LQCD for few-nucleon systems

Assumpta Parreño
Marc Illa

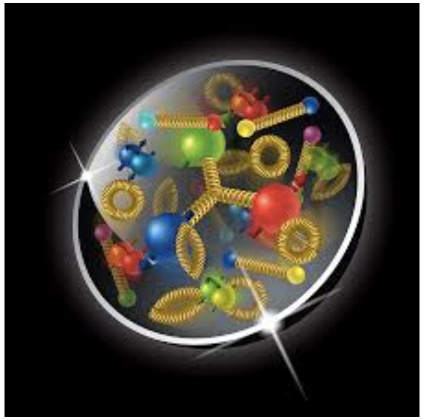
Universitat de Barcelona

NPLQCD Collaboration

Neutrini and nuclei, challenges and opportunities for nuclear theory

ECT, Villazzano (Trento), May 27-31, 2019*

Lattice QCD

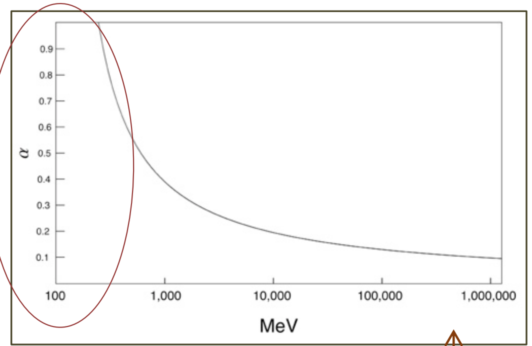


$$\mathcal{L}_{QCD} = \bar{q}_{ij}\left(i\gamma^{u}\partial_{u} - m_{j}\right)q_{ij} + g(\bar{q}_{ij}\gamma^{u}\lambda_{a}q_{ij})F_{u}^{a} - \frac{1}{4}F_{uv}^{a}F_{a}^{uv}$$

con $i = r,g,b \quad j = u,d,c,s,t,b \quad q$: quark spinor

Nuclear physics, the low-energy regime

strong coupling constant *vs* energy



Perturbation theory applicable

Absence of analytic solutions of QCD in the non-perturbative regime

"The effective action"

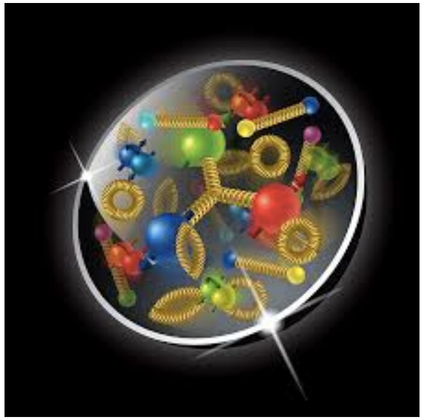
$$\int [Dq][D\bar{q}][DG]\, e^{\int i\, d^4x\, \mathcal{L}_{QCD}[q,\bar{q},G;\bar{J}]}$$

↑

fundamental degrees of freedom

Lattice QCD

$L_x \times L_y \times L_z \times T \;\rightarrow\; (N_s \times N_s \times N_s) \times N_t$



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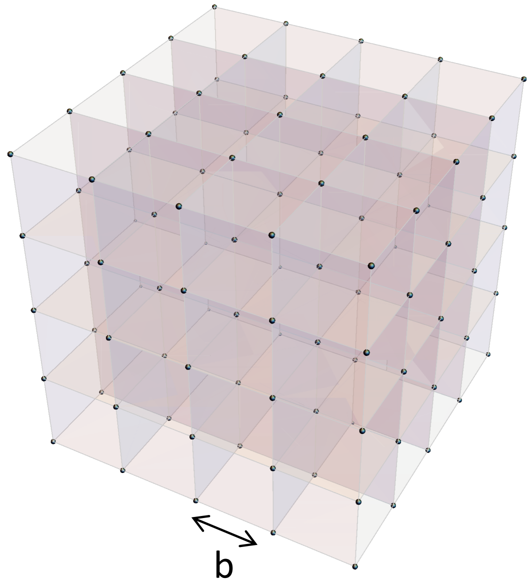
Nuclear physics, the low-energy regime



$x = b\,(n_1, n_2, n_3, n_4) \qquad n_j \in \mathbb{Z}$

Absence of analytic solutions of QCD in the non-perturbative regime

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↑

fundamental degrees of freedom

$$\int D\psi\; D\bar{\psi} \int D\,A \longrightarrow \prod_{n_\mu} d\psi\,(bn_\mu)\; d\psi\,(bn_\mu) \int dU(bn_\mu)$$

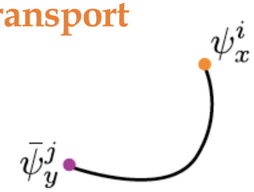
Path integral formalism
Richard P. Feynman, 1948

Lattice QCD

Lattice QCD

Continuum

Gauge-invariant form for the **color transport**



$$\bar{\psi}^j_y \left[e^{ig \int_x^y dz_\mu A_\mu(z)} \right]_{ji} \psi^i_x$$

"Wilson line"

© Jozef Dudek lectures

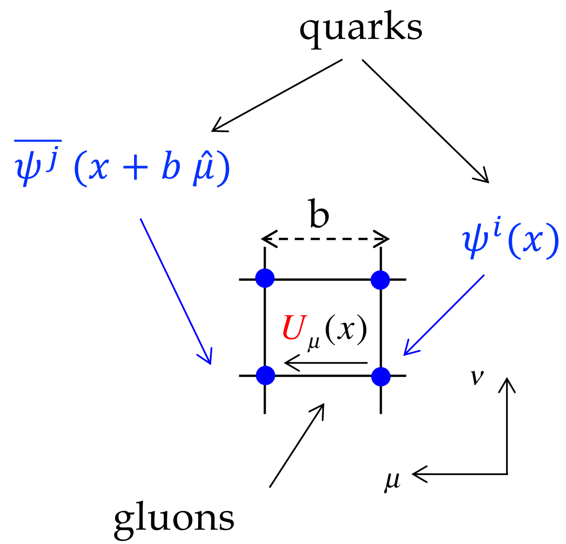
Finite volume

SU(3) matrix

$$\bar{\psi}^j_{x+b\,\hat{\mu}} \left[e^{igbA_{x\mu}} \right]_{ji} \psi^i_x$$

$$\bar{\psi}_{x+b\,\hat{\mu}}\, U_\mu(x)\, \psi_x$$

SU(3) matrix for the link



$$\int [Dq][D\bar{q}][DG]\, e^{\int i d^4x \mathscr{L}_{QCD}[q,\bar{q},G;\bar{J}]}$$

fundamental degrees of freedom

$$\int D\psi\ D\bar{\psi} \int D\, A \longrightarrow \prod_{n_\mu} d\psi\,(bn_\mu)\, d\psi\,(bn_\mu) \int dU(bn_\mu)$$

Path integral formalism
Richard P. Feynman, 1948

Lattice QCD

Lattice QCD

Solve a linear system of equations: $D^{\dagger}(U)[m]\, D(U)[m]\, \chi = \phi$

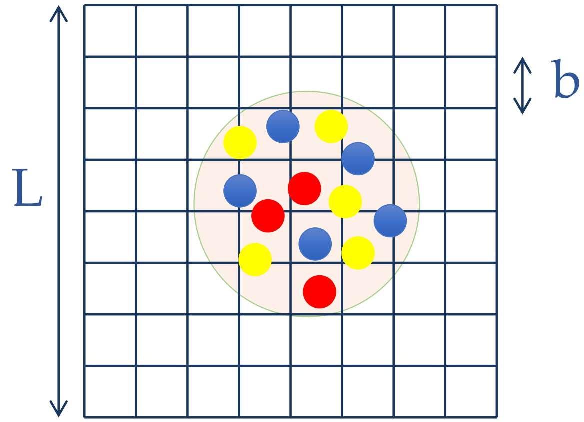
→ Condition number ≈ 1/m

→ Finite L, b and unphysical m_q

$$\text{Cost} \approx \left[\frac{1}{m_q}\right] [L]^{a} \left[\frac{1}{\mathrm{b}}\right]^{\gamma}$$

USE UNPHYSICAL VALUES
OF THESE PARAMETERS
(LATTICE ARTIFACTS)

source of systematic errors
in lattice QCD simulations



$$L >> \text{ relevant scales } >> b$$

$$\left(\frac{1}{L} << m_{\pi} << \Lambda_{\chi} << \frac{1}{b}\right)$$

$$b << M_N^{-1}$$

Extrapolate to physical results:
Use of Effective Field Theory

Lattice QCD

Solve a linear system of equations: $D^{\dagger}(U)[m]\, D(U)[m]\, \chi = \phi$

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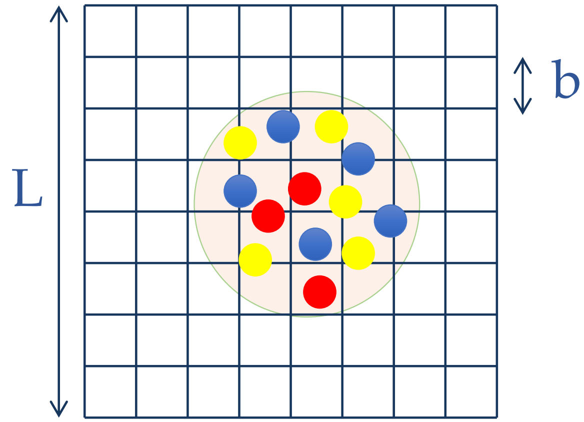
source of systematic errors in lattice QCD simulations

Extrapolations to connect with real life

$L \to \infty$ | $b \to 0$ | $m_q \to m_q^{physical}$

Published Works	
World	**NPLQCD**
L ~ 2 - 8 fm	**L ~ 3 - 7 fm**
b ~ 0.066 - 0.145 fm	**b ~ 0.117 – 0.145 fm**
m_π ~ 146 – 1100 MeV	**m_π ~ 230 – 806 MeV**



$L >>$ relevant scales $>> b$

$$\left(\frac{1}{L} << m_\pi << \Lambda_\chi << \frac{1}{b}\right)$$

$$b << M_N^{-1}$$

Extrapolate to physical results:
Use of Effective Field Theory

Lattice QCD

World @ 800 MeV

Solve a linear system of equations: $D^{\dagger}(U)[m]\, D(U)[m]\, \chi = \phi$

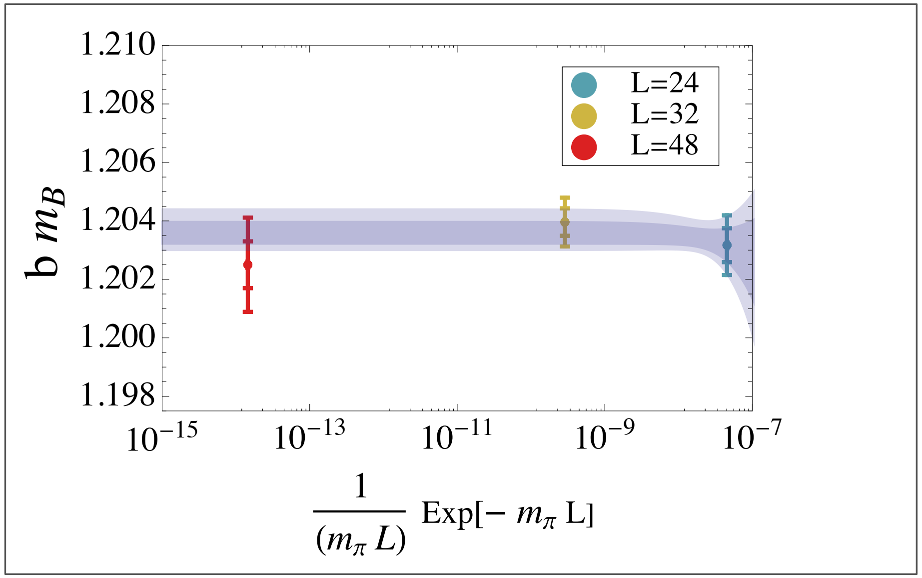
Condition number ≈ 1/m

UNPHYSICAL NUCLEI

Ensemble	$\|\mathbf{n}\| = 0$
$24^3 \times 48$	1.20317(58)(84)
$32^3 \times 48$	1.20396(47)(69)
$48^3 \times 64$	1.2032(07)(11)
$L = \infty$	1.20359(41)(61)

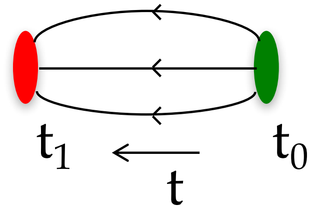
$m_\pi \sim 806$ MeV

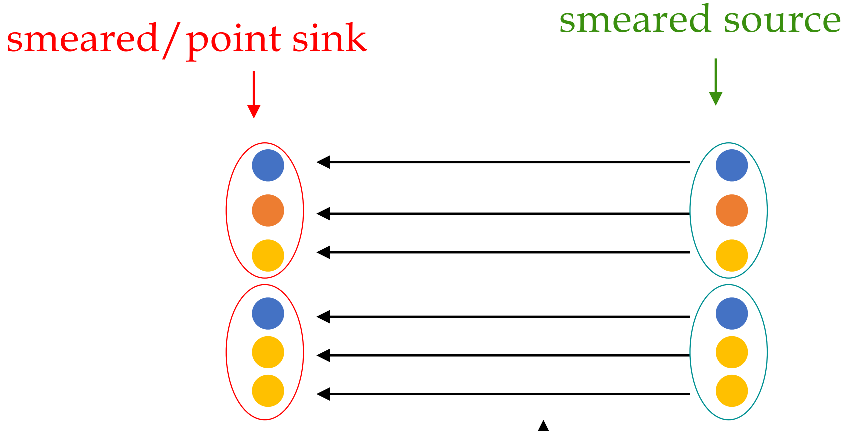
$$M_B^{(V)}(m_\pi L) = M_B^{(\infty)} + c_B^{(V)} \frac{e^{-m_\pi L}}{m_\pi L} + \cdots$$



$m_B = 1.634(0)(0)(18)$ GeV

Direct Lattice QCD extraction $\longleftrightarrow$ Compute correlation functions





$$\sum_n |n\rangle\langle n|$$

construction of
SS/SP correlation functions

example:
interpolating operator
for the proton

$$p_\alpha(\mathbf{x},t) = \epsilon^{ijk} u^i_\alpha(\mathbf{x},t)\left(u^{j\top}(\mathbf{x},t)C\gamma_5 d^k(\mathbf{x},t)\right)$$

$$C_{\hat{O},\hat{O}'}(t,\vec{d}) = \sum_{\vec{x}} e^{2\pi i \vec{d}\vec{x}/L} \langle 0|\hat{O}'(\vec{x},t)\hat{O}^\dagger(\vec{0},0)|0\rangle$$

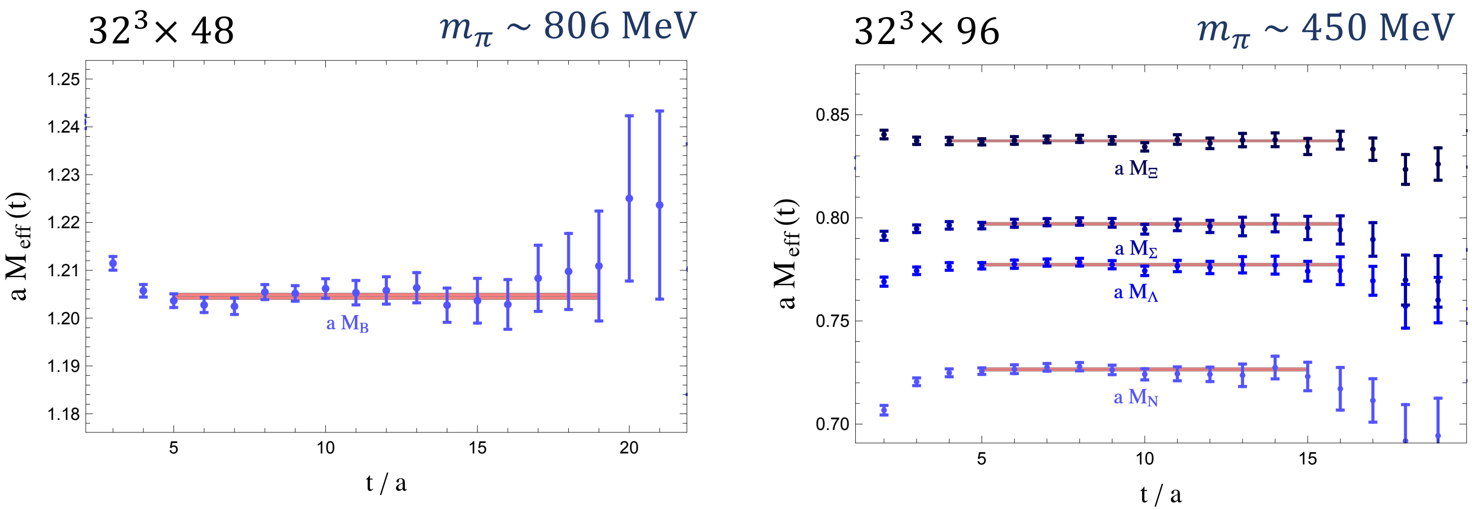
$$= \underbrace{Z_0^{snk} Z_0^{\dagger\, src} e^{-E^{(0)}t}}_{\text{dominates at large t}} + Z_1^{snk} Z_1^{\dagger\, src} e^{-E^{(1)}t} + \cdots$$

projected into total momentum $\frac{2\pi\vec{d}}{L}$

Determination of the energy levels. Fitting strategies.

- Use a linear combination of SS/SP correlation functions to remove the excited-state contamination of the lowest lying state at earlier times (Matrix-Prony method/GPoF method)
- Perform a correlated χ^2 fit to single or two-exponential forms
- Work with an effective mass(energy) function:

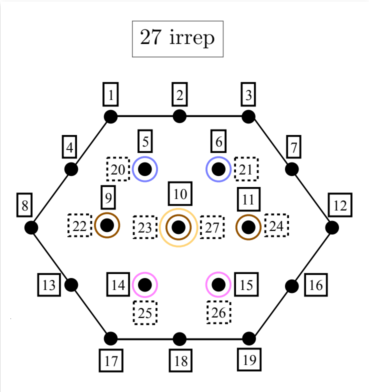
$$C_{\hat{O},\hat{O}'}(\tau;\ \vec{d},\tau_J) = \frac{1}{\tau_J}\log\left[\frac{C_{\hat{O},\hat{O}'}(\tau;\vec{d})}{C_{\hat{O},\hat{O}'}(\tau+\tau_J;\vec{d})}\right] \rightarrow E_0 \quad \text{at large times}$$



A. Parreño et al (NPLQCD), PRD 95 (2017), 114513

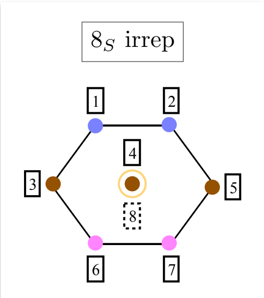
Ratios of single and two-body correlation functions → energy shift of the system resulting from two-body interactions

SU(3) decomposition

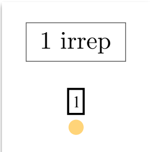


Two-baryon states

	Flavor channel		Flavor channel
1	nn	14	$-\sqrt{\frac{2}{3}}\Sigma^0\Xi^- + \sqrt{\frac{1}{3}}\Sigma^-\Xi^0$
2	$\frac{1}{\sqrt{2}}(np+pn)$	15	$\sqrt{\frac{1}{3}}\Sigma^+\Xi^- + \sqrt{\frac{2}{3}}\Sigma^0\Xi^0$
3	pp	16	$\Sigma^+\Xi^0$
4	Σ^-n	17	$\Xi^-\Xi^-$
5	$\sqrt{\frac{2}{3}}\Sigma^0 n + \sqrt{\frac{1}{3}}\Sigma^- p$	18	$\frac{1}{\sqrt{2}}(\Xi^-\Xi^0 + \Xi^0\Xi^-)$
6	$-\sqrt{\frac{1}{3}}\Sigma^+ n + \sqrt{\frac{2}{3}}\Sigma^0 p$	19	$\Xi^0\Xi^0$
7	Σ^+p	20	$\Lambda n / -\sqrt{\frac{1}{3}}\Sigma^0 n + \sqrt{\frac{2}{3}}\Sigma^- p$
8	$\Sigma^-\Sigma^-$	21	$\Lambda p / \sqrt{\frac{2}{3}}\Sigma^+ n + \sqrt{\frac{1}{3}}\Sigma^0 p$
9	$\frac{1}{\sqrt{2}}(\Sigma^-\Sigma^0 + \Sigma^0\Sigma^-)$	22	$\Lambda\Sigma^- / \Xi^- n$
10	$\frac{1}{\sqrt{6}}(\Sigma^-\Sigma^+ - 2\Sigma^0\Sigma^0 + \Sigma^+\Sigma^-)$	23	$\Lambda\Sigma^0 / \frac{1}{\sqrt{2}}(\Xi^- p - \Xi^0 n)$
11	$\frac{1}{\sqrt{2}}(\Sigma^0\Sigma^+ + \Sigma^+\Sigma^0)$	24	$\Lambda\Sigma^+ / \Xi^0 p$
12	$\Sigma^+\Sigma^+$	25	$\Lambda\Xi^- / \sqrt{\frac{1}{3}}\Sigma^0\Xi^- + \sqrt{\frac{2}{3}}\Sigma^-\Xi^0$
13	$\Sigma^-\Xi^-$	26	$\Lambda\Xi^0 / -\sqrt{\frac{2}{3}}\Sigma^+\Xi^- + \sqrt{\frac{1}{3}}\Sigma^0\Xi^0$
27	$\frac{1}{\sqrt{3}}(\Sigma^+\Sigma^- + \Sigma^0\Sigma^0 + \Sigma^-\Sigma^+) / \frac{1}{\sqrt{2}}(\Xi^0 n + \Xi^- p) / \Lambda\Lambda$		



	Flavor channel
1	$\Lambda n / -\sqrt{\frac{1}{3}}\Sigma^0 n + \sqrt{\frac{2}{3}}\Sigma^- p$
2	$\Lambda p / \sqrt{\frac{2}{3}}\Sigma^+ n + \sqrt{\frac{1}{3}}\Sigma^0 p$
3	$\Lambda\Sigma^- / \Xi^- n$
4	$\Lambda\Sigma^0 / \frac{1}{\sqrt{2}}(\Xi^- p - \Xi^0 n)$
5	$\Lambda\Sigma^+ / \Xi^0 p$
6	$\Lambda\Xi^- / \sqrt{\frac{1}{3}}\Sigma^0\Xi^- + \sqrt{\frac{2}{3}}\Sigma^-\Xi^0$
7	$\Lambda\Xi^0 / -\sqrt{\frac{2}{3}}\Sigma^+\Xi^- + \sqrt{\frac{1}{3}}\Sigma^0\Xi^0$
8	$\frac{1}{\sqrt{3}}(\Sigma^+\Sigma^- + \Sigma^0\Sigma^0 + \Sigma^-\Sigma^+) / \frac{1}{\sqrt{2}}(\Xi^0 n + \Xi^- p) / \Lambda\Lambda$



	Flavor channel
1	$\frac{1}{\sqrt{3}}(\Sigma^+\Sigma^- + \Sigma^0\Sigma^0 + \Sigma^-\Sigma^+) / \frac{1}{\sqrt{2}}(\Xi^0 n + \Xi^- p) / \Lambda\Lambda$

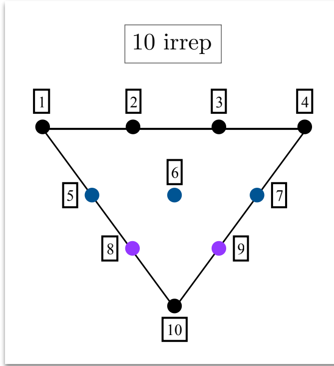
strangeness

I_3

J=0

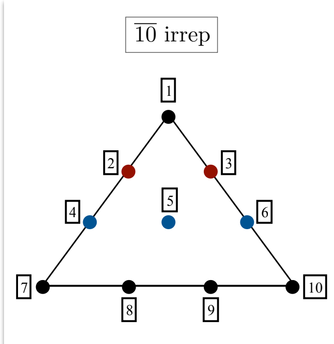
M.L. Wagman et al (NPLQCD)
ARXIV:1706.06550

SU(3) decomposition

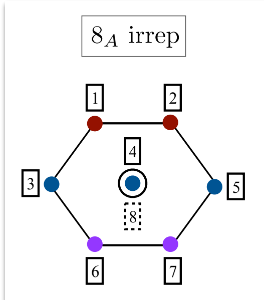


Two-baryon states

	Flavor channel
1	$\Sigma^- n$
2	$\sqrt{\frac{2}{3}}\Sigma^0 n + \sqrt{\frac{1}{3}}\Sigma^- p$
3	$-\sqrt{\frac{1}{3}}\Sigma^+ n + \sqrt{\frac{2}{3}}\Sigma^0 p$
4	$\Sigma^+ p$
5	$\frac{1}{\sqrt{2}}(\Sigma^-\Sigma^0 - \Sigma^0\Sigma^-) / \Xi^- n / \Lambda\Sigma^-$
6	$\frac{1}{\sqrt{2}}(\Sigma^-\Sigma^+ - \Sigma^+\Sigma^-) / \frac{1}{\sqrt{2}}(\Xi^- p - \Xi^0 n) / \Lambda\Sigma^0$
7	$\frac{1}{\sqrt{2}}(\Sigma^0\Sigma^+ - \Sigma^+\Sigma^0) / \Xi^0 p / \Lambda\Sigma^+$
8	$\sqrt{\frac{1}{3}}\Sigma^0\Xi^- + \sqrt{\frac{2}{3}}\Sigma^-\Xi^0 / \Lambda\Xi^-$
9	$-\sqrt{\frac{2}{3}}\Sigma^+\Xi^- + \sqrt{\frac{1}{3}}\Sigma^0\Xi^0 / \Lambda\Xi^0$
10	$\frac{1}{\sqrt{2}}(\Xi^0\Xi^- \text{-} \Xi^-\Xi^0)$



	Flavor channel
1	$\frac{1}{\sqrt{2}}(pn - np)$
2	$-\sqrt{\frac{1}{3}}\Sigma^0 n + \sqrt{\frac{2}{3}}\Sigma^- p / \Lambda n$
3	$\sqrt{\frac{2}{3}}\Sigma^+ n + \sqrt{\frac{1}{3}}\Sigma^0 p / \Lambda p$
4	$\frac{1}{\sqrt{2}}(\Sigma^-\Sigma^0 - \Sigma^0\Sigma^-) / \Xi^- n / \Lambda\Sigma^-$
5	$\frac{1}{\sqrt{2}}(\Sigma^-\Sigma^+ - \Sigma^+\Sigma^-) / \frac{1}{\sqrt{2}}(\Xi^- p - \Xi^0 n) / \Lambda\Sigma^0$
6	$\frac{1}{\sqrt{2}}(\Sigma^0\Sigma^+ - \Sigma^+\Sigma^0) / \Xi^0 p / \Lambda\Sigma^+$
7	$\Sigma^-\Xi^-$
8	$-\sqrt{\frac{2}{3}}\Sigma^0\Xi^- + \sqrt{\frac{1}{3}}\Sigma^-\Xi^0$
9	$\sqrt{\frac{1}{3}}\Sigma^+\Xi^- + \sqrt{\frac{2}{3}}\Sigma^0\Xi^0$
10	$\Sigma^+\Xi^0$



	Flavor channel
1	$-\sqrt{\frac{1}{3}}\Sigma^0 n + \sqrt{\frac{2}{3}}\Sigma^- p / \Lambda n$
2	$\sqrt{\frac{2}{3}}\Sigma^+ n + \sqrt{\frac{1}{3}}\Sigma^0 p / \Lambda p$
3	$\frac{1}{\sqrt{2}}(\Sigma^-\Sigma^0 - \Sigma^0\Sigma^-) / \Xi^- n / \Lambda\Sigma^-$
4	$\frac{1}{\sqrt{2}}(\Sigma^-\Sigma^+ - \Sigma^+\Sigma^-) / \frac{1}{\sqrt{2}}(\Xi^- p - \Xi^0 n) / \Lambda\Sigma^0$
5	$\frac{1}{\sqrt{2}}(\Sigma^0\Sigma^+ - \Sigma^+\Sigma^0) / \Xi^0 p / \Lambda\Sigma^+$
6	$\sqrt{\frac{1}{3}}\Sigma^0\Xi^- + \sqrt{\frac{2}{3}}\Sigma^-\Xi^0 / \Lambda\Xi^-$
7	$-\sqrt{\frac{2}{3}}\Sigma^+\Xi^- + \sqrt{\frac{1}{3}}\Sigma^0\Xi^0 / \Lambda\Xi^0$
8	$\frac{1}{\sqrt{2}}(\Xi^0 n + \Xi^- p)$

J=1

TWO-BARYON STATES AT THE SU(3) FLAVOR-SYMMETRIC POINT

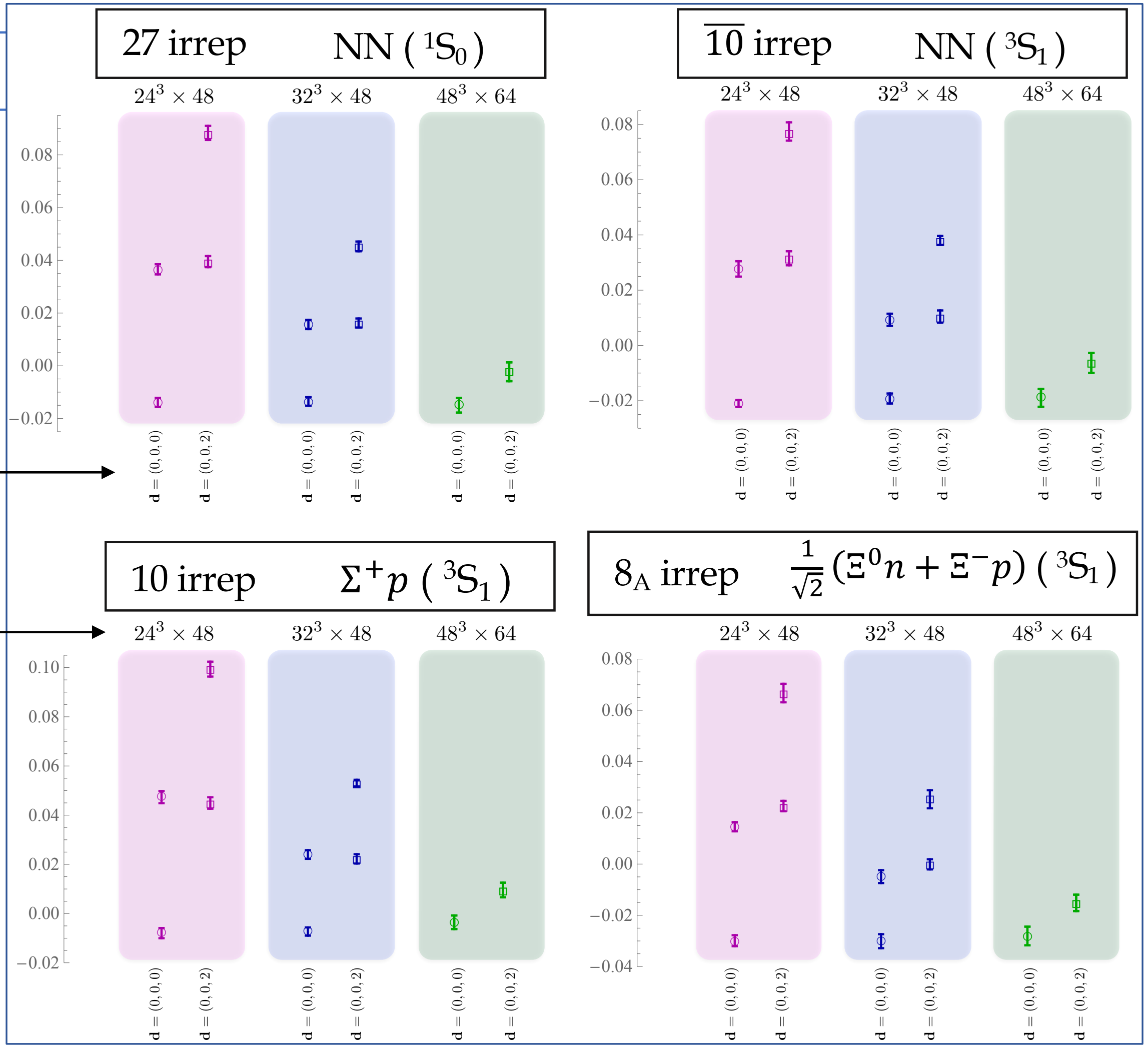
Energy shifts

$$\overline{\Delta E} = E_{BB'} - 2M_B$$

10 kinematical points for each irrep

total CoM momentum

volume



L=24 l.u. (3.4 fm)
L=32 l.u. (4.5 fm)
L=48 l.u. (6.7 fm)

M.L. Wagman et al (NPLQCD), PRD, ARXIV:1706.06550

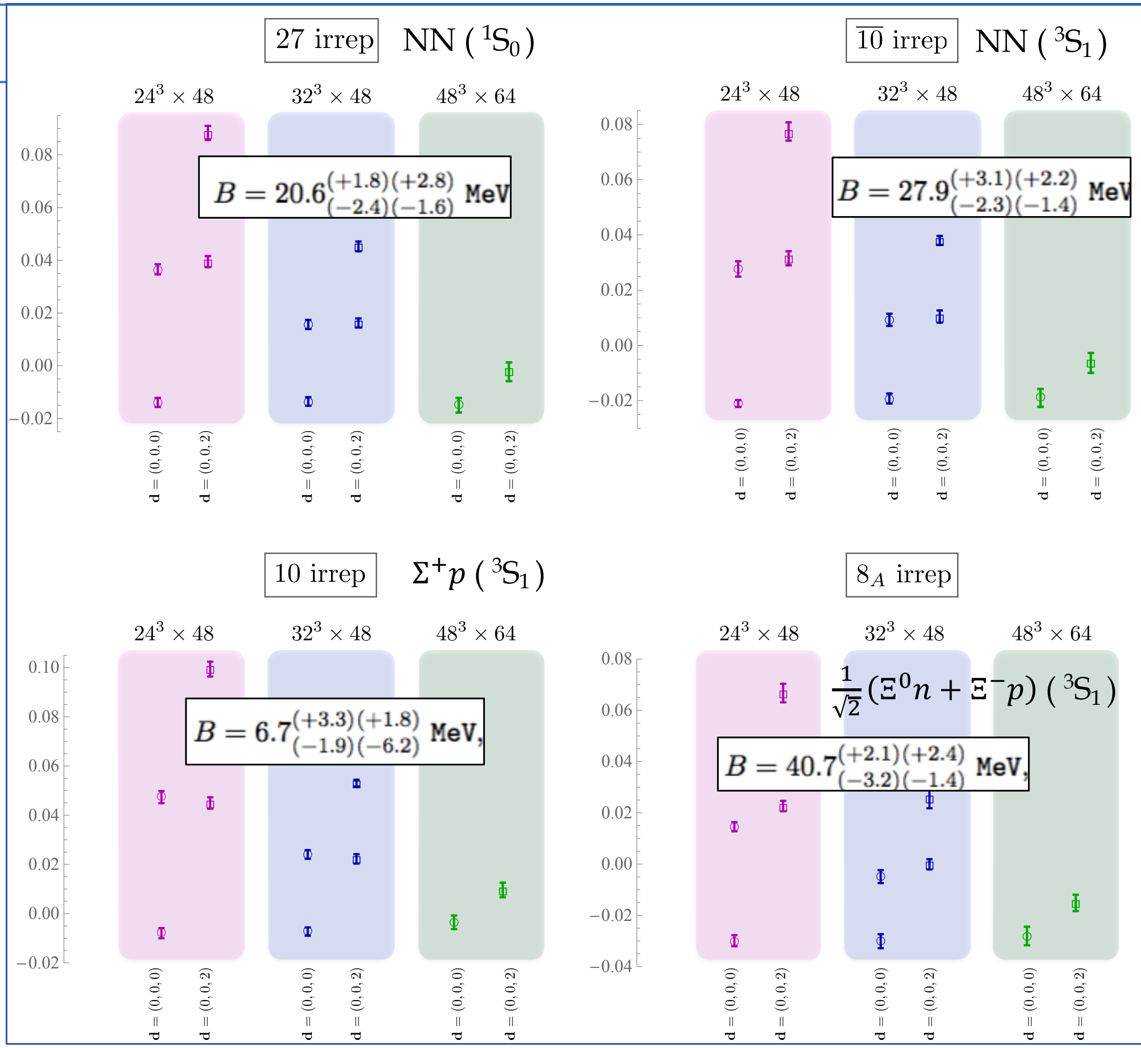
$m_\pi \sim 806$ MeV

Binding energies

$m_\pi \sim 806$ MeV

$$\overline{\Delta E} = E_{BB'} - 2M_B$$

10 kinematical points
for each irrep



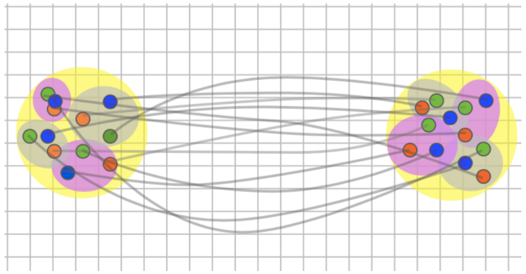
M.L. Wagman et al (NPLQCD), PRD, ARXIV:1706.06550

$$B = -2\sqrt{-\kappa^{(\infty)}{}^2 + M_B^2} + 2M_B$$

$$|k^*| = \kappa^{(\infty)} + \frac{Z^2}{L}\left[6e^{-\kappa^{(\infty)}L} + \frac{12}{\sqrt{2}}e^{-\sqrt{2}\kappa^{(\infty)}L} + \frac{8}{\sqrt{3}}e^{-\sqrt{3}\kappa^{(\infty)}L}\right] + \mathcal{O}\left(\frac{e^{-2\kappa^{(\infty)}L}}{L}\right)$$

Lüscher's quantization condition

World @ 800 MeV

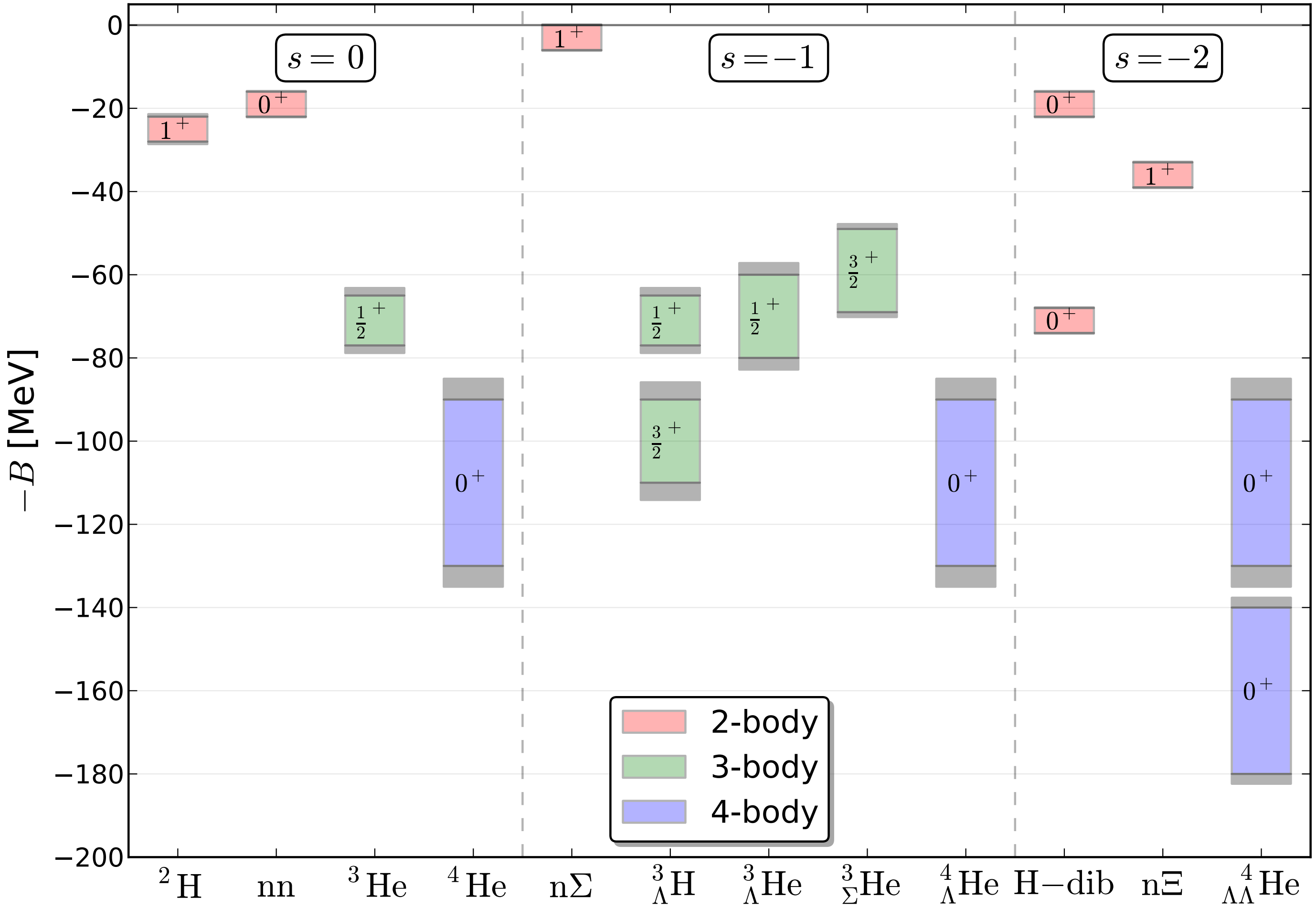


factorial growth in the number of contractions

NPLQCD Phys. Rev. D87 (2013) 3, 034506

$m_\pi \sim 800$ MeV

$SU(3)_f$



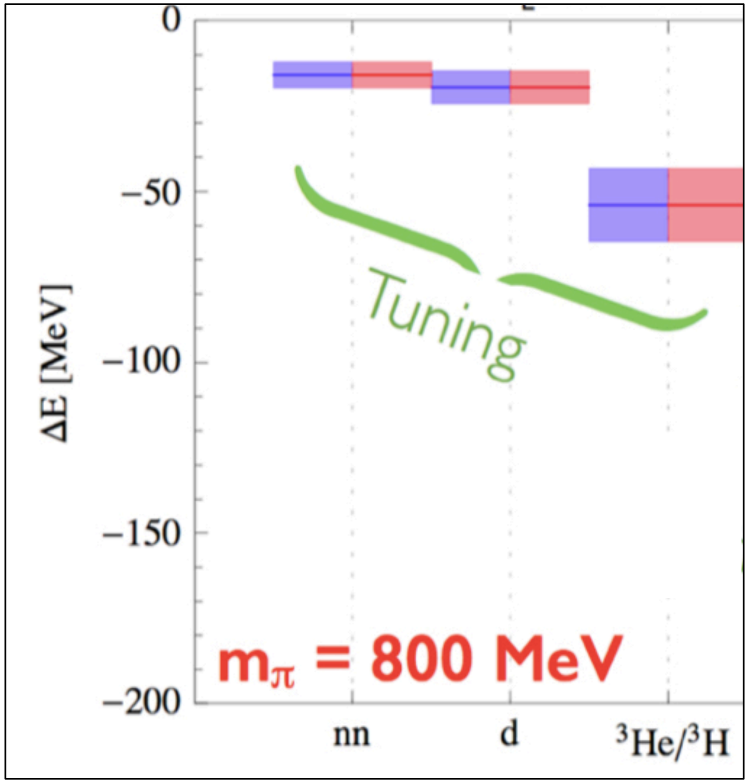
no e.m. interactions

(hadronic labels for (J^π, I, s, A) states)

LQCD → FEW-BODY CALCS → NUCLEAR MANY-BODY CALCS

EFT

$$V^{LO} = \begin{cases} C_0 + C_1\, \vec{\sigma_1}\, \vec{\sigma_2} & \text{2B (attractive)} \\ + D_0 & \text{3B (repulsive)} \end{cases}$$



NPLQCD, PRD 2013

EFT ($\not\pi$) at LO
Barnea et al. PRL 2015

N. Barnea, L. Contessi, D. Gazit, F. Pederiva, and U. van Kolck, PRL **114** (2015) 052501

L. Contessi, A. Lovato, F. Pederiva, A. Roggero, J. Kirscher and U. van Kolck, PLB **772** (2017) 839
Ground-state properties of ^{4}He and ^{16}O extrapolated from lattice QCD with pionless EFT

EFT

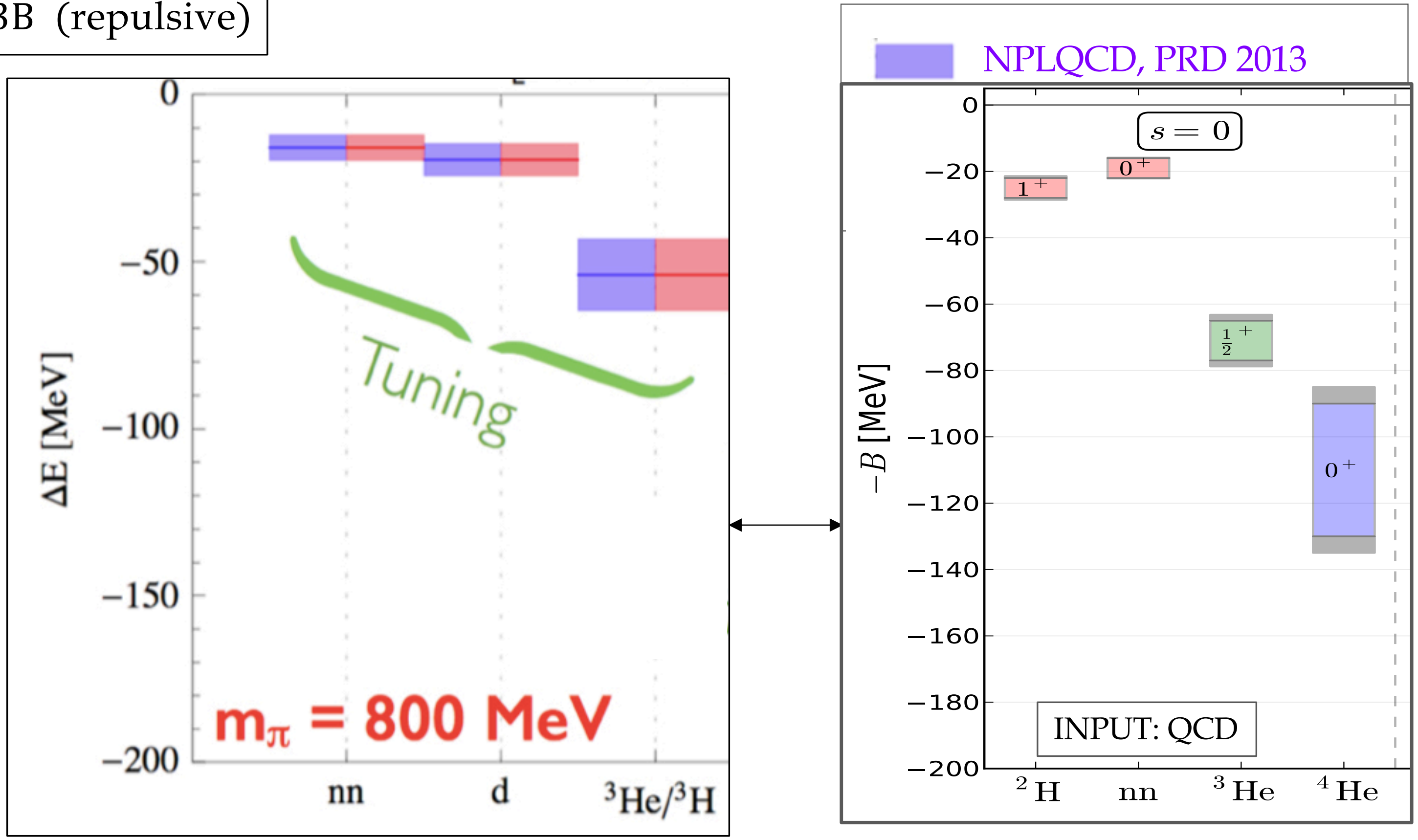
$$V^{LO} = \begin{cases} C_0 + C_1\, \vec{\sigma_1}\, \vec{\sigma_2} & \text{2B (attractive)} \\ + D_0 & \text{3B (repulsive)} \end{cases}$$

LQCD → FEW-BODY CALCS → NUCLEAR MANY-BODY CALCS

matching

↓

C_0 , C_1 , D_0 values



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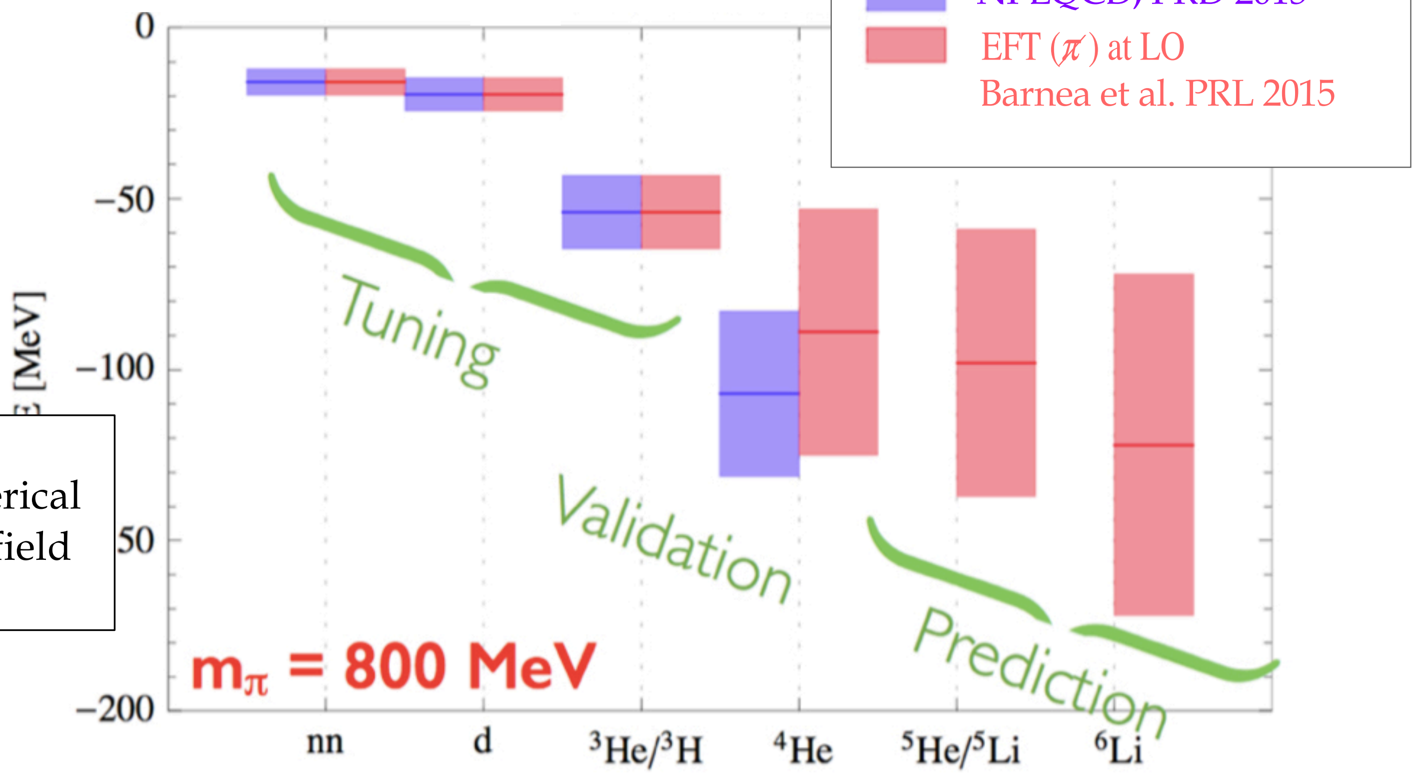
LQCD → FEW-BODY CALCS → NUCLEAR MANY-BODY CALCS

EFT

$$V^{LO} = \begin{cases} C_0 + C_1\, \vec{\sigma_1}\, \vec{\sigma_2} & \text{2B (attractive)} \\ + D_0 & \text{3B (repulsive)} \end{cases}$$

many body: EFT + EIHH/AFDMC



nuclear structure:
effective-interaction hyperspherical harmonics (A≤ 6) / auxiliary-field diffusion monte carlo (A≥ 4)

N. Barnea, L. Contessi, D. Gazit, F. Pederiva, and U. van Kolck, PRL **114** (2015) 052501

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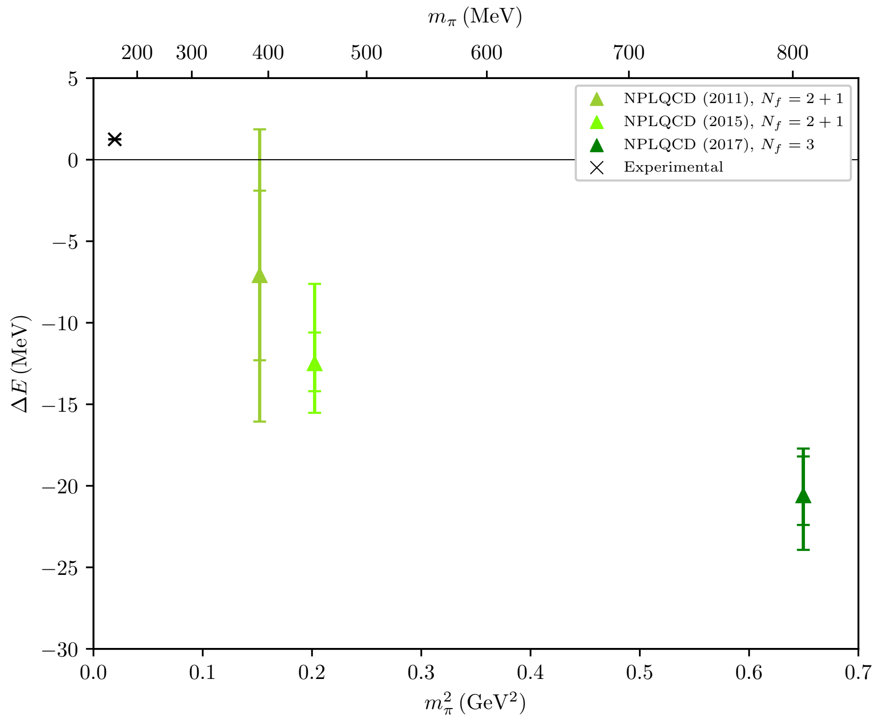
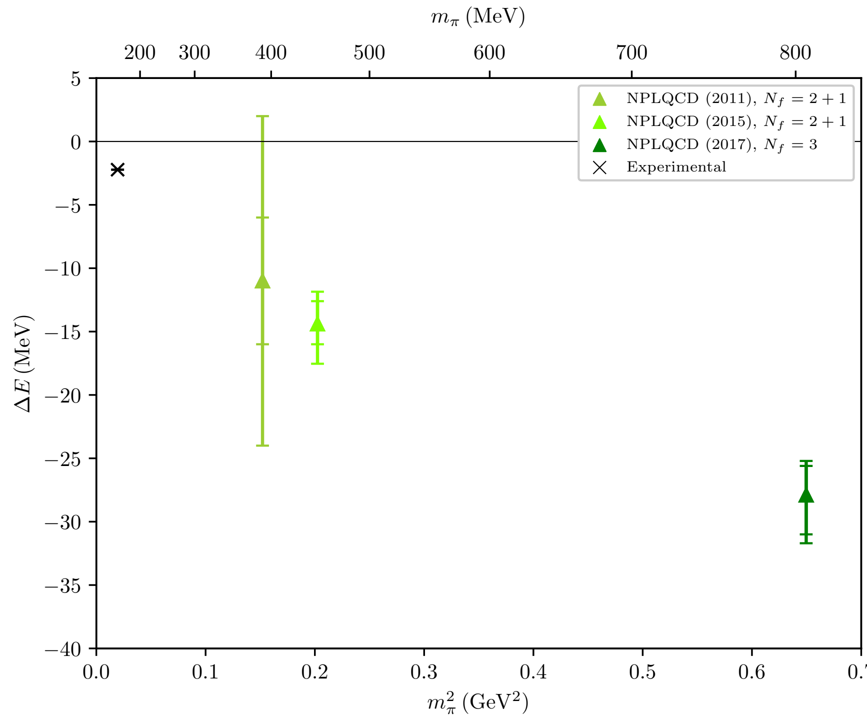
Ground-state properties of ^{4}He and ^{16}O extrapolated from lattice QCD with pionless EFT

Calculations at lighter quark masses - Nucleon-Nucleon sector

NN ($^1S_0$) NPLQCD NN ($^3S_1$)

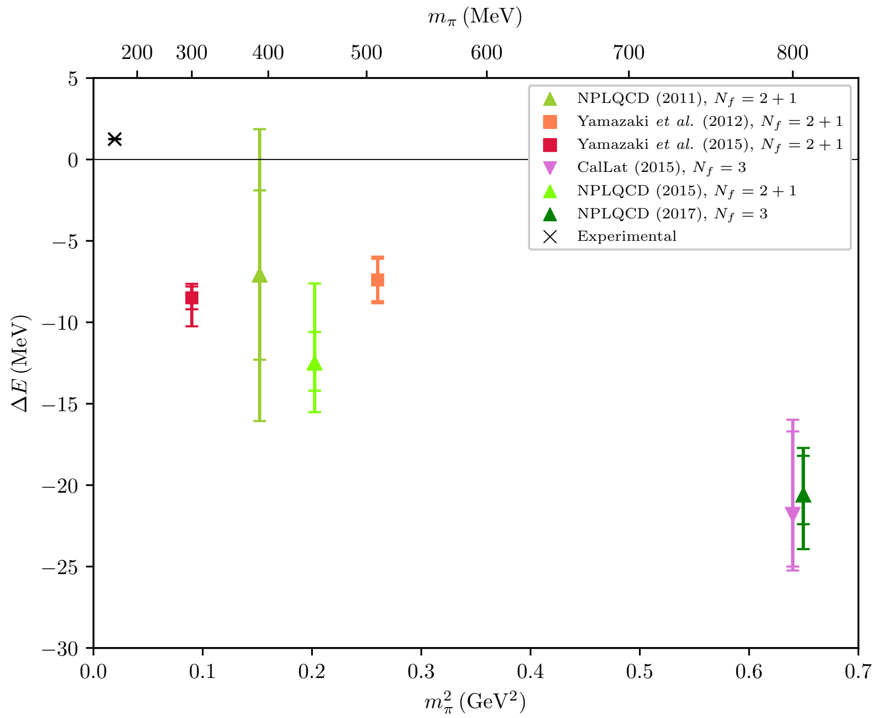




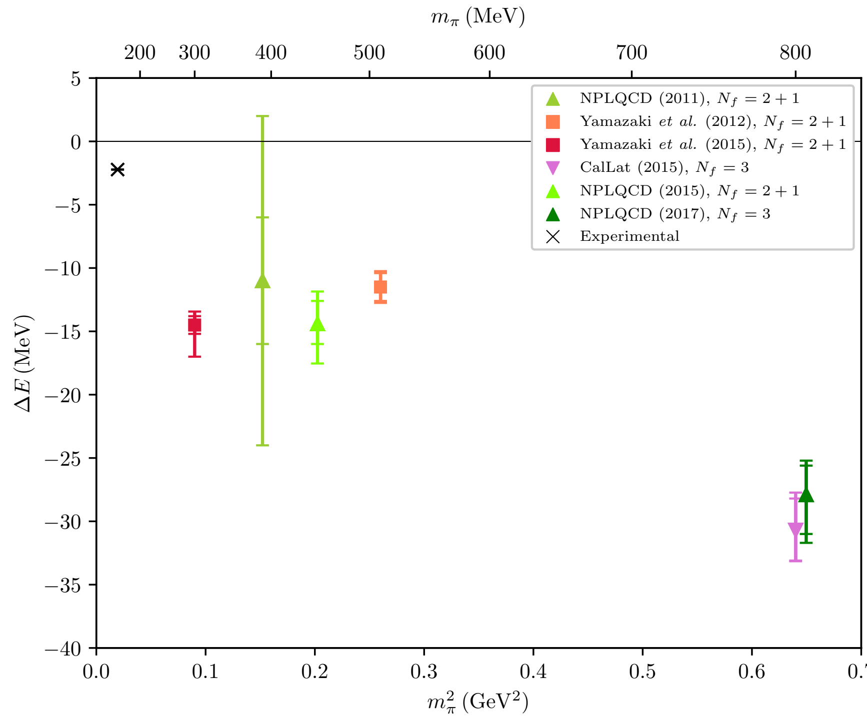
compiled by Marc Illa, UB

Calculations at lighter quark masses - Nucleon-Nucleon sector

NN ($^1S_0$)

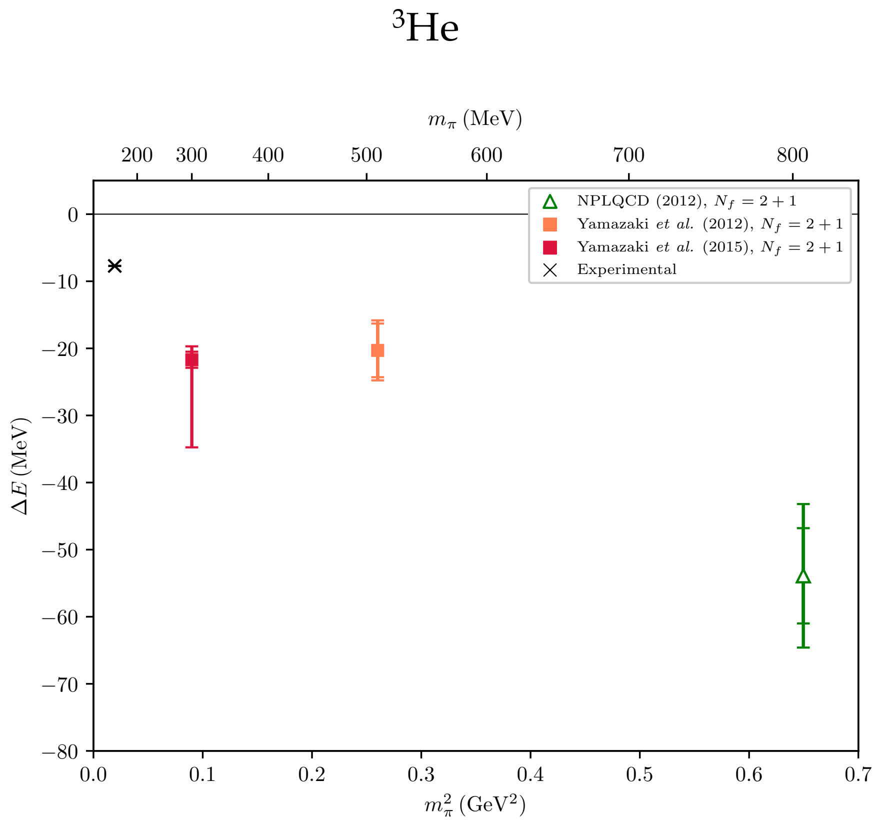


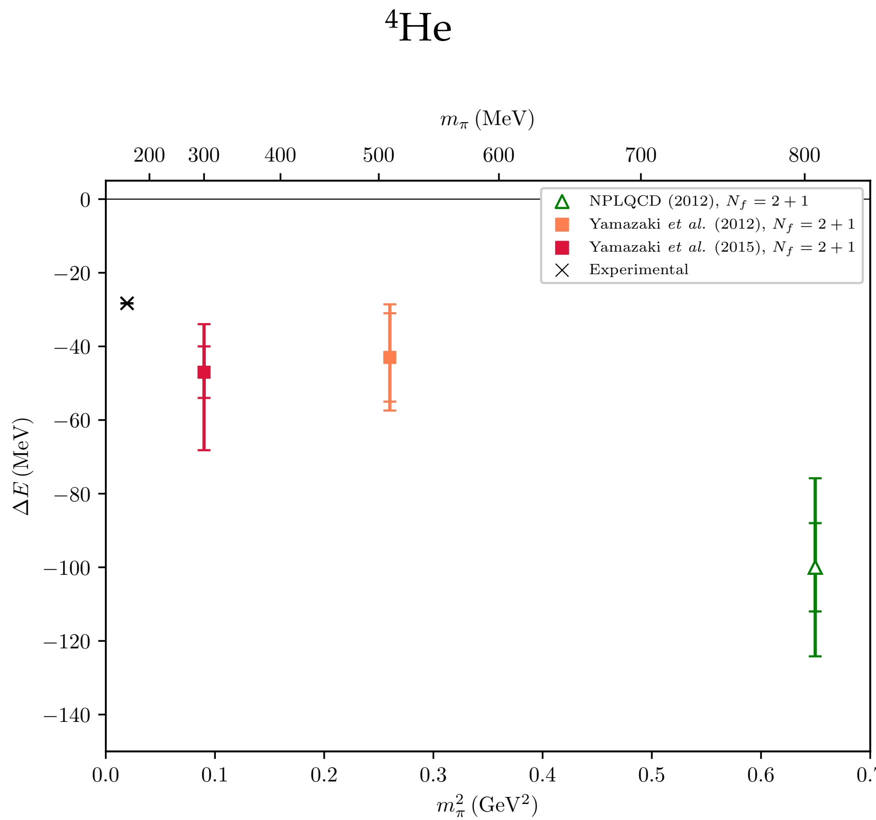
NN ($^3S_1$)



compiled by Marc Illa, UB

Calculations at lighter quark masses - Nucleon-Nucleon sector



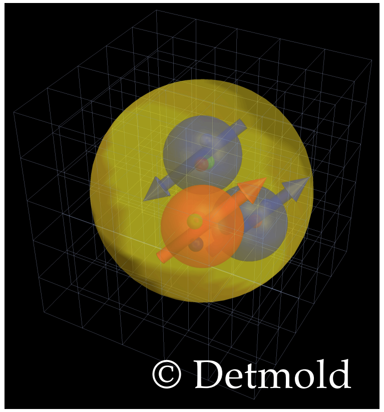


compiled by Marc Illa, UB

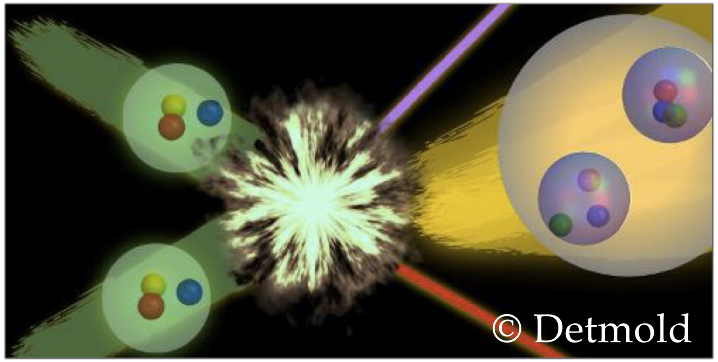
Calculations on baryons and light nuclei

Interaction of nucleons/nuclei with external currents

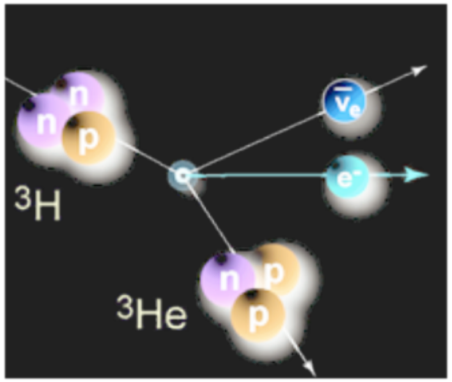
Magnetic Moments



PRD **95**, 114513 (2017)
PRL **116**, 112301 (2016)
PRD **92**, 114502 (2015)
PRL **113**, 252001 (2014)



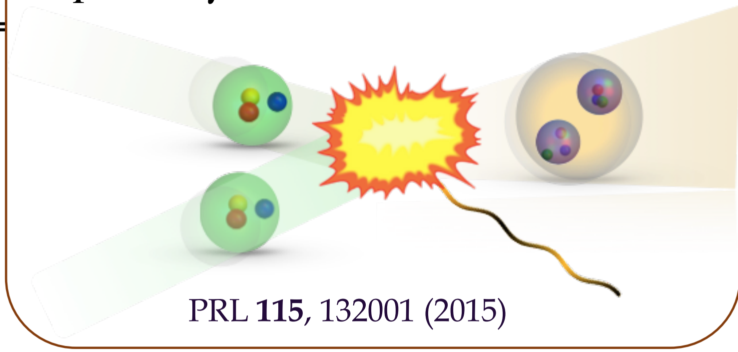
Proton-Proton Fusion



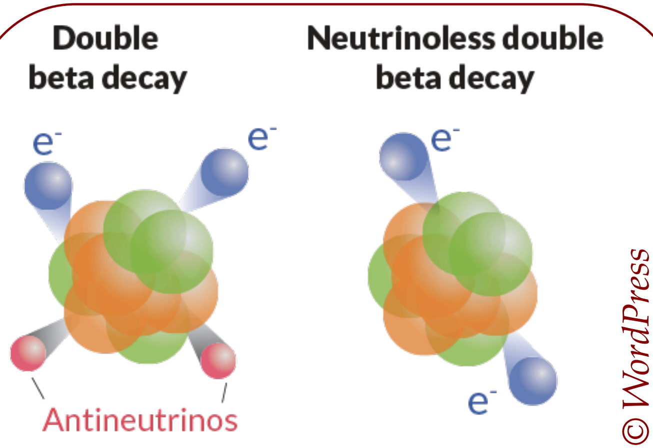
PRL **119**, 062002 (2017)

Tritium β Decay

$np \to d\gamma$ cross section

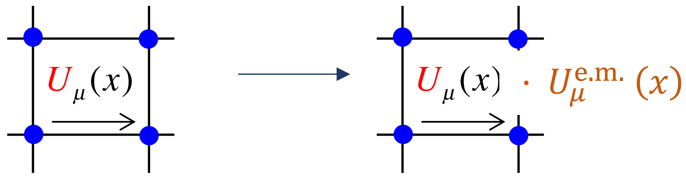


PRL **115**, 132001 (2015)

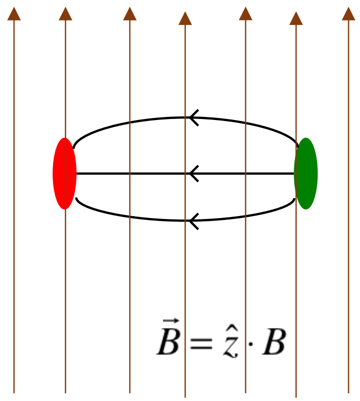


PRD 96, 054505 (2017)
PRL 119, 062003 (2017)

post-multiplication of the SU(3) color gauge links by fixed U(1) e.m. links



$$U_\mu^{\text{e.m.}}(x) = e^{iqA_\mu(x)} \in U(1)$$
$$= e^{-i\,qx_2 B\delta_{\mu 1}}\ e^{+i\,qx_1 BN\delta_{\mu 2}\delta_{x2.N-1}}$$



G. t'Hooft, 1979

U(1) flux through each plaquette = $e^{iQeF_{\mu\nu}}$,
with $F_{12} = -F_{21} = B_z$

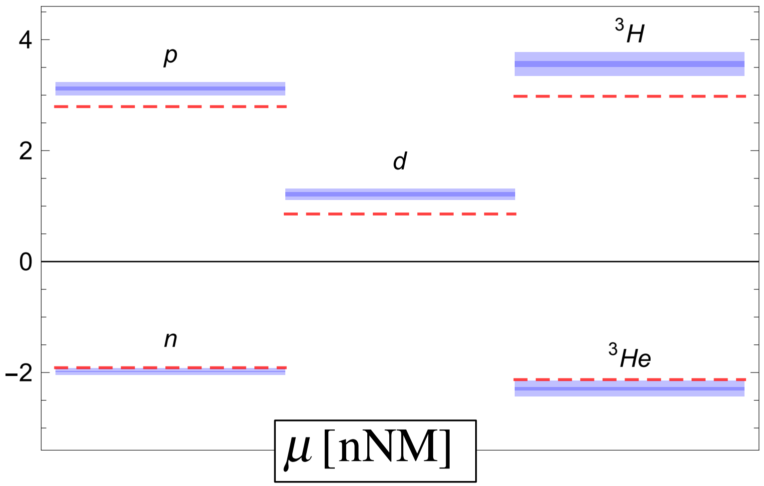
$$QeB_z = \frac{2\pi}{L^2} n$$

$$E_B^{(s)}(B_z) = M_B + \frac{|Q_B e B_z|}{M_B}\left(n_L + \frac{1}{2}\right) - 2\,\mu_B s\, B_z - 2\pi\,\beta_B^{(M0)}|B|^2 - 2\pi\,\beta_B^{(M2)}\langle \hat{T}_{ij} B_i B_j \rangle \ + \cdots$$

$O(B^2)$ (polarizabilities)

$$\boldsymbol{\delta E} \equiv E_B^{(+1/2)}(\vec{B}) - E_B^{(-1/2)}(\vec{B}) = -2\,\mu_B\, B_z + \cdots$$

NPLQCD, Phys. Rev .Lett. 113 (2014)



$$\text{nNM} = \frac{e}{2M_N^{\text{latt}}} = \frac{e}{2M_N(m_\pi^{\text{latt}})}$$

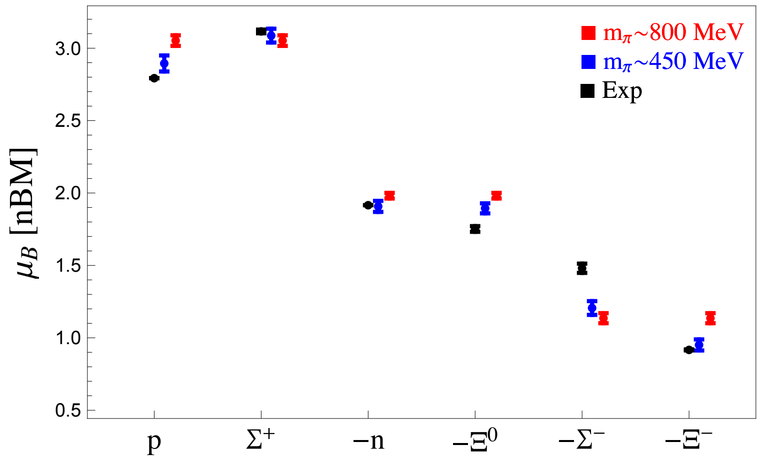
LQCD @ $m_\pi \sim 800$ MeV

experiment

Shell-model predictions

$$\mu(^3H) = \mu_p$$
$$\mu(^3He) = \mu_n$$
$$\mu_d = \mu_n + \mu_p$$

NPLQCD, PRD95, 114513 (2017)



$(\mu_{-B} \equiv -\mu_B)$

Octet baryon magnetic moments

@ ~800 MeV

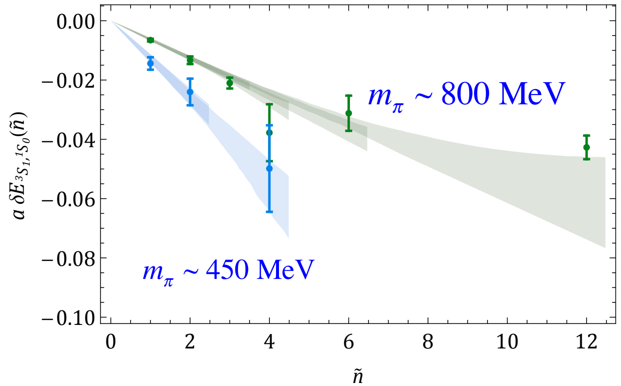
@ ~450 MeV

$np \to d\gamma$ cross section

PRL 115 (2015) 13, 132001

A magnetic field mixes the $J_z=I_z=0$ components of the NN $^3S_1$ and $^1S_0$

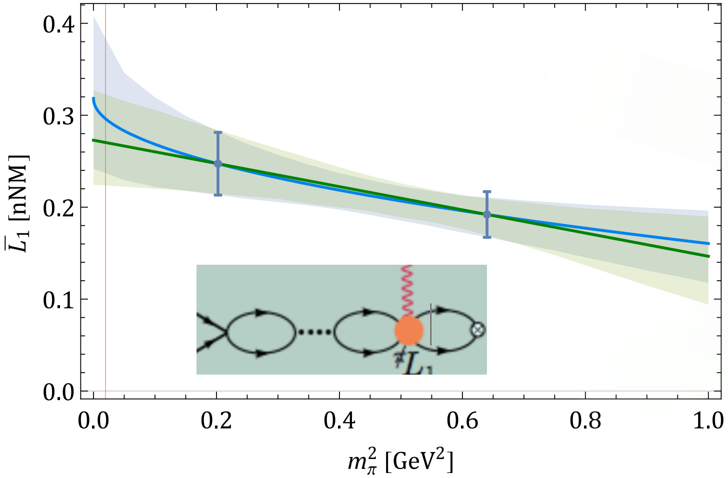
$$\mathbf{C}(t;\mathbf{B}) = \begin{pmatrix} C^{^3S_1,^3S_1}(t;\mathbf{B}) & C^{^3S_1,^1S_0}(t;\mathbf{B}) \\ C^{^1S_0,^3S_1}(t;\mathbf{B}) & C^{^1S_0,^1S_0}(t;\mathbf{B}) \end{pmatrix}$$



$$\Delta E_{^3S_1,^1S_0}(\mathbf{B}) = 2\left(\kappa_1 + \gamma_0 Z_d^2 \tilde{l}_1\right)\frac{e}{M}|\mathbf{B}| + \mathcal{O}(|\mathbf{B}|^2)$$

isovector nucleon magnetic moment (κ_1); $\overline{L}_1$ ($\gamma_0 Z_d^2 \tilde{l}_1$)

$$\delta E_{^3S_1,^1S_0} \equiv \Delta E_{^3S_1,^1S_0} - [E_{p,\uparrow} - E_{p,\downarrow}] + [E_{n,\uparrow} - E_{n,\downarrow}]$$
$$\to 2\overline{L}_1|e\mathbf{B}|/M + \mathcal{O}(\mathbf{B}^2)$$



$$\overline{L}_1^{\text{lqcd}} = 0.285\left(\begin{smallmatrix}+63\\-60\end{smallmatrix}\right)\ \text{nNM}$$

$$\sigma(np \to d\gamma) = \frac{e^2\left(\gamma_0^2 + |\vec{p}|^2\right)^3}{M^4\gamma_0^3|\vec{p}|}\left|\tilde{X}_{M1}\right|^2 + \ldots$$

$\sigma^{lqcd} = 332.4^{+5.4}_{-4.7}$ mb

($\sigma^{\exp} = 334.2 \pm 0.5$ mb)

Nuclear matrix elements in LQCD

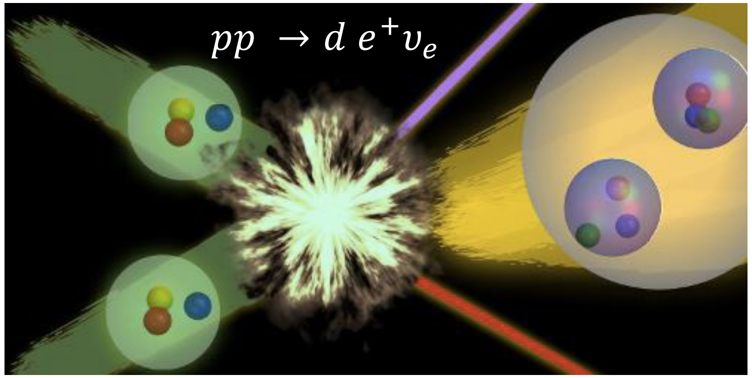
Proton-Proton Fusion and Tritium β Decay from Lattice Quantum Chromodynamics, Phys. Rev. Lett. 119, 062002 (2017)

$SU(3)_f$

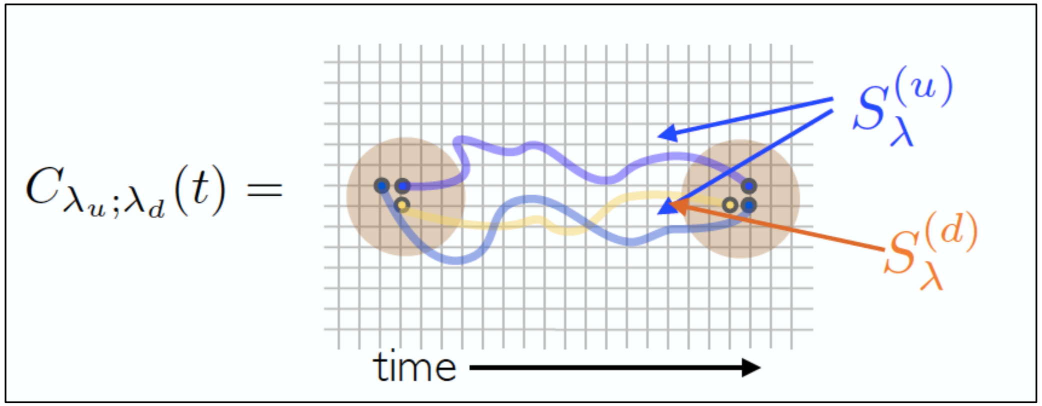
$L^3 \times T = 32^3 \times 64$

$a \approx 0.145$ fm

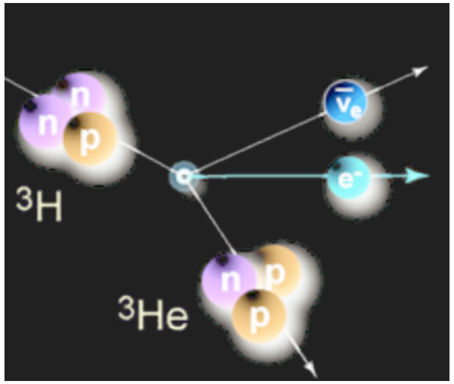
$m_\pi \approx 806$ MeV



© W. Detmold, MIT

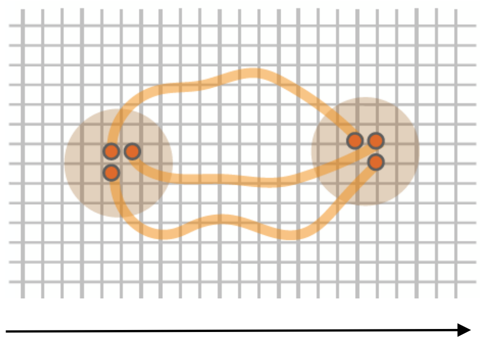


courtesy of P. Shanahan

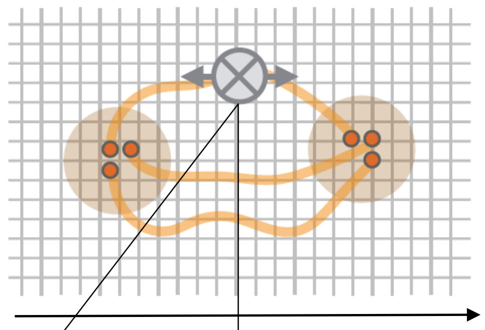


Gamow-Teller m.e.
(axial current)

nuclear matrix elements ⟷ linear response in the compound correlator



$+ \quad \lambda$



$+ \ldots$

implicit sum

courtesy of P. Shanahan

$$C^{(h\sigma)}_{\lambda_q}(t) = \Gamma_{\beta\alpha} \sum_{\boldsymbol{x}} \left(\langle 0 | \chi^h_\alpha(\boldsymbol{x}, t) \overline{\chi}^h_\beta(0) | 0 \rangle + \lambda_q \sum_{\boldsymbol{y}} \sum_{\tau=0}^{t} \langle 0 | \chi^h_\alpha(\boldsymbol{x}, t) O^{(q)}(\boldsymbol{y}, \tau) \overline{\chi}^h_\beta(0) | 0 \rangle \right) + \mathcal{O}(\lambda_q^2)$$

parity and spin projection

h={p, n, d, nn, np ($^1S_0$), pp, ^{3}H, ^{3}He}

constant background field strength parameter

$$O^{(q)} = \overline{q} \Upsilon q$$

$$\Upsilon = \{1, \gamma_\mu \gamma_5, i\sigma_{\mu\nu}\}$$

scalar, axial, tensor

(polynomial of maximum order $\lambda_u^{Nu}\ \lambda_d^{Nd}$)

nuclear matrix elements

$$C^{(h\sigma)}_{\lambda_q}(t) = \Gamma_{\beta\alpha} \sum_{\boldsymbol{x}} \left(\langle 0|\chi^h_\alpha(\boldsymbol{x},t)\overline{\chi}^h_\beta(0)|0\rangle + \lambda_q \sum_{\boldsymbol{y}} \sum_{\tau=0}^{t} \langle 0|\chi^h_\alpha(\boldsymbol{x},t) O^{(q)}(\boldsymbol{y},\tau)\overline{\chi}^h_\beta(0)|0\rangle \right) + \mathcal{O}(\lambda_q^2)$$

$$C^{(h\sigma)}_{\rm 2pt}(t,\boldsymbol{p}) \xrightarrow{t\gg 0} \mathcal{F}^{\circ}_{\rm 2pt}(\Gamma) e^{-E_{\boldsymbol{p}} t}$$

$$C^{(h\sigma)}_{\lambda_q}(t,\boldsymbol{p},\boldsymbol{p}')\Big|_{\mathcal{O}(\lambda_q)} \xrightarrow{t\to\infty} e^{-E_{\boldsymbol{p}} t}\left[c + t\mathcal{F}^{\circ,\circ}_{\rm 3pt}(\Gamma,\Upsilon)\underbrace{G_\Upsilon(Q^2=0)}_{g_\Upsilon} + \mathcal{O}(e^{-\delta E\, t})\right]$$

$$\Upsilon = \{1, \gamma_\mu\gamma_5, i\sigma_{\mu\nu}\}$$

$$R_h(t) = \sum_\sigma \frac{C^{(h\sigma)}_{\lambda_q}(t)\Big|_{\mathcal{O}(\lambda_q)}}{C^{(h\sigma)}_{\lambda_q=0}(t)}$$

$$\boxed{\overline{R}_h(t,\Upsilon) \equiv R_h(t+1) - R_h(t) \xrightarrow{t\to\infty} \frac{\mathcal{F}_{\rm 3pt}(\Gamma,\Upsilon)}{\mathcal{F}_{\rm 2pt}(\Gamma)} g_\Upsilon}$$

nuclear matrix elements

$$\overline{R}_h(t,\Upsilon) \equiv R_h(t+a) - R_h(t) \xrightarrow{t\to\infty} \frac{\mathcal{F}_{3\mathrm{pt}}(\Gamma,\Upsilon)}{\mathcal{F}_{2\mathrm{pt}}(\Gamma)} g_\Upsilon \qquad \Upsilon = \{1, \gamma_\mu\gamma_5, i\sigma_{\mu\nu}\}$$

For a polarized hadron: $\mathcal{F}_{2\mathrm{pt}}(\Gamma_{\mathrm{pol}}) = Z\tilde{Z}^\dagger$

$$\mathcal{F}_{3\mathrm{pt}}(\Gamma,\Upsilon) \begin{cases} \mathcal{F}_{3\mathrm{pt}}(\Gamma_{\mathrm{pol}},1) = Z\tilde{Z}^\dagger \\ \mathcal{F}_{3\mathrm{pt}}(\Gamma_{\mathrm{pol}},\gamma_4\gamma_5) = 0\ , \quad \mathcal{F}_{3\mathrm{pt}}(\Gamma_{\mathrm{pol}},\gamma_k\gamma_5) = Z\tilde{Z}^\dagger \cdot i s_k \\ \mathcal{F}_{3\mathrm{pt}}(\Gamma_{\mathrm{pol}},i\sigma_{i4}) = 0\ , \quad \mathcal{F}_{3\mathrm{pt}}(\Gamma_{\mathrm{pol}},i\sigma_{ij}) = Z\tilde{Z}^\dagger \cdot (-i\epsilon_{ijk4}s^k) \end{cases}$$

For a polarized hadron in the z-direction :

$$\frac{\mathcal{F}_{3\mathrm{pt}}(\Gamma_{\mathrm{pol}},1)}{\mathcal{F}_{2\mathrm{pt}}(\Gamma_{\mathrm{pol}})} g_S = g_S\ , \quad \frac{\mathcal{F}_{3\mathrm{pt}}(\Gamma_{\mathrm{pol}},\gamma_3\gamma_5)}{\mathcal{F}_{2\mathrm{pt}}(\Gamma_{\mathrm{pol}})} g_A = i s_z g_A\ , \quad \frac{\mathcal{F}_{3\mathrm{pt}}(\Gamma_{\mathrm{pol}},\gamma_1\gamma_2)}{\mathcal{F}_{2\mathrm{pt}}(\Gamma_{\mathrm{pol}})} g_T = i s_z g_T$$

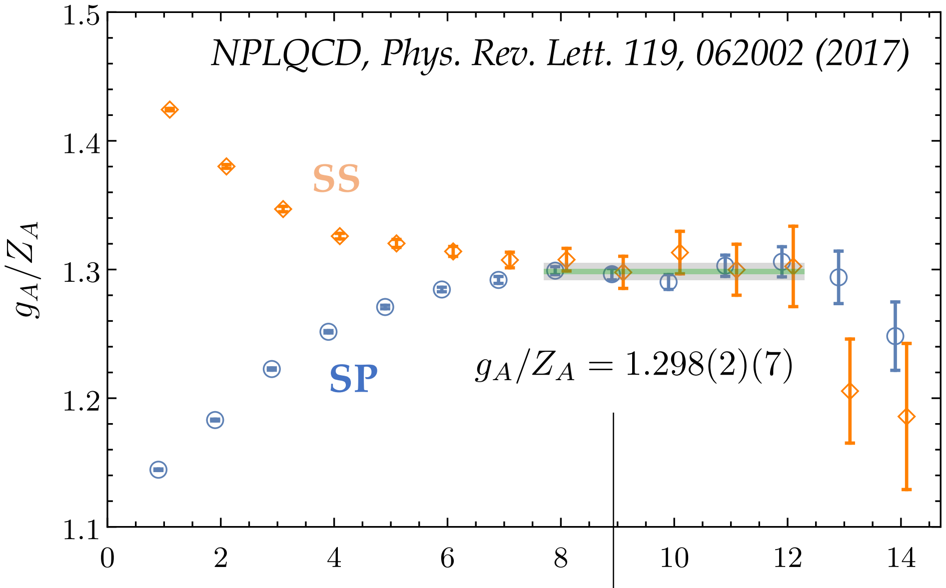
proton axial charge

$$C_{\lambda_q}^{(h\sigma)}(t) = \Gamma_{\beta\alpha} \sum_{\boldsymbol{x}} \left(\langle 0|\chi_\alpha^h(\boldsymbol{x},t)\overline{\chi}_\beta^h(0)|0\rangle + \lambda_q \sum_{\boldsymbol{y}} \sum_{\tau=0}^{t} \langle 0|\chi_\alpha^h(\boldsymbol{x},t) O^{(q)}(\boldsymbol{y},\tau)\overline{\chi}_\beta^h(0)|0\rangle \right) + \mathcal{O}(\lambda_q^2)$$

$$R_p(t) = \frac{C^{(p)}_{\lambda_u;\lambda_d=0}(t)\Big|_{\mathcal{O}(\lambda_u)} - C^{(p)}_{\lambda_u=0;\lambda_d}(t)\Big|_{\mathcal{O}(\lambda_d)}}{C^{(p)}_{\lambda_u=0;\lambda_d=0}(t)}$$

$$\overline{R}_p(t) \equiv R_p(t+1) - R_p(t) \xrightarrow{t\to\infty} \frac{g_A}{Z_A}$$

unrenormalized isovector axial charge of the proton



Z_A=0.867(43)

$$g_A = 1.13(2)(7)$$

Gamow-Teller matrix element for $^3\mathrm{H} \to {}^3\mathrm{He}\ e^- \bar{\nu}$

*Schiavilla, Viviani et al. PRC **58**, 1263 (1998)*

NPLQCD, Phys. Rev. Lett. 119, 062002 (2017)

$$\frac{(1+\delta_R) f_V}{K/G_V^2} t_{1/2} = \frac{1}{\langle \mathbf{F} \rangle^2 + f_A/f_V\, g_A^2 \langle \mathbf{GT} \rangle^2}$$

known (exp/theory)

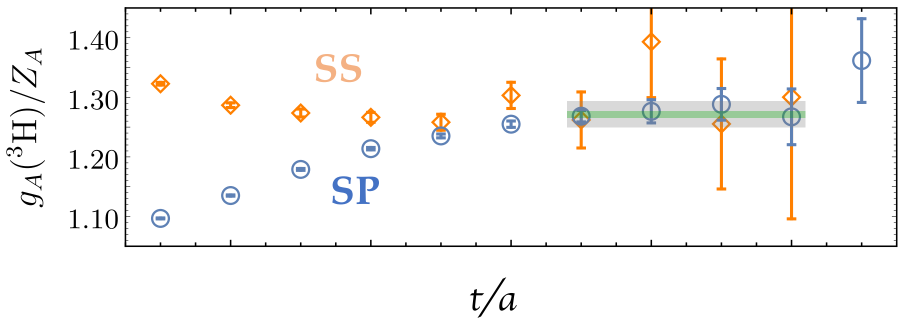
Fermi reduced m.e. (vector current)

Gamow-Teller m.e. (axial current)

$m_u = m_d$

~~e.m.~~

$$\overline{R}_{^3\mathrm{H}}(t) \xrightarrow{\text{large t}} g_A(^3\mathrm{H})/Z_A$$

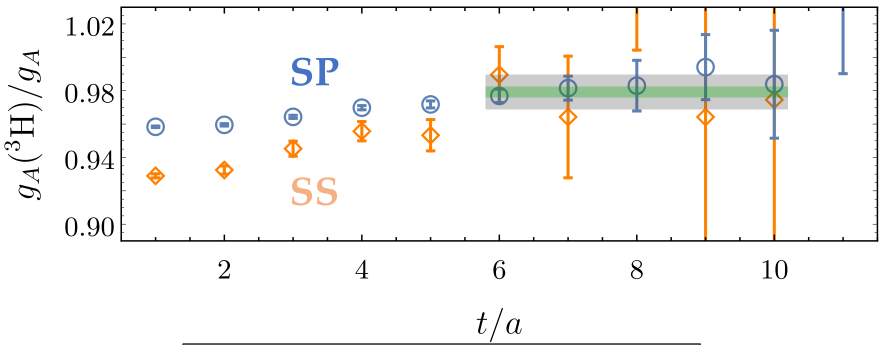


$$\frac{g_A(^3\mathrm{H})}{Z_A} = 1.272(6)(22)$$

Gamow-Teller matrix element for $^3\mathrm{H} \to {^3\mathrm{He}}\, e^- \bar{\nu}$

NPLQCD, Phys. Rev. Lett. 119, 062002 (2017)

$$\langle \mathbf{GT} \rangle(t) = \overline{R}_{^3H}(t)/\overline{R}_p(t)$$

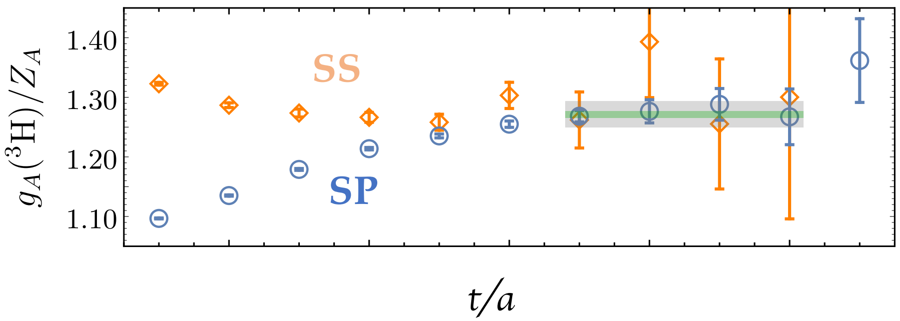


$$\boxed{\frac{g_A(^3\mathrm{H})}{g_A} = 0.979(3)(10)}$$

$m_u = m_d$

~~e.m.~~

$$\overline{R}_{^3\mathrm{H}}(t) \xrightarrow{\text{large t}} g_A(^3\mathrm{H})/Z_A$$



$$\boxed{\frac{g_A(^3\mathrm{H})}{Z_A} = 1.272(6)(22)}$$

$\langle \mathbf{GT} \rangle_{\mathrm{exp}} = 0.9511(13)$ *Baroni, Girlanda, Kievsky, Marcucci, Schiavilla, Viviani, PRC **94**, 024003 (2016)*

proton-proton fusion cross-section $pp \to de^+\nu$

NPLQCD, Phys. Rev. Lett. 119, 062002 (2017)

Background isovector axial-vector field

$$A^{a,\mu} = \bar{q}\gamma^\mu\gamma^5\frac{\tau^a}{2}q$$

$\Delta I = 1$ and $\Delta J = 1$

mixes the J_z=I_z=0 components of the NN $^3S_1$ and $^1S_0$

access to the $pp \to de^+\nu$ matrix element

$$C^{(h\sigma)}_{\lambda_q}(t) = \Gamma_{\beta\alpha}\sum_{\boldsymbol{x}}\left(\langle 0|\chi^h_\alpha(\boldsymbol{x},t)\overline{\chi}^h_\beta(0)|0\rangle + \lambda_q\sum_{\boldsymbol{y}}\sum_{\tau=0}^{t}\langle 0|\chi^h_\alpha(\boldsymbol{x},t)O^{(q)}(\boldsymbol{y},\tau)\overline{\chi}^h_\beta(0)|0\rangle\right) + \mathcal{O}(\lambda_q^2)$$

$$C^{(^3S_1,^1S_0)}_{\lambda_u,\lambda_d=0}(t) = \lambda_u\sum_{\boldsymbol{x},\boldsymbol{y}}\sum_{\tau=0}^{t}\langle 0|\chi_{^3S_1}(\boldsymbol{x},t)J_3^{(u)}(\boldsymbol{y},\tau)\chi^\dagger_{^1S_0}(0)|0\rangle + c_2\lambda_u^2 + c_3\lambda_u^3$$

$$C^{(^3S_1,^1S_0)}_{\lambda_u=0,\lambda_d}(t) = \lambda_d\sum_{\boldsymbol{x},\boldsymbol{y}}\sum_{\tau=0}^{t}\langle 0|\chi_{^3S_1}(\boldsymbol{x},t)J_3^{(d)}(\boldsymbol{y},\tau)\chi^\dagger_{^1S_0}(0)|0\rangle + d_2\lambda_d^2 + d_3\lambda_d^3$$

to extract the desired matrix element, one computes the correlator @ three (or more) values of λ (λ_u , λ_d)

proton-proton fusion cross-section $pp \to de^+\nu$

relevant matrix element

NPLQCD, Phys. Rev. Lett. 119, 062002 (2017)

$$\left|\langle d; j \left| A_k^- \right| pp\rangle\right| \equiv g_A C_\eta \sqrt{\frac{32\pi}{\gamma^3}}\, \Lambda(p)\, \delta_{jk},$$

M. Butler and J.-W. Chen, PLB 520, 87 (2001)

$\not{\pi}$EFT, N^{2}LO

$$\overline{L}_{1,A} = \frac{1}{2g_A}\frac{1-\gamma\rho}{\gamma} L_{1,A}^{sd-2b} - \frac{1}{2}\sqrt{r_1\rho}$$

$$\Lambda(0) = \frac{1}{\sqrt{1-\gamma\rho}}\{e^\chi - \gamma a_{pp}[1 - \chi e^\chi \Gamma(0,\chi)] + \frac{1}{2}\gamma^2 a_{pp}\sqrt{r_1\rho}\} - \frac{1}{2g_A}\gamma a_{pp}\sqrt{1-\gamma\rho}\, L_{1,A}^{sd-2b}$$

$$\chi = \alpha M_p/\gamma$$

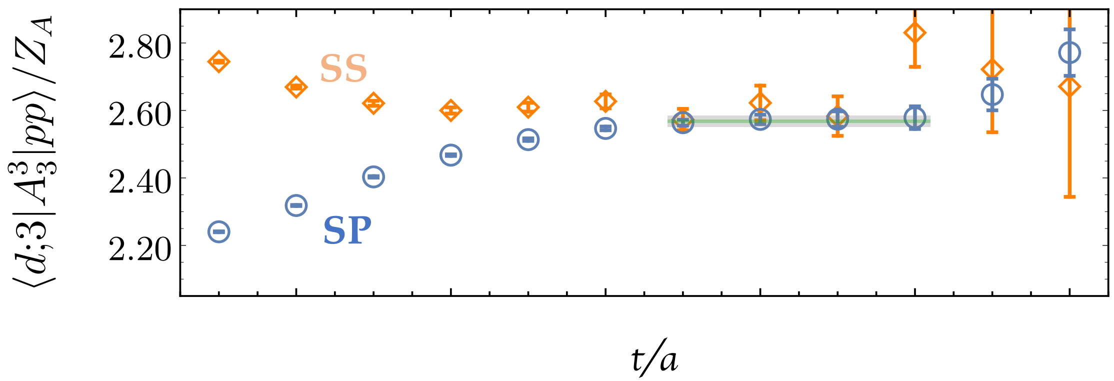
two-body short-distance

proton-proton fusion cross-section $pp \to de^+\nu$

NPLQCD, Phys. Rev. Lett. 119, 062002 (2017)

$$R_{^3S_1,^1S_0}(t) = \frac{C^{(^3S_1,^1S_0)}_{\lambda_u,\lambda_d=0}(t)\Big|_{\mathcal{O}(\lambda_u)} - C^{(^3S_1,^1S_0)}_{\lambda_u=0,\lambda_d}(t)\Big|_{\mathcal{O}(\lambda_d)}}{\sqrt{C^{(^3S_1,^3S_1)}_{\lambda_u=0,\lambda_d=0}(t)\,C^{(^1S_0,^1S_0)}_{\lambda_u=0,\lambda_d=0}(t)}}$$

$$\overline{R}_{^3S_1,^1S0}(t) \equiv R_{^3S_1,^1S0}(t+1) - R_{^3S_1,^1S_0}(t) \xrightarrow{t\to\infty} \frac{\left\langle ^3S_1; J_z=0 \left| A_3^3 \right| {}^1S_0; I_z=0 \right\rangle}{Z_A} = \boxed{2.568(5)(31)}$$



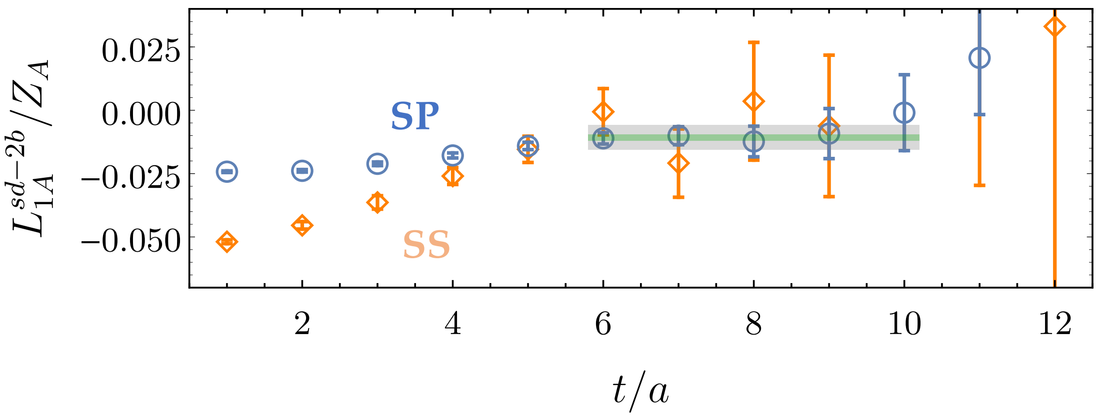
proton-proton fusion cross-section $pp \to de^+\nu$

NPLQCD, Phys. Rev. Lett. 119, 062002 (2017)

$$R_{^3S_1,^1S_0}(t) = \frac{C^{(^3S_1,^1S_0)}_{\lambda_u,\lambda_d=0}(t)\Big|_{\mathcal{O}(\lambda_u)} - C^{(^3S_1,^1S_0)}_{\lambda_u=0,\lambda_d}(t)\Big|_{\mathcal{O}(\lambda_d)}}{\sqrt{C^{(^3S_1,^3S_1)}_{\lambda_u=0,\lambda_d=0}(t)\,C^{(^1S_0,^1S_0)}_{\lambda_u=0,\lambda_d=0}(t)}}$$

$$L^{sd-2b}_{1,A}(t)/Z_A = [\overline{R}_{^3S_1,^1S_0}(t) - 2\overline{R}_p(t)]/2$$

$$\xrightarrow{t\to\infty} \frac{\left\langle ^3S_1; J_z=0 \left| A_3^3 \right| {}^1S_0; I_z=0 \right\rangle - 2g_A}{2Z_A} = \frac{L^{sd-2b}_{1,A}}{Z_A} = \boxed{-0.011(01)(15)}$$



proton-proton fusion cross-section $pp \to de^+\nu$

Estimation @ physical quark mass

NPLQCD, Phys. Rev. Lett. 119, 062002 (2017)

$$\Lambda(0) = 2.659(2)(9)(5)$$

$$\Lambda(0) = 2.65(1)$$

E. G. Adelberger et al., Rev. Mod. Phys. **83**, 195 (2011)

N^2LO

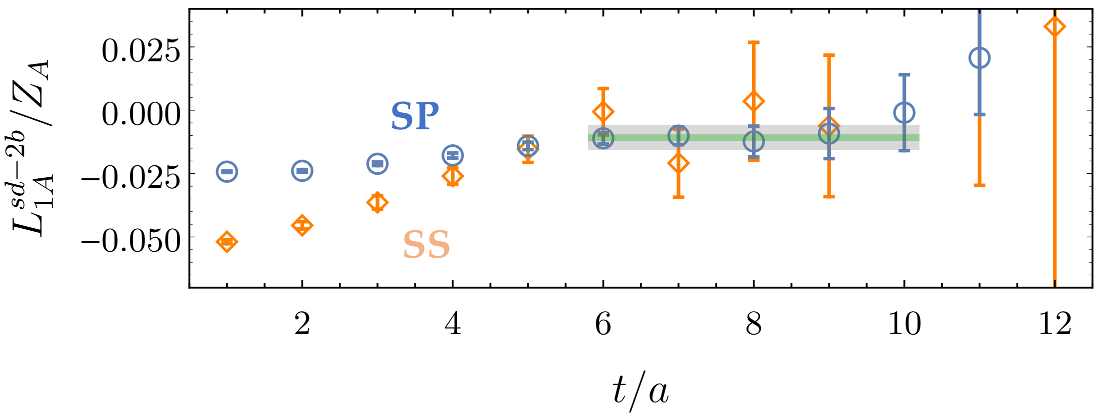
M. Butler and J.-W. Chen, Phys. Lett. B **520**, 87 (2001)

$$\Lambda(0) = 2.62(1) + 0.0105(1)L_{1,A}$$

$$\frac{L_{1,A}^{sd-2b}}{Z_A} = -0.011(01)(15)$$

$$L_{1,A} = 3.9(0.2)(1.0)(0.4)(0.9)\ \text{fm}^3$$

EFT ($\not\pi$)

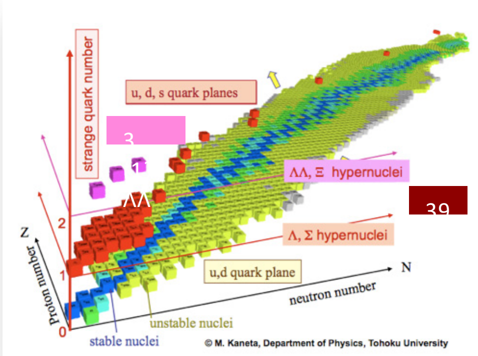


On going investigations

- ○ *Spectroscopy @ lighter quark masses*
  - *ongoing analysis of baryon interactions and light nuclear systems @ ~ 450 MeV , including strangeness*
  - *starting production @ ~ 170 MeV*

- ○ *LQCD calculations would be specially of interest for systems that are not accessible experimentally*

  Complementary information to experimental programs (JPARC/KEK, GSI/FAIR, JINR, BNL, JLAB, MAMI)



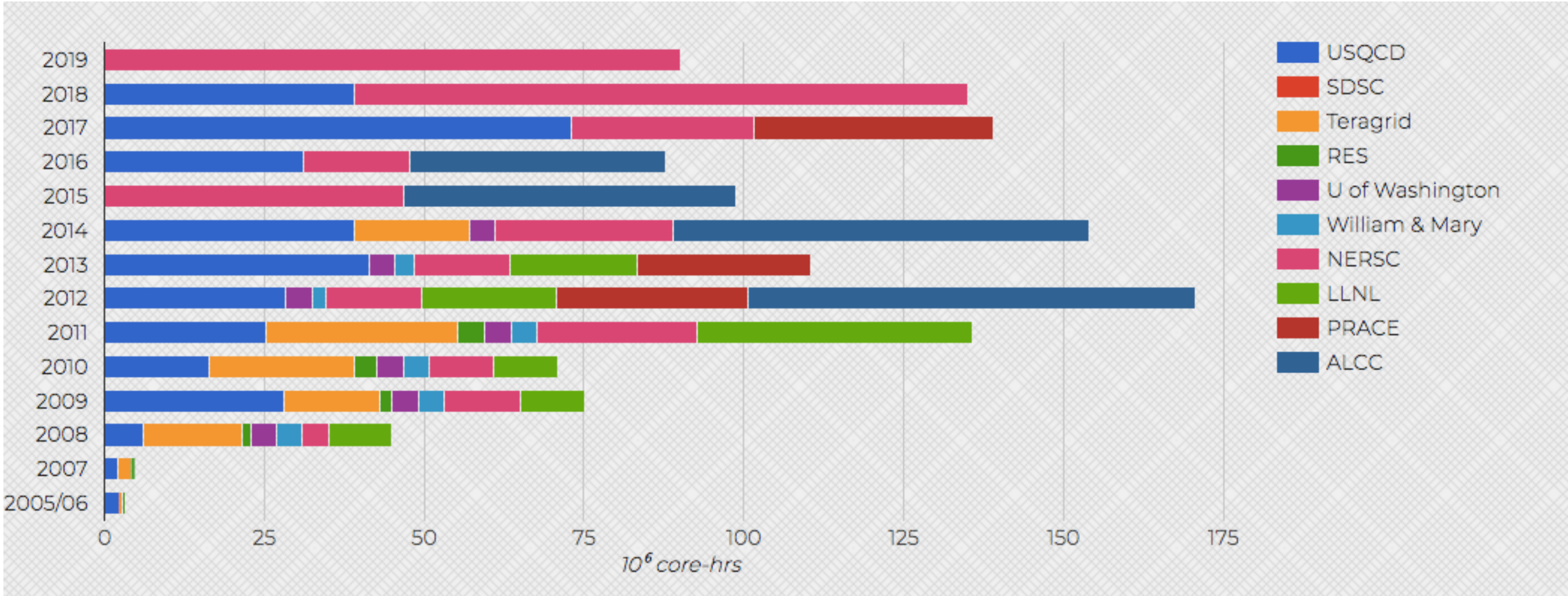
Weak transition amplitudes in few-nucleon systems can be studied directly from the fundamental quark and gluon degrees of freedom. The study at lighter quark masses is feasible and it is ongoing.

Lattice computations are relevant for the experimental program (SNO, MuSun, double-β decay, Electron-Ion Collider, Nuclear electric dipole moments, dark matter direct detection)

Acknowledgments

NPLQCD
Nuclear Physics with Lattice QCD

Computational resources, in units of 10^6 core-hrs



http://nplqcd.ub.edu

compiled by Marc Illa