$\gamma^*\gamma \to \eta, \eta'$ transition form factors

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Neutral Pseudoscalars

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- Axial-vector Ward-Takahashi identity

$$P_\mu \Gamma^a_{5\mu}(k;P) = \mathcal{S}^{-1}(k_+) i\gamma_5 \mathcal{F}^a + i\gamma_5 \mathcal{F}^a \mathcal{S}^{-1}(k_-) - 2i\mathcal{M}^{ab}\Gamma^b_5(k;P) - \mathcal{A}^a(k;P)\,, \quad (1)$$

  - P: total momentum, k: relative momentum.
  - $\{\mathcal{F}^a | a = 0, \ldots, N_f^2 - 1\}$: generators of $U(N_f)$, $\mathrm{tr}\mathcal{F}^a\mathcal{F}^b = \frac{1}{2}\delta^{ab}$
  - Quark propagator: $\mathcal{S} = diag[S_u, S_d, S_s, S_c, S_b, \ldots]$
  - $\mathcal{M}^{ab} = \mathrm{tr}[\{\mathcal{F}^a, \mathcal{M}\}\mathcal{F}^b]$, $\mathcal{M} = diag[m_u, m_d, m_s, m_c, m_b, \ldots]$
  - Axial-vector vertex: $\Gamma^a_{5\mu}(k;P)$, pseudoscalar vertex: $\Gamma^b_5(k;P)$
  - Non-Abelian axial anomaly: $\mathcal{A}^a(k;P)$
- Neutral Pseudoscalars: π^0, η, η', η_c, η_b

Neutral Pseudoscalars: π^0

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- Chiral limit: $\mathcal{M}^{ab} = 0$; anomaly: $\mathcal{A}^a(k;P) = 0$.

$$P_\mu \Gamma^a_{5\mu}(k;P) = \mathcal{S}^{-1}(k_+) i\gamma_5 \mathcal{F}^a + i\gamma_5 \mathcal{F}^a \mathcal{S}^{-1}(k_-) \quad (2)$$

- Quark propagator: $S^{-1}(k) = i\gamma\cdot k A(k^2) + B(k^2)$
- Axial vector vertex:

$$\lim_{P^2\to 0} P_\mu \Gamma_{5\mu}(k,P) = i\gamma_5 B(k^2) \neq 0 \quad (3)$$

- Massless pion pole: $\Gamma_{5\mu}(k,P) \xrightarrow{P^2=-M_\pi^2} \frac{f_\pi P_\mu}{P^2+M^2}\Gamma_\pi(k;P)$

$$\lim_{P^2\to 0} P_\mu \Gamma_{5\mu}(k,P=0) = i\gamma_5 f_\pi E_\pi(k;P=0)\,. \quad (4)$$

- Compare Eq.(3) & Eq.(4)

$$f_\pi E_\pi(k;P=0) = B(k^2)\,, \quad (5)$$

Neutral Pseudoscalars: π^0

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- Pion's Goldberger-Treiman relation

$$f_\pi E_\pi(k; P = 0) = B(k^2)\,, \tag{6}$$

- Pion's Bethe-Salpeter amplitude, solution of the Bethe-Salpeter equation

$$\Gamma_\pi(k; P) = \gamma_5[iE_\pi(k; P) + \gamma\cdot PF(k; P) + \gamma\cdot kk\cdot PG(k; P) + \sigma_{\mu\nu}k_\mu P_\nu H(k; P)] \tag{7}$$

- Dressed-quark propagator

$$S^{-1}(k) = i\gamma\cdot kA(k^2) + B(k^2) \tag{8}$$

- Dynamical chiral symmetry breaking (DCSB) $\Leftrightarrow$ Goldstone theorem
  - Pion exists if, and only if, mass is dynamically generated
  - Algebraically explain why pion is massless in the chiral limit
  - two body problem solved, almost completely, once solution of one body problem is known

Neutral Pseudoscalars $\eta\&\eta'$

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- Current quark mass: $\mathcal{M}^{ab} \neq 0$; anomaly: $\mathcal{A}^a(k;P) \neq 0$.

$$P_\mu \Gamma^a_{5\mu}(k;P) = \mathcal{S}^{-1}(k_+) i\gamma_5 \mathcal{F}^a + i\gamma_5 \mathcal{F}^a \mathcal{S}^{-1}(k_-) - 2i\mathcal{M}^{ab}\Gamma^b_5(k;P) - \mathcal{A}^a(k;P)\,, \tag{9}$$

- Pseudoscalar pole in vertices:

$$\Gamma_{5\mu}(k,P)|_{P^2=-m^2_{\pi_i}} = \frac{f^a_{\pi_i} P_\mu}{P^2+m^2_{\pi_i}} \Gamma_{\pi_i}(k;P) + \Gamma^{a\,reg}_{5\mu}(k;P)\,, \tag{10a}$$

$$i\Gamma_5(k,P)|_{P^2=-m^2_{\pi_i}} = \frac{\rho^a_{\pi_i}(\zeta)}{P^2+m^2_{\pi_i}} \Gamma_{\pi_i}(k;P) + i\Gamma^{a\,reg}_5(k;P)\,. \tag{10b}$$

- Pseudoscalar pole in anomaly: $\mathcal{A}^a(k;P)$:

$$\mathcal{A}^0(k;P)|_{P^2=-m^2_{\pi_i}} = \frac{n_{\pi_i}}{P^2+m^2_{\pi_i}} \Gamma_{\pi_i}(k;P) + \mathcal{A}^{0\,reg}(k;P)\,, \tag{11}$$

- Compare Eq.(10) & Eq.(11)

$$m^2_{\pi_i} f^a_{\pi_i} = \delta^{a0} n_{\pi_i} + 2\mathcal{M}^{ab}\rho^b_{\pi_i} \tag{12}$$

- Topological charge density: $n_{\pi_i} = \sqrt{\frac{N_f}{2}}\, v_{\pi_i}$, $v_{\pi_i} = \langle 0|\mathcal{Q}|\pi_i\rangle$.

Neutral Pseudoscalars $\eta\&\eta'$

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- General mass formula

$$m^2_{\eta,\eta'} f^a_{\eta,\eta'} = \delta^{a0} n_{\eta,\eta'} + 2\mathcal{M}^{ab}\rho^b_{\eta,\eta'} \tag{13}$$

- isospin limit

$$m^2_{\eta,\eta'} \begin{bmatrix} f^8_{\eta,\eta'} \\ f^0_{\eta,\eta'} \end{bmatrix} = \begin{bmatrix} 0 \\ n_{\eta,\eta'} \end{bmatrix} + 2 \begin{bmatrix} \frac{1}{3} m_{12} & \frac{\sqrt{2}}{3} m_{1-1} \\ \frac{\sqrt{2}}{3} m_{1-1} & \frac{1}{3} m_{21} \end{bmatrix} \begin{bmatrix} \rho^8_{\eta,\eta'} \\ \rho^0_{\eta,\eta'} \end{bmatrix}$$

- Flavor $U(N_f)$ limit

$$m^2_{\eta'} f^0_{\eta'} = n_{\eta'} + 2m_\zeta \rho^0_{\eta'} \tag{14}$$

- η' is split from the octet pseudoscalars by topological susceptibility

Neutral Pseudoscalars: $\eta_c \& \eta_b$

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- General mass formula

$$m^2_{\eta_{c,b}} f_{\eta_{c,b}} = 2 m_{c,b} \rho_{\eta_{c,b}} \tag{15}$$

- Non-relativistic-QCD (NRQCD):

$$m_{\eta_{c,b}} = 2 M^S_{c,b} (1 + \mathcal{E}_{c,b} / M^S_{c,b}) \tag{16}$$

  - $M^S_{c,b}$: heavy quark pole-mass; $\mathcal{E}_{c,b}$: binding-energy not grow with $M^S_{c,b}$
- Renormalisation-group-invariance of $m_{c,b} \rho_{\eta_{c,b}}$

$$\rho_{\eta_{c,b}} \overset{m_{c,b} \to \infty}{=\!=\!=\!=\!=} f_{\eta_{c,b}} m_{\eta_{c,b}} \tag{17}$$

- Identity between the $\rho_{\eta_{c,b}}$ & $f_{\eta_{c,b}}$.
  - pseudoscalar projection of the $\eta_{c,b}$ Bethe-Salpeter wave function
  - pseudovector projection of the $\eta_{c,b}$ Bethe-Salpeter wave function

Neutral Pseudoscalars:

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- Axial-vector Ward-Takahashi identity
- π^0:
  - $f_\pi E_\pi(k; P=0) = B(k^2)$
  - Dynamical chiral symmetry breaking (DCSB) $\Leftrightarrow$ Goldstone theorem
  - Pion exists if, and only if, mass is dynamically generated
- $\eta \& \eta'$:
  - $m^2_{\eta'} f^0_{\eta'} = n_{\eta'} + 2 m_\zeta \rho^0_{\eta'}$
  - η' is split from the octet pseudoscalars by topological susceptibility
- η_c, η_b:
  - $\rho_{\eta_{c,b}} \xrightarrow{m_{c,b}\to\infty} f_{\eta_{c,b}} m_{\eta_{c,b}}$
  - Identity between the pseudoscalar projection and pseudovector projection of the Bethe-Salpeter wave function .
- Parton structure of π^0, η, η', η_c, η_b with two photon transition form facotor.

Motivation

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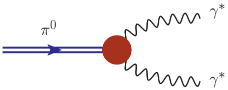
- transition form factors: $\gamma^*\gamma \to M$[1], Strong-QCD prediction.

$$Q^2 F_M^q(Q^2) \overset{Q^2 > Q_0^2}{\approx} 4\pi^2 f_M^q e_q^2 \tilde{w}_M^q(Q^2)$$



  - $q\bar{q}$ component of M; $Q_0 > \Lambda_{QCD}$; f_M^q, $q\bar{q}$-component leptonic decay constant; e, electric charge of quark q.

$$\tilde{w}_M^q(Q^2) = \int_0^1 dx \frac{1}{x} \varphi_M^q(x, Q^2)\,. \tag{18}$$

  - $\varphi_M^q(x, Q^2)$, dressed-valence q-parton contribution to the meson's distribution amplitude (DA).

- Complete transition form factor is a sum over the various $q\bar{q}$ subcomponent contributions

$$F_M = \sum_{q \in M} \psi_M^q F_M^q \tag{19}$$

  - ψ_M^q, flavour weighting factor

[1] G.P.Lepage and S.J.Brodsky. Exclusive processes in perturbative quantum chromodynamics. Phys. Rev. D 22, 2157 (1980).

Motivation

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- Asymptotic DA: $\tau^2 := \Lambda_{QCD}^2/Q^2$

$$\varphi_M(x,Q^2) \overset{\tau\simeq 0}{\approx} \varphi_\infty = 6x(1-x)\,, \tag{20}$$

- $\gamma^*\gamma\to M$ transition form factor exhibits simple power-law scaling

$$Q^2 F_M^q(Q^2) \overset{\tau\simeq 0}{\approx} 12\pi^2 f_M^q e_q^2 \tag{21}$$

- Questions:
  - π^0, η_c, η_b
    - ★ Is experimental data consistent with Eq.(21)?
    - ★ TFFs with Q^2 is small or medium
    - ★ DAs of π^0, η_c, η_b with hadronic scale
  - η, η': $F_M = \sum_{q\in M} \psi_M^q F_M^q$
    - ★ Non-Abelian axial anomaly, will Eq.(21) be amended?
    - ★ TFFs with Q^2 is small or medium
    - ★ Flavour structure of η, η'
    - ★ DA of η, η' with hadronic scale

TFFs in Experiment

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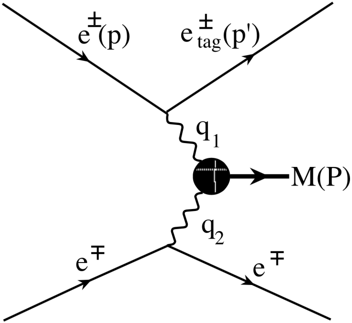

- No-tagged mode: two photon decay widths[1].
  - $q_1^2 \approx q_2^2 \approx 0$.
  - $\Gamma^{\pi^0 \to \gamma\gamma} = \frac{\alpha_{em}^2}{64\pi^3} \frac{m_\pi^3}{f_\pi^2} = 7.8$ eV.
    Calculated exactly relate to chiral anomaly.
  - $\Gamma^{\chi_\eta \to \gamma\gamma} = 0.516 \pm 0.018$ keV.
  - $\Gamma^{\chi_{\eta'} \to \gamma\gamma} = 4.36 \pm 0.14$ keV.
  - $\Gamma^{\eta_c \to \gamma\gamma} = 5.0 \pm 0.4$ keV.
- Single-tagged mode:
  transition form factors (TFFs).
  - $q_1^2 \neq 0, q_2^2 \approx 0$.
  - CELLO, (1991).
  - CLEO, (1998).
  - BaBar, (2009, 2010, 2011).
  - Belle, (2012).
- Doubel-tagged mode:
  - $q_1^2 \neq 0, q_2^2 \neq 0$.
  - BaBar, (2018)[2].

[1] M. Tanabashi et al. (Particle Data Group) Phys. Rev. D 98, 030001 (2018).

[2] BaBar Collaboration. Measurement of the $\gamma^*\gamma^* \to \eta'$ transition form factor. arXiv:1808.08038 [hep-ex].

TFFs in Experiment

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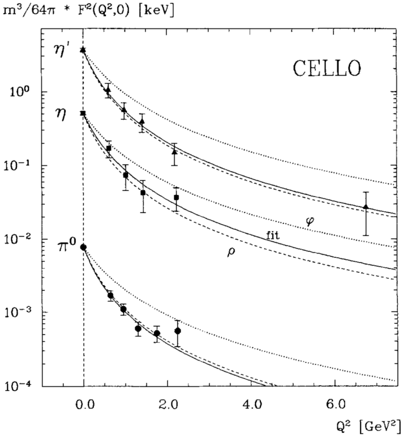
- CELLO, (1991)[1]: Singly-virtual transition form factors: $q_1^2 \neq 0, q_2^2 \approx 0$.

- $\gamma\gamma^* \to \pi^0, \eta, \eta'$.
- $Q^2 \in [0.62, 2.23]$ GeV^2
- The π^0 form factor first time observed in the space-like region.
- Agree well with ρ-pole predicted by the vector meson dominance (VMD) model. $F(Q^2) = A/(1 + Q^2/\Lambda_P^2)$
- Agree with the QCD inspired Brodsky-Lepage model, $F_{\gamma\gamma^* P}(Q^2) \sim \frac{1}{4\pi^2 f_P} \frac{1}{1+(Q^2/8\pi^2 f_P^2)}$.
  (i) $F(Q^2 = 0) \propto 1/f_P$
  (ii) $Q^2 F(Q^2 \to \infty) \propto f_P$



[1] CELLO Collaboration, A Measurement of the π^0, η and η' electromagnetic form factors, Z.Phys. C49 (1991) 401-410.

TFFs in Experiment

- CLEO, (1998)[1]

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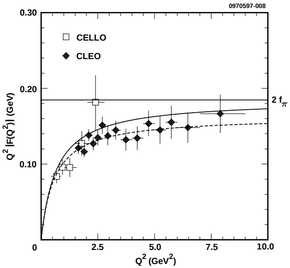
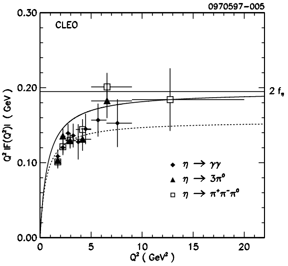
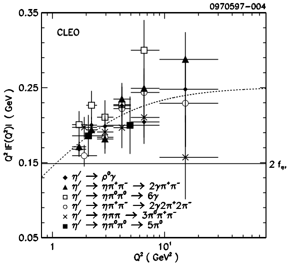
- $\gamma\gamma^* \to \pi^0, \eta, \eta'$.
- π^0 : $Q^2 \in [1.5, 9]$ GeV^2.
- η : $Q^2 \in [1.5, 20]$ GeV^2.
- η' : $Q^2 \in [1.5, 30]$ GeV^2.
- Dashed line: pole-mass formula
  $F_{\gamma\gamma^*\mathcal{R}}(Q^2) = \frac{1}{4\pi\alpha}\sqrt{\frac{64\pi\Gamma(\mathcal{R}\to\gamma\gamma)}{M_{\mathcal{R}}^3}}\frac{1}{1+Q^2/\Lambda_{\mathcal{R}}^2}$.
- Solid line: Brodsky-Lepage model, $F_{\gamma\gamma^*\mathcal{R}}(Q^2) \sim \frac{1}{4\pi^2 f_{\mathcal{R}}}\frac{1}{1+(Q^2/8\pi^2 f_{\mathcal{R}}^2)}$
  satisfying
  $F_{\gamma\gamma^*\mathcal{R}}(Q^2=0) = \frac{1}{4\pi^2 f_{\mathcal{R}}}$,
  $Q^2 F_{\gamma\gamma^*\mathcal{R}}(Q^2\to\infty) = 2f_{\mathcal{R}}$.
- data lie blow pQCD limit: $2f_\pi$





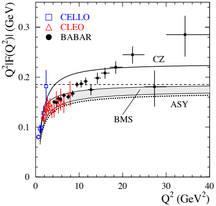


[1] CLEO Collaboration, Measurements of the meson photon transition form factors of light pseudoscalar mesons at large momentum transfer, Phys.Rev. D57 (1998) 33-54.

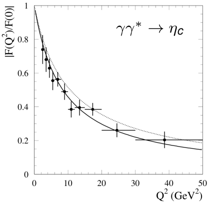
TFFs in Experiment

- BaBar, (2009, 2010, 2011)[1,2,3]

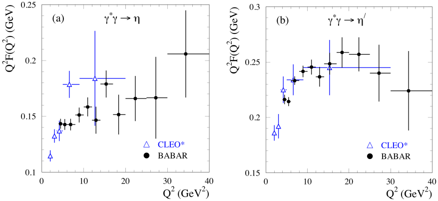
- $\gamma\gamma^* \to \pi^0$.
  - π^0 : $Q^2 \in [4, 40]$ GeV^2.
  - $Q^2 F_{\gamma\gamma^*\pi}(Q^2) = A\left(\frac{Q^2}{10GeV^2}\right)^{\beta}$, $A_\pi = 0.182$GeV, $\beta = 0.25$.
  - data grow with Q^2 above 10 GeV^2, Chernyak-Zhitnitsky DA (CZ).



- $\gamma\gamma^* \to \eta_c$.
  - η_c : $Q^2 \in [2, 50]$ GeV^2.
  - Monopole form $F_{\gamma\gamma^*\eta_c}(Q^2) = \frac{F_{\eta_c}(0)}{1+Q^2/\Lambda}$
  - data lie below leading-order pQCD prediction



- $\gamma\gamma^* \to \eta, \eta'$.
  - η, η' : $Q^2 \in [4, 40]$ GeV^2.
  - Agreement with CLEO, improve precision.



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[1] BaBar Collaboration. Measurement of the $\gamma\gamma^* \to \pi^0$ transition form factor. Phys.Rev. D80 (2009) 052002.

[2] BaBar Collaboration. Measurement of the $\gamma\gamma^* \to \eta_c$ transition form factor. Phys.Rev. D81 (2010) 052010.

[3] BaBar Collaboration. Measurement of the $\gamma\gamma^* \to \eta$ and $\gamma\gamma^* \to \eta'$ transition form factors. Phys.Rev.D84 (2011) 052001.

TFFs in Experiment

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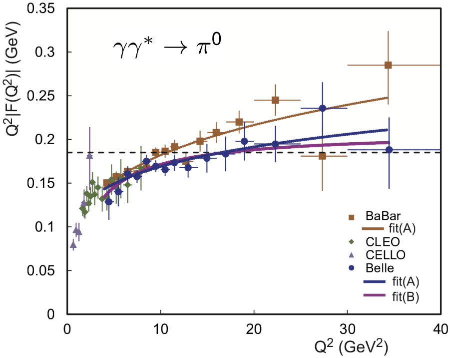
- Belle, (2012)[1]

- $\gamma\gamma^* \to \pi^0$.
- π^0 : $Q^2 \in [4, 40]$ GeV^2.
- Consistency with BaBar up to 9 GeV^2.
- Belle do not confirm auxetic BaBar form factor behavior above 10 GeV^2.
- data not always lie blow pQCD limit: $2f_\pi$.
- fit(A): $Q^2 F_{\gamma\gamma^*\pi}(Q^2) = A\left(\frac{Q^2}{10GeV^2}\right)^\beta$,
  $A_{Babar} = 0.182\text{GeV}$, $\beta_{Babar} = 0.25$,
  $A_{Belle} = 0.169\text{GeV}$, $\beta_{Belle} = 0.18$
- ♣ fit(B): $Q^2 F_{\gamma\gamma^*\pi}(Q^2) = \frac{BQ^2}{Q^2+C}$,
  $B_{Belle} = 0.209\text{GeV}$, $C_{Belle} = 2.2\text{GeV}^2$.
- pQCD: $Q^2 F(Q^2 \to \infty) = 2f_\pi$,
  $B_{pQCD} = 0.185\text{GeV}$.



[1] Belle Collaboration. Measurement of the $\gamma\gamma^* \to \pi^0$ transition form factor at Belle. Phys.Rev. D86 (2012) 092007.

TFFs in Experiment

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- $\gamma\gamma^* \to \pi^0$
  - CELLO, (1991):
    - ★ $Q^2 \in [0.68, 2.17]$ GeV^2, first time observed
    - ★ agree well with ρ-pole VMD model, $F(Q^2) = A/(1 + Q^2/\Lambda_\pi^2)$
    - ★ agree with Brodsky-Lepage model, $F(Q^2) \approx \frac{1}{4\pi^2 f_\pi} \frac{1}{1+(Q^2/8\pi^2 f_\pi^2)}$
  - CLEO, (1998):
    - ★ $Q^2 \in [1.5, 9]$ GeV^2
    - ★ VMD-like model describes the data very well
    - ★ data lie blow pQCD limit: $2f_\pi$
    - ★ VMD-like model $\leftrightsquigarrow$ Brodsky-Lepage pQCD asymptotic prediction
  - BaBar, (2009):
    - ★ $Q^2 \in [4, 40]$ GeV^2
    - ★ data grow with Q^2 above 10 GeV^2
    - ★ $Q^2 F_{\gamma\gamma^*\pi}(Q^2) = A\left(\frac{Q^2}{10GeV^2}\right)^\beta$, $A_\pi = 0.182$GeV, $\beta = 0.25$
    - ★ Chernyak-Zhitnitsky (CZ) distribution amplitude (DA).
  - Belle, (2012):
    - ★ $Q^2 \in [4, 40]$ GeV^2
    - ★ Consistency with BaBar up to 9 GeV^2.
    - ★ Belle do not confirm auxetic BaBar form factor behavior above 10 GeV^2.
    - ★ data not always lie blow pQCD limit: $2f_\pi$

TFFs in Experiment

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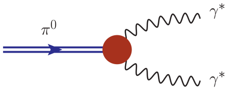
- $\gamma\gamma^* \to \eta_c$
  - BaBar, (2010):
    - ★ $Q^2 \in [2, 50]$ GeV^2
    - ★ Monopole form $F_{\gamma\gamma^*\eta_c}(Q^2) = \frac{F_{\eta_c}(0)}{1+Q^2/\Lambda}$, $\Lambda_{Babar} = 8.5\text{GeV}^2$
    - ★ agreement with J/Ψ-pole VMD model: $\Lambda = m^2_{J/\Psi} = 9.6\text{GeV}^2$.
    - ★ data lie below leading-order pQCD prediction
- $\gamma\gamma^* \to \eta, \eta'$
  - CELLO, (1991):
    - ★ $Q^2 \in [0.62, 2.23]$ GeV^2
    - ★ Agree well ρ-pole VMD model, $\Lambda_\eta = 0.84\text{GeV}$, $\Lambda_{\eta'} = 0.81\text{GeV}$
  - CLEO, (1998):
    - ★ $Q^2 \in [1.5, 20\&30]$ GeV^2
    - ★ Agree well pole-mass model, $\Lambda_\eta = 0.77\text{GeV}$, $\Lambda_{\eta'} = 0.86\text{GeV}$
    - ★ TFF shapes of π^0 and η are nearly identical
    - ★ Non-perturbative properties of η' differ substantially from π^0 and η.
  - BaBar, (2011):
    - ★ $Q^2 \in [4, 40]$ GeV^2
    - ★ Agreement with CLEO, improve precision.
    - ★ Relation between TFFs with theoretical meson DA.

- Questions:
  - Empirical effective of VMD assumption
  - Brodsky-Lepage pQCD asymptotic prediction
  - $\gamma\gamma^* \to \pi^0$:
    - ★ Conflict between BaBar and Belle above 10 GeV^2
    - ★ Belle data not always lie blow pQCD limit: $2f_\pi$
    - ★ Distribution amplitude (DA) of π^0 at hadronic scale
  - $\gamma\gamma^* \to \eta_c$:
    - ★ Data lie below leading-order pQCD prediction
    - ★ DA of η_c at hadronic scale
  - $\gamma\gamma^* \to \eta, \eta'$:
    - ★ Non-perturbative properties of η, η'
    - ★ DA of η, η' at hadronic scale

TFFs in theory

- Defined by pseudoscalar-vector-vector (PVV) vertex

$$\Lambda_{\mu\nu}^{\gamma^*\gamma^*\pi^0} = \frac{2i\alpha_{em} g_{\gamma\gamma\pi}}{\pi \tilde{f}_\pi} \epsilon_{\mu\nu\rho\sigma} Q_{1\rho} Q_{2\sigma} F_{\gamma^*\gamma^*\pi^0}(Q_1^2, Q_2^2)$$



  - two photon momenta Q_1 and Q_2, $\tilde{f}_\pi = f_\pi/\sqrt{2} = 93$MeV, pion decay constant.

- Normalization fixed by the axial anomaly

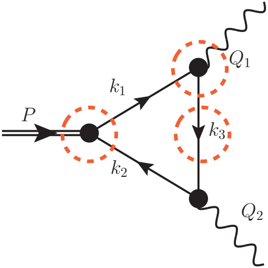
$$\Gamma^{\pi^0\to\gamma\gamma} = \frac{g_{\gamma\gamma\pi}^2 \alpha_{em}^2 m_\pi^3}{64\pi^3 \tilde{f}_\pi^2} \tag{22}$$

  - $F_{\gamma^*\gamma^*\pi^0}(0,0) = 1$, $g_{\gamma\gamma\pi} = 1/2$, $\Gamma^{\pi^0\to\gamma\gamma} = 7.7$ eV.

TFFs with DSE

- Transition form factor in impulse approximation[1]:

$$\Lambda^{q}_{\alpha\beta}(Q_1^2, Q_2^2) = e^2 N_c \int_{dk}^{\Lambda} \mathrm{tr}\left[S(k_1)\Gamma_M^{q\bar{q}}(k;P)S(k_2)\right.$$
$$\left.\times i\Gamma_\beta(k+Q_1/2;Q_2)S(k_3)i\Gamma_\alpha(k-Q_2/2;Q_1)\right], \quad (23)$$



- $k_1 = k + P/2$, $k_2 = k - P/2$,
  $k_3 = k + (Q_1 - Q_2)/2$,
  Q_1, Q_2 momentum of the photons.
- $S(k_i)$, dressed quark propagator.
- $\Gamma_M^{q\bar{q}}$, meson Bethe-Salpeter amplitude.
- $\Gamma_{\alpha/\beta}$, quark-photon vertex.

$$\Lambda^{\pi^0\to\gamma\gamma^*}_{\alpha\beta}(Q_1;Q_2) = \frac{2}{\sqrt{2}}\left((\hat{Q}^u)^2 - (\hat{Q}^d)^2\right)\Lambda^{u}_{\alpha\beta}(Q_1;Q_2) \quad (24)$$

[1] P. Maris and P. C. Tandy, Electromagnetic transition form factors of light mesons, Phys.Rev. C65 (2002) 045211

TFFs with DSEs

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- $S(k_i)$, dressed quark propagator.

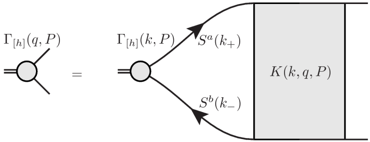
$$S(p)^{-1} = Z_2 i\gamma \cdot p + Z_4 m_q(\mu) + \frac{4}{3} Z_2^2 \int_{dq}^{\Lambda} \mathcal{G}((p-q)^2) \times D_{\alpha\beta}(p-q)\gamma_\alpha S(q)\gamma_\beta \quad (25)$$

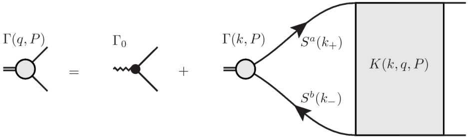
- $\Gamma_M^{q\bar{q}}$, meson Bethe-Salpeter amplitude.

$$\Gamma(q;P) = -\frac{4}{3} Z_2^2 \int_{dk}^{\Lambda} \Big[\mathcal{G}((k-q)^2) \times D_{\alpha\beta}(k-q)\gamma_\alpha S(k_+)\Gamma(k;P)S(k_-)\gamma_\beta\Big], \quad (26)$$

- $\Gamma_{\alpha/\beta}$, quark-photon vertex.

$$\Gamma_\mu^q(q;P) = Z_2\gamma_\mu - \frac{4}{3} Z_2^2 \int_{dk}^{\Lambda} [\mathcal{G}((k-q)^2) \times D_{\alpha\beta}(k-q)\gamma_\alpha S^q(k_+)\Gamma_\mu^q(k;P)S^q(k_-)\gamma_\beta]. \quad (27)$$





TFFs with DSEs

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- Static frame for the meson:

$$P = (0, 0, 0, im_H) \,,$$
$$Q_1 = \Big(0, 0, \frac{m_H}{2} + \frac{q^2}{2m_H}, i(\frac{q^2}{2m_H} - \frac{m_H}{2})\Big),$$
$$Q_2 = \Big(0, 0, -\frac{m_H}{2} - \frac{q^2}{2m_H}, -i(\frac{q^2}{2m_H} + \frac{m_H}{2})\Big), \tag{28}$$

  - satisfying the constraint $P^2 = -m_H^2,\ Q_1^2 = q^2,\ Q_2^2 = 0.$

- Continuation of the Bethe-Salpeter amplitude:

$$\Gamma(k; P) = \Gamma(k^2, z) = \sum_{m=0, j=0}^{N,M} a_{mj} P_m(k^2) U_j(z) \,, \tag{29}$$

- $P(m)$-Legendre polynomial, $U_j(z)$-Chebyshev polynomial. $N = 20, M = 4$. is enough for convergence.

Two photon decay width with DSEs

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- $\Gamma^{\pi^0\to\gamma\gamma} = 7.7\text{eV}$.

	$\omega = 0.5$	$\omega = 0.65$	$\omega = 0.8$	exp
$\Gamma^{\eta_c\to\gamma\gamma}$	6.32	6.36	6.39	5.1±0.4
$\Gamma^{\chi_{c0}\to\gamma\gamma}$	2.06	2.20	2.39	2.33±0.42
$\Gamma^{\chi_{c2}\to\gamma\gamma}$	0.401	0.464	0.655	0.63±0.10
$\sigma_{0/2}$	0.0185	0.00813	0.00413	0.00±0.04
$\frac{\Gamma^{\chi_{c2}\to\gamma\gamma}}{\Gamma^{\chi_{c0}\to\gamma\gamma}}$	0.191	0.211	0.274	0.27±0.06

$\Gamma^{\eta_b\to\gamma\gamma}$	$\Gamma^{\chi_{b0}\to\gamma\gamma}$	$\Gamma^{\chi_{b2}\to\gamma\gamma}$	$f_{0/2}$	$\frac{\Gamma^{\chi_{b2}\to\gamma\gamma}}{\Gamma^{\chi_{b0}\to\gamma\gamma}}$
0.469	0.0600	0.0143	0.000198	0.238

- $\Gamma^{\eta_c\to\gamma\gamma}$:[1]

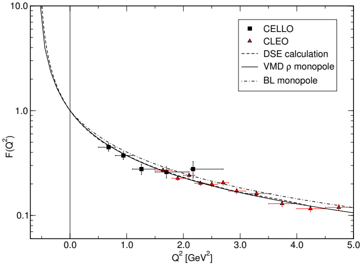
$$\Gamma^{\eta_c\to\gamma\gamma} = 8\pi\left(\frac{2}{3}\right)^4 \frac{\alpha_{em}^2}{m_{\eta_c}} \frac{f_{\eta_c}^2}{(1+\delta)^2}\,, \tag{30}$$

- $f_{\eta_c} = 0.24\text{GeV}$, $\delta = 0.03$, $\Gamma^{\eta_c\to\gamma\gamma} = 4.8\text{keV}$.

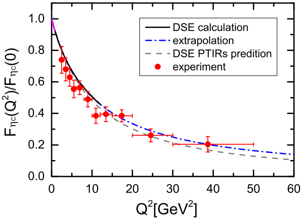
[1] J. Chen, M. Ding, L. Chang and Y.X. Liu, Two photon transition form factor of $c\bar{c}$ quarkonia, Phys.Rev. D95 (2017) no.1, 016010.

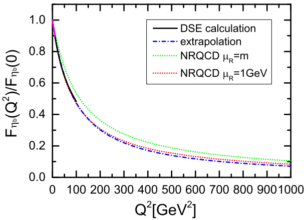
TFFs with DSEs

- Pion TFF[1]:



- η_c and η_b TFFs[2]:





- Singularity of quark propagator,
  F_{π^0} up to 5GeV^2 $\leftrightsquigarrow$ Belle $Q^2 \in [4, 40]$ GeV^2
  F_{η_c} up to 12GeV^2 $\leftrightsquigarrow$ BaBar $Q^2 \in [2, 50]$ GeV^2
  F_{η_b} up to 100GeV^2.
- Low Q^2: VMD-like model
  High Q^2: Brodsky-Lepage asymptotic form
- Need to do extrapolation.

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1 P. Maris and P. C. Tandy, Electromagnetic transition form factors of light mesons, Phys.Rev. C65 (2002) 045211

2 J. Chen, M. Ding, L. Chang and Y.X. Liu, Two photon transition form factor of $c\bar{c}$ quarkonia, Phys.Rev. D95 (2017) no.1, 016010.

TFFs with DSEs

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- TFF extrapolation methods:
  - Method based on Distribution amplitude (DA):
  - Off-shell method:
    - ★ Introducing a virtuality eigenvalue $\lambda(\upsilon)$ into the Bethe-Salpeter equations.
    - ★ Considering the υ-dependence of the pointwise behaviour of the Bethe-Salpeter amplitude.
- $\gamma\gamma^* \to \pi^0$ TFF with large momentum transfer[1,2]:

$$Q^2 F_\pi(Q^2) \overset{Q^2 \geq Q_0^2}{=} 4\pi^2 f_\pi \frac{1}{3} \int_0^1 dx \frac{\phi_\pi(x; Q^2)}{x} \tag{31}$$

- $\phi_\pi(x; Q^2)$, two-particle twist-two DA.

[1] G.P.Lepage and S.J.Brodsky. Exclusive processes in perturbative quantum chromodynamics. Physical Review D, 1980.
[2] A.Efremov and A.Radyushkin. Factorization and asymptotic behaviour of pion form factor in QCD. Physics Letters B, 1980.

Leading twist DAs

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- Matrix elements:

$$\langle 0|\overline{\psi}(-z)\hat{O}\psi(z)|\pi(P)\rangle \tag{32}$$

- Twist-2 operator: $\overline{\psi}_+\hat{O}\psi_+$, and $\hat{O}\in\{\gamma_+,\gamma_+\gamma_5,\sigma_{+\perp},\sigma_{+\perp}\gamma_5\}$

$$\langle 0|\overline{u}(-z)\gamma_\mu\gamma_5 d(z)|\pi(P)\rangle = f_\pi P_\mu \int_0^1 dx e^{-i(2x-1)z\cdot P}\phi_\pi(x) \tag{33}$$

- G-parity transform:

$$\hat{G}\overline{u}(-z)\gamma_\mu\gamma_5 d(z)\hat{G}^\dagger = -\overline{u}(z)\gamma_\mu\gamma_5 d(-z)$$

$$\begin{aligned}&\langle 0|\overline{u}(-z)\gamma_\mu\gamma_5 d(z)|\pi(P)\rangle\\ =&\langle 0|\hat{G}^\dagger\left(\hat{G}\overline{u}(-z)\gamma_\mu\gamma_5 d(z)\hat{G}^\dagger\right)\hat{G}|\pi(P)\rangle\\ =&\langle 0|\hat{G}^\dagger\left(-\overline{u}(z)\gamma_\mu\gamma_5 d(-z)\right)\hat{G}|\pi(P)\rangle\end{aligned}$$

- π: $I^G = 1^- \to \hat{G}|\pi(P)\rangle = -|\pi(P)\rangle$

- Pion DA: $\phi_\pi(x)$

$$\begin{aligned}&\langle 0|\overline{u}(-z)\gamma_\mu\gamma_5 d(z)|\pi(P)\rangle\\ =&\langle 0|\overline{u}(z)\gamma_\mu\gamma_5 d(-z)|\pi(P)\rangle\\ =&f_\pi P_\mu\int_0^1 dx e^{-i(2x-1)z\cdot P}\phi_\pi(x)\\ =&f_\pi P_\mu\int_0^1 dx e^{i(2x-1)*(-z)\cdot P}\phi_\pi(1-x)\end{aligned} \tag{34}$$

- $\Longrightarrow \phi_\pi(x) = \phi_\pi(1-x)$

Leading twist DAs

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- Matrix elements:

$$\langle 0|\psi(-z)\gamma_5\gamma.n\psi(z)|\pi(P)\rangle = f_\pi n\cdot P\int_0^1 dx e^{-i(2x-1)z\cdot P}\phi(x)\,,$$
$$= tr_{CD}Z_2\int_{dq}^{\Lambda} e^{-iz\cdot q-iz\cdot(q-P)}\gamma_5\gamma.n\chi(q;P)\,. \quad (35)$$

- Projecting Bethe-Salpeter wave function onto the light front:

$$f_\pi\phi(x) = tr_{CD}Z_2\int_{dq}^{\Lambda}\delta(n\cdot q_+ - xn\cdot P)\gamma_5\gamma\cdot n\chi(q;P)\,. \quad (36)$$

  - n, light-like four-vector, $n^2 = 0$.
  - f_π, decay constant.
  - $\chi(q;P)$, Bethe-Salpeter wave function, the solution of Bethe-Salpeter equation.
- S: dressed-quark propagator; Γ: Bethe-Salpter amplitude.

$$\chi(k,P) = S(k_+)\Gamma(k,P)S(k_-)\,, \quad (37)$$

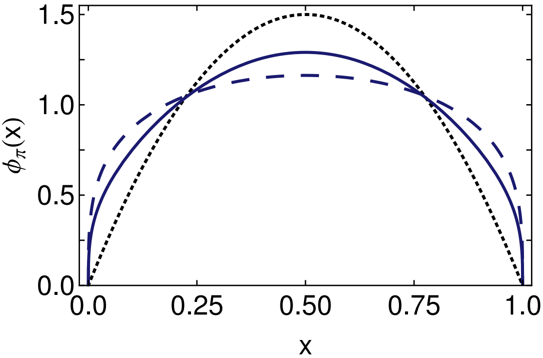
Methods computing DAs with DSEs

- Moments: $\langle x^m \rangle = \int_0^1 dx x^m \phi(x)$
  - Perturbation theory integral representations (PTIRs):
    - ★ Infinite number of Mellin moments.
    - ★ Combine denominators $\Rightarrow$ the integral over feynman parameters.
    - ★ Represent the Bethe-Sapeter wave function with parameters.
  - "Brute-force" approach:
    - ★ Limited number of Mellin moments.

- Spectral function: $\chi(q,P) = \int_{-1}^{1} dz \int_0^\infty d\gamma \frac{g(z,\gamma)}{(q^2+zq\cdot P+\frac{1}{4}P^2+M^2+\gamma)^3}$
  - Maximum entropy method (MEM):
    - ★ Well-known method to solve the ill-posed inversion problem.
    - ★ Extract the weight function of Bethe-Salpeter wave function.

Pion DA

- Pion DA from DSE[1]:



- Properties:
  - Solid line: Broad, concave function at hadronic scale.
  - Dotted line: Brodsky-Lepage asymptotic prediction: $\phi_\pi(x) = 6x(1-x)$.

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[1] L. Chang, I.C. Cloet, J.J. Cobos-Martinez, C.D. Roberts, S.M. Schmidt, and P.C. Tandy, Phys. Rev. Lett. 110, 132001 (2013).

Pion TFF

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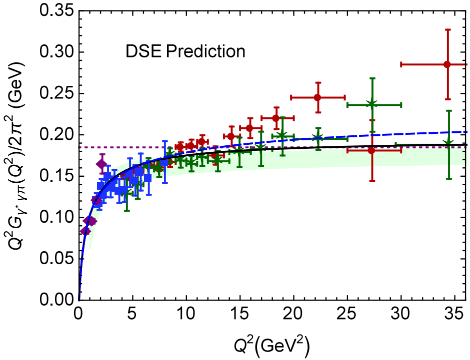
- Questions:
  - Conflict between BaBar and Belle above 10 GeV^2
  - Belle data not always lie blow pQCD limit: $2f_\pi$
  - Distribution amplitude (DA) of π^0
- Answers:
  - Consistent with all non-BaBar data
  - Approach the limit in pQCD limit $2f_\pi$ from above
  - Broad concave function

$$Q^2 F_\pi(Q^2) \overset{Q^2 > Q_0^2}{=} 4\pi^2 f_\pi \frac{1}{3}\int_0^1 dx \frac{\phi_\pi(x;Q^2)}{x} \quad (38)$$

- Pion TFF[1]: $\gamma\gamma^* \to \pi^0$:



- BaBar: circles (red), Belle: stars (green)
DSE with ERBL: solid (black),
DSE without evolution: dashed (blue).

[1] K. Raya, L. Chang, A. Bashir, J.J. Cobos-Martinez, L.X. Gutiérrez-Guerrero, C.D. Roberts, and P.C. Tandy. Phys.Rev. D93 (2016) no.7, 074017.

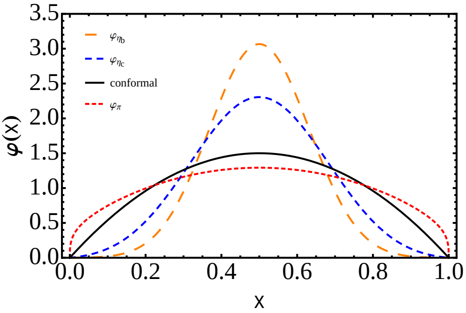
DAs of $\eta_c \& \eta_b$

- Piecewise convex-concave-convex function
- Ordering of DAs peak heights and widths: ($<_N$ means narrower than) $\phi^{asy}(x)$.

$$\phi_{\eta_b} <_N \phi_{\eta_c} <_N \phi^{asy} <_N \phi_\pi$$

- $\Lambda_{QCD}/m_q(\zeta) \to 0$, $\phi(x) \to \delta(x - 1/2)$.
- Critical current quark mass $m_q^c(\zeta = 2GeV) = 0.15\text{GeV}$, $\phi(x) = \phi^{asy}(x)$.

- DAs of $\eta_c \& \eta_b$ from DSE[1]

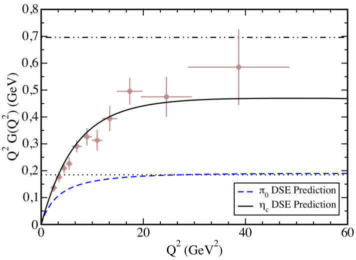


- $\phi_{NRQCD}(x) = \delta(x - 1/2)$, $\phi^{asy}(x) = 6x(1 - x)$.

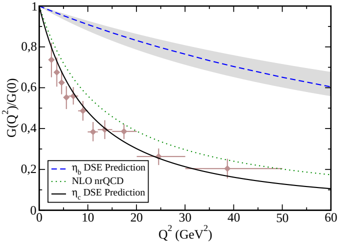
[1] M. Ding, F. Gao, L. Chang, Y.-X. Liu, and C. D. Roberts, Phys. Lett. B 753, 330 (2016).

TFF of $\eta_c \& \eta_b$

- $\eta_c \& \eta_b$ TFF[1]:



- Questions:
  - Data lie below leading-order pQCD prediction
  - DA of η_c
- Answers:
  - Consistent with BaBar data
  - Approach the limit in pQCD limit $\frac{8}{3} f_{\eta_c}$ from below
  - Narrow Piecewise convex-concave-convex function: $\phi_{\eta_c} <_N \phi^{asy}$



$$\lim_{Q^2 \to \infty} Q^2 F_{\eta_c}(Q^2) = 4\pi^2 \int_0^1 dx \, \frac{\frac{4}{9} f_{\eta_c} \phi_{\eta_c}(x)}{1 - x} \quad (39)$$

[1] K. Raya, M. Ding, A. Bashir, L. Chang and C.D. Roberts. Partonic structure of neutral pseudoscalars via two photon transition form factors, Phys.Rev. D95 (2017) 074014.

Rainbow-Ladder: π, η_c, η_b

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- Axial-vector Ward-Takahashi identity: π, η_c, η_b: $\mathcal{A}^a(k;P) = 0$

$$P_\mu \Gamma^a_{5\mu}(k;P) + 2i\mathcal{M}^{ab}\Gamma^b_5(k;P) = \mathcal{S}^{-1}(k_+) i\gamma_5 \mathcal{F}^a + i\gamma_5 \mathcal{F}^a \mathcal{S}^{-1}(k_-) \quad (40)$$

- $\Gamma^a_{5\mu}(k;P), \Gamma^b_5(k;P), \mathcal{S}^{-1}(k)$

$$\Gamma_{5\mu}(p,P) = \gamma_5\gamma_\mu + \int_k K(p,k,P) S^a(k_+) \Gamma_{5\mu}(k,P) S^b(k_-)\,, \quad (41a)$$

$$\Gamma_5(p,P) = \gamma_5 + \int_k K(p,k,P) S^a(k_+) \Gamma_5(k,P) S^b(k_-) \quad (41b)$$

$$S^{-1}(p) = i\gamma\cdot p + m + \Sigma(p) \quad (41c)$$

- Relation between self-energy and Bethe-Salpeter kernel:

$$\Sigma(p_+)\, i\gamma_5 + i\gamma_5 \Sigma(p_-) = -\int_k K(p,k,P) \left(S^a(k_+) i\gamma_5 + i\gamma_5 S^a(k_-)\right)\,. \quad (42)$$

- Rainbow-Ladder: $\Sigma(p) = \int_{dk} g^2 D_{\mu\nu}(p-k)\gamma_\mu S(k)\gamma_\nu$

$$K_{RL}(p,k,P) = -g^2 D_{\mu\nu}(p-k) \frac{\lambda^a}{2}\frac{\lambda^a}{2}\gamma_\mu\gamma_\nu\,. \quad (43)$$

Bethe-Salpeter kernel $\eta\&\eta'$

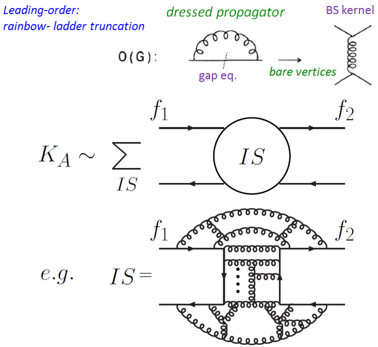
- Axial-vector Ward-Takahashi identity: $\eta\&\eta'$: $\mathcal{A}^a(k;P) \neq 0$

$$P_\mu \Gamma^a_{5\mu}(k;P) = \mathcal{S}^{-1}(k_+) i\gamma_5 \mathcal{F}^a + i\gamma_5 \mathcal{F}^a \mathcal{S}^{-1}(k_-) - 2i\mathcal{M}^{ab}\Gamma^b_5(k;P) - \mathcal{A}^a(k;P)\,, \quad (44)$$

- No simple relation between self-energy and Bethe-Salpeter kernel:

- "hairpin" structure: intermediate state (IS) involve infinitely many lines
- connect external (anti-)quark with internal (anti-)quark

$$[K_A(k,q,P)]^{\alpha\beta}_{\rho\sigma} = \xi(s)\cos^2(\theta)_\xi [zi\gamma_5]_{\alpha\beta}[zi\gamma_5]_{\rho\sigma} + \frac{1}{\chi^2}\xi(s)\sin^2(\theta)_\xi [zi\gamma_5\gamma\cdot P]_{\alpha\beta}[zi\gamma_5\gamma\cdot P]_{\rho\sigma} \quad (45)$$



- $\xi(s) = 0 \longrightarrow$ anomaly kernel: $K_A = 0$

Masses with anomaly $\eta\&\eta'$

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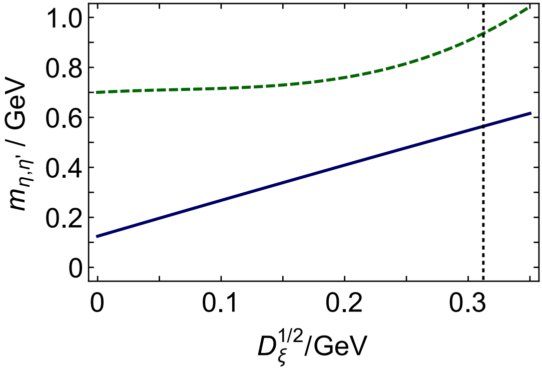

- Ideal mixing: no anomaly. $m_\eta = m_\pi$, $m_{\eta'} = m_{s\bar{s}} = 0.7$GeV.
- Meson mass grow with the anomaly strength, reach the empirical values $m_\eta = 0.56$GeV, $m_{\eta'} = 0.96$GeV.

Leptonic decay constants and mixing: $\eta\&\eta'$

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- Pseudovector projection of the Bethe-Salpeter wave function

$$f^{l,s}_{\eta,\eta'} P_\mu = Z_2 tr \int_{dk}^{\Lambda} \gamma_5 \gamma_\mu \chi^{l,s}_{\eta,\eta'}(k;P) \quad (46)$$

	f^l_η	f^s_η	$f^l_{\eta'}$	$f^s_{\eta'}$
herein-direct	0.072	-0.092	0.070	0.104
herein-fit	0.074	-0.094	0.068	0.101
phen.[1] [2]	0.090(13)	-0.093(28)	0.073(14)	0.094(8)

- Use decay constants to define flavor mixing angle

$$\begin{pmatrix} f^l_\eta & f^s_\eta \\ f^l_{\eta'} & f^s_{\eta'} \end{pmatrix} = \begin{pmatrix} f^l \cos\theta & -f^s \sin\theta \\ f^l \sin\theta & f^s \cos\theta \end{pmatrix},$$

$$\begin{pmatrix} f^8_\eta & f^0_\eta \\ f^8_{\eta'} & f^0_{\eta'} \end{pmatrix} = \begin{pmatrix} f^8 \cos\theta_8 & -f^0 \sin\theta_0 \\ f^8 \sin\theta_8 & f^0 \cos\theta_0 \end{pmatrix}$$

$\theta = 42.8^\circ$.

$f^l = 0.101 GeV = 1.08 f_\pi$

$f^s = 0.138 GeV = 1.49 f_\pi$.

$\theta_8 = -21^\circ \ \theta_0 = -2.8^\circ$.

$f^8 = 1.34 f_\pi$

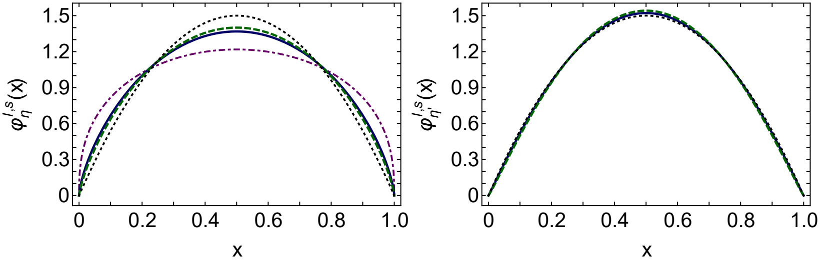
$f^0 = 1.26 f_\pi$.

[1] T. Feldmann, P. Kroll, and B. Stech, Phys. Lett. B 449, 339 (1999).

[2] M. Benayoun, L. DelBuono and H. B. O'Connell, Eur. Phys. J. C 17, 593 (2000).

DAs of $\eta\&\eta'$

- DAs of $\eta\&\eta'$ from DSE



- η DAs: $\phi^{asy} <_N \phi_\eta <_N \phi_\pi$; η' DAs: $\phi_{\eta'} \sim \phi^{asy}$
- light- and s- quark component DAs have similar profiles

TFFs evolution of $\eta\&\eta'$

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- Axial-vector Ward-Takahashi identity.

$$P_\mu \Gamma^a_{5\mu}(k;P) = \mathcal{S}^{-1}(k_+) i\gamma_5 \mathcal{F}^a + i\gamma_5 \mathcal{F}^a \mathcal{S}^{-1}(k_-) - 2i\mathcal{M}^{ab}\Gamma^b_5(k;P) - \mathcal{A}^a(k;P)\,, \tag{47}$$

- anomaly: $\mathcal{A}^a(k;P) = \mathcal{S}^{-1}(k_+)\delta^{a0}\mathcal{A}_{\mathcal{U}}(k;P)\mathcal{S}^{-1}(k_-)$.
- Flavor Octet current: $\mathcal{A}^8(k;P) = 0$, f^8 is independent of the renormalisation scale
- Flavor singlet current: $\mathcal{A}^0(k;P) \neq 0$, f^0 depend on the renormalisation scale[1]

$$f^0_{\eta,\eta'}(\zeta^2) = f^0_{\eta,\eta'}(\zeta_2^2)\left(1 + \frac{2N_f}{\pi\beta_0}[\alpha_S(\zeta^2) - \alpha_S(\zeta_0^2)]\right) \tag{48}$$

- Should be considered into Bethe-Salpeter wave function and transition form factors

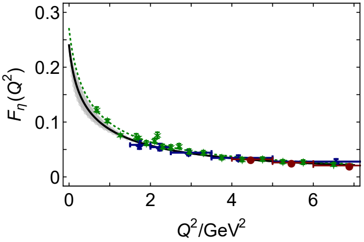
[1] S. S. Agaev, V. M. Braun, N. Offen, F. A. Porkert, A. Schafer, Transition form factors $\gamma^*\gamma \to \eta$ and $\gamma^*\gamma \to \eta'$ in QCD. Phys.Rev. D90 (2014) no.7, 074019

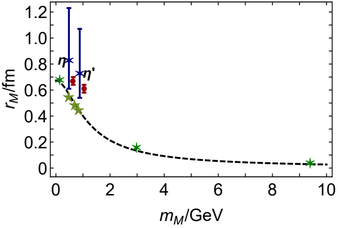
TFFs of $\eta\&\eta'$: Low Q^2

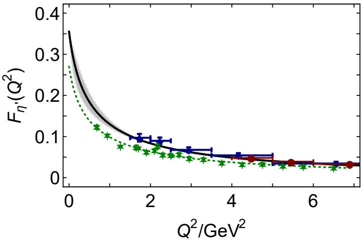
- TFFs agree with data
- radii: $r_\eta = 0.83^{+0.40}_{-0.22}\,fm\,, \quad r_{\eta'} = 0.73^{+0.34}_{-0.19}\,fm$

$$r^2_{\eta,\eta'} := \frac{-6}{F_{\eta,\eta'}(0)} \frac{d}{dQ^2} F_{\eta,\eta'}(Q^2)|_{Q^2=0}$$

- radii: $\pi^0, \eta, \eta', \eta_c, \eta_b \& m_{0-}$ /GeV= 0.47, 0.69, 0.83
- Non-Abelian anomaly: η, η' larger than a fit by 24% and 48%.







TFFs of $\eta\&\eta'$: Large Q^2

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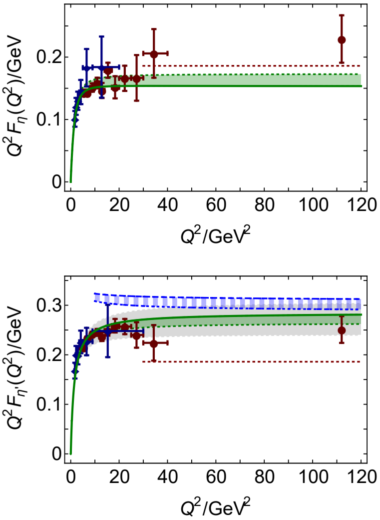
- $\eta\&\eta'$ TFF[1]:



- Questions:
  - Non-perturbative properties of η, η'
  - DA of η, η'
- Answers:
  - $Q^2 F_\eta(Q^2 \to \infty) = 0.15$GeV, $Q^2 F_{\eta'}(Q^2 \to \infty) = 0.30$GeV
  - Omission of $\varphi^0_{\eta,\eta'} - \varphi^g_{\eta,\eta'}$ mixing

$$\lim_{Q^2\to\infty} Q^2 F_{\eta,\eta'}(Q^2)$$
$$=6\left[\frac{5}{9} f^l_{\eta,\eta'}(Q^2) + \frac{\sqrt{2}}{9} f^s_{\eta,\eta'}(Q^2)\right]$$

[1] M. Ding, K. Raya, A. Bashir, D. Binosi, L. Chang, M. Chen and C.D. Roberts. $\gamma^*\gamma \to \eta, \eta'$ transition form factors. Phys.Rev. D99 (2019) no.1, 014014.

Topological charge: $\eta\&\eta'$

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- General mass formula

$$m^2_{\eta,\eta'}\begin{bmatrix} f^8_{\eta,\eta'} \\ f^0_{\eta,\eta'}\end{bmatrix} = \begin{bmatrix} 0 \\ n_{\eta,\eta'}\end{bmatrix} + 2\begin{bmatrix} \frac{1}{3}m_{12} & \frac{\sqrt{2}}{3}m_{1-1} \\ \frac{\sqrt{2}}{3}m_{1-1} & \frac{1}{3}m_{21}\end{bmatrix}\begin{bmatrix} \rho^8_{\eta,\eta'} \\ \rho^0_{\eta,\eta'}\end{bmatrix}$$

- Topological charge density operator: $\mathcal{Q}(x)$

  $n_{\eta,\eta'} = \sqrt{\frac{3}{2}}\nu_{\eta,\eta'}, \; \nu_{\eta,\eta'} = \langle 0|\mathcal{Q}(x) = i\frac{\alpha_s}{4\pi}tr_C[\epsilon_{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma(x)}]|\eta,\eta'\rangle$
- Pseudoscalar projection of the Bethe-Salpeter wave function:

$$i\rho^{l,s}_{\eta,\eta'} = Z_4 tr\int^{\Lambda}_{dk}\gamma_5\chi^{l,s}_{\eta,\eta'}(k;P)\,, \tag{49}$$

- Topological charge content of the η' is 2.1 times that of η

ρ^8_η	ρ^0_η	$\rho^8_{\eta'}$	$\rho^0_{\eta'}$
0.50^2	0.033^2	-0.37^2	0.57^2

$$\nu_\eta = (0.29GeV)^3$$
$$\nu_{\eta'} = (0.37GeV)^3\,.$$

Summary

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- Summary: Two photon transition form facotors
  - $\gamma\gamma^* \to \pi^0$
    - ★ Consistent with all non-BaBar data
    - ★ Approach the limit in pQCD limit $2f_\pi$ from above
    - ★ Pion DA: broad concave function
  - $\gamma\gamma^* \to \eta_c, \eta_b$
    - ★ Consistent with BaBar data
    - ★ Approach the limit in pQCD limit $\frac{8}{3} f_{\eta_c}$ from below
    - ★ DAs: Narrow Piecewise convex-concave-convex function: $\phi_{\eta_b} <_N \phi_{\eta_c} <_N \phi^{asy}$
  - $\gamma\gamma^* \to \eta, \eta'$
    - ★ TFFs agree with Babar data
    - ★ η DAs: $\phi^{asy} <_N \phi_\eta <_N \phi_\pi$; η' DAs: $\phi_{\eta'} \sim \phi^{asy}$
    - ★ Non-Abelian anomaly: η, η' larger than a non-mixing pseudoscalar radii fit by 24% and 48%.
    - ★ $Q^2 F_\eta(Q^2 \to \infty) = 0.15\text{GeV}$, $Q^2 F_{\eta'}(Q^2 \to \infty) = 0.30\text{GeV}$
    - ★ Topological charge content of the η' is 2.1 times that of η
- Thanks!