# Threading the needle with Schwinger-Dyson equations

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Continuum Functional Methods for QCD at New Generation Facilities

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ECT* - Villa Tambosi

*Emergence:* *low-level rules producing high-level phenomena with enormous apparent complexity*

*Start from the QCD Lagrangian :*

$$\mathcal{L}_{QCD} = -\frac{1}{4}G_a^{\mu\nu}G^a_{\mu\nu} + \frac{1}{2\xi}(\partial^\mu A^a_\mu)^2 + \partial^\mu \bar{c}^a \partial_\mu c^a + g f^{abc}(\partial^\mu \bar{c}^a) A^b_\mu c^c$$

+ Quarks

*SDE, lattice, BSE …*

*and obtain*

*Dynamical generation of a fundamental mass scale in pure Yang-Mills (gluon mass)*

*Quark constituent masses and chiral symmetry breaking*

*Bound state formation: mesons, hadrons, glueballs, hybrids, exotics …*

*Signals of Confinement*

…

# *Off-shell Green's (correlation) functions*


- *Green's functions: Propagators and vertices*

*Even though they are:*

- *Gauge-dependent*
- *Renormalization point (μ) and scheme-dependent*

*However*

- *They capture characteristic features of the underlying dynamics, both perturbative and non-perturbative.*
- *When appropriately combined they give rise to physical observables*

*Crucial pieces for completing the QCD puzzle*

# Schwinger-Dyson equations

C. D. Roberts and A.G.Williams, Prog. Part. Nucl.Phys.33, 477 (1994)
P. Maris and C. D. Roberts, Int. J. Mod. Phys. E12, 297 (2003)
I. C. Cloet and C. D. Roberts, Prog. Part. Nucl. Phys.77, 1 (2014)

- Insightful computational framework
- Equations of motion for off-shell Green's functions, derived formally from the generating functional
- Inherently non-perturbative, but at the same time captures the perturbative behavior ⟹ accommodates the full range of physical momenta.

## However

- Infinite system of coupled non-linear integral equations
- No obvious expansion parameter, so, no formal way of estimating the size of the omitted terms. But, it seems that the "projection" of higher Green's functions on the lower ones is "small".
- Casual truncation interferes with the symmetries encoded in the form of the SDEs

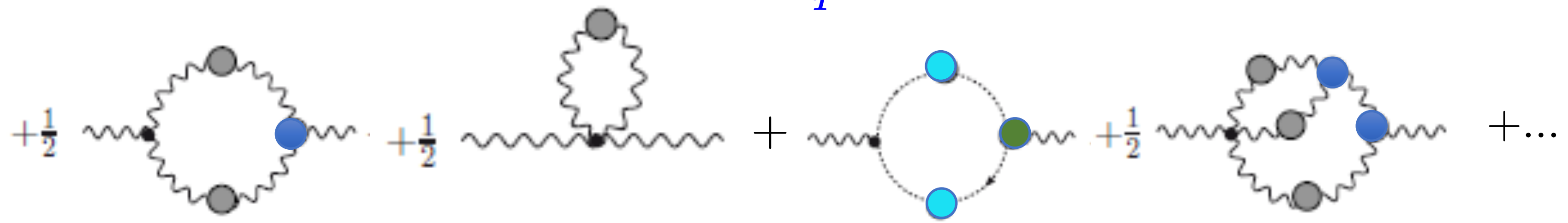
⟹ Symmetry-preserving truncation scheme is preferable, whenever possible

# *Three main ingredients*

- *Gluon propagator (Landau gauge)*

$$\Delta_{\mu\nu}(q) = -i\Delta(q^2)P_{\mu\nu}(q) \quad \textit{with} \quad P_{\mu\nu}(q) = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}$$

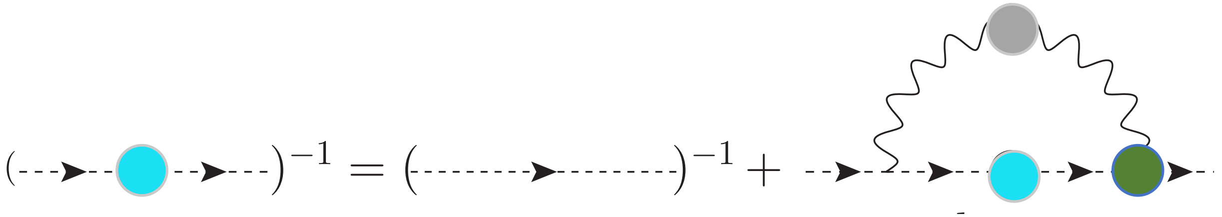
$$\Delta^{-1}(q^2) = \Delta_0^{-1}(q^2)$$



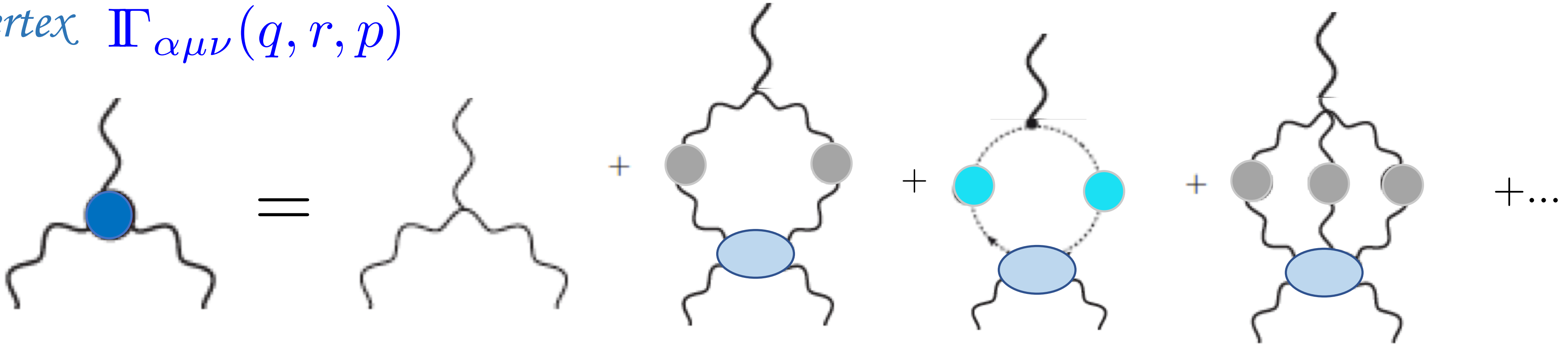
- *Ghost propagator*

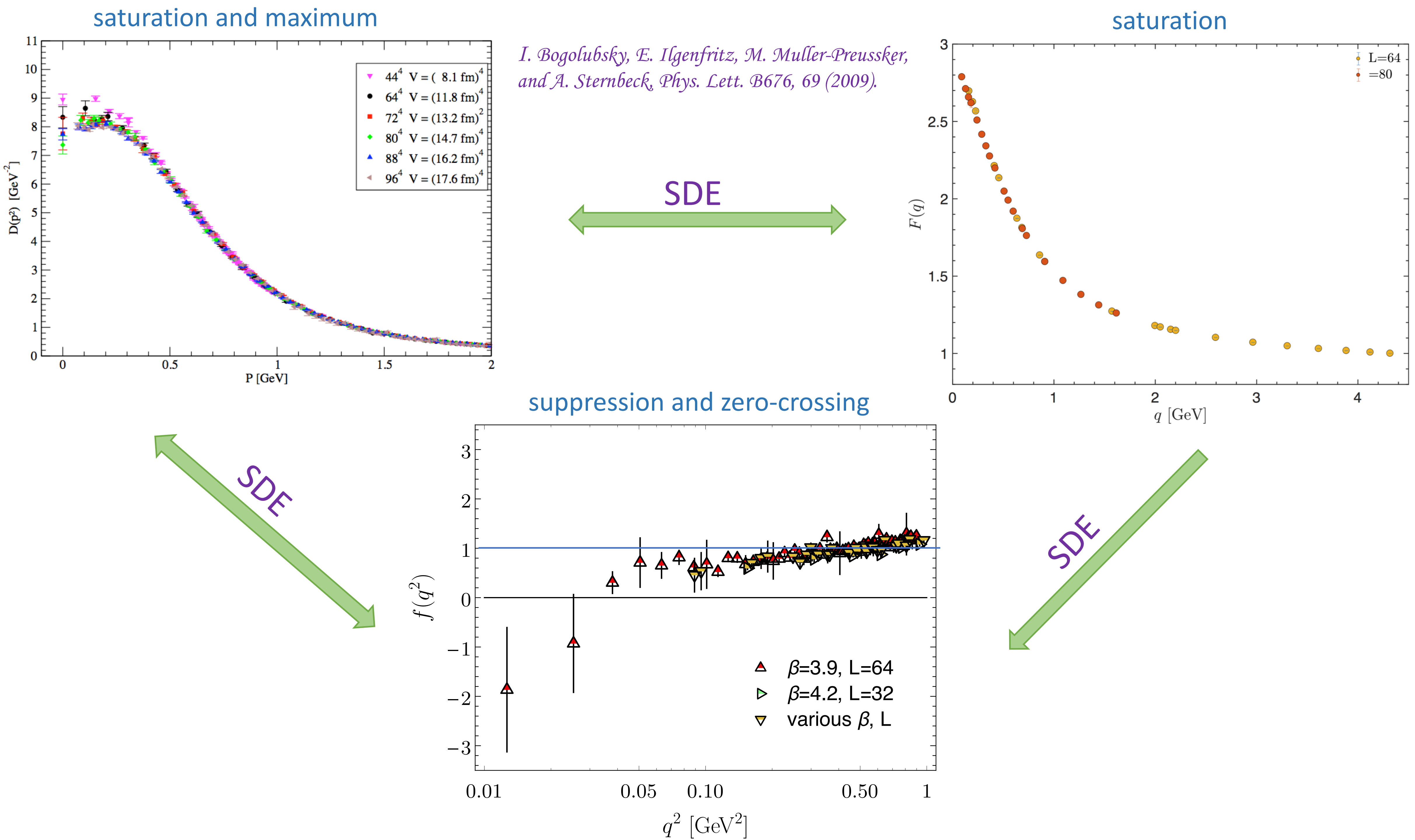
$$D(q^2) = \frac{iF(q^2)}{q^2}$$

← *dressing function*



- *Three-gluon vertex* $\mathrm{I\Gamma}_{\alpha\mu\nu}(q,r,p)$




saturation and maximum
D(p²) [GeV⁻²]
P [GeV]
44⁴ V = ( 8.1 fm)⁴
64⁴ V = (11.8 fm)⁴
72⁴ V = (13.2 fm)²
80⁴ V = (14.7 fm)⁴
88⁴ V = (16.2 fm)⁴
96⁴ V = (17.6 fm)⁴
I. Bogolubsky, E. Ilgenfritz, M. Muller-Preussker, and A. Sternbeck, Phys. Lett. B676, 69 (2009).
SDE
saturation
F(q)
q [GeV]
L=64
=80
suppression and zero-crossing
SDE
SDE
f(q²)
q² [GeV²]
β=3.9, L=64
β=4.2, L=32
various β, L

# *Emergent mass scale in the gauge sector*

*J. M. Cornwall, Phys. Rev. D26, 1453 (1982)*
*A.C. Aguilar, D. Binosi, J.P. (various works)*

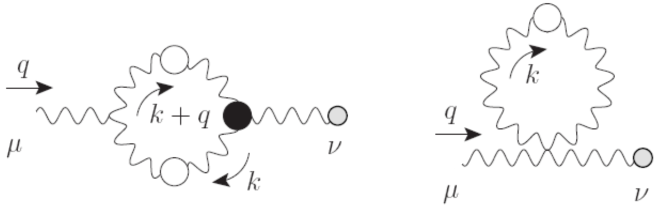
- *Saturation of* $\Delta(0)$ ⟷ *generation of a gluon mass scale*

- *Natural parametrization:* $\Delta^{-1}(q) = \underbrace{q^2 J(q)}_{\text{kinetic term}} + \underbrace{m^2(q)}_{\text{running mass}}$ *with* $\Delta^{-1}(0) = m^2(0)$

## *A mechanism is needed*

*"Ward-Takahashi" identity:* $q^\alpha \mathrm{I\Gamma}_{\alpha\mu\nu}(q,r,p) = i\Delta^{-1}_{\mu\nu}(r) - i\Delta^{-1}_{\mu\nu}(p)$ $\xrightarrow[\text{expansion}]{\text{Taylor}}$ $\mathrm{I\Gamma}_{\alpha\mu\nu}(0,r,-r) = -i\dfrac{\partial\Delta^{-1}_{\mu\nu}(r)}{\partial r^\alpha}$

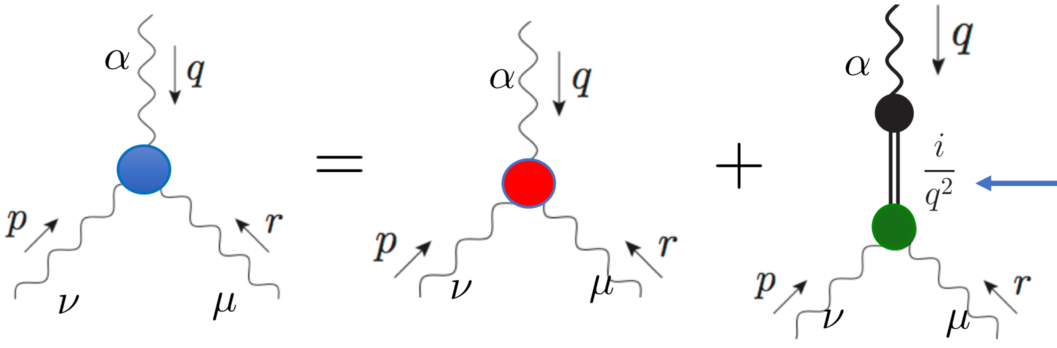
⟹ $\Delta^{-1}(0) = \lim_{q\to 0} \mathrm{Tr}\Big\{$




$\Big\} \sim \underbrace{\int_k k^2 \frac{\partial\Delta(k^2)}{\partial k^2} + \frac{d}{2}\int_k \Delta(k)}_{\text{Seagull identity}} = 0$

$\Delta^{-1}(0) = 0$ ⟹ *No "gluon mass"*

## Existence of "massive" solution requires non-analyticity of $\mathbb{\Gamma}$




$\mathbb{\Gamma}_{\alpha\mu\nu}(q,r,p) \qquad \Gamma^{\mathbf{np}}_{\alpha\mu\nu}(q,r,p) \qquad \Gamma^{\mathbf{p}}_{\alpha\mu\nu}(q,r,p)$

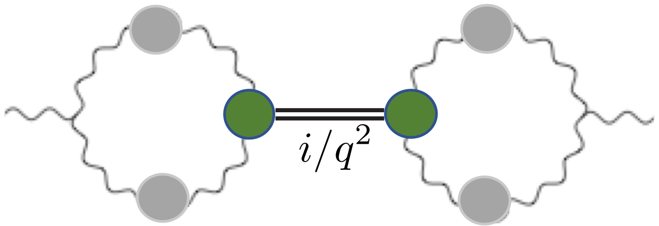
*Dynamically generated massless excitations*

*R. Jackiw and K. Johnson*, Phys. Rev. D8, 2386 (1973)
*E. Eichten and F. Feinberg*, Phys. Rev. D10, 3254 (1974)

$\Gamma^{\mathbf{p}}_{\alpha\mu\nu}(q,r,p)$

*Triggers the "Schwinger mechanism"*

*J.S. Schwinger*, Phys. Rev.125, 397 (1962); Phys.Rev.128, 2425 (1962)




→ *Makes possible* $\Delta^{-1}(0) \neq 0$

*" Longitudinally coupled "* → 

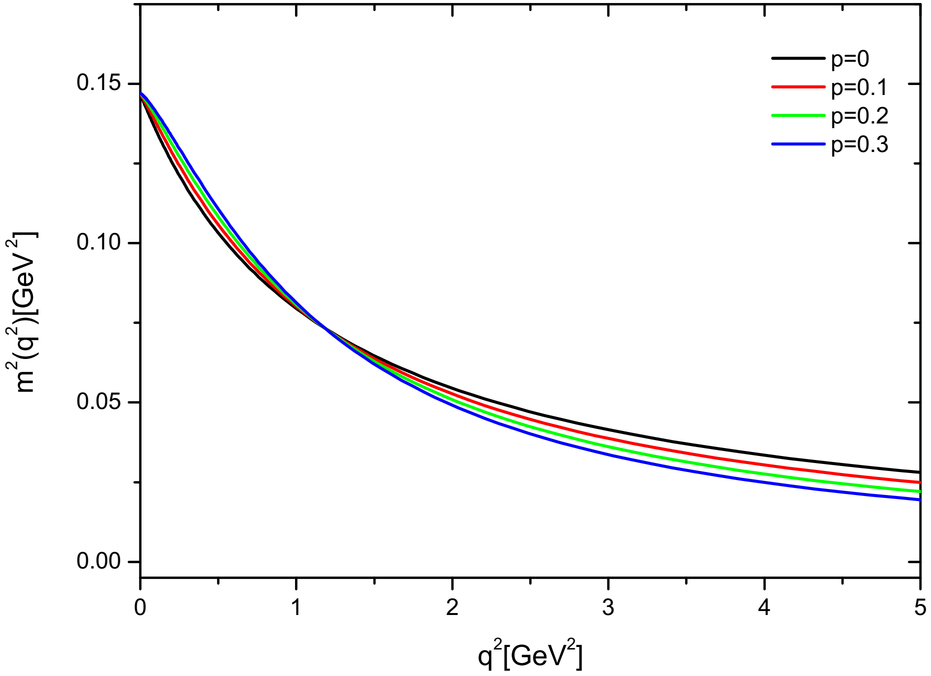
$$P_{\alpha\alpha'}(q)P_{\mu\mu'}(r)P_{\nu\nu'}(p)\,\Gamma^{\mathbf{p}}_{\alpha\mu\nu}(q,r,p) = 0$$

*Drops out from "lattice observables" and transversely projected Green's functions*

## Integral equation for $m^2(q)$

$$m^2(q^2) = \alpha_s \int_k m^2(k^2)\, \mathcal{K}(q,k,\Delta,\Gamma^{\mathbf{np}})$$

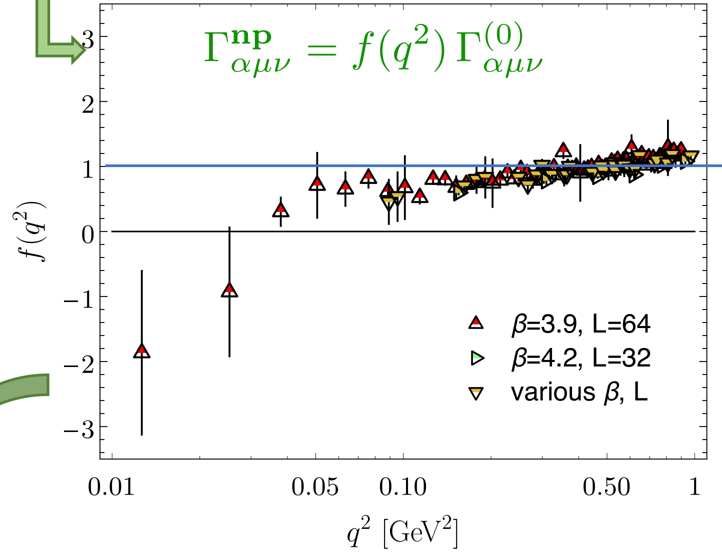
$$m^2(q^2) = \frac{m^2(0)}{1+(q^2/\lambda^2)^{1+p}}$$




0.24 - 0.28

$(\mu = 4.3\,\mathrm{GeV})$

$$\Gamma^{\mathbf{np}}_{\alpha\mu\nu} = f(q^2)\,\Gamma^{(0)}_{\alpha\mu\nu}$$




*Infrared suppression makes everything numerically compatible*

*Dramatic confirmation that all the pieces fit together !*

# *The "kinetic" term*

- *Complicated integral equation*

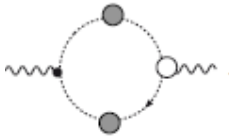
$$J(q^2) = 1 + \alpha_s \int_k \mathcal{K}(q, k, J, m^2, \Gamma_3, \Gamma_4)$$

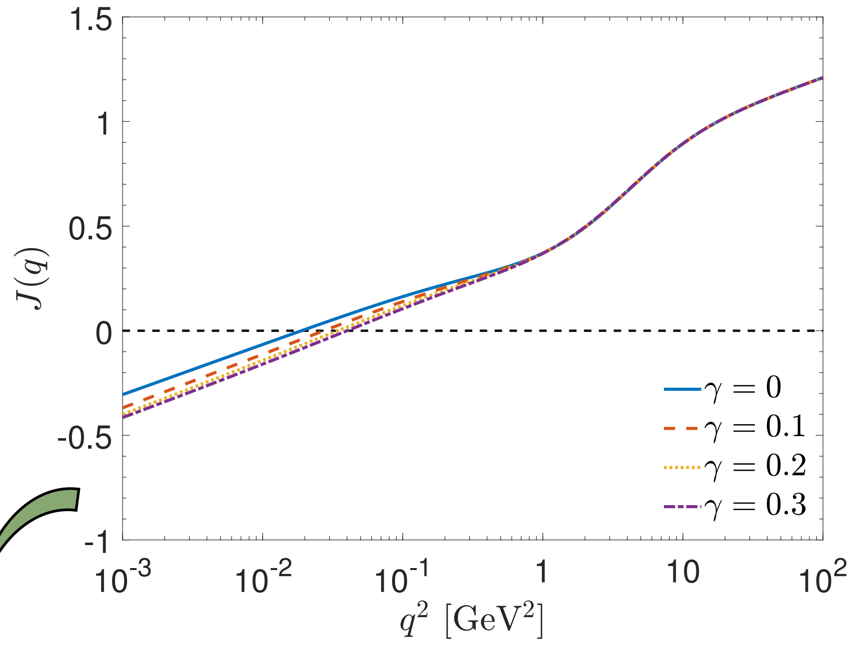
*solution*

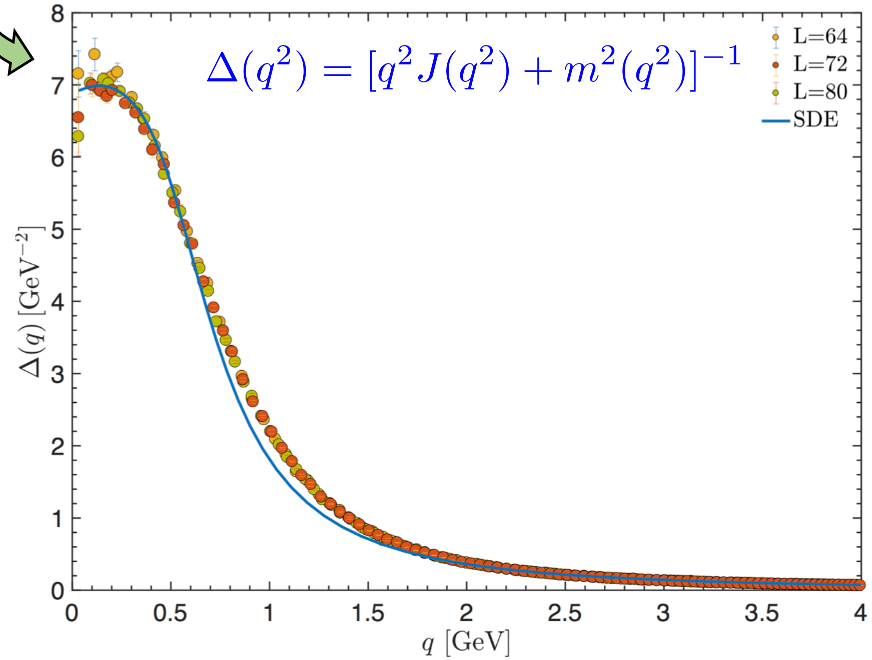
$$J(q^2) = 1 + \frac{C_{\rm A}\alpha_s}{4\pi}\left[2\ln\left(\frac{q^2+\rho m^2(q)}{\mu^2}\right) + \frac{1}{6}\ln\left(\frac{q^2}{\mu^2}\right)\right]$$

*"protected"*

*"unprotected"*





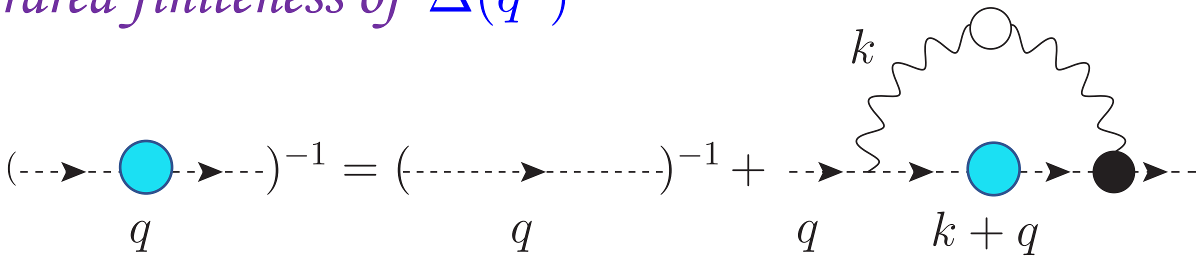

- $q^2 \gg m^2 \Longrightarrow J(q^2) \to 1 + \frac{C_{\rm A}\alpha_s}{4\pi}\left(\frac{13}{6}\right)\ln\left(\frac{q^2}{\mu^2}\right)$

*one-loop*

- $\frac{\text{unprotected}}{\text{protected}} = \frac{1}{12}$ *"Imperfect protection"* ⟹ *Important consequences …*

# *Ghost dressing function*

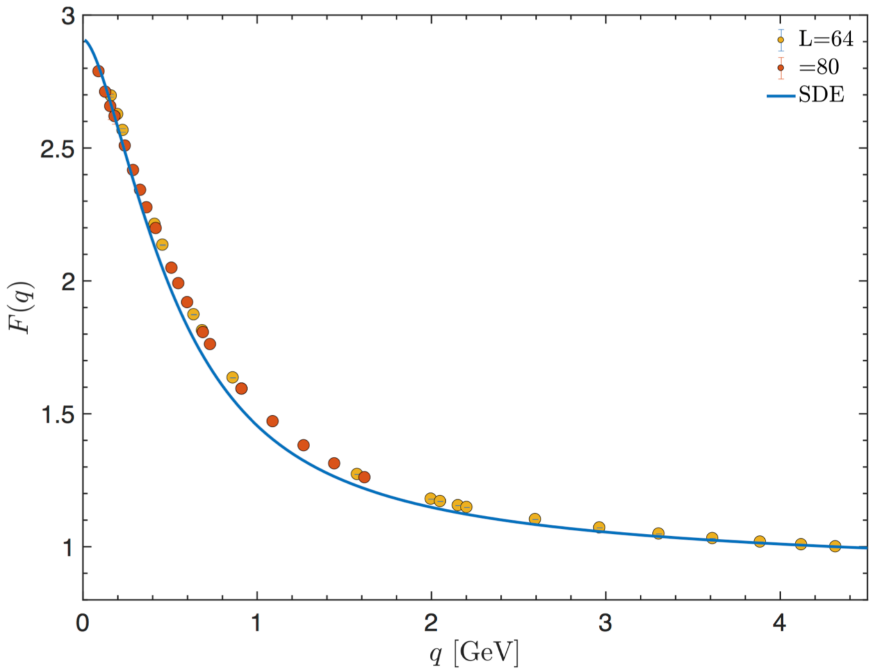
1. *The infrared finiteness of* $F(q^2)$ *is a direct consequence of the infrared finiteness of* $\Delta(q^2)$

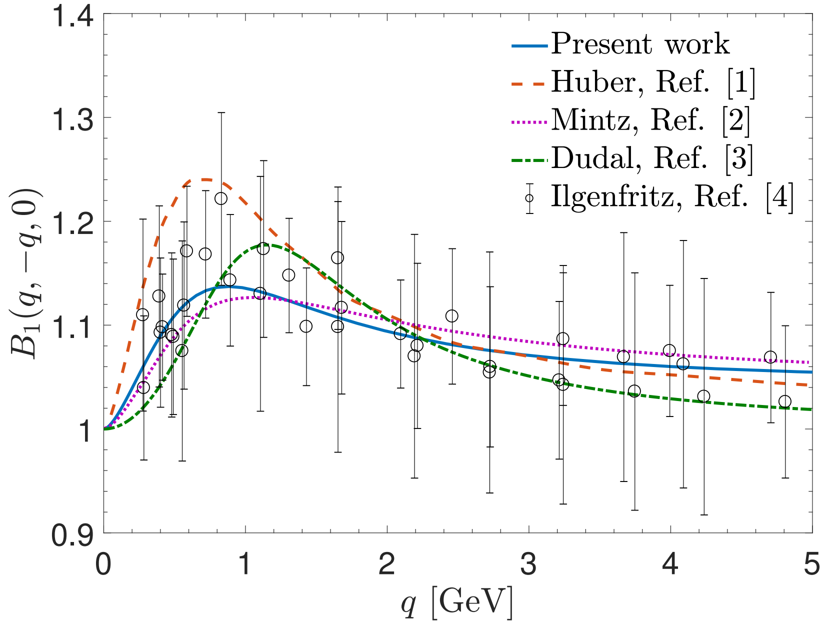
2. 


$$F^{-1}(q^2) = 1 + g^2 C_{\rm A} \int_k \left[1 - \frac{(k\cdot q)^2}{k^2 q^2}\right] D(k+q)\Delta(k) B_1(q,k)$$

*ghost-gluon vertex* ($B_1(q,k)$)

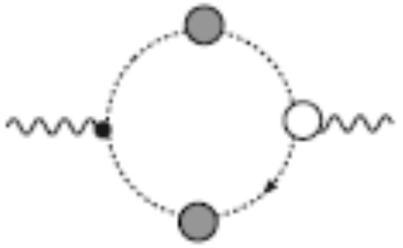
3. $$F^{-1}(q^2) = 1 + \frac{9 C_{\rm A}\alpha_s}{48\pi}\left[1 + c_1 \exp\left(-c_2 q^2\right)\right] \ln\left(\frac{q^2 + c_3 m^2(q)}{\mu^2}\right)$$



# Maximum of gluon propagator

*Nonperturbative masslessnesss of ghost:* $\lim_{q^2\to 0} D(q^2) = \frac{F(0)}{q^2}$

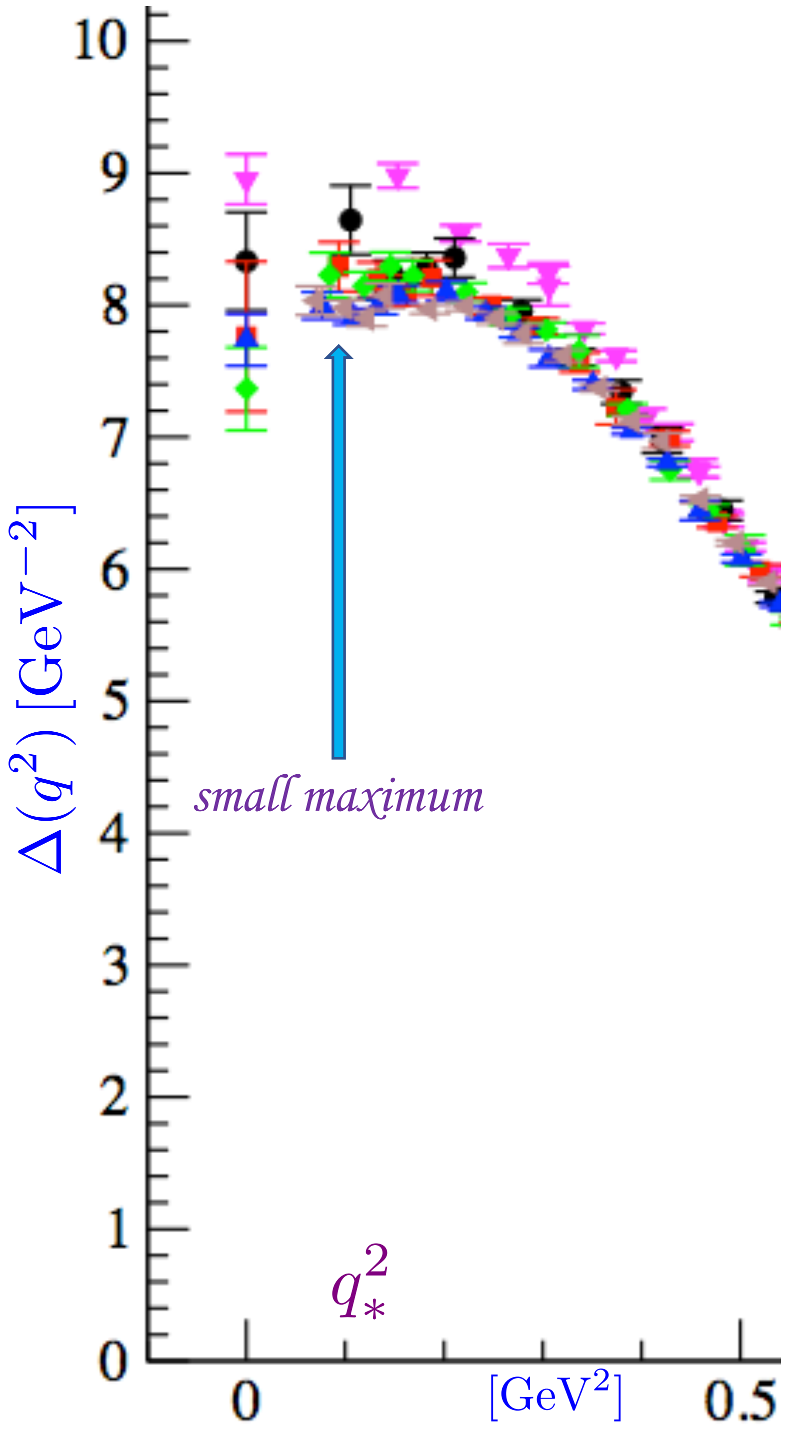


$$\sim q^2 \int_k \frac{F(k^2)}{k^2(k+q)^2} \sim q^2 \ln(q^2/\mu^2)$$

$$\Delta^{-1}(q^2) = \underbrace{q^2 + m^2(q^2) + c_1 q^2 \ln\left(\frac{q^2+\rho m^2(q)}{\mu^2}\right)}_{f(q^2,m^2)} + c_2 q^2 \ln(q^2/\mu^2)$$

$$[\Delta^{-1}(q^2)]' = \underbrace{f'(q^2,m^2)}_{\text{finite for } \forall q^2 \geq 0} + c_2[1 + \underbrace{\ln(q^2/\mu^2)}_{\text{takes over in the IR}}]$$

$$\exists \quad q_*^2 : [\Delta^{-1}(q_*^2)]' = 0 \qquad (q_* \approx 150\,\text{MeV})$$



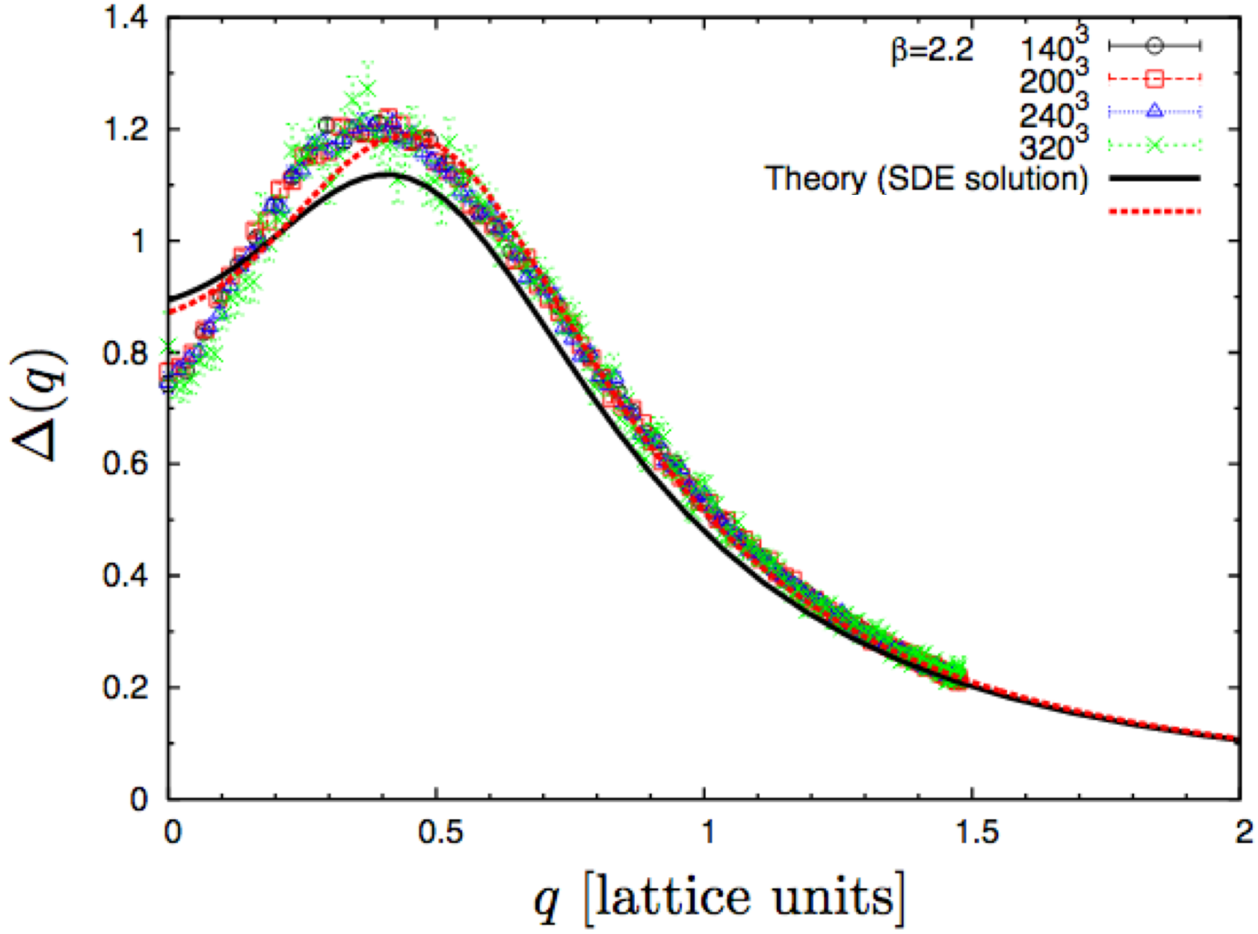
# *Much more "visible" in 3-D*

- *Same "physics" but "enhanced"*

$$\ln(q^2/\mu^2) \to 1/q$$

*Sharper maximum*

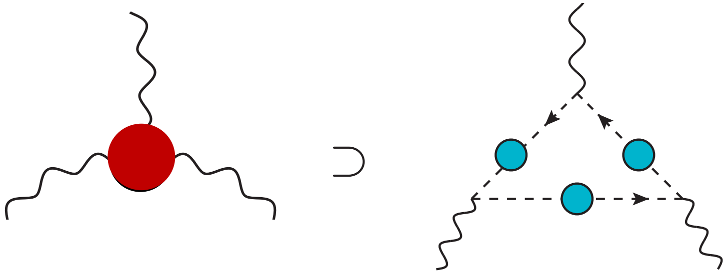
*Located more towards the intermediate region*




*A. Cucchieri, T. Mendes and A. R. Taurines*, *Phys. Rev. D 67, 091502 (2003);*
*A. Cucchieri and T. Mendes*, *PoS(QCD-TNT09)026 (2010)*

# *Whence the suppression of the three-gluon vertex?*

*A.C.Aguilar, D.Binosi, D.Ibañez, J.P., Phys. Rev. D 89, no. 8, 085008 (2014)*



*ghost loop*

*massless ghost propagators*

↓

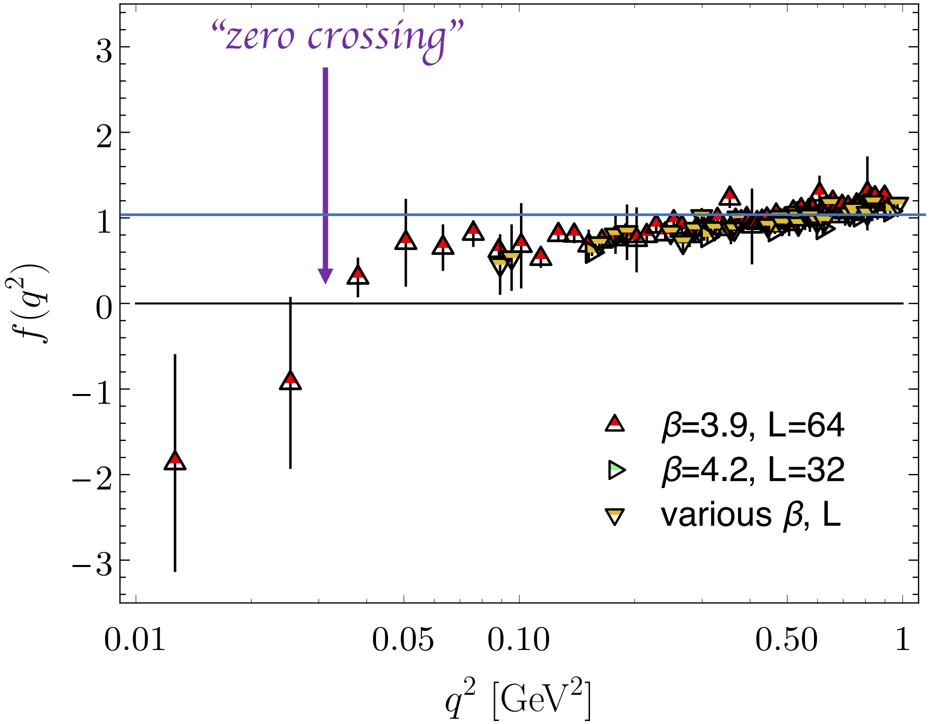
*log-divergences in the infrared*

$$\Gamma^{\mathbf{np}}_{\alpha\mu\nu} = f(q,r,p)\,\Gamma^{(0)}_{\alpha\mu\nu}$$

*symmetric point:* $q^2 = p^2 = r^2$

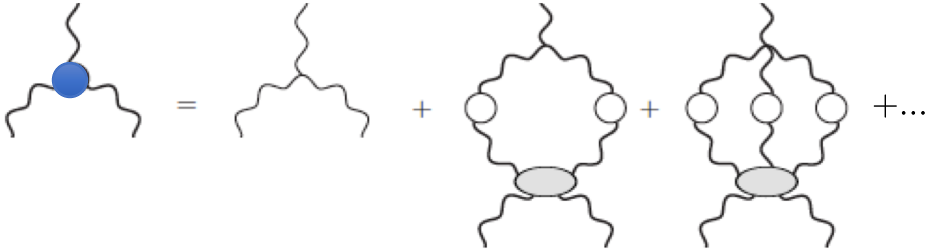
$$f(q^2) = a\left[1 + b\ln\frac{q^2+m^2}{\mu^2} + c\ln\frac{q^2}{\mu^2}\right]$$

*"protected"* *"unprotected"*



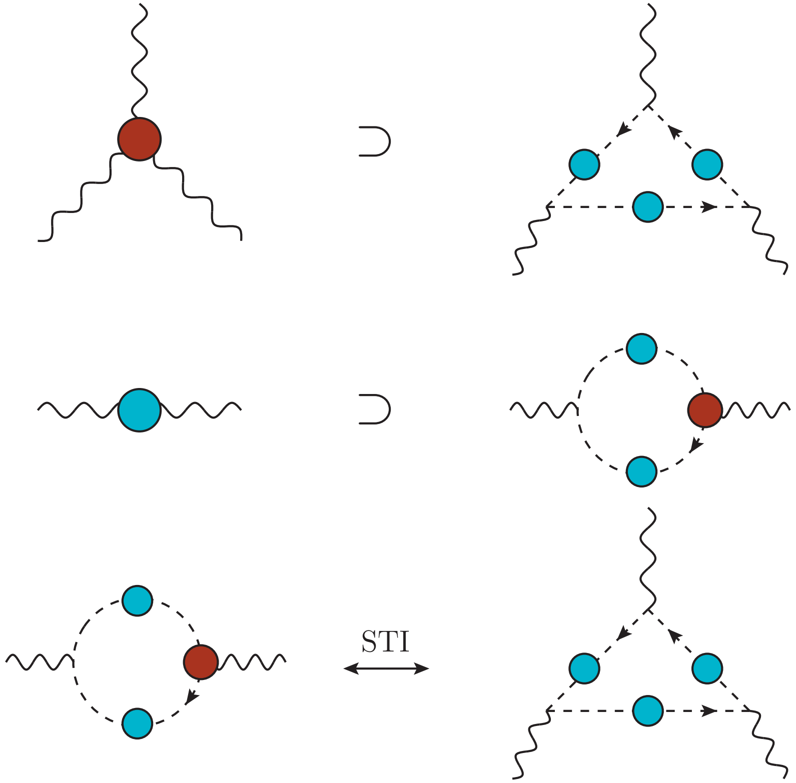

⟹ $f(0) \to -\infty$ ⟹ *behaves like the* $J(q^2)$ **!**

## *"Conventional" approach: solve the SDE of the three-gluon vertex*



*R. Alkofer, M.Q. Huber, K. Schwenzer, Eur. Phys. J. C62 , 761 (2009); Phys. Rev. D81, 105010 (2010)*
*G. Eichmann, R. Williams, R. Alkofer, M. Vujinovic, Phys. Rev. D89, 105014 (2014)*
*A. Blum, M.Q. Huber, M. Mitter, L. von Smekal, Phys. Rev. D 89, 061703(R) (2014)*
*A. K. Cyrol, L. Fister, M. Mitter, J. M. Pawlowski, N. Strodthoff, Phys. Rev. D94, 054005 (2016)*
*R. Williams, C. S. Fischer, and W.Heupel, Phys. Rev. D 93, no. 3, 034026 (2016)*

## *Instead …*




## *Reconstruct the three-gluon vertex from its Slavnov-Taylor identity (STI)*

*The general idea: a sophisticated version of the Gauge Technique*

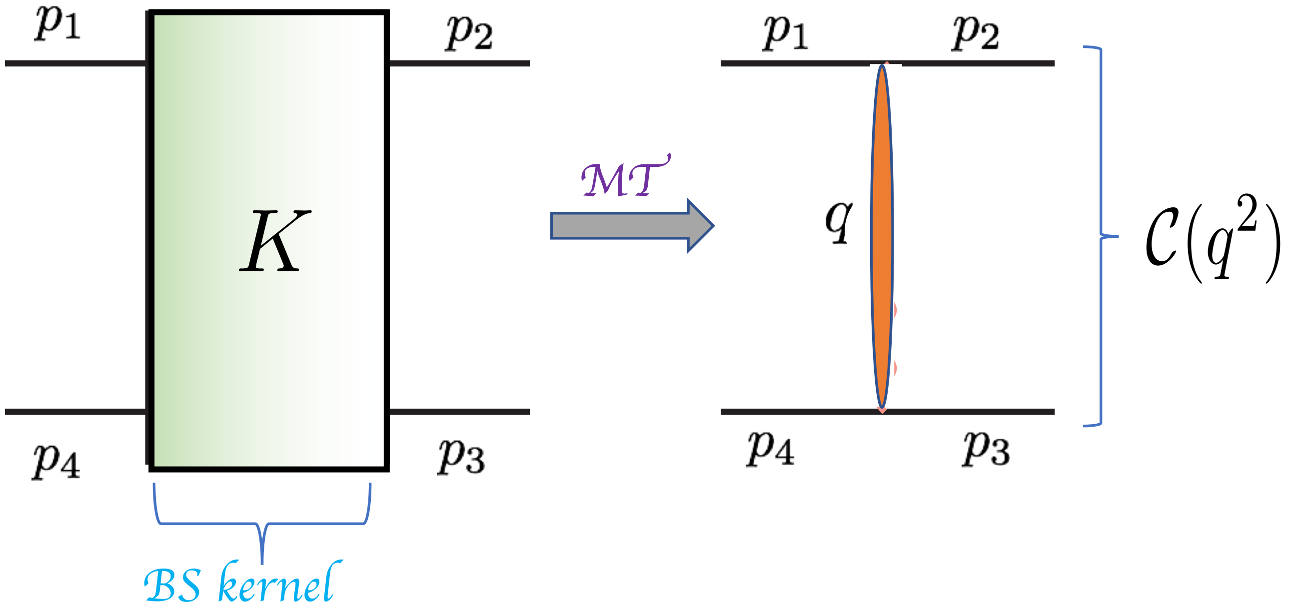
*A.C.Aguilar, M.N.Ferreira, C.T.Figueiredo, J.P, Phys. Rev. D 99, no. 3, 034026 (2019)*
*arXiv:1903.01184; Phys.Rev. D, in press*

*(see Cristina's talk)*

*How to get phenomenology using these correlation functions as ingredients*

*Some examples …*

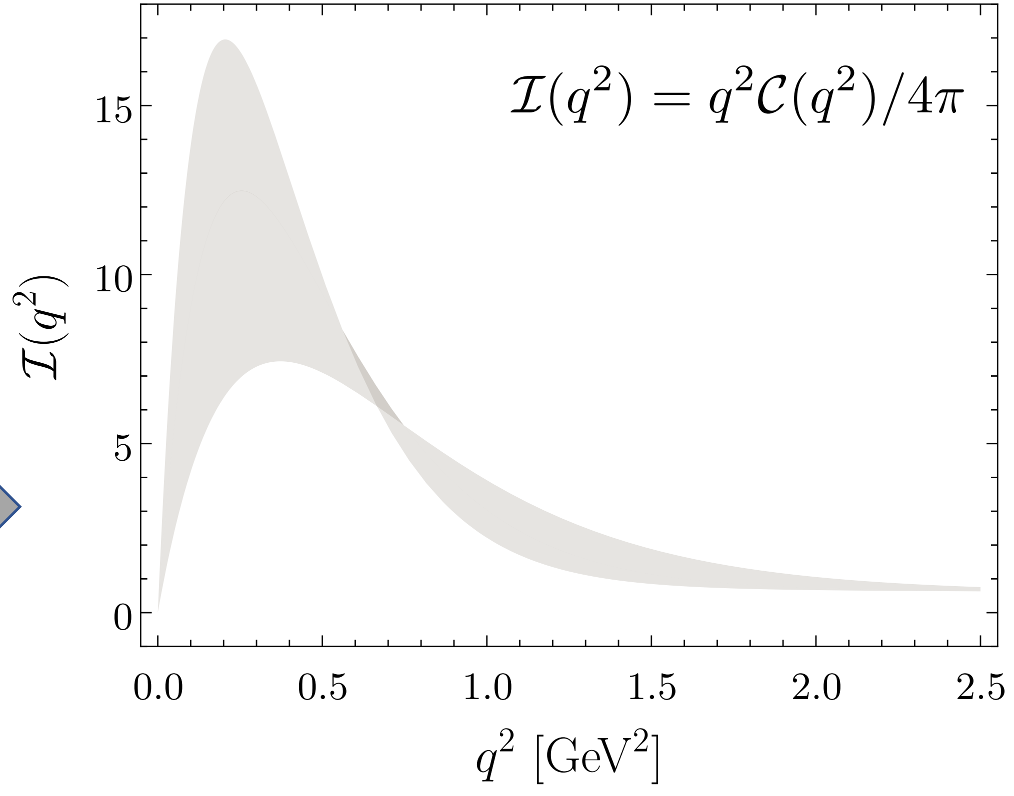
# "top-down" derivation of the Maris-Tandy interaction




## Phenomenological Ansatz

$$\mathcal{C}(q^2) = \underbrace{\frac{8\pi^2}{\omega^4} D e^{-q^2/\omega^2}}_{\text{IR}} + \underbrace{\frac{f(q^2)}{\ln[\tau + (1 + q^2/\Lambda_{QCD}^2)^2]}}_{\text{UV}}$$

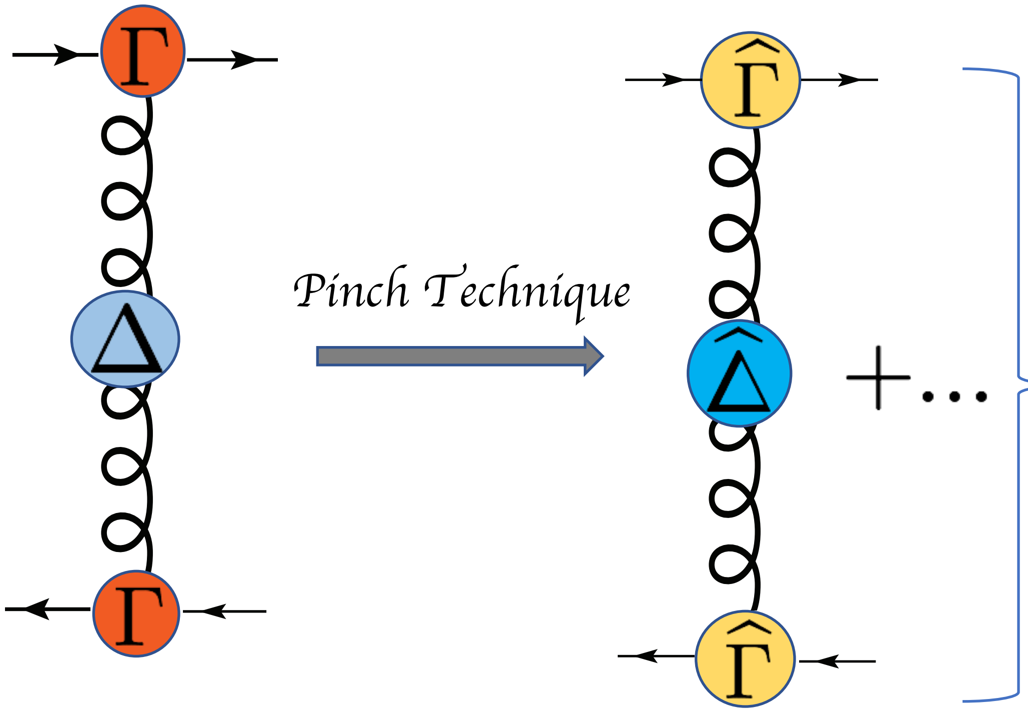
$$D\omega = (0.87)^3\ \mathrm{GeV}^3$$




P. Maris and P.C. Tandy, Phys. Rev. C 60, 055214 (1999).

# PT-BFM framework

D.Binosi and J. P, Phys. Rept. 479, 1 (2009)




Effective "Abelianization"

$$q^{\mu}\widehat{\Gamma}_{\mu}(q,p_1,p_2) = S^{-1}(p_2) - S^{-1}(p_1)$$

QED-like Ward-Takahashi identity

$$\Delta(q^2) = \widehat{\Delta}(q^2)[1+G(q^2)]^2$$

Ghost-related function

Captures the renormalization-group logs just as the photon vacuum polarization

$$Z_g = \widehat{Z}_A^{-1/2}$$

(as in QED !)

$$\widehat{d}(q^2) = \alpha_s \widehat{\Delta}(q^2) = \frac{\alpha_s \Delta(q^2)}{[1+G(q^2)]^2}$$

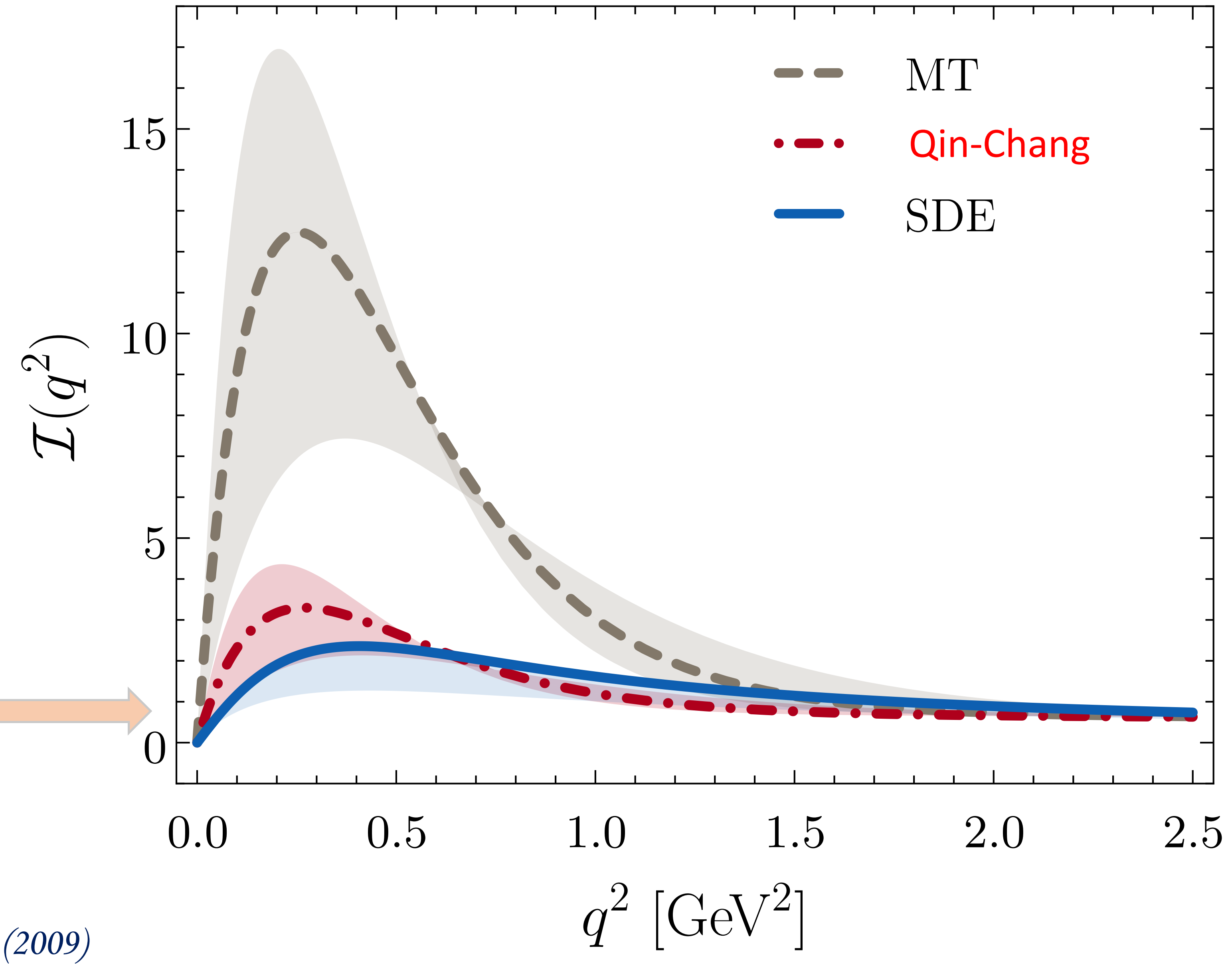
Process-independent and renormalization-group invariant combination

# The $\mu$ –independent interaction strength

D. Binosi, L. Chang, J.P. and C.D. Roberts, Phys. Lett. B742, 183 (2015)

To make contact with the MT interactions set

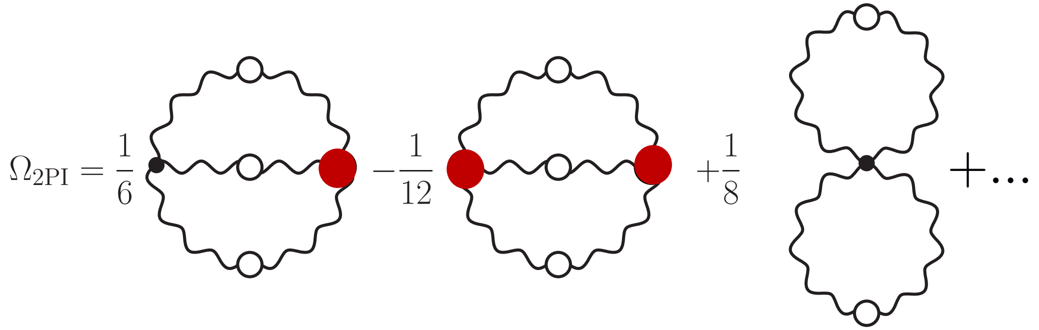
$$\mathcal{I}(q^2) = q^2\, \widehat{d}(q^2)$$




L. Chang and C.D. Roberts, Phys. Rev. Lett. 103, 081601 (2009)

# QCD vacuum

*CJT effective action*




*$\Gamma^{\mathbf{np}}$ must be suppressed in order to obtain the "right" range of values for the gluon condensate*

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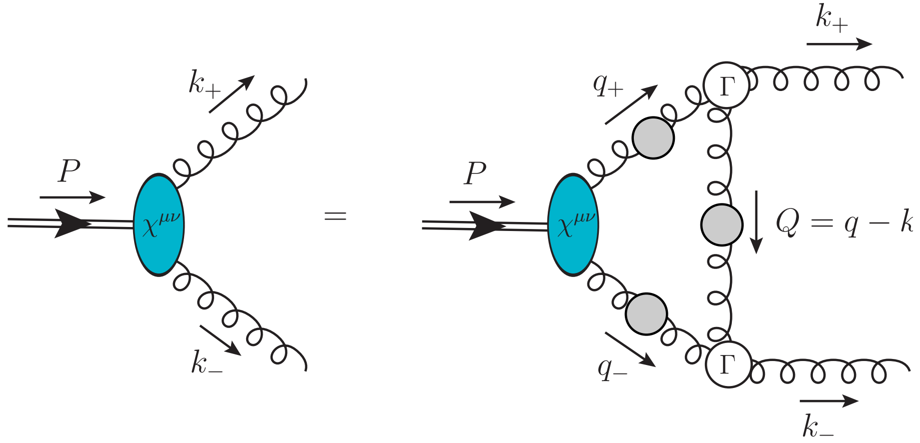
# Glueballs

*The infrared suppression of $\Gamma^{\mathbf{np}}$ seems crucial for getting "correct" glueball masses from the BSE*

J. Meyers, E.S. Swanson, Phys. Rev. D 87, 036009 (2013)

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# Hybrids and Exotics

*Systems with a "valence" gluon $G$ :* $Q\bar{Q}G,\ QQQG$ (E.I.C.)

*$G$ plays a dual role : both force field and matter*

*The suppression of $\Gamma^{\mathbf{np}}$ affects their appearance in the specturm*

S.S.Xu, Z.F.Cui, L.Chang, J.P., C.D.Roberts, H.S.Zong, arXiv:1805.06430