From light-front wavefunctions to parton distributions (part I)

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Continuum Functional Methods for QCD @ New Generation Facilities

ECT*, Trento. May 7-10, 2019

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In Collaboration with...

J. Rodriguez-Quintero, C.D. Roberts...

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Motivation

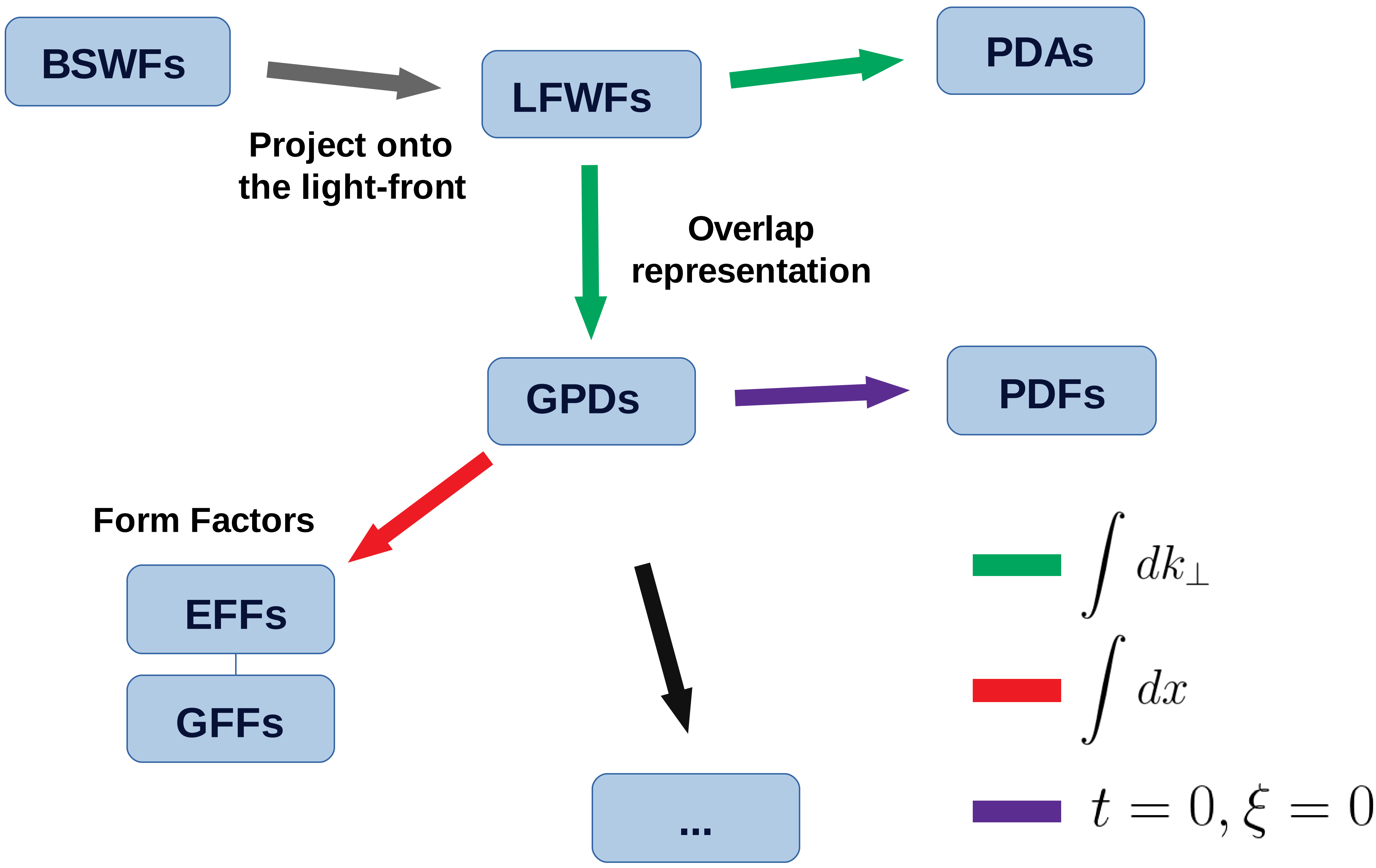
- Decades after the formulation of the fundamental theory of quarks and gluons, **Quantum Chromodynamics (QCD)**, understanding the strong interactions is still being a challenge.

- QCD is characterized by two emergent phenomena: **confinement** and **dynamical chiral symmetry breaking (DCSB)**, which have far reaching consequences in the hadron spectrum and their properties.

- Due to the non perturbative nature of QCD, unraveling the hadron structure, from the fundamental degrees of freedom, is an outstanding problem.

- I shall present an approach, based on **Dyson-Schwinger equations (DSEs)**, to compute a choice of parton distributions within hadrons (pions and kaons).

The idea...


BSWFs
Project onto the light-front
LFWFs
PDAs
Overlap representation
GPDs
PDFs
Form Factors
EFFs
GFFs
...
$\int dk_\perp$
$\int dx$
$t = 0, \xi = 0$

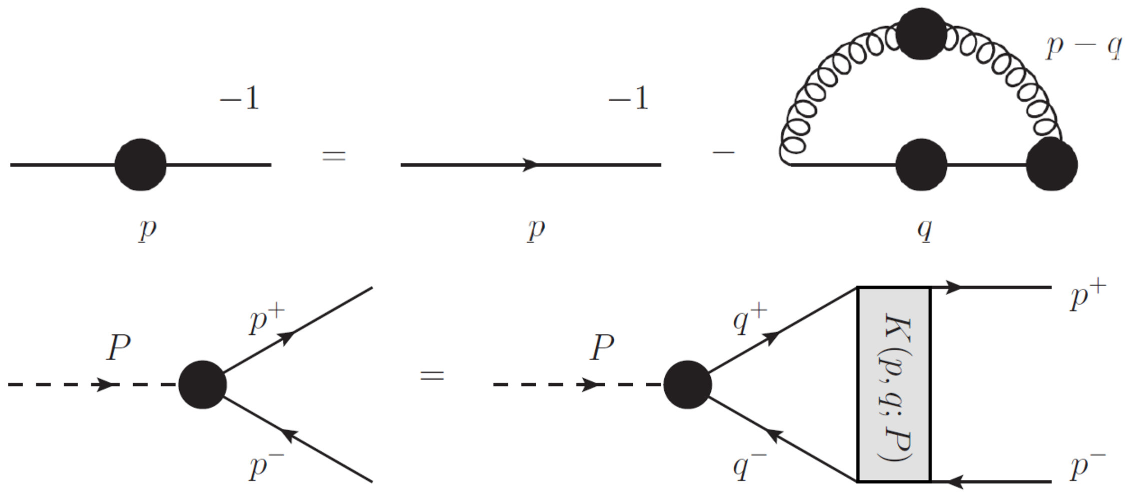
Bethe-Salpeter WF

- The BSWF is the sandwich of the Bethe-Salpeter amplitude and the quark/antiquark propagators:

$$\chi_H(k_-^H; P_H) = S_q(k)\Gamma_H(k_-^H; P_H)S_{\bar{q}}(k - P_H) \,,\; k_-^H = k - P_H/2 \,.$$

$P_H^2 = -m_H^2$: **meson's mass**; Γ_H:**BS amplitude**; $S_{q(\bar{q})}$: **quark (antiquark) propagator**

- Quark propagator and BSA should come from solutions of:



Quark DSE

Meson BSE

- **Alternative** first step: **construct** an *educated* ***ansätz***.

Bethe-Salpeter WF

- Starting with the Kaon as a example, we employ a Nakanishi-like representation of the BSWF:

$$n_K \chi_K^{(2)}(k_-^K; P_K) = \underbrace{\mathcal{M}(k; P_K)}_{1} \int_{-1}^{1} d\omega \; \underbrace{\rho_K(\omega)}_{2} \underbrace{\mathcal{D}(k; P_K)}_{3} \; ,$$

1: Leading twist contribution to PDA (only γ_5 BSA):

$$\mathcal{M}(k; P_K) = -\gamma_5[\gamma \cdot P_K M_u + \gamma \cdot k(M_u - M_s) + \sigma_{\mu\nu} k_\mu P_{K\nu}] \; ,$$

2: Sprectral weight: To be chosen later.

3: Product of 3 quadratic forms in the denominator:

$$\mathcal{D}(k; P_K) = \Delta(k^2, M_u^2)\Delta((k - P_K)^2, M_s^2)\hat{\Delta}(k_{\omega-1}^2, \Lambda_K^2) \; ,$$

$$\text{where: } \Delta(s,t) = [s + t]^{-1}, \; \hat{\Delta}(s,t) = t\Delta(s,t) \; .$$

Phys.Rev. D97 (2018) no.9, 094014. S-S Xu et al.

Bethe-Salpeter WF

- Combining denominators and **rearranging** the order of integration, we arrive at:

$$\chi_K^{(2)}(k_-^K; P_K) = \mathcal{M}(k; P_K) \int_0^1 d\alpha \, 2\chi_K(\alpha; \sigma^3(\alpha)) \; , \; \sigma = (k - \alpha P_K)^2 + \Omega_K^2 \; ,$$

where Ω_K^2 depends on the model and Feynman parameters and:

$$\chi_K(\alpha; \sigma^3) = \left[\int_{-1}^{1-2\alpha} d\omega \int_{1+\frac{2\alpha}{\omega-1}}^{1} dv + \int_{1-2\alpha}^{1} d\omega \int_{\frac{\omega-1+2\alpha}{\omega+1}}^{1} dv \right] \frac{\rho_K(\omega)}{n_K} \frac{\Lambda_K^2}{\sigma^3} \; .$$

- As we will see $\rho_K(\omega)$ **plays a crucial role** in determining the meson's properties.

- The explicit form will be discussed later.

Light-Front WF

- The pseudoscalar LFWF can be written:

$$f_K \psi_K^{\uparrow\downarrow}(x, k_\perp^2) = \mathrm{tr}_{CD} \int_{dk_{||}} \delta(n \cdot k - x n \cdot P_K) \gamma_5 \gamma \cdot n \chi_K^{(2)}(k_-^K; P_K) \, .$$

- The moments of the distribution:

$$< x^m >_{\psi_K^{\uparrow\downarrow}} = \int_0^1 dx x^m \psi_K^{\uparrow\downarrow}(x, k_\perp^2) = \frac{1}{f_K n \cdot P} \int_{dk_{||}} \left[\frac{n \cdot k}{n \cdot P}\right]^m \gamma_5 \gamma \cdot n \chi_K^{(2)}(k_-^K; P_K)$$

$$\int_0^1 d\alpha \alpha^m \left[\frac{12}{f_K} \mathcal{Y}_K(\alpha; \sigma^2)\right] \, , \quad \mathcal{Y}_K(\alpha; \sigma^2) = [M_u(1-\alpha) + M_s \alpha] \mathcal{X}(\alpha; \sigma_\perp^2) \, .$$

Uniqueness of Mellin moments ⟶

$$\psi_K^{\uparrow\downarrow}(x, k_\perp^2) = \frac{12}{f_K} \mathcal{Y}_K(x; \sigma_\perp^2)$$

❖ Compactness of this result is a merit of the algebraic model.

Light-Front WF

- Notably, the LFWF is determined by the Nakanishi weight:

$$\psi_K^{\uparrow\downarrow}(x,k_\perp^2) = \frac{12}{f_K}\mathcal{Y}_K(x;\sigma_\perp^2)\ , \quad \mathcal{Y}_K(\alpha;\sigma^2) = [M_u(1-\alpha)+M_s\alpha]\mathcal{X}_K(\alpha;\sigma_\perp^2)\ ,$$

$$\chi_K(\alpha;\sigma^3) = \left[\int_{-1}^{1-2\alpha} d\omega \int_{1+\frac{2\alpha}{\omega-1}}^{1} dv + \int_{1-2\alpha}^{1} d\omega \int_{\frac{\omega-1+2\alpha}{\omega+1}}^{1} dv\right] \frac{\rho_K(\omega)}{n_K}\frac{\Lambda_K^2}{\sigma^3}\ .$$

$$\Rightarrow \psi_K^{\uparrow\downarrow}(x,k_\perp^2) \sim \int d\omega\ \cdots \rho_K(\omega) \cdots$$

- The spectral density can be determined from realistic solutions of the BSE.

---- Our medium term goal

- Alternatively, we constructed an *educated* ansätz based on contemporary DSE predictions.

Nakanishi weight

- The spectral density is chosen as:

$$u_G \rho_G(\omega) = \frac{1}{2b_0^G}\left[\operatorname{sech}^2\left(\frac{\omega - \omega_0^G}{2b_0^G}\right) + \operatorname{sech}^2\left(\frac{\omega + \omega_0^G}{2b_0^G}\right)\right][1 + \omega\ v_G]\ ,$$

and, the parameters:

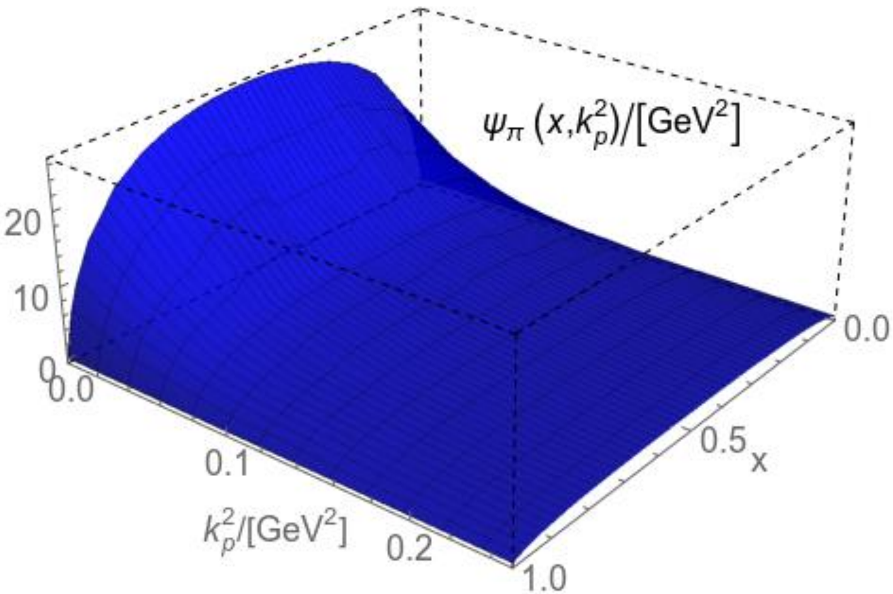
Λ_π	b_0^π	ω_0^π	ν_π	Λ_K	b_0^K	ω_0^K	ν_K
M_u	0.275	1.23	0	$2\Lambda_\pi$	b_0^π	0.95	0.16

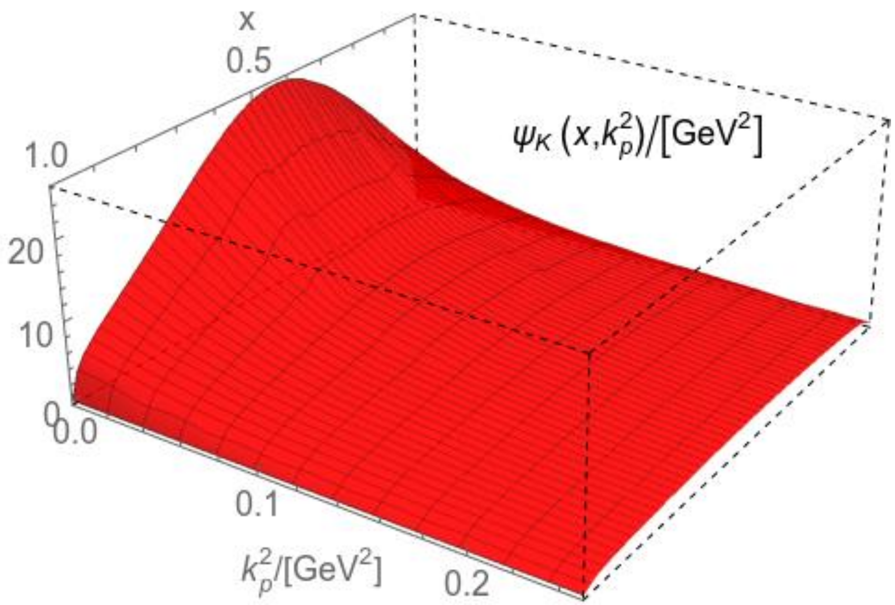
$$M_u = 0.31\ \text{GeV},\ m_\pi = 0.14\ \text{GeV},\ m_K = 0.49\ \text{GeV}\ .$$

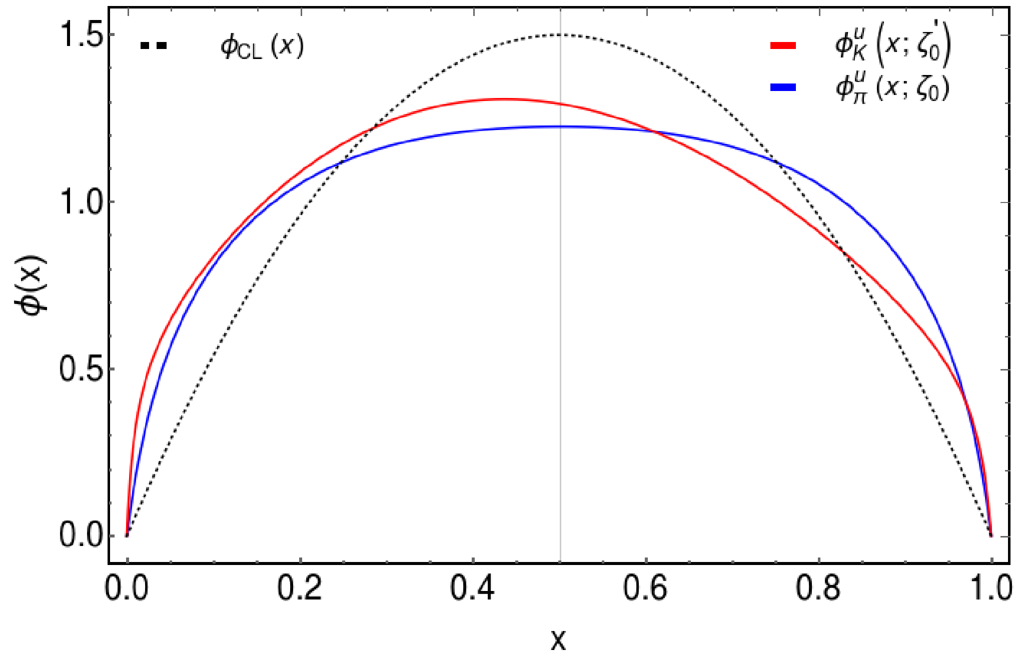
- The empirical values of the decay constants are faithfully reproduced.

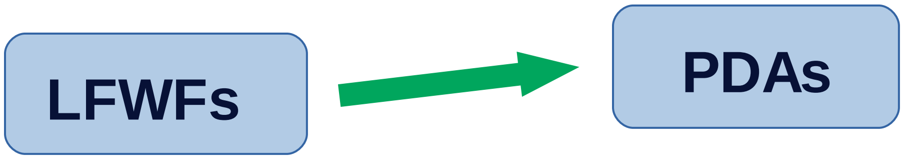
- We have modified the set of pion parameters from those shown in **Phys.Rev. D97 (2018) no.9.**

Ligth-front WF and PDAs



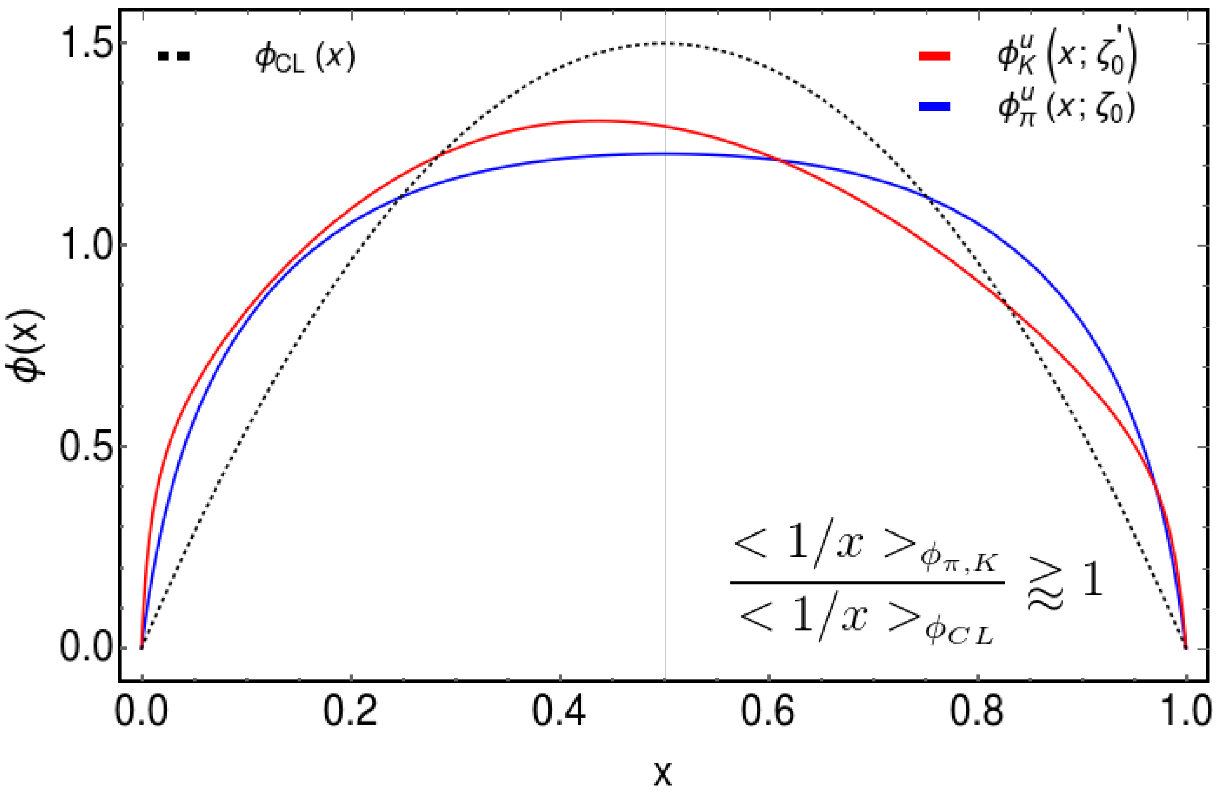






$$\phi_M(x) = \frac{1}{16\pi^3}\int d^2\vec{k}_\perp \psi_M^{\uparrow\downarrow}(x,k_\perp^2)$$

Valence-quark DAs



PDAs: Broad, concave functions of x.

Dilation: Signal of DCSB

Symmetric PDA: Isospin symmetric limit.

Endpoints: Smooth fall at the endpoints.

First non-trivial moments: in agreement with sophisticated DSE results*.

$$< (2x-1)^2 >_{\phi_\pi^q} \approx 0.26 \;,\; < (2x-1) >_{\phi_\pi^q} = 0 \;.$$

$$< (2x-1)^2 >_{\phi_K^u} \approx 0.25 \;,\; < (2x-1) >_{\phi_K^u} \approx -0.036 \;.$$

Phys.Rev.Lett. 110 (2013) no.13, 132001*
Phys.Lett. B731 (2014) 13-18

Asymptotic model — Side note

- **Note**: If the spectral density is chosen as: $\rho(\omega;\nu) \sim (1-\omega^2)^\nu$,

one obtains closed algebraic forms of PDAs and PDFs:

$$\phi(x;\nu) \sim [x(1-x)]^\nu \ , \quad q(x;\nu) \sim [x(1-x)]^{2\nu} \ .$$

- In particular, asymptotic PDA corresponds to ν=1.

Sketching the pion's valence-quark generalised parton distribution

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[b]*CSSM, School of Chemistry and Physics University of Adelaide, Adelaide SA 5005, Australia*
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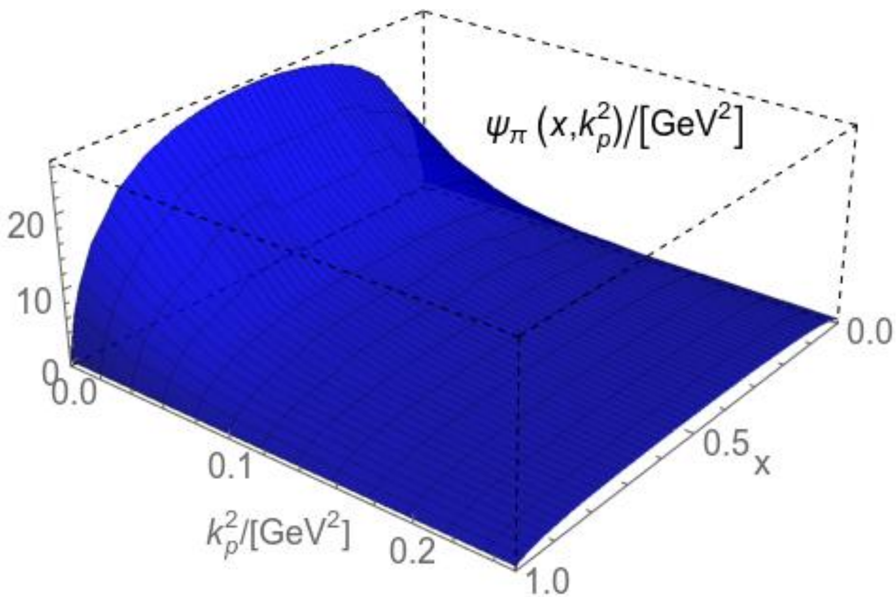
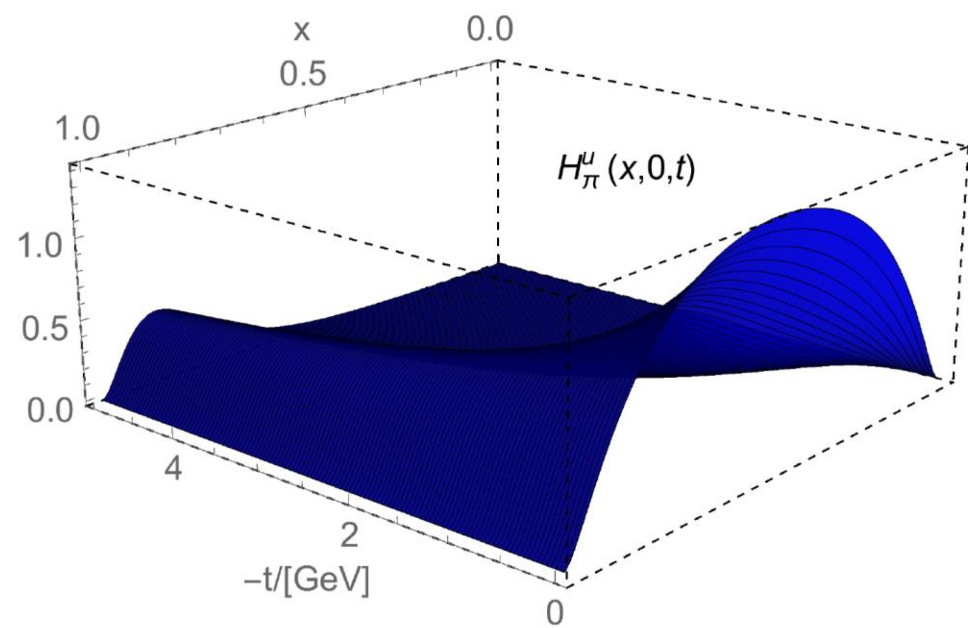
Phys.Lett. B741 (2015) 190-196. C. Mezrag et al.

GPD - DGLAP

- A two-particle truncated expression for the Pion and Kaon **GPDs**, in the **DGLAP** kinematic **domain**, is obtained from the **overlap of the LFWF**:

$$H_M^q(x,\xi,t) = \int \frac{d^2\mathbf{k}_\perp}{16\,\pi^3} \Psi^*_{u\bar{f}}\left(\frac{x-\xi}{1-\xi}, \mathbf{k}_\perp + \frac{1-x}{1-\xi}\frac{\Delta_\perp}{2}\right) \Psi_{u\bar{f}}\left(\frac{x+\xi}{1+\xi}, \mathbf{k}_\perp - \frac{1-x}{1+\xi}\frac{\Delta_\perp}{2}\right).$$





LFWFs → **GPDs**

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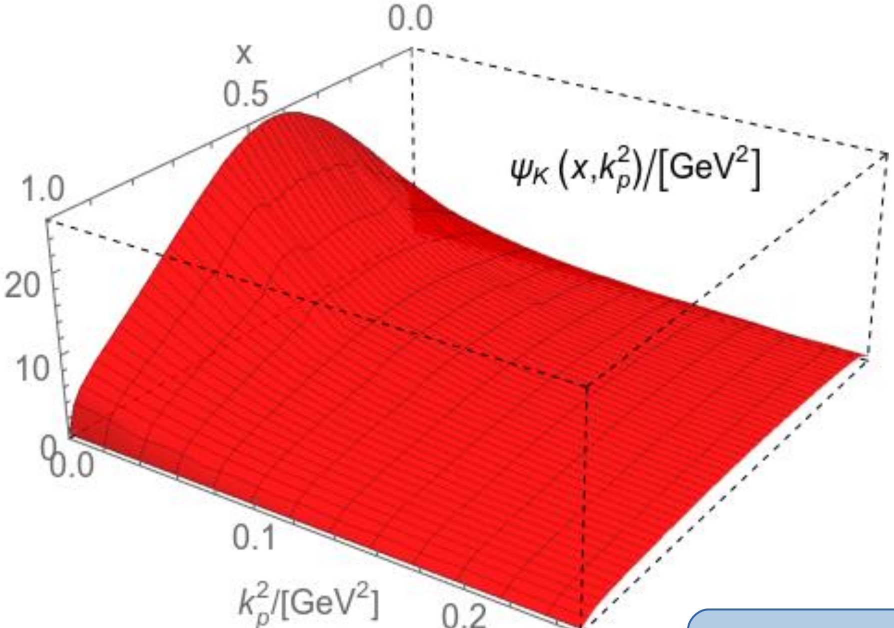
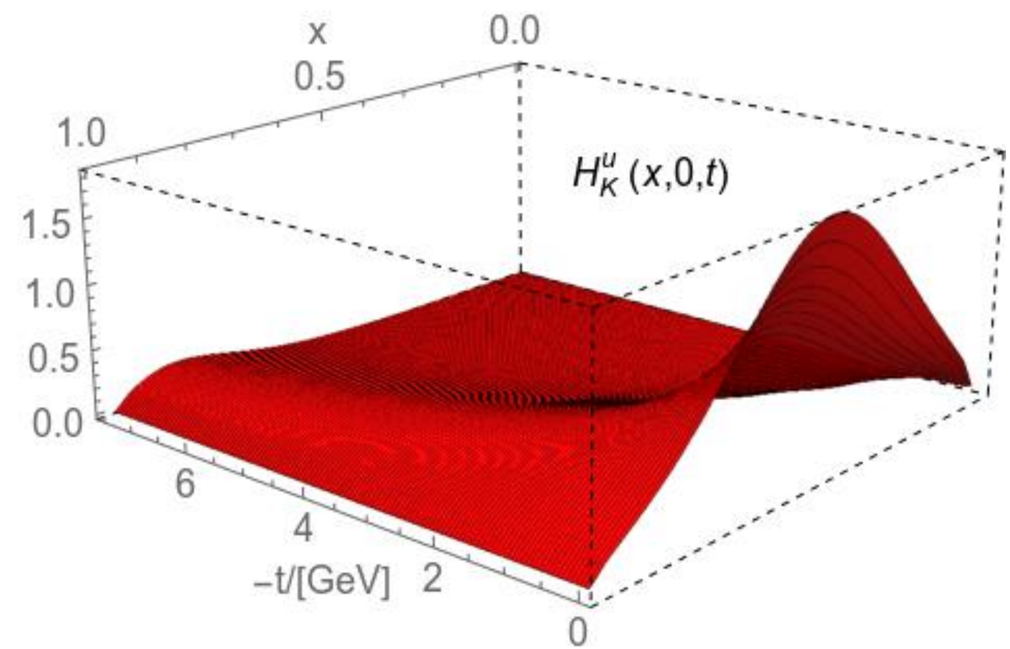


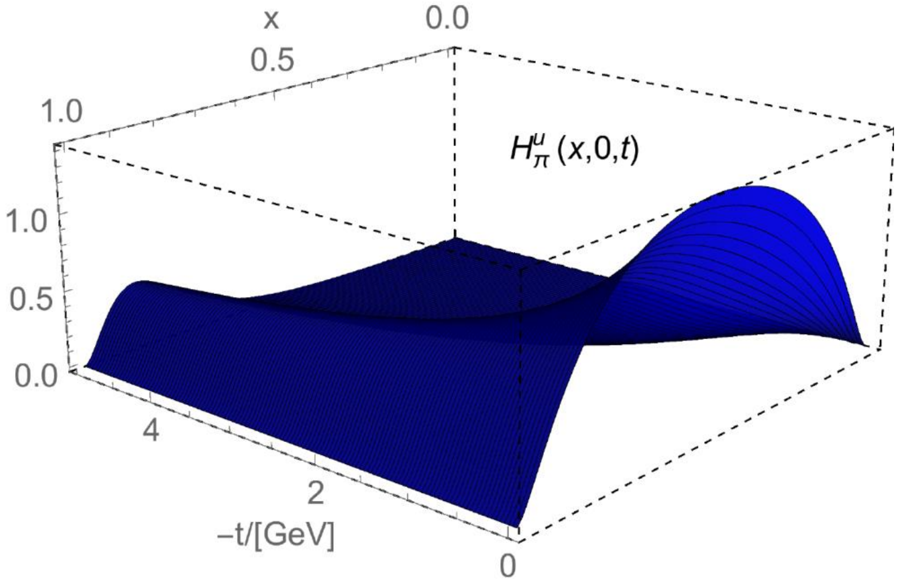
LFWFs → **GPDs**



Valence PDFs

GPDs → PDFs

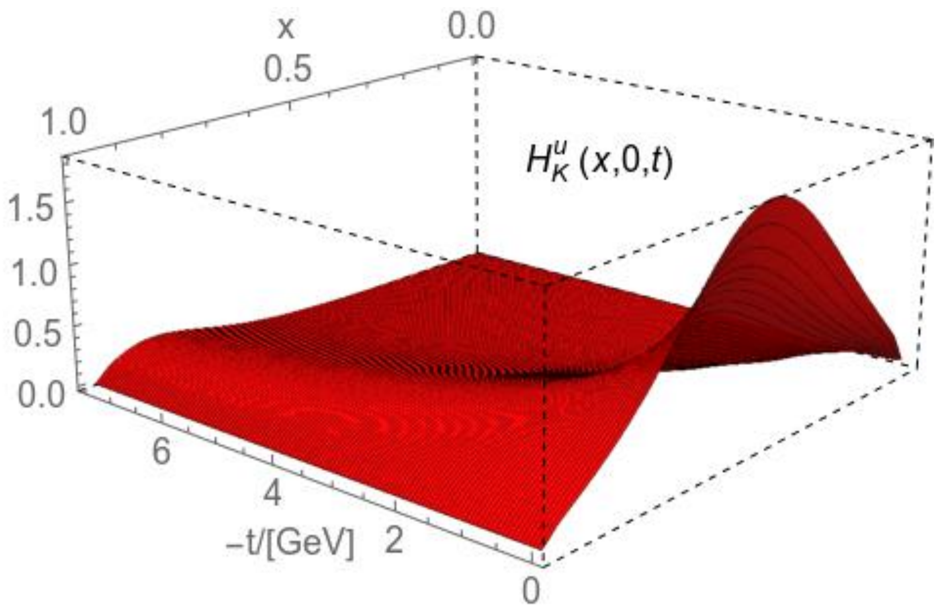
- At zero skewness and t = 0, the valence GPD defines the PDF.

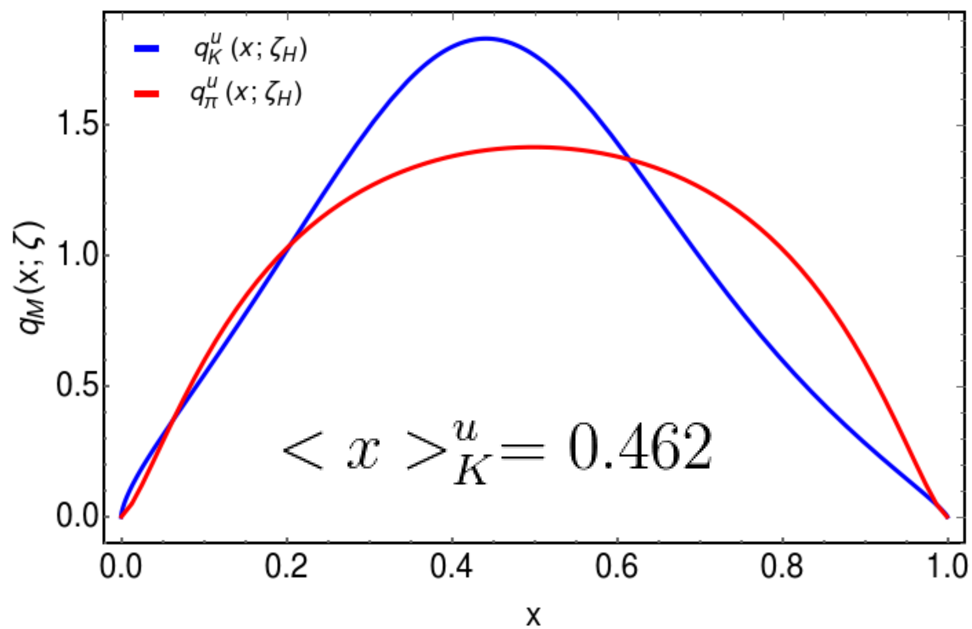


Hadron scale: valence quarks carry all the momentum.

Unambiguously defined from the PI coupling.

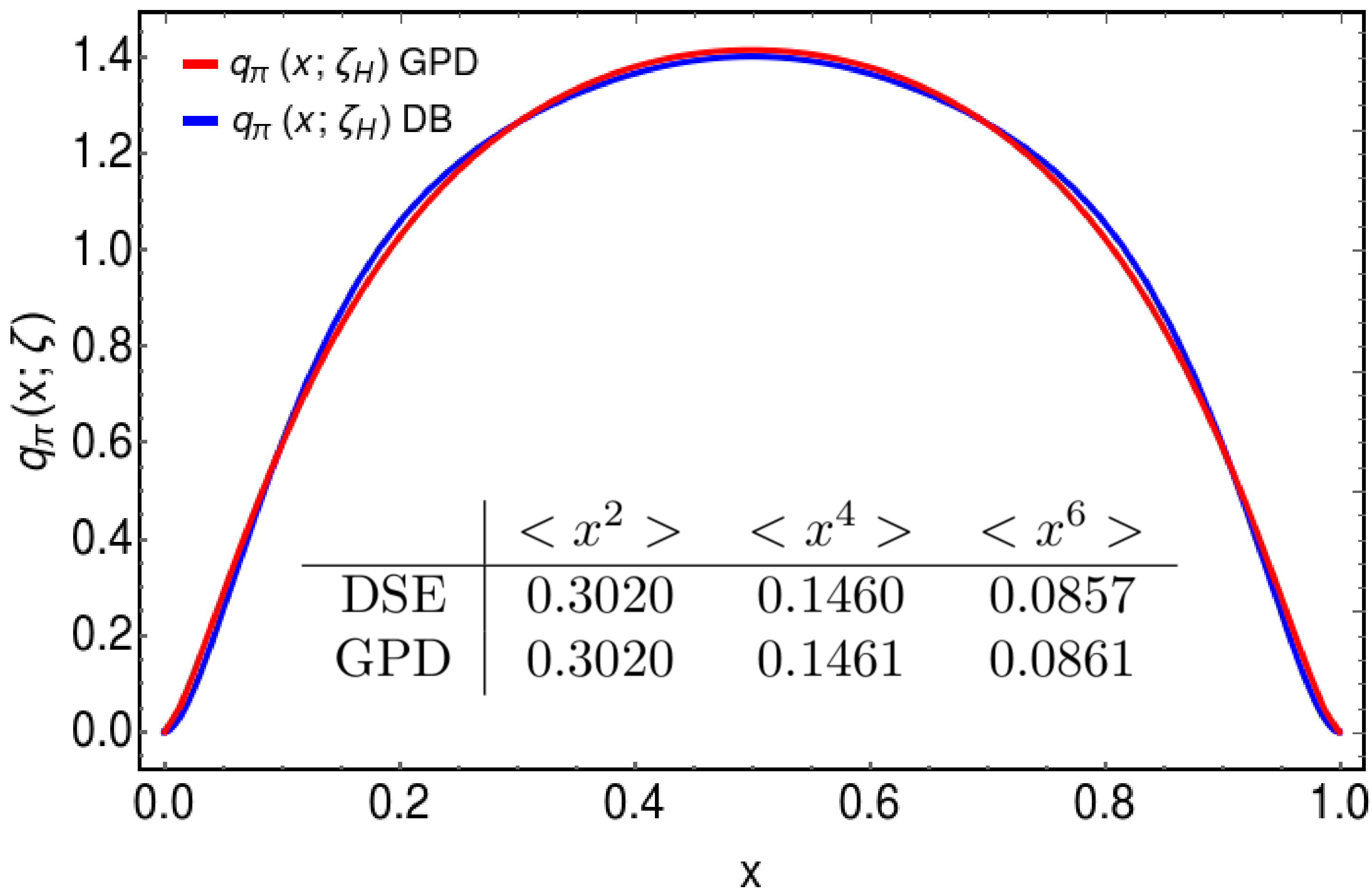
Evolution: DGLAP evolution incorporates sea and gluon content.





PDF: LFWF model and DSE

- PDF obtained from the overlap of the LFWF is practically indistinguishable from the most sophisticated DSE prediction.



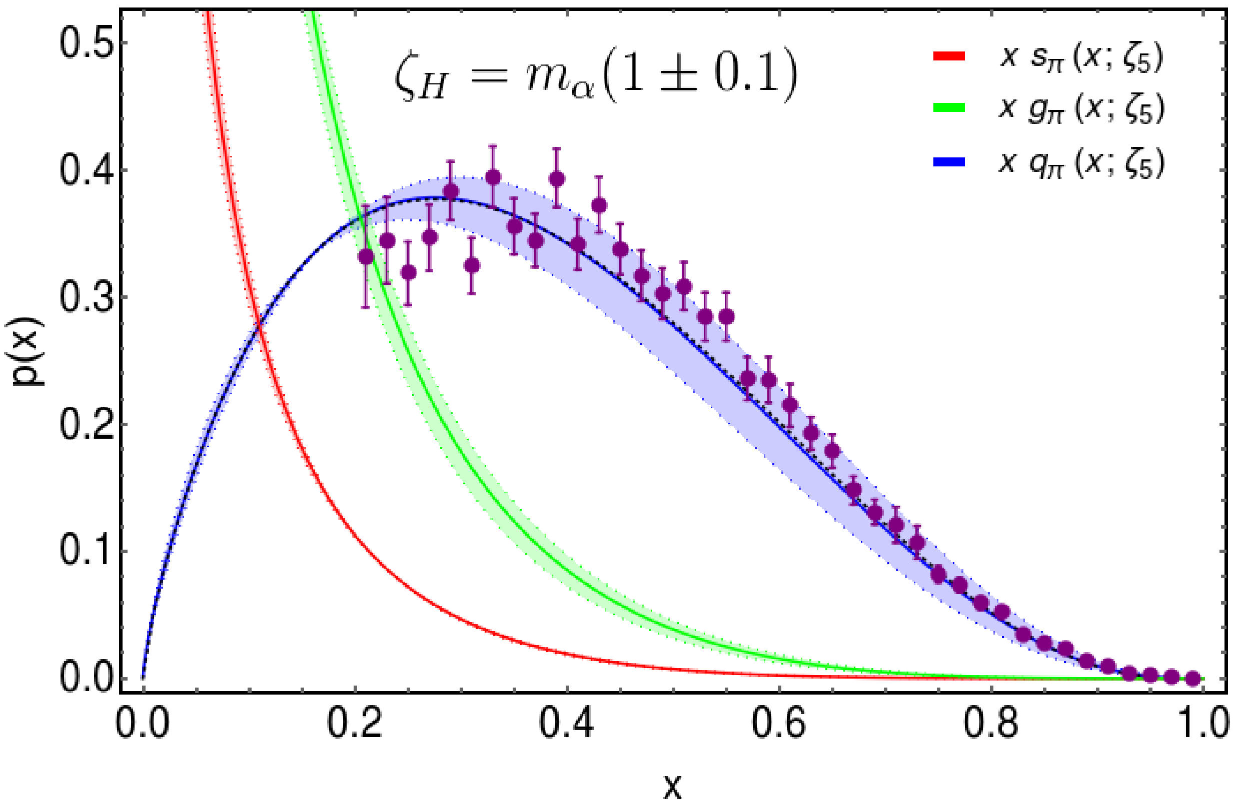
- DSE: Ding et al. In preparation

See Craig's talk

PDF: LFWF model and DSE

- Hadron scale defined from the PI coupling:

$$\alpha_{\rm PI}(k^2) = \frac{4\pi}{\beta_0 \ln[(m_\alpha^2 + k^2)/\Lambda_{\rm QCD}^2]}\ , \ m_\alpha = 0.3 \text{ GeV} \sim \Lambda_{\rm QCD}$$



Momentum fractions (5.2 GeV):

$$< x >_{\rm val} = 0.21(2)$$

$$< x >_{\rm sea} = 0.14(1)$$

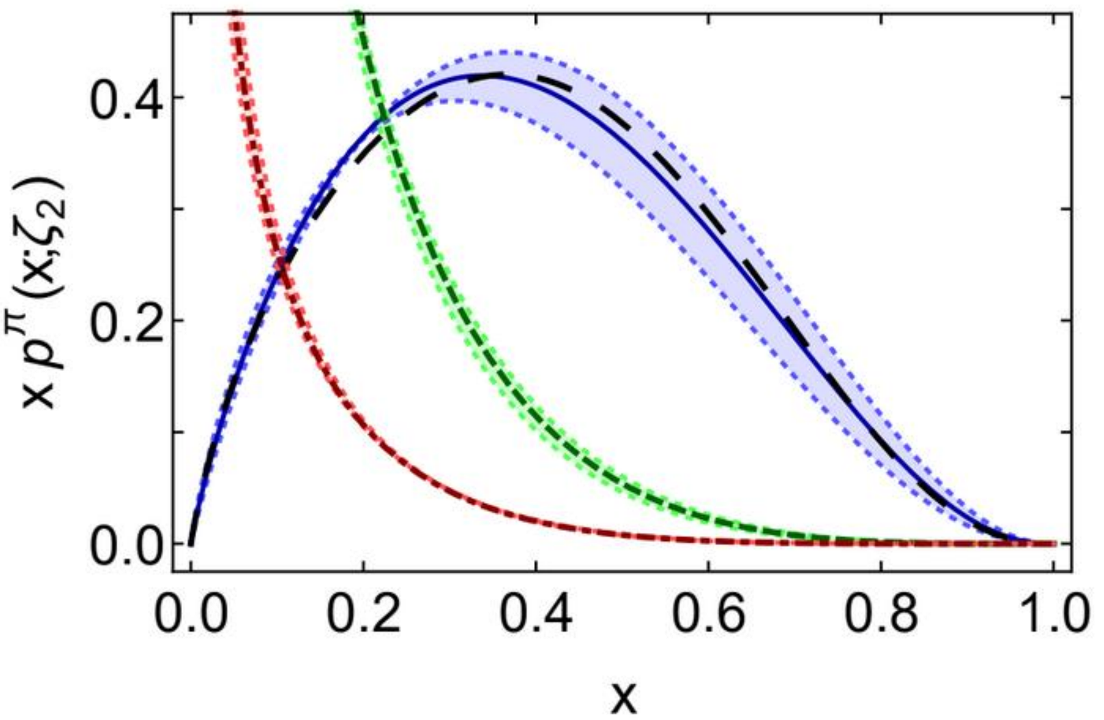
$$< x >_{\rm gluon} = 0.45(1)$$

Singlet evolution allowed us to compute sea and gluon distributions.

DSE [Black, dotted]: Ding et al. In preparation
PI coupling: Phys.Rev. D96 (2017) no.5, 054026

PDF: LFWF model and DSE

- Any small differences between the GPD infered PDF and the recent DSE prediction* vanishes when we evolve with DGLAP.



ζ_2	$\langle x \rangle_u^\pi$	$\langle x^2 \rangle_u^\pi$	$\langle x^3 \rangle_u^\pi$
Ref. [33]	0.24(2)	0.09(3)	0.053(15)
Ref. [34]	0.27(1)	0.13(1)	0.074(10)
Ref. [35]	0.21(1)	0.16(3)	
average	0.24(2)	0.13(4)	0.064(18)
Herein	0.24(2)	0.098(10)	0.049(07)

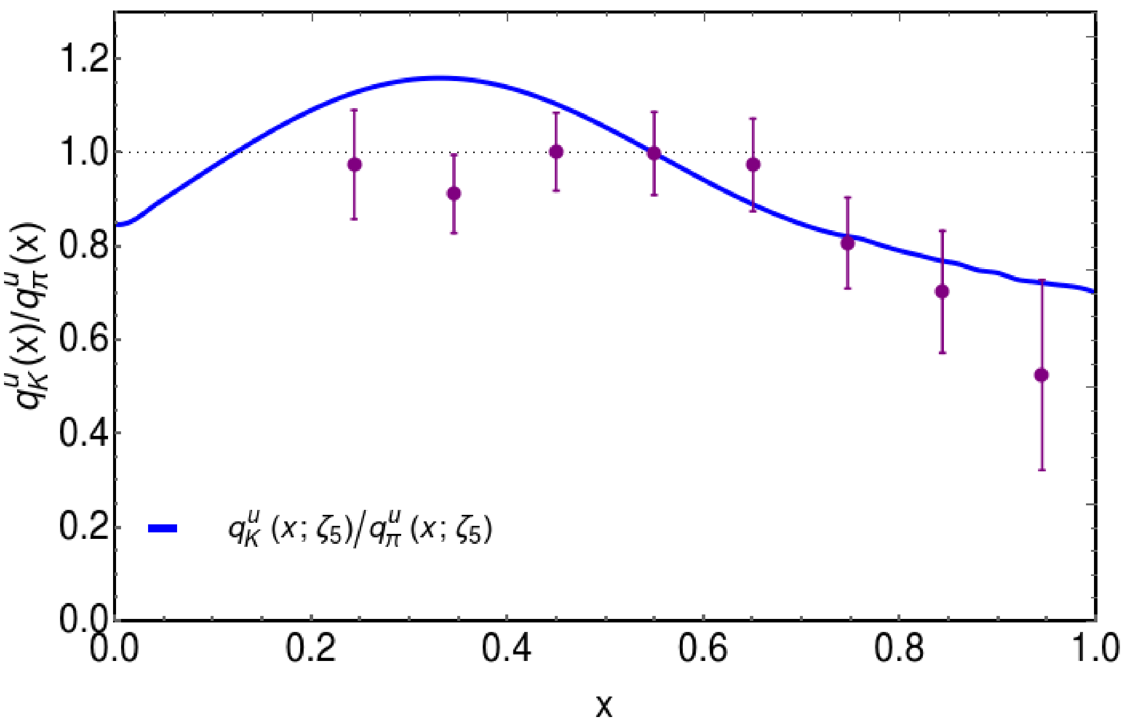
$$\langle x \rangle_g^\pi = 0.41(2)\,, \quad \langle x \rangle_{\text{sea}}^\pi = 0.11(2)\,.$$

[33] W. Detmold, W. Melnitchouk and A. W. Thomas, Phys. Rev. D **68**, 034025 (2003).
[34] D. Brommel *et al.*, PoS **LAT2007**, 140 (2007).
[35] M. Oehm *et al.*, Phys. Rev. D **99**, 014508 (2019).

•DSE: Ding et al. In preparation*

See Craig's talk

PDF: Kaon?... not now



$$< x >_K^u = 0.218 \qquad < x >_K^s = 0.254$$

$$< x >_K^{\text{sea+glue}} = 0.528 \qquad < x >_\pi^{\text{sea+glue}} = 0.59(2)$$

- *Naive* DGLAP evolution of the kaon PDF yields to the **same sea and gluon** distribution of the pion.
- This is **unrealistic**. More massive quarks emit less gluons. *
- For the moment, we mimic this effect by increasing the hadron scale (kaon) by 15%.
- Nevertheless, DGLAP evolution must be adapted to account for the more massive s quark.

Phys.Rev. D93 (2016) no.7, 074021

Electromagnetic Form Factors

- The **electromagnetic form factor** can be computed form the GPD **zero-th Mellin moment**, such that:

$$F_M(\Delta^2) = e_u F_M^u(\Delta^2) + e_f F_M^f(\Delta^2)\ ,\ F_M^q(-t=\Delta^2) = \int_{-1}^{1} dx\ H_M^q(x,\xi,t)\ .$$

Electric charges

- Due to the polinomiality of the GPD, it is irrespective of the value of skewness. Thus, one can simply take it to zero.

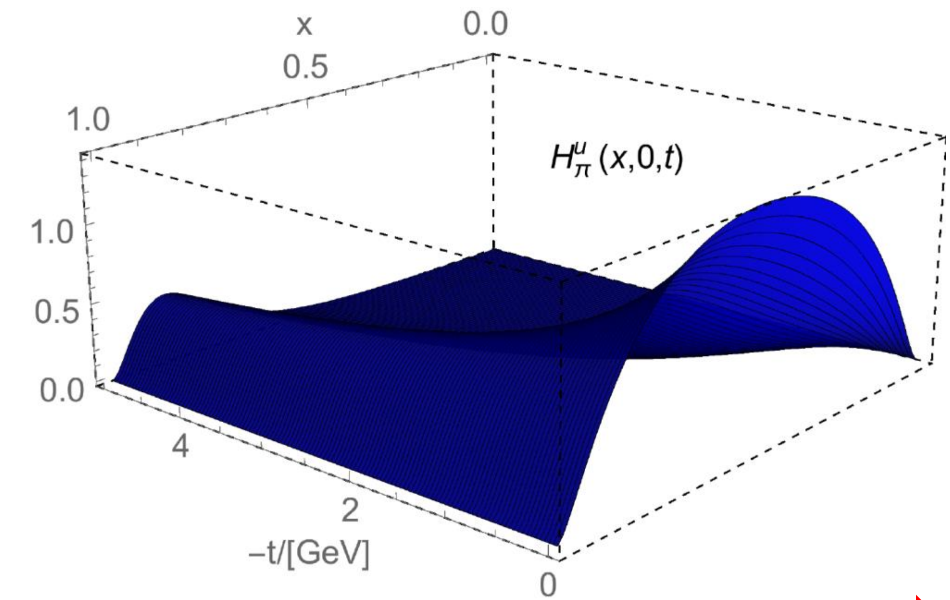
- For the pion, charge conjugation + isospin symmetry entail:

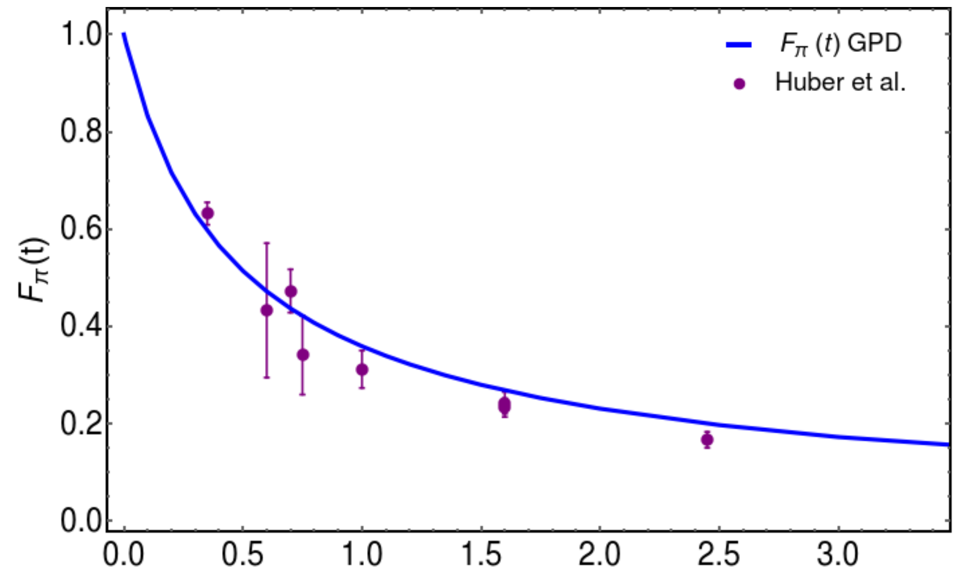
$$F_{\pi^+}^u(\Delta^2) = -F_{\pi^+}^d(\Delta^2) \Rightarrow F_{\pi^+}(\Delta^2) = \frac{2}{3}F_{\pi^+}^u(\Delta^2) - \frac{1}{3}F_{\pi^+}^d(\Delta^2) = F_{\pi^+}^u(\Delta^2)\ .$$

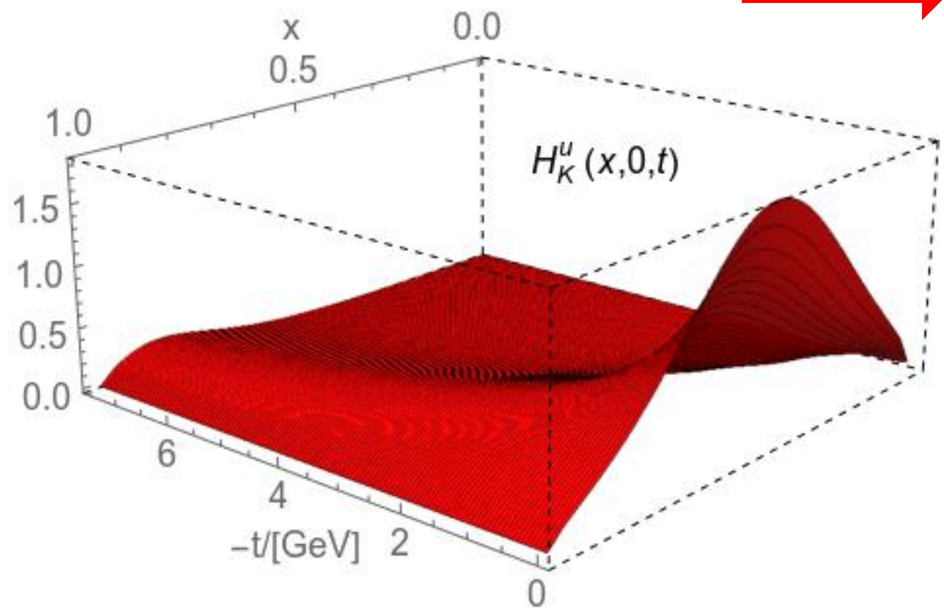
- For the kaon, the form factors should be taken separately for each quark/antiquark flavor.

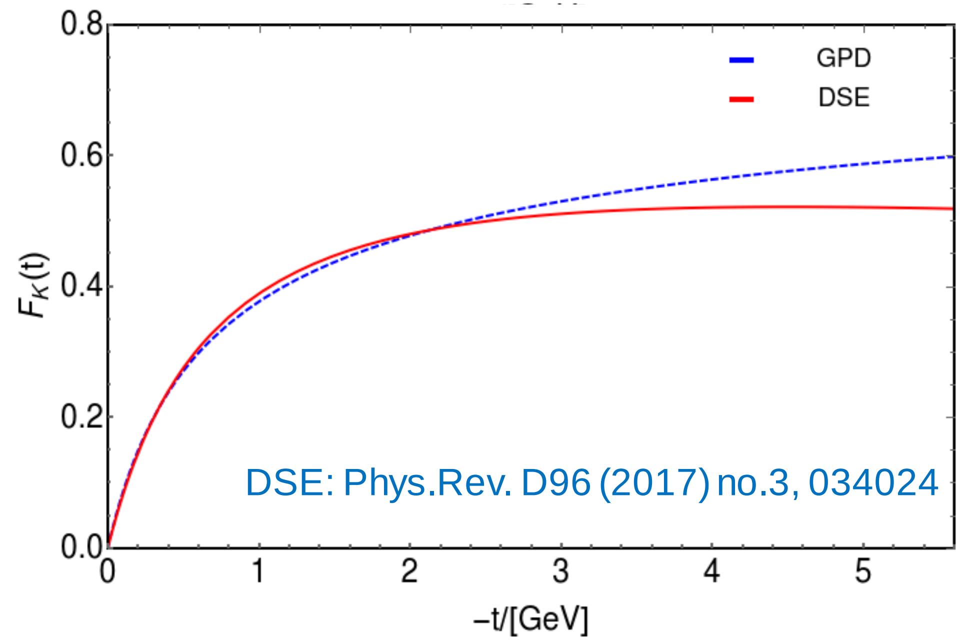
GPDs → **EFFs**

Electromagnetic Form Factors



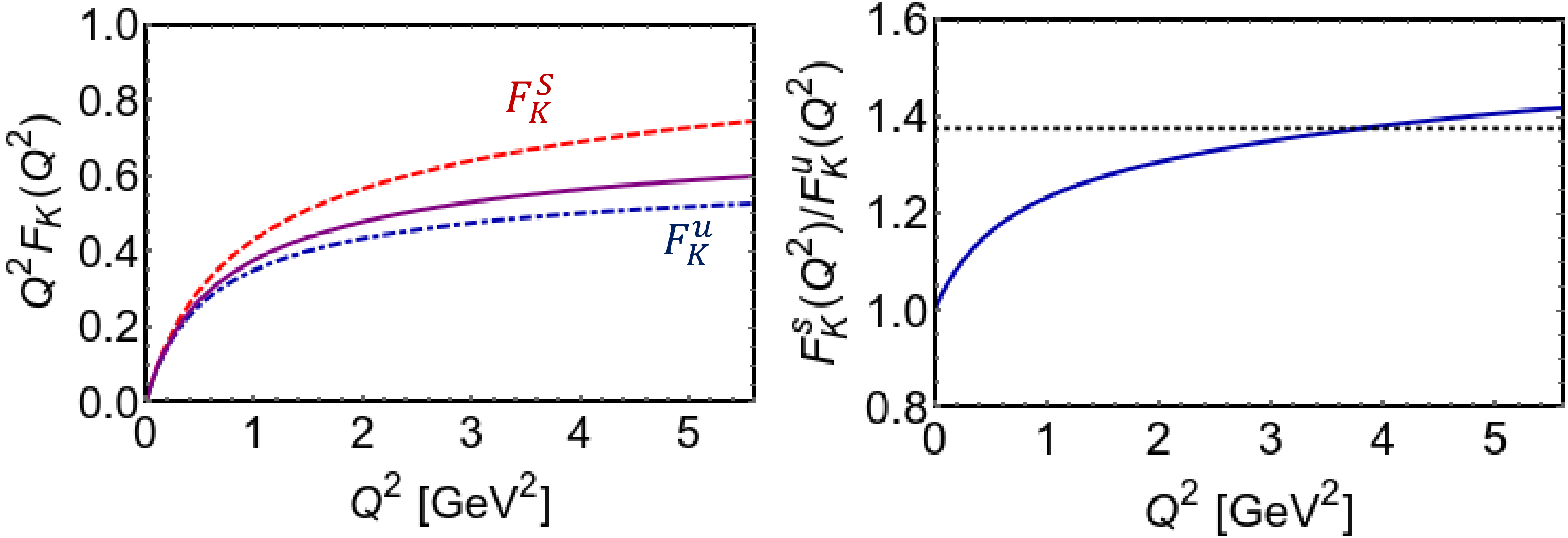






Electromagnetic form factors

- Flavor separation of the kaon form factors.



$$\tilde{r}_K = \frac{F_K^s(Q^2)}{F_K^u(Q^2)} \overset{Q^2 \gg \Lambda_{QCD}^2}{\approx} \left(\frac{\omega_K^s}{\omega_K^u}\right)^2 \approx 1.37$$

$$\omega_\phi = \frac{1}{3}\int_0^1 \frac{\phi(x;\zeta)}{x}\, dx$$

Asymptotically, flavor symmetry is restored.

Proper **QCD evolution is necessary** to accomplish that.

See Pepe's talk

Gravitational Form Factors

- Pion gravitational form factors are defined as:

$$J_{\pi^+}(-t,\xi) \equiv \int_{-1}^{1} dx\; x H_{\pi^+}(x,\xi,t) = \Theta_2(t) - \Theta_1(t)\xi^2 \; .$$

- At zero skewness, one can readily compute

$$\Theta_2(t) = \int_0^1 dx\; x[H^u_{\pi^+}(x,0,t) + H^d_{\pi^+}(x,0,t)] = \int_0^1 dx\; 2x H^u_{\pi^+}(x,0,t) \; .$$

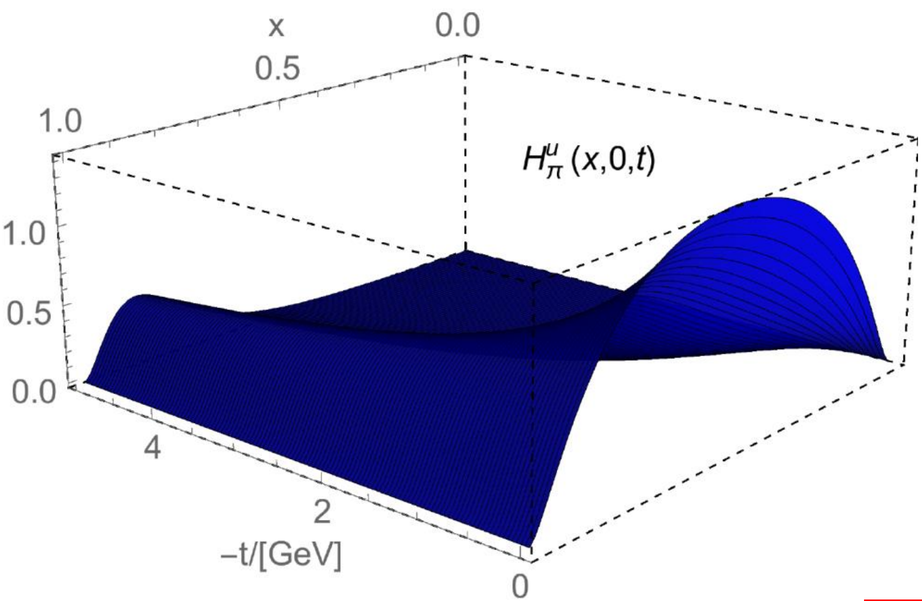
- In principle, $\Theta_1(t)$ can be obtained from the expression:

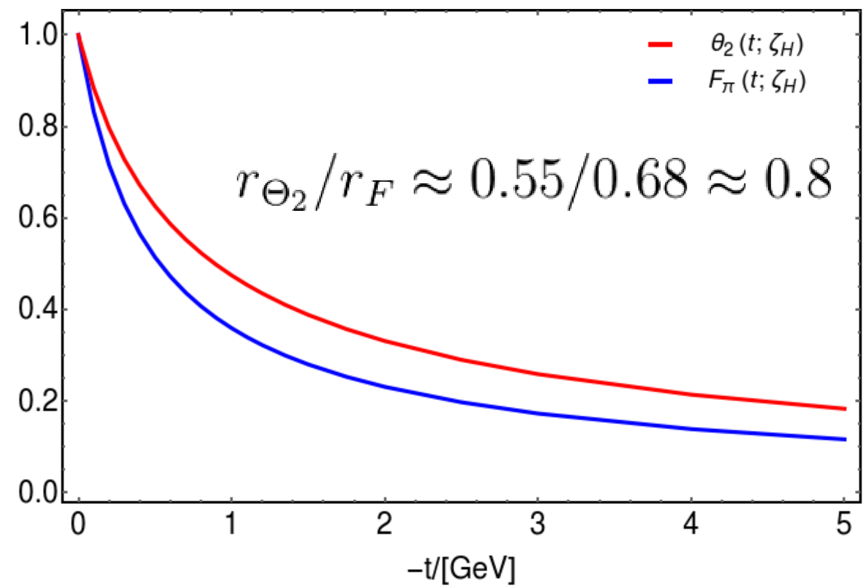
$$\Delta_J(t,\xi) \equiv J(t,0) - J(t,\xi) = \xi^2\Theta_1(t)$$

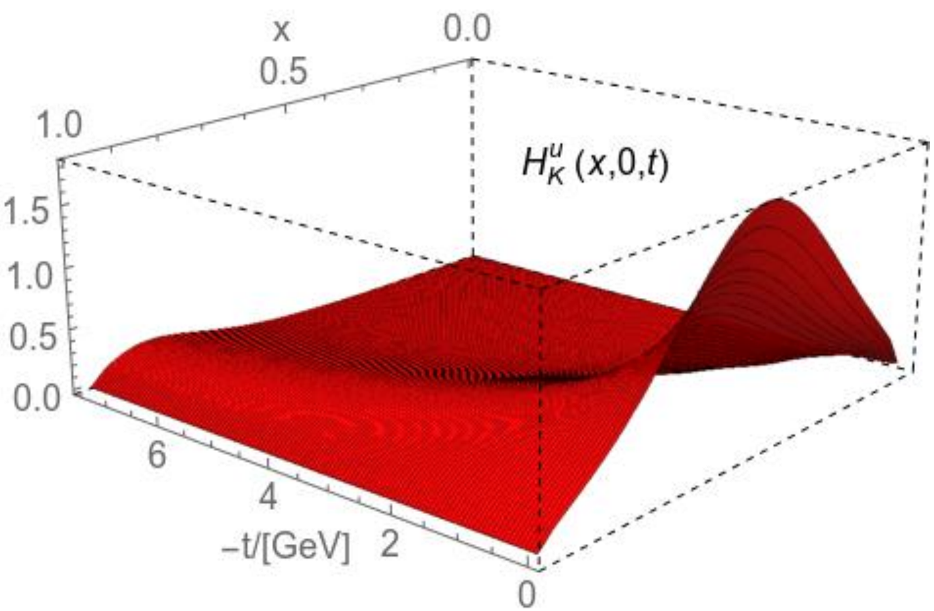
- In practice, it doesn't. Even if we take the limit $\xi^2 \to 0$

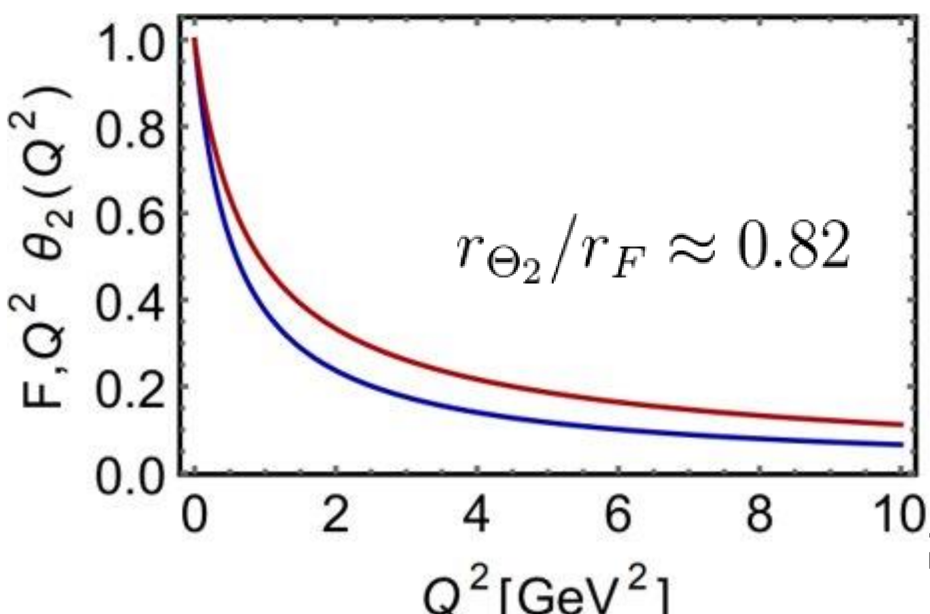
GPDs → EFFs

'Off-forward' moments

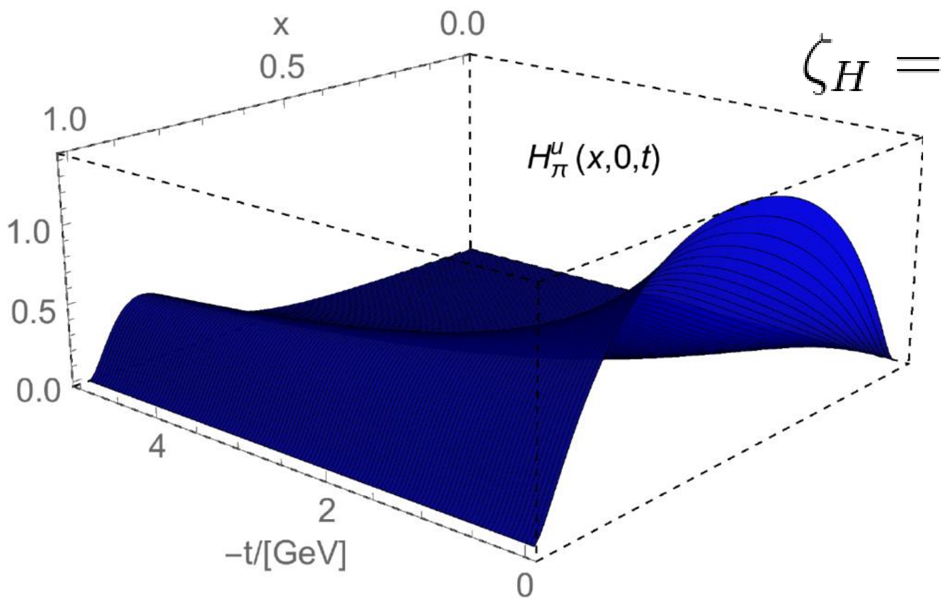




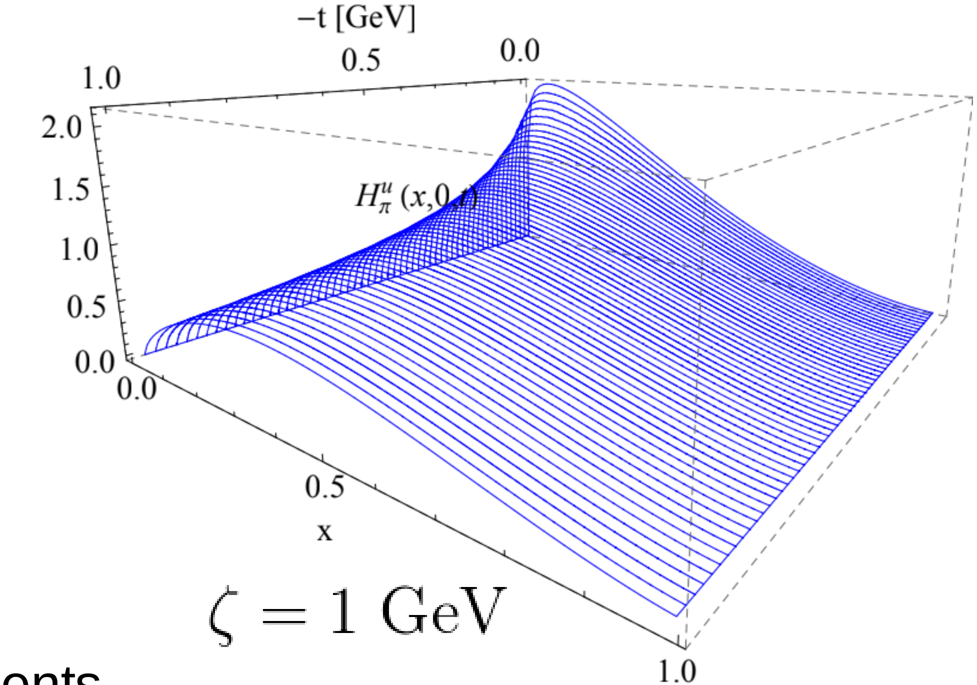




Evolved GPD



$\zeta_H = 0.3\ \mathrm{GeV}$



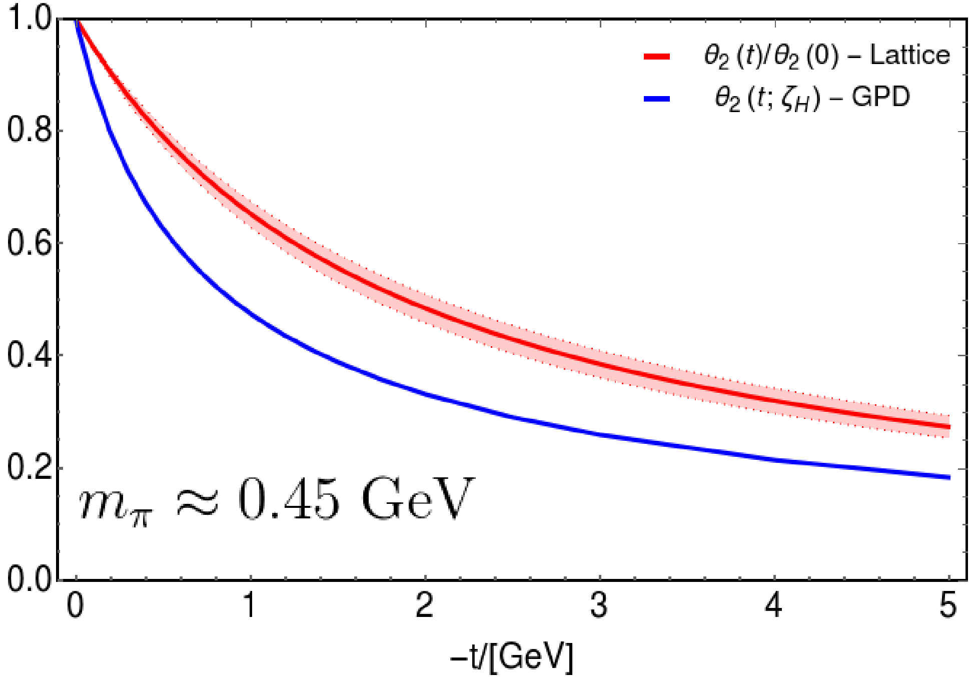
$\zeta = 1\ \mathrm{GeV}$

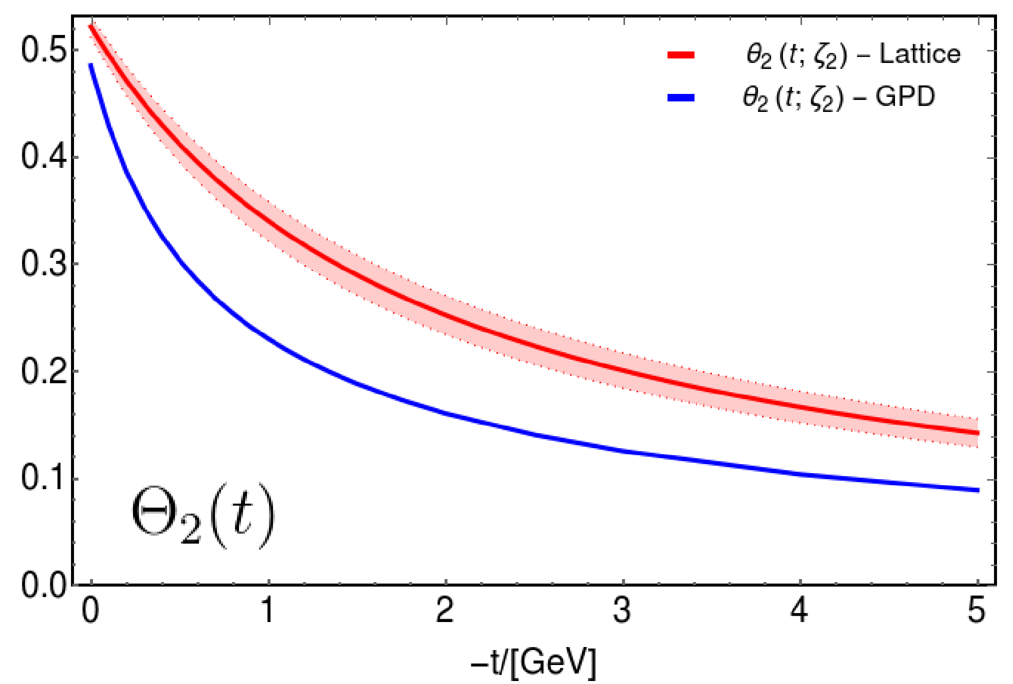
DGLAP evolution:

- No affects 0-th moment (EFF)
- **Important to compare** other moments (GFFs, PDFs)
- **Crucial** to incorporate sea-glue contributions.

Gravitational Form Factors

- In order to compare with lattice available results, we evolve $\Theta_2(t)$ from the hadron scale to 2 GeV.



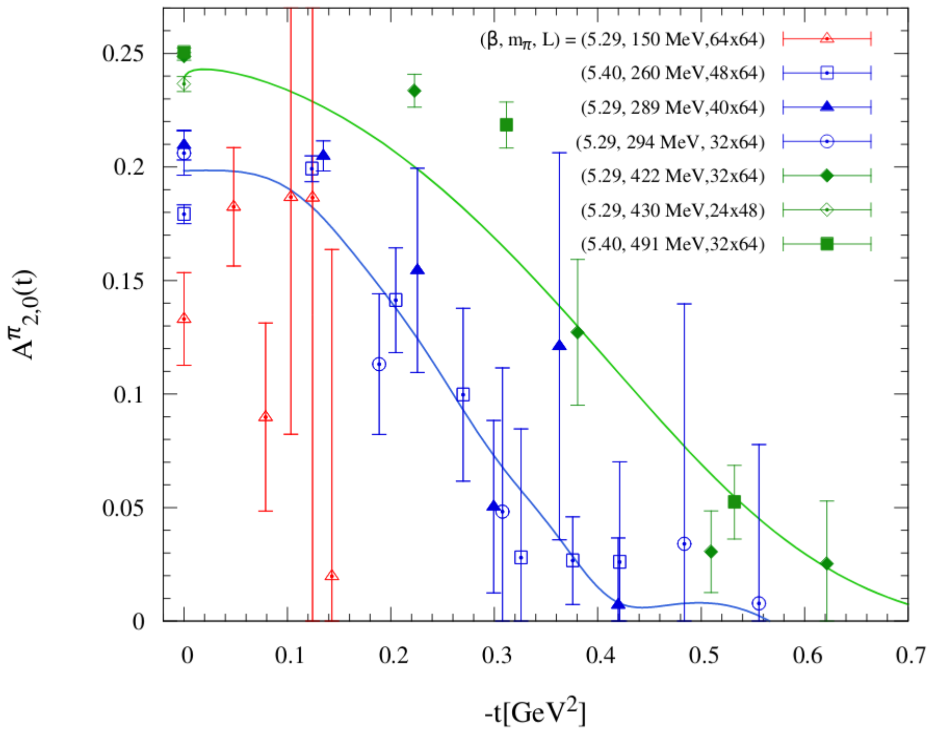


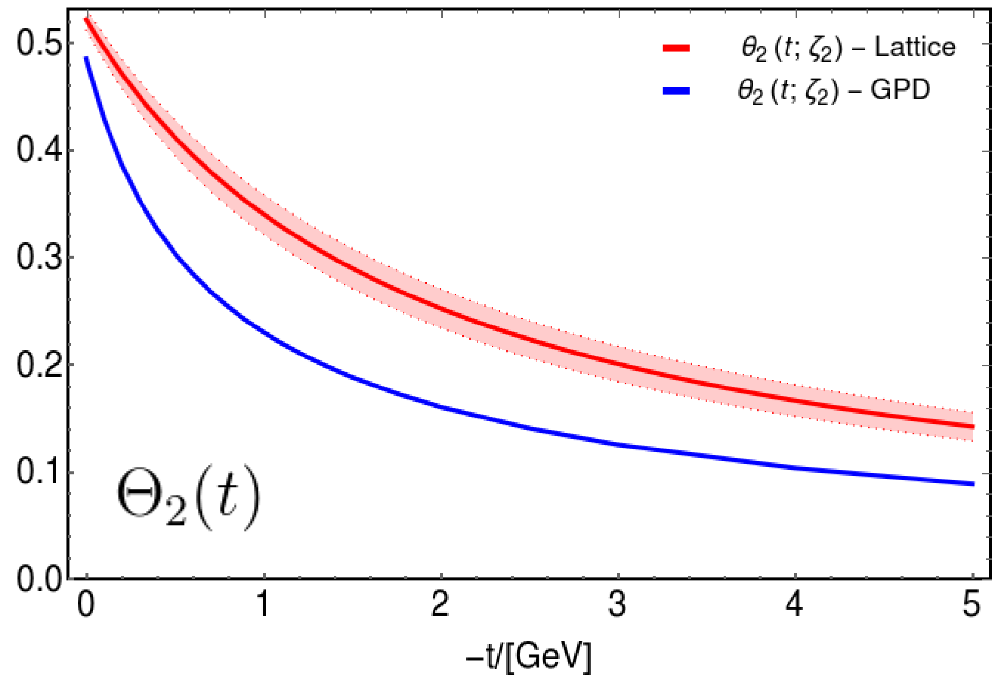
Lattice: (2007) Brömmel's dissertation. $\Theta_2(0)/2 = < x > = 0.261(5)$

GPD + Ding et al. $\Theta_2(0)/2 = < x > = 0.242(20)$

Gravitational Form Factors

- We need to understand the meson mass dependence.





Lattice: G. Bali et al. (2013)

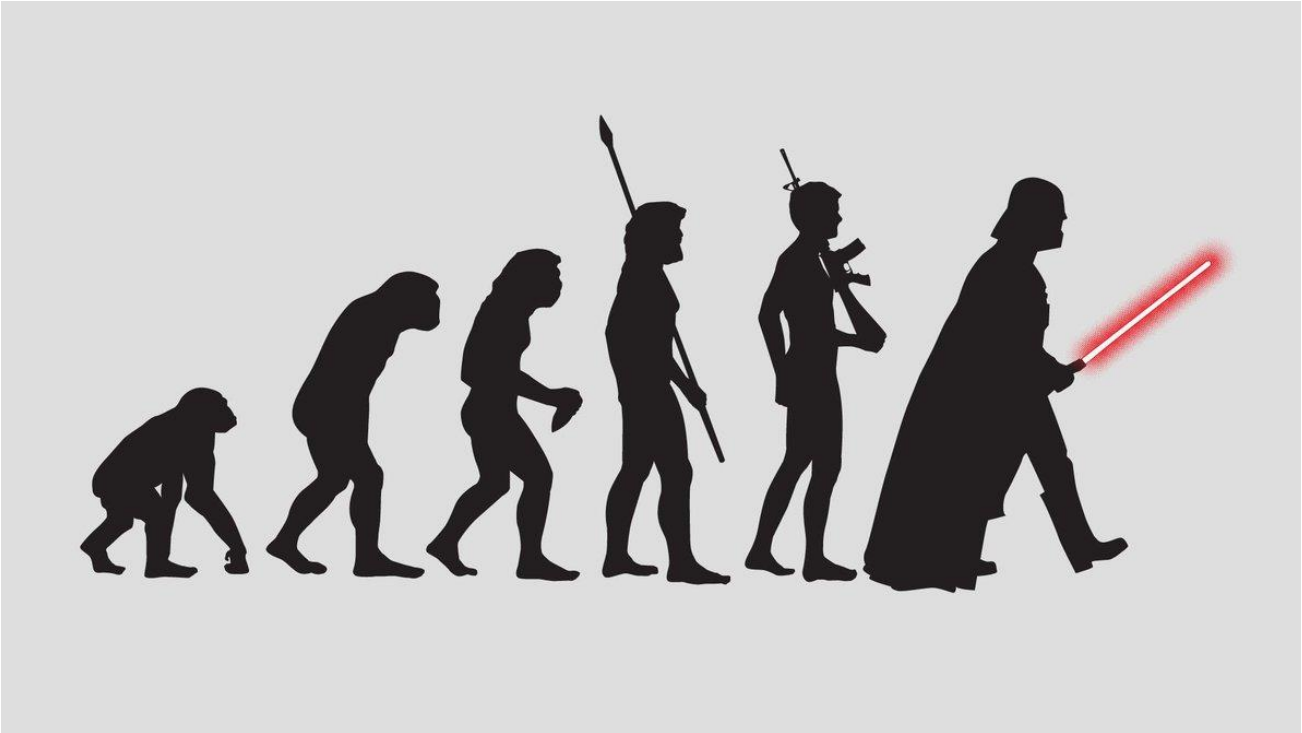
Lattice: (2007) Brömmel's dissertation.

GPD/Ding et al.

Lattice average/GPD/DSE: $\Theta_2(0)/2 = A_{20}(t) = 0.24(2)$

See Pepe's talk

(QCD) Evolution

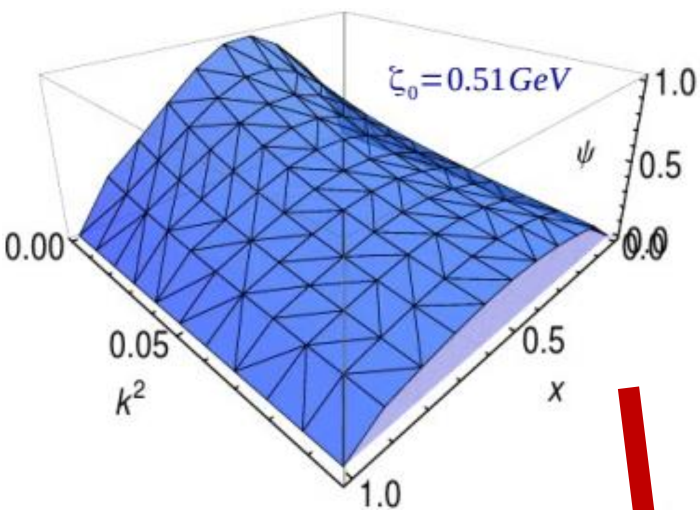


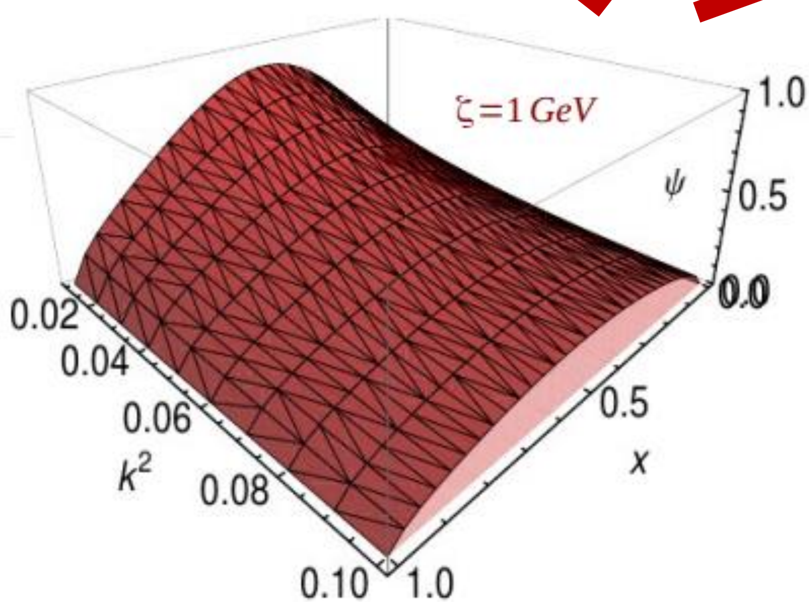
If the LFWF evolves as the PDA..., a.k.a. it factorises as

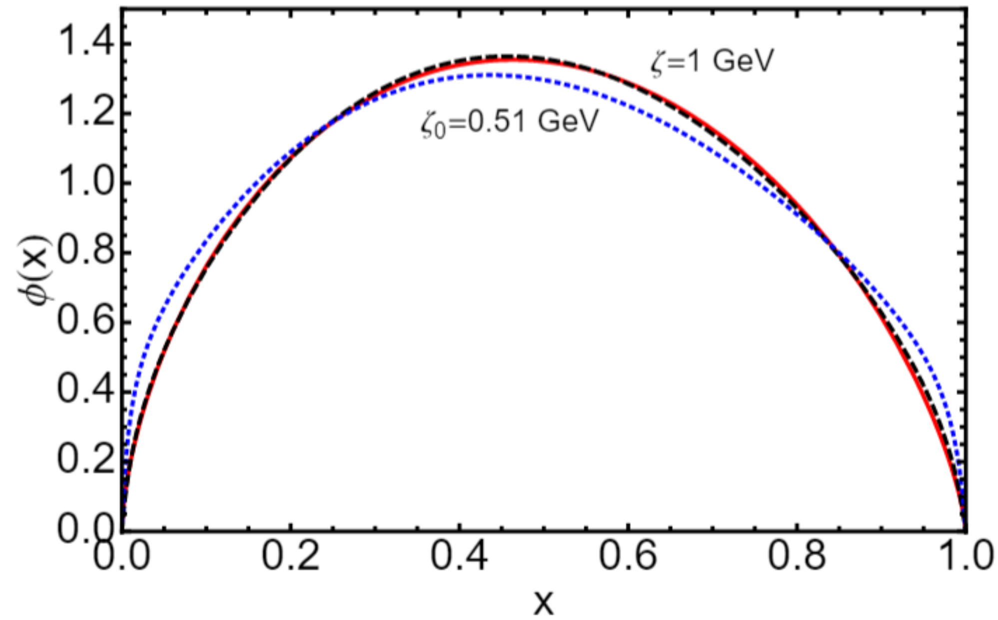
$$\psi(x, k_\perp^2; \zeta) \equiv \phi(x; \zeta)\, \chi(k_\perp^2)$$

Proven sufficiently valid in Phys.Rev. D97 (2018) no.9, 094014. S-S Xu et al.

LFWF and PDA evolution





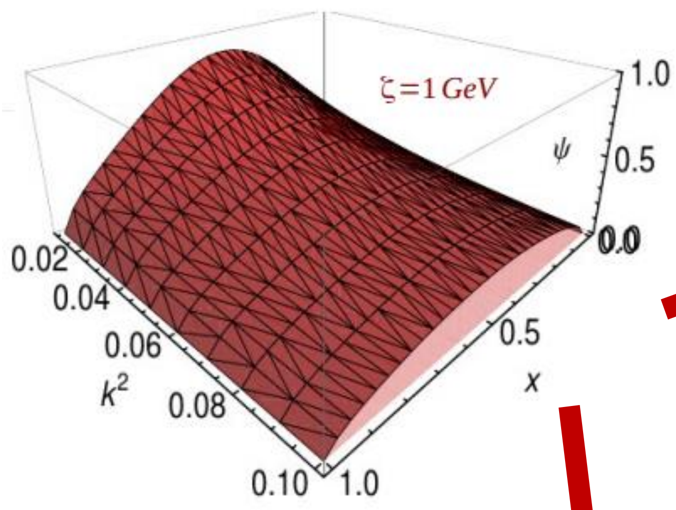


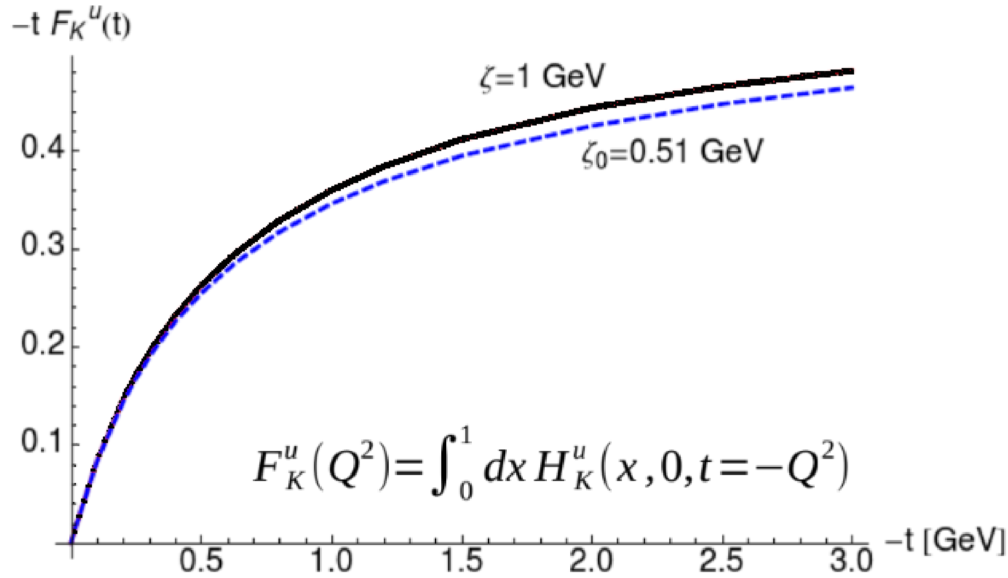
1) Compute LFWF and ERBL running of PDA

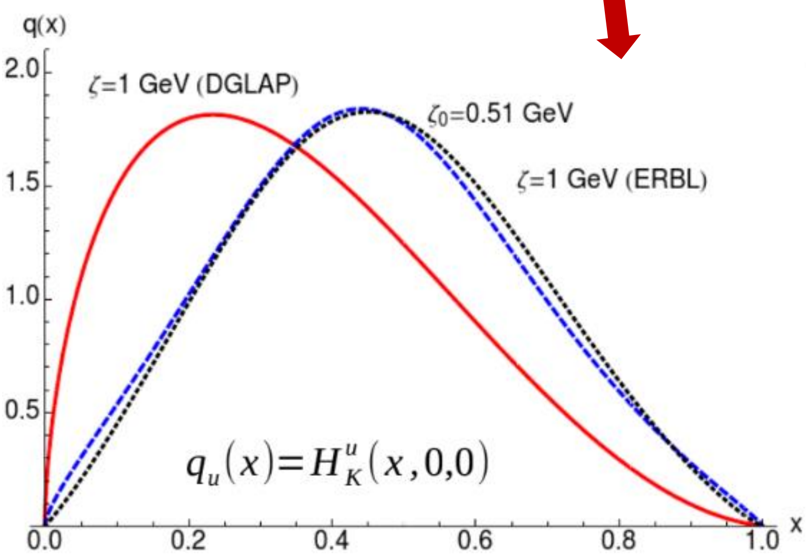
2) ERBL running of LFWF and compute PDA

Notably, **1)** and **2)** are **equivalent**. Factorization assumption and evolution seem reasonable.

LFWF and PDA evolution







1) Obtained from ERBL evolution of LFWF

2) Obtained from DGLAP evolution of GPD

Clearly, **1)** and **2)** are **not equivalent**.

LFWF is purely q-qbar!!!

See Pepe's talk

Gravitational Form Factors

- Pion gravitational form factors are defined as:

$$J_{\pi^+}(-t,\xi) \equiv \int_{-1}^{1} dx\; x H_{\pi^+}(x,\xi,t) = \Theta_2(t) - \Theta_1(t)\xi^2 \;.$$

- At zero skewness, one can readily compute

$$\Theta_2(t) = \int_0^1 dx\; x[H^u_{\pi^+}(x,0,t) + H^d_{\pi^+}(x,0,t)] = \int_0^1 dx\; 2x H^u_{\pi^+}(x,0,t) \;.$$

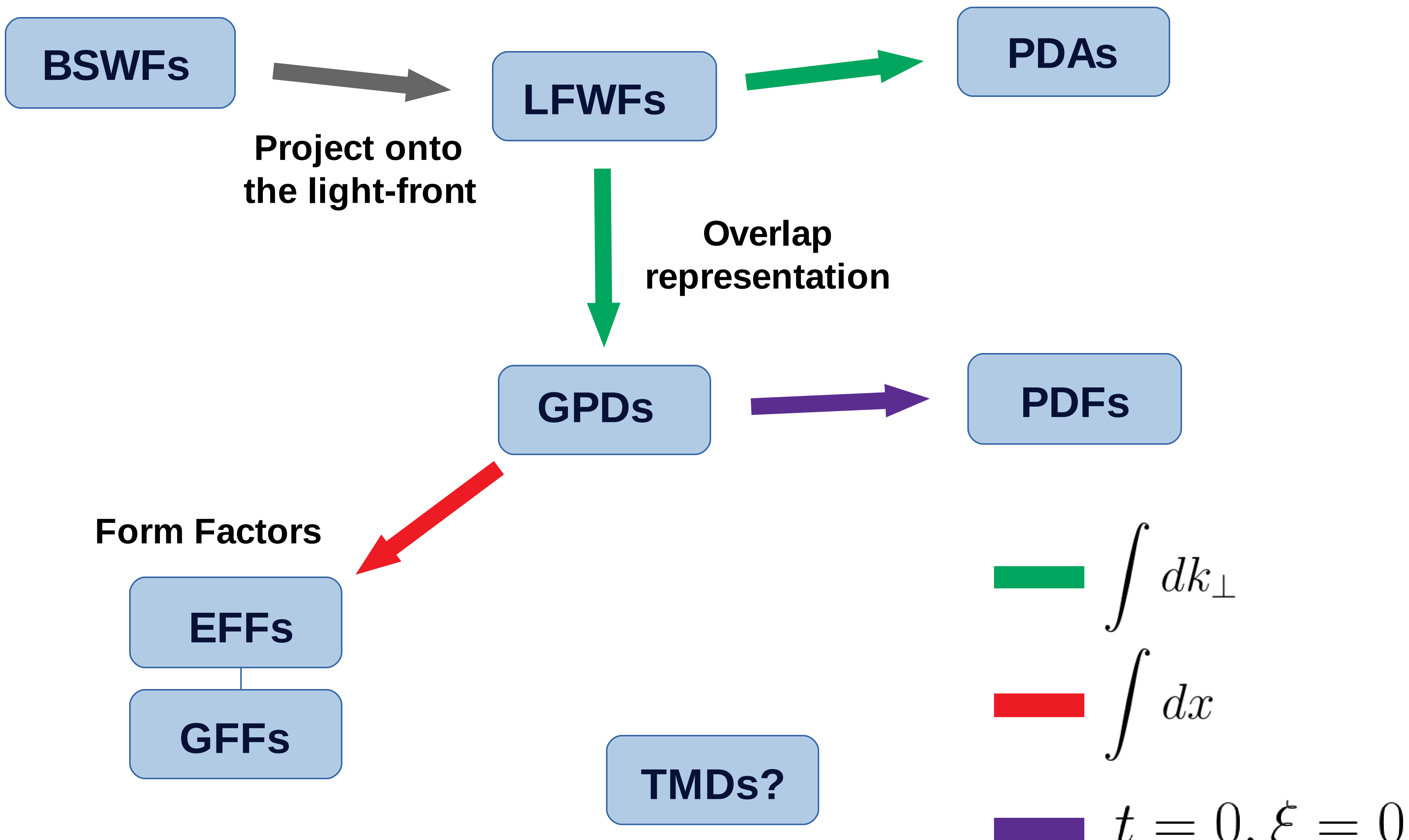
- In principle, $\Theta_1(t)$ can be obtained from the expression:

$$\Delta_J(t,\xi) \equiv J(t,0) - J(t,\xi) = \xi^2\Theta_1(t)$$

- In practice, it doesn't. Even if we take the limit $\xi^2 \to 0$, the missing part of the ERBL region is necessary.

See Pepe's talk

The idea...


BSWFs
Project onto the light-front
LFWFs
PDAs
Overlap representation
GPDs
PDFs
Form Factors
EFFs
GFFs
TMDs?
$\int dk_\perp$
$\int dx$
$t = 0, \xi = 0$

Remarks

- Employing a **QCD-connected** algebraic model, **we compute** pion and kaon LFWF and **all sorts of distributions**:
  - Valence-quark distribution amplitudes
  - Parton distribution functions
  - Electromagnetic and gravitational form factors
  - Generalized parton distributions

- Our results exhibits **keen agreement** with modern computations/predictions, when available, **with DSEs** and **experiments**.
- In particular, the pion **parton distrubution** functions **from DSEs** are **accurately reproduced**, while also being in agreement with the experiment.
- Thus, this work represents a big step towards the computation of LFWF and GPDs **from realistic solutions** of the DSE and Bethe-Salpeter equations.

To do...

- In order to compute $\Theta_1(t)$. One must **obtain** the **ERBL GPD** [numerically demanding] .. **or**... **invent** some consistent model.

- **Refine** the **kaon model** **and** the DGLAP **evolution**. This could be a benchmark for the realistic computation of the kaon PDF.

- In general, the **mass dependence** of the PDF.

- Eventually, **calculate** the **pion LFWF from** the actual **solutions** of the quark propagator and BSE. (Most of the numerical machinery is ready).

- **Eye-catching plots** of the GPD: evolved GPD, impact parameter GPD and distributions.