Toward Baryon Distributions Amplitudes

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May 8^{th}, 2019

In collaboration with:
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Hadrons seen as Fock States

- Lightfront quantization allows to expand hadrons on a Fock basis:

$$|P,\pi\rangle \propto \sum_\beta \Psi_\beta^{q\bar{q}}|q\bar{q}\rangle + \sum_\beta \Psi_\beta^{q\bar{q},q\bar{q}}|q\bar{q},q\bar{q}\rangle + \dots$$

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- Non-perturbative physics is contained in the N-particles Lightfront-Wave Functions (LFWF) Ψ^N
- Schematically a distribution amplitude φ is related to the LFWF through:

$$\varphi(x) \propto \int \frac{\mathrm{d}^2 k_\perp}{(2\pi)^2} \Psi(x, k_\perp)$$

S. Brodsky and G. Lepage, PRD 22, (1980)

Nucleon Distribution Amplitudes

- 3 bodies matrix element:

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$$|P,\uparrow\rangle = \int \frac{[\mathrm{d}x]}{8\sqrt{6x_1x_2x_3}}|uud\rangle \otimes [\varphi(x_1,x_2,x_3)|\uparrow\downarrow\uparrow\rangle$$
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- Isospin symmetry:

$$2T(x_1,x_2,x_3) = \varphi(x_1,x_3,x_2) + \varphi(x_2,x_3,x_1)$$

Evolution and Asymptotic results

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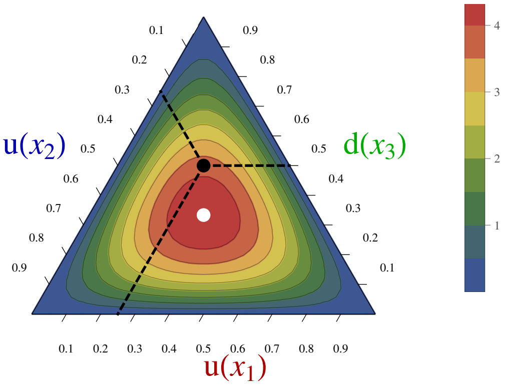
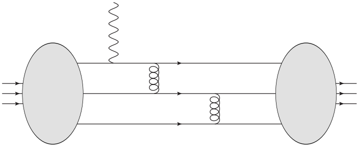
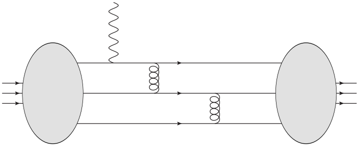
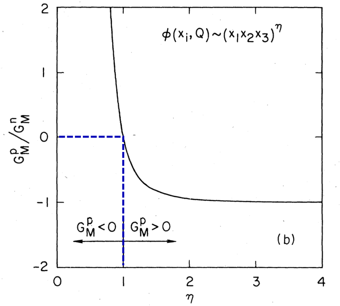


Form Factors: Nucleon case







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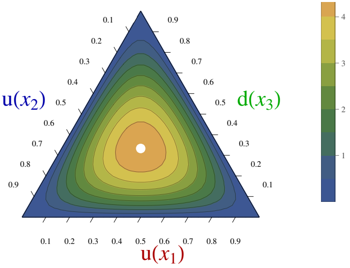
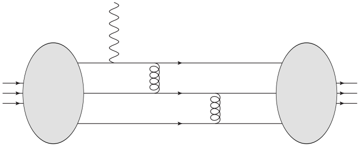
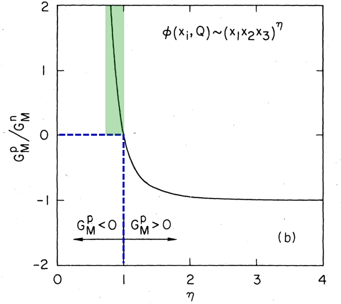


$\eta = 1$





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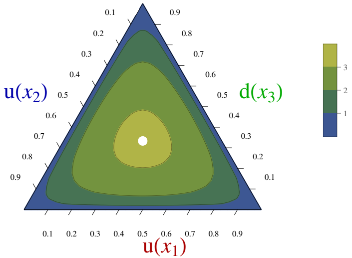
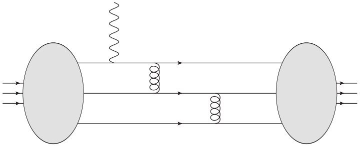
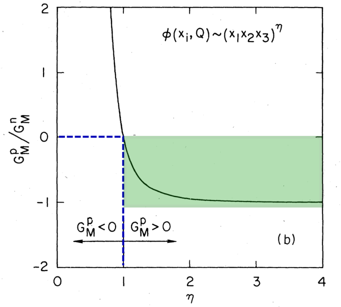
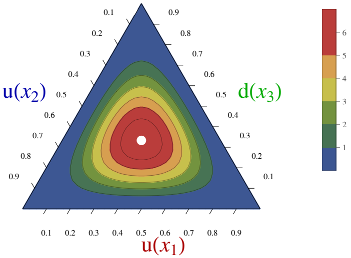


$\eta = 0.5$







$\eta = 2$

S. Brodsky and G. Lepage, PRD 22, (1980)

Some previous studies of DA

- QCD Sum Rules
  - V. Chernyak and I. Zhitnitsky, Nucl. Phys. B 246 (1984)
- Relativistic quark model
  - Z. Dziembowski, PRD 37 (1988)
- Scalar diquark clustering
  - Z. Dziembowski and J. Franklin, PRD 42 (1990)
- Phenomenological fit
  - J. Bolz and P. Kroll, Z. Phys. A 356 (1996)
- Lightcone quark model
  - B. Pasquini *et al.*, PRD 80 (2009)
- Lightcone sum rules
  - I. Anikin *et al.*, PRD 88 (2013)
- Lattice Mellin moment computation
  - G. Bali *et al.*, JHEP 2016 02

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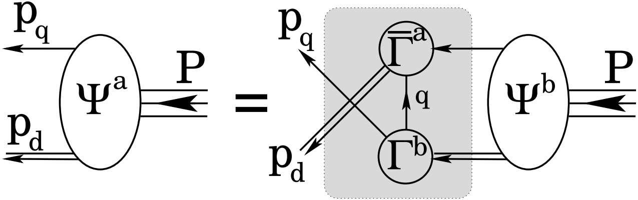
- It predicts the existence of strong diquarks correlations inside the nucleon.



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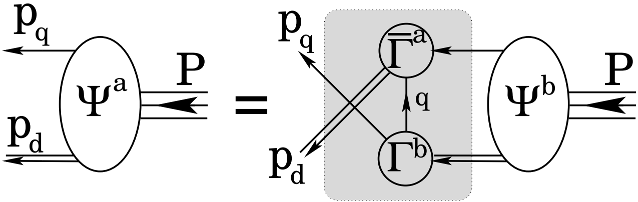

- Mostly two types of diquark are dynamically generated by the Faddeev equation:
  - Scalar diquarks,
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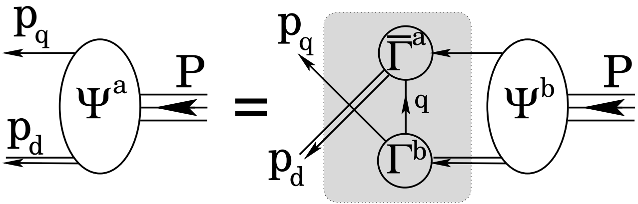

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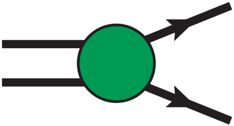
- Can we understand the nucleon structure in terms of quark-diquarks correlations?

Faddeev WF Model

- Algebraic parametrisation inspired by the results obtained from DSEs and Faddeev equations.
- It is based on Nakanishi representation, which is proved to be a good parametrisation of Green functions at all order of perturbation theory.
- We also assume the dynamical diquark correlations, both scalar and AV, and compare in the end with Lattice QCD results.
- This is a work in progress, an update of the previous baryon PDA work toward more realistic results



At all order of perturbation theory, one can write (Euclidean space):

$$\Gamma(k,P) = \mathcal{N}\int_0^\infty \mathrm{d}\gamma \int_{-1}^{1} \mathrm{d}z \frac{\rho_n(\gamma,z)}{(\gamma + (k+\frac{z}{2}P)^2)^n}$$

We use a "simpler" version of the latter as follow:

$$\tilde{\Gamma}(q,P) = \mathcal{N}\int_{-1}^{1} \mathrm{d}z \frac{\rho_n(z)}{(\Lambda^2 + (q+\frac{z}{2}P)^2)^n}$$

Nucleon Distribution Amplitude

- Operator point of view for every DA (and at every twist):

$$\langle 0|\epsilon^{ijk}\left(u_\uparrow^i(z_1)C\not{n}u_\downarrow^j(z_2)\right)\not{n}d_\uparrow^k(z_3)|P,\lambda\rangle \rightarrow \varphi(x_1,x_2,x_3),$$

Braun *et al.*, Nucl.Phys. B589 (2000)

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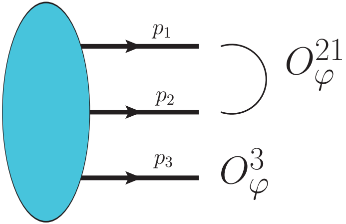

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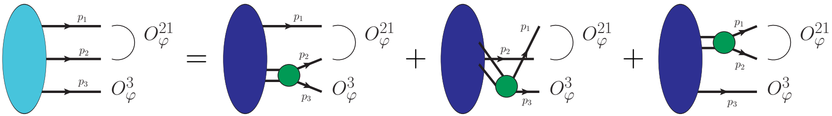

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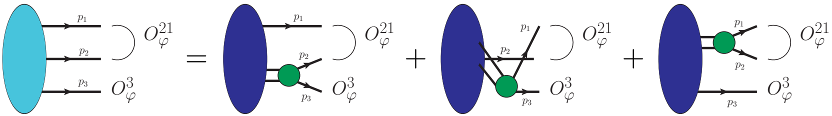

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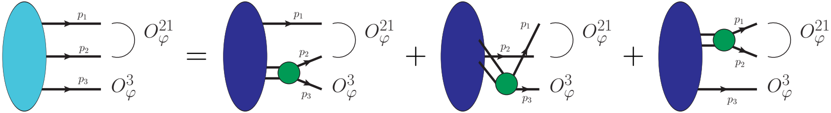
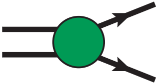


- The operator then selects the relevant component of the wave function.
- Our ingredients are:
  - Perturbative-like quark and diquark propagator
  - Nakanishi based diquark Bethe-Salpeter-like amplitude (green disks)
  - Nakanishi based quark-diquark amplitude (dark blue ellipses)

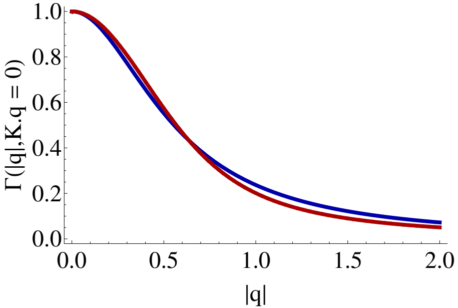
Scalar Diquark BSA

The model used:



$$= \mathcal{N} \int_{-1}^{1} \mathrm{d}z \frac{(1-z^2)}{(\Lambda^2 + (q + \frac{z}{2}K)^2)}$$

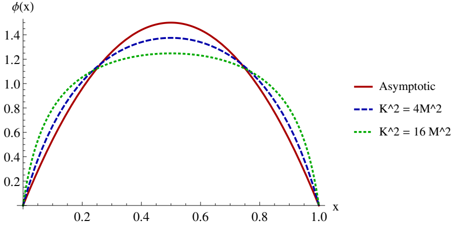
Comparable to scalar diquark amplitude previously used:



red curve from Segovia *et al.*,Few Body Syst. 55 (2014) 1185-1222

$$\phi(x) \propto 1 - \frac{M^2}{K^2}\frac{\ln\left[1+\frac{K^2}{M^2}x(1-x)\right]}{x(1-x)}$$

Scalar diquark



Diquark DA

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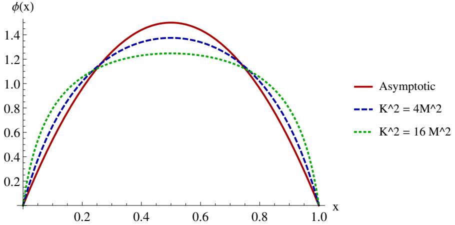
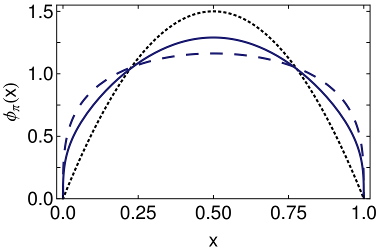


Pion



Pion figure from L. Chang *et al.*, PRL 110 (2013)

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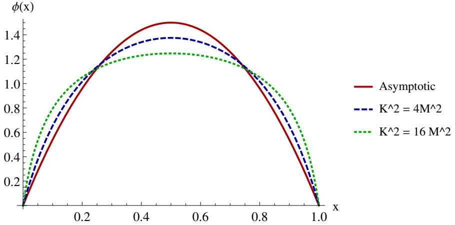

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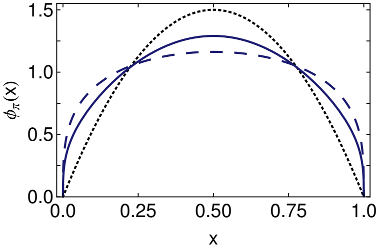
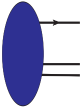


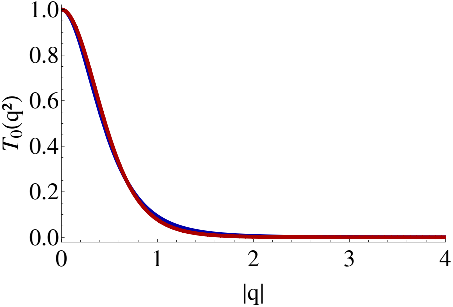
Pion figure from L. Chang *et al.*, PRL 110 (2013)

- This results provide a broad and concave meson DA parametrisation
- The endpoint behaviour remains linear



$$= \mathcal{N}\int_{-1}^{1} \mathrm{d}z \frac{(1-z^2)\tilde{\rho}(z)}{(\Lambda^2+(\ell-\frac{1+3z}{6}P)^2)^3}, \quad \tilde{\rho}(z)=\prod_j (z-a_j)(z-\bar{a}_j)$$

Fits of the parameters through comparison to Chebychev moments:



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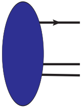

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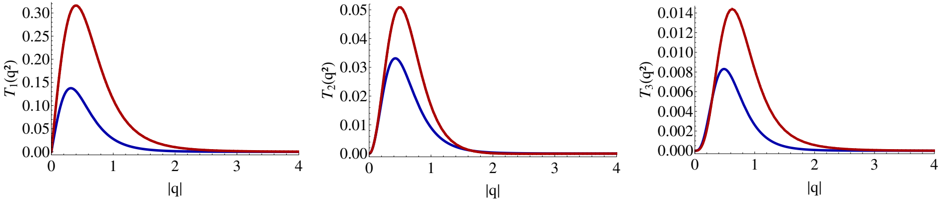
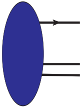


red curves from Segovia *et al.*,

Nucleon Quark-Diquark Amplitude

Scalar diquark case



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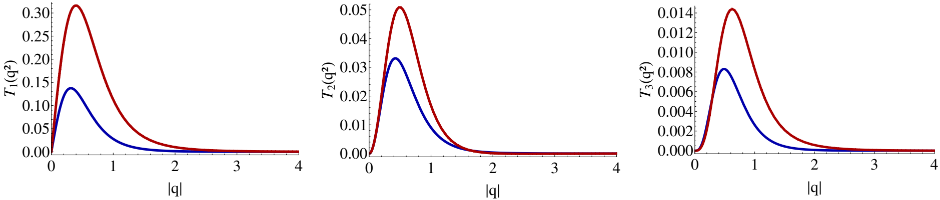

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Modification of the $\tilde{\rho}$ Ansatz ?

Axial-Vector Diquark

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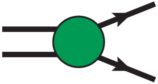

$$= \mathcal{N}\left(\tau_1^\mu + \tau_6^\mu\right) \int_{-1}^{1} \mathrm{d}z \frac{(1-z^2)}{(\lambda_q^2 + (q + \frac{z}{2}K)^2)^\nu}$$

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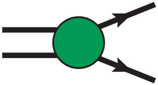

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- τ_1^μ and τ_6^μ contain the leading contributions to the longitudinal and transverse PDAs.

Axial-Vector Diquark

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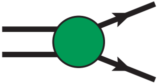

$$= \mathcal{N} \left(\tau_1^\mu + \tau_6^\mu\right) \int_{-1}^{1} \mathrm{d}z \frac{(1-z^2)}{(\lambda_q^2 + (q + \frac{z}{2}K)^2)^\nu}$$

- τ_1^μ and τ_6^μ contain the leading contributions to the longitudinal and transverse PDAs.
- For $\nu \to 1$, the PDA is logarithmically divergent
  Regularisation : $\mathcal{N} \to \mathcal{N}(\nu)$ such that $\int_0^1 \mathrm{d}x \varphi(x) = 1 \ \forall \ \nu$
  - Advantages: simple + keeps only leading contributions
  - Drawbacks: kill the K^2 dependence of the DA

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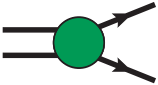

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  - Advantages: simple + keeps only leading contributions
  - Drawbacks: kill the K^2 dependence of the DA
- We therefore add K^2 dependence by hand, by "copying" the scalar diquark result
  - This should be consider as an additional modeling hypothesis
  - Renormalization needs to be performed in the future

Normalisation of diquark BSA

- Canonical normalisation:

$$2K_\mu = \left[\frac{\partial}{\partial Q_\mu}\mathrm{Tr}\left(\int \frac{\mathrm{d}^4 q}{(2\pi)^4}\bar{\Gamma}(q,-K)S(q_+)\Gamma(q,K)S^T(-q_-)\right)\right]_{Q=K}^{K^2=-m_{J^P}^2}$$

$$q_\pm = q \pm \frac{Q}{2}$$

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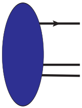
- Lightfront PDA normalisation :

$$K\cdot n = Tr\left[\int \frac{\mathrm{d}q^4}{(2\pi)^4}S(q_+)\Gamma(q,K)S^T(-q_-)O_\varphi\right]_{Q=K}^{K^2=0}$$

We normalise the PDA to be the asympotic one when the diquark is flying along the lightcone.



$$= \sum_{i=1}^{6} \gamma_5 A_i^\mu(\ell, P) p_i(\ell, P)$$

- We keep only two structures, which are the ones independent of ℓ.
  $\rightarrow$ Contact interaction-like tensorial structures.
- Our model is therefore:

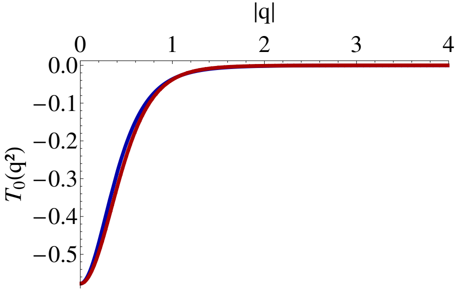
$$\mathcal{A}^\mu = \gamma_5 \left(A_2^\mu \int_{-1}^{1} \mathrm{d}z \frac{(1-z^2)\tilde{\rho}_2(z)}{(\Lambda_2^2 + (\ell - \frac{1+3z}{6}P)^2)^3} + A_5^\mu \int_{-1}^{1} \mathrm{d}z \frac{(1-z^2)\tilde{\rho}_5(z)}{(\Lambda_5^2 + (\ell - \frac{1+3z}{6}P)^2)^3} \right)$$

$$A_2^\mu = -i\frac{P^\mu}{M_N}, \quad A_5^\mu = \gamma^\mu - \frac{\gamma \cdot P P^\mu}{P^2}$$

$$\tilde{\rho}(z) = \prod_j (z - a_j)(z - \bar{a}_j)$$

$$p_5(\ell, P) = \mathcal{N}\int_{-1}^{1} \mathrm{d}z \frac{(1-z^2)\tilde{\rho}_5(z)}{(\Lambda_5^2 + (\ell - \frac{1+3z}{6}P)^2)^3}, \quad \tilde{\rho}_5(z) = \prod_j (z - a_j)(z - \bar{a}_j)$$

Fits of the parameters through comparison to Chebychev moments:



red curve from Segovia *et al.*,

$$p_5(\ell, P) = \mathcal{N} \int_{-1}^{1} \mathrm{d}z \frac{(1-z^2)\tilde{\rho}_5(z)}{(\Lambda_5^2 + (\ell - \frac{1+3z}{6}P)^2)^3}, \quad \tilde{\rho}_5(z) = \prod_j (z-a_j)(z-\bar{a}_j)$$

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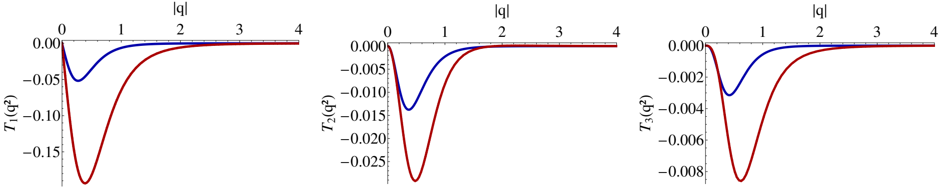

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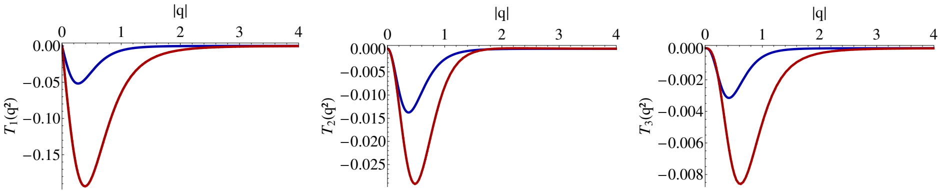
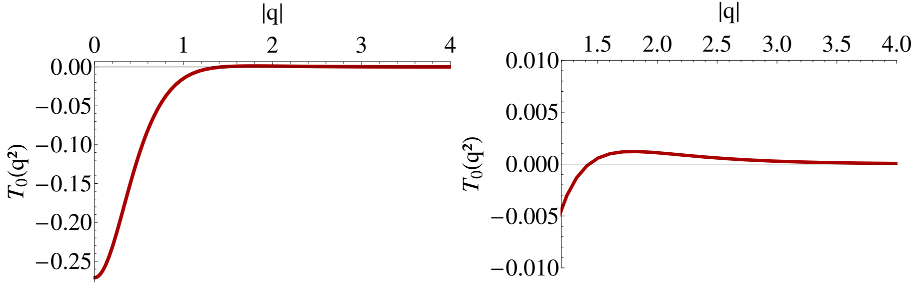


red curves from Segovia *et al.*,

Same issue than in the scalar diquark case.

$$p_2(\ell, P) \stackrel{?}{=} \mathcal{N} \int_{-1}^{1} \mathrm{d}z \frac{(1-z^2)\tilde{\rho}_2(z)}{(\Lambda_2^2 + (\ell - \frac{1+3z}{6}P)^2)^3}, \quad \tilde{\rho}_2(z) \stackrel{?}{=} \prod_j (z - a_j)(z - \bar{a}_j)$$

0^{th} Chebychev moments have a zero crossing (but much farther away than in the Roper case):



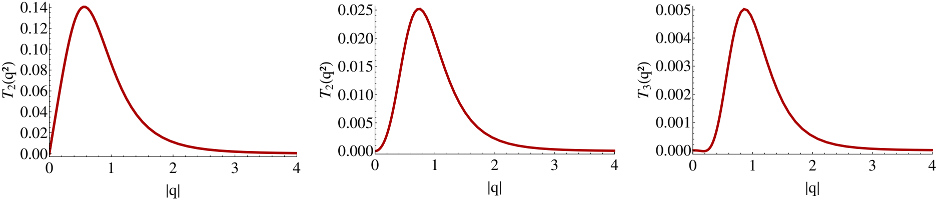
red curve from Segovia *et al.*,

Nucleon Quark-Diquark Amplitude

Axial-Vector Case III

$$p_2(\ell, P) \stackrel{?}{=} \mathcal{N} \int_{-1}^{1} \mathrm{d}z \frac{(1-z^2)\tilde{\rho}_2(z)}{(\Lambda_2^2 + (\ell - \frac{1+3z}{6}P)^2)^3}, \quad \tilde{\rho}_2(z) \stackrel{?}{=} \prod_j (z-a_j)(z-\bar{a}_j)$$

Higer moments have opposite sign wrt the 0^{th}*one*:



red curves from Segovia *et al.*,

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red curves from Segovia *et al.*,

$$\rho_2(z,\gamma) \to \tilde{\rho}_2(z)\delta(\Lambda_{UV} - \gamma) + \tilde{\omega}_2(z)\delta^{(n)}(\Lambda_{IR} - \gamma)$$

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- Work in progress

$$p_2(\ell, P) \stackrel{?}{=} \mathcal{N} \int_{-1}^{1} \mathrm{d}z \frac{(1-z^2)\tilde{\rho}_2(z)}{(\Lambda_2^2 + (\ell - \frac{1+3z}{6}P)^2)^3}, \quad \tilde{\rho}_2(z) \stackrel{?}{=} \prod_j (z-a_j)(z-\bar{a}_j)$$

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$$\rho_2(z,\gamma) \to \tilde{\rho}_2(z)\delta(\Lambda_{UV}-\gamma) + \tilde{\omega}_2(z)\delta^{(n)}(\Lambda_{IR}-\gamma)$$

- Work in progress
- "Back propagate" this modifications to A_5 and S_1

Mellin Moments

- We do not compute the PDA directly but Mellin moments of it:

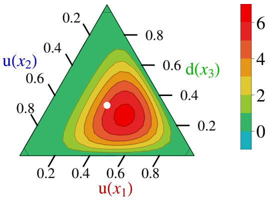
$$\langle x_1^m x_2^n \rangle = \int_0^1 \mathrm{d}x_1 \int_0^{1-x_1} \mathrm{d}x_2 \; x_1^m x_2^n \varphi(x_1, x_2, 1 - x_1 - x_2)$$

- For a general moment $\langle x_1^m x_2^n \rangle$, we change the variable in such a way to right down our moments as:

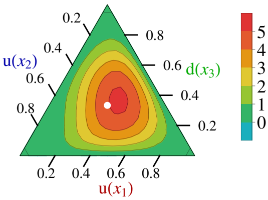
$$\langle x_1^m x_2^n \rangle = \int_0^1 \mathrm{d}\alpha \int_0^{1-\alpha} \mathrm{d}\beta \; \alpha^m \beta^n f(\alpha, \beta)$$

- f is a complicated function involving the integration on 6 parameters
- Uniqueness of the Mellin moments of continuous functions allows us to identify f and φ

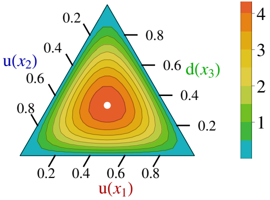
Preliminary Results



Nucleon DA
(Scalar Diquark only)



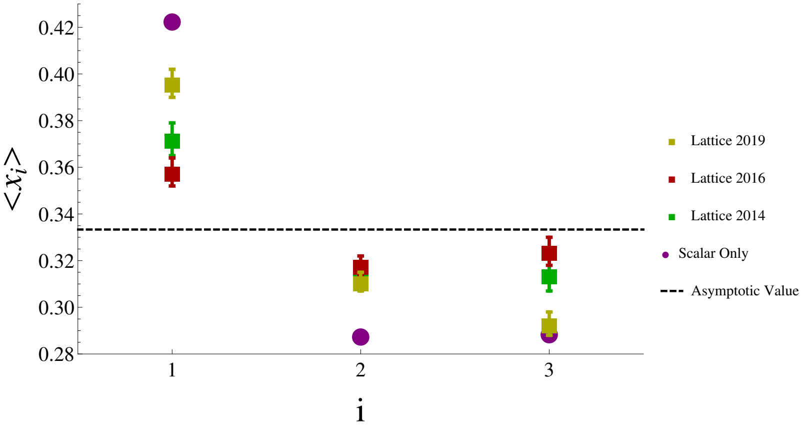
Nucleon DA
S_1 and A_5



Asymptotic DA

- Nucleon DA is skewed compared to the asymptotic one
- These properties are consequences of our quark-diquark picture
- Relative strength : 70% S_1, 30% A_5

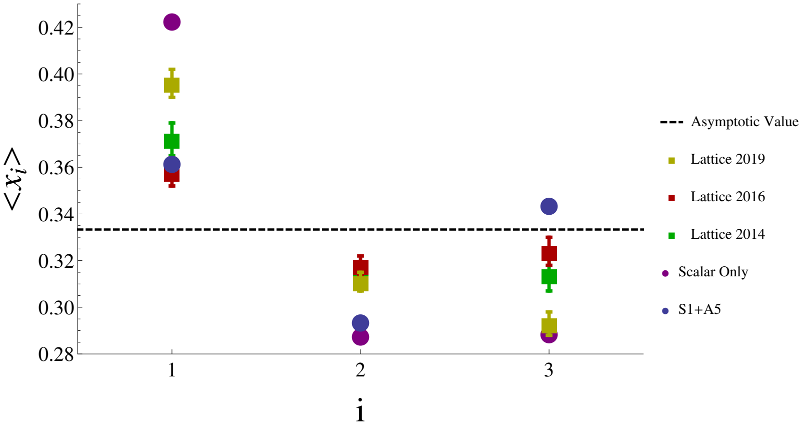
$$< x_i >_\varphi = \int \mathcal{D}x \; x_i \varphi(x_1, x_2, x_3)$$



Lattice data from V.Braun *et al*, PRD 89 (2014)
G. Bali *et al.*, JHEP 2016 02
G. Bali *et al.*, arXiv:1903.12590

Preliminary comparison with lattice

$$< x_i >_\varphi = \int \mathcal{D}x \; x_i \varphi(x_1, x_2, x_3)$$



Lattice data from V.Braun *et al*, PRD 89 (2014)
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Summary

Baryon PDA

- **DSE compatible** framework for Baryon PDAs.
- Based on the Nakanishi representation.
- Preliminary results.

For the Future

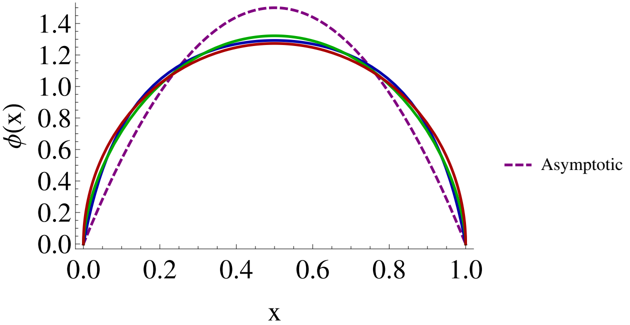
- **Priority**: Stabilise the Nakanishi Ansätze.
- Improvement of our various components (ie propagators, renormalisations)
- Calculation of the Dirac form factor.
- Higher-twist PDA.
- Light-front wave functions.

Addendum:
Meson Form Factors and beyond

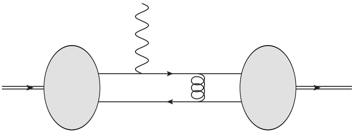
$n = -1$ Mellin Moment

$$\langle x^{-1} \rangle = \int_0^1 \mathrm{d}x \frac{\varphi(x)}{1-x} \qquad \phi_{\ln}(x) \propto 1 - \frac{\ln\left[1 + \kappa x(1-x)\right]}{\kappa x(1-x)}$$

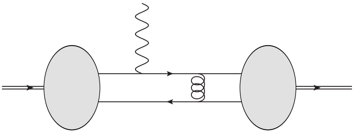
	$x(1-x)$	$\phi_{\ln}(x)$	$(x(1-x))^{\nu}$	$\sqrt{x(1-x)}$
$\langle x^{-1} \rangle$	3	3.41	3.66	4
$\frac{\langle x^{-1} \rangle}{\langle x^{-1} \rangle_{As}}$	1	1.14	1.22	1.33



Form Factors



$$Q^2F(Q^2) = \mathcal{N}\int [\mathrm{d}x_i][\mathrm{d}y_i]\varphi(x,\zeta_x^2)T(x,y,Q^2,\zeta_x^2,\zeta_y^2)\varphi(y,\zeta_y^2)$$



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- LO Kernel and NLO kernels are known
- $T_0 \propto \frac{\alpha_S(\mu_R^2)}{(1-x)(1-y)}$
- $T_1 \propto \frac{\alpha_s^2(\mu_R^2)}{(1-x)(1-y)}(f_{UV}(\mu_R^2) + f_{IR}(\zeta^2) + f_{finite})$

R Field *et al.*, NPB 186 429 (1981)
F. Dittes and A. Radyushkin, YF 34 529 (1981)
B. Melic *et al.*, PRD 60 074004 (1999)

Pion FF

- The UV scale dependent term behaves like:

$$f_{UV}(\mu_R^2) \propto \beta_0 \left(5/3 - \ln((1-x)(1-y)) + \ln\left(\frac{\mu_R^2}{Q^2}\right)\right)$$

- Here I take two examples:
  - the standard choice of $\zeta_x^2 = \zeta_y^2 = \mu^2 = Q^2/4$
  - the regularised BLM-PMC scale $\zeta_x^2 = \zeta_y^2 = \mu^2 = e^{-5/3}Q^2/4$

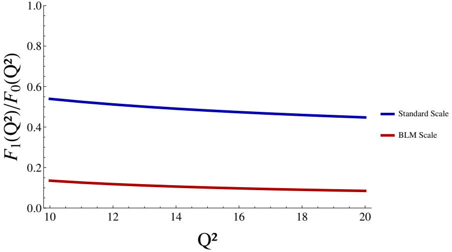
S. Brodsky *et al.*, PRD 28 228 (1983)
S. Brodsky and L. Di Giustino, PRD 86 085026 (2011)

- Take the PDA model coming from the scalar diquark:

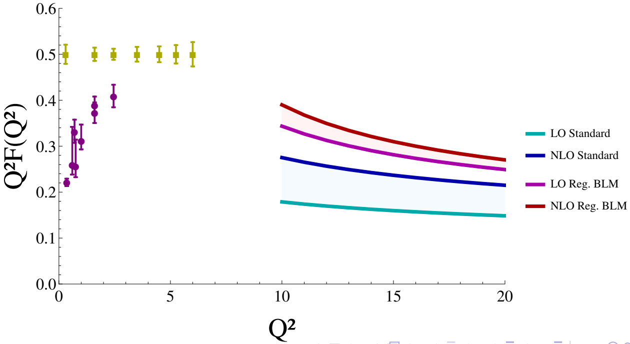
$$\phi(x) \propto 1 - \frac{\ln\left[1 + \kappa x(1-x)\right]}{\kappa x(1-x)}$$

κ is fitted to the lattice Mellin Moment

Pion FF



1.0
0.8
0.6
0.4
0.2
0.0
$F_1(Q^2)/F_0(Q^2)$
10
12
14
16
18
20
Q^2
Standard Scale
BLM Scale



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0.1
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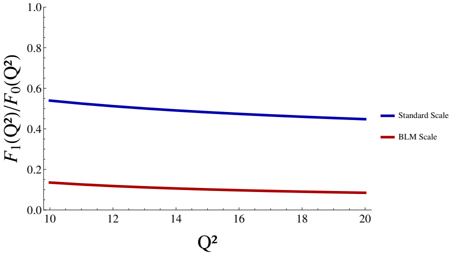
Q^2
LO Standard
NLO Standard
LO Reg. BLM
NLO Reg. BLM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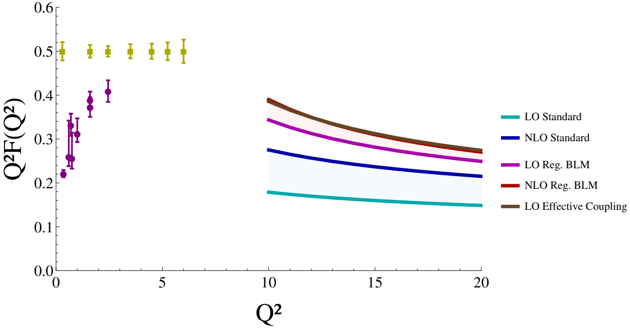
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LO Standard
NLO Standard
LO Reg. BLM
NLO Reg. BLM
LO Effective Coupling

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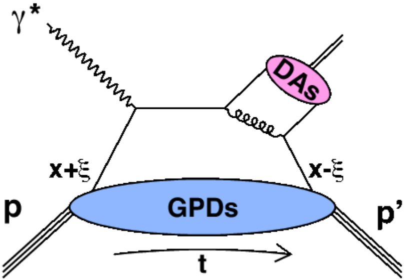
S. Brodsky *et al.*, PRD 28 228 (1983)
S. Brodsky and L. Di Giustino, PRD 86 085026 (2011)

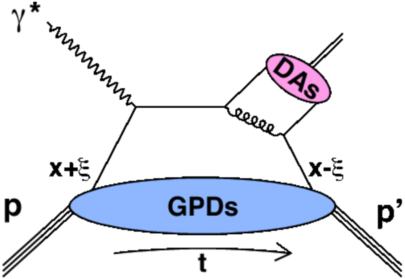
- BLM scale reduces significantly the impact of the NLO corrections and increase dramatically the LO one.



- LO Transition Form Factor $\propto \langle x^{-1} \rangle$
- At NLO : $g_{UV} \propto \beta_0 \left(5/3 - \ln((1-u)(1-v)) + \ln\left(\frac{\mu_R^2}{Q^2}\right)\right)$
- Shape effects are also magnified

D. Müller *et al.*, Nucl.Phys. B884 (2014) 438-546

DVMP



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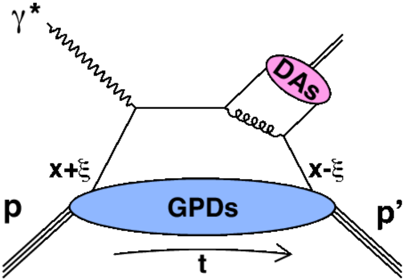
Bottom Line

A good knowledge of the PDA is a key point to perform reliable extraction of GPDs though DVMP



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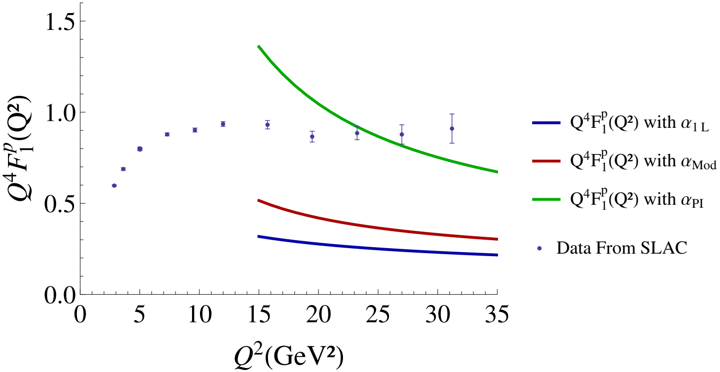
Optimism

Our understanding of PDA is much better today than 10 years ago

- Unfortunately, only the LO treatment has been performed
  $\Rightarrow$ BLM scale is therefore unknown
- We use the Chernyak-Zhitnitsky formalism to compute the nucleon for factor with:
  - the CZ scale setting $\rightarrow \alpha_s(Q^2/9)\alpha_s(4Q^2/9)$
  - the pion BLM factor $\rightarrow \alpha_s(Q^2/9\, e^{-5/3})\alpha_s(4Q^2/9\, e^{-5/3})$

  and using both perturbative and effective couplings.

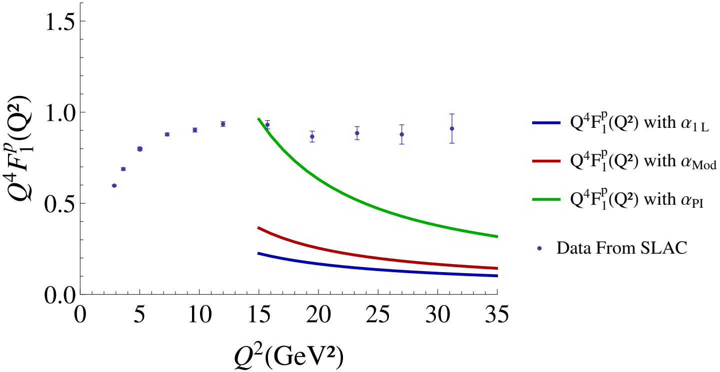
CZ scale setting with frozen PDA at $1\mathrm{GeV}^2$



Data from Arnold et al. PRL 57

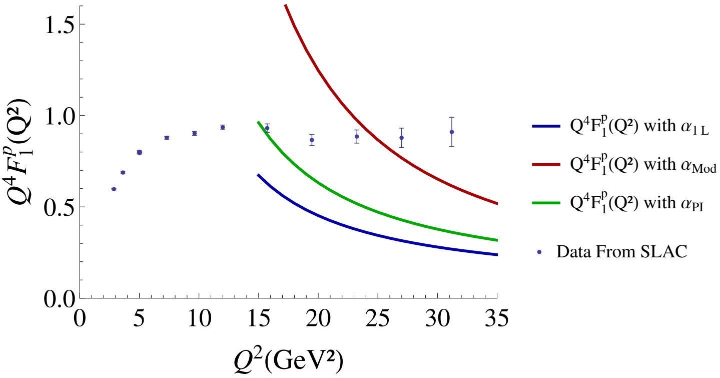
Proton case

CZ scale setting + evolution



Data from Arnold et al. PRL 57

Pion BLM Factor + evolution



Data from Arnold et al. PRL 57

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  and using both perturbative and effective couplings.
- The data remain flat while the perturbative running show a logarithmic decreasing.
- More work are required to conclude on the validity of the perturbative approach:
  - Theory side : NLO + higher-twists?
  - Experimental side : more precise data to spot a logarithmic decreasing

Thank you for your attention

Back up slides