z)\rangle$ (s-shell nuclei): Jastrow wave function, fully antisymmetric

$S \prod_{i<j}$: represents a symmetrized product

$U_{ij} = \sum_{p=2,6} u_p(r_{ij})\, O^p_{ij}$: pair correlation operators

$U_{ijk} = \sum_x \epsilon_x\, V^x_{ijk}$: three-body correlation operators

$|\Psi_T\rangle$ are spin-isospin vectors in 3A dimension with $2^A \begin{pmatrix} A \\ Z \end{pmatrix}$

The search in the parameter space is made using COBYLA (Constrained Optimization BY Linear Approximations) algorithm available in NLopt library

QMC: Diffusion Monte Carlo (DMC)

The diffusion Monte Carlo (DMC) method (ex. GFMC or AFDMC) overcomes the limitations of VMC by using a projection technique to determine the true ground-state

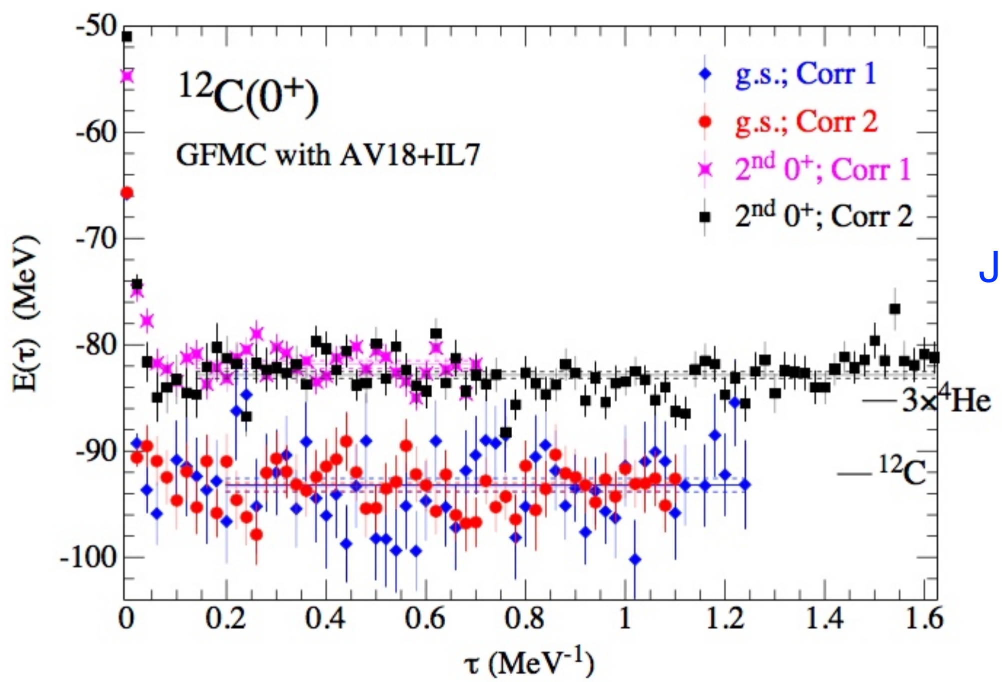
The method relies on the observation that Ψ_T can be expanded in the complete set of eigenstates of the Hamiltonian according to

$$|\Psi_T\rangle = \sum_n c_n|\Psi_n\rangle \qquad H|\Psi_n\rangle = E_n|\Psi_n\rangle$$

$$\lim_{\tau\to\infty} |\Psi(\tau)\rangle = \lim_{\tau\to\infty} e^{-(H-E_0)\,\tau}\,|\Psi_T\rangle = c_0|\Psi_0\rangle \qquad |\Psi(\tau=0)\rangle = |\Psi_T\rangle$$

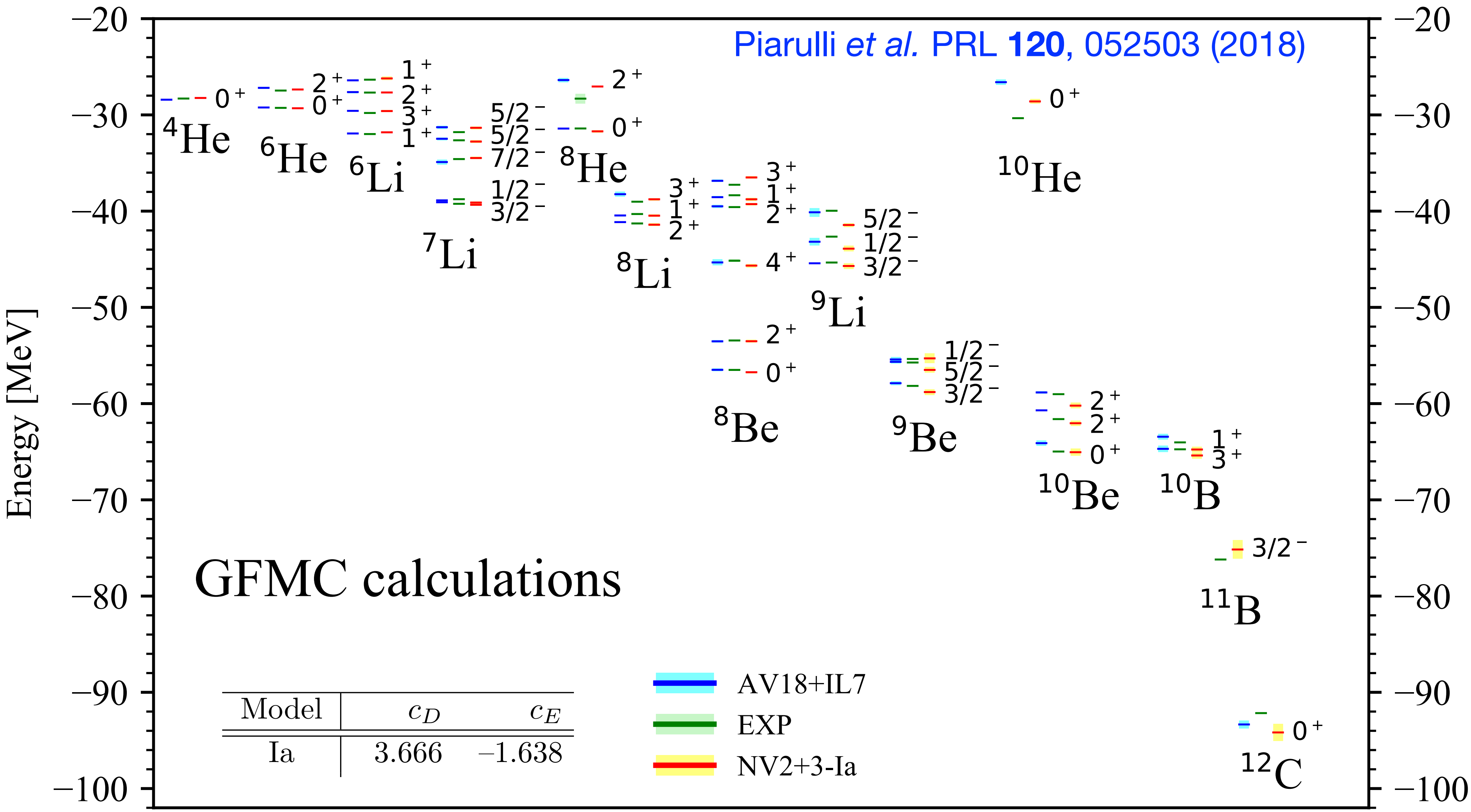
where τ is the imaginary time

The evaluation of $\Psi(\tau)$ is done stochastically in small time steps $\Delta\tau$ ($\tau = n\,\Delta\tau$) using a Green's function formulation



J. Carlson et al., RMP. **87**, 1067 (2015)

Spectra of Light Nuclei: Phenomenology vs χEFT

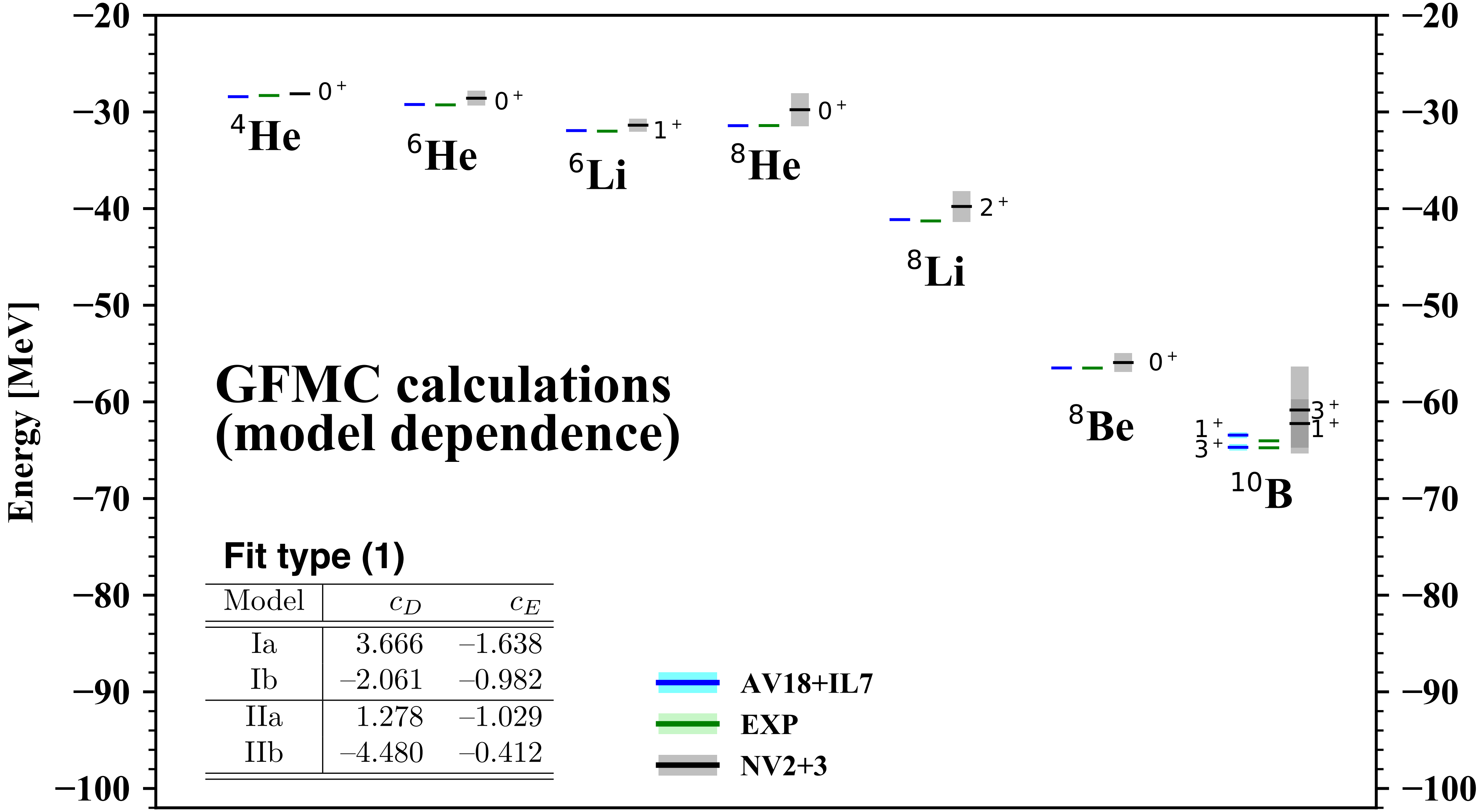


The rms from experiment is 0.72 MeV for NV2+3-Ia compared to 0.80 MeV for AV18+IL7

$c_E < (>) 0$: repulsion (attraction) in light-nuclei (the opposite effect in PNM)

$c_D < (>) 0$: repulsion (attraction) in light-nuclei (same effect in PNM but very small)

Energies of Light Nuclei: Model-dependence



Fit type (1)

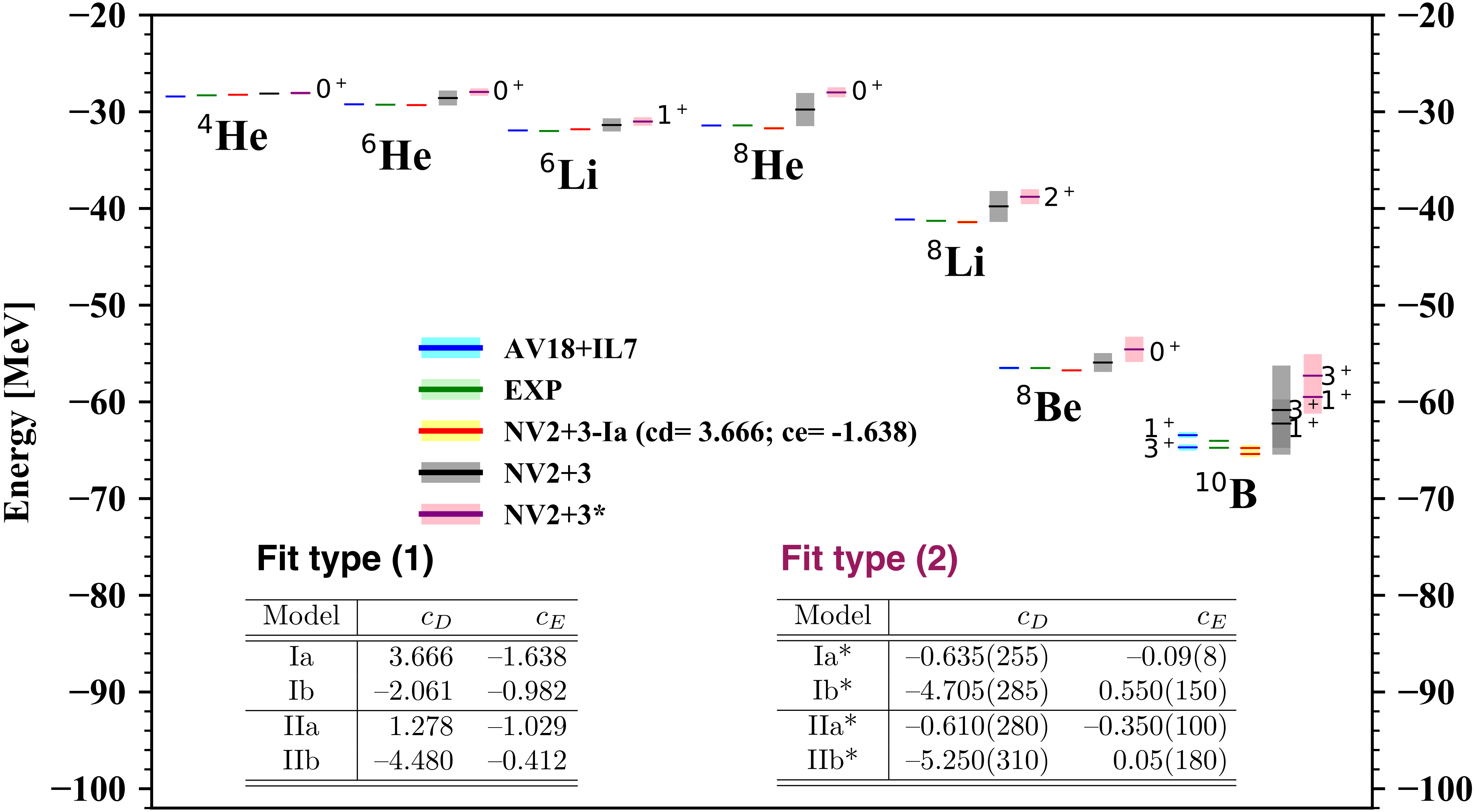
Model	c_D	c_E
Ia	3.666	–1.638
Ib	–2.061	–0.982
IIa	1.278	–1.029
IIb	–4.480	–0.412

Model-dependence for NV2+3 up to 7-8% of the total binding energy

$c_E < (>) 0$: repulsion (attraction) in light-nuclei (the opposite effect in PNM)

$c_D < (>) 0$: repulsion (attraction) in light-nuclei (same effect in PNM but very small)

Energies of Light Nuclei: Model-dependence



Fit type (1)

Model	c_D	c_E
Ia	3.666	−1.638
Ib	−2.061	−0.982
IIa	1.278	−1.029
IIb	−4.480	−0.412

Fit type (2)

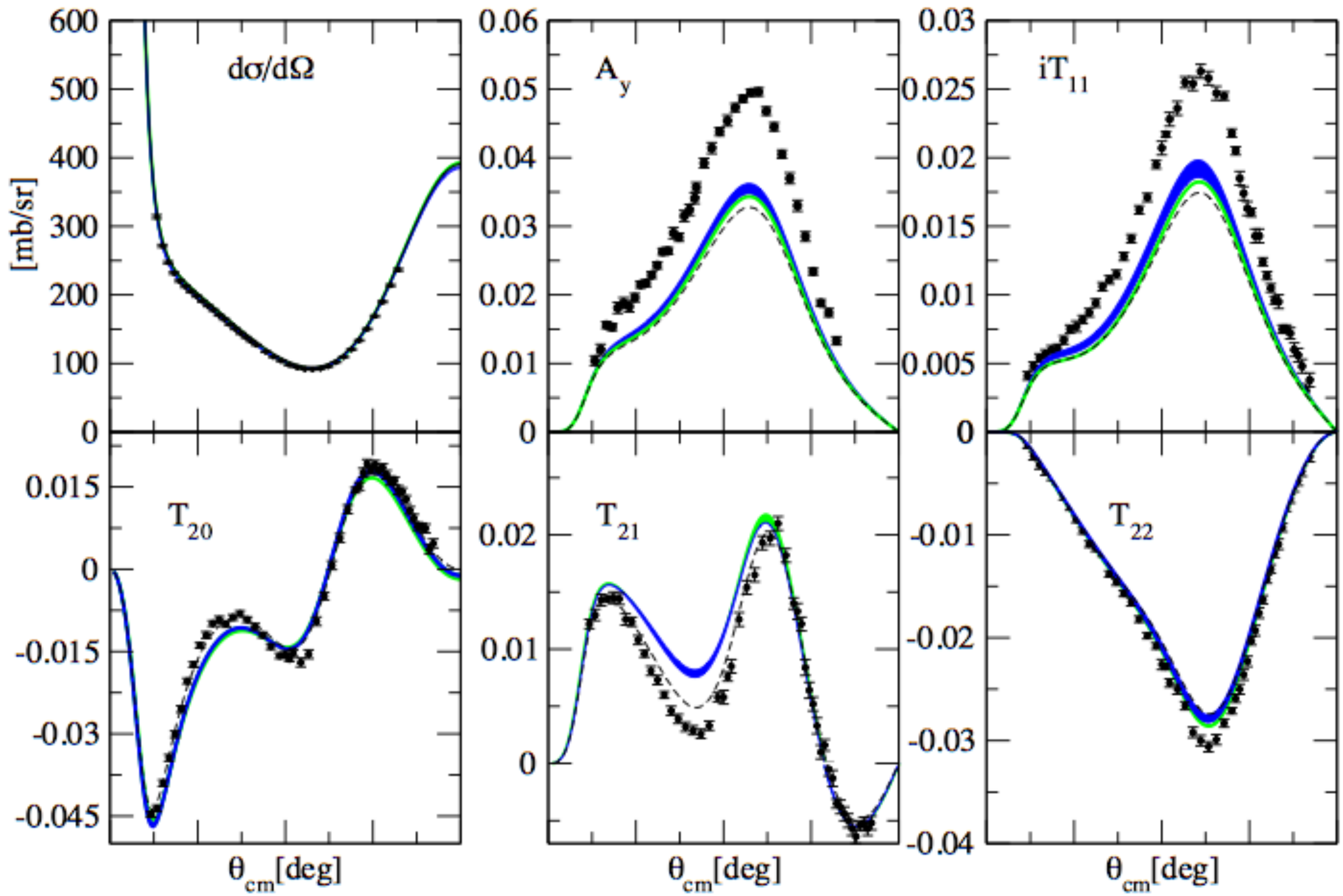
Model	c_D	c_E
Ia*	−0.635(255)	−0.09(8)
Ib*	−4.705(285)	0.550(150)
IIa*	−0.610(280)	−0.350(100)
IIb*	−5.250(310)	0.05(180)

Model-dependence for NV2+3 up to 7-8% of the total binding energy

Model-dependence for NV2+3* up to 2-3% of the total binding energy

Polarization observables in pd elastic scattering at 3 MeV: HH calculations with the NV2+3 models Ia-Ib (IIa-IIb), are shown by the green (blue) band. The black dashed line are results obtained with only the two-body interaction NV2-Ia

Girlanda, Kievsky, Marcucci, Viviani



More sophisticated 3N force??? Different way to fix the 3N??? subleading contact terms in 3N interaction???

3N subleading contact terms

There are 146 3N contact operators with two derivatives; but Fierz identities lead to 10 independent operator structure; a possible choice

Girlanda et al., PRC 84 014001 (2011)

$$\sum_{n=1}^{4} V_{ijk}^{(n)} = (E_1 + E_2\,\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j + E_3\,\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j + E_4\,\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j\,\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j) \times \left[C''_{R_S}(r_{ij}) + 2\,\frac{C'_{R_S}(r_{ij})}{r_{ij}}\right] C_{R_S}(r_{jk}) + (j \rightleftharpoons k)$$

$$\sum_{n=5}^{6} V_{ijk}^{(n)} = (E_5 + E_6\,\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j)\,S_{ij}\left[C''_{R_S}(r_{ij}) - \frac{C'_{R_S}(r_{ij})}{r_{ij}}\right] C_{R_S}(r_{jk}) + (j \rightleftharpoons k)$$

$$\sum_{n=7}^{8} V_{ijk}^{(n)} = -2\,(E_7 + E_8\,\boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_k)\,\frac{C'_{R_S}(r_{ij})}{r_{ij}}\left\{(\mathbf{L}\cdot\mathbf{S})_{ij}\,,\,C_{R_S}(r_{jk})\right\} + (j \rightleftharpoons k)$$

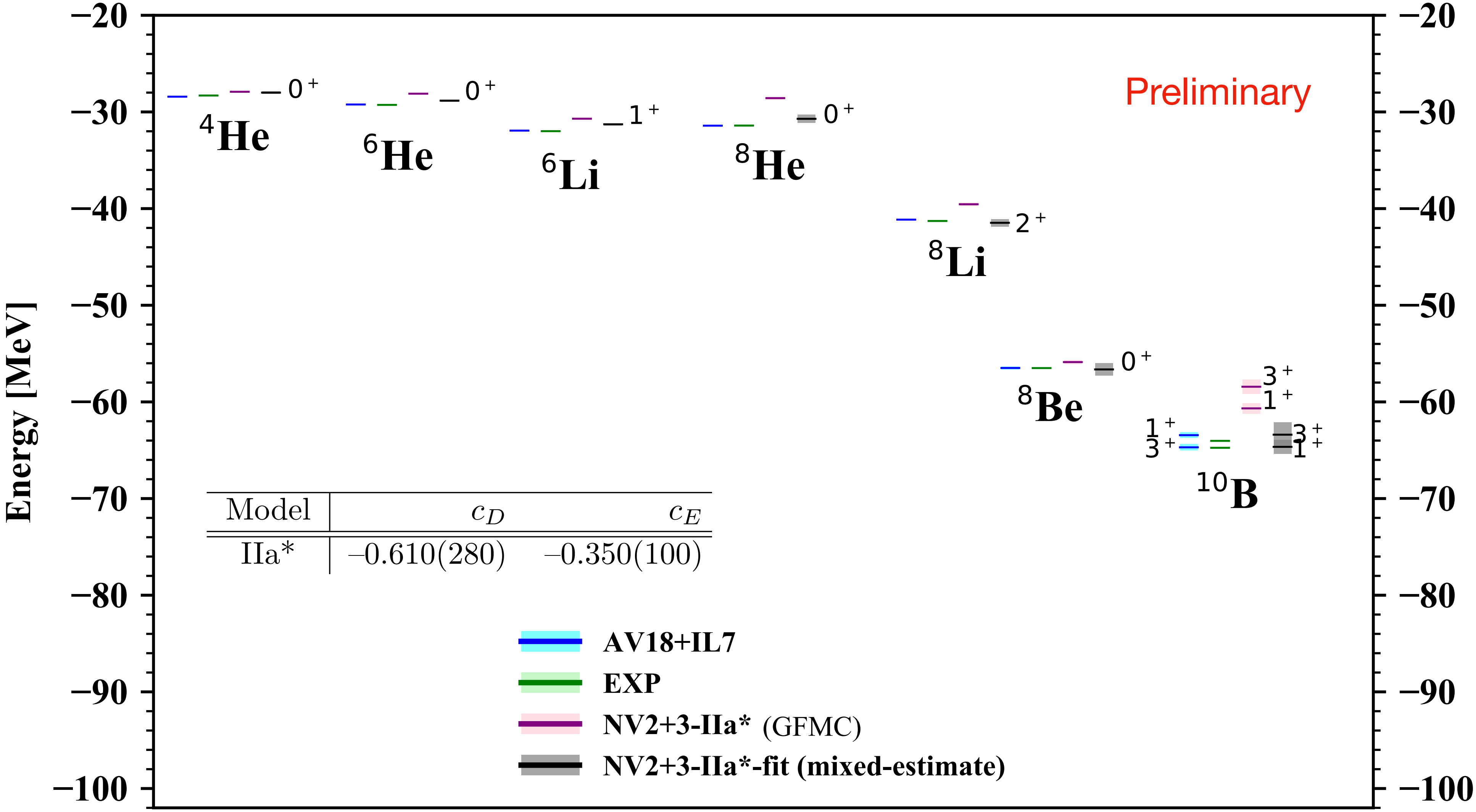
$$\sum_{n=9}^{10} V_{ijk}^{(n)} = (E_9 + E_{10}\,\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j)\,\boldsymbol{\sigma}_i\cdot\hat{\mathbf{r}}_{ik}\,\boldsymbol{\sigma}_j\cdot\hat{\mathbf{r}}_{jk}\,C'_{R_S}(r_{ik})\,C'_{R_S}(r_{jk}) + (j \rightleftharpoons k)$$

For consistency these operators should go along with NN[1] and (multi-pion exchange) 3N potentials at N4LO[2]

However it is worth the effort to test them in calculations of few-body reactions (p-d and p-^{3}He A_y) and spectra of light-nuclei

[1]Entem et al. (2015) and Epelbaum et al. (2015); [2]Bernard et al. (2008) and (2011)

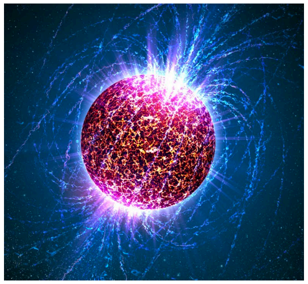
Energies of Light Nuclei: inclusion-subleadings



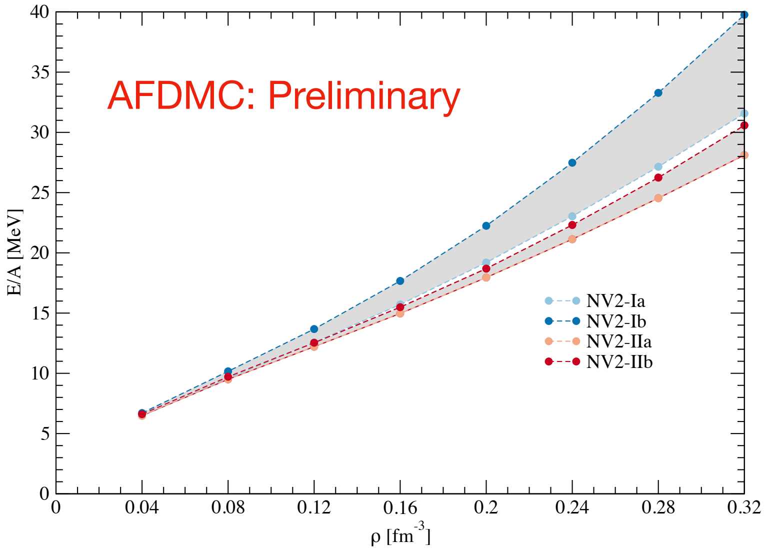
- E5 and E6 are helping mostly with 10B even if the splitting is not quite solved
- E7 and E8 are helping mostly to get A=8 nuclei

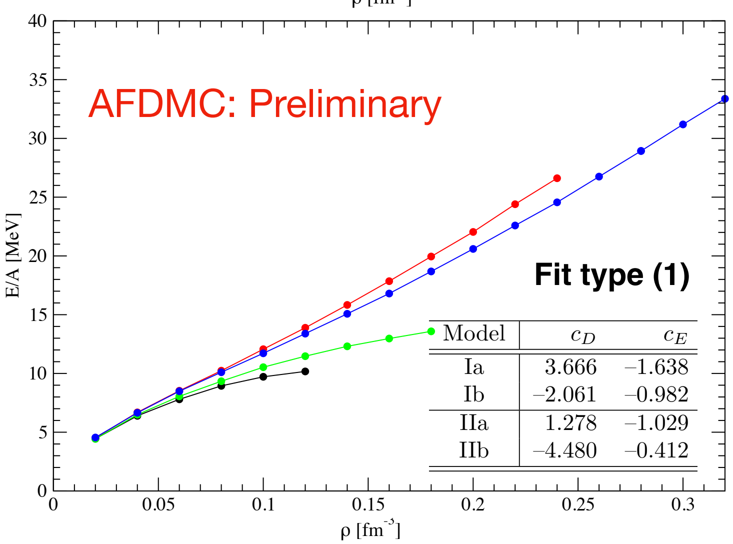
Equation of State of Pure Neutron Matter in χEFT

The EoS of pure neutron matter (PNM): neutrons stars



- Compact objects: R ~ 10km, $M_{\rm max}^{\rm obs} \sim 2M_{\odot}$
- Composed predominantly of neutrons between the inner crust and the outer core
- NS from gravitational collapse of a massive star after a supernova explosion





Model	c_D	c_E
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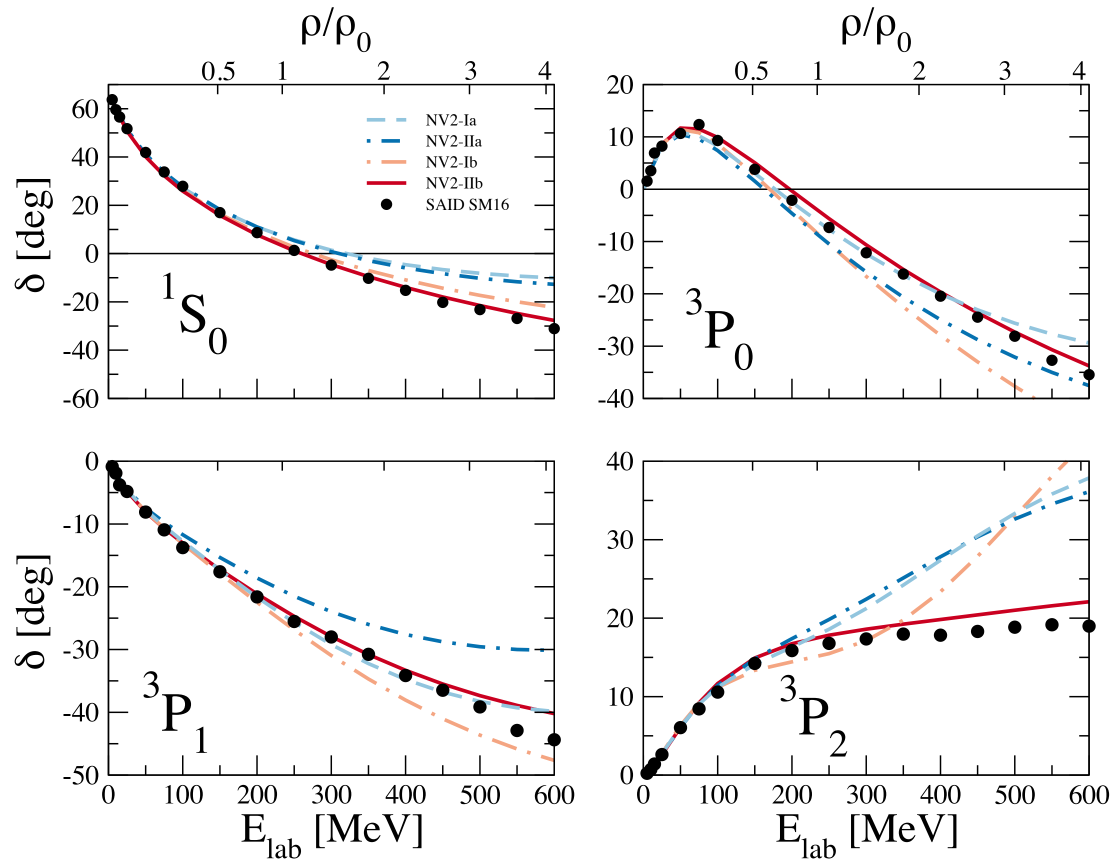


Logoteta, Piarulli, Bombaci, Lovato, Wiringa in preparation

Cutoff sensitivity: modest in NV2 models; very large in NV2+3 models

Nuclear axial currents and beta-decays in light-nuclei

$(Z,N) \rightarrow (Z+1, N-1) + e + \bar{\nu}_e$

Matrix Element $< \Psi_f | \mathrm{GT} | \Psi_i > \sim g_A$ and decay rate $\sim g_A^2$

Understanding "quenching" of $\sim g_A$

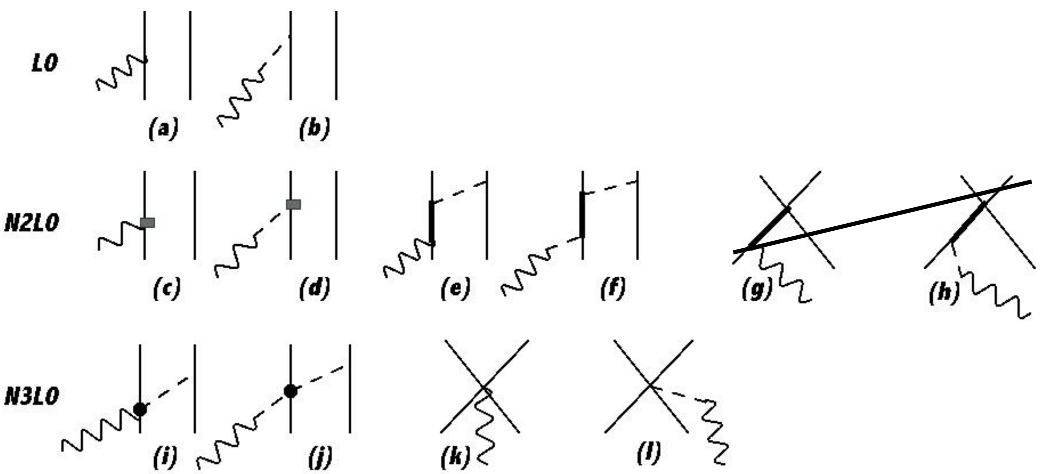
Relevant for neutrinoless double beta decay since rate $\sim g_A^4$

Nuclear astrophysics (Sun chain reaction)

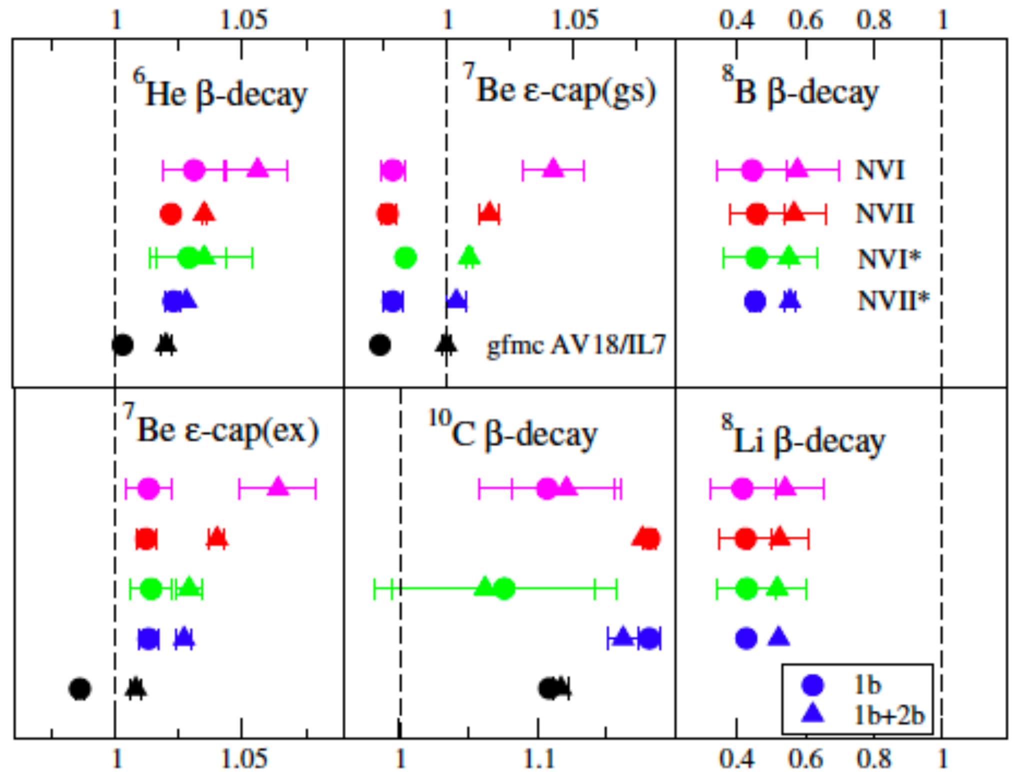
Schiavilla et al. PRC 99, 034005 (2019)

Baroni et al. PRC 93, 015501 (2016)

Pastore et al. PRC 78, 064002 (2008)



A single unknown LEC in the axial contact current fixed in 3H beta-decay



NVI - database fitted up to 125 MeV - c_D, c_E fitted to B.E. and *nd*-scattering length (VMC calculations)

NVII - database fitted up to 200 MeV - c_D, c_E fitted to B.E. and *nd*-scattering length (VMC calculations)

NVI* - database fitted up to 125 MeV - c_D, c_E fitted to B.E. and GT triton (VMC calculations)

NVII* - database fitted up to 200 MeV - c_D, c_E fitted to B.E. and GT triton (VMC calculations)

Pastore, Piarulli, Schiavilla, Wiringa, Baroni, Carlson, Gandolfi, in preparation

PRELIMINARY

AV18+IL7 - database fitted up to 350 MeV - c_D fitted to GT triton (GFMC calculations)

Pastore et al. PRC **97** 022501 (2018)

Conclusions

We are testing our models of NN+3N interactions with Δ-isobar based on chiral EFT framework in both light-nuclei and infinite nuclear matter

We mainly focused our attention on studying properties of nuclei up to A=12 and EoS of infinite neutron matter

For the time being, we are interested in studying the model-dependence of the nuclear observables by exploring different cutoffs and range of energies used to fit the NN interactions as well as analyzing different strategies fo fit the TNI

It looks like that the formulation of the TNI with only c_D and c_E terms is too simplistic if we want to have a good descriptions of spectra, properties of light-nuclei, infinite nuclear matter, three-body observables with a certain degree of accuracy

We are investigating the effect of subleading 3N contact interactions in light-nuclei (we will do so also for infinite nuclear matter)

THANK YOU

Washington University in St. Louis

FRIB Theory Alliance

Argonne NATIONAL LABORATORY

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Robert Wiringa, Argonne National Lab, USA

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