Spectrum of Hadrons with heavy quarks from Contact Interaction

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Continuum Functional Methods for QCD at New Generation Facilities
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Outline

1 Introduction
- QCD
- DSEs

2 Masses of ground-state mesons and baryons
- Contact Interaction
- Meson
- Baryon

3 Summary

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3 Summary

- In 1961, Gell-Mann and Ne'eman developed *eightfold way* to classify a large number of light hadrons.
- In 1964, Gell-Mann and Zweig proposed *quark model* to explain the classification scheme for hadrons.
- In 1964, Greenberg and Han-Nambu introduced *color* degree of freedom to obtain an antisymmetric wave function for Δ^{++}.
- In 1973, Gross, Politzer and Wilczek discovered *asymptotic freedom* and QCD was formally established.

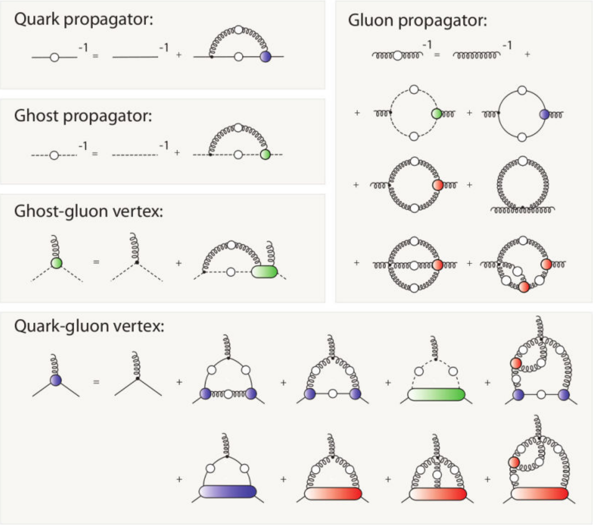
QCD

- At large momentum transfers, the interactions between quarks can be well described with perturbative calculations.
- But at small momentum transfers, the perturbation theory is not valid and QCD is phenomenology governed by DCSB and confinement.
- Unraveling the nonperturbative properties of QCD in low energy regime occupies a central place in nuclear physics.
- Mesons and baryons provide a platform to understand the nonperturbative properties of QCD.

DSEs

- DSEs are the equations of motion for a quantum field theory.
  - an infinite tower of coupled integral equations.
  - a solution is only possible after a symmetry preserving truncation.
- Some key features of the DSEs.
  - treats hadrons as bound states of quarks and gluons.
  - Poincare covariance.
  - renormalizable.
  - exhibits DCSB and confinement.
- provides a nonperturbative and continuum approach to QCD.

DSEs


Quark propagator:
Ghost propagator:
Ghost-gluon vertex:
Gluon propagator:
Quark-gluon vertex:

- Axial-vector Ward-Takahashi identity encodes the chiral symmetry properties of QCD and relates the kernel in the meson BSE to that in the quark DSE.
- In studies of the hadron spectrum, it is critical that a computational approach satisfy this identity.
- In the chiral limit, it reads

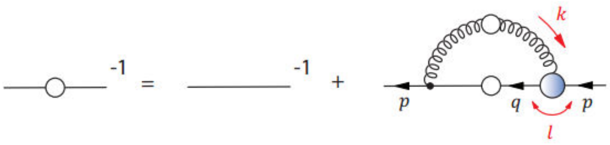
$$P_\mu \Gamma_{5\mu}(k, P) = S^{-1}(k_+) i\gamma_5 + i\gamma_5 S^{-1}(k_-). \quad (1)$$

- The axial-vector vertex is determined by

$$\Gamma_{5\mu}(k; P) = \gamma_5\gamma_\mu + \int_q K(k, q, P) S(q_+) \Gamma_{5\mu}(q, P) S(q_-). \quad (2)$$

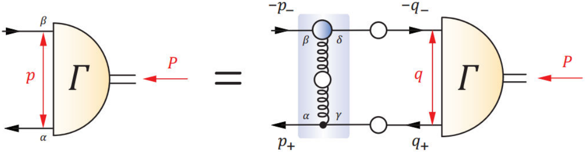
- The Gap equation describes quark propagator that is a basic quantity that appears in any of the bound-state equations.
- Quark Gap equation:

$$S(p)^{-1} = Z_2(i\not{p} + m_0) + \frac{4g^2}{3} Z_1 \int_q D_{\mu\nu}(k)\gamma_\mu S(q)\Gamma_\nu(q,p). \quad (3)$$



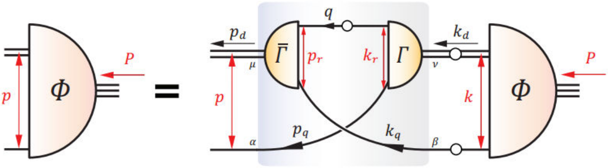
- The BSE describes the meson as a bound state of a quark and an antiquark.
- Meson BSE:

$$\Gamma(p,P)=\int_q K(p,q,P)S(q_+)\Gamma(q,P)S(q_-). \quad (4)$$



- The Faddeev equation describes the baryon as a bound state of three spin-1/2 valence quarks where the interaction kernel comprises two- and three-quark contributions.
- Baryon Faddeev equation (quark-diquark picture):

$$\Phi(p,P) = \int_k K(p,k,P)S(k_+)\Phi(k,P)D(k_-). \tag{5}$$



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- A contact interaction model mediated by a vector-vector interaction is employed.
- This model provides a simple scheme to explore the DCSB and its consequences like:
  - Dynamical mass generation.
  - Spectrum of hadrons.
  - Electromagnetic elastic and transition form factors.
- Series of work:
  - Pion form factor from a contact interaction, L. X. Gutiérrez-Guerrero et al., Phys. Rev. C 81, 065202 (2010).

- - Masses of Ground- and Excited-State Hadrons, H. L. L. Roberts et al., Few-Body Syst. 5:11-25 (2011).
  - Spectrum of Hadrons with Strangeness, C. Chen et al., Few-Body Syst. 53:293C326 (2012).
  - Nucleon and Roper Electromagnetic Elastic and Transition Form Factors, D. J. Wilson et al., Phys. Rev. C 85. 3, 025205 (2012).
  - Electric dipole moment of the ρ meson, M. Pitschmann et al., Phys. Rev. C 87, 015205 (2013).
  - Elastic and Transition Form Factors of the $\Delta(1232)$, J. Segovia et al., Few-Body Syst. 55:1C33 (2014).

- Gluon propagator:

$$g^2 D_{\mu\nu}(p-q) = \delta_{\mu\nu} \frac{4\pi\alpha_{\rm IR}}{m_G^2}. \tag{6}$$

- Quark-gluon vertex:

$$\Gamma_\nu(p,q) = \gamma_\nu. \tag{7}$$

- Quark-antiquark kernel:

$$K(p,q,P) = -\frac{16\pi\alpha_{\rm IR}}{m_G^2}\gamma_\mu\gamma_\mu. \tag{8}$$

- Axial-Vector Ward-Takahashi identity:

$$0 \;=\; \int \frac{d^4q}{(2\pi)^4}\left[\frac{P\cdot q_+}{q_+^2+M^2} - \frac{P\cdot q}{q^2+M^2}\right], \tag{9}$$

- Gap equation:

$$
\begin{aligned}
S(p)^{-1} &= i\gamma\cdot p + m + \frac{16\pi\alpha_{\rm IR}}{3m_G^2}\int\frac{d^4q}{(2\pi)^4}\,\gamma_\mu\,S(q)\,\gamma_\mu \\
S(p)^{-1} &= i\gamma\cdot p + M. \qquad (10)
\end{aligned}
$$

- Proper time regularization:

$$
\begin{aligned}
\frac{1}{s+M^2} &= \int_0^\infty d\tau\,{\rm e}^{-\tau(s+M^2)} \\
&\to \int_{\tau_{\rm uv}^2}^{\tau_{\rm ir}^2} d\tau\,{\rm e}^{-\tau(s+M^2)} \\
&= \frac{{\rm e}^{-(s+M^2)\tau_{\rm uv}^2}-{\rm e}^{-(s+M^2)\tau_{\rm ir}^2}}{s+M^2}. \qquad (11)
\end{aligned}
$$

- BSE:

$$\begin{aligned}
\Gamma_{f\bar{g}}(k,P) &= -\frac{16\pi\alpha_{\rm IR}}{3m_G^2}\int_q \gamma_\mu S_f(q+P)\Gamma_{f\bar{g}}(q,P)S_g(q)\gamma_\mu \\
\Gamma_{0^-}(P) &= i\gamma_5\, E_{0^-}(P) + \frac{1}{2M_R}\gamma_5\gamma\cdot P\, F_{0^-}(P) \\
\Gamma_\mu^{1^-}(P) &= \gamma_\mu^\perp E_{1^-}(P) \qquad (12)
\end{aligned}$$

- Normalization condition:

$$\begin{aligned}
\mathcal{N}^2 P_\mu &= N_c \int_q {\rm tr}\left[\Gamma_{0^-}(-P)\frac{\partial}{\partial P_\mu}S_f(q+P)\Gamma_{0^-}(P)S_g(q)\right] \\
P_\mu &= \frac{N_c}{3}\int_q {\rm tr}[\Gamma_\alpha^{1^-}(-P)\frac{\partial}{\partial P_\mu}S_f(q+P)\Gamma_\alpha^{1^-}(P)S_g(q)] \\
&. \qquad (13)
\end{aligned}$$

- Decay constants:

$$f_{0^-} P_\mu = N_c \int \frac{\mathrm{d}^4 q}{(2\pi)^4} \mathrm{tr}[\gamma_5 \gamma_\mu S_f(q+P) \Gamma_{0^-}(P) S_g(q)]$$

$$f_{1^-} m_{1^-} = \frac{N_c}{3} \int \frac{\mathrm{d}^4 q}{(2\pi)^4} \mathrm{tr}[\gamma_\mu S_f(q+P) \Gamma^{1^-}_\mu(P) S_g(q)]. \quad (14)$$

- Model parameters:
  - Light meson:

Meson	α_{IR}	Λ_{uv}	m	M	m_{0^-}	f_{0^-}
$\pi(u\bar{d})$	0.93π	0.905	0.007	0.367	0.14	0.10
$K(u\bar{s})$	0.93π	0.905	0.17	0.533	0.50	0.11

  - Heavy meson:

$$\alpha'_{\mathrm{IR}} \Lambda'^2_{\mathrm{uv}} \ln \frac{\Lambda'_{\mathrm{uv}}}{\Lambda_{\mathrm{ir}}} = \alpha_{\mathrm{IR}} \Lambda^2_{\mathrm{uv}} \ln \frac{\Lambda_{\mathrm{uv}}}{\Lambda_{\mathrm{ir}}}. \quad (15)$$

- Heavy meson:

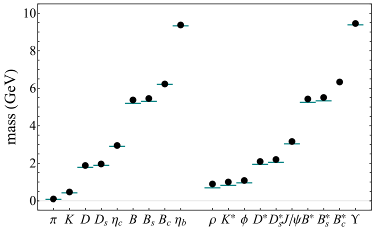
Meson	$\alpha_{\rm IR}$	$\Lambda_{\rm uv}$	m	M	m_{0^-}	f_{0^-}
$\eta_c(c\bar{c})$	0.438	1.878	1.235	1.603	2.98	0.24
$\eta_b(b\bar{b})$	0.097	3.495	4.669	4.829	9.40	0.41

- Light-Heavy meson:

$$\alpha_{\rm IR}(m_{0^-}) = 0.149/\ln[1.042 + 0.041 m_{0^-}^2]. \qquad (16)$$

Meson

- The masses of pseudoscalar and vector mesons:



- The mean relative-difference between theory and experiment or lQCD is just 5(5)%.
- The computed masses fit neatly within a pattern prescribed by ESRs.

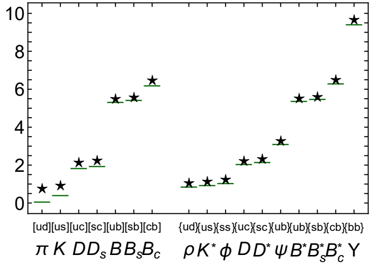
- Diquark BSE:

$$\begin{aligned}
\Gamma^{C}_{f\bar{g}}(k,P) &= -\frac{1}{2}\frac{16\pi\alpha_{\rm IR}}{3m_G^2}\int_q \gamma_\mu S_f(q+P)\Gamma^{C}_{f\bar{g}}(q,P)S_g(q)\gamma_\mu \\
\Gamma^{C}_{0^+}(P) &= i\gamma_5\, E_{0^+}(P) + \frac{1}{2M_R}\gamma_5\gamma\cdot P\, F_{0^+}(P) \\
\Gamma^{C1^+}_{\mu}(P) &= \gamma^{\perp}_{\mu} E_{1^+}(P) \qquad (17)
\end{aligned}$$

- Normalization condition:

$$\begin{aligned}
\mathcal{N}^2 P_\mu &= \frac{2}{3}N_c\int_q {\rm tr}\left[\Gamma^{C}_{0^+}(-P)\frac{\partial}{\partial P_\mu}S_f(q+P)\Gamma^{C}_{0^+}(P)S_g(q)\right] \\
P_\mu &= \frac{2}{3}\frac{N_c}{3}\int_q {\rm tr}[\Gamma^{C1^+}_{\alpha}(-P)\frac{\partial}{\partial P_\mu}S_f(q+P)\Gamma^{C1^+}_{\alpha}(P)S_g(q)] \\
&. \qquad (18)
\end{aligned}$$

- The masses of scalar and axial-vector diquarks:



- The level ordering of diquark correlations is precisely the same as that for mesons.
- The meson mass bounds the partner diquark's mass from below.

- Faddeev equation:

$$\Phi(P) = \frac{4g_B^2}{M}\int_q \tau\Gamma(K)\tau^{\mathrm{T}}\bar{\Gamma}(-K)S(q)\Delta(K)\Phi(P). \tag{19}$$

  - Scalar diquark:

$$\begin{aligned} \Phi(P) &= s(P)I_D \\ \Delta(K) &= \frac{1}{K^2+m_{0^+}^2}. \end{aligned} \tag{20}$$

  - Axial-vector diquark:

$$\begin{aligned} \Phi_\mu(P) &= a_1(P)i\gamma_5\gamma_\mu + a_2(P)\gamma_5\hat{P}_\mu \\ \Delta_{\mu\nu}(K) &= \frac{1}{K^2+m_{1^+}^2}\left(\delta_{\mu\nu}+\frac{K_\mu K_\nu}{m_{1^+}^2}\right). \end{aligned} \tag{21}$$

- The flavour structure of spin-1/2 baryons are

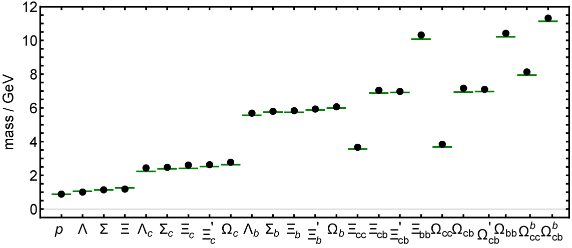
$$
\begin{aligned}
u_p = \begin{bmatrix} [ud]u \\ \{uu\}d \\ \{ud\}u \end{bmatrix}, \qquad & u_{\Sigma^+} = \begin{bmatrix} [us]u \\ \{uu\}s \\ \{us\}u \end{bmatrix}, \\
u_{\Xi^0} = \begin{bmatrix} [us]s \\ \{us\}s \\ \{ss\}u \end{bmatrix}, \quad & u_{\Lambda} = \frac{1}{\sqrt{2}} \begin{bmatrix} \sqrt{2}[ud]s \\ [us]d - [ds]u \\ \{us\}d - \{ds\}u \end{bmatrix}.
\end{aligned} \tag{22}
$$

Baryon

- The flavour structure of spin-3/2 baryons are

$$u_\Delta = \left[\ \{uu\}u\ \right], \quad u_\Omega = \left[\ \{ss\}s\ \right],$$
$$u_{\Sigma^*} = \begin{bmatrix} \{uu\}s \\ \{us\}u \end{bmatrix}, \quad u_{\Xi^*} = \begin{bmatrix} \{us\}s \\ \{ss\}u \end{bmatrix}. \tag{23}$$

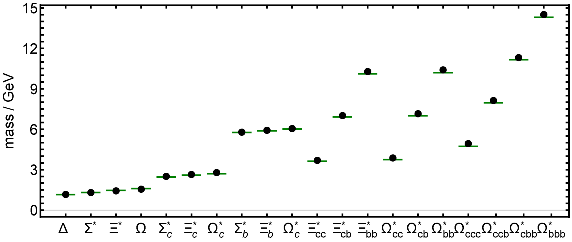
- The masses of spin-1/2 baryons:



- The mean relative-difference between theory and experiment or lQCD is 2.2%.
- This compares well with the fully-covariant three-body calculation.

Baryon

- The masses of spin-3/2 baryons:



- The mean relative-difference between theory and experiment or lQCD is 1.0%.
- Once again, this compares well with the fully-covariant three-body calculation.

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- A symmetry-preserving contact-interaction is employed to calculate spectra of ground-state mesons, diquarks, and baryons.
- The contact-interaction is extend to systems involving heavy quarks by considering the feedback between masses and decay constants.
- The computed masses can be compared well with the fully-covariant three-body calculation.

Thank You !