Neutrino crossings and flavor correlation instabilities

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Introduction

evolution of the topic:

- self-induced flavor conversion
  Chakraborty, Hansen, Izzaguirre, Raffelt (2016) [arXiv:1602.00698]
- classification of instabilities
  Cappozzi, Dasgupta, Lisi, Marrone, Mirizzi (2017) [arXiv:1706.03360]
- critical points on the branch
  Yi, Ma, Martin, Duan (2019) [arXiv:1901.01546]

Equation of motion

setting: 1D system with two neutrino flavors

interpretation: ultra-relativistic neutrinos collectively source a flavor matrix field

linearized EoM for flavor correlation Q_u:

$$(\omega - uk)\, Q_u(\omega,\, k) = -\mu \int \mathrm{d}v\, (1 - uv)\, G_v Q_v(\omega,\, k) \qquad (1)$$

with "lepton number" $G_v \sim \int \mathrm{d}E \left(f_{\nu_e} - f_{\bar{\nu}_e} - f_{\nu_\mu} + f_{\bar{\nu}_\mu}\right)$

Dispersion relation

usually dispersion relation for collective mode $\omega(k)$ derived
$\rightarrow$ critical points leading to instabilities

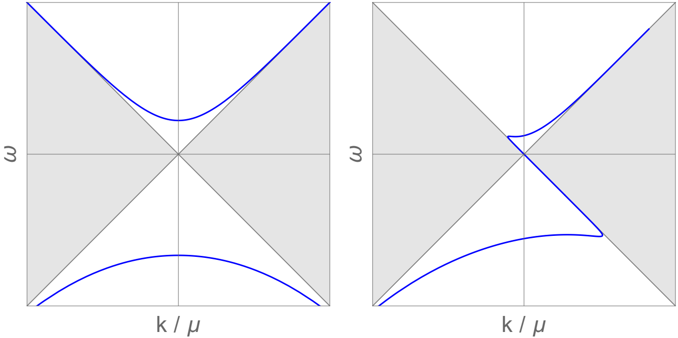


Figure: DR for no and small crossing of G_v, cf. Yi, Ma, Martin, Duan (2019)

Eigenfunction ansatz

eigenfunction connects collective and non-collective modes

non-collective modes densely fill out the "forbidden" region
$\rightarrow$ EV known, EF not

eigenfunction $Q_u = A_1 \left[\frac{\sin\varphi}{\pi(\omega - uk)} + \cos\varphi\,\delta(\omega - uk)\right] + A_2$

mutual consistency for $\mathcal{O}(1)$ and $\mathcal{O}(u)$ leads to matrix equation

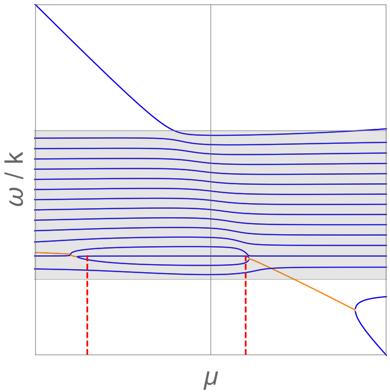
$$\hat{M}\left(\frac{\omega}{k},\ \mu,\ G_u,\ \varphi\right)\mathbf{A} = 0 \qquad (2)$$

Merging points

crossing at u_0, i.e.
$G_{u_0} = 0$

$G_{u_0} \cos\varphi \stackrel{!}{=} 0$ gives polynomial for μ

$\rightarrow$ no real μ
$\hat{=}$ no complex DR branch



Single crossing

choose $G_u = (u - u_0)\, P(u)$ with $\mathrm{sign}[P(u)] = \mathrm{const.}$

equation for μ:

$$\left(P_1^2 - P_0 P_2\right) \mu^2 + \left(P_0 - P_2\right) \mu + 1 = 0 \qquad (3)$$

with $P_i = \int_{-1}^{1} \mathrm{d}u\, u^i P(u)$

real solution for μ if

$$\int \mathrm{d}u\, (1+u)^2 P(u) \int \mathrm{d}u' \left(1 - u'\right)^2 P(u') > 0 \qquad (4)$$

Double crossing

ansatz: $G_u = (u - u_1)(u - u_2)$

use condition for real solution to derive condition on u_2

demand positive discriminant $\text{Disk}_\mu(u_2) = u_2^2 - \frac{1}{4} > 0$
$\rightarrow$ no instability for $u_2 \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

Summary

non-collective modes can help to understand the system

complex collective branches start where two non-collective modes merge

single crossing always sources complex branch in dispersion relation

in general: additional criterion needs to be satisfied