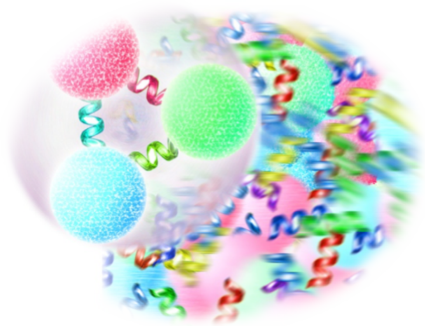

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Nonpertubative structure of the three-gluon vertex

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Based on

A.C.A, M. N. Ferreira, C. T. Figueiredo and J. Papavassiliou: Phys. Rev. D99, no. 3, 034026 (2019)
and arXiv:1903.01184 [hep-ph] - PRD in press

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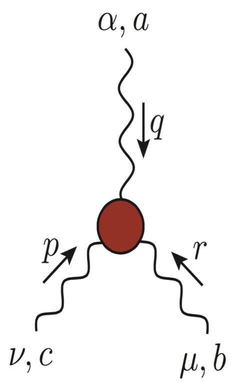
Continuum Functional Methods for QCD at New Generation Facilities -Trento, Italy
May 7-10, 2019

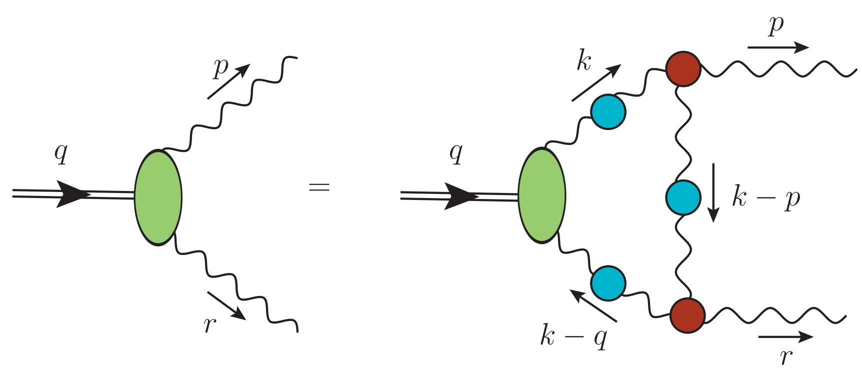
Motivation

✓ Determine the nonperturbative structure of *three-gluon vertex is essential for the understanding the gluon mass generation.*



$$= g f^{abc} \mathrm{I}\Gamma_{\alpha\mu\nu}(q, r, p)$$

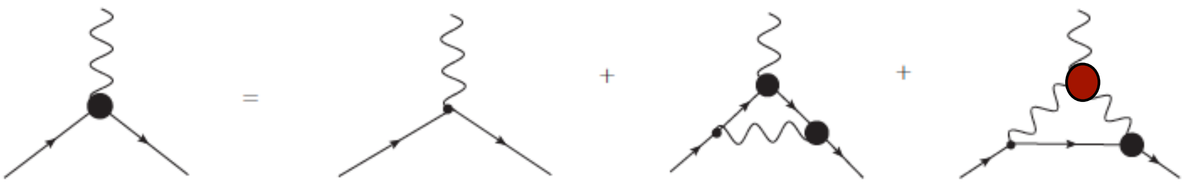
Key ingredient of the BSE for massless poles, crucial for the dynamical gluon mass generation



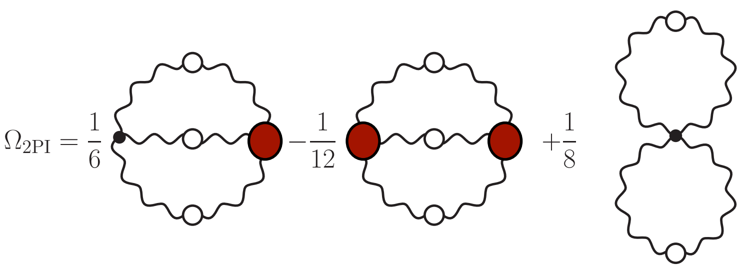
(c)

D.Binosi and J. Papavassiliou, Phys. Rev. D97, no. 5, 054029 (2018);
A.C.A, D. Binosi and J.Papavassiliou, Phys. Rev. D95, 034017 (2017);
A.C.A, D. Binosi and J.Papavassiliou, Phys. Rev. D78, 025010 (2008);
A.C.A and J.Papavassiliou, JHEP 0612, 012 (2006).

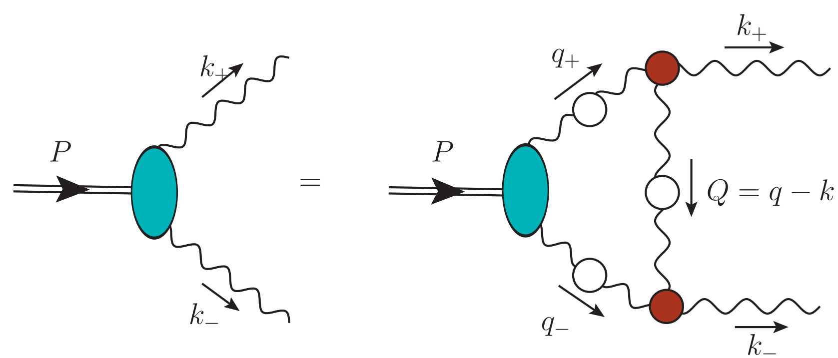
✓ *Not only that, it appears in the nonperturbative skeleton expansion of others vertices (as the quark-gluon vertex), effective potential, BSE for the glueballs, hybrids etc.*



Quark-gluon vertex



Effective potential



glueball

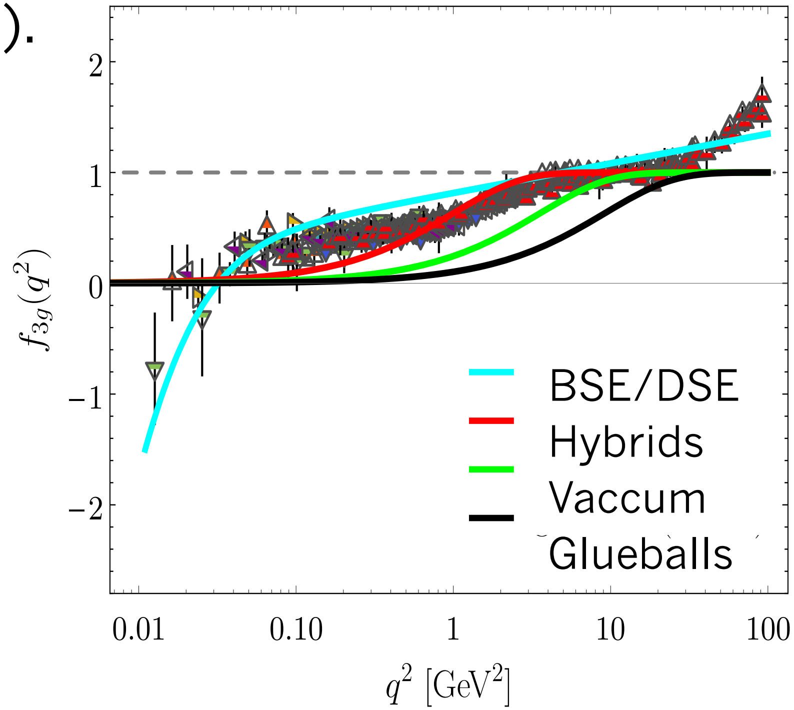
Strong evidences of IR suppression

✓ Suppression with respect to its tree-value.

✓ In the previous examples, *it is crucial to use* a model for the three-gluon vertex, $f_{3g}(q^2)$, which displays an IR suppression for a particular kinematic configuration→ (symmetric).

$$f_{3g}(q^2) = 1 - \exp(-q^2/\omega_{3g}^2)$$

✓ Lattice also seems this suppression!



Lattice:
A.Athenodorou, D.Binosi, P. Boucaud, F.De Soto, J.Papavassiliou, J.Rodriguez-Quintero and S.Zafeiropoulos, Phys.Lett.B761, 444 (2016)
A. G. Duarte, O. Oliveira and P.J.Silva, Phys. Rev. D94, 074502 (2016)

Hybrids:
S.S.Xu, Z.F.Cui, L.Chang, J.Papavassiliou, C.D.Roberts and H.S.Zong arXiv:1805.06430 [nucl-th].

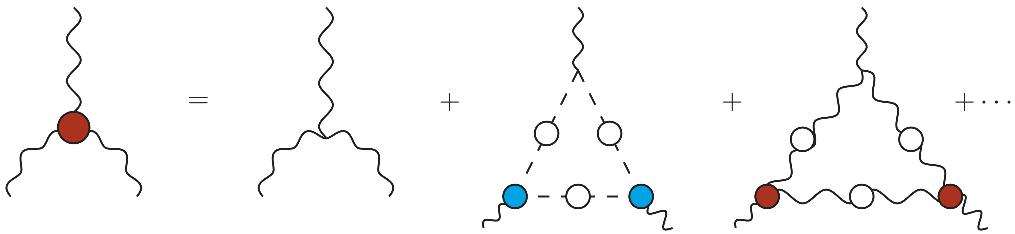
Variations in the value of ω_{3g}
→ most likely each equation probes a different kinematics

What is the origin of the IR suppression?

- It is the outcome of the competition between the IR contributions originating from diagrams containing "massless" ghosts diagrams containing "massive" gluons"

massless ghost propagators

log-divergences in the infrared



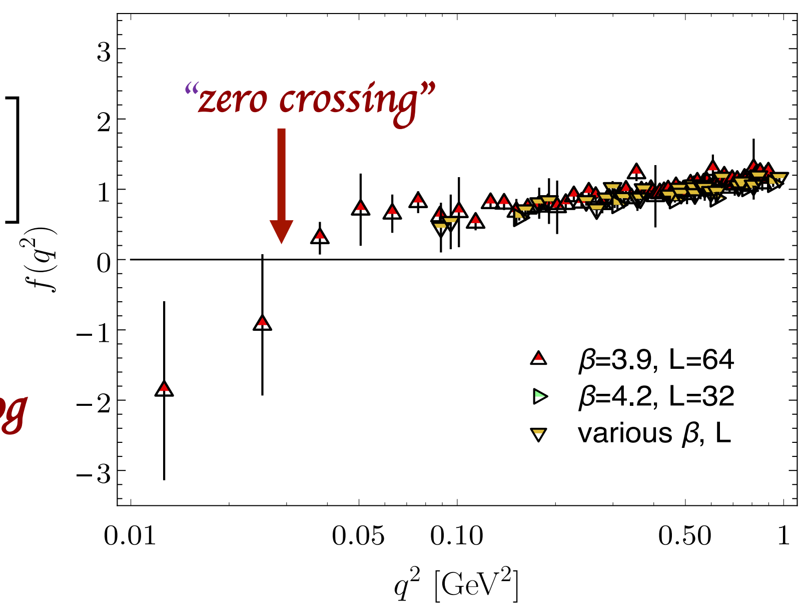
Symmetric point:
$q^2 = p^2 = r^2$

$$f(q^2) = a\left[1 + b\ln\left(\frac{q^2}{\mu^2}\right) + c\ln\left(\frac{q^2+m^2}{\mu^2}\right)\right]$$

massless loops
IR unprotected log

massive loops
IR protected log

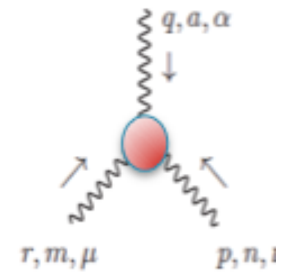
$$f(0) \to -\infty$$



A.C.A, D.Binosi, D.Ibañez, J.Papavassiliou, Phys. Rev. D 89, no. 8, 085008 (2014)
A.Athenodorou, D.Binosi, P. Boucaud, F.De Soto, J.Papavassiliou, J.Rodriguez-Quintero and S.Zafeiropoulos, Phys.Lett.B761, 444 (2016)

- Despite its physical importance, its nonperturbative behavior is still only partially known, mainly due to a variety of serious technical difficulties

1. ***Rich tensorial structure*** leads to a proliferation of form factors:
   - ✓ 10 non-transverse tensors – constrained by STI
   - ✓ 4 transverse tensors **J. S. Ball and T. W. Chiu**, Phys. Rev. D 22, 2550 (1980)



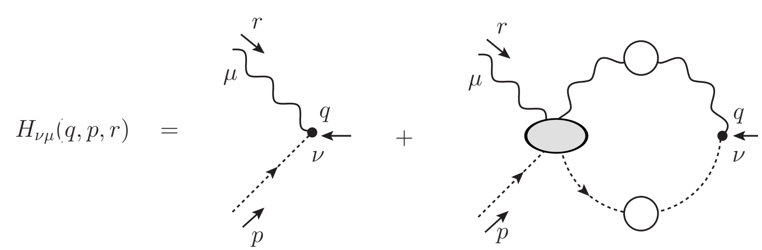
2. Form factors ***depend on three kinematic variables***
   - ✓ The moduli of two momenta, q, r and their relative angle θ .

3. ***Satisfies STIs instead of simple QED-like WIs*** → contribution of the ghost sector → ***gluon-ghost scattering kernel.***

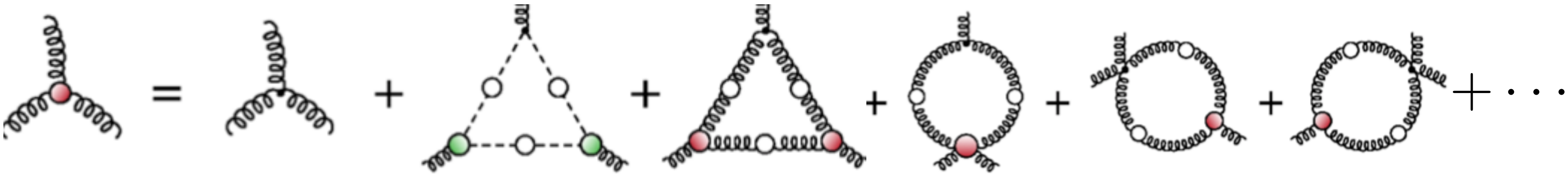
$$q^\alpha \Pi_{\alpha\mu\nu}(q,r,p) = F(q)[\Delta^{-1}(p)P_\nu^\alpha(p)H_{\alpha\mu}(p,q,r) - \Delta^{-1}(r)P_\mu^\alpha(r)H_{\alpha\nu}(r,q,p)]$$

$$D(q^2) = \frac{iF(q^2)}{q^2}$$

Ghost dresssing function



- *Conventional approach*: Solve the SDE for the three-gluon vertex



R. Alkofer, M.Q. Huber, K. Schwenzer, Eur. Phys. J. C62 , 761 (2009),
G. Eichmann, R. Williams, R. Alkofer, M. Vujinovic,Phys. Rev. D89,105014 (2014)
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A. K. Cyrol, L. Fister, M. Mitter, J. M. Pawlowski, N. Strodthoff, Phys. Rev. D94, 054005 (2016)
R. Williams, C. S. Fischer, and W.Heupel, Phys. Rev. D 93, no. 3, 034026 (2016)

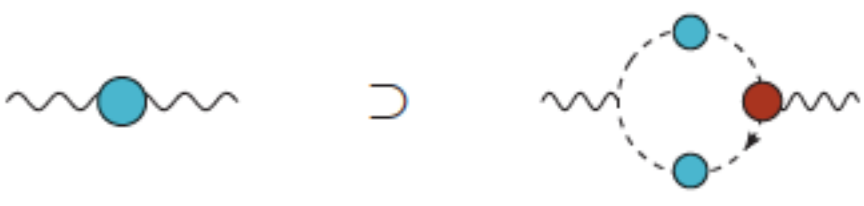
- Instead, we reconstruct $\mathrm{I\Gamma}_{\alpha\mu\nu}$ from its

***Slavnov-Taylor identity* (STI)**

✓ Notice that in the three-gluon vertex, $\mathrm{I\Gamma}_{\alpha\mu\nu}$, we have

← *massless ghost loop (unprotected log)*

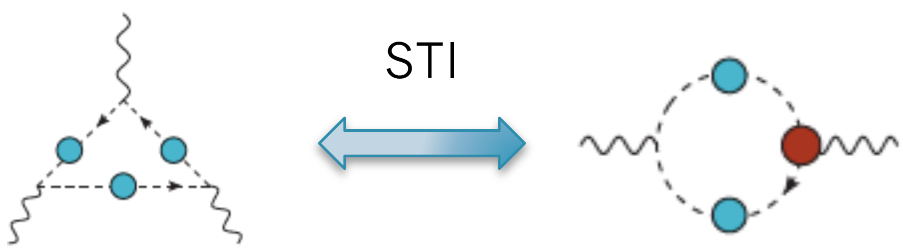
whereas in the gluon propagator, $\Delta(q^2)$



← *massless ghost loop (unprotected log)*

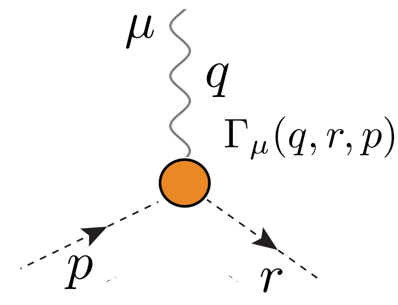
✓ Therefore the STI connects

$$q^\alpha \mathrm{I\Gamma}_{\alpha\mu\nu}(q,r,p) = F(q)[\Delta^{-1}(p)P^\alpha_\nu(p)H_{\alpha\mu}(p,q,r) - \Delta^{-1}(r)P^\alpha_\mu(r)H_{\alpha\nu}(r,q,p)]$$



A sophisticated version of the gauge technique

- Prototype: Three-particle vertex of scalar QED



- It satisfies the Ward-Takahashi identity

$$q^\mu \Gamma_\mu(q,r,p) = \mathcal{D}^{-1}(p^2) - \mathcal{D}^{-1}(r^2)$$

write the vertex as
$$\Gamma_\mu(q,r,p) = \underbrace{\Gamma^{\mathrm{L}}_\mu(q,r,p)}_{\text{“longitudinal”}} + \underbrace{\Gamma^{\mathrm{T}}_\mu(q,r,p)}_{\text{“transverse”}}$$

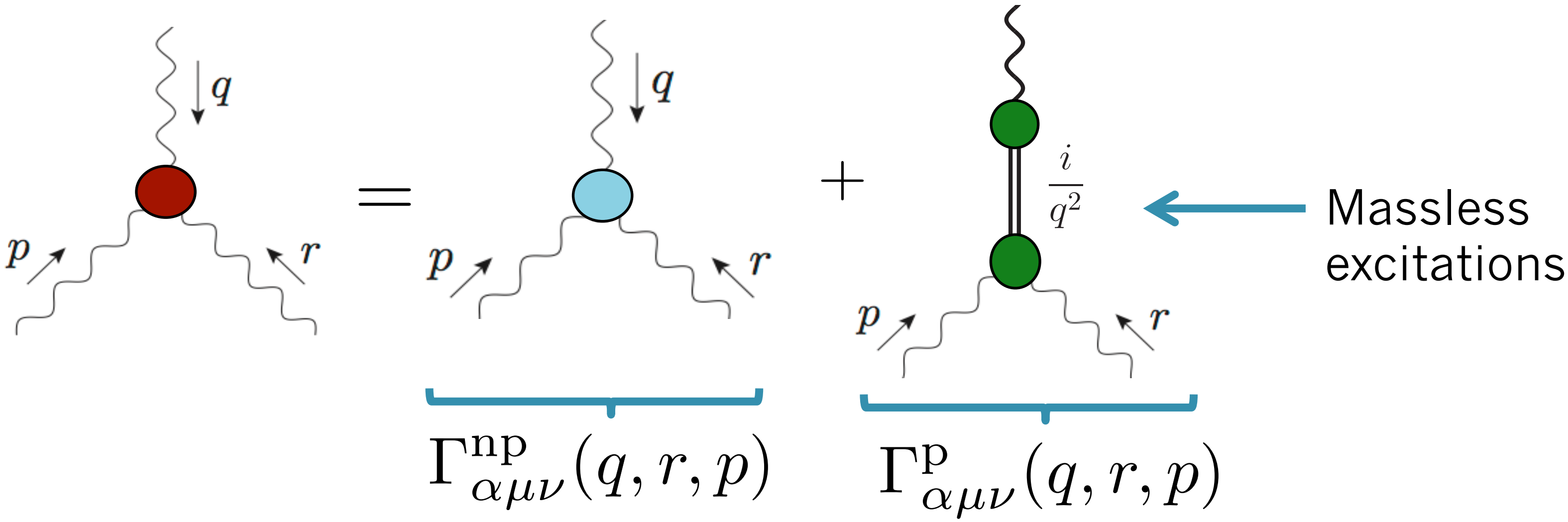
- The longitudinal piece satisfies the WTI

$$\Gamma^L_\mu(q,r,p) = \frac{(r-p)_\mu}{p^2-r^2}[\mathcal{D}^{-1}(p^2) - \mathcal{D}^{-1}(r^2)]$$

while the transversal part is automatically conserved

$q^\mu \Gamma^{\mathrm{T}}_\mu(q,r,p) = 0$ → ***undertermined***

But non-perturbative aspects make the construction rather subtle



$$\mathrm{I\!\Gamma}_{\alpha\mu\nu}(q,r,p) = \Gamma^{\rm NP}_{\alpha\mu\nu}(q,r,p) + \Gamma^{\rm P}_{\alpha\mu\nu}(q,r,p)$$

$$\Delta^{-1}(q^2) = q^2 J(q^2) + m^2(q^2)$$

Massless excitations → **trigger** → Schwinger mechanism → **generates** → dynamical gluon mass

Piece-wise realization of the STI

$$q^\alpha \mathrm{I\Gamma}_{\alpha\mu\nu}(q,r,p) = F(q)[\Delta^{-1}(p)P^\alpha_\nu(p)H_{\alpha\mu}(p,q,r) - \Delta^{-1}(r)P^\alpha_\mu(r)H_{\alpha\nu}(r,q,p)]$$

Turn off ghost sector
$F(q)=1; \quad H_{\alpha\mu}=g_{\alpha\mu}$

$$q^\alpha \mathrm{I\Gamma}_{\alpha\mu\nu}(q,r,p) = [\Delta^{-1}(p)P_{\mu\nu}(p) - \Delta^{-1}(r)P_{\mu\nu}(r)]$$

$$\mathrm{I\Gamma}_{\alpha\mu\nu}(q,r,p) = \Gamma^{\mathrm{NP}}_{\alpha\mu\nu}(q,r,p) + \Gamma^{\mathrm{P}}_{\alpha\mu\nu}(q,r,p)$$

$$\Delta^{-1}(q^2) = q^2 J(q^2) + m^2(q^2)$$

The sum recovers the original STI

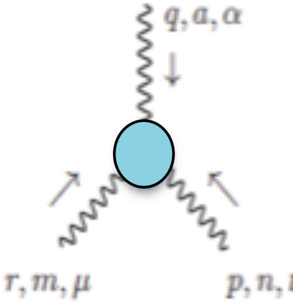
Piece-wise realization :

$$\left\{ \begin{array}{l} q^\alpha \Gamma^{\mathrm{NP}}_{\alpha\mu\nu}(q,r,p) = [p^2 J(p^2)P_{\mu\nu}(p) - r^2 J(r^2)P_{\mu\nu}(r)] \\ q^\alpha \Gamma^{\mathrm{P}}_{\alpha\mu\nu}(q,r,p) = [m^2(p^2)P_{\mu\nu}(p) - m^2(r^2)P_{\mu\nu}(r)] \end{array} \right.$$

$$P_{\alpha\alpha'}(q)P_{\mu\mu'}(r)P_{\nu\nu'}(p)\Gamma^{\mathrm{p}}_{\alpha\mu\nu}(q,r,p) = 0$$

Drops out from observables

Constructing the three-gluon vertex



- The most general decomposition of the full three-gluon vertex has 14 tensorial structures.
- It can be separated in "longitudinal" and "transverse" part

$$\Gamma^{\rm NP}_{\alpha\mu\nu}(q,r,p) = \Gamma^{\rm L}_{\alpha\mu\nu}(q,r,p) + \Gamma^{\rm T}_{\alpha\mu\nu}(q,r,p)$$

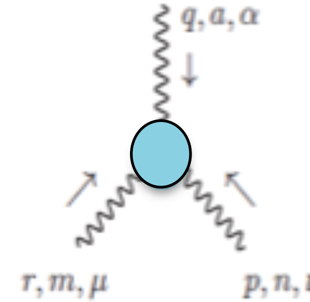
✓ The transverse part (4 tensorial structures) is automatically conserved:

$$q^\alpha \Gamma^T_{\alpha\mu\nu}(q,r,p) = 0\,, \quad r^\mu \Gamma^T_{\alpha\mu\nu}(q,r,p) = 0\,, \quad p^\nu \Gamma^T_{\alpha\mu\nu}(q,r,p) = 0$$

✓ While the 10 "longitudinal" structures saturate the STI:

$$q^\alpha \Gamma^{\rm L}_{\alpha\mu\nu}(q,r,p) = F(q)[p^2 J(p^2) P^\alpha_\nu(p) H_{\alpha\mu}(p,q,r) - r^2 J(r^2) P^\alpha_\mu(r) H_{\alpha\nu}(r,q,p)]$$

Tensorial basis



- The most general Lorentz decomposition of the non-transverse part in the Ball-Chiu basis is

$$\Gamma_{\mathrm{L}}^{\alpha\mu\nu}(q,r,p) = \sum_{i=1}^{10} X_i(q,r,p)\,\ell_i^{\alpha\mu\nu}$$

J. S. Ball and T. W. Chiu, Phys. Rev. D 22, 2550 (1980)
D. Binosi and J. Papavassiliou, JHEP 1103, 121 (2011)

where the tensors

$$\ell^1_{\alpha\mu\nu} = (q-r)_\nu g_{\alpha\mu}\,, \qquad \ell^2_{\alpha\mu\nu} = -p_\nu g_{\alpha\mu}\,, \qquad \ell^3_{\alpha\mu\nu} = (q-r)_\nu[q_\mu r_\alpha - (q\cdot r)g_{\alpha\mu}]\,,$$

$$\ell^4_{\alpha\mu\nu} = (r-p)_\alpha g_{\mu\nu}\,, \qquad \ell^5_{\alpha\mu\nu} = -q_\alpha g_{\mu\nu}\,, \qquad \ell^6_{\alpha\mu\nu} = (r-p)_\alpha[r_\nu p_\mu - (r\cdot p)g_{\mu\nu}]\,,$$

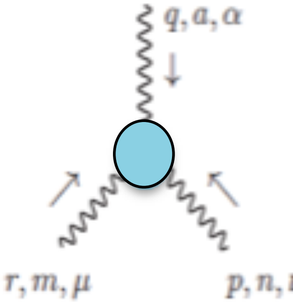
$$\ell^7_{\alpha\mu\nu} = (p-q)_\mu g_{\alpha\nu}\,, \qquad \ell^8_{\alpha\mu\nu} = -r_\mu g_{\alpha\nu}\,, \qquad \ell^9_{\alpha\mu\nu} = (p-q)_\mu[p_\alpha q_\nu - (p\cdot q)g_{\alpha\nu}]\,,$$

$$\ell^{10}_{\alpha\mu\nu} = q_\nu r_\alpha p_\mu + q_\mu r_\nu p_\alpha\,.$$

- At tree level the form factors reduce to

$$X_1(q,r,p) = 1 \quad X_4(q,r,p) = 1 \quad X_7(q,r,p) = 1 \qquad X_i(q,r,p) = 0$$

Bose Symmetry



- Bose symmetry implies that some form factors are related to each other by cyclic permutations of the legs

$$X_4(q,r,p) = X_1(r,p,q)\,, \qquad X_5(q,r,p) = X_2(r,p,q)\,, \qquad X_6(q,r,p) = X_3(r,p,q)\,,$$

$$X_7(q,r,p) = X_1(p,q,r)\,, \qquad X_8(q,r,p) = X_2(p,q,r)\,, \qquad X_9(q,r,p) = X_3(p,q,r)\,,$$

As consequence, only 4 out of the 10 form factors should be determined → X_1, X_2, X_3 *and* X_{10}.

Abelianized solution

- Turn of the ghost sector → $F(q) = 1; \quad H_{\alpha\mu} = g_{\alpha\mu}$
- We are "solving"

$$q^{\alpha}\Gamma^{\mathrm{L}}_{\alpha\mu\nu}(q,r,p) = [p^2 J(p^2) P_{\mu\nu}(p) - r^2 J(r^2) P_{\mu\nu}(r)]$$

$$\Gamma_{\mathrm{L}}^{\alpha\mu\nu}(q,r,p) = \sum_{i=1}^{10} X_i(q,r,p)\ell_i^{\alpha\mu\nu}$$

- Then

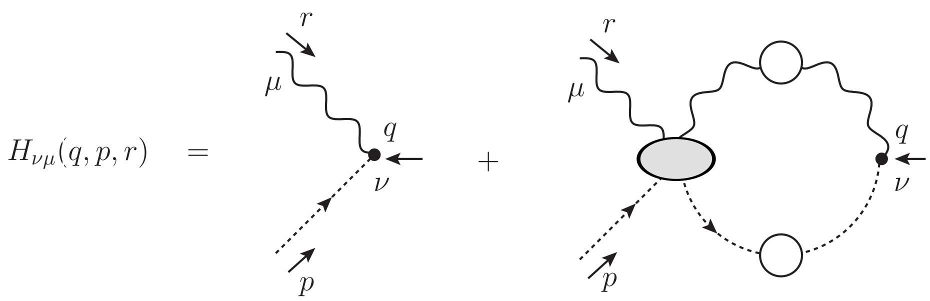
$$\widehat{X}_1(q,r,p) = \frac{1}{2}[J(r) + J(q)]\,, \qquad \widehat{X}_3(q,r,p) = \frac{[J(q) - J(r)]}{q^2 - r^2}\,,$$

$$\widehat{X}_2(q,r,p) = \frac{1}{2}[J(q) - J(r)]\,. \qquad \widehat{X}_{10}(q,r,p) = 0\,,$$

But the full STI has…

The STI solution

...the ***gluon-ghost scattering kernel,*** H



✓ which has the following Lorentz decomposition

$$H_{\nu\mu}(q,p,r) = g_{\mu\nu} A_1 + q_\mu q_\nu A_2 + r_\mu r_\nu A_3 + q_\mu r_\nu A_4 + r_\mu q_\nu A_5 \,,$$

$A_i \equiv A_i(q,p,r),$ → form factors (function of two momenta and the angle between them)

✓ At tree level:

$$H^{(0)}_{\nu\mu}(q,p,r) = g_{\mu\nu} \,, \quad \longrightarrow \quad A_1^{(0)} = 1 \qquad A_i^{(0)} = 0, \text{ for } i = 1,2,3,4.$$

The STI solution for $\Gamma^{\mathrm{L}}_{\alpha\mu\nu}(q,r,p)$

- Replacing in the STI the tensorial decompositions for the three-gluon vertex and gluon-ghost scattering kernel

$$H_{\nu\mu}(q,p,r) = g_{\mu\nu}A_1 + q_\mu q_\nu A_2 + r_\mu r_\nu A_3 + q_\mu r_\nu A_4 + r_\mu q_\nu A_5\,,$$

$$\Gamma_{\mathrm{L}}^{\alpha\mu\nu}(q,r,p) = \sum_{i=1}^{10} X_i(q,r,p)\ell_i^{\alpha\mu\nu}$$

$$q^\alpha\Gamma^{\mathrm{L}}_{\alpha\mu\nu}(q,r,p) = F(q)[p^2 J(p^2)P^\alpha_\nu(p)H_{\alpha\mu}(p,q,r) - r^2 J(r^2)P^\alpha_\mu(r)H_{\alpha\nu}(r,q,p)]$$

We obtain…

- The following expressions for the longitudinal three-gluon form factors

$$X_1(q,r,p) = \frac{1}{4}[2(a_{pqr} + a_{prq}) + p^2(b_{qrp} + b_{rqp}) + 2(\, q\cdot p\, d_{prq} + r\cdot p\, d_{pqr}) + (q^2 - r^2)(b_{rpq} + b_{pqr} - b_{qpr} - b_{prq})]\,,$$

$$X_2(q,r,p) = \cdots$$
$$X_3(q,r,p) = \cdots$$
$$X_{10}(q,r,p) = \cdots$$

where

$$a_{qrp} \equiv F(r)J(p)A_1(p,r,q)\,,$$
$$b_{qrp} \equiv F(r)J(p)A_3(p,r,q)\,,$$
$$d_{qrp} \equiv F(r)J(p)[A_4(p,r,q) - A_3(p,r,q)]\,.$$

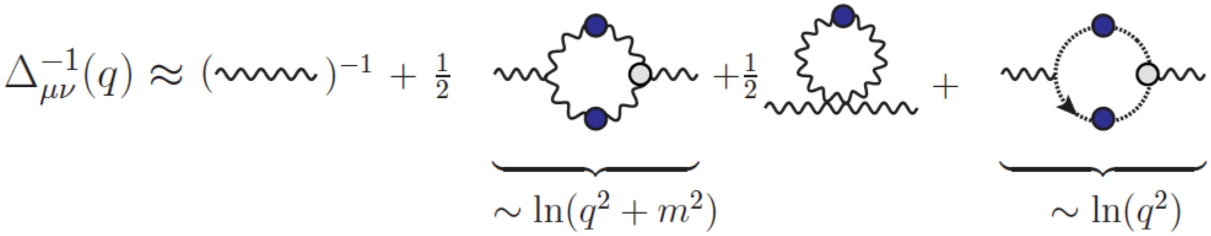
The longitudinal form factors may be construct from:

$$J(p), F(r), A_1, A_3 \text{ and } A_4$$

J. S. Ball and T. W. Chiu, Phys. Rev. D 22, 2550 (1980)

Ingredients

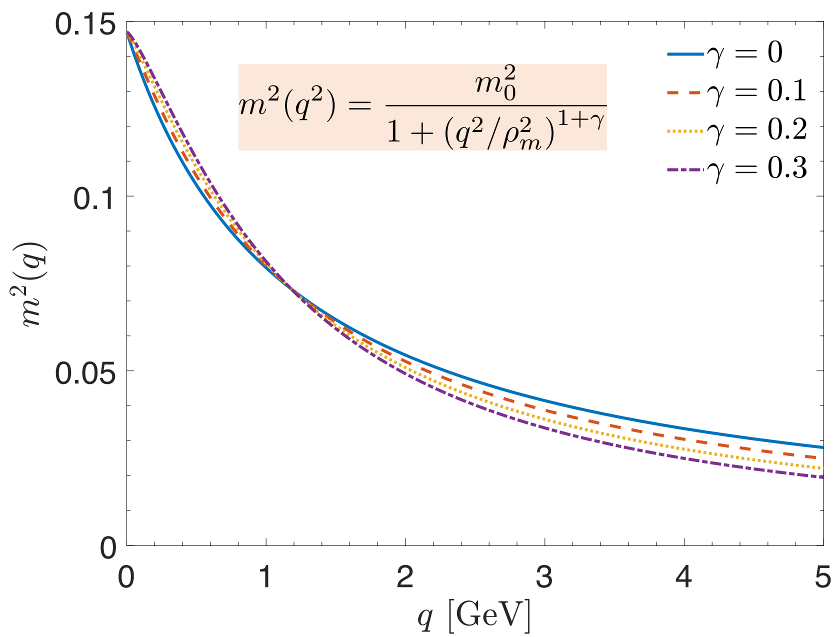
- Solving the dynamical equations for the gluon propagator

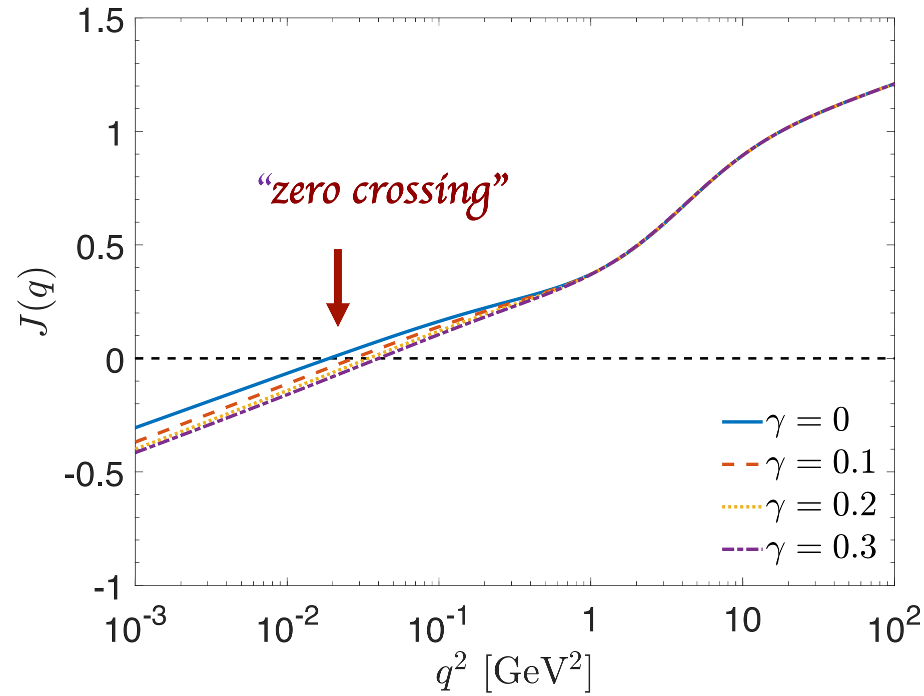


Ghost loop Creates the zero crossing in J(q)

$$m^2(q^2) = \int_k \mathcal{K}_2(q,k,m^2,J)$$

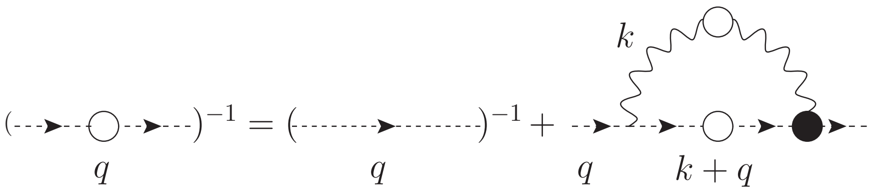
$$J(q^2) = 1 + \int_k \mathcal{K}_1(q,k,m^2,J)$$

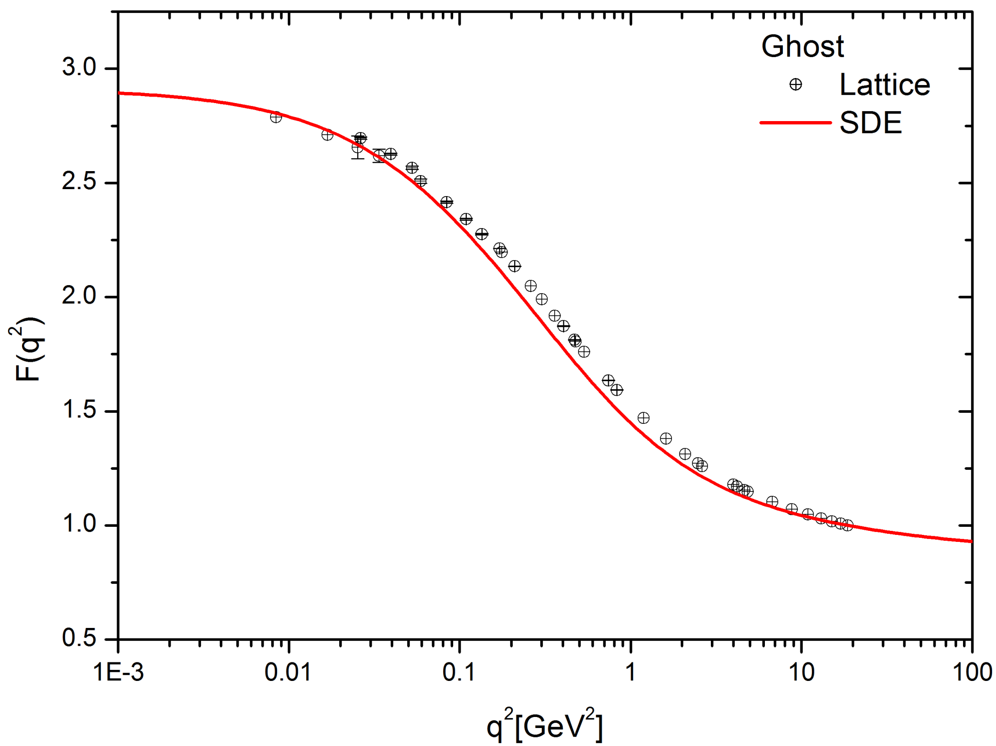




Ghost dressing function

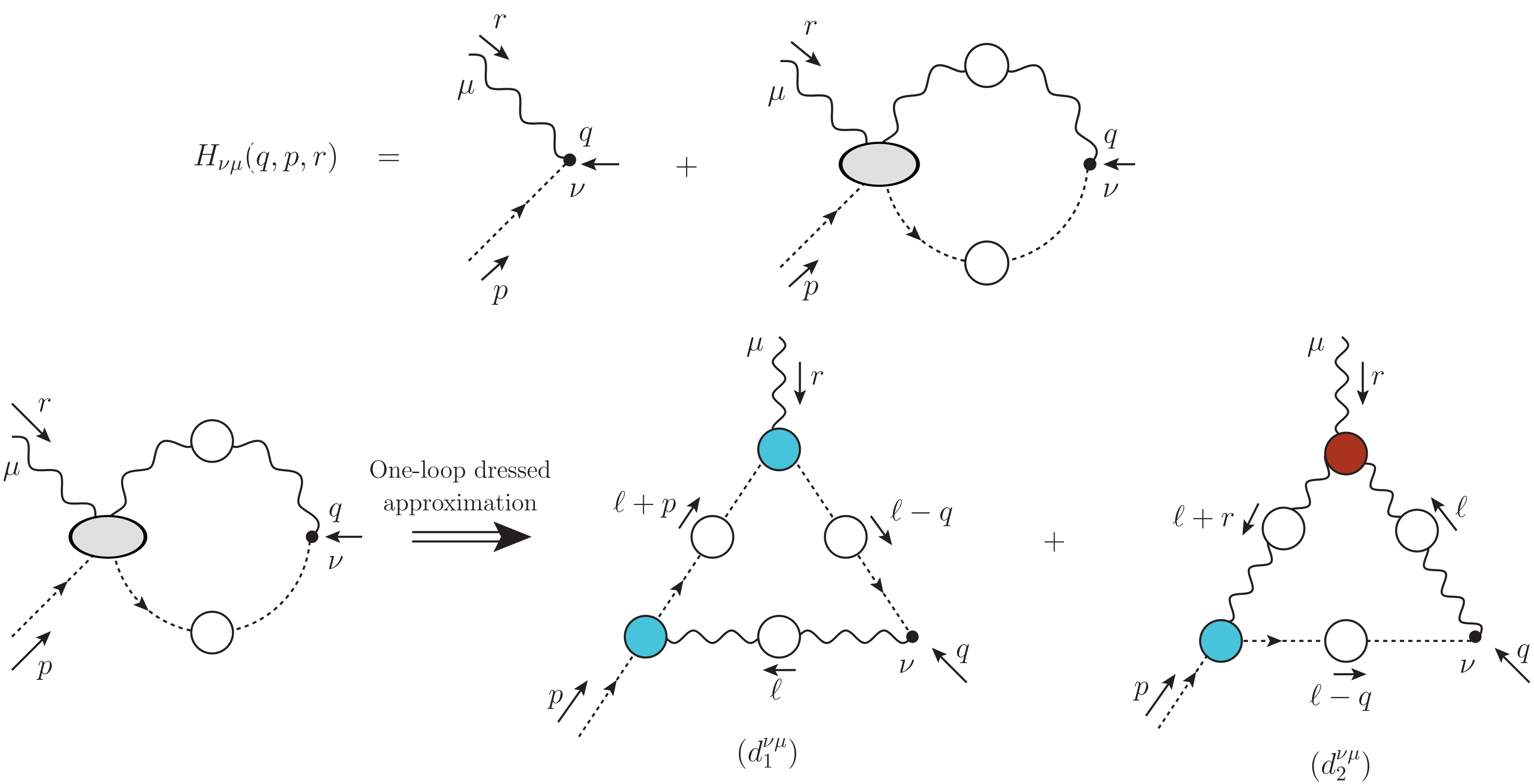
$$(\cdots\blacktriangleright\cdots\bigcirc\cdots\blacktriangleright\cdots)^{-1} = (\cdots\cdots\blacktriangleright\cdots\cdots)^{-1} + \ldots$$





I. L. Bogolubsky, et al. PoS LATTICE, 290 (2007).
A. C. A., D. Ibáñez, and J. Papavassiliou, Phys.Rev. D87, 114020 (2013).

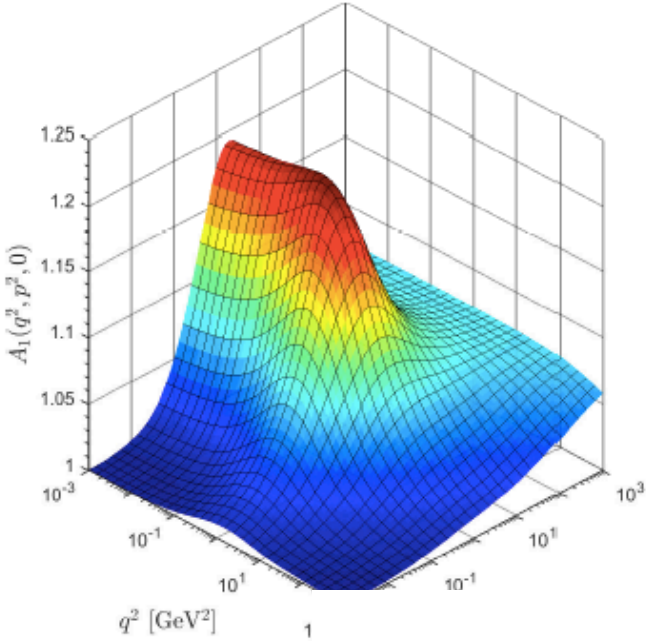
Scattering gluon-ghost kernel

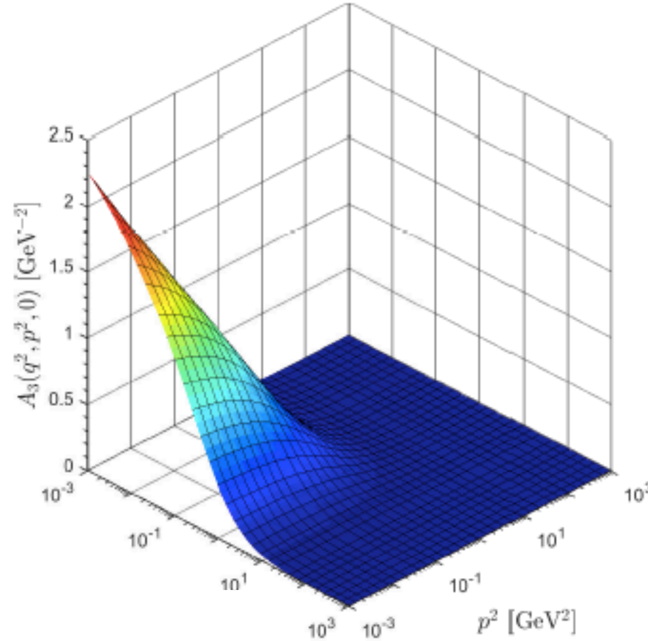


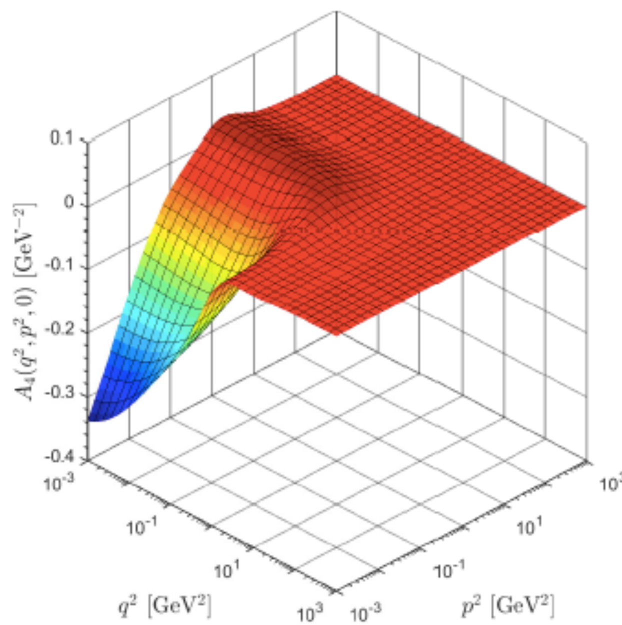
Contribution of 2 dressed diagrams

$$A_1, A_3 \text{ and } A_4$$

A.C.A, M. N. Ferreira, C. T. Figueiredo and J. Papavassiliou: Phys. Rev. D99, no. 3, 034026 (2019)







Sizable deviation from their tree level values!

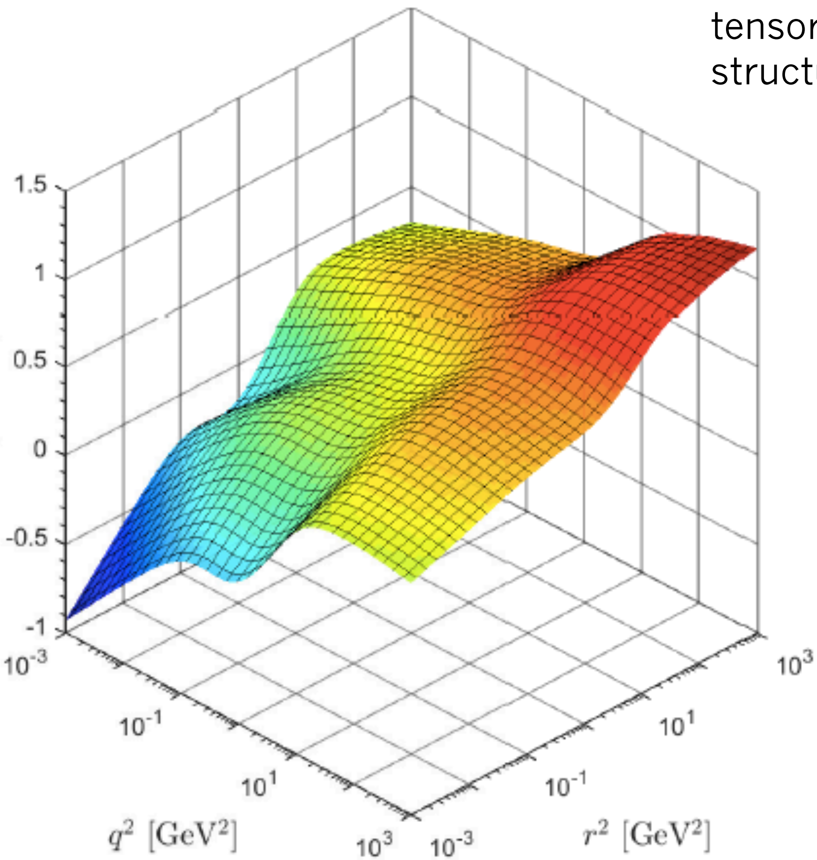
A.C.A, M. N. Ferreira, C. T. Figueiredo and J. Papavassiliou: Phys. Rev. D99, no. 3, 034026 (2019)

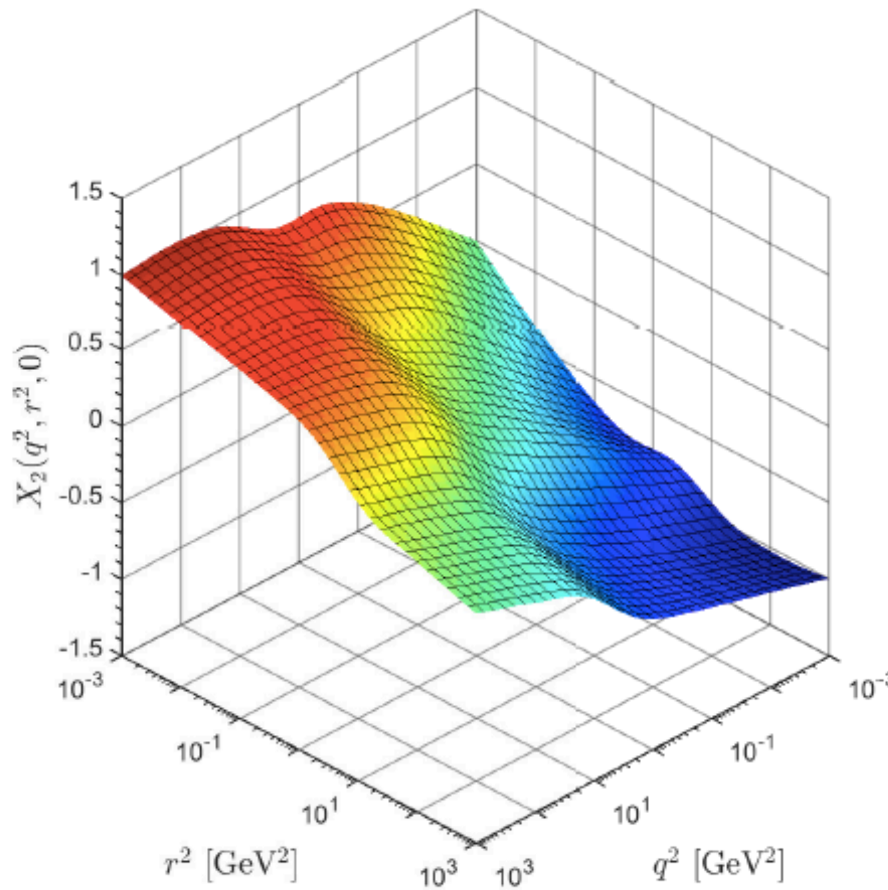
Numerical results for the three-gluon vertex

✓ The angular dependence is weak and barely visible in the 3D plots

$X_1(q^2, r^2, \theta = 0)$ ← Related to the tree level tensorial structure

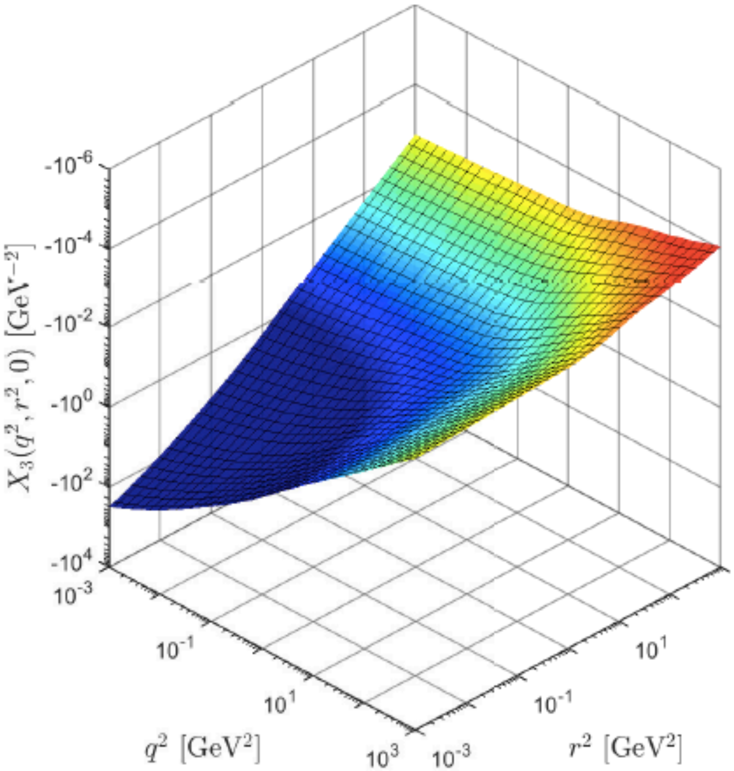
$X_2(q^2, r^2, \theta = 0)$





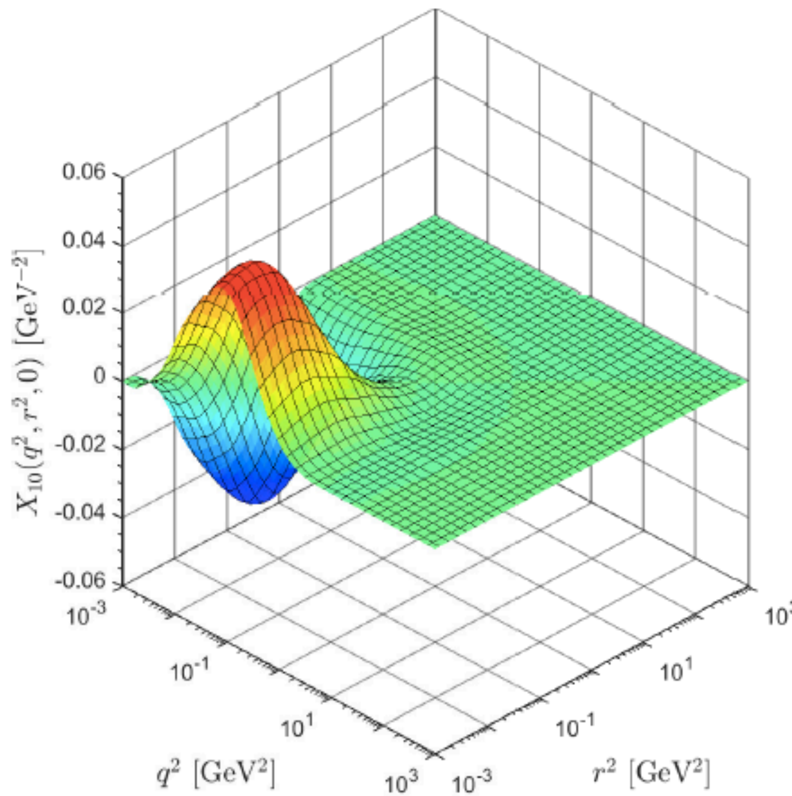
A.C.A, M. N. Ferreira, C. T. Figueiredo and J. Papavassiliou: arXiv:1903.01184 [hep-ph]

$$X_3(q^2, r^2, \theta = 0)$$

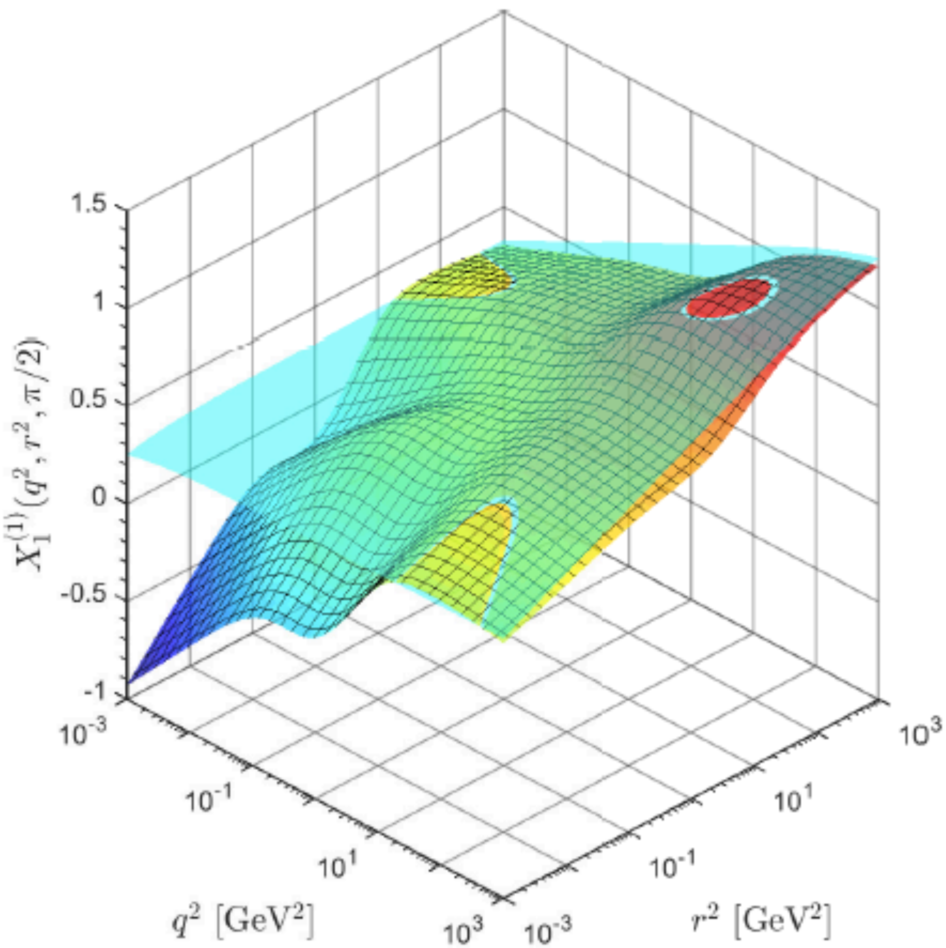


In the abelian approximation is identically zero

$$X_{10}(q^2, r^2, \theta = 0)$$



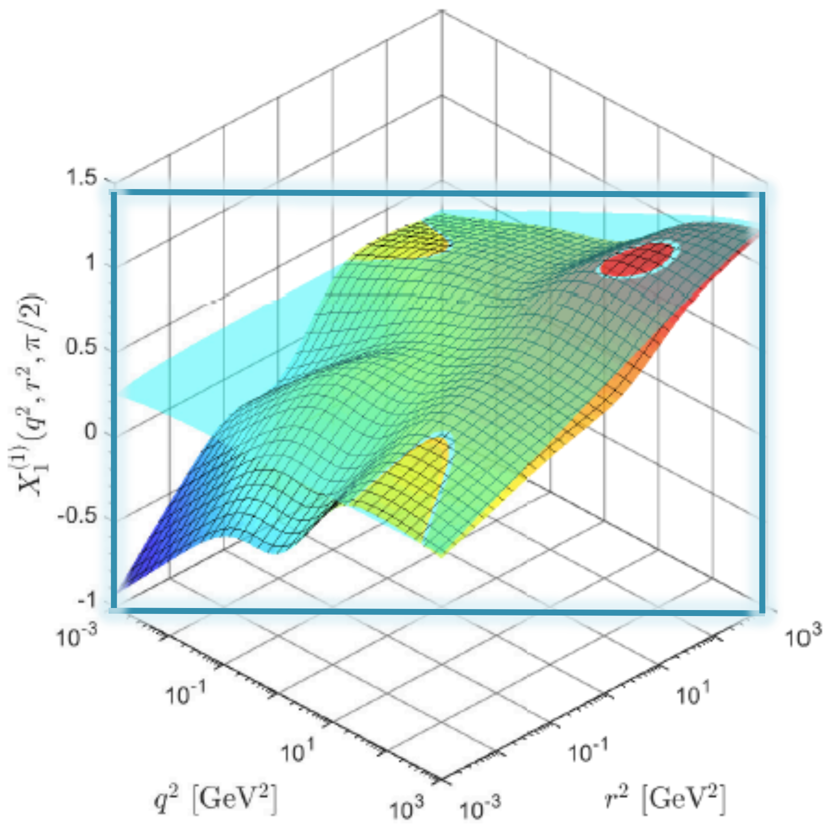
Remark 1:
Suppression is a nonperturbative effect

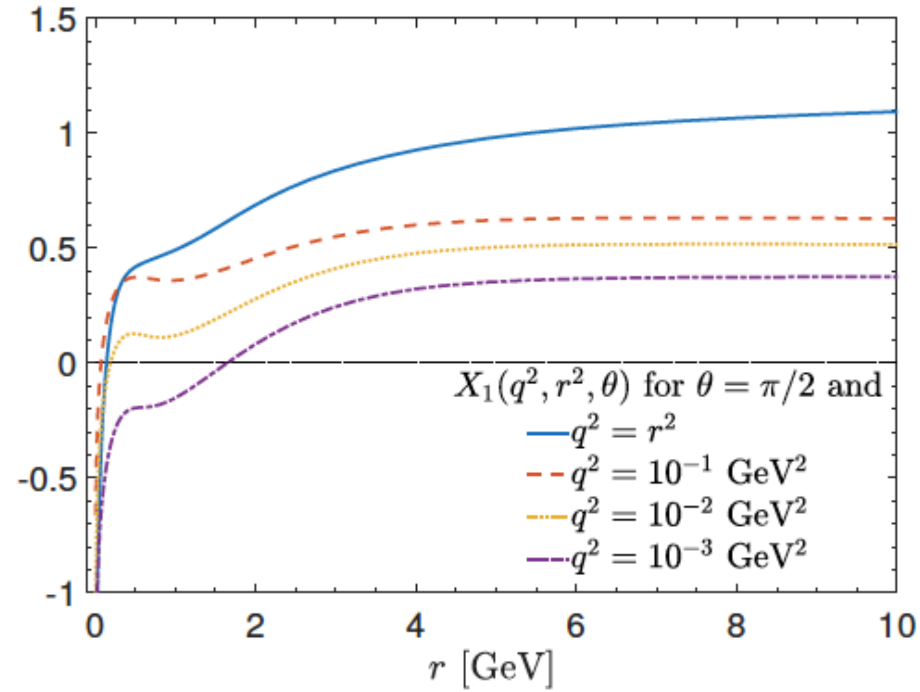


- ✓ Cyan surface is the one-loop result
- ✓ Nonperturbative result is more tilted towards the IR → presence of the crossing

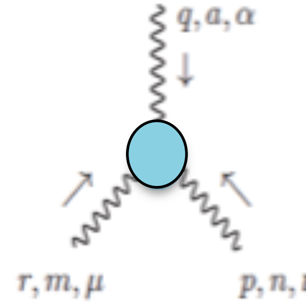
Remark 2:

- The configuration where $q^2 = r^2$ *is less suppressed* then others configurations





Comparison with the lattice results



- In the totally symmetric configuration

$$q^2 = r^2 = p^2 \text{ and } q \cdot r = q \cdot p = r \cdot p = -q^2/2.$$

$$\theta = 120^\circ$$

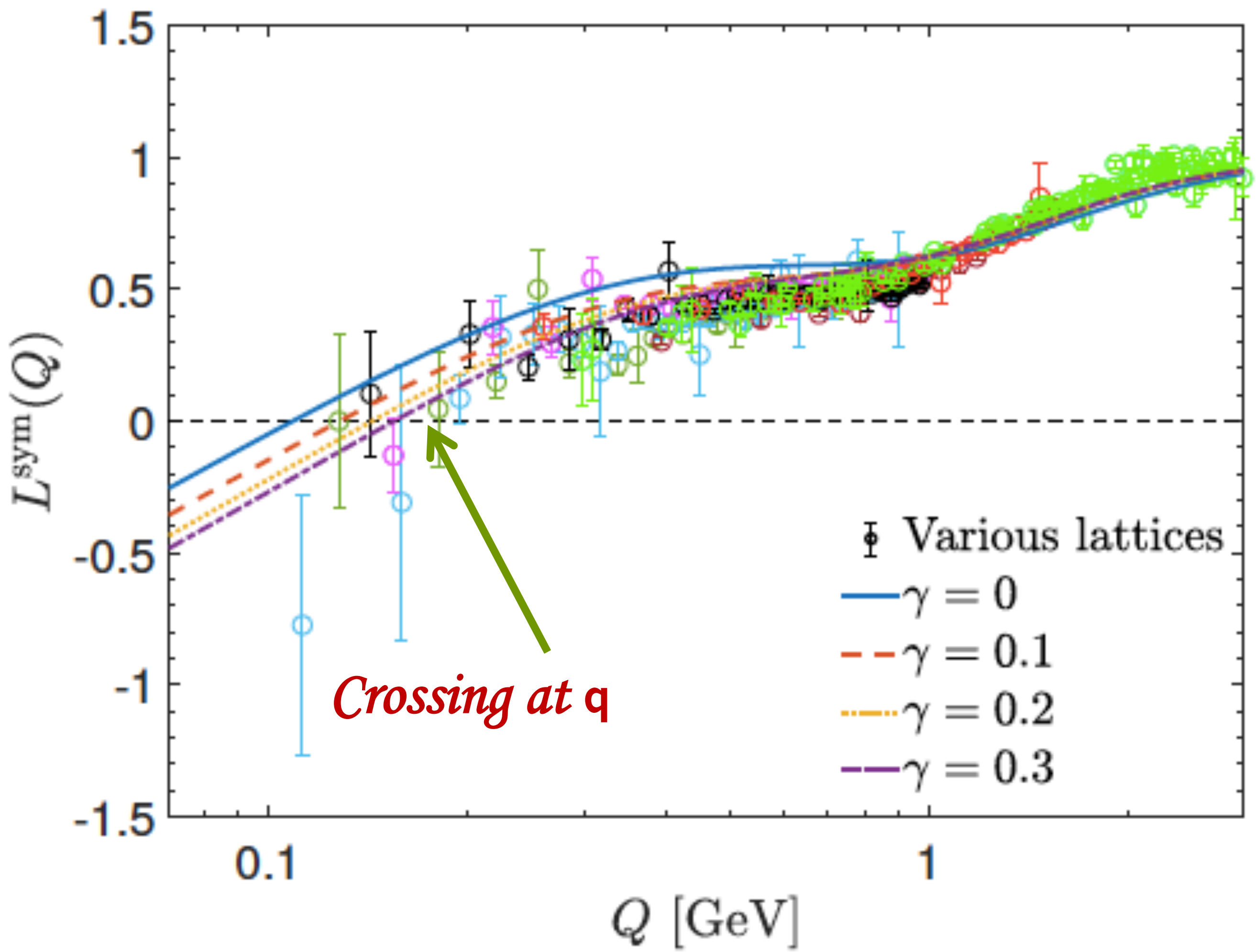
- Lattice has access to the following combination of form factors

$$L^{\text{sym}}(Q) = X_1(Q) - \frac{Q^2}{2} X_3(Q) + \frac{Q^4}{4} Y_1(Q) - \frac{Q^2}{2} Y_4(Q)$$

0

- In our approach ***we can not determine the transverse form factors*** using the STI, then we will consider in the above expression T_i=0

Comparison with the lattice results: Symmetric



Good agreement
In the IR two sources of errors:

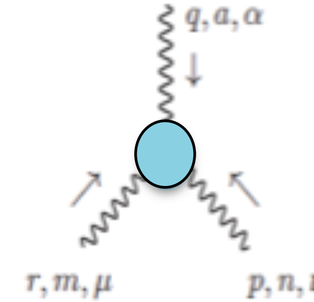
- ✓ *Truncation*
- ✓ *Transverse form factors*

γ	$q_0^{\rm sym}$ [in MeV]
0	109
0.1	128
0.2	143
0.3	155

A.C.A, M. N. Ferreira, C. T. Figueiredo and J. Papavassiliou, 1903.01184 [hep-ph]
Lattice data:
A. Athenodorou, D. Binosi, P. Boucaud, F. De Soto, J. Papavassiliou, J. Rodriguez-Quinteros and S. Zafeiropoulos, Phys. Lett. B 761, 444 (2016).

- In the asymmetric configuration :

$$p = 0 \text{ and } r = -q$$



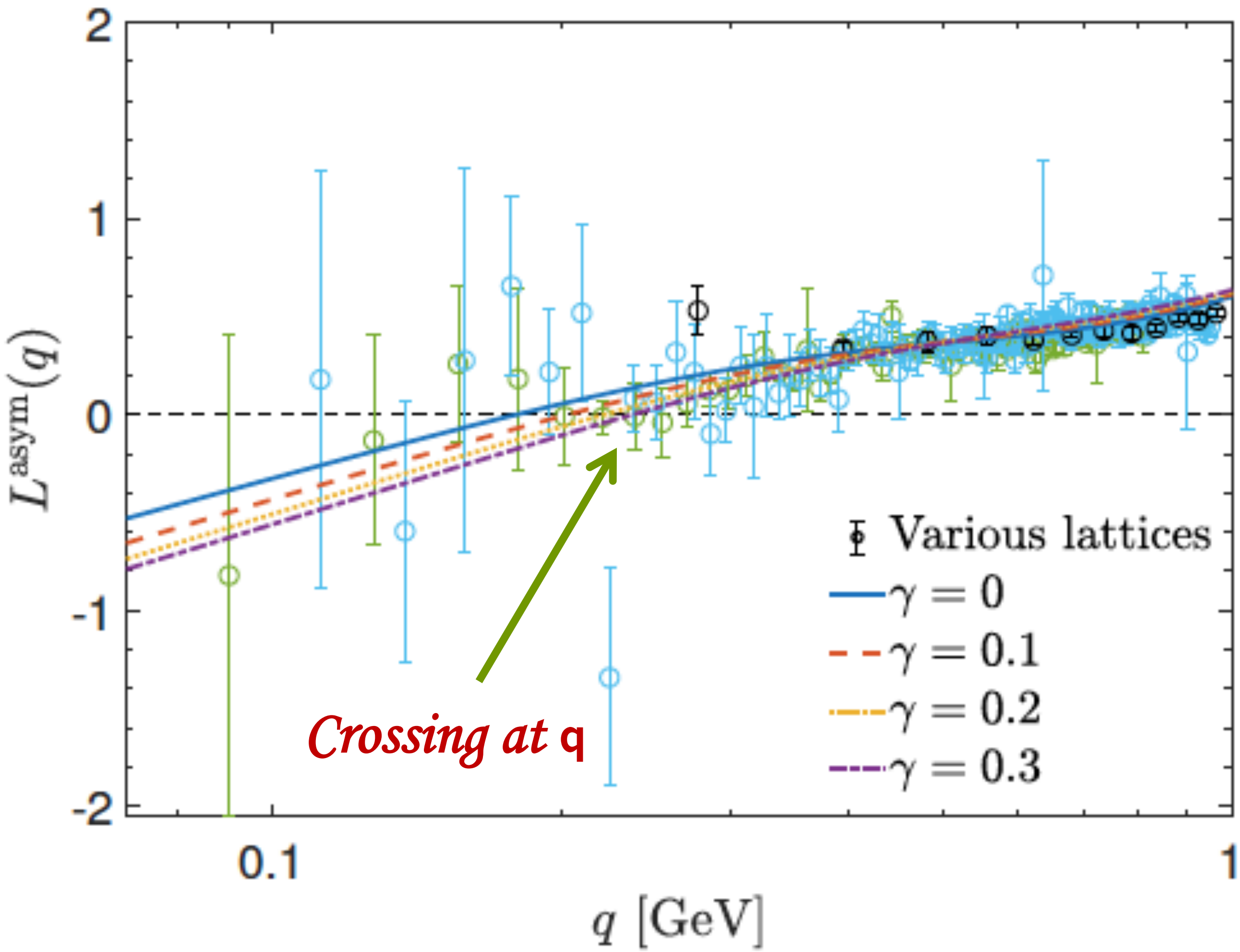
$$\theta = \pi$$

- We have

$$L^{\text{asym}}(q) = X_1(q^2, q^2, \pi) - q^2 X_3(q^2, q^2, \pi)\,.$$

No transversal contamination!

Comparison with the lattice results: Asymmetric



Good agreement!

γ	q_0^{asym} [in MeV]
0	180
0.1	204
0.2	221
0.3	237

A.C.A, M. N. Ferreira, C. T. Figueiredo and J. Papavassiliou, 1903.01184 [hep-ph]
Lattice data:
A. Athenodorou, D. Binosi, P. Boucaud, F. De Soto, J. Papavassiliou, J. Rodriguez-Quinteros and S. Zafeiropoulos, Phys. Lett. B 761, 444 (2016).

Conclusions

- We have determined the **longitudinal form factors of the three-gluon vertex** for **general values of the Euclidean momenta**.

- **STI fixes completely its 10 longitudinal form factors** in terms of the gluon propagator, the ghost dressing function, and the **gluon-ghost scattering kernel.**

- **Due to the Bose symmetry out of 10 form factors we have to computed only 4.**

- The form factors of **the gluon-ghost scattering kernel** have been computed within the **``one-loop dressed" approximation**

- There is a **sizable suppression in the IR** caused by the presence of the massless ghost loops

- In agreement with **lattice simulations** and **phenomenological requirements**