Neutrino induced one- and two-pion production on nucleons

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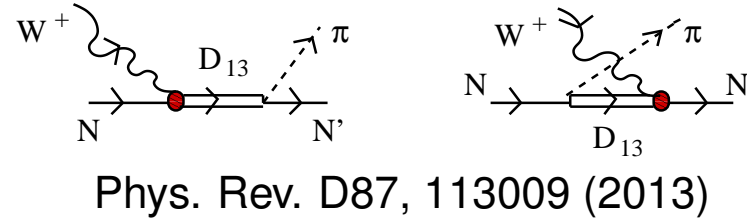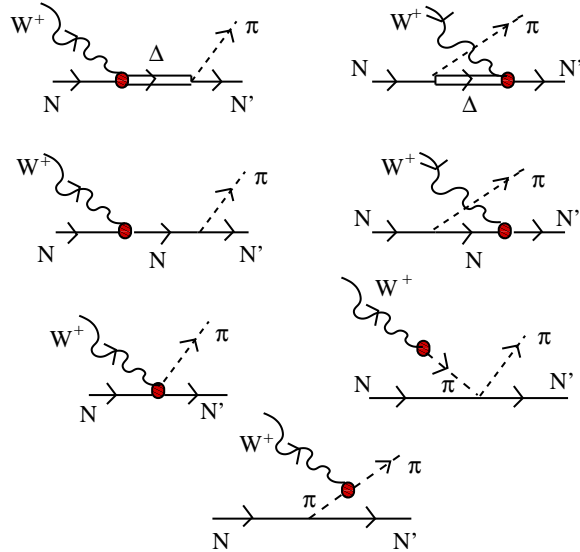
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HNV pion production model for one pion production in one slide

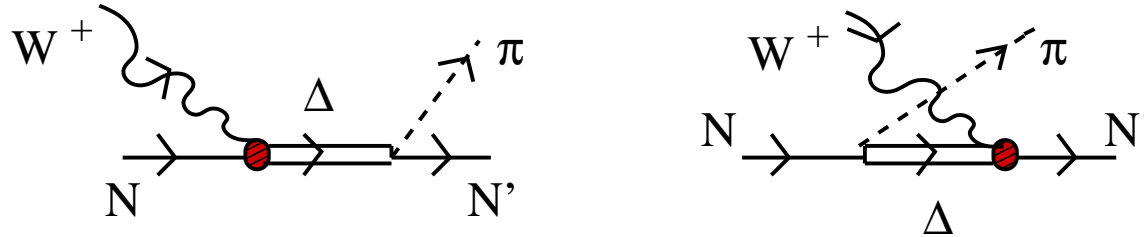Phys. Rev. D87, 113009 (2013)

Phys. Rev. D76, 033005(2007)

Partial unitarization through Olsson's method: $T_B + T_{\Delta P} \to T_B + e^{i\delta_V} T^V_{\Delta P} + e^{i\delta_A} T^A_{\Delta P}$
Phys. Rev. D93, 014016 (2016)

Modified Delta propagator:

$$\frac{P_{\mu\nu}}{p_\Delta^2 - M_\Delta^2 + iM_\Delta\Gamma_\Delta} \to \frac{P_{\mu\nu}}{p_\Delta^2 - M_\Delta^2 + iM_\Delta\Gamma_\Delta} + c\,\delta P_{\mu\nu}, \quad \delta P_{\mu\nu} = \frac{P_{\mu\nu} - \frac{p_\Delta^2}{M_\Delta^2} P_{\mu\nu}^{\frac{3}{2}}}{p_\Delta^2 - M_\Delta^2}$$

We get better results for $\nu_\mu n \to \mu^- n\pi^+$ by fitting c [Phys. Rev. D95, 053007 (2017)]

Δ contribution to one pion production

$$j^{\mu}_{cc+}\Big|_{\Delta P} = \mathrm{i}\, C^{\Delta} \frac{f^*}{m_\pi} \sqrt{3} \cos\theta_C \frac{k^{\alpha}_{\pi}}{p^2_{\Delta} - M^2_{\Delta} + iM_{\Delta}\Gamma_{\Delta}} \bar{u}(\vec{p}\,')P_{\alpha\beta}(p_\Delta)\Gamma^{\beta\mu}(p,q)u(\vec{p}\,), \quad p_\Delta = p + q,$$

$$\text{and } C^{\Delta} = \begin{pmatrix} 1 & \text{for} & p\pi^+ \\ 1/3 & \text{for} & n\pi^+ \end{pmatrix}$$

$$j^{\mu}_{cc+}\Big|_{C\Delta P} = \mathrm{i}\, C^{C\Delta} \frac{f^*}{m_\pi} \frac{1}{\sqrt{3}} \cos\theta_C \frac{k^{\beta}_{\pi}}{p^2_{\Delta} - M^2_{\Delta} + iM_{\Delta}\Gamma_{\Delta}} \bar{u}(\vec{p}\,')\hat{\Gamma}^{\mu\alpha}(p',q)P_{\alpha\beta}(p_\Delta)u(\vec{p}\,), \quad p_\Delta = p' - q,$$

$$\text{and } C^{C\Delta} = \begin{pmatrix} 1 & \text{for} & p\pi^+ \\ 3 & \text{for} & n\pi^+ \end{pmatrix}, \quad \hat{\Gamma}^{\mu\alpha}(p',q) = \gamma^0 \left[\Gamma^{\alpha\mu}(p',-q)\right]^{\dagger}\gamma^0$$

Δ contribution to one pion production II

$$P^{\mu\nu}(p_\Delta) = -(\not{p}_\Delta + M_\Delta)\left[g^{\mu\nu} - \frac{1}{3}\gamma^\mu\gamma^\nu - \frac{2}{3}\frac{p_\Delta^\mu p_\Delta^\nu}{M_\Delta^2} + \frac{1}{3}\frac{p_\Delta^\mu\gamma^\nu - p_\Delta^\nu\gamma^\mu}{M_\Delta}\right]$$

$$\Gamma^{\alpha\mu}(p,q) = \left[\frac{C_3^V}{M}\left(g^{\alpha\mu}\not{q} - q^\alpha\gamma^\mu\right) + \frac{C_4^V}{M^2}\left(g^{\alpha\mu}q\cdot p_\Delta - q^\alpha p_\Delta^\mu\right) + \frac{C_5^V}{M^2}\left(g^{\alpha\mu}q\cdot p - q^\alpha p^\mu\right) + C_6^V g^{\mu\alpha}\right]\gamma_5$$

$$+ \left[\frac{C_3^A}{M}\left(g^{\alpha\mu}\not{q} - q^\alpha\gamma^\mu\right) + \frac{C_4^A}{M^2}\left(g^{\alpha\mu}q\cdot p_\Delta - q^\alpha p_\Delta^\mu\right) + C_5^A g^{\alpha\mu} + \frac{C_6^A}{M^2}q^\mu q^\alpha\right]$$

Vector form factors taken from O. Lalakulich et al., PRD74,014009 (2006)

$$C_3^V = \frac{2.13}{(1 - q^2/M_V^2)^2} \times \frac{1}{1 - \frac{q^2}{4M_V^2}}, \quad C_4^V = \frac{-1.51}{(1 - q^2/M_V^2)^2} \times \frac{1}{1 - \frac{q^2}{4M_V^2}},$$

$$C_5^V = \frac{0.48}{(1 - q^2/M_V^2)^2} \times \frac{1}{1 - \frac{q^2}{0.776 M_V^2}}, \quad C_6^V = 0 \,(CVC) \qquad \text{with } M_V = 0.84\,\text{GeV}$$

For the axial form factors we use Adler's, model [Ann. Phys. 50, 189 (1968)] in which
$C_4^A(q^2) = -\frac{C_5^A(q^2)}{4}, C_3^A(q^2) = 0$, and we further take

$$C_5^A(q^2) = \frac{C_5^A(0)}{(1 - q^2/M_{A\Delta}^2)^2}, \quad C_6^A(q^2) = C_5^A(q^2)\frac{M^2}{m_\pi^2 - q^2} \quad (\text{PCAC})$$

From our latest fit we obtain $C_5^A(0) = 1.18 \pm 0.07$, $M_{A\Delta} = 950 \pm 60\,\text{MeV}$.

Nonresonant production

Interaction Lagrangian

We use a SU(2) nonlinear sigma model. Up to $\mathcal{O}(1/f_\pi^3)$ the interaction Lagrangian reads

$$
\begin{aligned}
\mathcal{L}_{\mathrm{int}}^\sigma &= \frac{g_A}{f_\pi} \bar{\Psi}\gamma^\mu\gamma_5 \frac{\vec{\tau}}{2}(\partial_\mu\vec{\phi})\Psi - \frac{1}{4f_\pi^2}\bar{\Psi}\gamma_\mu\vec{\tau}\left(\vec{\phi}\times\partial^\mu\vec{\phi}\right)\Psi \\
&\quad - \frac{1}{6f_\pi^2}\left(\vec{\phi}^2\partial_\mu\vec{\phi}\partial^\mu\vec{\phi} - (\vec{\phi}\partial_\mu\vec{\phi})(\vec{\phi}\partial^\mu\vec{\phi})\right) \\
&\quad + \frac{m_\pi^2}{24f_\pi^2}(\vec{\phi}^2)^2 - \frac{g_A}{6f_\pi^3}\bar{\Psi}\gamma^\mu\gamma_5\left[\vec{\phi}^2\frac{\vec{\tau}}{2}\partial_\mu\vec{\phi} - (\vec{\phi}\partial_\mu\vec{\phi})\frac{\vec{\tau}}{2}\vec{\phi}\right]\Psi \, ,
\end{aligned}
$$

where $\Psi = \begin{pmatrix} p \\ n \end{pmatrix}$ is the nucleon field, $\vec{\phi}$ is the isovector pion field.

To evaluate the different contributions we also need the coupling to the W boson, i.e. we need the vector and axial currents.

Nonresonant production

Vector and axial currents I

To the corresponding order the vector and axial currents are

$$
\begin{aligned}
\vec{V}^\mu &= \vec{\phi} \times \partial^\mu \vec{\phi} + \bar{\Psi}\gamma^\mu \frac{\vec{\tau}}{2}\Psi + \frac{g_A}{2f_\pi}\bar{\Psi}\gamma^\mu\gamma_5(\vec{\phi}\times\vec{\tau})\Psi - \frac{1}{4f_\pi^2}\bar{\Psi}\gamma^\mu\left[\vec{\tau}\vec{\phi}^{\,2} - \vec{\phi}(\vec{\tau}\cdot\vec{\phi})\right]\Psi \\
&\quad - \frac{\vec{\phi}^{\,2}}{3f_\pi^2}(\vec{\phi}\times\partial^\mu\vec{\phi}) + \mathcal{O}(\frac{1}{f_\pi^3})
\end{aligned}
$$

$$
\begin{aligned}
\vec{A}^\mu &= f_\pi\partial^\mu\vec{\phi} + g_A\bar{\Psi}\gamma^\mu\gamma_5\frac{\vec{\tau}}{2}\Psi + \frac{1}{2f_\pi}\bar{\Psi}\gamma^\mu(\vec{\phi}\times\vec{\tau})\Psi + \frac{2}{3f_\pi}\left[\vec{\phi}(\vec{\phi}\cdot\partial^\mu\vec{\phi}) - \vec{\phi}^{\,2}\partial^\mu\vec{\phi}\right] \\
&\quad - \frac{g_A}{4f_\pi^2}\bar{\Psi}\gamma^\mu\gamma_5\left[\vec{\tau}\vec{\phi}^{\,2} - \vec{\phi}(\vec{\tau}\cdot\vec{\phi})\right]\Psi + \mathcal{O}(\frac{1}{f_\pi^3})
\end{aligned}
$$

The normalization is such that $-\sqrt{2}\cos\theta_C(V^\mu_{+1} - A^\mu_{+1})$ provides the $W^+ n \to p$ vertex

Nonresonant production

Vector and axial currents II

The pure nucleonic part of the vector and axial currents are further modified by the inclusion of form factors. For the neutron to proton transition which fixes our normalization we have

$$V_N^\alpha(q) = 2 \times \left(F_1^V(q^2)\gamma^\alpha + \mathrm{i}\mu_V \frac{F_2^V(q^2)}{2M}\sigma^{\alpha\nu}q_\nu \right),$$

$$A_N^\alpha(q) = G_A(q^2) \times \left(\gamma^\alpha\gamma_5 + \frac{\slashed{q}}{m_\pi^2 - q^2}q^\alpha\gamma_5 \right).$$

The magnetic part in $V_N^\alpha(q)$ is not provided by the sigma model neither the q^2 dependence of the form factors.

Vector current conservation requires that all other terms in the vector current are multiplied by the $F_1^V(q^2)$ form factor.

Nonresonant production

Vector and axial currents III

For the vector form factors we use the parametrization of S. Galster et al in NPB 32, 221 (1971)

$$F_1^N = \frac{G_E^N + \kappa G_M^N}{1 + \kappa}, \qquad \mu_N F_2^N = \frac{G_M^N - G_E^N}{1 + \kappa},$$

$$G_E^p = \frac{G_M^p}{\mu_p} = \frac{G_M^n}{\mu_n} = -(1 + \lambda_n \kappa)\frac{G_E^n}{\mu_n \kappa} = \left(\frac{1}{1 - q^2/M_D^2}\right)^2$$
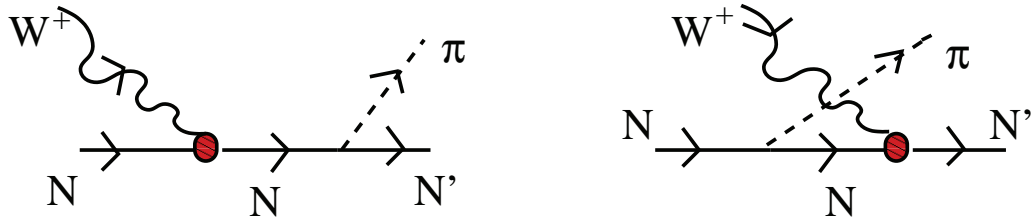
with $\kappa = -q^2/4M^2$, $M_D = 0.843$ GeV, $\mu_p = 2.792847$, $\mu_n = -1.913043$ and $\lambda_n = 5.6$.
Besides,

$$F_1^V(q^2) = \frac{1}{2}\left(F_1^p(q^2) - F_1^n(q^2)\right), \qquad \mu_V F_2^V(q^2) = \frac{1}{2}\left(\mu_p F_2^p(q^2) - \mu_n F_2^n(q^2)\right).$$

The axial form factor we take from the book of Ericson & Weise (The International Series of Monographs on Physics 74)

$$G_A(q^2) = \frac{g_A}{(1 - q^2/M_A^2)^2}, \qquad g_A = 1.26, \quad M_A = 1.05 \text{ GeV}$$
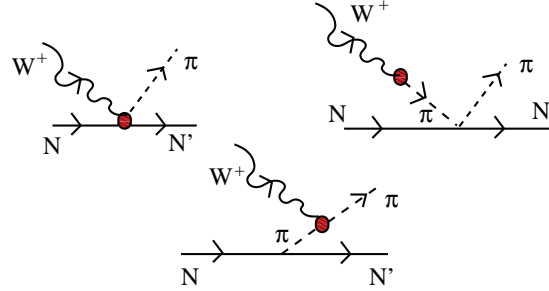
One-pion nonresonant production

$$j^\mu_{cc+}\Big|_{NP} = -i\, C^{NP}\, \frac{g_A}{\sqrt{2}f_\pi}\, \cos\theta_C\; \bar{u}(\vec{p}\,')\slashed{k}_\pi \gamma_5\, \frac{\slashed{p}+\slashed{q}+M}{(p+q)^2 - M^2 + i\epsilon}\, \left[V^\mu_N(q) - A^\mu_N(q)\right] u(\vec{p}),$$

$$C^{NP} = \begin{pmatrix} 0 & \text{for} & p\pi^+ \\ 1 & \text{for} & n\pi^+ \end{pmatrix}$$

$$j^\mu_{cc+}\Big|_{CNP} = -i\, C^{CNP}\, \frac{g_A}{\sqrt{2}f_\pi}\, \cos\theta_C\; \bar{u}(\vec{p}\,')\left[V^\mu_N(q) - A^\mu_N(q)\right] \frac{\slashed{p}' - \slashed{q} + M}{(p'-q)^2 - M^2 + i\epsilon}\slashed{k}_\pi\gamma_5 u(\vec{p}),$$

$$C^{CNP} = \begin{pmatrix} 1 & \text{for} & p\pi^+ \\ 0 & \text{for} & n\pi^+ \end{pmatrix}$$

One-pion nonresonant production II

$$j^{\mu}_{cc+}\Big|_{CT} = -i\,C^{CT}\,\frac{1}{\sqrt{2}f_{\pi}}\cos\theta_C\,\bar{u}(\vec{p}^{\,\prime})\gamma^{\mu}\left(g_A F^{V}_{CT}(q^2)\gamma_5 - F_{\rho}\left((q-k_{\pi})^2\right)\right)u(\vec{p}),$$

$$C^{CT} = \begin{pmatrix} 1 & \text{for} & p\pi^+ \\ -1 & \text{for} & n\pi^+ \end{pmatrix}$$

$$j^{\mu}_{cc+}\Big|_{PP} = -i\,C^{PP}\,F_{\rho}\left((q-k_{\pi})^2\right)\frac{1}{\sqrt{2}f_{\pi}}\cos\theta_C\,\frac{q^{\mu}}{q^2-m^2_{\pi}}\,\bar{u}(\vec{p}^{\,\prime})\,\slashed{q}\,u(\vec{p}),$$
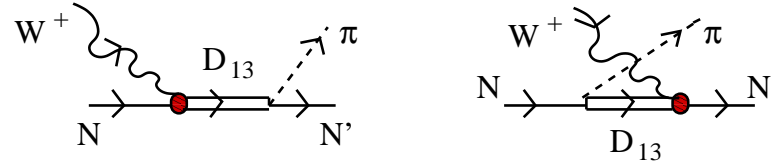
$$C^{PP} = \begin{pmatrix} 1 & \text{for} & p\pi^+ \\ -1 & \text{for} & n\pi^+ \end{pmatrix}$$

$$j^{\mu}_{cc+}\Big|_{PF} = -i\,C^{PF}\,F_{PF}(q^2)\frac{g_A}{\sqrt{2}f_{\pi}}\cos\theta_C\,\frac{(2k_{\pi}-q)^{\mu}}{(k_{\pi}-q)^2-m^2_{\pi}}\,2M\bar{u}(\vec{p}^{\,\prime})\gamma_5 u(\vec{p}),$$

$$C^{PF} = \begin{pmatrix} 1 & \text{for} & p\pi^+ \\ -1 & \text{for} & n\pi^+ \end{pmatrix}$$

$F^{V}_{CT} = F_{PF} = 2F^{V}_1$, $F_{\rho}(1) = 1/(1 - t/m^2_{\rho})$ with $m_{\rho} = 0.7758$ GeV.

D_{13} contribution to one pion production

$$j^\mu_{cc+}|_{DP} = iC^{DP}g_D\sqrt{\frac{2}{3}}\cos\theta_C \frac{k^\alpha_\pi}{p^2_D - M^2_D + iM_D\Gamma_D}\bar{u}(\vec{p}\,')\gamma_5 P^D_{\alpha\beta}(p_D)\Gamma^{\beta\mu}_D(p,q)u(\vec{p}), \quad p_D = p+q,$$

$$C^{DP} = \begin{cases} 0 & p\pi^+ \\ 1 & n\pi^+ \end{cases}$$

$$j^\mu_{cc+}|_{CDP} = -iC^{CDP}g_D\sqrt{\frac{2}{3}}\cos\theta_C \frac{k^\alpha_\pi}{p^2_D - M^2_D + iM_D\Gamma_D}\bar{u}(\vec{p}\,')\hat{\Gamma}^{\mu\beta}_D\left(p',-q\right) P^D_{\beta\alpha}(p_D)\gamma_5 u(\vec{p}), \quad p_D = p'-q$$

$$C^{CDP} = \begin{cases} 1 & p\pi^+ \\ 0 & n\pi^+ \end{cases}$$

$M_D = 1520\,\mathrm{MeV}, g_D = 20\,\mathrm{GeV}^{-1}.\ \Gamma_D = \Gamma^{N\pi}_D + \Gamma^{\Delta\pi}_D$

$$\Gamma^{N\pi}_D = \frac{g^2_D}{8\pi}\frac{1}{3s}[(\sqrt{s}-M)^2 - m^2_\pi]|\vec{p}_\pi|^3\theta(\sqrt{s}-M-m_\pi),$$

$$\Gamma^{\Delta\pi}_D = 0.39 \times 115\,\mathrm{MeV}\frac{|\vec{p}\,'_\pi|}{|\vec{p}\,'^{\,o-s}_\pi|}\theta(\sqrt{s}-M-m_\pi),$$

with $|\vec{p}\,'_\pi| = \dfrac{\lambda^{1/2}(s,M^2_\Delta,m^2_\pi)}{2\sqrt{s}}$ and $|\vec{p}\,'^{\,o-s}_\pi| = \dfrac{\lambda^{1/2}(M^2_D,M^2_\Delta,m^2_\pi)}{2M_D}.$

D_{13} contribution to one pion production II

$$\Gamma^{\beta\mu}(p,q) = \left[\frac{\tilde{C}_3^V}{M}\left(g^{\beta\mu}\slashed{q} - q^\beta\gamma^\mu\right) + \frac{\tilde{C}_4^V}{M^2}\left(g^{\beta\mu}q\cdot p_D - q^\beta p_D^\mu\right) + \frac{\tilde{C}_5^V}{M^2}\left(g^{\beta\mu}q\cdot p - q^\beta p^\mu\right) + \tilde{C}_6^V g^{\beta\mu}\right]$$

$$+ \left[\frac{\tilde{C}_3^A}{M}\left(g^{\beta\mu}\slashed{q} - q^\beta\gamma^\mu\right) + \frac{\tilde{C}_4^A}{M^2}\left(g^{\beta\mu}q\cdot p_D - q^\beta p_D^\mu\right) + \tilde{C}_5^A g^{\beta\mu} + \frac{\tilde{C}_6^A}{M^2}q^\beta q^\mu\right]\gamma_5.$$

The axial form factors are taken from O. Lalakulich et al., PRD74, 014009 (2006)

$$\tilde{C}_3^A = \tilde{C}_4^A = 0, \quad \tilde{C}_5^A = \frac{-2.1}{(1 - q^2/M_A^2)^2}\frac{1}{1 - q^2/(3M_A^2)}, \quad \tilde{C}_6^A(q^2) = \tilde{C}_5^A(q^2)\frac{M^2}{m_\pi^2 - q^2}, \quad M_A = 1\,\text{GeV},$$

while for the vector ones we fitted the form factor results in the Thesis of T. Leitner, from where we got

$$\tilde{C}_3^V = \frac{-2.98}{[1 - q^2/(1.4M_V^2)]^2}, \quad \tilde{C}_4^V = \frac{4.21/D_V}{1 - q^2/(3.7M_V^2)}, \quad \tilde{C}_5^V = \frac{-3.13/D_V}{1 - q^2/(0.42M_V^2)}, \quad \tilde{C}_6^V = 0,$$

with $M_V = 0.84\,\text{GeV}$ and $D_V = (1 - q^2/M_V^2)^2$

A copy of T. LeitnerThesis can be retrieved from

"https://www.uni-giessen.de/fbz/fb07/fachgebiete/physik/einrichtungen/theorie/inst/theses/dissertation/neutrino-nucleus-

interactions-in-acoupled-channel-hadronic-transport-model"

Δ propagator modification

The $\nu_\mu n \to \mu^- n\pi^+$ channel gets a large contribution from the crossed Delta term being very sensitive to the spin 1/2 components in the Δ propagator. In the zero width limit, the Δ propagator is given by

$$G_{\mu\nu}(p_\Delta) = \frac{P_{\mu\nu}(p_\Delta)}{p_\Delta^2 - M_\Delta^2 + i\epsilon}$$

$$
\begin{aligned}
P^{\mu\nu}(p_\Delta) &= -(\not{p}_\Delta + M_\Delta)\left[g^{\mu\nu} - \frac{1}{3}\gamma^\mu\gamma^\nu - \frac{2}{3}\frac{p_\Delta^\mu p_\Delta^\nu}{M_\Delta^2} + \frac{1}{3}\frac{p_\Delta^\mu \gamma^\nu - p_\Delta^\nu \gamma^\mu}{M_\Delta} \right] \\
&= P_{\mu\nu}^{\frac{3}{2}}(p) + (p^2 - M_\Delta^2)\underbrace{\left[\frac{2}{3M_\Delta^2}(\not{p} + M_\Delta)\frac{p_\mu p_\nu}{p^2} - \frac{1}{3M_\Delta}\left(\frac{p^\rho p_\nu \gamma_{\mu\rho}}{p^2} + \frac{p^\rho p_\mu \gamma_{\rho\nu}}{p^2} \right) \right]}_{\text{spin}-1/2},
\end{aligned}
$$

with the true spin-3/2 projection operator being

$$P_{\mu\nu}^{\frac{3}{2}}(p) = -(\not{p} + M_\Delta)\left[g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{1}{3p^2}\left(\not{p}\gamma_\mu p_\nu + p_\mu\gamma_\nu\not{p} \right) \right].$$

Due to the $(p^2 - M_\Delta^2)$ factor that cancels the corresponding factor in the propagator denominator, the spin-1/2 component do not propagate giving rise to contact interactions. Its contribution is small for the direct-Δ term while it is large for the crossed-Δ term.

For some authors [See for instance V. Pascalutsa, Phys. Lett. B 503, 85 (2001)] the use of this spin-1/2 part should be avoided. One way of achieving this goal is by the use of "consistent couplings"

$\triangle$ propagator modification. Consistent $\triangle$ couplings

Consistent couplings are the ones that respect the gauge symmetry

$$\Psi_\mu \to \Psi_\mu + \partial_\mu \epsilon$$

present in the free-massless Rarita-Schwinger lagrangian. This symmetry requires that in any linear interaction term the $\triangle$ field couples only to conserved currents

$$\mathcal{L}_{\text{int}} = g\bar{\Psi}_\beta J^\beta + H.c., \qquad \partial_\beta J^\beta = 0.$$

Couplings not respecting that symmetry are called inconsistent.

As shown in V. Pascalutsa, Phys. Lett. B 503, 85 (2001), inconsistent couplings can be transformed into consistent ones by a redefinition of the $\triangle$ field resulting in a new consistent interaction lagrangian

$$\mathcal{L}'_{\text{int}} = g\,\bar{\Psi}_\beta \mathcal{J}^\beta + H.c.,$$

plus an additional contact interaction lagrangian. Thus, as far as all relevant contact interactions are taken into account, descriptions using consistent or inconsistent couplings are equivalent. Is is only the coupling constants of the contact terms that differ. But those constants have to be fitted to experimental data.

Δ propagator modification. Consistent Δ couplings II

For process mediated by an intermediate Δ
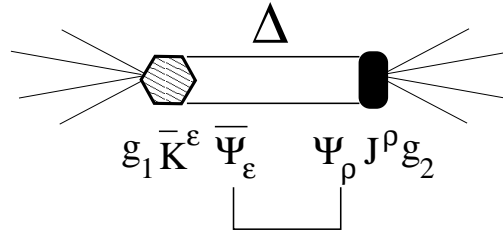
$$T = g_1 g_2 \bar{K}^\epsilon \frac{P_{\epsilon\rho}}{p_\Delta^2 - M_\Delta^2} J^\rho , \rightarrow T_{consistent} = g_1 g_2 \bar{\mathcal{K}}^\epsilon \frac{P_{\epsilon\rho}}{p_\Delta^2 - M_\Delta^2} \mathcal{J}^\rho = g_1 g_2 \bar{K}^\epsilon \frac{p_\Delta^2}{M_\Delta^2} \frac{P_{\epsilon\rho}^{\frac{3}{2}}}{p_\Delta^2 - M_\Delta^2} J^\rho .$$

Then, one has $T = T_{consistent} + \delta T$ with $\delta T = g_1 g_2 \bar{K}^\epsilon \dfrac{P_{\epsilon\rho} - \dfrac{P_\Delta^2}{M_\Delta^2} P_{\epsilon\rho}^{\frac{3}{2}}}{p_\Delta^2 - M_\Delta^2} J^\rho$. Since,

$$P_{\epsilon\rho} - \frac{P_\Delta^2}{M_\Delta^2} P_{\epsilon\rho}^{\frac{3}{2}} = (p_\Delta^2 - M_\Delta^2) \, \delta P_{\epsilon\rho},$$

$$\delta P_{\epsilon\rho} = \frac{1}{M_\Delta^2} (\not{p}_\Delta + M_\Delta) \left(g_{\epsilon\rho} - \frac{1}{3} \gamma_\epsilon \gamma_\rho \right) + \frac{1}{3M_\Delta^2} \left(p_{\Delta\,\epsilon} \gamma_\rho - p_{\Delta\,\rho} \gamma_\epsilon \right)$$

the δT contributions amount to a contact (nonpropagating) interaction.

Δ propagator modification. Consistent Δ couplings III

Aiming at improving the description of the $\nu_\mu n \to \mu^- n\pi^+$ channel we supplement the model with additional contact terms by modifying

$$\frac{P_{\mu\nu}(p_\Delta)}{p_\Delta^2 - M_\Delta^2 + i\epsilon} \quad \to \quad \frac{P_{\mu\nu}(p_\Delta) + c\left(P_{\mu\nu}(p_\Delta) - \frac{p_\Delta^2}{M_\Delta^2} P_{\mu\nu}^{\frac{3}{2}}(p_\Delta)\right)}{p_\Delta^2 - M_\Delta^2 + i\epsilon} = \frac{P_{\mu\nu}(p_\Delta)}{p_\Delta^2 - M_\Delta^2 + i\epsilon} + c\,\delta P_{\mu\nu}(p_\Delta)$$

$$\to \quad \frac{P_{\mu\nu}(p_\Delta)}{p_\Delta^2 - M_\Delta^2 + iM_\Delta\Gamma_\Delta} + c\,\delta P_{\mu\nu}(p_\Delta)$$

$$= \quad \frac{p_\Delta^2}{M_\Delta^2} \frac{P_{\mu\nu}^{\frac{3}{2}}(p_\Delta)}{p_\Delta^2 - M_\Delta^2 + iM_\Delta\Gamma_\Delta} + \frac{(1+c)(p_\Delta^2 - M_\Delta^2) + icM_\Delta\Gamma_\Delta}{p_\Delta^2 - M_\Delta^2 + iM_\Delta\Gamma_\Delta}\,\delta P_{\mu\nu}(p_\Delta)$$

Due to the presence of Γ_Δ, a value of $c = -1$ does not corresponds exactly to the use of a consistent $\pi N\Delta$ coupling.

Our final value is $c = 1.11 \pm 0.21$

Partial unitarization

Writing the S matrix as

$$S = I - iT$$

the unitarity condition $S^\dagger S = I$ that guarantees the conservation of probability implies

$$i(T - T^\dagger) = T^\dagger T$$

For a given transition between asymptotic states $|I\rangle$, $|F\rangle$ one has

$$i\Big(\langle F|T|I\rangle - \langle F|T^\dagger|I\rangle\Big) \;=\; \langle F|T^\dagger T|I\rangle = \sum_N \langle F|T^\dagger|N\rangle\langle N|T|I\rangle = \sum_N \langle N|T|F\rangle^* \langle N|T|I\rangle$$

For the case of $|I\rangle = |F\rangle$ one has

$$\mathrm{Im}\langle I|T|I\rangle = -\frac{1}{2}\sum_N |\langle N|T|I\rangle|^2$$

which constitutes the optical theorem

Partial unitariztion II

Time reversal invariance states that (Time reversal operator $\mathcal{T}$ is antilinear)

$$\langle F|S|I\rangle = \langle I_{\mathcal{T}}|S|F_{\mathcal{T}}\rangle = \langle F_{\mathcal{T}}|S^{\dagger}|I_{\mathcal{T}}\rangle^{*} = \langle F|\mathcal{T}^{\dagger}S^{\dagger}\mathcal{T}|F\rangle$$

from where

$$\mathcal{T}^{\dagger}S^{\dagger}\mathcal{T} = S \implies \mathcal{T}^{\dagger}T^{\dagger}\mathcal{T} = T$$

and then

$$
\begin{aligned}
\sum_{N}\langle N|T|F\rangle^{*}\langle N|T|I\rangle &= i\Big(\langle F|T|I\rangle - \langle F|T^{\dagger}|I\rangle\Big) = i\Big(\langle F|T|I\rangle - \langle I|T|F\rangle^{*}\Big) \\
&= i\Big(\langle F|T|I\rangle - \langle I|\mathcal{T}^{\dagger}T^{\dagger}\mathcal{T}|F\rangle^{*}\Big) = i\Big(\langle F|T|I\rangle - \langle I_{\mathcal{T}}|T^{\dagger}|F_{\mathcal{T}}\rangle\Big) \\
&= i\Big(\langle F|T|I\rangle - \langle F_{\mathcal{T}}|T|I_{\mathcal{T}}\rangle^{*}\Big)
\end{aligned}
$$

For the case in which $\langle F|T|I\rangle = \langle F_{\mathcal{T}}|T|I_{\mathcal{T}}\rangle$ and there is only one intermediate state $|N\rangle = |F\rangle$ contributing to the sum one arrives at

$$\langle N|T|N\rangle^{*}\langle N|T|I\rangle = -2\mathrm{Im}\langle F|T|I\rangle \in \mathbb{R}$$

so that the phases of $\langle N|T|I\rangle$ and $\langle N|T|N\rangle$ coincide. This result constitutes Watson theorem.
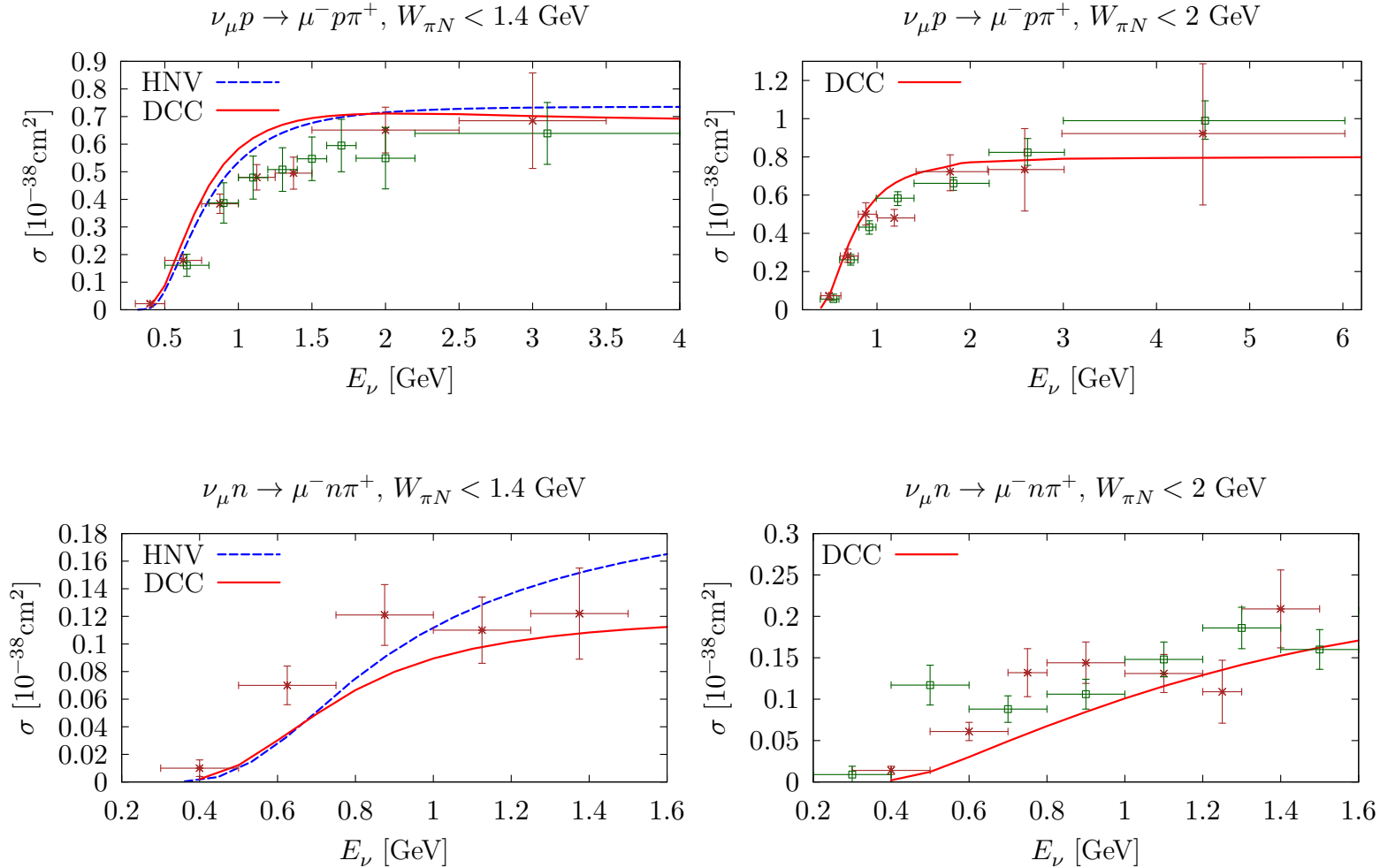
Partial unitarization II

We partially implement Watson theorem. We follow the procedure suggested by M.G. Olsson in NPB78,55 (1974) and change

$$T_B + T_{\Delta P} \rightarrow T_B + e^{i\varphi_V} T^V_{\Delta P} + e^{i\varphi_A} T^A_{\Delta P}$$

The idea was to choose φ_V, φ_A in such a way that each multipole contributing to the total amplitude had the right πN strong phase.

In practice, we were only able to recover Watson theorem for the dominant vector and the dominant axial multipoles with the Δ quatum numbers.

Total cross section results

$\nu_\mu p \to \mu^- p \pi^+$, $W_{\pi N} < 1.4$ GeV

$\nu_\mu p \to \mu^- p \pi^+$, $W_{\pi N} < 2$ GeV

$\nu_\mu n \to \mu^- n \pi^+$, $W_{\pi N} < 1.4$ GeV

$\nu_\mu n \to \mu^- n \pi^+$, $W_{\pi N} < 2$ GeV

Data from P. Rodrigues, C. Wilkinson, and K. McFarland, Eur. Phys. J. C76, 474 (2016) and C. Wilkinson, P. Rodrigues, S. Cartwright, L. Thompson, and K. McFarland, Phys. Rev. D90, 112017 (2014).

DCC stands for the dynamical coupled channel model of described in A. Matsuyama, T. Sato, and T. S. H. Lee, Phys. Rep. 439, 193 (2007) and H. Kamano, S. X. Nakamura, T. S. H. Lee, and T. Sato, Phys. Rev. C 88, 035209 (2013).

Total cross section results II

$\nu_\mu n \to \mu^- p \pi^0$, $W_{\pi N} < 1.4$ GeV

$\nu_\mu n \to \mu^- p \pi^0$, $W_{\pi N} < 2$ GeV

HNV
DCC

DCC

Data P. Rodrigues, C. Wilkinson, and K. McFarland, Eur. Phys. J. C76, 474 (2016).

$\nu_\mu n \to \nu_\mu p \pi^-$

HNV $W_{\pi N} < 1.4$ GeV
DCC $W_{\pi N} < 1.4$ GeV
DCC $W_{\pi N} < 2$ GeV
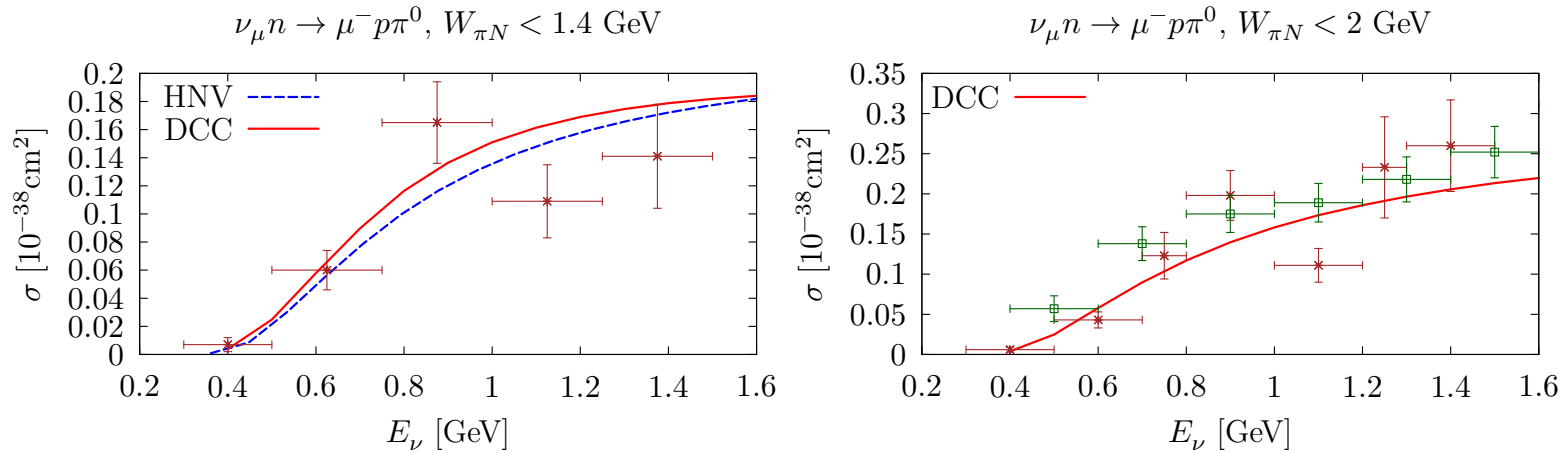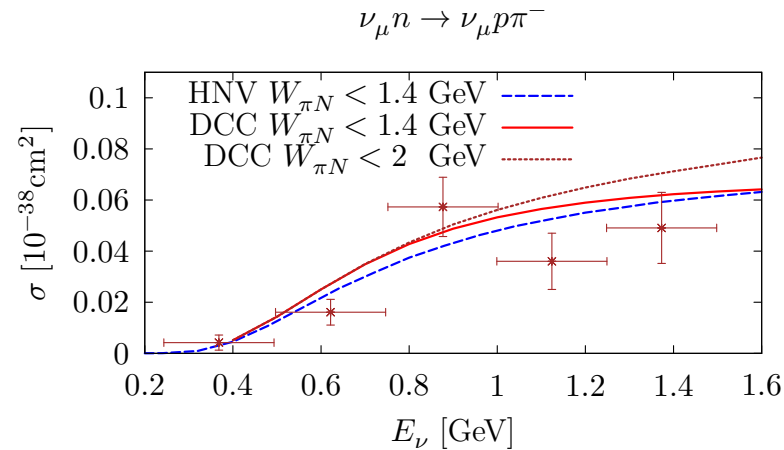
Data from M. Derrick et al., Phys. Lett. 92B, 363 (1980).

Pion angular distribution
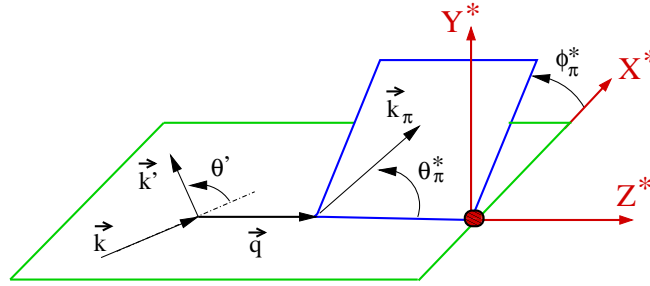
Using Lorentz covariance one can write

$$\frac{d\sigma_{\mathrm{CC}\pm}}{dQ^2\,dW_{\pi N}\,d\Omega_\pi^*} = \frac{G_F^2\,W_{\pi N}}{4\pi M|\vec{k}|^2}\left(A^*\cdot 1 + B^*\cdot\cos\phi_\pi^* + C^*\cdot\cos 2\phi_\pi^* + D^*\cdot\sin\phi_\pi^* + E^*\cdot\sin 2\phi_\pi^*\right)$$

$$A^* = \int \frac{|\vec{k}_\pi^*|^2 d|\vec{k}_\pi^*|}{E_\pi^*}\left[L^{00}W_{00}^{(s)} + 2L^{03}W_{03}^{(s)} + L^{33}W_{33}^{(s)} + \frac{1}{2}(L^{11}+L^{22})\left(W_{11}^{(s)}+W_{22}^{(s)}\right) + 2iL^{12}W_{12}^{(a)}\right]_{\phi_\pi^*=0},$$

$$B^* = \int \frac{|\vec{k}_\pi^*|^2 d|\vec{k}_\pi^*|}{E_\pi^*}\,2[L^{01}\,W_{01}^{(s)} + L^{13}\,W_{13}^{(s)} + iL^{02}\,W_{02}^{(a)} + iL^{23}\,W_{23}^{(a)}]_{\phi_\pi^*=0},$$

$$C^* = \int \frac{|\vec{k}_\pi^*|^2 d|\vec{k}_\pi^*|}{E_\pi^*}\,\frac{1}{2}\left[(L^{11}-L^{22})\left(W_{11}^{(s)}-W_{22}^{(s)}\right)\right]_{\phi_\pi^*=0},$$

$$D^* = \int \frac{|\vec{k}_\pi^*|^2 d|\vec{k}_\pi^*|}{E_\pi^*}\,2[-L^{01}\,W_{02}^{(s)} - L^{13}\,W_{23}^{(s)} + iL^{02}\,W_{01}^{(a)} + iL^{23}\,W_{13}^{(a)}]_{\phi_\pi^*=0},$$

$$E^* = \int \frac{|\vec{k}_\pi^*|^2 d|\vec{k}_\pi^*|}{E_\pi^*}\,\left[(L^{22}-L^{11})\,W_{12}^{(s)}\right]_{\phi_\pi^*=0},$$

s and a stand for symmetric and antisymmetric.

Besides, B^*, D^* have a multiplicative $\sin\theta_\pi^*$ factor and E^* a $\sin^2\theta_\pi^*$ one.

Parity violation
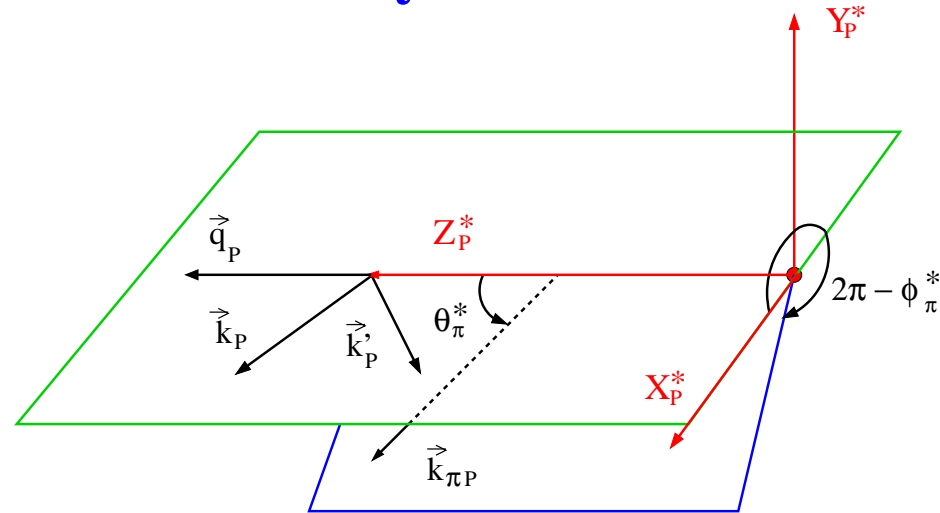
Under parity all three-momenta get reversed but so do the Z^* and X^* axes. As a result the components of k_P^*, $k_P'^*$, q_P^*, p_P^* in the new system remain unchanged. Only $k_{\pi P}^*$ is modified

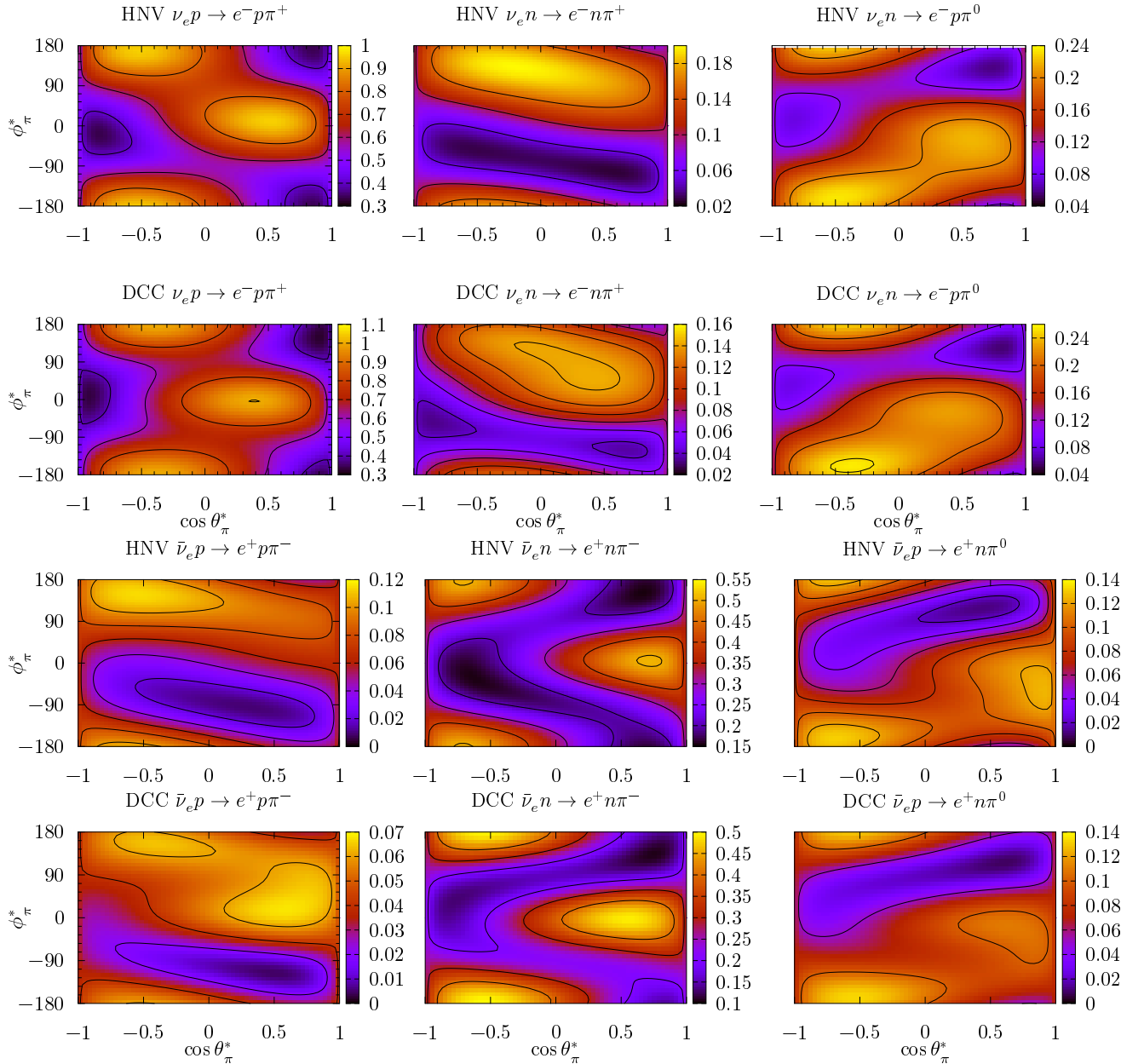$$\theta_\pi^* \to \theta_\pi^* \quad , \quad \phi_\pi^* \to 2\pi - \phi_\pi^*.$$

Thus, the terms in $sin\phi_\pi^*$, $sin2\phi_\pi^*$ change sign.

Parity violation reflects itself in the fact that the pion distributions measured above and below the scattering plane are different.

It originates from the interference between multipoles that have different phases. Below the two pion threshold, all multipoles with given total angular momentum, parity and pion orbital angular momentum quantum numbers have the same phase fixed by Watson theorem. Thus, a proper unitarization is essential to get a good reproduction of the parity violation observables.

Pion angular distribution III

CC processes

HNV $\nu_e p \to e^- p \pi^+$

HNV $\nu_e n \to e^- n \pi^+$

HNV $\nu_e n \to e^- p \pi^0$

DCC $\nu_e p \to e^- p \pi^+$

DCC $\nu_e n \to e^- n \pi^+$

DCC $\nu_e n \to e^- p \pi^0$

HNV $\bar{\nu}_e p \to e^+ p \pi^-$

HNV $\bar{\nu}_e n \to e^+ n \pi^-$

HNV $\bar{\nu}_e p \to e^+ n \pi^0$

DCC $\bar{\nu}_e p \to e^+ p \pi^-$

DCC $\bar{\nu}_e n \to e^+ n \pi^-$

DCC $\bar{\nu}_e p \to e^+ n \pi^0$

$d\sigma/dQ^2 dW_{\pi N} d\Omega_\pi^*$
$[10^{-38} \mathrm{cm}^2 c^2 / \mathrm{GeV}^2]$

$E_\nu = 1\,\mathrm{GeV}, \; Q^2 = 0.1\,\mathrm{GeV}^2/c^2$

$W_{\pi N} = 1.23\,\mathrm{GeV}$

Pion angular distribution IV

Same as before for NC processes
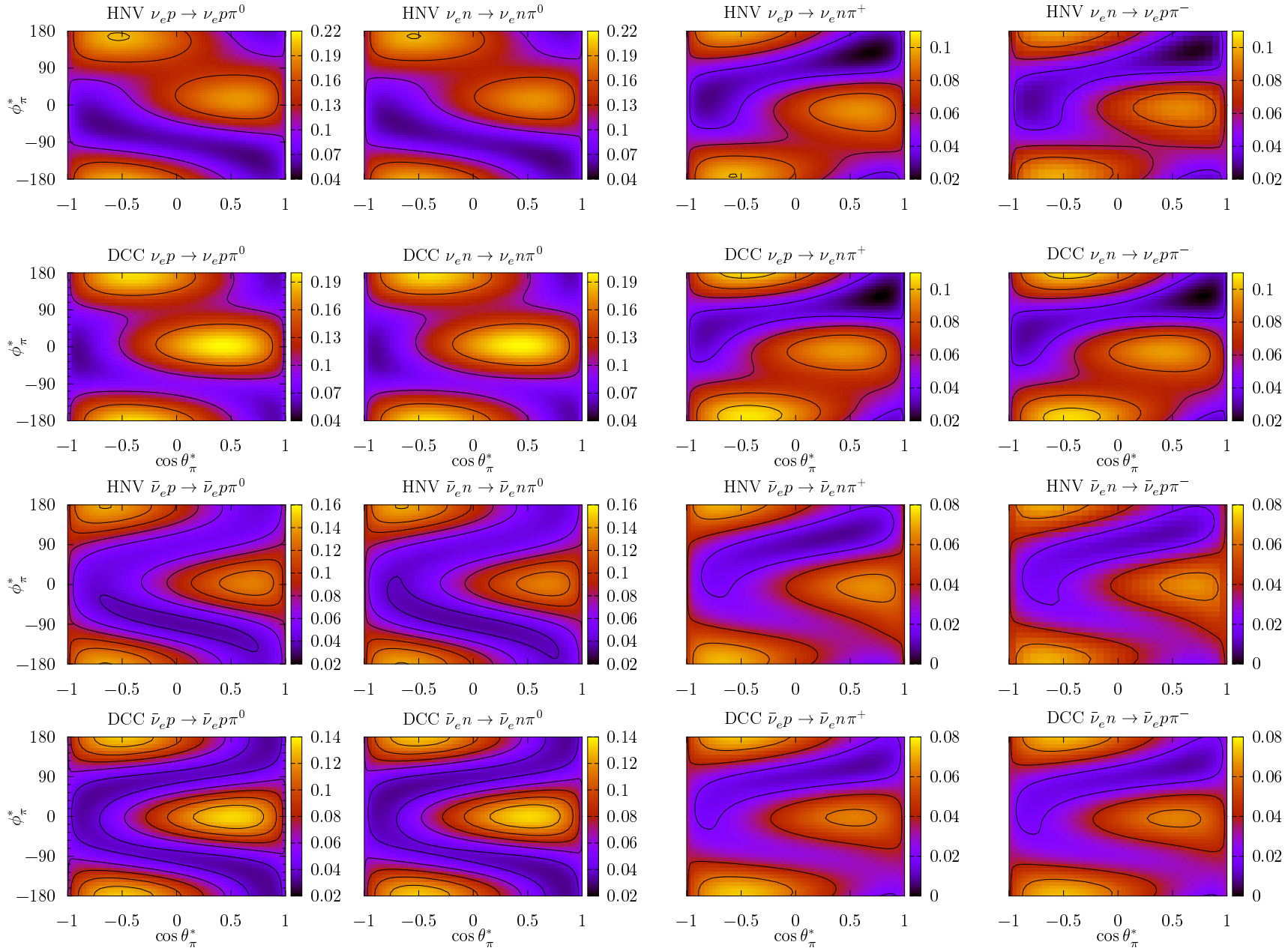
Pion angular distributions V
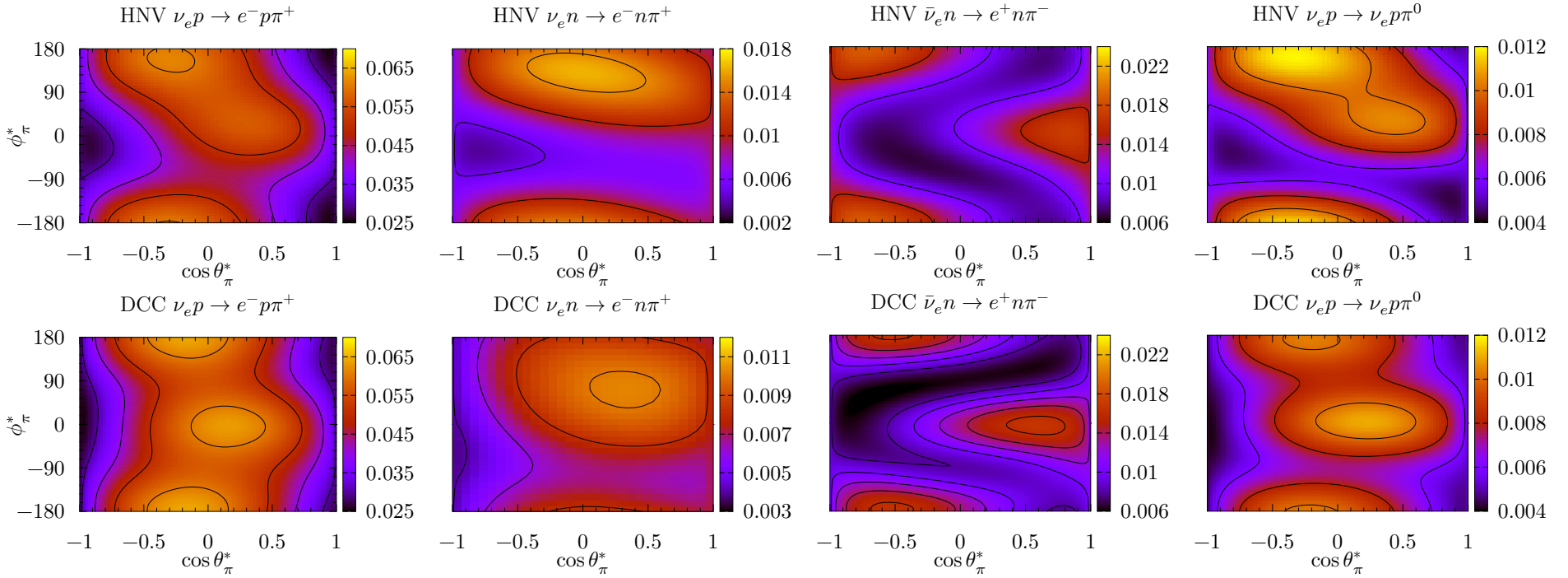
$d\sigma/d\Omega_\pi^*[10^{-38}\mathrm{cm}^2]$, evaluated at $E_\nu = 1\,\mathrm{GeV}$ and with a $W_{\pi N} < 1.4\,\mathrm{GeV}$ cut.

A clear anisotropy is still seen and the distribution is channel dependent

Pion angular distribution VI
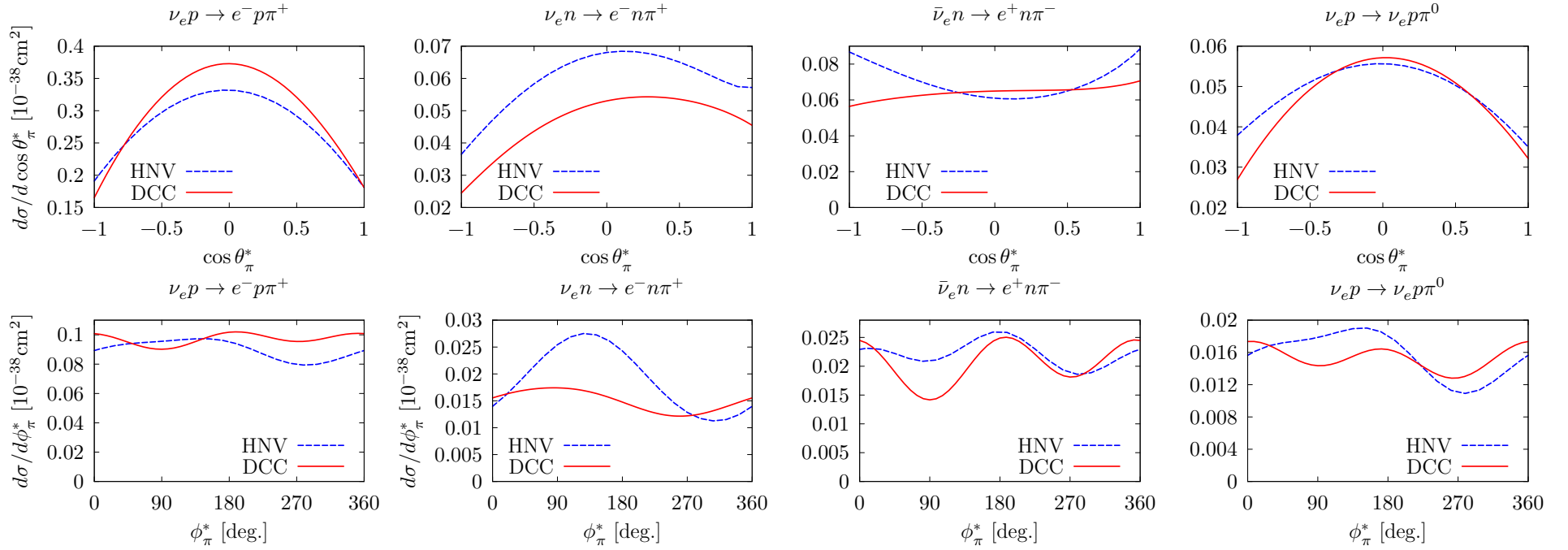
$d\sigma/d\cos\theta_\pi^*$ and $d\sigma/d\phi_\pi^*$ in units of $10^{-38}\mathrm{cm}^2$ at $E_\nu = 1\,\mathrm{GeV}$ and with $W_{\pi N} < 1.4\,\mathrm{GeV}$

Pion angular distribution VII

ANL $\nu_\mu p \to \mu^- p \pi^+$

BNL $\nu_\mu p \to \mu^- p \pi^+$

HNV ------
HNV1 —·—·—
HNV2 ········
DCC ——
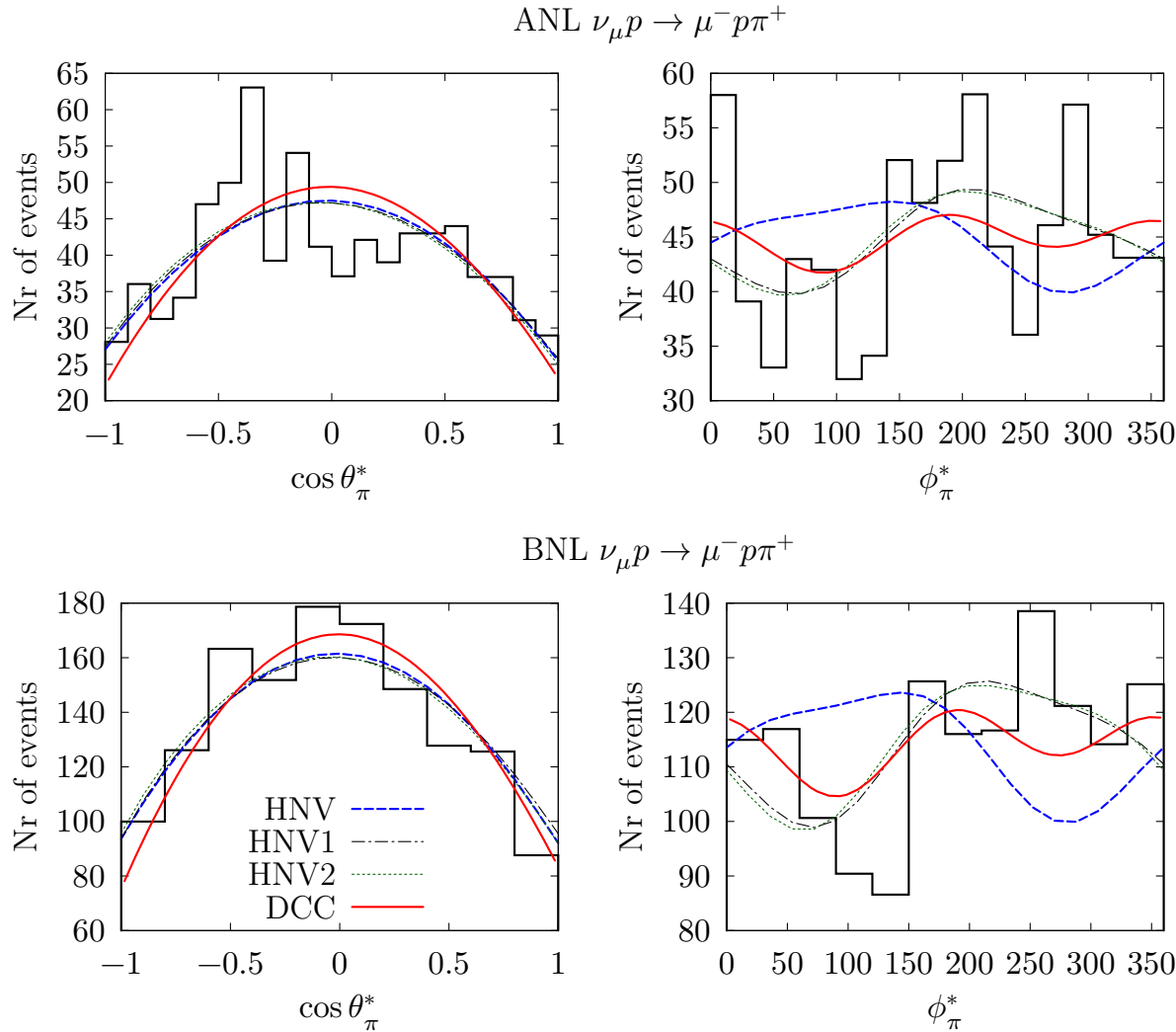
Unnormalized, flux-averaged $d\sigma/d\cos\theta_\pi^*$ and $d\sigma/d\phi_\pi^*$. $W_{\pi N} < 1.4\,\text{GeV}$.

ANL data: G. M. Radecky et al, Phys. Rev. D 25, 1161 (1982).

BNL data: T. Kitagaki, Phys. Rev. D 34, 2554 (1986).

Incoherent pion production inside the nucleus I

PRD 87,113009 (2013)

We assume the nucleus can be described by its density and we shall use the local density approximation

The cross section at the nucleus level for initial pion production (prior to any FSI) is then

$$\frac{d\sigma}{d\cos\theta_\pi dE_\pi} = \int d^3r \sum_{N=n,p} 2 \int \frac{d^3 p_N}{(2\pi)^3} \theta(E_F^N(r) - E_N) \theta(E_N + q^0 - E_\pi - E_F^{N'}(r))$$

$$\times \frac{d\sigma(\nu N \to l^- N'\pi)}{d\cos\theta_\pi dE_\pi}$$

To compare with experiment, we have to convolute it with the neutrino flux $\Phi(|\vec{k}|)$

$$\frac{d\sigma}{d\cos\theta_\pi dE_\pi} = \int d|\vec{k}|\, \Phi(|\vec{k}|)\, 4\pi \int dr r^2 \sum_{N=n,p} 2 \int \frac{d^3 p_N}{(2\pi)^3} \theta(E_F^N(r) - E_N) \theta(E_N + q^0 - E_\pi - E_F^{N'}(r))$$

$$\times \frac{d\sigma(\nu N \to l^- N'\pi)}{d\cos\theta_\pi dE_\pi}$$

Incoherent pion production inside the nucleus II

From there we obtain

$$\frac{d\sigma}{d|\vec{k}|\,4\pi r^2 dr\, d\cos\theta_\pi\, dE_\pi} = \Phi(|\vec{k}|) \sum_{N=n,p} 2 \int \frac{d^3 p_N}{(2\pi)^3}\, \theta(E_F^N(r) - E_N)\, \theta(E_N + q^0 - E_\pi - E_F^{N'}(r))$$

$$\times \frac{d\sigma(\nu N \to l^- N' \pi)}{d\cos\theta_\pi\, dE_\pi}$$

Apart from modifications discussed in what follows, the above differential cross section is used in our simulation code to generate, in a given point inside the nucleus and by neutrinos of a given energy, pions with a certain charge, energy and momentum direction.

Defining $P = q - k_\pi$ (the four momentum transferred to the nucleus) and writing $d^3 p_N = d\cos\vartheta_N\, d\phi_N\, |\vec{p}_N| E_N dE_N$, where the angles are referred to a system in which the Z axis is along $\vec{P}$, we can integrate in the ϑ_N variable using the energy delta function present in $\frac{d\sigma(\nu N \to l^- N' \pi)}{d\cos\theta_\pi\, dE_\pi}$

Incoherent pion production inside the nucleus III

The final result is

$$
\frac{d\sigma}{d|\vec{k}|4\pi r^2\, dr\, d\cos\theta_\pi\, dE_\pi} = \Phi(|\vec{k}|) \int d\Omega'\, dE'\, |\vec{k}'| \Bigg\{ \sum_{N=n,p} \frac{G_F^2}{512\pi^7} \frac{|\vec{k}_\pi|}{|\vec{P}|\,|\vec{k}|} \theta(E_F^N(r)-\mathcal{E})\, \theta(-P^2)\, \theta(P^0)\, \mathcal{L}_{\mu\sigma}(k,k')
$$

$$
\int_0^{2\pi} d\phi_N \int_{\mathcal{E}}^{E_F^N(r)} dE_N\, \mathcal{W}^{\mu\sigma}(p_N,q,k_\pi) \Bigg|_{\cos\vartheta_N = \cos\vartheta_N^0} \Bigg\},
$$

where

$$
\cos\vartheta_N^0 = \frac{P^2 + 2E_N P^0}{2|\vec{p}_N||\vec{P}|}, \quad \mathcal{E}' = \frac{-P^0 + |\vec{P}|\sqrt{1-4M^2/P^2}}{2}, \quad \mathcal{E} = \max\{M, E_F^{N'} - P^0, \mathcal{E}'\}.
$$

To speed up the computational time, we approximate the last two integrals by

$$
\int_0^{2\pi} d\phi_N \int_{\mathcal{E}}^{E_F^N(r)} dE_N\, \mathcal{W}^{\mu\sigma}(p_N,q,k_\pi) \Bigg|_{\cos\vartheta_N = \cos\vartheta_N^0} \approx 2\pi(E_F^N(r)-\mathcal{E})\mathcal{W}^{\mu\sigma}(\tilde{p}_N,q,k_\pi) \Bigg|_{\cos\vartheta_N = \cos\vartheta_N^0}
$$

where $\tilde{p}_N$ is evaluated at the value $\tilde{E}_N = (E_F^N(r) + \mathcal{E})/2$, (middle of the integration interval), with the corresponding $\cos\tilde{\vartheta}_N^0$ value, and $\tilde{\phi}_N$ is set to zero.

Similar approximations were done, in the works of Carrasco et al. and Gil et al. to study pion photo- and electroproduction in nuclei.

Medium corrections I

Δ properties are strongly modified in the nuclear medium.

Its imaginary part changes due to

- Pauli blocking of the final nucleon which reduces the free width.

- In medium modification of the pionic decay width other than Pauli blocking

- Absorption processes $\Delta N \rightarrow NN$ and $\Delta NN \rightarrow NNN$.

We thus modify the Δ propagator of the direct Δ contribution approximating

$$\frac{1}{p_\Delta^2 - M_\Delta^2 + iM_\Delta\Gamma_\Delta} \approx \frac{1}{\sqrt{p_\Delta^2} + M_\Delta} \frac{1}{\sqrt{p_\Delta^2} - M_\Delta + i\Gamma_\Delta/2}$$

and substituting

$$\frac{\Gamma_\Delta}{2} \rightarrow \frac{\Gamma_\Delta^{\text{Pauli}}}{2} - \text{Im}\,\Sigma_\Delta$$

while keeping M_Δ in the propagator unchanged.

Medium corrections II

Delta width in a nuclear medium
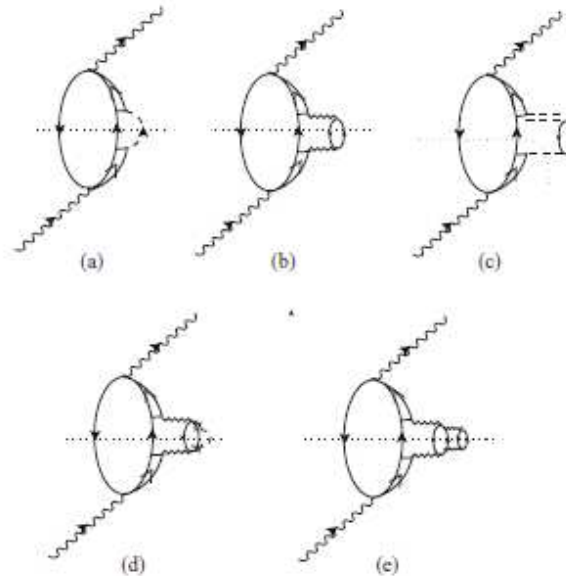
The double dashed line represents the effective spin-isospin interaction originated by π and ρ exchange in the presence of short range correlations.

The wavy line includes an RPA sum with particle-hole and Delta-hole excitations.

The evaluation of $\mathrm{Im}\,\Sigma_\Delta$ was done by E. Oset and L.L. Salcedo [Nuc. Phys. A468 (1987) 631].

Medium corrections III

The imaginary part can be parameterized as

$$-\mathrm{Im}\,\Sigma_\Delta = C_Q \left(\frac{\rho}{\rho_0}\right)^\alpha + C_{A2} \left(\frac{\rho}{\rho_0}\right)^\beta + C_{A3} \left(\frac{\rho}{\rho_0}\right)^\gamma$$

with $\rho_0 = 0.17\,\mathrm{fm}^{-3}$.

- The C_Q term corrects the pionic decay in the medium.

- The C_{A2} term corresponds to the process $\Delta N \to NN$

- The C_{A3} term corresponds to the $\Delta NN \to NNN$ process

The C_Q, α, C_{A2}, β and C_{A3}, γ coefficients are parametrized as a function of the kinetic energy of a pion that would excite a Δ of the corresponding invariant mass and are valid in the range $85\,\mathrm{MeV} < T_\pi < 315\,\mathrm{MeV}$.

Below 85 MeV the contributions from C_Q and C_{A3} are rather small and we take them from Nieves et al. [Nuc. Phys. A 554 (1993) 554], where the model was extended to lower energies. The term with C_{A2} shows a very mild energy dependence and we still use the original parameterization even at low energies.

For T_π above 315 MeV, we have kept these self-energy terms constant and equal to their values at the bound. The uncertainties in these pieces are not very relevant there because the $\Delta \to N\pi$ decay becomes very large and dominant.

Incoherent pion production inside the nucleus IV

The C_Q term not only modifies the Δ propagator but it also gives rise to a new source of pion production in the nuclear medium that has to be taken into account.

This new contribution has to be added incoherently and we implement it in a approximate way by taking as amplitude square for this process the amplitude square of the ΔP contribution multiplied by

$$\frac{C_Q(\rho/\rho_0)^\alpha}{\Gamma_\Delta^{\text{Free}}/2}$$

Its effect increases the total pion production cross section by less that 10%

Final state interaction

Once the pions are produced, we follow their path on its way out of the nucleus.

We use, with slight modifications, the model of L.L. Salcedo et al. [Nuc. Phys. A484 (1988) 557]

- P and S-wave pion absorption.

- P and S-wave quasielastic scattering on a nucleon.
  - Pions change energy and direction.
  - Pions could change charge.

- Pion propagate on straight lines in between collisions.

The P- wave interaction is mediated by the Δ resonance excitation where the different contributions to the imaginary part of its self-energy give rise to pion two- and three-nucleon absorption and quasielastic processes.

The intrinsic probabilities for each of the above mentioned reactions are evaluated microscopically as a function of the density and we use the local density approximation to evaluate them in finite nuclei.

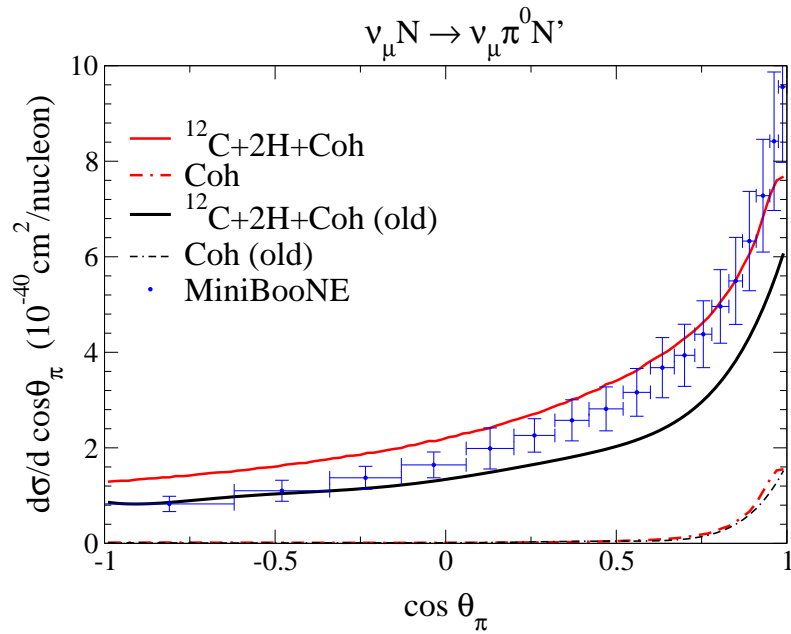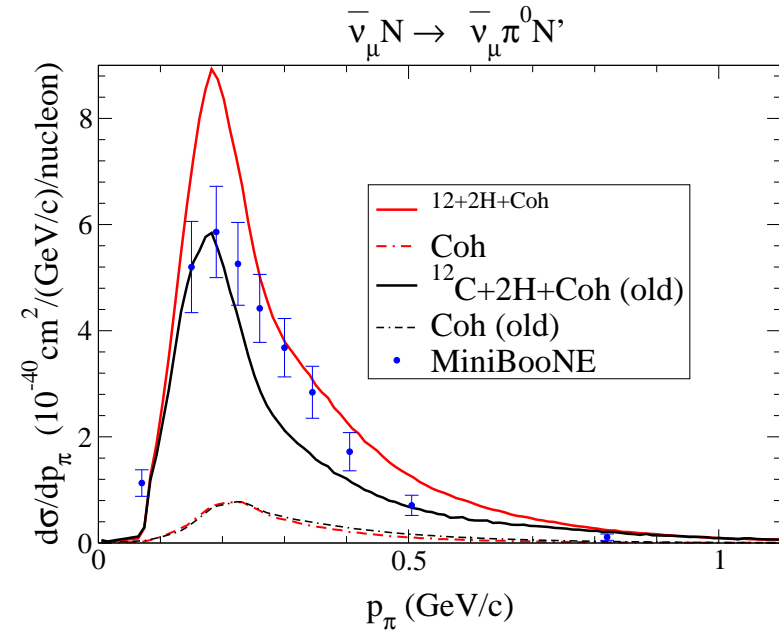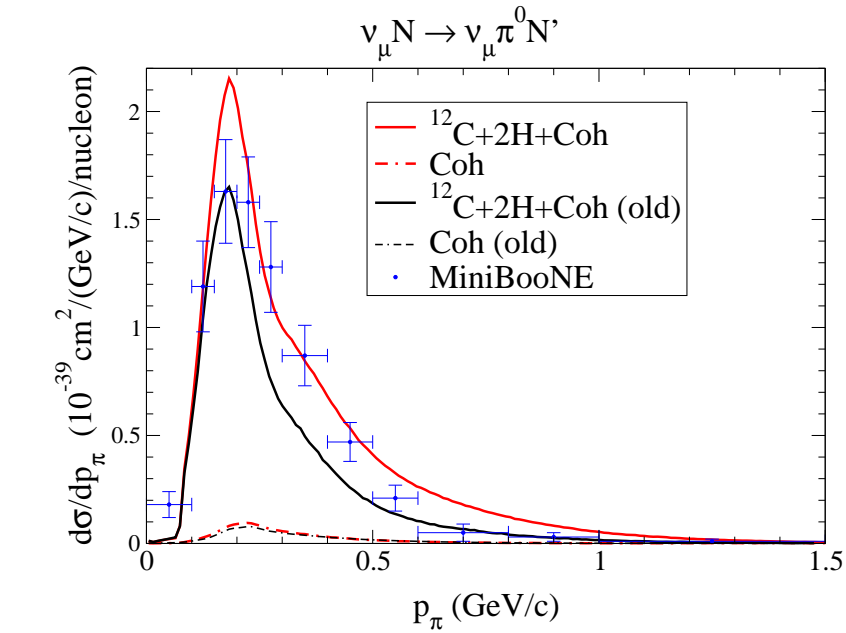# (preliminary) Results for MiniBooNE (CH_2 target) I
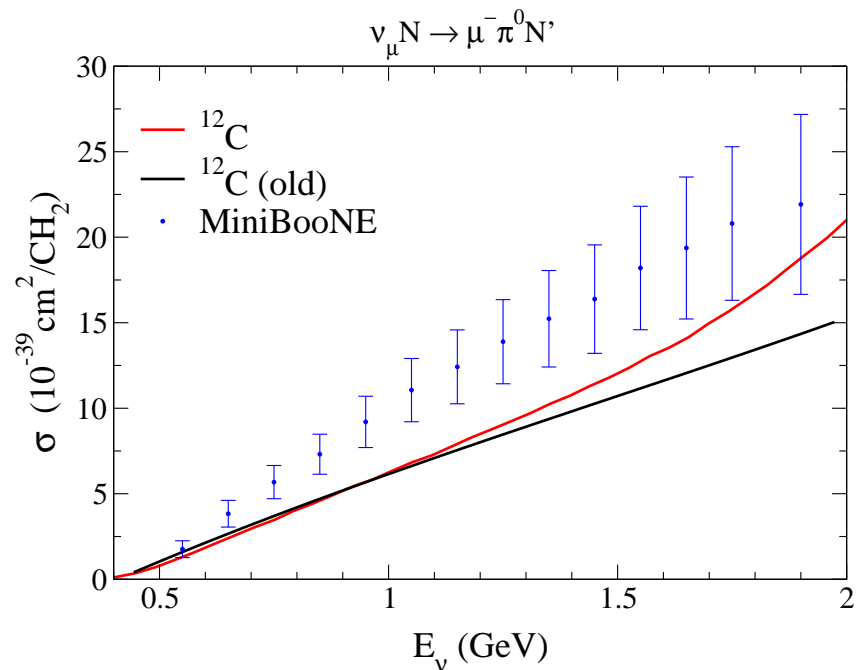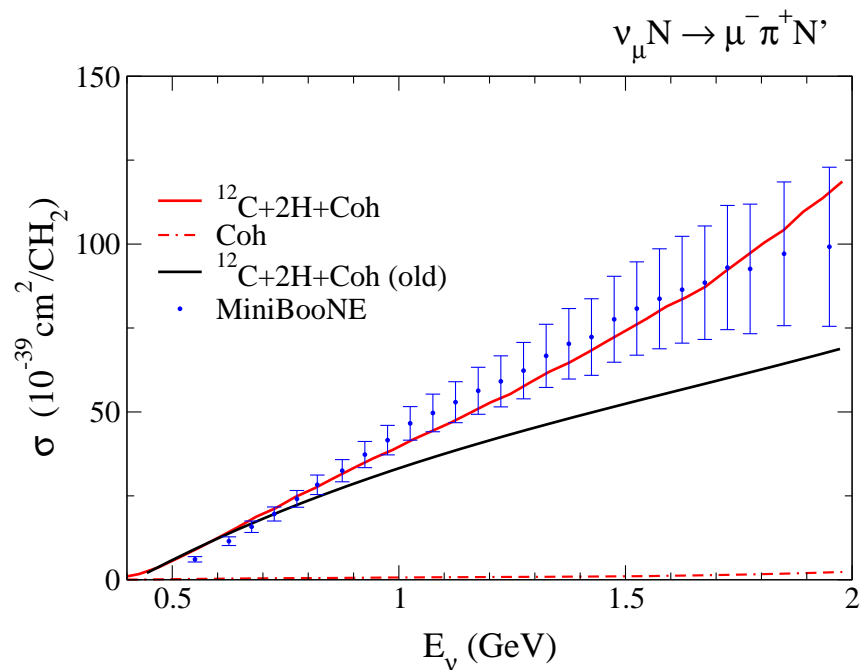
We get a better agreement at higher energies than before.
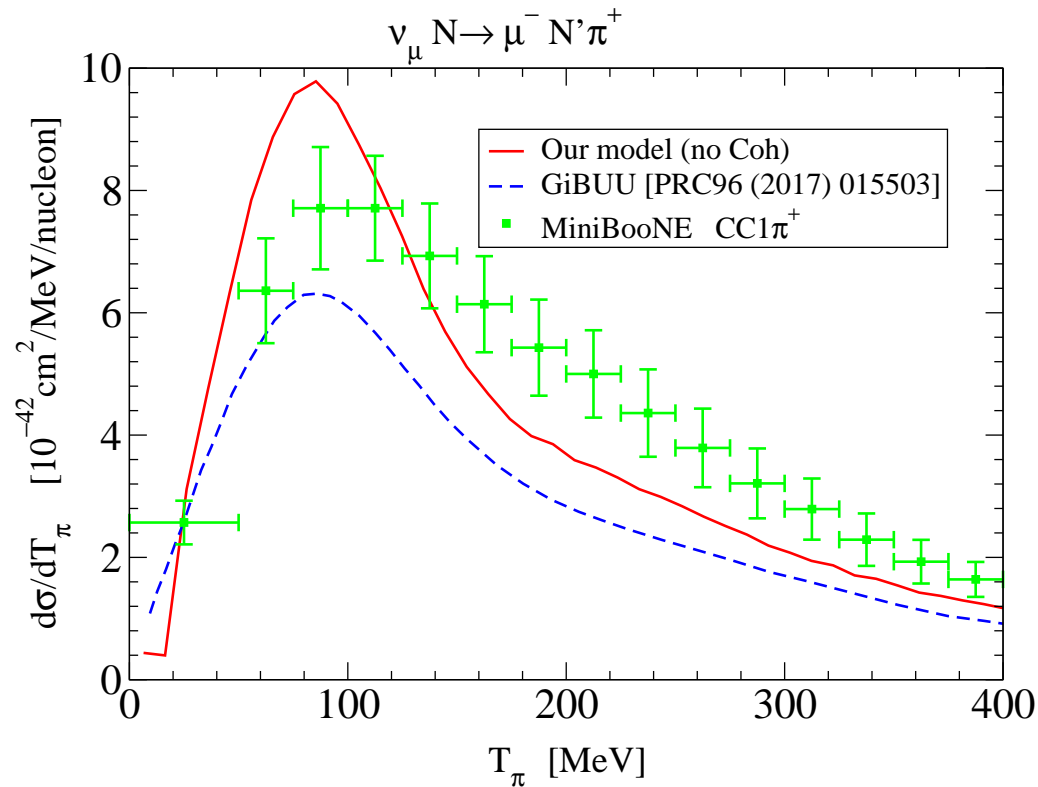
(preliminary) Results for MiniBooNE (CH_2 target) II
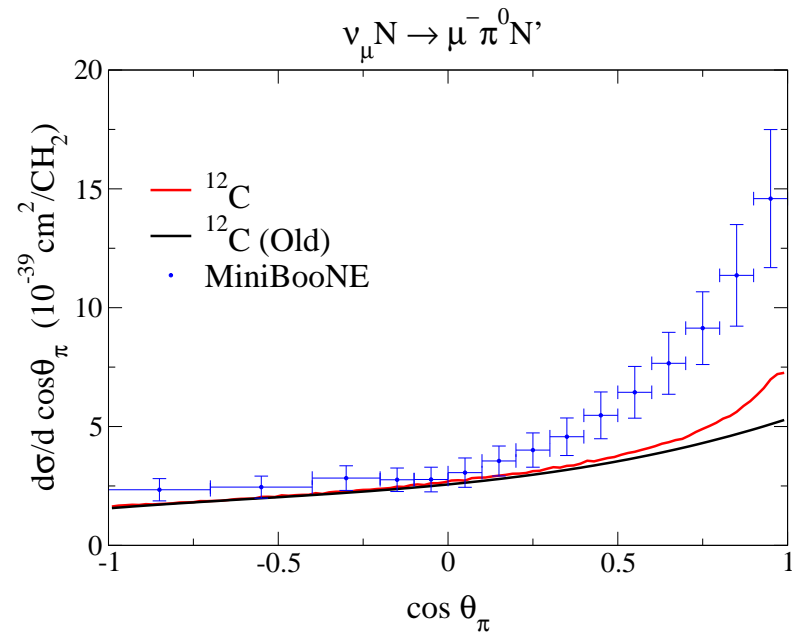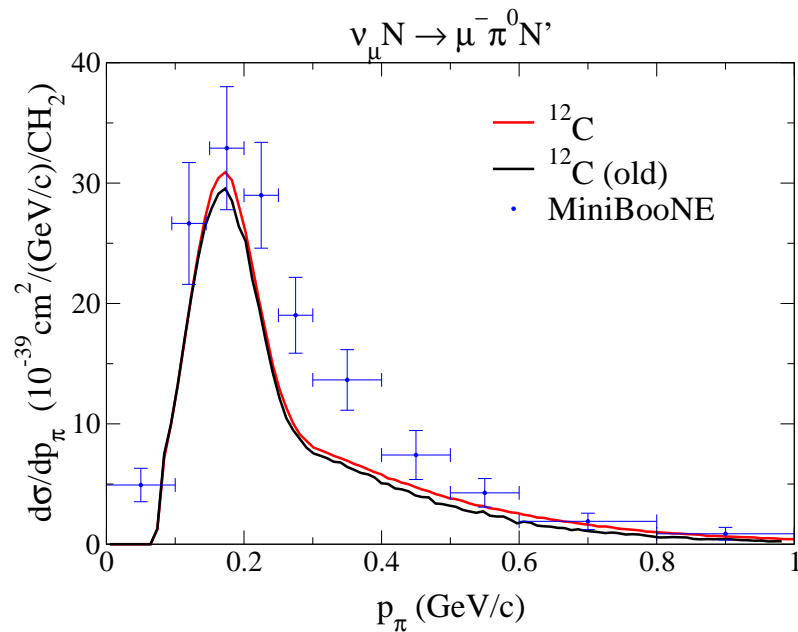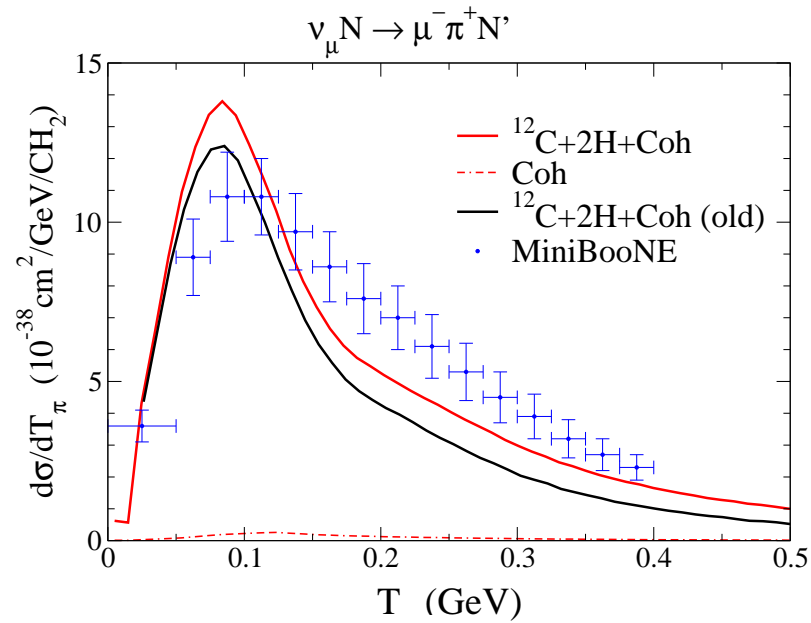
(preliminary) Results for MiniBooNE (CH_2 target) II
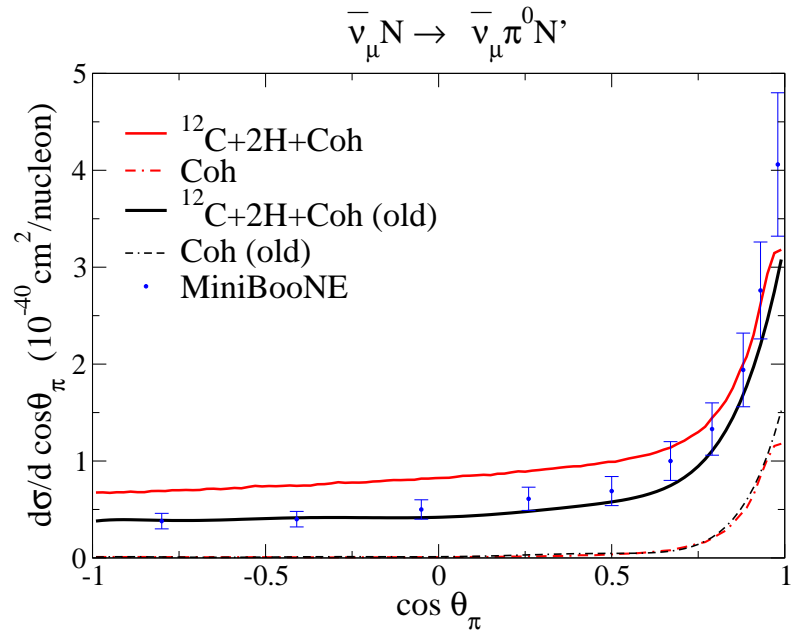
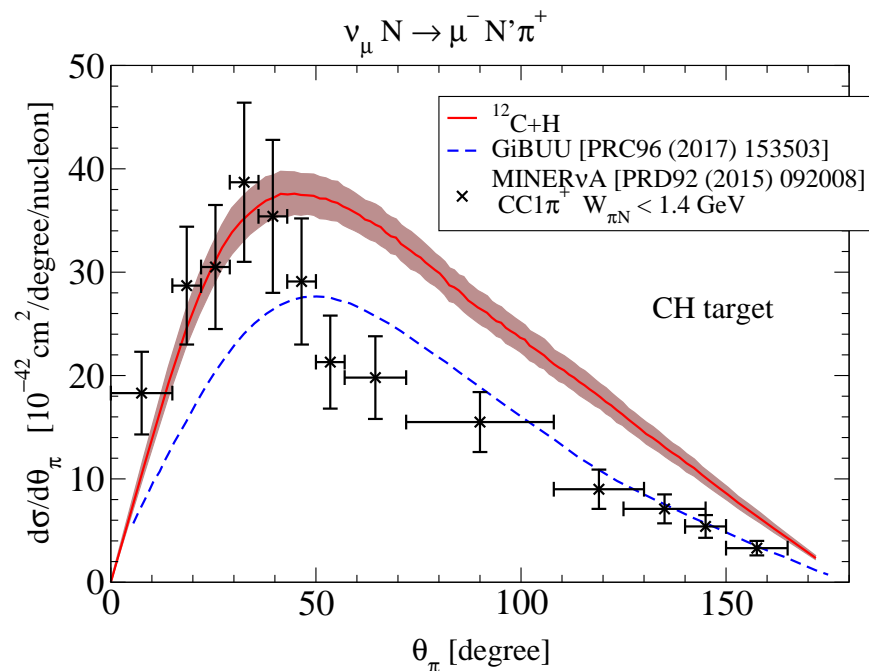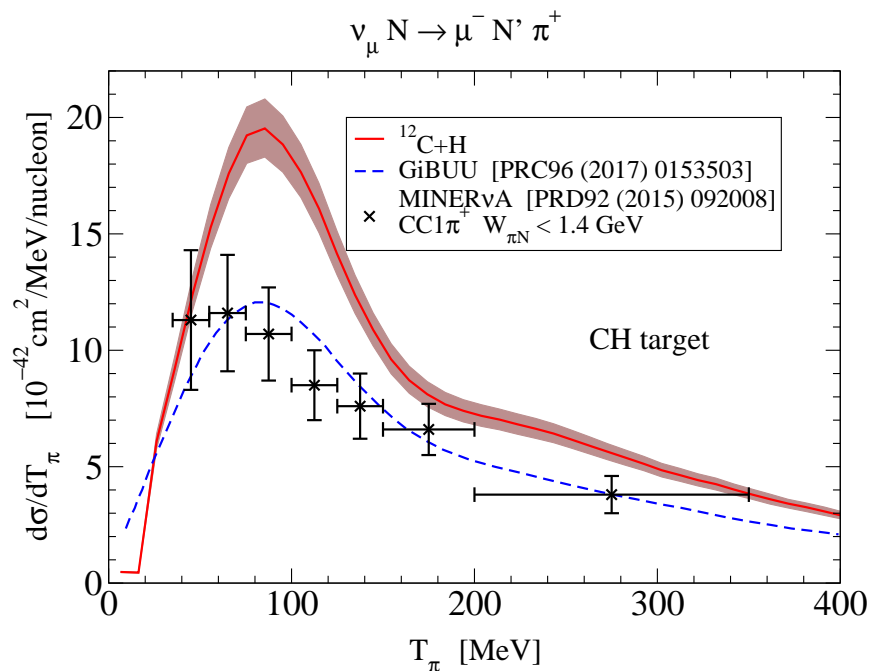# (preliminary) Results for MiniBooNE (CH_2 target)III

Comparison with GiBUU [PRC96 (2017) 015503]

Our results for high energy pions are in better agreement with experiment.

(preliminary) Results for MINERνA (CH target)

We integrate the MINERνA flux up to $E_\nu = 5\,\text{GeV}$.

We produce too many pions in the backward direction while GiBUU underestimates the production of forward pions.

GiBUU gives a good reproduction of the $d\sigma/dT_\pi$ differential cross section

(preliminary) Results for T2K (H_2O target)

We integrate the T2K flux up to $E_\nu = 2\,\text{GeV}$.

We implement the cuts on the muon momentum on production (neglecting FSI for the muon)
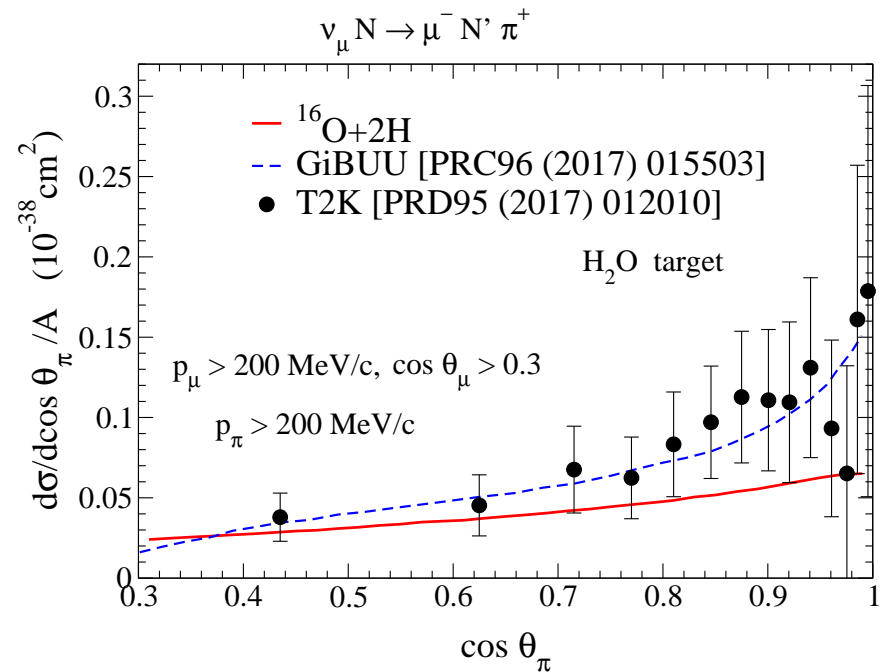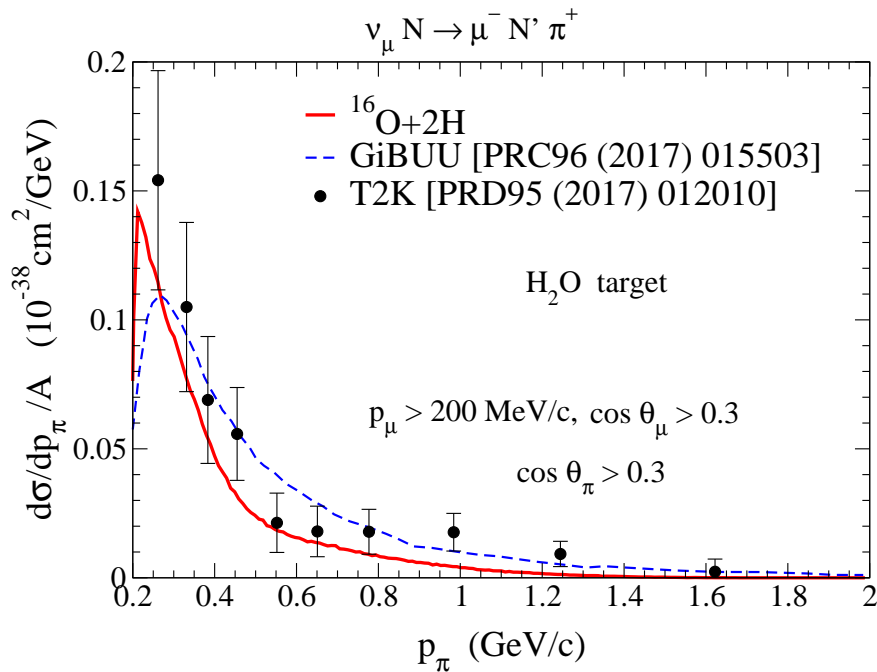
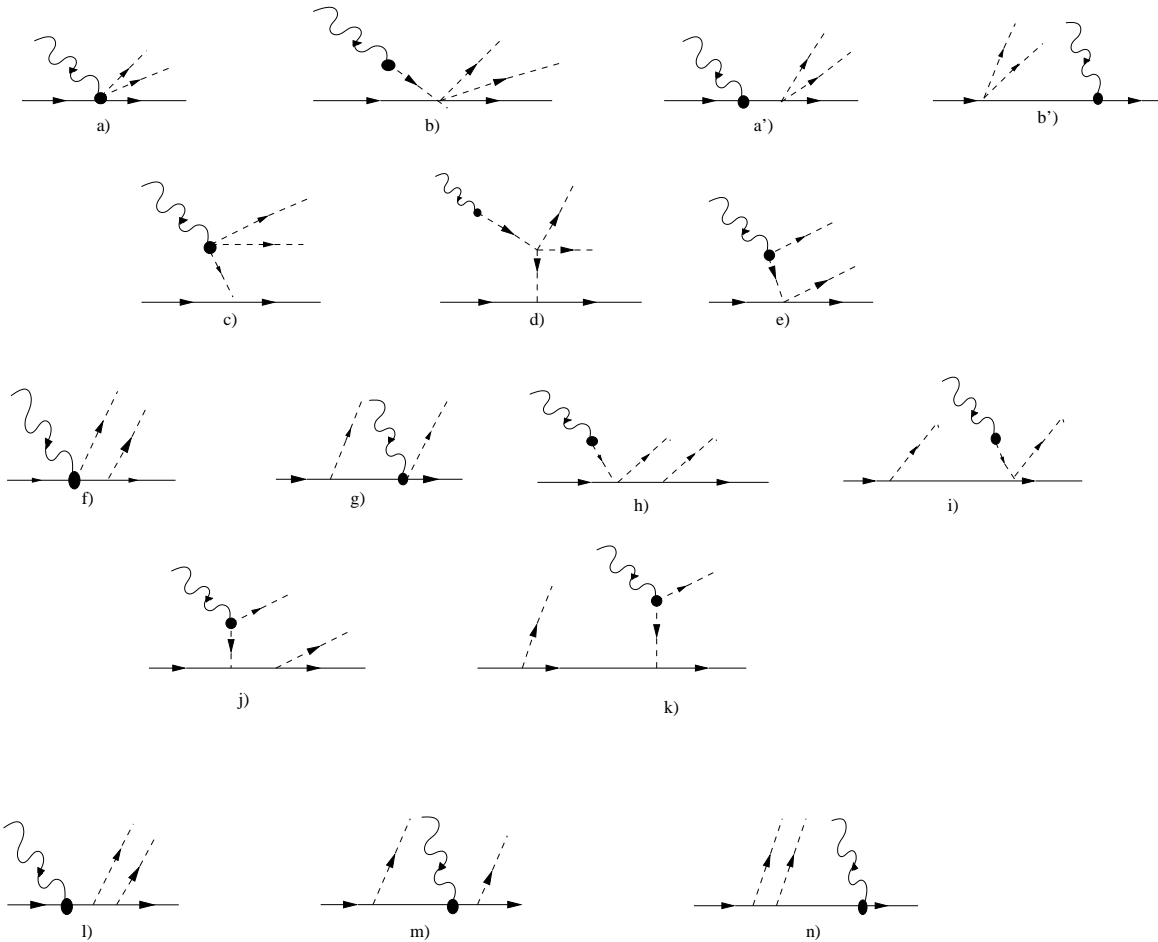$$\langle \sigma \rangle_\phi^{\text{exp.}} = 4.25 \pm 0.48 \pm 1.56 \times 10^{-40}\text{cm}^2$$

$$\langle \sigma \rangle_\phi^{\text{GiBUU}} \approx 4.0 \times 10^{-40}\text{cm}^2$$

$$\langle \sigma \rangle_\phi^{\text{ours}} \approx 2.8 \times 10^{-40}\text{cm}^2$$

Two-pion production close to threshold. Nonresonant production

We have in total 16 different contributions corresponding to the Feynman diagrams

Two-pion production close to threshold. Resonant production

Since the Δ does not couple to two pions in s-wave, Δ excitation contributions are expected to be small when only slow pions are produced (close to threshold).

The Roper resonance $N^*(1440)$ on the other hand has a sizeable decay into a scalar pion pair and it is very wide so that its contribution could be large. In fact, the $N^*(1440)$ plays a major role in $\pi N \to \pi \pi N$ and $NN \to \pi \pi NN$ reactions for certain channels and close to threshold.

Our work was the first attempt to include the $N^*(1440)$ in two pion production in weak processes, the study of which could serve to study electroweak nucleon to Roper transition form factors.

Two-pion production close to threshold.

$N^*(1440)$ contribution

In this case we have two contributions coming form direct an crossed Roper excitation

For the s-wave $N^* \to N\pi\pi$ decay we use the Lagrangian

$$\mathcal{L}_{N^* N\pi\pi} = -c_1^* \frac{m_\pi^2}{f_\pi^2} \bar{\psi}_{N^*} \vec{\phi}^2 \Psi + c_2^* \frac{1}{f_\pi^2} \bar{\psi}_{N^*} (\vec{\tau}\partial_0\vec{\phi})(\vec{\tau}\partial_0\vec{\phi})\Psi + h.c. \, .$$

Assuming a branching ratio of 7.5% for the $N(\pi\pi)_{J=0}^{I=0}$ decay mode and a total decay width of $\Gamma_{\text{tot}} = 350$ MeV, the best agreement with $NN \to \pi\pi NN$ and the $\pi N \to \pi\pi N$ data was obtained by L. Alvarez-Ruso et al. [NPA633, 519 (1998)] using

$$c_1^* = -7.27\,\text{GeV}^{-1}, \ c_2^* = 0\,\text{GeV}^{-1}$$

Since in the above Lagrangian the pions are coupled to isospin 0 the Roper excitation contributes only for final $\pi^+\pi^-$ and $\pi^0\pi^0$ channels

Two-pion production close to threshold.

$N \to N^*(1440)$ weak current

The $W^+ n \to N^{*+}(1440)$ weak transition current is given by

$$\cos\theta_C \, \bar{u}_*(\vec{p}_*) J^\alpha_{cc*} u(\vec{p}) \, ,$$

with

$$J^\alpha_{cc*} = \frac{F_1^{V*}(q^2)}{\mu^2}(q^\alpha \slashed{q} - q^2\gamma^\alpha) + i\frac{F_2^{V*}(q^2)}{\mu}\sigma^{\alpha\nu}q_\nu - G_A\gamma^\alpha\gamma_5 - \frac{G_P}{\mu}q^\alpha\slashed{q}\gamma_5 - \frac{G_T}{\mu}\sigma^{\alpha\nu}q_\nu\gamma_5$$

with $\mu = M + M_*$.

The vector part is the most general compatible with current conservation.

Although the G_T term is in principle allowed by G-parity invariance, we shall assume it is zero.

Two-pion production close to threshold

$N \to N^*(1440)$ axial form factors

Assuming the pseudoscalar coupling G_P is dominated by the pion pole contribution, and imposing partial conservation of the axial current (PCAC) hypothesis

$$G_P(q^2) = \frac{\mu}{m_\pi^2 - q^2} G_A(q^2) \,.$$

$G_A(0)$ can be related with the $N^* N \pi$ coupling constant at $q^2 = 0$ using the corresponding non-diagonal Goldberger–Treiman relation

$$G_A(0) = 2 f_\pi \frac{\tilde{f}}{m_\pi} = 0.63$$

with $\tilde{f}/m_\pi$ the $N^* N \pi$ strong coupling determined from the $\Gamma_{N^* \to N\pi}$ decay width.

For the q^2 dependence of $G_A(q^2)$ we assumed a dipole form

$$G_A(q^2) = \frac{G_A(0)}{(1 - q^2/M_{A*}^2)^2} \,,$$

with an axial mass $M_{A*} = 1$ GeV

Two-pion production close to threshold

$N \to N^*(1440)$ vector form factors I

The vector-isovector form factors F_1^{V*}/μ^2 and F_2^{V*}/μ can be related to the isovector part of the electromagnetic form factors, $F_i^{V*} = F_i^{p*} - F_i^{n*}$. The experimental information is given in terms of helicity amplitudes

$$A_{1/2}^N = \sqrt{\frac{2\pi\alpha}{k_R}} \langle N^* \uparrow | \sum_{\text{pol}} \epsilon \cdot j_{\text{e.m.}}(0) | N \downarrow \rangle \, \xi$$

$$S_{1/2}^N = \sqrt{\frac{2\pi\alpha}{k_R}} \frac{|\vec{q}|}{\sqrt{-q^2}} \langle N^* \uparrow | \sum_{\text{pol}} \epsilon \cdot j_{\text{e.m.}}(0) | N \uparrow \rangle \, \xi \, ,$$

$\alpha = 1/137$, $k_R = (W^2 - M^2)/2W$ with W the energy of the Roper in its center of mass and ϵ stands for the polarization vectors.

ξ is the relative sign between the $NN\pi$ and $N^*N\pi$ couplings which we have taken to be positive (I shall come back to this later).

Two-pion production close to threshold

$N \to N^*(1440)$ vector form factors II

Following quark models predictions [L.A. Copley et al., PLB29, 117 (1969)] we assume

$$A^n_{1/2} = -2/3 A^p_{1/2}, \ \ S^n_{1/2} = 0$$

We fit the $\gamma p \to N^*$ electromagnetic form factors to experiment [I.G. Aznauryan et al., PRC71, 015201 (2005); L. Tiator et al., EPJA19, 55 (2004)] using a parametrization from O. Lalakulich et al. [PRD74, 014009 (2006)]

$$F_1^{p*}(q^2) = \frac{g_1^p / D_V}{1 - q^2 / X_1 M_V^2},$$

$$F_2^{p*}(q^2) = \frac{g_2^p}{D_V} \left(1 - X_2 \ln\left(1 - \frac{q^2}{1\,\mathrm{GeV}^2}\right)\right)$$

where $D_V = \left(1 - q^2 / M_V^2\right)^2$ with $M_V = 0.84$ GeV.
The best fit parameters are:

$$g_1^p = -5.7 \pm 0.9, \quad g_2^p = -0.64 \pm 0.04, \quad X_1 = 1.4 \pm 0.5, \quad X_2 = 2.47 \pm 0.12,$$

This defines our FF1 set of vector form factors.

Two-pion production close to threshold

$N \to N^*(1440)$ vector form factors III

The $\gamma p \to N^*$ data have large uncertainties so that other fits are possible.
We shall consider three more sets of vector form factors:

- Set FF2. Quark model calculation of U. Meyer et al., PRC 64, 035203 (2001)

- Set FF3. This we take from O. Lalakulich et al., PRD74, 014009 (2005)

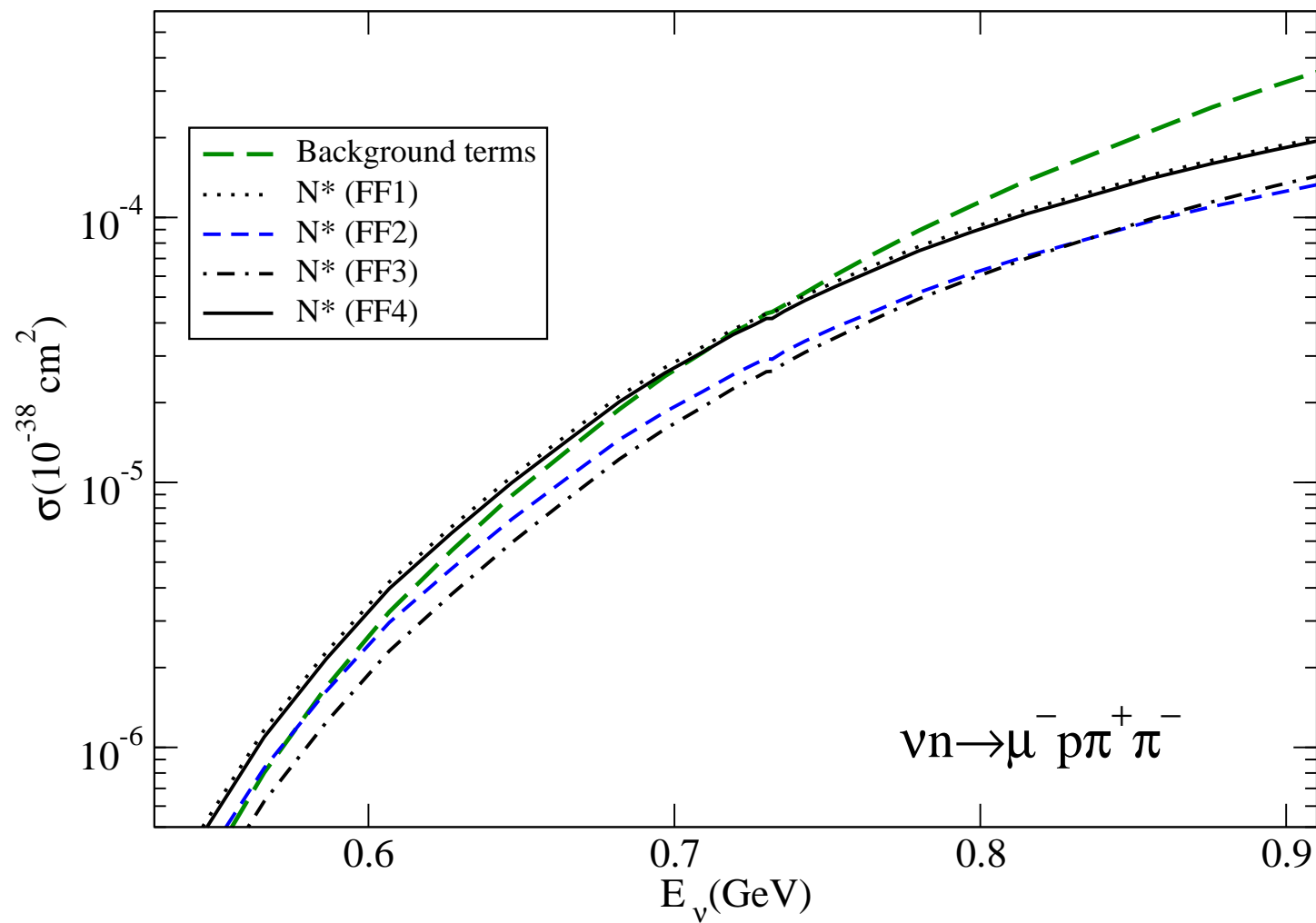- Set FF4. MAID analysis from D. Drechsel et al., EPJA34, 69 (2007)

Two-pion production close to threshold
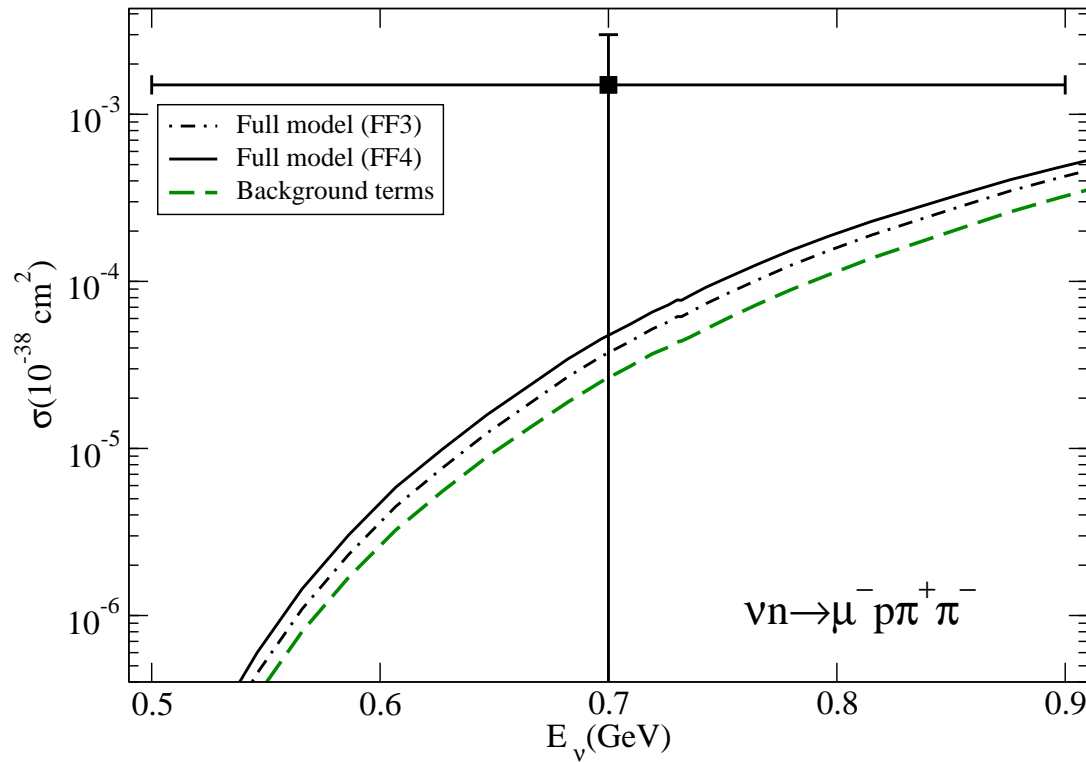
$N \to N^*(1440)$ and background relative sign

- All relative phases in the background terms are fixed by the nonlinear sigma model Lagrangian.

- We have assumed the sign of the $N^*N\pi$ coupling to be the same as the $NN\pi$ coupling. This fixes the vector part of the $N \to N^*$ weak current. The axial part is also fixed through the nondiagonal Goldberger-Treiman relation.
  Besides, it also fixes the sign of the c_1^*, c_2^* $N^*N\pi\pi$ coupling constants.

Taking the opposite sign for the $N^*N\pi$ coupling does not affect the results since both the $N \to N^*$ weak current and the c_1^*, c_2^* coupling would change sign leaving the Roper contribution unaltered.

Two-pion production close to threshold. Results I

$\nu n \rightarrow \mu^- p \pi^+ \pi^-$

Legend:
- – – Background terms
- ⋯ N* (FF1)
- – – N* (FF2)
- –·– N* (FF3)
- —— N* (FF4)

Axes: $\sigma(10^{-38}\ cm^2)$ versus $E_\nu(GeV)$

Two-pion production close to threshold. Results II

- BNL data by T. Kitagaki et al., PRD34, 2554 (1986)

Better results for this channel are obtained within the DCC model, PRD92, 074024 (2015)

For other channels there is no data in the region of validity of our model determined by $\pi\pi N$ invariant masses less than $1.4\,\text{GeV}$ and thus $E_{\nu_\mu} < 0.75\,\text{GeV}$.

Two-pion production close to threshold. Results III

In order to keep data close to threshold, S.A. Adjei et al. [PRD23,672 (1981)] suggested to implement the following phase space kinematical cuts

(1) $$q_\pi^2 \le ((1 + \eta/2)m_\pi)^2 \,,$$

(2) $$2p \cdot q_\pi \le (M + (1 + \eta)m_\pi)^2 - M^2 - m_\pi^2$$

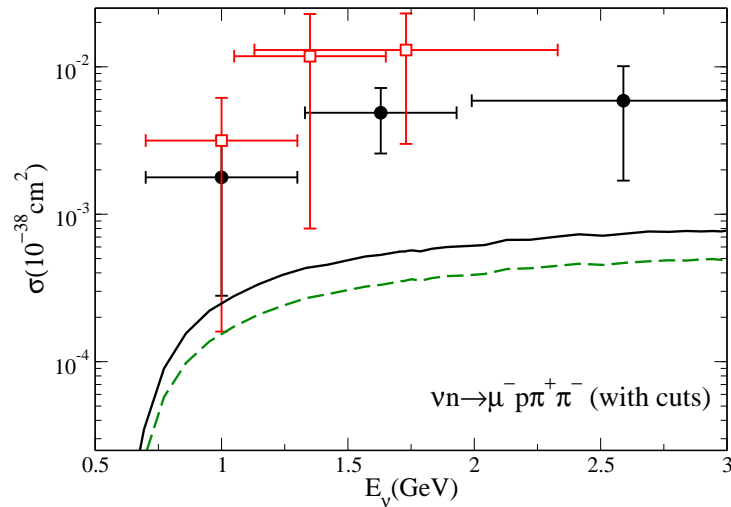(3) $$2p' \cdot q_\pi \le (M + (1 + \eta)m_\pi)^2 - M^2 - m_\pi^2 \,,$$

with $q_\pi = (k_{\pi_1} + k_{\pi_2})/2$. $\eta = 1/4, 2/4, 3/4$ were proposed.

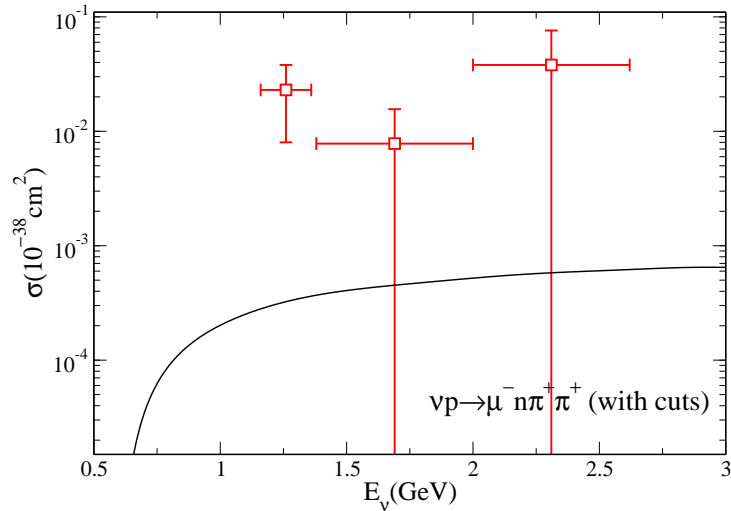(1)$\rightarrow$ keeps individual pion momenta close to average pion momentum.
(2) and (3)$\rightarrow$ pole terms should dominate.

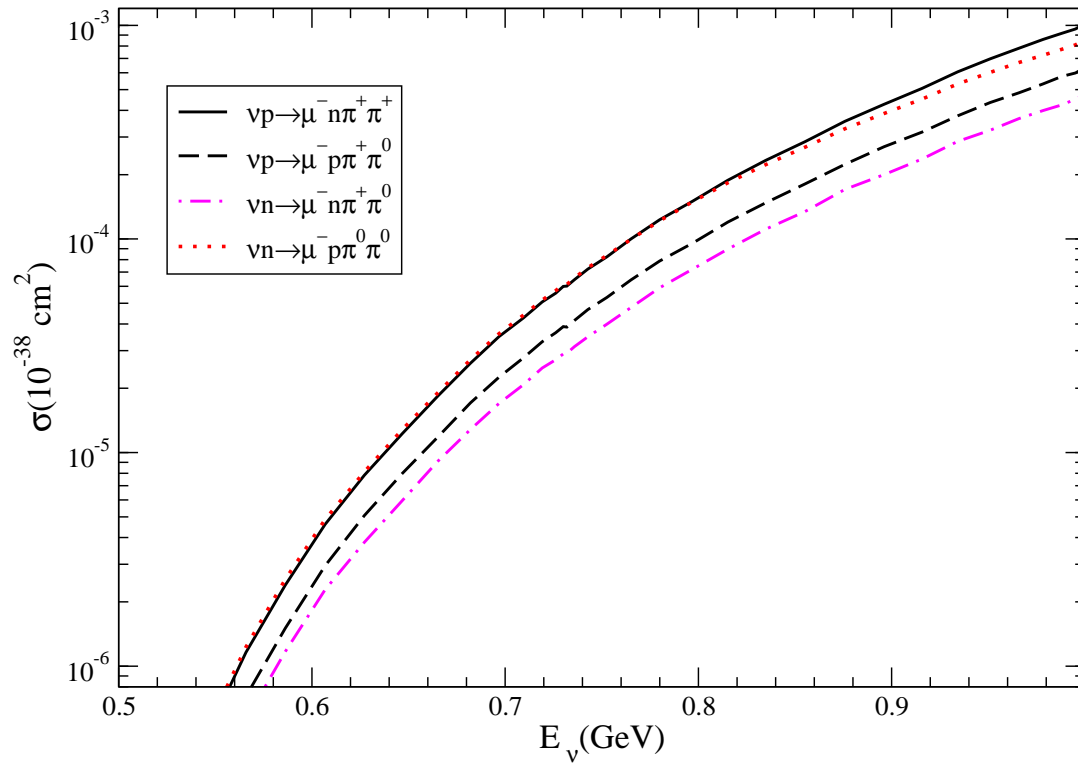# Two-pion production close to threshold. Results IV

$\eta = 3/4$

Dashed line: Background terms alone
Solid line: Full calculation with FF1 set

Solid line: Background terms alone

- BNL data by T. Kitagaki et al., PRD34, 2554 (1986)
- ANL data by D. Day et al., PRD28, 2714 (1983)

Two-pion production close to threshold. Results V

$\pi^0\pi^0$: Full model with FF1 set.

Other channels: Background contribution.

Summary

- I have shown our models for one- and two-pion production by neutrinos.

- For the one-pion production case we get good agreement with data at the nucleon level.

- In good qualitative agreement with the DCC model, we predict anisotropic pion distributions (measured in the final πN center of mass).

- For the case of one-pion production in nuclei the situation is not so clear. We get a resonable reproduction of MiniBooNE data but we overestimate the production in the case of Minerνa data.

- In the case of two-pion production close to threshold, our results are below the data by one order of magnitude. Better agreement is found by the DCC model with the exception of the $\pi^+\pi^+$ channel.