Hybrid states in the framework of Dyson-Schwinger equations

Shu-Sheng Xu

（徐书生）

Nanjing University of Posts and Telecommunications

Based on arXiv: 1805.06430 [nucl-th]

Continuum Functional Methods for QCD at New Generation Facilities @ ECT*, Trento, Italy, May 10, 2019
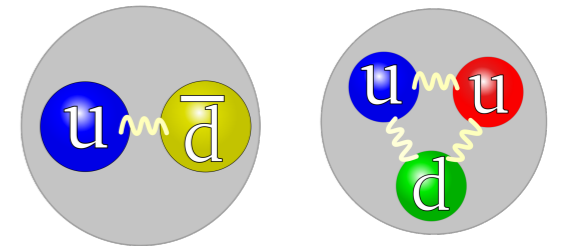
Spectrum of light hadrons

❖ The well-known light hadron is simple

  ◦ It is qualitatively matches the constituent quark model by Gell-Mann and Zweig (1964).

    ◦ Mesons built from a constituent-quark-antiquark ($Q\bar{Q}$) pair

    ◦ Baryons constituted from three constituent quarks (QQQ)

  where Q is u, d, s-quarks
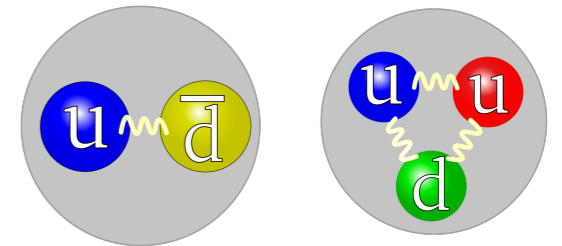
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❖ Gell-Mann and Zweig also raised possibility of multi-quark state

- Tetraquark: $QQ\bar{Q}\bar{Q}$

- Pentaquark: $Q\bar{Q}QQQ$

No candidate were then known, and they didn't know gluon

After ~50 years, in heavy quark systems, that now has changed

X, Y, Z,... pentaquark appears.

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- hybrid baryons - GQQQ

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❖ A few plausible hybrid-meson candidates below 2 GeV

  ◉ Searches for such states are underway at modern facilities (e.g. COMPASS @ CERN, GlueX @ JLab)

Model studies of hybrids

❖ Numerous models have employed to study spectrum of light hybrid mesons

- Approaches are distinguished by their treatment of constituent gluon
- Their spectrum disagree each other

Model	$J^{PC}_{q\bar{q}'}$	J^{PC}_{g}	J^{PC}	Mass (GeV/c^2)
Bag [2, 3]	0^{-+}	1^{+-} (TE)	1^{--}	~ 1.7
	1^{--}	1^{+-} (TE)	$(0,\mathbf{1},2)^{-+}$	$\sim 1.3, 1.5, 1.9$
	0^{-+}	1^{--} (TM)	1^{+-}	heavier
	1^{--}	1^{--} (TM)	$(0,1,2)^{++}$	heavier
Flux tube [4, 5]	0^{-+}	1^{+-}	1^{--}	1.7-1.9
	1^{--}	1^{+-}	$(0,\mathbf{1},2)^{-+}$	1.7-1.9
	0^{-+}	1^{-+}	1^{++}	1.7-1.9
	1^{--}	1^{-+}	$(\mathbf{0},1,2)^{+-}$	1.7-1.9
Constituent gluon	0^{-+}	1^{--}	1^{+-}	1.3-1.8 / 2.1
[6]/[7]	1^{--}	1^{--}	$(0,1,2)^{++}$	1.3-1.8 / 2.2
	1^{+-}	1^{--}	$(0,\mathbf{1},2)^{-+}$	1.8-2.2 / 2.2
	$(0,1,2)^{++}$	1^{--}	$1^{--},(\mathbf{0},1,2)^{--},(1,2,3)^{--}$	1.8-2.2 / 2.3
Constituent gluon /	0^{-+}	1^{+-}	1^{--}	(2.3)
LQCD [8, 9]	1^{--}	1^{+-}	$(0,\mathbf{1},2)^{-+}$	(2.1, 2.0, 2.4)
	1^{+-}	1^{+-}	$(0,1,2)^{++}$	(> 2.4)
	$(0,1,2)^{++}$	1^{+-}	$1^{+-},(\mathbf{0},1,\mathbf{2})^{+-},(1,\mathbf{2},3)^{+-}$	(> 2.4)

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- ❖ Development of a reliable continuum method for calculating hybrid meson properties would be valuable
  - ○ For interpretation of empirical observations
  - ○ Provide insights into results obtained via the numerical simulation of LQCD

Exotic mesons

❖ QQ̄ mesons in quantum mechanics can't possess exotic quantum numbers: JPC=0+-, 0--,1-+ , etc.

L	S	J^{PC}
0	0	0-+
0	1	1--
1	0	1+-
1	1	0++
1	1	1+-

❖ Nevertheless, exotic quantum numbers are allowed in relativistic two-body bound state

0++	0+-	0-+	0--
1++	1+-	1-+	1--
2++	2+-	2-+	2--
3++	3+-	3-+	3--

❖ Studies of exotic mesons using simple truncation for Bethe-Salpeter kernel produce unrealistic spectra

Si-xue Qin, et al., Phys.Rev. C85 (2012) 035202

ω	0.4	0.5	0.6
m_{0--}	0.814	0.940	1.053
m_{0+-}	1.186	1.252	1.323
m_{1-+}	1.234	1.277	1.318

❖ More sophisticated kernel can not remedied, it signal that exotic may contain explicit valence gluon degree of freedom

New perspective on hybrid mesons

❖ Can one produce sound treatment of hybrids using Poincaré-covariant Faddeev equation?

- Treat these systems as bound states of valence-gluon, -quark and antiquark.

- Each constituent is massive in their infrared

  region

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❖ Recall DSEs for quark propagator and gluon propagator

- Quark is massive in its infrared region

- Running gluon mass

$$d(k^2) = \frac{\alpha(\zeta)}{k^2 + m_g^2(k^2;\zeta)}$$

$$m_g^2(k^2) \approx \frac{\mu_g^4}{\mu_g^2 + k^2}$$

- It implies gluon is massive in it's infrared region
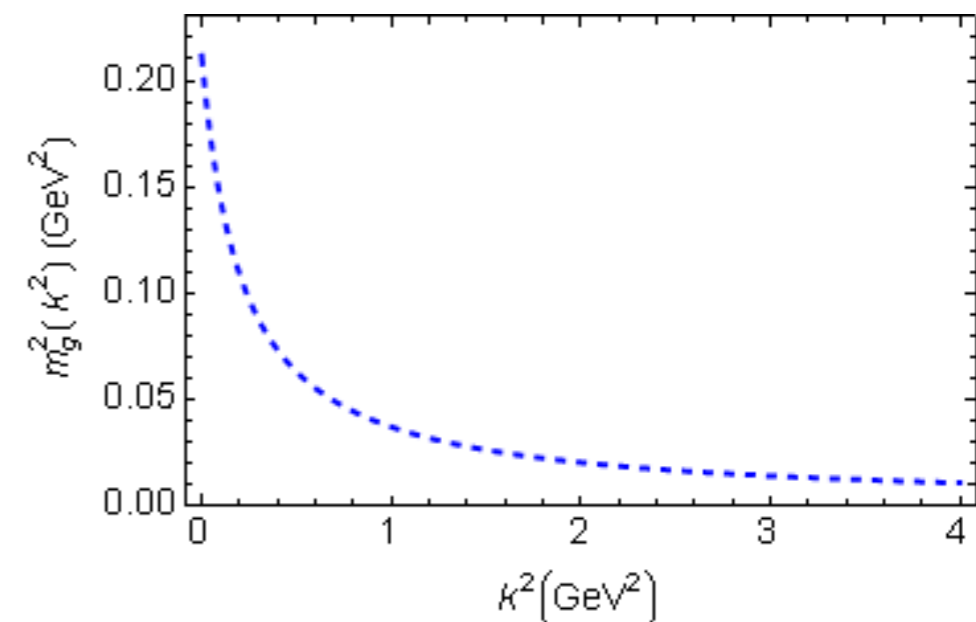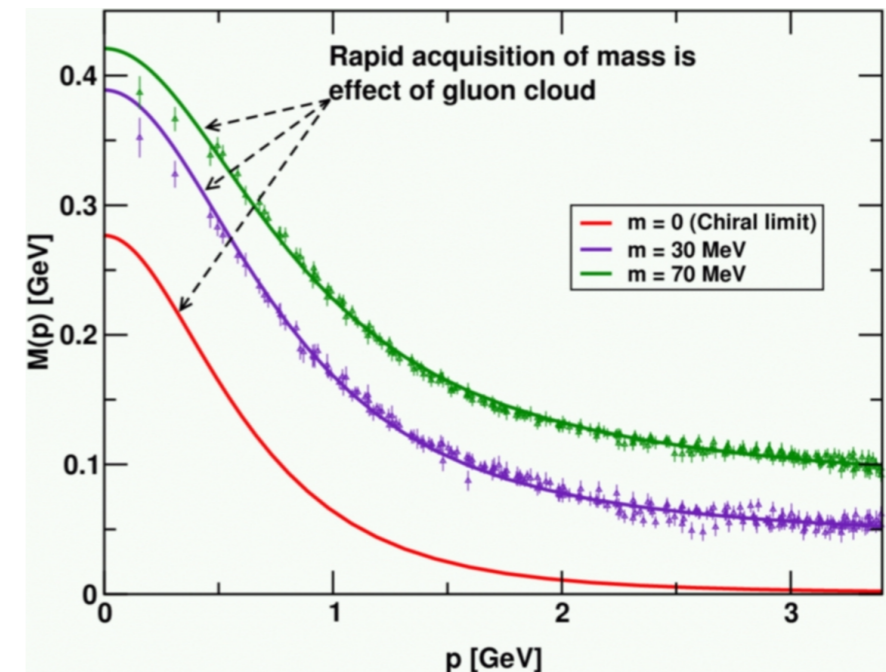
Hints from baryons

❖ Baryon is a bound state of three valence quarks

- The anti-triplet coloured diquark correlations play in simplifying the baryon three body problem
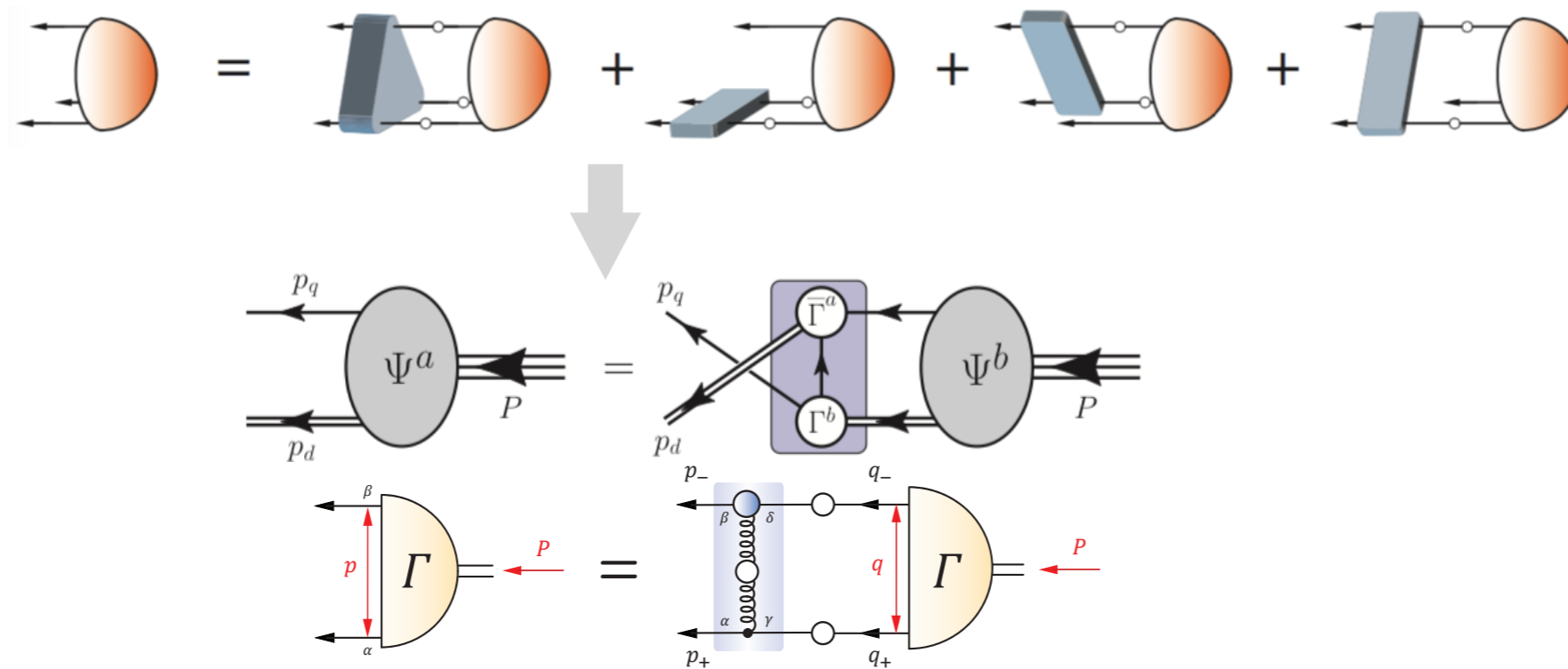
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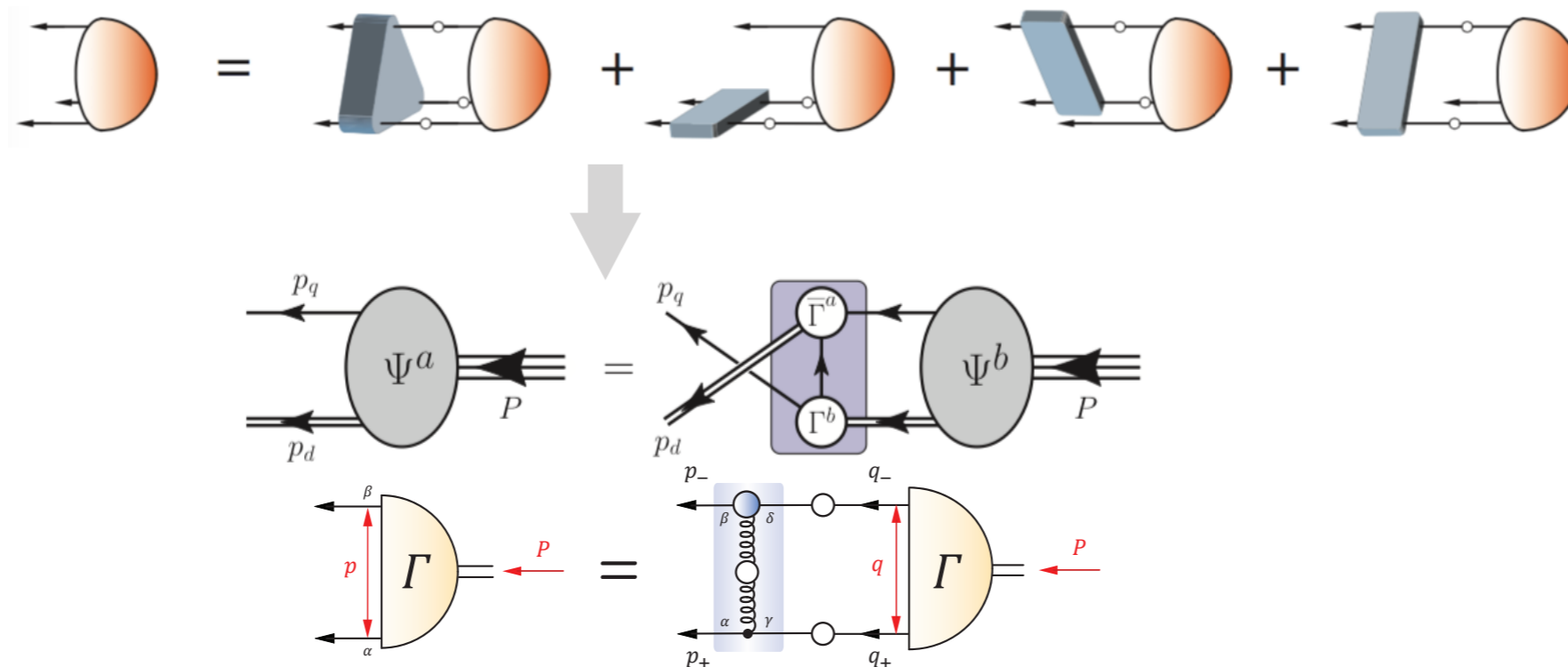
   ● The spectrum obtained from quark-diquark picture is almost same as full 3-body Faddeev equation

❖ Can hybrid states be solved in this way?

The idea towards hybrids

❖ **Suppose** strong q_g and $\overline{q}_g$ correlation exist, then

  ○ Hybrids explained by coupled channel Faddeev-like bound state equation

  Ψ=Ψ₁+Ψ₂, where Ψ₁ is Faddeev amplitude for $q_g\overline{q}$ and Ψ₂ is that for $q\overline{q}_g$
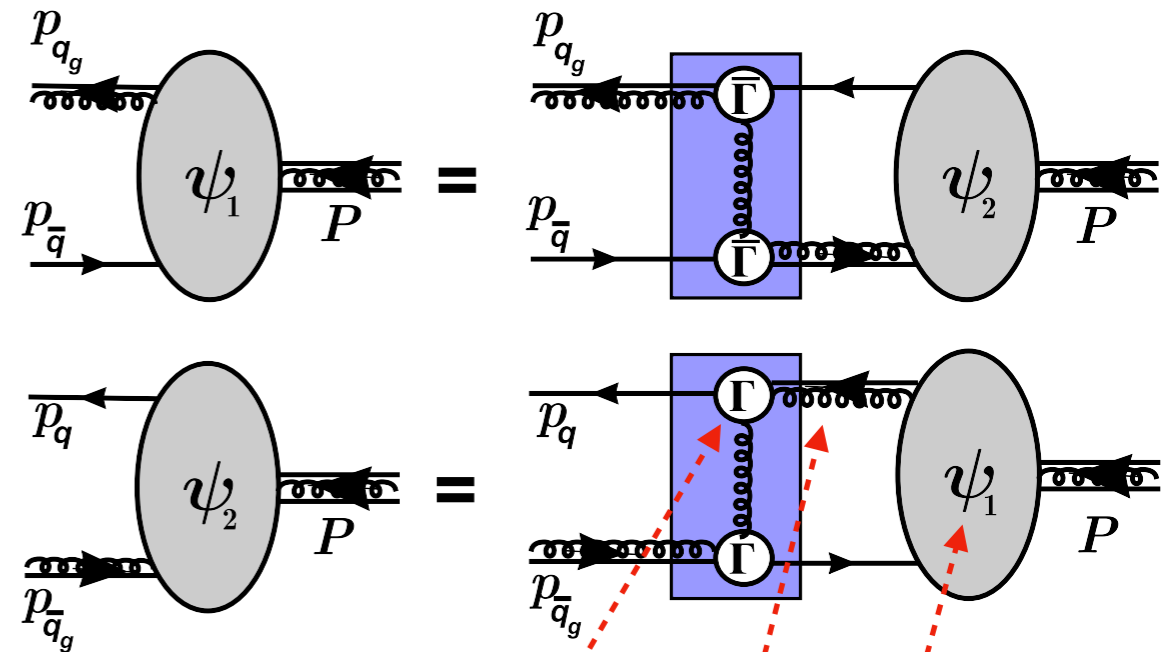
$$\Psi_1 = \Gamma^a_\mu(l;p_{qg})\, S_{gq}(p_{qg})\, \psi_1(p;P)$$

gq correlation amplitude

gq correlation propagator

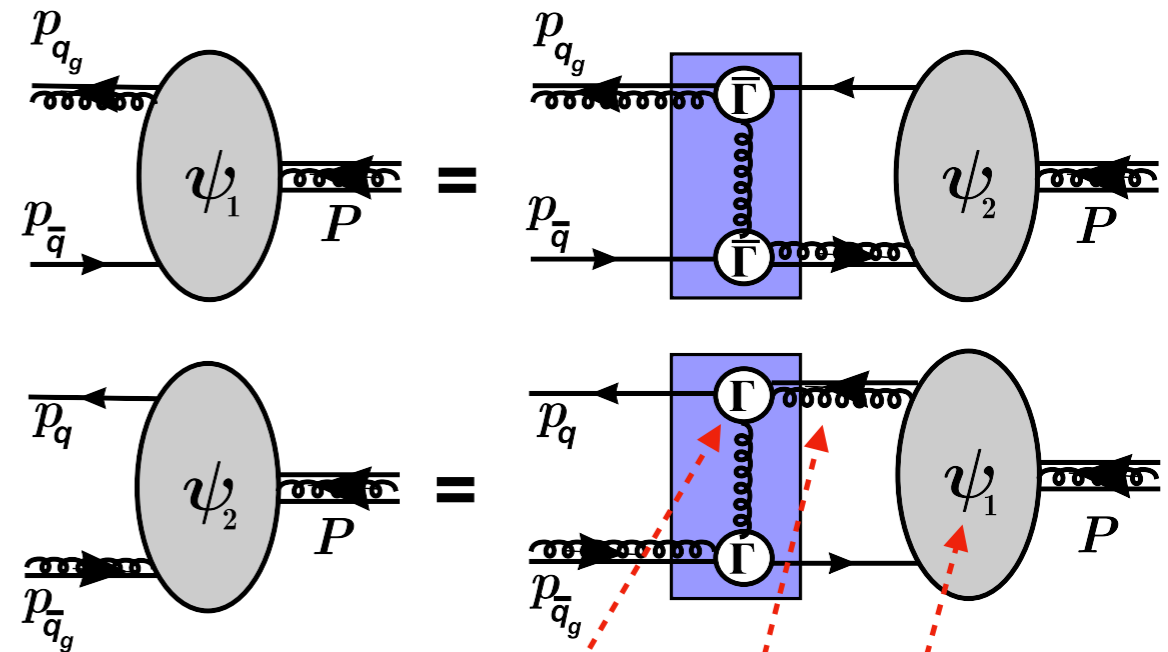bystander+correlation Faddeev amplitude

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  Ψ=Ψ$_1$+Ψ$_2$, where Ψ$_1$ is Faddeev amplitude for $q_g\overline{q}$ and Ψ$_2$ is that for $q\overline{q}_g$

❖ **Challenge:**

  ○ confirm existence of tight gluon-quark correlations

  ○ determine their properties

$$\Psi_1 = \Gamma^a{}_\mu(l;p_{qg})\, S_{gq}(p_{qg})\, \psi_1(p;P)$$
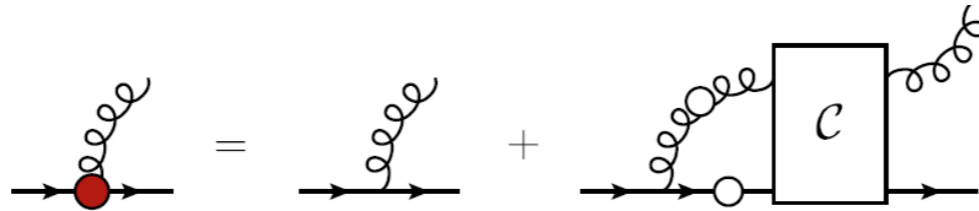
gq correlation amplitude

gq correlation propagator

bystander+correlation Faddeev amplitude

Gluon-quark correlations

❖ Using rainbow-ladder truncation for gluon-quark Bethe-Salpeter equation, and search for a pole solution
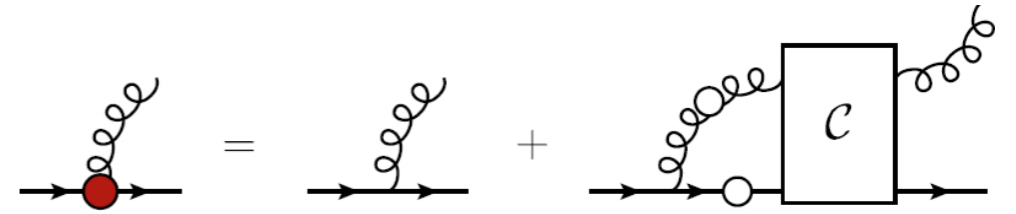

$$t^a \Gamma_\mu(p;Q) = -\int_{d\ell} \mathcal{G}(k^2) t^d \gamma_\rho S(\ell_+)$$

valence quark

bare 3-gluon vertex

$$\times\, t^c \Gamma_\lambda(\ell;Q) D_{\lambda\tau}(\bar{\ell}_-) f_{3g}(k^2)\, {}_0V^{bca}_{\rho\tau\mu}(k,\bar{\ell},-\bar{p}_-)$$

valence gluon

3g vertex dressing factor
continuum & lattice: 3g vertex greatly suppressed on $k^2 < 1$ GeV^2

- The gluon infrared mass ~1/2 m_{proton}

- The quark infrared mass ~1/3 m_{proton}

- The pole of gluon-quark corralation located at m_{qg} ~m_{proton}~1.0 GeV.

Gluon-quark correlations

❖ [gq] correlation behave like a dressed quark

- Colour-triplet fermion-like object
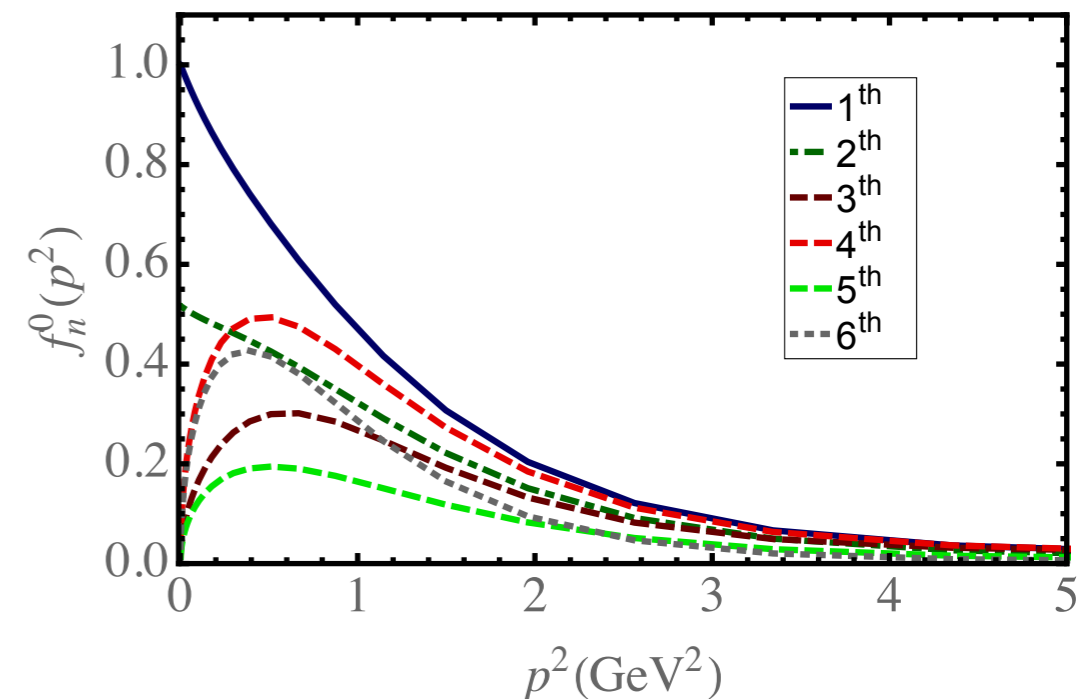
- Propagator takes the standard form

  $S_{gq}(p) = - i \, \gamma \cdot p \, \sigma_V(p^2) + \sigma_S(p^2)$

  $$\sigma_V(s) = \mathcal{E}(s, s_V), \quad \sigma_S(s) = \frac{m_{gq}}{s}[1 - s_S \mathcal{E}(s, s_S)]$$

  $E(s,s0) = (1-Exp[-s/s0])/s$

❖ The behavior of [gq] propagator

- free-particle like in UV

- infrared behavior is controlled by s_V & s_S

Hybrid spectrum in Rainbow-Ladder

JPC	0-+	1-+	1--	0+-	0--
m(GeV)RL	1.21(5)	1.78(7)	1.60(6)	1.71(7)	1.72(2)
LQCD$_R$-16^3	1.72(2)	1.73(2)	1.84(2)	2.03(1)	
LQCD$_R$-20^3	1.69(2)	1.72(2)	1.77(6)	1.99(2)	
LQCD$_R$-16^3	2.14(1)	2.15(2)	2.26(2)	2.45(1)	
LQCD$_R$-20^3	2.12(2)	2.16(2)	2.21(6)	2.43(2)	

LQCD. Row 4,5: m_π>0.4 GeV…Dudek, et al. ePrint: arXiv:1004.4930 [hep-ph]

These simulations overestimate mass of pion's first radial excitation by $\delta\pi_1$ = 0.43 GeV

LQCD. Row 2,3 = Row 4,5 - $\delta\pi_1$

- Bound states exist in all channels

- 0-+ and 1-- hybrids are structurally distinct from those accessible using the 2-body Bethe-Salpeter equation in these channel

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❖ In comparison with LQCD predictions:

- all states are too light, especially 0-+, and 1^{-+}-1^{--} ordering is reversed.

- wide variations of model parameters do not alter this outcome.

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We must reconsider each element in our formulation of hybrid mesons.
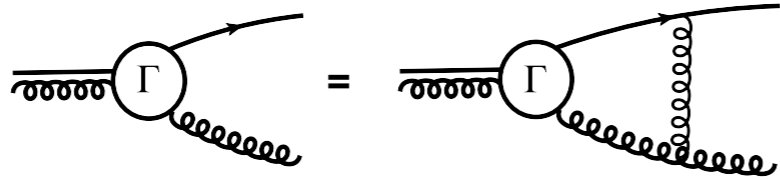
Hybrid spectrum

❖ Mismatch between RL-direct and LQCD results

❖ RL truncation can be improved

- ◉ [gq] correlation amplitude is computed in RL truncation

- ◉ RL truncation underestimates DCSB in bound state amplitudes

❖ Consequently, anomalous chromomagnetic moment (ACM) associated with this correlation is greatly underestimated

- ◉ ACM enhencement essential to explain a_1-ρ splitting.

❖ Introduce a correction factor

- ◉ Multiplication of ACM term by constant κ_{gq}

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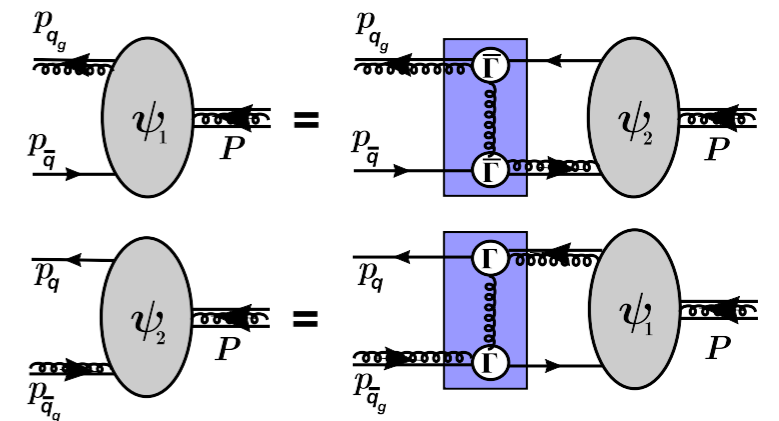❖ Can any value of k_{gq} yield match with LQCD?

Hybrid spectrum

❖ The RL truncation underestimated contributions from angular momentum.

$$\Gamma_\mu(p;Q) = \sum_{i=1}^{6} g_i(p;Q) t_\mu^i(p,Q),$$

$$t^1 = \gamma_\mu, \quad t^2 = i\hat{p}_\mu, \quad t^3 = \hat{Q}_\mu,$$

$$t^4 = i\gamma \cdot \hat{p}\,\hat{Q}_\mu, \quad t^5 = i\gamma_\mu \gamma \cdot \hat{p}, \quad t^6 = \gamma \cdot \hat{p}\,\hat{p}_\mu$$


❖ We find t^5 raised by 2.5, and omit the spin-independent coupling t^3, the hybrid spectrum will be significantly changed.

JPC	0-+	1-+	1—	0+-	0—
m(GeV)RL	1.21	1.78	1.60	1.71	1.72
m(GeV)ACM-improved	1.60	1.74	1.85	1.86	1.90
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Hybrid spectrum

❖ Beyond RL spectrum agreement with refined spectrum of LQCD

❖ Agreement is non-trivial

❖ Magnitude of our results set by

  ◦ infrared values of the running gluon and quark masses

  ◦ π and ρ meson properties

  ◦ unrelated to hybrid channels

❖ 0-- state deserves special attention

❖ LQCD predict lightest 0-- state above m_ρ+2GeV

❖ We confirm 0-- is ground-state heaviest hybrid, but probably too light.

  ◦ Large angular momentum

  ◦ Simple corrected RL truncation may not be adequate.

  ◦ DCSB-enhancement
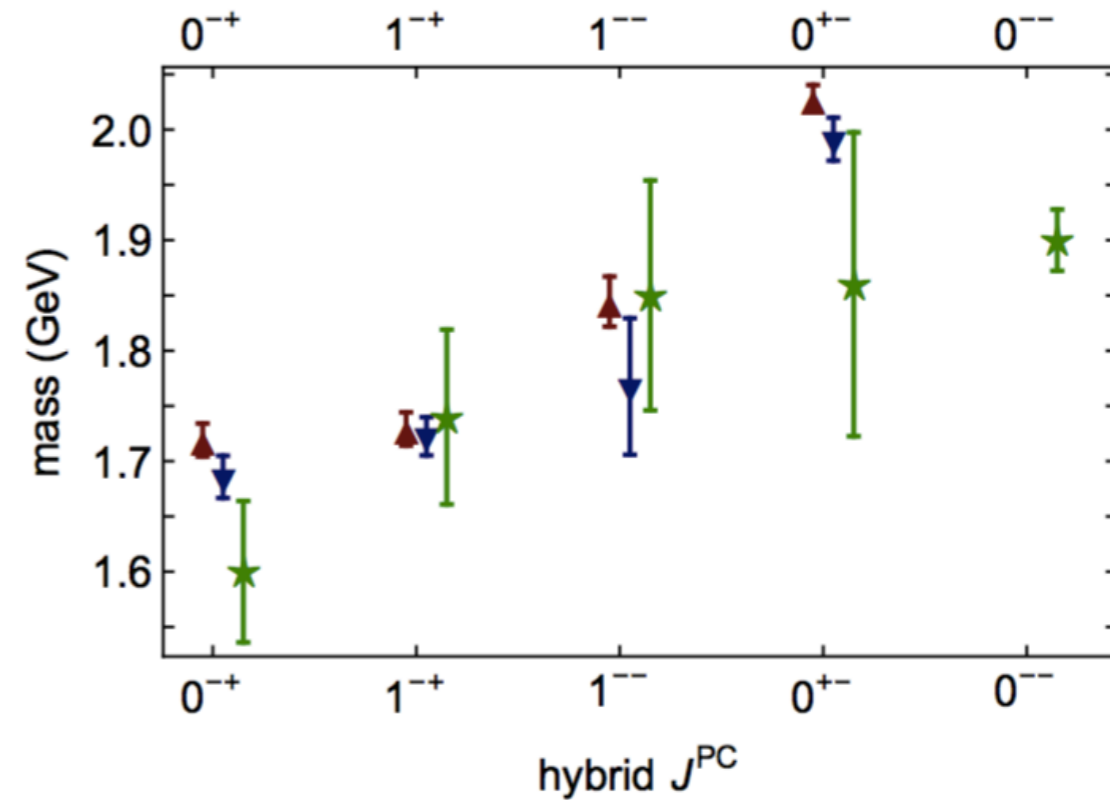
Summary

❖ We introduced a novel approach to the valence-gluon+quark+antiquark bound-state problem in quantum field theory

❖ Strong correlations exist in $[q_g = qg]$ & $[\bar{q}_g = g\bar{q}]$, and hence that a simpler, coupled pair of effectively two-body equations can provide the basis for a realistic description of hybrid mesons

❖ It reproduce the mass and ordering of ground-state light-quark hybrids obtained via LQCD

❖ It should serve as a guide for subsequent continuum treatments of the hybrid-meson three-body problem

❖ The hybrids wave functions can be used to predict a range of hybrid decays and other processes