Theoretical Models of Neutrino-Nucleus Scattering. Status and Perspectives

Omar Benhar

INFN and Department of Physics, “Sapienza” University
I-00185 Roma, Italy

Neutrino-Nucleus Scattering: Status and Opportunities
ECT*, May 27, 2019

PREAMBLE

- ⋆ The present understanding of ν-nucleus cross sections is not adequate to the needs of neutrino experiments
- ⋆ A great deal of effort is being devoted to both theoretical and experimental studies of neutrino-nucleus interactions
- ⋆ The large body of accurate of electron scattering data provides valuable complementary information
- ⋆ The long-term goal, strongly advocated by Eligio Lisi in his opening talk at NUINT 2018, is a unified model for the nuclear response to electroweak probes

- ⋆ This talk will attempt to provide an admittedly biased review of theoretical models of neutrino-nucleus interactions in the kinematical regime relevant to long-baseline neutrino experiments

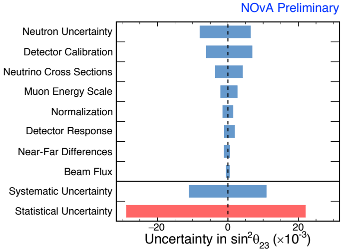
Event Rate Distribution

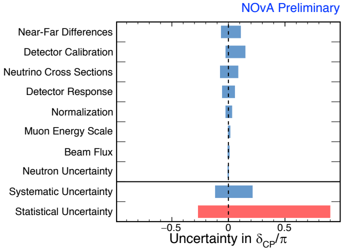
⋆ Neutrino experiments measure event rate distributions as a function of the *visible energy*, E_{vis}

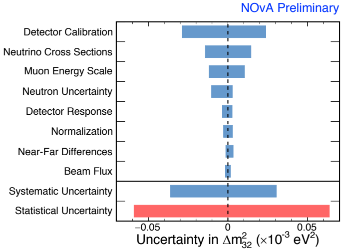
$$R_{\alpha\to\beta}(E_{\text{vis}}) = \mathcal{N} \int dE_\nu \Phi_\alpha(E_\nu) P_{\nu_\alpha\to\nu_\beta}(E_\nu)\, \sigma_\beta(E_\nu, E_{\text{vis}})\, \epsilon_\beta(E_\nu)$$

- $\mathcal{N}$ is a normalization factor
- $\Phi_\alpha(E_\nu)$ is the neutrino flux as a function of neutrino energy
- $P_{\nu_\alpha\to\nu_\beta}(E_\nu)$ is the oscillation probability as function of the neutrino energy
- the differential cross section $\sigma_\beta(E_\nu, E_{\text{vis}})$ describes the probability that a neutrino of flavor β and energy E_ν produces a distribution of visible energies E_{vis} in the detector
- $\epsilon_\beta(E_\nu)$ is the detection efficiency

Systematic Uncertainties





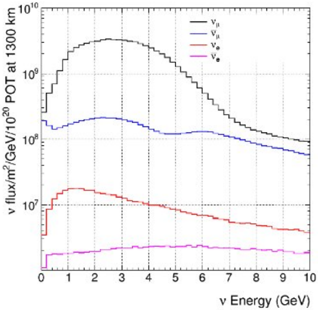


Most important systematics:

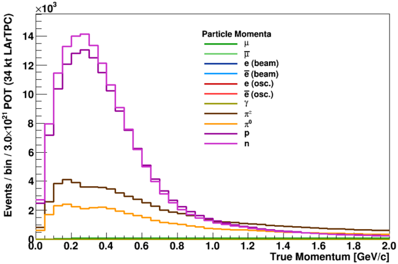
- Detector Calibration
  - Will be improved by the 2019 test beam program
- Neutrino cross sections
  - Particularly nuclear effects (RPA, MEC)
- Muon energy scale
- Neutron uncertainty – **new** with $\bar{\nu}$'s

THE CHALLENGE

- Predicted neutrino fluxes at SURF, for the DUNE beam in “neutrino” mode.



- Single-particle momentum spectra from LBNF beam neutrino interactions (sample obtained from reverse horn-current running).



- Meeting this challenge will require the development of a theoretical framework providing a consistent description of a variety of reaction channels over a wide range of neutrino energy.

THE NEUTRINO-NUCLEUS X-SECTION

- ⋆ Consider, for example, the inclusive cross section of the charged-current reaction

$$\nu_\mu + A \to \mu^- + X$$

at fixed beam energy

$$d\sigma_A \propto L_{\mu\nu} W_A^{\mu\nu}$$

  - $L_{\mu\nu}$ is fully specified by the lepton kinematical variables
  - The determination of the nuclear response

$$W_A^{\mu\nu} = \sum_X \langle 0 | J_A^{\mu\,\dagger} | X \rangle \langle X | J_A^\nu | 0 \rangle \delta^{(4)}(P_0 + k - P_X - k')$$

  involves

    - the ground state of the target nucleus, $|0\rangle$
    - all relevant hadronic final states, $|X\rangle$
    - the nuclear current operator ($q \equiv (\omega, \mathbf{q}) = P_X - P_0$)

$$J_A^\mu(q) = \sum_i j_i^\mu + \sum_{j>i} j_{ij}^\mu + \ldots$$

Modeling Nuclear Dynamics

- At low to moderate $|\mathbf{q}|$—typically $\lesssim 0.5$ GeV—the initial and final states can be obtained from the non relativistic nuclear Hamiltonian

$$H = \sum_i \frac{\mathbf{p_i}^2}{2m} + \sum_{j>i} v_{ij} + \sum_{k>j>i} V_{ijk}$$

  where v_{ij} and V_{ijk} account for the properties of the two- and three-nucleon system by construction

- the nuclear current operators J_A^μ, consistent with the Hamiltonian H, can be approximated by the leading terms of an expansion in powers of the ratio $|\mathbf{q}|/m$
- In this kinematical regime, accurate parameter free calculations of the neutrino-nucleus cross section can be performed within the framework of Nuclear Many-body Theory (NMBT)
- At larger q and ω the non relativistic descritpion breaks down, and degrees of freedom other than nucleons play a role. Further approximations are needed!

A Somewhat Arbitrary Classification

Theoretical models of neutrino-nucleus interactions can be classified according to the underlying description of nuclear dynamics

- ⋆ Approaches based on microscopic models of nuclear dynamics, constrained by observed properties of the two- and three-nucleon systems
  - Quantum Monte Carlo
  - Spectral Function Formalism
- ⋆ Diagrammatic approaches based on simplified models of nuclear dynamics
  - Valencia model
  - Lyon-CERN model
- ⋆ Semi-phenomenological models
  - Superscaling model and its extensions

Green's Function Monte Carlo

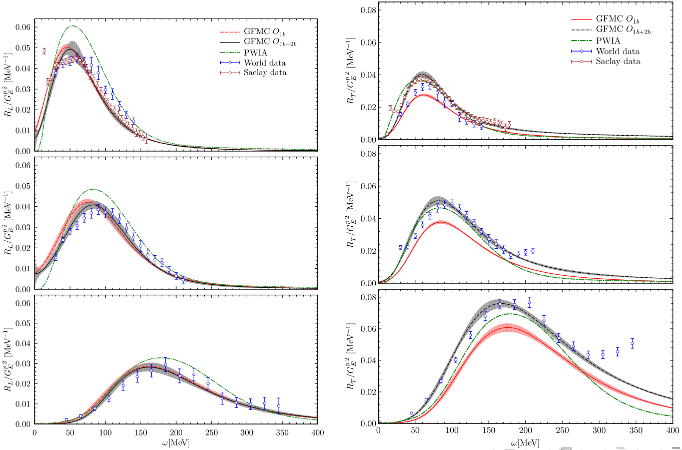
- In the non relativistic regime, the Laplace transform of the response functions of isospin-symmetric nuclei with $A \leq 12$ have been obtained from imaginary-time evolution
- The euclidean response tensor can be cast in the form

$$E_A^{\mu\nu}(|\mathbf{q}|, \tau) \propto \int_{\omega_{el}}^{\infty} d\omega e^{-\omega\tau} W_A^{\mu\nu}(q) = \frac{\langle 0|J_A^{\mu\,\dagger} e^{-(H-E_0)\tau} J_A^{\nu}|0\rangle}{\langle 0|e^{-(H-E_0)\tau}|0\rangle}$$

- Inversion of the euclidean response, which amounts to performig the transformation $E_A^{\mu\nu}(|\mathbf{q}|, \tau) \to W_A^{\mu\nu}(|\mathbf{q}|, \omega)$, is a long standing problem, involving non trivial issues
- An efficient inversion procedure, based on the maximum entropy principle, has been recently developed and applied to study both the electromagnetic and weak nuclear responses

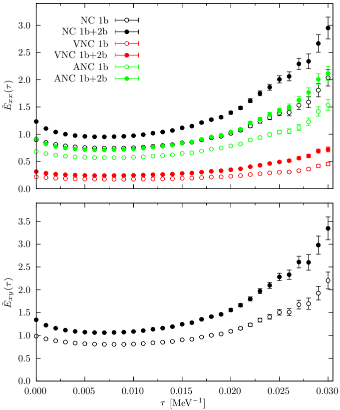
Electron Scattering as a Testing Ground

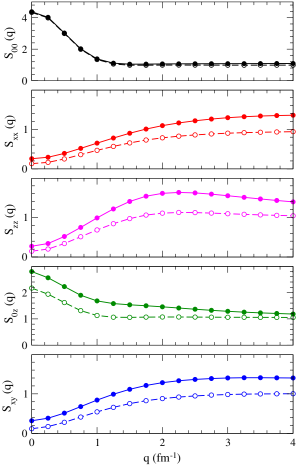
- ⋆ GFMC longitudinal (left) and transverse (right) electromagnetic response functions of ^{12}C [PRL 117, 082501 (2016)], compared to the results of J. Jourdan's analysis of the world data. The momentum transfer ranges from 300 MeV (top) to 570 MeV (bottom)



Weak Euclidean Response and Sum Rules

- ⋆ Neutral current euclidean responses of ^{12}C at momentum transfer $|\mathbf{q}| = 570$ MeV [left, PRC 91, 062501 (2015)] and sum rules [right, PRL 112, 182502 (2014)]





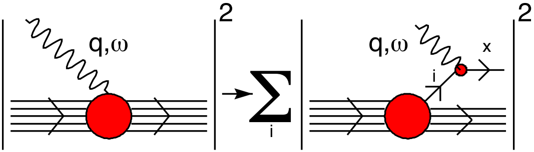
Breakdown of the Non Relativistic Approximation

- the bad news: at large momentum transfer, the initial and final states and the current operator can no longer be described within a fully consistent framework
- the good news: in the kinematical regime in which

$$\lambda \sim \frac{\pi}{|\mathbf{q}|} \ll d_{\rm NN} \ ,$$

where $d_{\rm NN}$ is the average NN distance in the target nucleus, nuclear scattering reduces to the incoherent sum of scattering processes involving individual nucleons

- Enter the Impulse Approximation (IA)



$$\left| \quad \right|^2 \rightarrow \sum_i \left| \quad \right|^2$$

Impulse Approximation and Factorization

⋆ The IA naturally leads to factorization of the nuclear transition amplitude. As a consequence the double differential cross section of the process $\nu_\mu + A \to \mu^- + X$ can be written in the simple form

$$\frac{d^2\sigma_{IA}}{d\Omega_\mu dE_\mu} = \int d^3k\, dE\, P(\mathbf{k}, E)\, \frac{d^2\sigma_{\nu N}}{d\Omega_\mu dE_\mu} \ ,$$

where

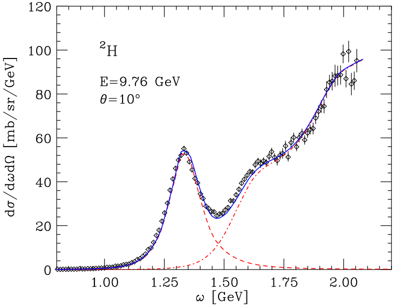
- the elementary cross section $d^2\sigma_{\nu N}$—written in terms of five nucleon structure functions W_i—can (at least in principle) be obtained from proton and deuteron data
- the spectral function $P(\mathbf{k}, E)$, describing the probability of removing a nucleon of momentum $\mathbf{k}$ from the target ground state, leaving the residual system with excitation energy E, is an intrinsic property of the target, independent of q
- factorization allows for a consistent treatment of all relevant reaction channels
- corrections arising from the effects of final-state interactions (FSI) and two-body currents (MEC) can be consistently taken into account

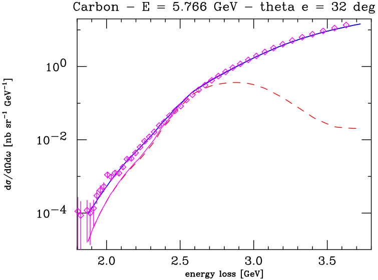
ELECTRON SCATTERING AS A TESTING GROUND

- ⋆ cross section of the process

$$e + A \to e' + X$$

- ⋆ Deuteron (SLAC data)
- ⋆ Carbon (JLab data)

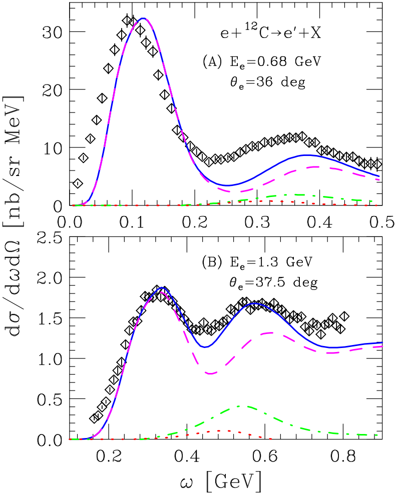


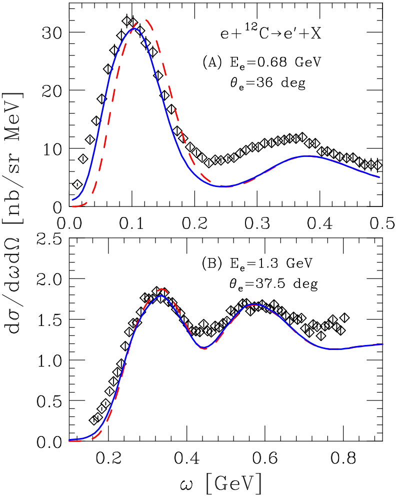


- ⋆ The formalism based on nuclear spectral functions provides a unified and remarkably accurate description of the data for a broad range of targets and kinematics

Electron Scattering as a Testing Ground (continued)

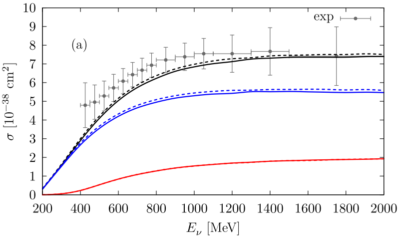
- N. Rocco, A. Lovato, and OB [PRL 116, 192501 (2015)]

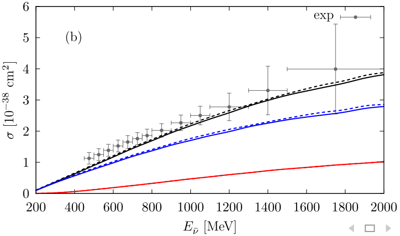




Flux-Unfolded Total Cross Section

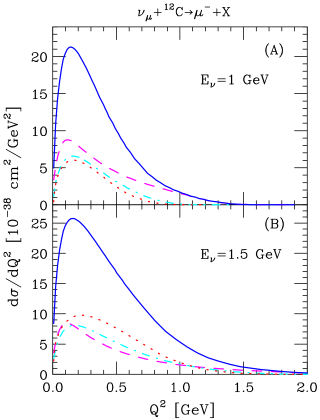
- Flux unfolded ν- (top panel) and $\bar{\nu}$-carbon (bottom panel) total cross sections in the quasi elastic channel, compared to MiniBooNE data [PRC 99 022502 (2019)]

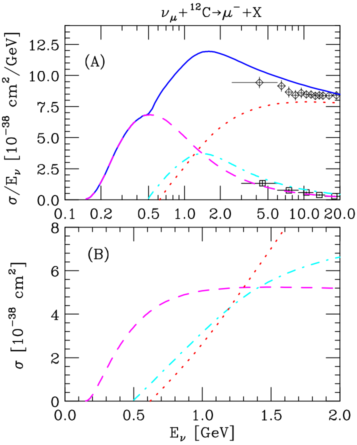




Extension to the Inelastic Sector

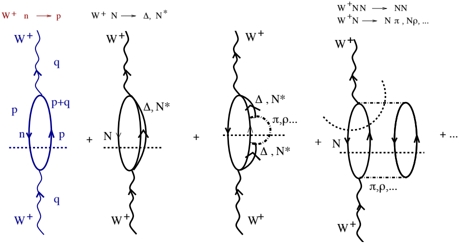
- ⋆ Comparison to NOMAD data [Vagnoni et al, PRL 118, 142502 (2017)]

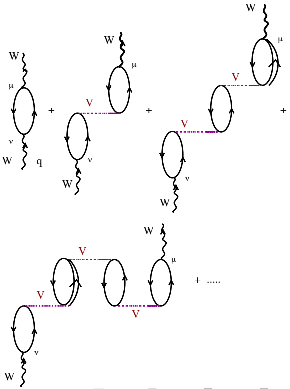




Diagrammatic Approaches: Valencia Model

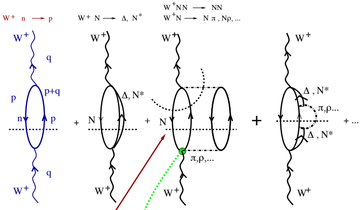
- A variety of reaction mechanisms contributing to the nuclear responses (quasi elastic scattering, resonance production, MEC, collective excitations ...) are taken into account using a simplified model of nuclear dynamics, typically based on the mean field approximation and meson exchange interactions
  - from J. Nieves' talk at NUFACT11

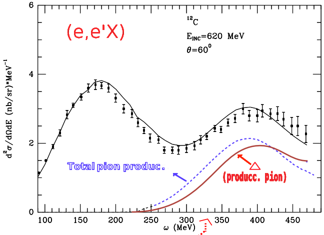




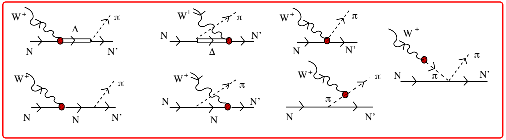
ELECTRON SCATTERING AS A TESTING GROUND

Above QE Region: π Production





MEC ⟶ QE like !



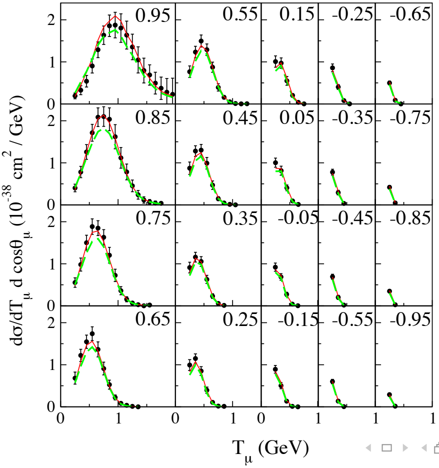
PRD D76 (2007) 033005 ⟶

NEUTRINO-CARBON CROSS SECTION

- ⋆ Double differential cross section of the process

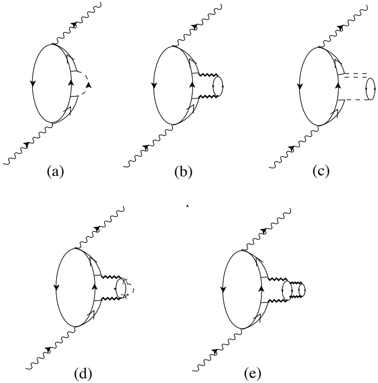
$$\nu_\mu +^{12} C \to \mu^- + X$$

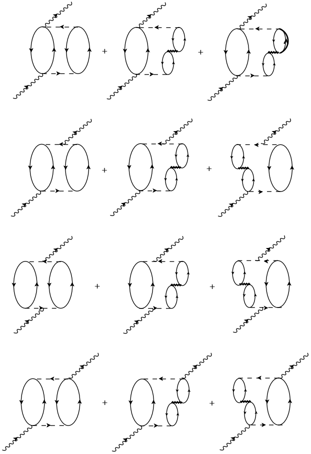
in the quasi elastic (CCQE) channel



LYON-CERN (MARTINI) MODEL

- ⋆ Conceptually similar to Valencia model. The main differences arise from a different treatment of the excitation of two-particle-two-hole final states





Neutrino-Carbon Cross Section

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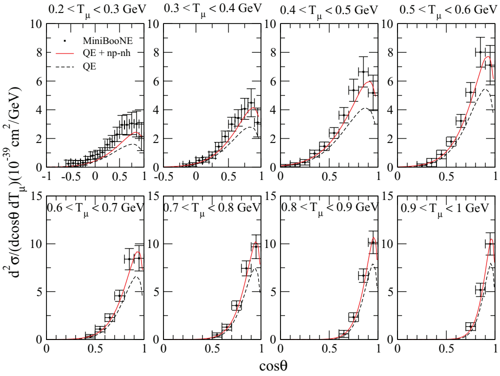
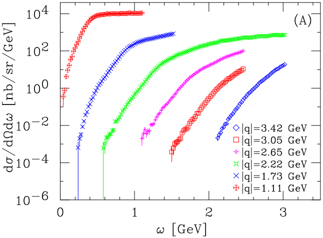
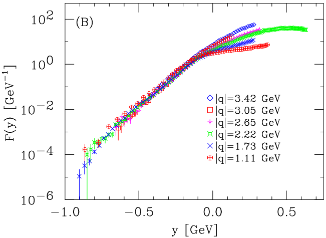


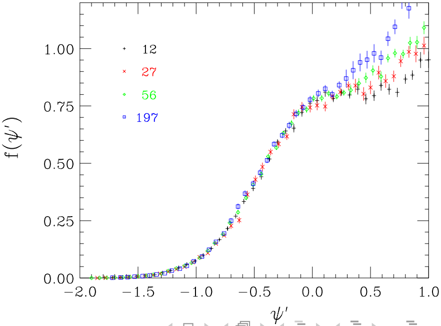
Scaling of First and Second Kind

- In the IA regime, the quasielastic nuclear response exhibits scaling in the variable $y(|\mathbf{q}|, \omega)$



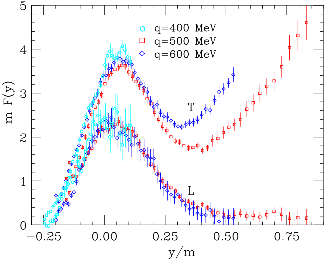


- The response functions of different nuclei scale in a different variable, whose definition involves a parameter referred to as Fermi momentum, parametrizing the A-dependence of nuclear effects



SUPERSCALING MODEL

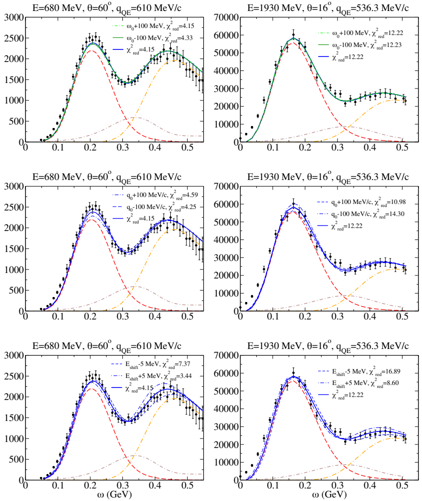
- Sumultaneous occurrence of scaling of first and second kind is referred to as superscaling
- The availability of longitudinal and transverse electromagnetic responses allows to extract a universal scaling function extending to the Δ-production region



- In principle, superscaling can be extended exploited to predict neutrino-nucleus cross sections. However, inclusion the inclusion of contributions from non-scaling mechanisms, such as FSI, MEC and inelastic scattering, involves a somewhat ad hoc procedure

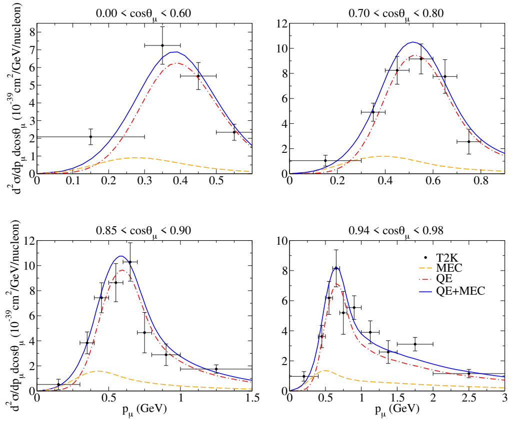
Superscaling Model

- Comparison to electron scattering data (G. Megias' talk at NUINT 2018)



Neutrino Cross Sections

- ⋆ Double differential cross section of CC0π ν_μ-C8H8 events measured by the T2K collaboration (G. Megias' talk at NUINT2018)



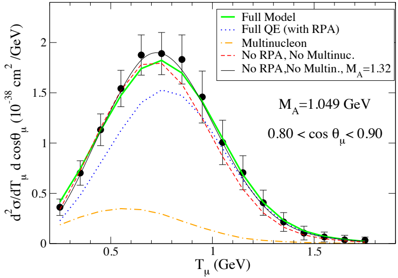
WHERE WE ARE

- ⋆ Even if we restrict ourselves to the 0π sector, the interpretation of the signals measured by neutrino detectors require the understanding of the different reaction mechanisms contributing to the neutrino-nucleus cross section: single-nucleon knock out, coupling to meson-exchange currents (MEC), and excitation of collective modes
- ⋆ Over the ~ 15 years since the first NuINT Workshop—that we may characterize as the post Fermi-gas age—a number of more advanced models have been developed
- ⋆ Electron scattering data, mainly *inclusive* cross sections, have been exploited to derive or validate the some of proposed models
- ⋆ Several models have achieved the degree of maturity required for a meaningful comparison between their predictions and the measured neutrino-nucleus cross sections

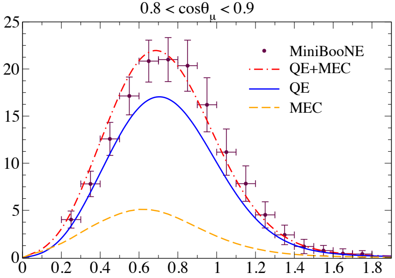
THE ISSUE OF DEGENERACY

- ⋆ Comparison to the flux-integrated CCQE cross section measured by the MiniBooNE collaboration

- ▶ Nieves *et al*



- ▶ Megias *et al*



- ⋆ Calculation based on different approximations and including different reaction mechanisms yield similar results!

Unraveling the CCQE Neutrino-Nucleus Cross Section

- ⋆ An accurate description of the 2p2h sector and collective excitations, providing a $\sim 20\%$ contribution to the nuclear cross section, is only relevant to the extent to which the remaining $\sim 80\%$, arising from processes involving 1p1h final states, is fully understood. The ability of the models to explain single-nucleon knock out needs to be assessed

- ⋆ Fifty years of $(e, e'p)$ experiments, in which the scattered electron and the outgoing proton are detected in coincidence, have provided a wealth of information on single nucleon knock-out processes associated with 1p1h final states

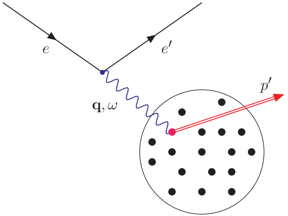
- ⋆ The large database of $(e, e'p)$ cross sections—measured mainly at Saclay, NIKHEF-K and Jefferson Lab—must be exploited to test the theoretical approaches employed to study neutrino-nucleus interaction, and assess their predictive power

THE $(e, e'p)$ REACTION

- Consider the process

$$e + A \to e' + p + (A-1)$$

in which both the outgoing electron and the proton, carrying momentum p', are detected in coincidence, and the recoiling nucleus can be left in a any (bound or continuum) state $|n\rangle$ with energy E_n



- In the absence of final state interactions (FSI)—which can be taken into acount as corrections—the the *measured* missing momentum and missing energy can be identified with the momentum of the knocked out nucleon and the excitation energy of the recoiling nucleus, $E_n - E_0$

$$\mathbf{p}_m = \mathbf{p}' - \mathbf{q} \quad , \quad E_m = \omega - T_{\mathbf{p}'} - T_{A-1} \approx \omega - T_{\mathbf{p}'}$$

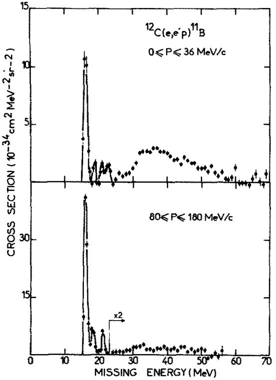
Pinning Down the 1p1h Sector

- At moderate missing energy—typically $E_m \lesssim 50$ MeV—the recoiling nucleus is left in a bound state
- The final state is a 1p1h state of the A-nucleon system
- The missing energy spectrum exhibits spectroscopic lines, corresponding to knock out from the shell model states. However the normalization of the shell model states is suppressed with respect to the predictions of the independent particle model.
- The momentum distributions of nucleons in the shell model states can be obtained measuring the missing momentum spectra at fixed missing energy
- Consider ${}^{12}\mathrm{C}(e, e'p){}^{11}\mathrm{B}$, as an example. The expected 1p1h final states are

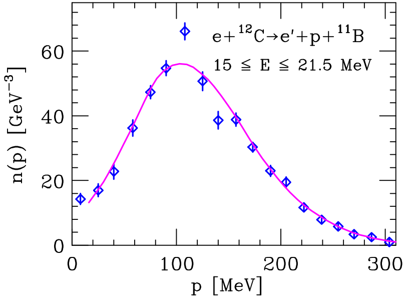
$$|{}^{11}\mathrm{B}(3/2^-),\ p\rangle \ , \ |{}^{11}\mathrm{B}(1/2^-),\ p\rangle \ , \ldots$$

$C(e, e'p)$ at Moderate Missing Energy

- Missing energy spectrum of ^{12}C measured at Saclay in the 1970s



- P- state momentum distribution. Solid line: LDA spectral function



MORE $(e, e'p)$ DATA IS ON ITS WAY

- ⋆ Jlab experiment E12-12-14-012 has measured the $\mathrm{Ar}, \mathrm{Ti}(e, e'p)$ cross section. These data will allow the determination of the spectral functions needed for the analysis of both ν and $\bar{\nu}$ interactions in liquid argon detectors

- ⋆ Collaboration involving 38 physicists, including few theorists, from 8 institutions

- ⋆ Approved by the Jefferson Lab PAC42 in July, 2014, with scientific grade A-

- ⋆ Experimental readiness review passed in July, 2016

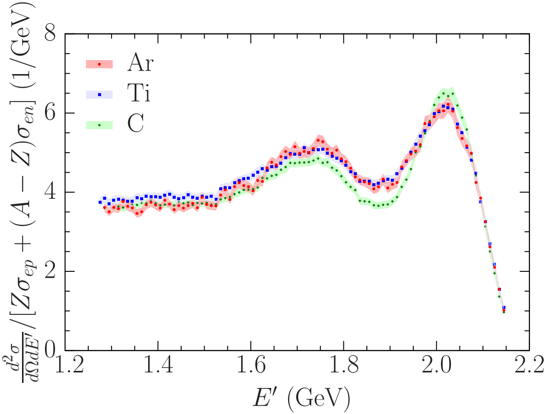
- ⋆ Data taking in February-March 2017

- ⋆ First results (inclusive) published in 2018 and 2019

- ⋆ First results of the exclusive analysis exoected in 2019

JLAB E12-14-012 DATA

- Inclusive cross sections at $E = 2.222$ GeV and $\theta = 15.54$ deg.
PRC 99, 054608 (2019)



OUTLOOK

- ⋆ In spite of the significant progress of the past two decades, substantial developments of the theoretical models are required to match the needs of ongoing and future neutrino experiments
- ⋆ Inclusive and exclusive electron scattering data should be fully exploited to validate the models and remove the existing degeneracies.
- ⋆ The potential of the Monte Carlo approach to perform accurate calculations of nuclear properties should be combined with the flexibility of the approach based on factorization
- ⋆ The results of lattice calculations could also be employed, to describe the interaction vertices
- ⋆ Being inherently modular, the factorization scheme is best suited for implementation in generators, and allows for an event-by-event analysis

Backup slides

Can a Near Detector Help?

- In principle, the determination of $\sigma_\beta(E_\nu, E_{\rm vis})$ could be avoided using a near detector to measure the unoscillated event rate, and exploiting the fact that in the ratio between near and far detector data many uncertainties cancel. In a disappearance experiment

$$\frac{R_{\alpha\to\alpha}(\text{far})\text{L}^2}{R_{\alpha\to\alpha}(\text{near})} = \frac{N_{\rm far}\Phi_\alpha\,\tilde\sigma_\alpha\,P(\nu_\alpha\to\nu_\alpha)}{N_{\rm near}\Phi_\alpha\,\tilde\sigma_\alpha 1} \approx \frac{N_{\rm far}}{N_{\rm near}}\,P(\nu_\alpha\to\nu_\alpha)$$

- For this cancellation to occur efficiently, it is essential that near and far detectors have nearly identical response

- Steve Manely's view on this issue (from a talk delivered at CERN on January 29, 2018)

The Good

If use the same target material and detectors are identical and flavor differences in lepton reconstruction are negligible and/or well understood, then nuclear and detector effects largely cancel in the systematic error.

The Bad

Detectors never perfect
Must unfold observations to get "truth"

Impulse Approximation & Factorization

- ⋆ Basic assumptions
  - ▷ $|X\rangle \approx |x, \mathbf{p}_x\rangle \otimes |R_{(A-1)}, \mathbf{p_R}\rangle$
  - ▷ $J^\mu_A(q) = \sum_i j^\mu_i(q) + \sum_{j>i} j^\mu_{ij}(q) \approx \sum_i j^\mu_i(q)$
- ⋆ As a zero-th order approximation, Final State Interactions (FSI) and processes involving two-nucleon Meson-Exchange Currents (MEC) are neglected (can be added as corrections)
- ⋆ The nuclear matrix element reduces to the simplified form

$$\langle X|J^\nu_A|0\rangle \approx N\sqrt{\frac{m}{E_{p_R}}} M_n(\mathbf{p}_R)\langle x, \mathbf{p}_x|j^\nu|n, -\mathbf{p}_R\rangle \ ,$$

where $N = A - Z$, $E_{p_R} = \sqrt{\mathbf{p}_R^2 + m^2}$,

$$M_n(\mathbf{k}) = \{\langle n, \mathbf{k}| \otimes \langle n_{(A-1)}, -\mathbf{k}|\}|0\rangle \ ,$$

$\mathbf{k}$ is the initial momentum of the struck nucleon and $|x, \mathbf{p}_x\rangle$ is the hadronic final state produced at the interaction vertex

- Within the factorization *ansatz* the target response reduces to

$$W_A^{\mu\nu} = N \int d^3k \, dE \, \frac{m}{E_k} P(\mathbf{k}, E) w^{\mu\nu}$$

$$w^{\mu\nu} = \sum_x \int d^3p_x \langle \mathbf{k}, n | j^\mu | x, \mathbf{p}_x \rangle \langle \mathbf{p}_x, x | j^\nu | n, \mathbf{k} \rangle \delta^{(4)}(k + \tilde{q} - p_x)$$

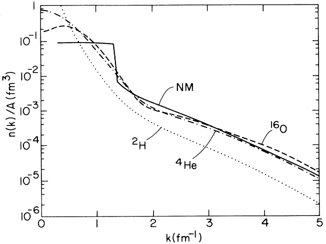
- $w^{\mu\nu}$ is the tensor describing the interaction of a free neutron of momentum $\mathbf{k}$ at four momentum transfer

$$\tilde{q} \equiv (\tilde{\omega}, \mathbf{q}) \quad , \quad \tilde{\omega} = \omega + M_A - E_R - E_k$$

- The substitution $\omega \to \tilde{\omega} < \omega$ accounts the fact that an amount $\delta\omega = \omega - \tilde{\omega}$ of the energy transfer goes into excitation energy of the residual system.
- The spectral function $P(\mathbf{k}, E)$ describes the probability of removing a nucleon of momentum $\mathbf{k}$ from the target nucleus, leaving the residual system with excitation energy E

THE NUCLEAR SPECTRAL FUNCTION

$$n(k) = \int dE\, P(\mathbf{k}, E)$$



- ⋆ Bottom line: the tail of the momentum distribution, arising from the continuum contribution to the spectral function, turns out to be largely A-independent for $A > 2$

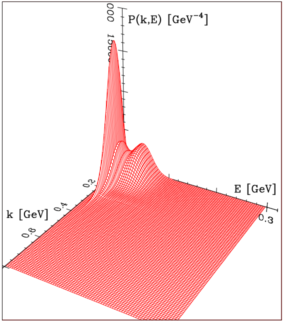
- ⋆ Spectral functions of isospin symmetric nuclei have been obtained within the Local Density Approximation (LDA)

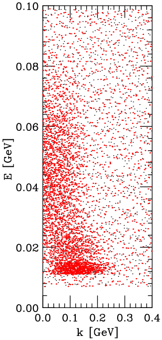
$$P_{\rm LDA}(\mathbf{k}, E) = P_{\rm MF}(\mathbf{k}, E) + \int d^3r\, \rho_A(r)\, P^{NM}_{corr}(\mathbf{k}, E; \rho = \rho_A(r))$$

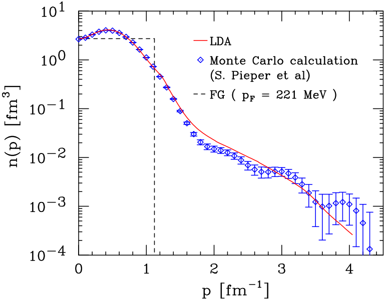
using the Mean Field (MF), or shell model, contributions extracted from $(e, e'p)$ data (more on this later)

- ⋆ The continuum contribution $P^{NM}_{corr}(\mathbf{k}, E)$ can be accurately computed in uniform nuclear matter at different densities

LDA Spectral Function of ^{16}O



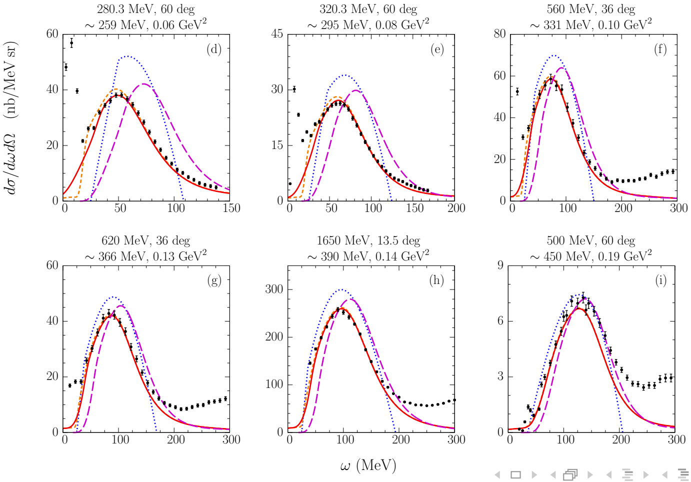




- ⋆ shell model states account for $\sim 80\%$ of the strenght
- ⋆ the remaining $\sim 20\%$, arising from NN correlations, is located at high momentum and large removal energy ($\mathbf{k} \gg k_F, E \gg \epsilon$)

Electron Scattering as a Testing Ground

- ⋆ $e + {}^{12}\mathrm{C} \to e' + X$ quasi elastic cross section computed within the IA including FSI. The predictions of the Relativistic Fermi Gas Model (RFGM) are also shown for comparison.



Kinematic Neutrino Energy Reconstruction

- In the charged current quasi elastic (CCQE) channel, assuming single nucleon single knock out, the relevant elementary process is

$$\nu_\ell + n \to \ell^- + p$$

- The *reconstructed* neutrino energy is

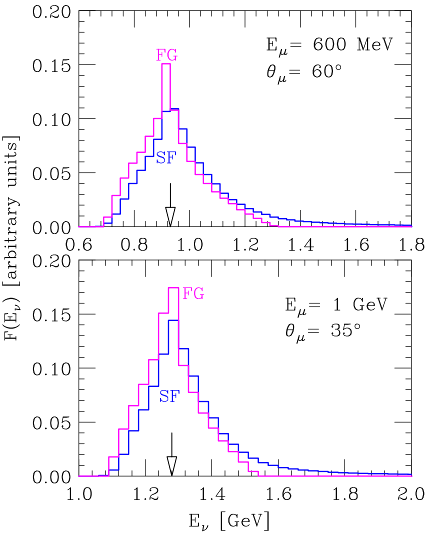
$$E_\nu = \frac{m_p^2 - m_\mu^2 - {E_n}^2 + 2E_\mu E_n - 2\mathbf{k}_\mu \cdot \mathbf{p}_n + |{\mathbf{p}_n}^2|}{2(E_n - E_\mu + |\mathbf{k}_\mu| \cos\theta_\mu - |\mathbf{p}_n| \cos\theta_n)} ,$$

  where $|\mathbf{k}_\mu|$ and θ_μ are measured, while $\mathbf{p}_n$ and E_n are the *unknown* momentum and energy of the interacting neutron

- Existing simulation codes routinely use $|\mathbf{p}_n| = 0$, $E_n = m_n - \epsilon$, with $\epsilon \sim 20$ MeV for carbon and oxygen, or the predictions of the Fermi gas model

Reconstructed neutrino energy in the CCQE channel

- Neutrino energy reconstructed using 2×10^4 pairs of $(|\mathbf{p}|, E)$ values sampled from LDA (SF) and Fermi gas oxygen spectral functions
- The average value $\langle E_\nu \rangle$ obtained from the realistic spectral function turns out to be shifted towards larger energy by ~ 70 MeV



The E12-14-012 Experiment at Jefferson Lab

- The reconstruction of neutrino and antineutrino energy in liquid argon detectors will require the understanding of the spectral functions describing both protons and neutrons
- The $\mathrm{Ar}(e, e'p)$ cross section only provides information on proton interactions. The information on neutrons can be obtained from the $\mathrm{Ti}(e, e'p)$, exploiting the pattern of shell model levels