DSEs in Hadroparticle Physics: Past, Present, … Future

Craig Roberts

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Sevilla Oct. 2016 - Nov. 2018

Students, Postdocs, Profs.

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7. *Minghui DING (ECT*, ANL, Nankai U.) ;*
8. *Fei GAO (Valencia, Peking U.) ;*
9. *Bo-Lin LI (Nanjing U.)*
10. *Cédric MEZRAG (INFN-Roma) ;*
11. *Khépani RAYA (Nankai U., U Michoácan);*
12. Adnan Bashir (U Michoácan);
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14. Volker Burkert (Jlab)
15. Lei Chang (Nankai U.) ;
16. Xiao-Yun Chen (Jinling Inst. Tech., Nanjing)
17. Tanja Horn (Catholic U. America)
18. Yu-Xin Liu (PKU);
19. Joannis Papavassiliou (U.Valencia)
20. Jia-Lun Ping (Nanjing Normal U.)
21. Si-xue Qin (Chongqing U.);
22. Jose Rodriguez Quintero (U. Huelva) ;
23. Jorge Segovia (U. Pablo de Olavide);
24. Sebastian Schmidt (IAS-FZJ & JARA);
25. Shaolong Wan (USTC) ;
26. Qing-Wu Wang (Sichuan U)
27. Shu-Sheng XU (NJUPT, Nanjing U.)
28. Pei-Lin Yin (NJUPT)
29. Hong-Shi Zong (Nanjing U)

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25. Jose Rodriguez Quintero (U. Huelva) ;
26. *Elena Santopinto (Genoa)*
27. Jorge Segovia (U. Pablo de Olavide = UPO);
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31. Shu-Sheng XU (NJUPT, Nanjing U.)
32. Pei-Lin Yin (NJUPT)
33. Hong-Shi Zong (Nanjing U)

SELECTED ARTICLES – 11/2018

i. *Scale-setting, flavour dependence and chiral symmetry restoration,* Daniele Binosi, Craig D. Roberts and Jose Rodríguez-Quintero, arXiv:1611.03523 [nucl-th], Phys. Rev. D. **95** (2017) 114009/1-10

ii. *Process-independent strong running coupling,* Daniele Binosi, Cédric Mezrag, Joannis Papavassiliou, Craig D. Roberts and Jose Rodríguez-Quintero, arXiv:1612.04835 [nucl-th], Phys. Rev. D **96** (2017) 054026/1-7

iii. *Off-shell persistence of composite pions and kaons,* Si-Xue Qin, Chen Chen, Cédric Mezrag and Craig D. Roberts, arXiv:1702.06100 [nucl-th], Phys. Rev. C **97** (2018) 015203/1-7

iv. *Exposing strangeness: projections for kaon electromagnetic form factors,* Fei Gao, Lei Chang, Yu-Xin Liu, C. D. Roberts …, arXiv:1703.04875 [nucl-th], Phys. Rev. D **96** (2017) 034024/1-8

v. *Parity partners in the baryon resonance spectrum,* Ya Lu, Chen Chen, Craig D. Roberts, Jorge Segovia, Shu-Sheng Xu and Hong-Shi Zong, arXiv:1705.03988 [nucl-th], Phys. Rev. C **96** (2017) 015208/1-11

vi. *Locating the Gribov horizon,* Fei Gao, Si-Xue Qin, Craig D. Roberts and Jose Rodríguez-Quintero, arXiv:1706.04681 [hep-ph], Phys. Rev. D **97** (2018) 034010/1-9

vii. *Roper resonance: Toward a solution to the fifty year puzzle,* Volker D. Burkert and Craig D. Roberts, arXiv:1710.02549 [nucl-ex], Rev. Mod. Phys. **91** (2019) 011003/1-18

viii. *Structure of the nucleon's low-lying excitations,* Chen Chen, Craig D. Roberts, Sebastian M. Schmidt, Jorge Segovia and Shaolong Wan, arXiv:1711.03142 [nucl-th], Phys. Rev. D **97** (2018) 034016/1-13

SELECTED ARTICLES – 11/2018

ix. *Parton distribution amplitudes: revealing diquarks in the proton and Roper resonance,* Cédric Mezrag, Jorge Segovia, Lei Chang and Craig D. Roberts, arXiv:1711.09101 [nucl-th], Phys. Lett. B **783** (2018) 263-267

x. *Light-meson masses in an unquenched quark model,* Xiaoyun Chen, Jialun Ping, Craig D. Roberts and Jorge Segovia, arXiv:1712.04457 [nucl-th], Phys. Rev. D **97** (2018) 094016/1-11

xi. *Poincaré-covariant analysis of heavy-quark baryons,* Si-Xue Qin, Craig D. Roberts and Sebastian M. Schmidt, arXiv:1801.09697 [nucl-th], Phys.Rev. D **97** (2018) 114017/1-13

xii. *Pion and kaon valence-quark parton quasidistributions,* Shu-Sheng Xu, Lei Chang, Craig D. Roberts and Hong-Shi Zong, arXiv:1802.09552 [nucl-th], Phys. Rev. D **97** (2018) 094014/1-10

xiii. *New Perspective on Hybrid Mesons,* Shu-Sheng Xu, Zhu-Fang Cui, Lei Chang, Joannis Papavassiliou, Craig D. Roberts and Hong-Shi Zong, arXiv:1805.06430 [nucl-th]

xiv. *Proton tensor charges from a Poincaré-covariant Faddeev equation,* Qing-Wu Wang, Si-Xue Qin, Craig D. Roberts and Sebastian M. Schmidt, arXiv:1806.01287 [nucl-th], Phys. Rev. D **98** (2018) 054019/1-10

xv. *Mass-dependence of pseudoscalar meson elastic form factors,* Muyang Chen, Minghui Ding, Lei Chang and Craig D. Roberts, arXiv:1808.09461 [nucl-th], Phys. Rev. D **98** (2018) 091505(R)/1-7

SELECTED ARTICLES – 05/2019

i. *$\gamma^* \gamma \to \eta, \eta'$ transition form factors*, Minghui Ding, Khepani Raya, Daniele Binosi, Adnan Bashir, Lei Chang, Muyang Chen and Craig D. Roberts, arXiv:1810.12313 [nucl-th], Phys. Rev. D **99** (2019) 014014/1-17

ii. *Nucleon-to-Roper electromagnetic transition form factors at large-Q^2*, Chen Chen, Ya Lu, Daniele Binosi, Craig D. Roberts, Jose Rodríguez-Quintero and Jorge Segovia, arXiv:1811.08440 [nucl-th], Phys. Rev. D 99 (2019) 034013/1-13

iii. *Distribution Amplitudes of Heavy-Light Mesons*, Daniele Binosi, Lei Chang, Minghui Ding, Fei Gao, Joannis Papavassiliou and Craig D. Roberts, arXiv:1812.05112 [nucl-th], Phys. Lett. B **790** (2019) 257-262

iv. *Spectrum and structure of octet and decuplet baryons and their positive-parity excitations*, Chen Chen, Gastão Krein, Craig D. Roberts, Sebastian M. Schmidt and Jorge Segovia, arXiv:1901.04305 [nucl-th]

v. *Spectrum of light- and heavy-baryons*, Si-Xue Qin, Craig D. Roberts and Sebastian M. Schmidt, arXiv:1902.00026 [nucl-th], Few Body Syst. **60** (2019) 26/1-18 (Contribution to a Special Issue dedicated to Ludwig Faddeev.)

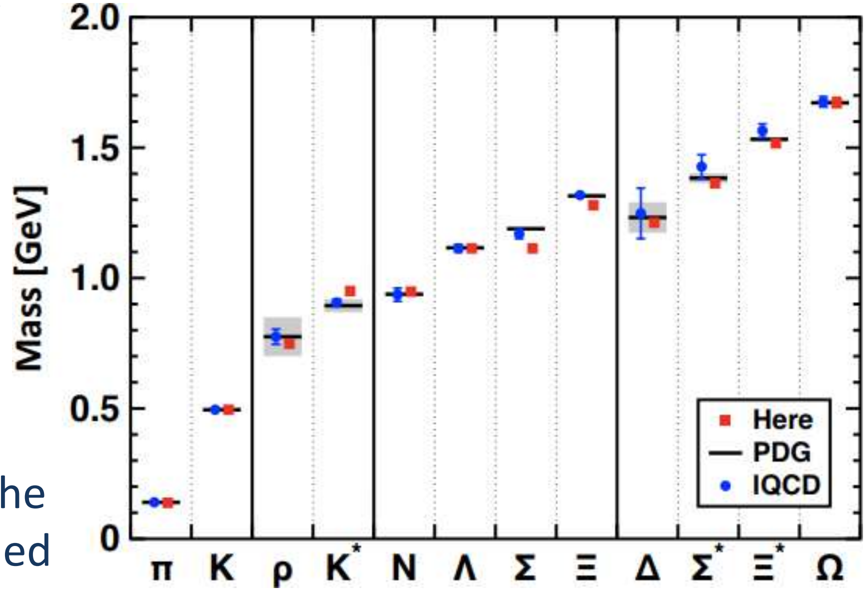
vi. *Masses of ground-state mesons and baryons, including those with heavy quarks*, Pei-Lin Yin, Chen Chen, Gastão Krein, Craig D. Roberts, Jorge Segovia and Shu-Sheng Xu, arXiv:1903.00160

vii. *Transition form factors: $\gamma* + p \to \Delta(1232), \Delta(1600)$*, Ya Lu, Chen Chen, Zhu-Fang Cui, Craig D Roberts, Sebastian M Schmidt, Jorge Segovia and Hong Shi Zong, arXiv:1904.03205 [nucl-th]

OPENING HIGHLIGHT

i. *Spectrum of light- and heavy-baryons*, Si-Xue Qin, C. D. Roberts and S.M. Schmidt, arXiv:1902.00026 [nucl-th], Few Body Syst. **60** (2019) 26/1-18 (Contribution to a Special Issue dedicated to Ludwig Faddeev.)

- ✓ Symmetry-preserving truncation of the strong-interaction bound-state equations = gap- and Faddeev-equations
  - ✓ No diquark approximation
- ✓ Calculate spectrum of
  - ✓ ground-state J=1/2$^+$, 3/2$^+$ (qq′q′′) -baryons, q,q′,q′′∈{u,d,s,c,b} ,
  - ✓ their first positive-parity excitations
  - ✓ parity partners.
- ✓ Using two parameters (RL), description of the known spectrum of 39 such states is obtained
  - ✓ with a mean-absolute-relative-difference between calculation and experiment of 3.6(2.7)%.



- ✓ The framework is subsequently used to predict the masses of 90 states not yet seen empirically.

SOME PROJECTS UNDERWAY

i. Computation of gluon in-hadron condensate … Binosi, Papavassiliou

ii. π & K TMDs … Ding, Binosi, Chang

iii. π & K GPDs and gravitational form factors … Raya, Rodríguez-Qintero

iv. π & K fragmentation functions … Li, Chang

v. PDFs for J=1 mesons … Chen-M., Chang

vi. DAs for heavy-light mesons … Binosi, Chang, Gao, Papavassiliou

vii. Nucleon PDFs … Gao, Chang, Papavassiliou

viii. Nucleon and Roper DAs = Big Picture … Mezrag, Binosi, Chang, Ding, Segovia

ix. Analysis of π & K mesons off-shell (TDIS at JLAB12 & EIC) … Chen-M, Chang

x. Timelike behaviour of neutral pseudoscalar meson transition form factors …Chen-M, Chang

xi. $\gamma\, n \to R^0(1440)$ … Chen-C, Lu, Binosi, Segovia

xii. $\gamma\, N \to N(1535)$ … Lu, Chen-C, Raya, Bashir, Segovia

xiii. $\gamma\, N \to \Delta(1600)$ … Lu, Chen-C, Segovia

xiv. $\gamma\, N \to N(1710)\ \frac{1}{2}^+$… Cui, Chen-C, Segovia

xv. Structure of $I = \frac{1}{2}$, $J = {}^{3}\!/_{2}$, $P = \pm$ baryons … Lu, Chen, Segovia

xvi. Insights into beyond-RL truncations for heavy-light mesons … Binosi, Gao, Papavassiliou

xvii. Spectrum of u, d, s, c, b baryons using contact-interaction … Yin, Chen

xviii. Spectrum of u, d, s, c, b baryons using Faddeev equation … Qin

xix. Nucleon axial form factors using Faddeev equation … Qin, Lovato

xx. Glueball spectrum … Aguilar, Papavassiliou, Xu

xxi. Spectrum of doubly-heavy tetraquarks … Lovato, Segovia

SOME PROJECTS UNDERWAY

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xix. Nucleon axial form factors using Faddeev equation … Qin, Lovato
xx. Glueball spectrum … Aguilar, Papavassiliou, Xu
xxi. Spectrum of doubly-heavy tetraquarks … Lovato, Segovia
xxii. Pion parton distribution functions … Ding, Raya, Binosi, Chang
xxiii. Radial excitations of mesons, including heavy-quarks using contact interaction … Bedolla, Santopinto
xxiv. Quadruply-heavy tetra quarks using contact interaction … Bedolla, Feretti, Santopinto

Why are we here?

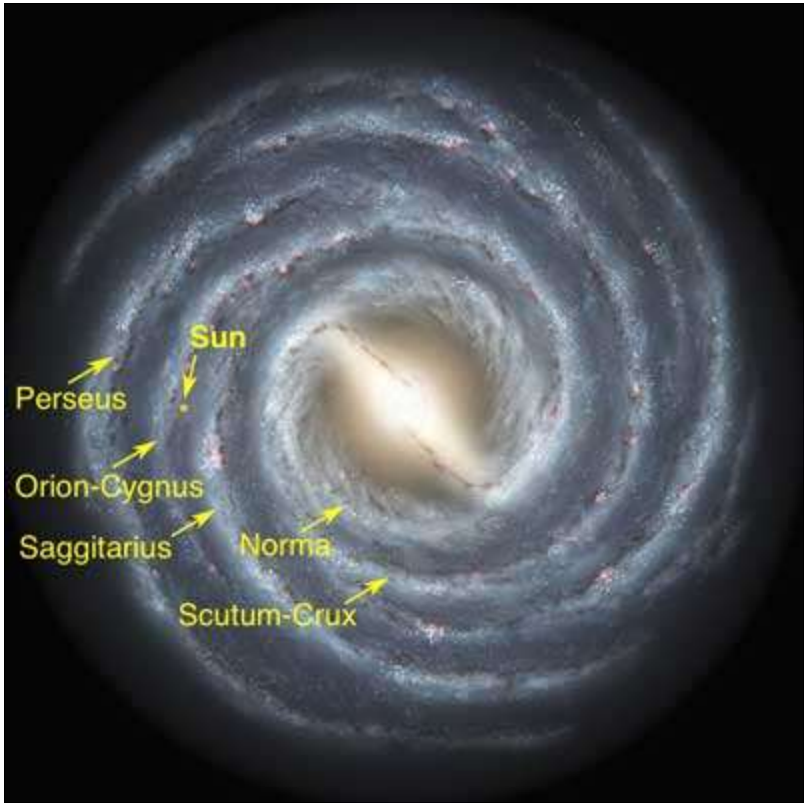
Why are we here?

- *Suppose* quantum field theories are the correct paradigm for understanding Nature
- We find ourselves in a Universe where time and space give us Four Dimensions (D = 4)
- D = 4 is a critical point
  - Quantum Field Theories with D ≠ 4 possess an explicit mass-dimension
  - Couplings are mass-dimensioned, setting scale for all quantities
    - D > 4 … uncontrollable ultraviolet divergences
    - D < 4 … super-convergent, but hierarchy problem with dynamical effects < 10% of explicit scale
- Standard Model is built from scale-invariant classical field theories (Ignoring Higgs couplings)
  - Such theories are renormalizable
  - Procedure introduces a mass scale
  - The size of the mass-scale is not determined by the theory
- What determines the natural mass-scale for visible matter?
- We know it is $m_{\text{Nature}} \approx m_p \approx 1$ GeV
  - How much tolerance exists? … We can exist so long as $m_{\text{Nature}} = 1 \pm ?$ GeV

Emergent Phenomena in the Standard Model

Existence of our Universe depends critically on the following empirical facts:

- Proton is massive
  - *i.e*. the mass-scale for strong interactions is vastly different to that of electromagnetism
- Proton is absolutely stable
  - Despite being a composite object constituted from three valence quarks
- Pion is unnaturally light (not massless, but lepton-like mass)
  - Despite being a strongly interacting composite object built from a valence-quark and valence antiquark



Emergence: low-level rules producing high-level phenomena, with enormous apparent complexity

Strong Interactions in the Standard Model

$$\mathcal{L}_{\rm QCD} = \bar{\psi}_i \left(i(\gamma^\mu D_\mu)_{ij} - m\, \delta_{ij} \right) \psi_j - \frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu}$$

- Only apparent scale in chromodynamics is mass of the quark field
- Quark mass is said to be generated by Higgs boson.
- In connection with everyday matter, that mass is 1/250th of the natural (empirical) scale for strong interactions,

  viz. more-than two orders-of-magnitude smaller
- Plainly, the Higgs-generated mass is very far removed from the natural scale for strongly-interacting matter
- *Nuclear physics mass-scale* – 1 GeV – is an *emergent feature of the Standard Model*
  - No amount of staring at $L_{\rm QCD}$ can reveal that scale
- Contrast with quantum electrodynamics, *e.g*. spectrum of hydrogen levels measured in units of m_e, which appears in $L_{\rm QED}$

$$\mathcal{L}_{\mathrm{QCD}} = \bar{\psi}_i \left(i(\gamma^\mu D_\mu)_{ij} \qquad\qquad \right) \psi_j - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$

Whence Mass?

- Classical chromodynamics … non-Abelian local gauge theory
- Remove the current mass … there's no energy scale left
- *No dynamics in a scale-invariant theory*; only kinematics … the theory looks the same at all length-scales … there can be no clumps of anything … *hence bound-states are impossible*.
- *Our Universe can't exist*
- *Higgs boson doesn't solve this problem* …
  - normal matter is constituted from light-quarks
  - the mass of protons and neutrons, the kernels of all visible matter, are 100-times larger than anything the Higgs can produce
- *Where did it all begin?*

  … becomes … Where did it all come from?

$$T_{\mu\mu} = \frac{1}{4}\beta(\alpha(\zeta))G^a_{\mu\nu}G^a_{\mu\nu}$$

Trace Anomaly

- Classically, in a scale invariant theory

  the *energy-momentum tensor must be traceless:* $T_{\mu\mu} \equiv 0$

- Regularisation and renormalisation of (ultraviolet) divergences in Quantum Chromodynamics introduces a mass-scale

  … *dimensional transmutation*: mass-dimensionless quantities become dependent on a mass-scale, ζ

- $\alpha \to \alpha(\zeta)$ in QCD's (massless) Lagrangian density, $\mathcal{L}(m=0)$

Trace anomaly

$$\Rightarrow \partial_\mu \mathcal{D}_\mu = \delta\mathcal{L}/\delta\sigma = \alpha\beta(\alpha)\, d\mathcal{L}/d\alpha = \boxed{\beta(\alpha)\, \tfrac{1}{4} G_{\mu\nu} G_{\mu\nu} = T_{\rho\rho} =: \Theta_0}$$

QCD β function

Quantisation of renormalisable four-dimensional theory forces nonzero value for trace of energy-momentum tensor

Where is the mass?

$$T_{\mu\mu} = \frac{1}{4}\beta(\alpha(\zeta))G^a_{\mu\nu}G^a_{\mu\nu}$$

Trace Anomaly

- Knowing that a trace anomaly exists does not deliver a great deal
  … Indicates only that a mass-scale must exist
- Can one compute and/or understand the magnitude of that scale?
- One can certainly *measure* the magnitude … consider proton:

$$\langle p(P)|T_{\mu\nu}|p(P)\rangle = -P_\mu P_\nu$$

$$\langle p(P)|T_{\mu\mu}|p(P)\rangle = -P^2 = m_p^2$$

$$= \langle p(P)|\Theta_0|p(P)\rangle$$

- In the chiral limit the entirety of the proton's mass is produced by the trace anomaly, *Θ_0*
  … In QCD, *Θ_0* measures the strength of gluon self-interactions
  … so, from one perspective,
  m_p is (somehow) completely generated by glue.

On the other hand …

$$T_{\mu\mu} = \frac{1}{4}\beta(\alpha(\zeta))G^a_{\mu\nu}G^a_{\mu\nu}$$

Trace Anomaly

- In the chiral limit

$$\langle\pi(q)|T_{\mu\nu}|\pi(q)\rangle = -q_\mu q_\nu \;\Rightarrow\; \langle\pi(q)|\Theta_0|\pi(q)\rangle = 0$$

- Does this mean that the scale anomaly vanishes trivially in the pion state, *i.e*. gluons contribute nothing to the pion mass?
- Difficult way to obtain "zero"!

- Easier to imagine that "zero" owes to cancellations between different operator contributions to the expectation value of Θ_0.
- Of course, such precise cancellation should not be an accident.

  It could only arise naturally because

  of some symmetry and/or symmetry-breaking pattern.

Whence “1” and yet “0” ?

$$\langle p(P)|\Theta_0|p(P)\rangle = m_p^2\,, \qquad \langle \pi(q)|\Theta_0|\pi(q)\rangle = 0$$

- *No statement of the question*

  “How does the mass of the proton arise?”

  is complete without the additional clause

  “How does the pion remain ***massless****?”*

- Natural visible-matter mass-scale must emerge simultaneously with apparent preservation of scale invariance in related systems
  - Expectation value of Θ_0 in pion is always zero, irrespective of the size of the natural mass-scale for strong interactions = m_p

Whence "1" and yet "0" ?

$$\langle p(P)|\Theta_0|p(P)\rangle = m_p^2\,, \quad \langle \pi(q)|\Theta_0|\pi(q)\rangle = 0$$

- *No statement of the question*

  "How does the mass of the proton arise?"

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- Natu usly
  with stems
  - Ex e of
    th = m_p

Elucidate the entire array
of empirical consequences
of the mechanism responsible
so that the theory can be validated

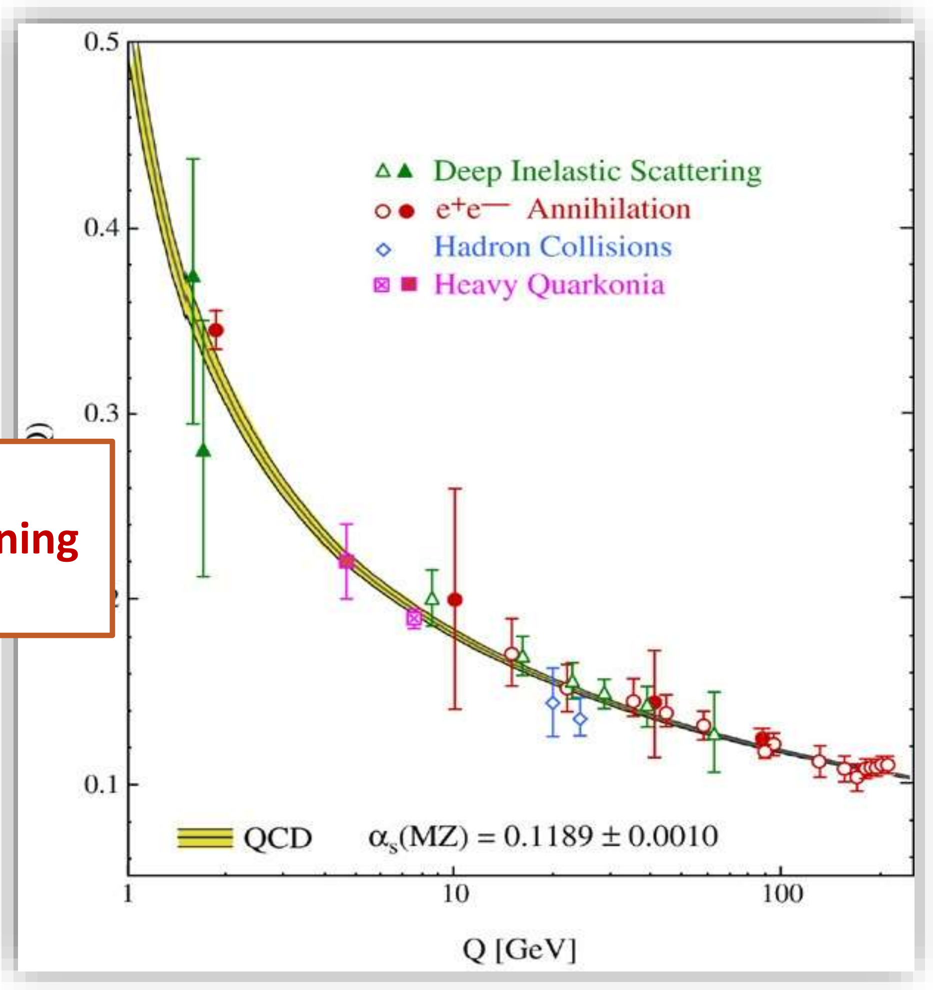
Ideas: Old & New

SELECTED HIGHLIGHT

ii. *Process-independent strong running coupling,* Daniele Binosi, Cédric Mezrag, Joannis Papavassiliou, Craig D. Roberts and Jose Rodríguez-Quintero, arXiv:1612.04835 [nucl-th], Phys. Rev. D **96** (2017) 054026/1-7

Process-independent effective coupling. From QCD Green functions to phenomenology, Jose Rodríguez-Quintero, Daniele Binosi, Cédric Mezrag, Joannis Papavassiliou and Craig D. Roberts, arXiv:1801.10164 [nucl-th]. Few Body Syst. **59** (2018) 121/1-9

QCD’s Running Coupling



⇐
What’s happening out here?!

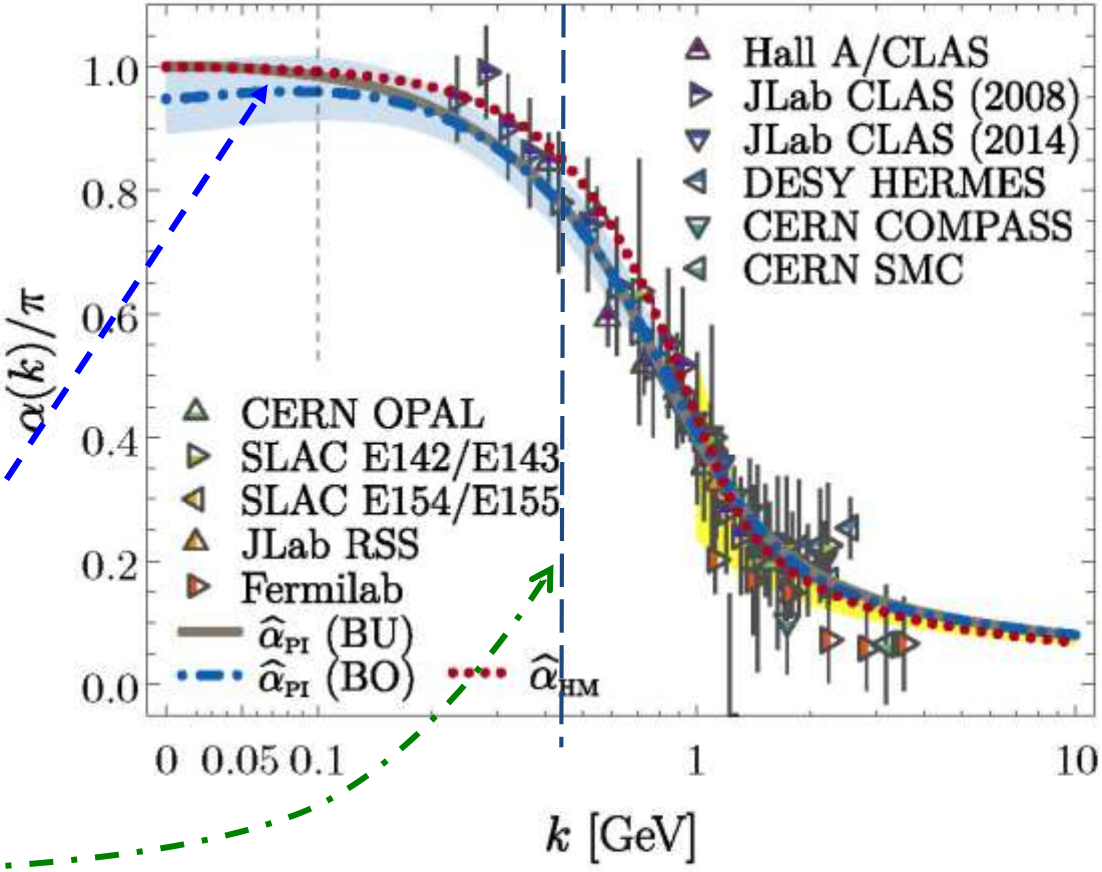
Process independent strong running coupling
Binosi, Mezrag, Papavassiliou, Roberts, Rodriguez-Quintero
arXiv:1612.04835 [nucl-th], Phys. Rev. D 96 (2017) 054026/1-7
The QCD Running Coupling,
A. Deur, S. J. Brodsky and G. F. de Teramond,
Prog. Part. Nucl. Phys. **90** (2016) 1-74

Process-independent effective-charge in QCD

- Modern continuum & lattice methods for analysing gauge sector enable

  "Gell-Mann – Low"

  running charge to be defined in QCD
- Combined continuum and lattice analysis of QCD's gauge sector yields a parameter-free prediction
- N.B. Qualitative change in $\hat{\alpha}_{PI}(k)$ at $k \approx ½\, m_p$



Process independent strong running coupling
Binosi, Mezrag, Papavassiliou, Roberts, Rodriguez-Quintero
arXiv:1612.04835 [nucl-th], Phys. Rev. D 96 (2017) 054026/1-7

The QCD Running Coupling,
A. Deur, S. J. Brodsky and G. F. de Teramond,
Prog. Part. Nucl. Phys. **90** (2016) 1-74

QCD Effective Charge

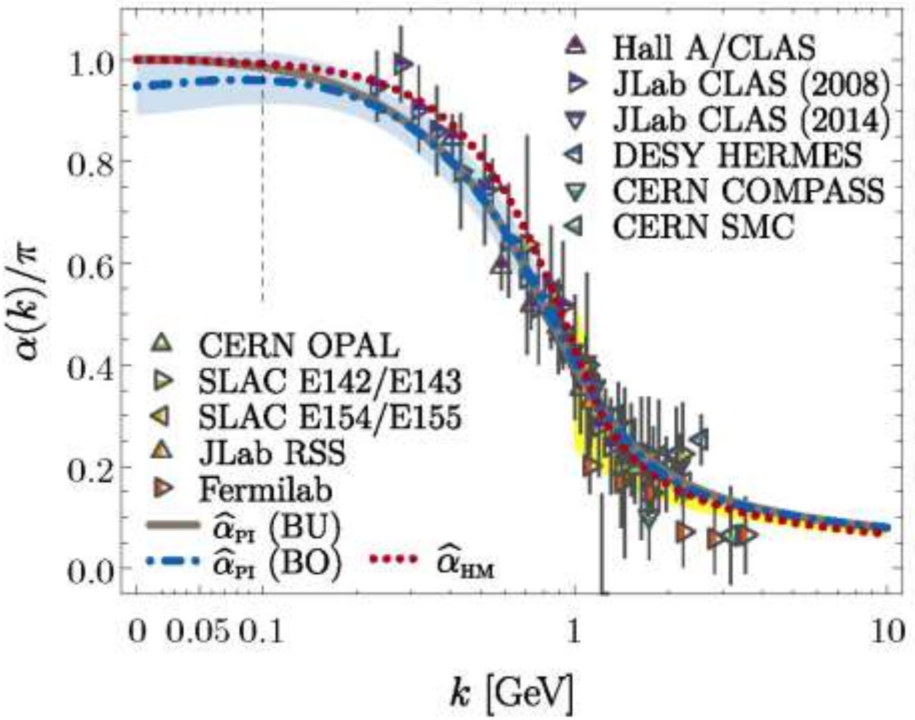
- Parameter-free prediction:
  - Curve completely determined by results obtained for gluon & ghost two-point functions using continuun and lattice-regularised QCD.
- Near precise agreement between process-independent

  $\hat{\alpha}_{PI}$ and α_{g1}
  & $\hat{\alpha}_{PI} \approx \alpha_{HM}$

- Perturbative domain:

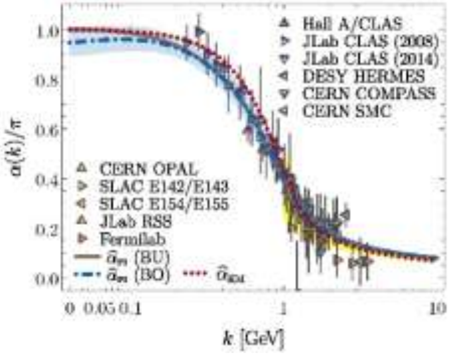
$$\alpha_{g_1}(k^2) = \alpha_{\overline{\rm MS}}(k^2)(1 + 1.14\, \alpha_{\overline{\rm MS}}(k^2) + \ldots)\,,$$
$$\widehat{\alpha}_{\rm PI}(k^2) = \alpha_{\overline{\rm MS}}(k^2)(1 + 1.09\, \alpha_{\overline{\rm MS}}(k^2) + \ldots)\,,$$

difference = (1/20) $\alpha_{\overline{MS}}^2$



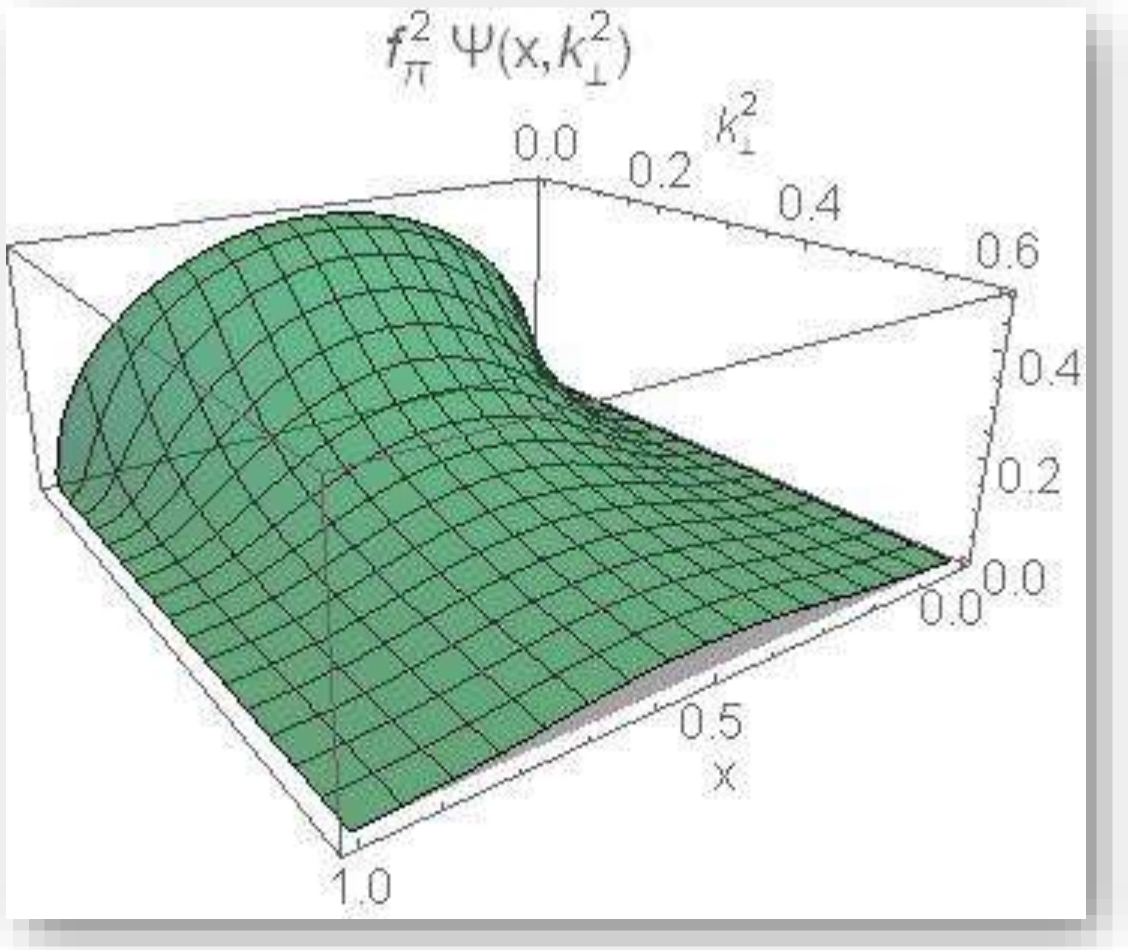
Data = process dependent effective charge [Grunberg:1982fw]:
α_{g1}, defined via Bjorken Sum Rule

QCD Effective Charge



- $\hat{\alpha}_{PI}$ is a new type of effective charge
  - direct analogue of the Gell-Mann–Low effective coupling in QED, *i.e.* completely determined by the gauge-boson two-point function.
- $\hat{\alpha}_{PI}$ is
  - process-independent
  - known to unify a vast array of observables
- $\hat{\alpha}_{PI}$ possesses an infrared-stable fixed-point
  - Nonperturbative analysis demonstrating absence of a Landau pole in QCD
- QCD is IR finite, owing to dynamical generation of gluon mass-scale, which also serves to eliminate the Gribov ambiguity
- Asymptotic freedom ⇒ QCD is well-defined at UV momenta
- ***QCD is therefore unique amongst known 4D quantum field theories***
  - ***Potentially, defined & internally consistent at all momenta***

Observing *Mass*

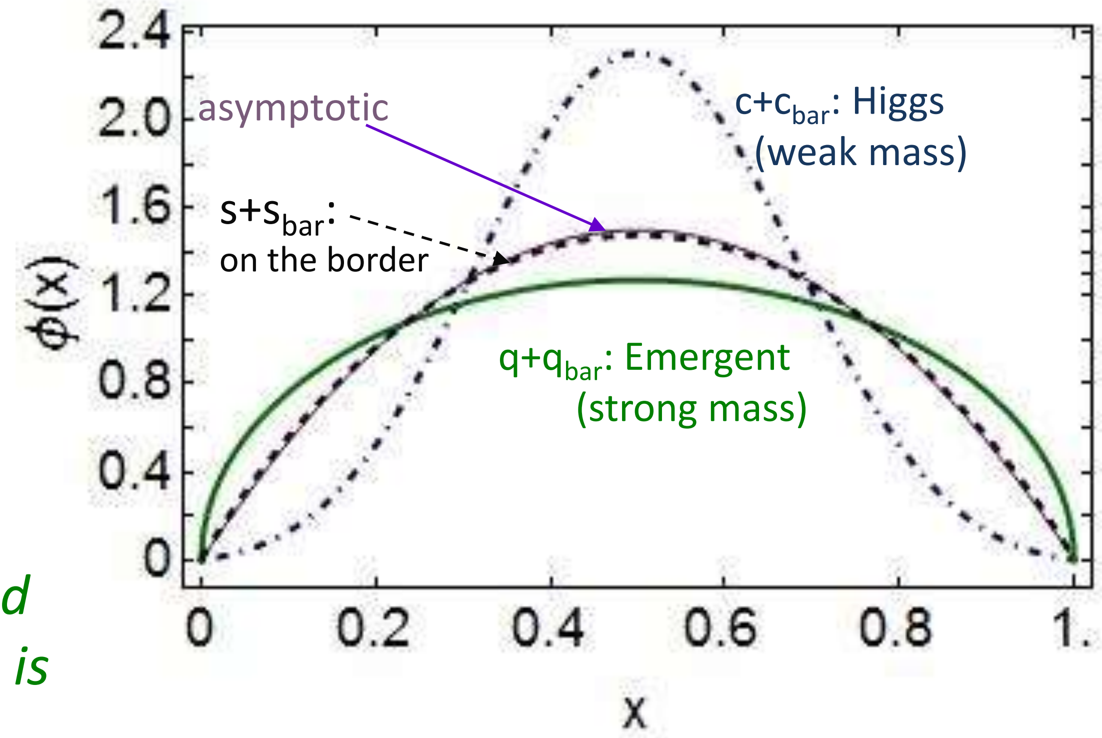


Meson Wave Functions

Parton distribution amplitudes of S-wave heavy-quarkonia
Minghui Ding, Fei Gao, Lei Chang, Yu-Xin Liu and Craig D. Roberts
arXiv:1511.04943 [nucl-th], Phys. Lett. B **753** (2016) pp. 330-335

Emergent Mass *vs*. Higgs Mechanism

- When does Higgs mechanism begin to influence mass generation?
- limit $m_{\text{quark}} \to \infty$
  $\varphi(x) \to \delta(x-½)$
- limit $m_{\text{quark}} \to 0$
  $\varphi(x) \sim (8/\pi)\,[x(1-x)]^{½}$
- Transition boundary lies just above m_{strange}
- *Comparison between distributions of light-quarks and those involving strange-quarks is good place to seek signals for strong-mass generation*

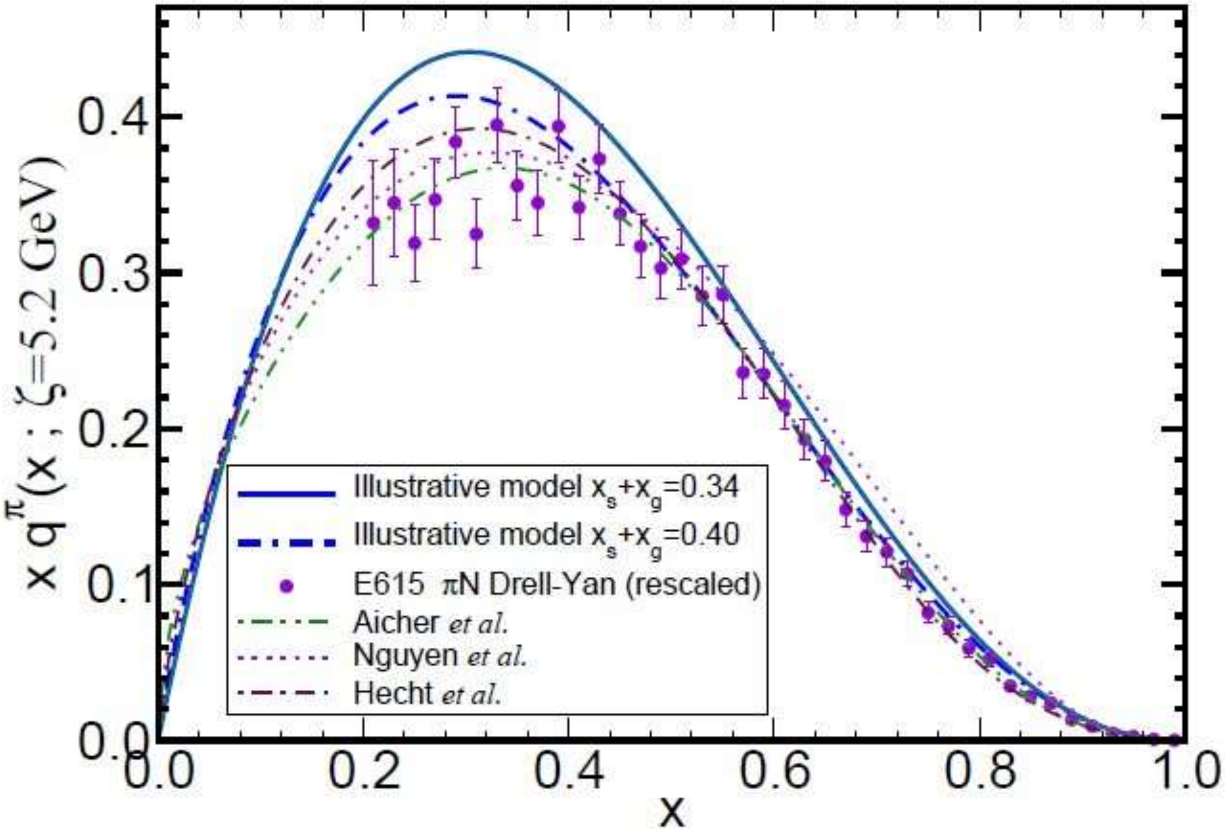


asymptotic

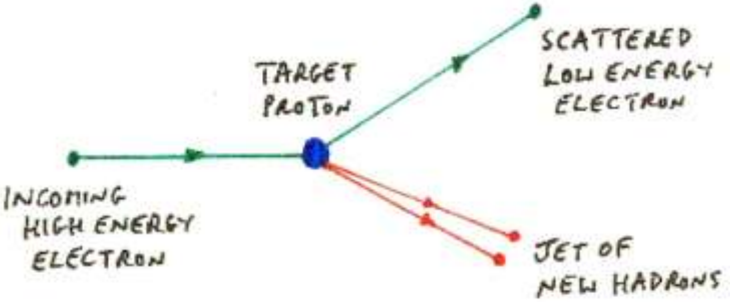
$c+c_{\text{bar}}$: Higgs (weak mass)

$s+s_{\text{bar}}$: on the border

$q+q_{\text{bar}}$: Emergent (strong mass)



π & *K* Valence-quark Distribution Functions



Deep inelastic scattering

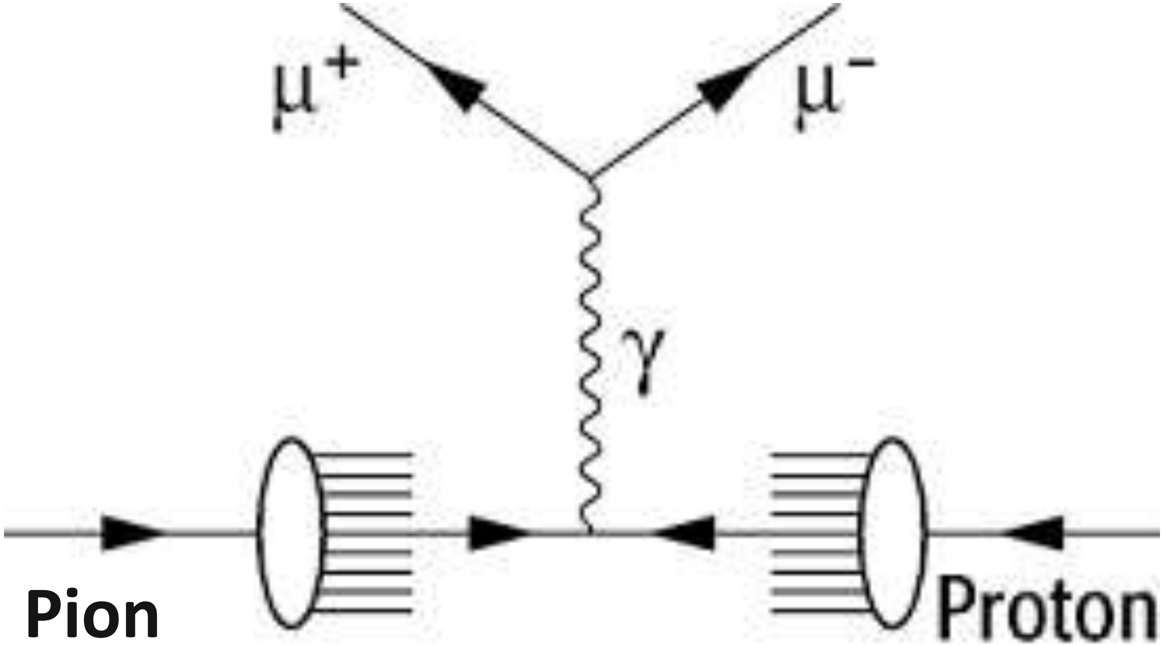
- Quark discovery experiment at SLAC (1966-1978, Nobel Prize in 1990)
- Completely different to elastic scattering
  - *Blow the target to pieces instead of keeping only those events where it remains intact.*
- Cross-section is interpreted as a measurement of the momentum-fraction probability distribution for quarks and gluons within the target hadron: $q(x)$, $g(x)$ ← *Probability that a quark/gluon within the target will carry a fraction x of the bound-state's light-front momentum*

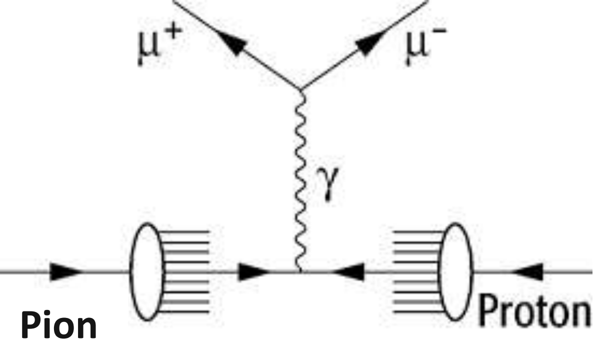
Distribution Functions of the Nucleon and Pion in the Valence Region, Roy J. Holt and Craig D. Roberts, arXiv:1002.4666 [nucl-th], Rev. Mod. Phys. **82** (2010) pp. 2991-3044

π Valence-quark Distribution Functions

Owing to absence of pion targets, the pion's valence-quark distribution functions are measured via the Drell-Yan process:

$$\pi^- p \rightarrow \mu^+ \mu^- X$$





Continuum QCD prediction of *π* valence-quark distributions

- Owing to absence of pion targets, the pion's valence-quark distribution functions are measured via the Drell-Yan process:

  $\pi\, p \to \mu^+ \mu^- X$

- Consider a theory in which quarks scatter via a vector-boson exchange interaction whose $k^2 >> m_G^2$ behaviour is $(1/k^2)^\beta$,
- Then at a resolving scale Q_0

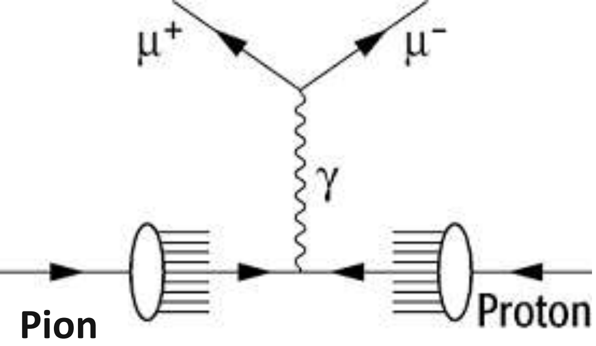
  $$u_\pi(x;Q_0) \sim (1-x)^{2\beta}$$

  namely, the large-x behaviour of the quark distribution function is a direct measure of the momentum-dependence of the underlying interaction.

- In QCD, β=1 and hence

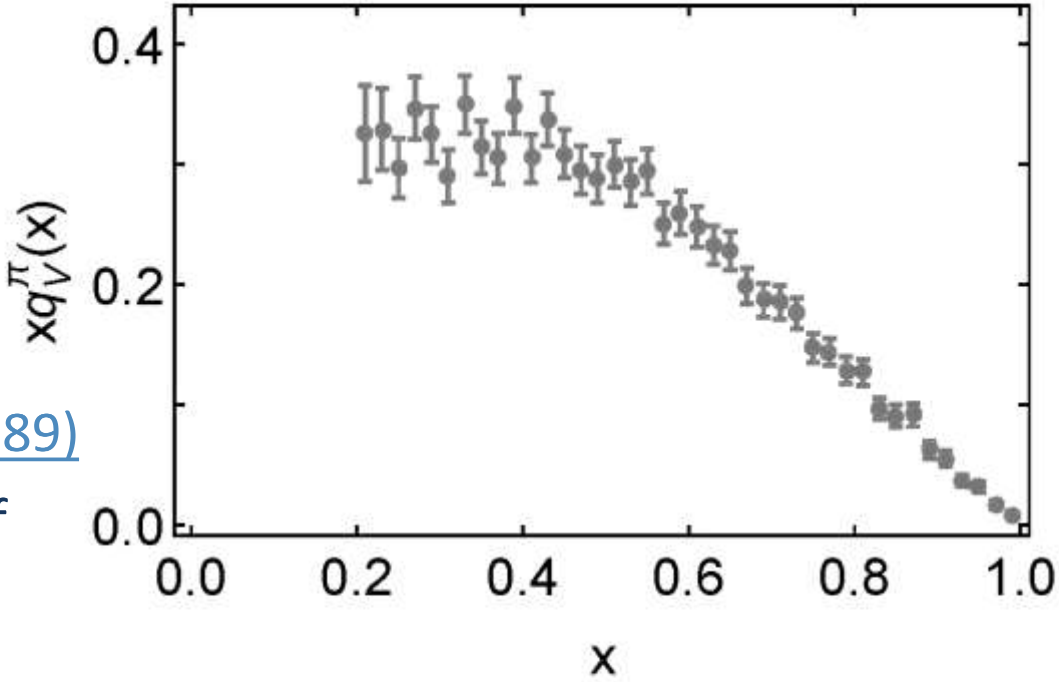
  QCD: $Q>Q_0 \Rightarrow 2 \to 2+\gamma,\ \gamma > 0$

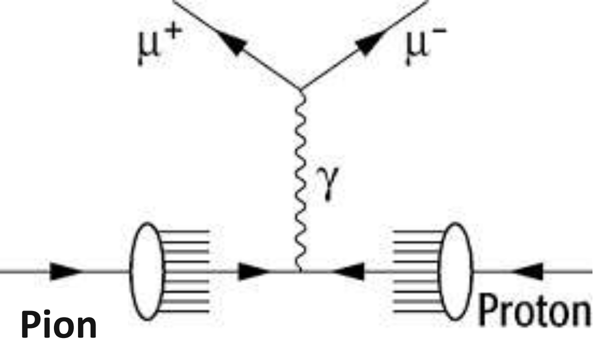
  $$^{QCD}u_\pi(x;Q_0) \sim (1-x)^2$$



Empirical status of the Pion's valence-quark distributions

- Owing to absence of pion targets, the pion's valence-quark distribution functions are measured via the Drell-Yan process:
  $\pi\, p \rightarrow \mu^+ \mu^- X$
- Three experiments:
  - CERN (1983 & 1985)
  - FNAL (1989).
- None more recent
- Conway *et al*.
  Phys. Rev. D **39**, 92 (1989)
  - Leading-order analysis of the Drell-Yan data
  - ~ 400 citations





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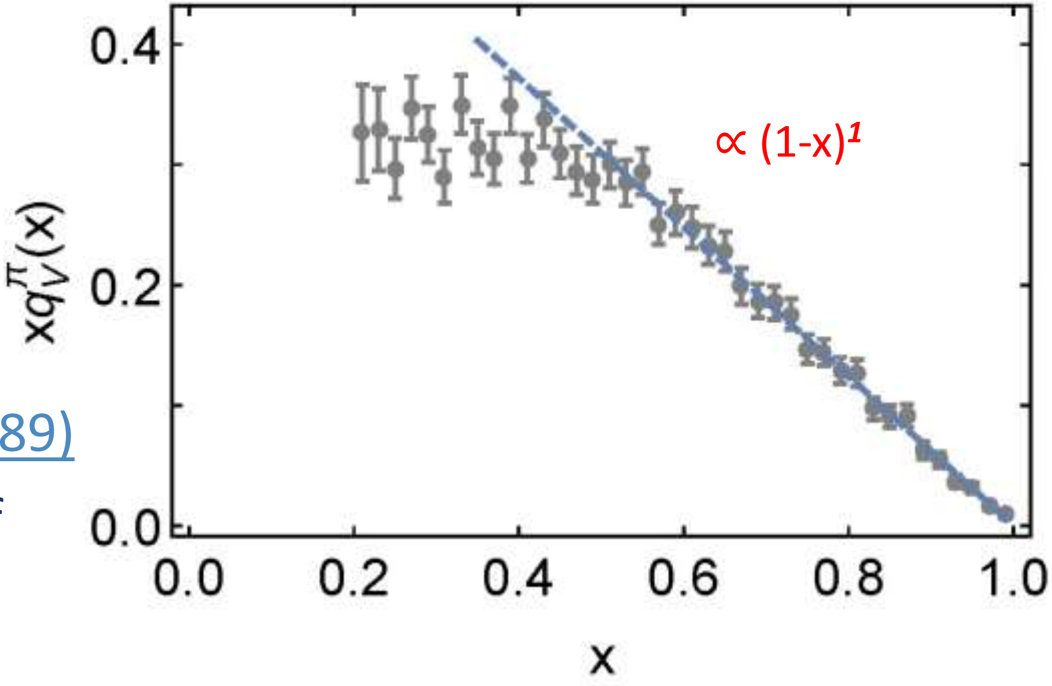
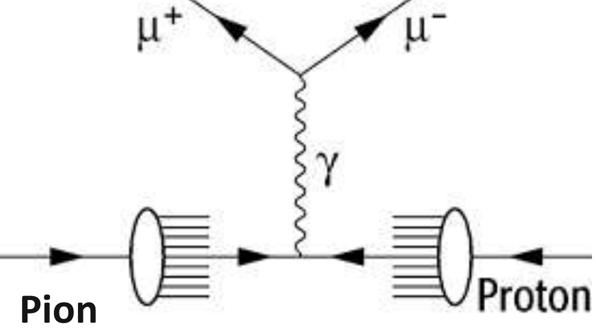
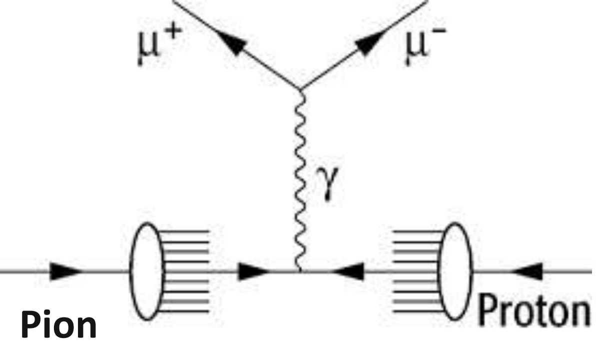
- None more recent
- Conway *et al.*
  Phys. Rev. D **39**, 92 (1989)
  - Leading-order analysis of the Drell-Yan data
- ***Controversial!***





Models of the Pion's valence-quark distributions

- *$(1-x)^\beta$ with $\beta=0$ (i.e., a* constant – any fraction is equally probable!)
  - Nambu–Jona-Lasinio models, when a translationally invariant regularization is used
- *$(1-x)^\beta$ with $\beta=1$*
  - *Nambu*–Jona-Lasinio NJL models with a hard cutoff
  - AdS/QCD models using light-front holography
  - Duality arguments produced by some theorists
- *$(1-x)^\beta$ with $0<\beta<2$*
  - Relativistic constituent-quark models, with power-law depending on the form of model wave function
- *$(1-x)^\beta$ with $1<\beta<2$*
  - Instanton-based models, all of which have incorrect large-k^2 behaviour



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Completely unsatisfactory. Impossible to suggest that there's even qualitative agreement!

π valence-quark distributions
20 Years of Evolution

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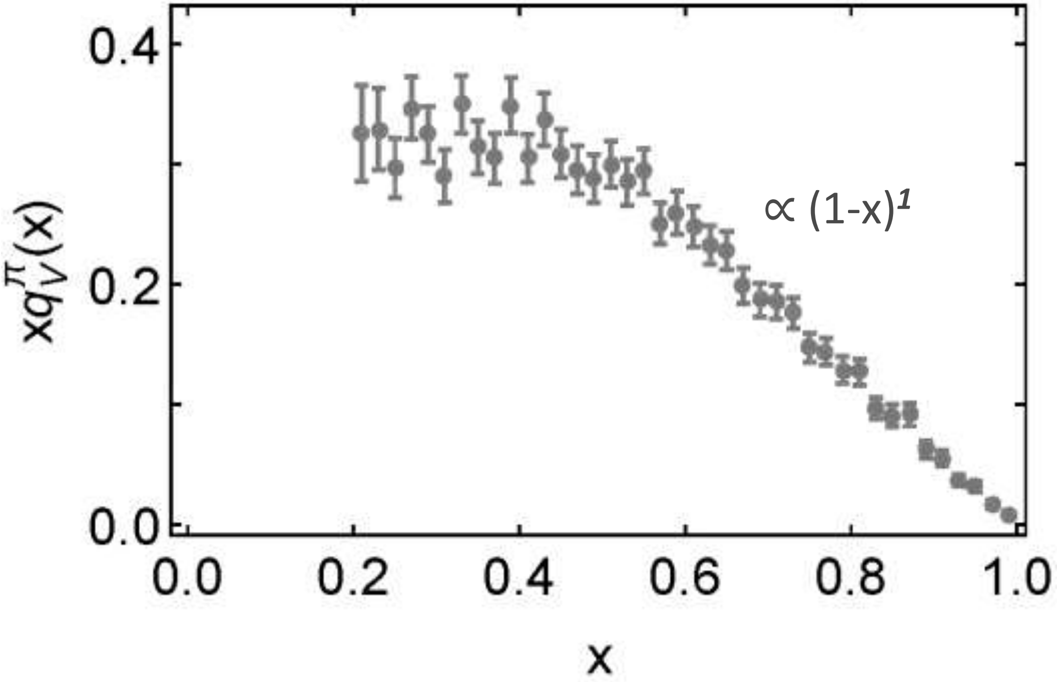

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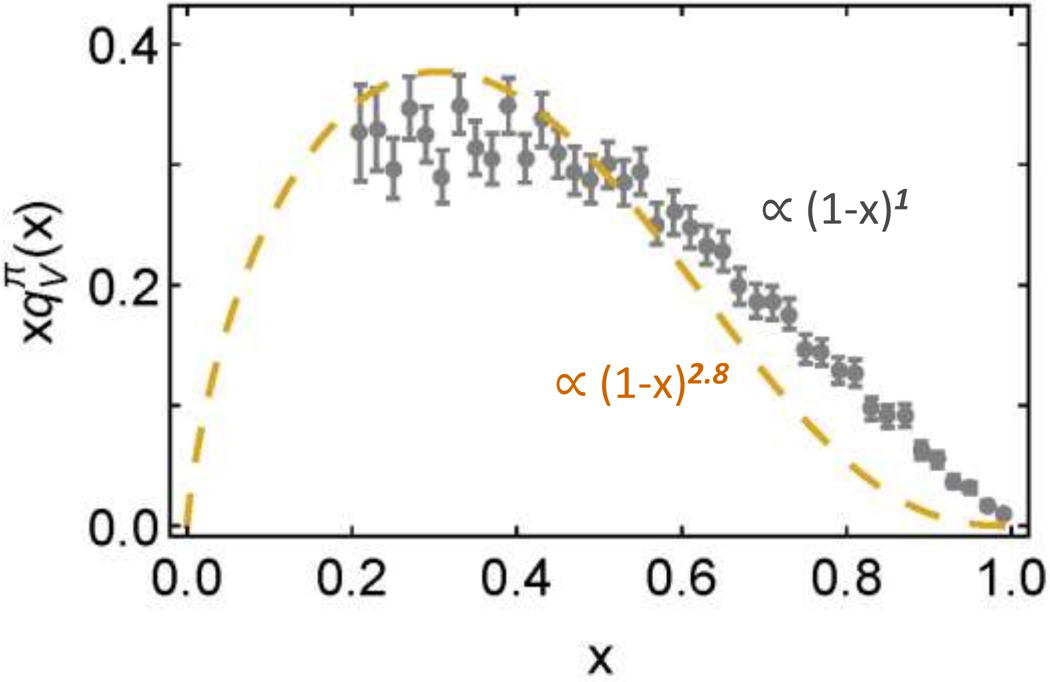

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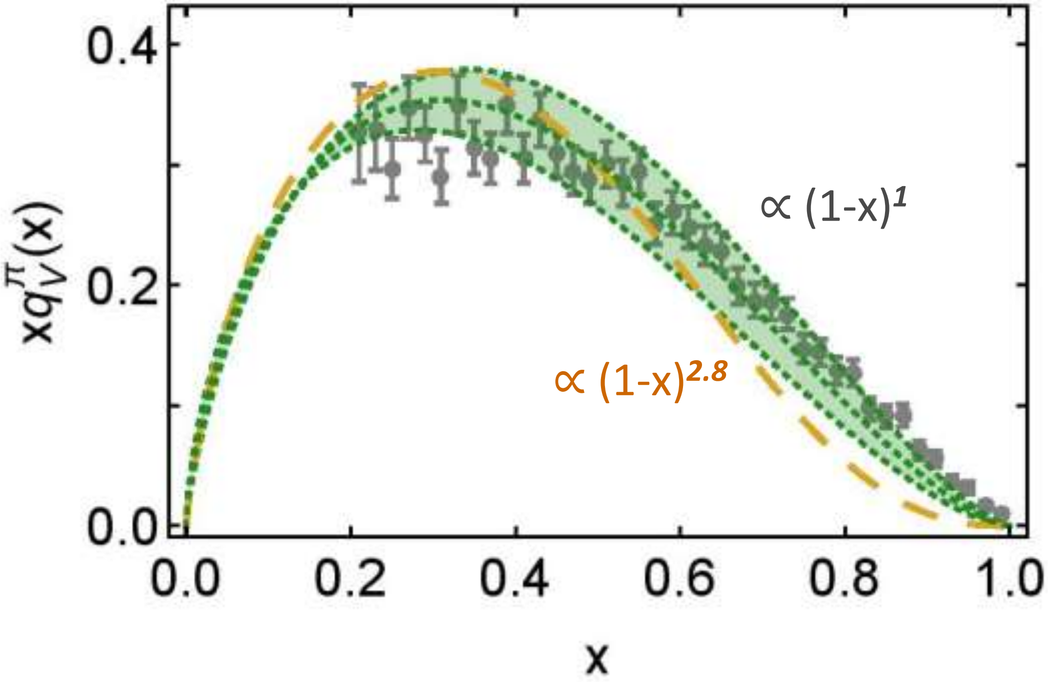
  - Large-*x* power-law → 1.54±0.08



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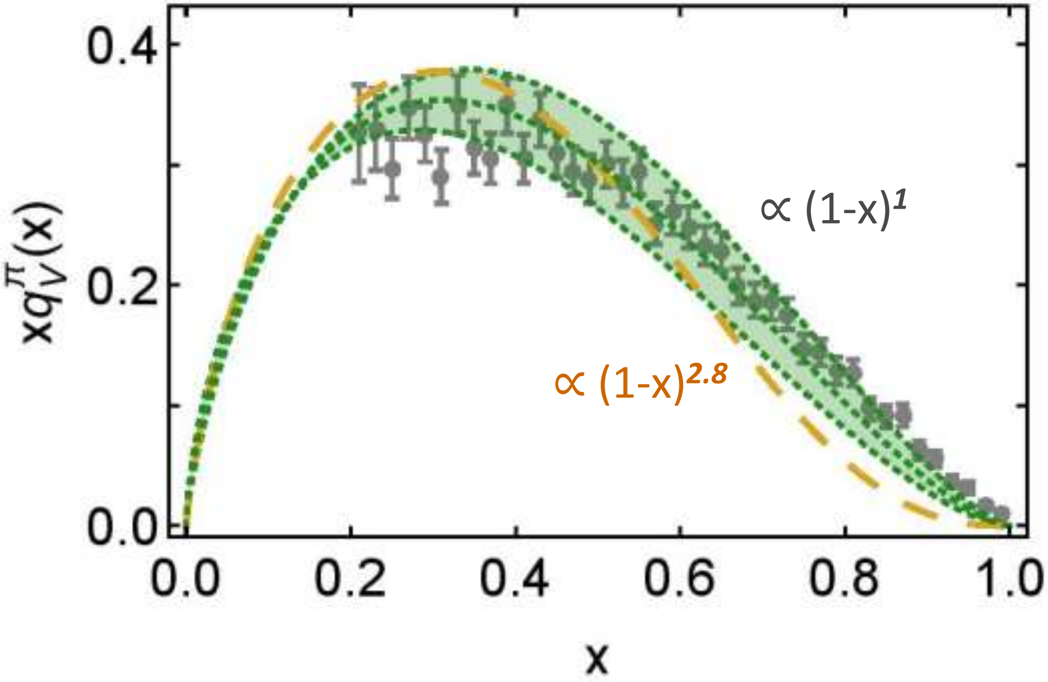

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- 2010/09 … Reconsideration of data: Aicher *et al*., Phys. Rev. Lett. **105** (2010) 252003
  - Consistent next-to-leading-order analysis
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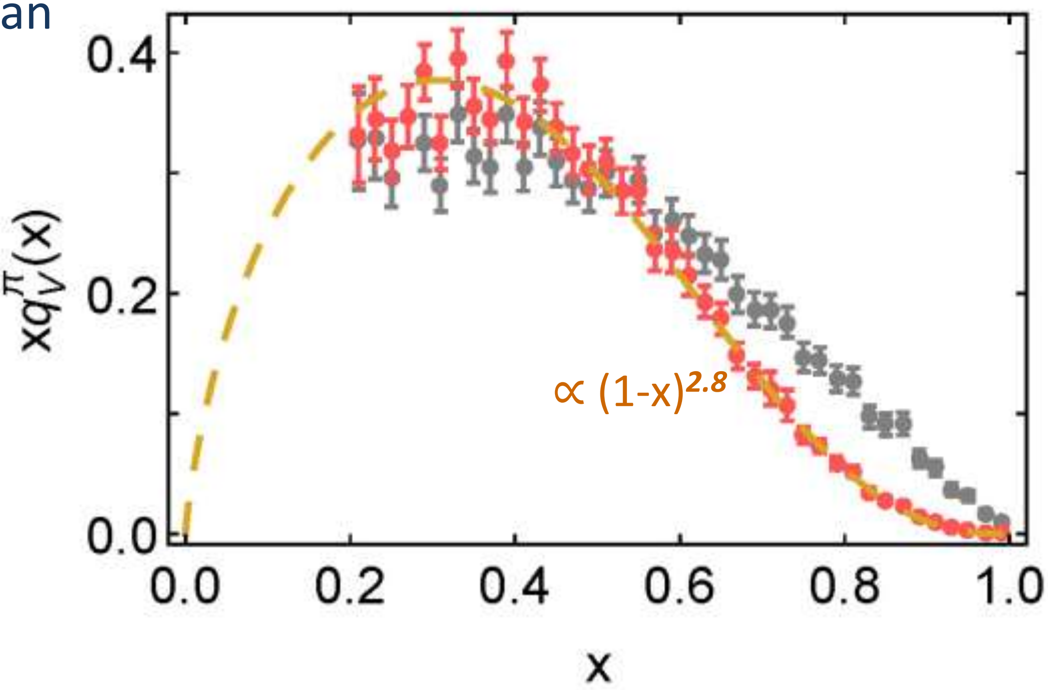

Craig Roberts. DSEs in Hadroparticle Physics: Past, Present, and Future

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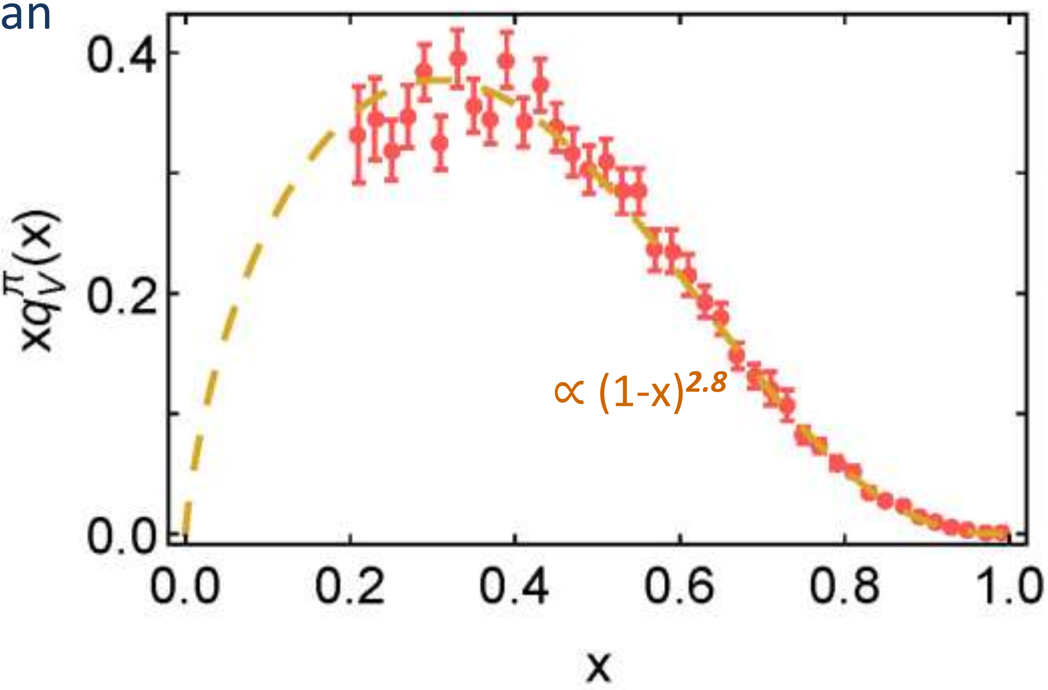
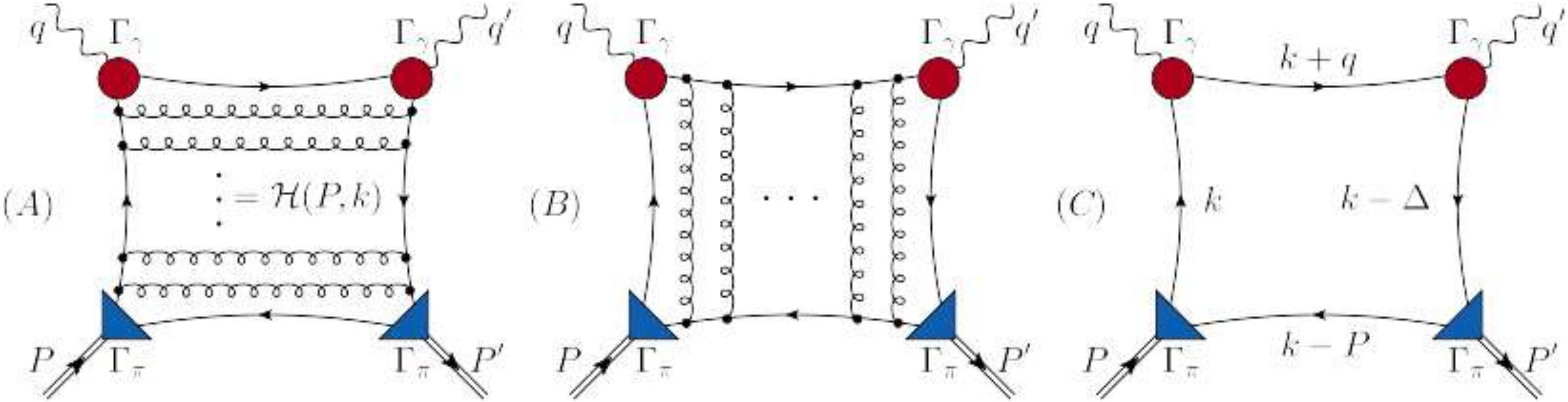




Symmetry, Symmetry Breaking, & Pion Structure

Symmetry, Symmetry Breaking, & Pion Parton Distributions

- Optical theorem

  ⇒ pion structure function is given by imaginary part of the virtual-photon – pion forward Compton scattering amplitude:

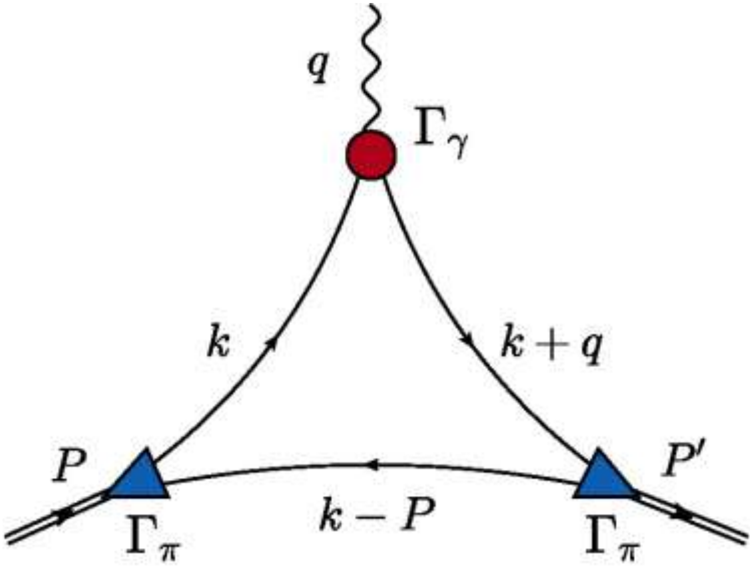
  $\gamma^*(q) + \pi(P) \to \gamma^*(q) + \pi(P)$

  "Four-point" function

- Any nonperturbative analysis will compute the structure function at an hadronic scale, ζ_H
- Using Rainbow-Ladder Truncation, what collection of diagrams is necessary and sufficient to preserve all Ward-Green-Takahashi identities?

Symmetry, Symmetry Breaking, & Pion Parton Distributions

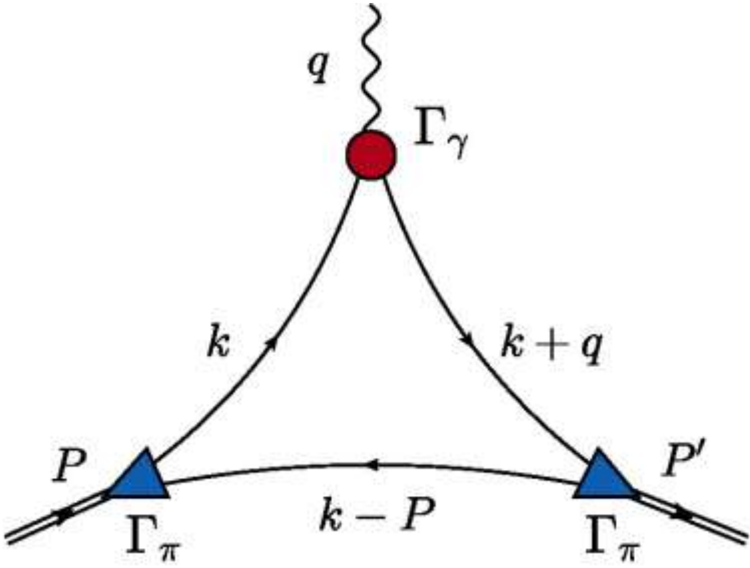
- Simpler case = pion form factor = "Three-point" function
  - line (*a*) = momentum *k*
  - line (*b*) = *k+q*
  - line (*c*) = *k-P*
- Suppose one were to add a gluon emitted from (*a*) and reabsorbed by (*a*)
  - over-counting because the contribution is already included in the rainbow truncation computation of *S*
- Suppose next one were to add a gluon emitted from (*a*) and reabsorbed by (*c*)
  - over-counting because the contribution is already included in RL-truncation of Γ_π



No Corrections in RL truncation

Symmetry, Symmetry Breaking, & Pion Parton Distributions

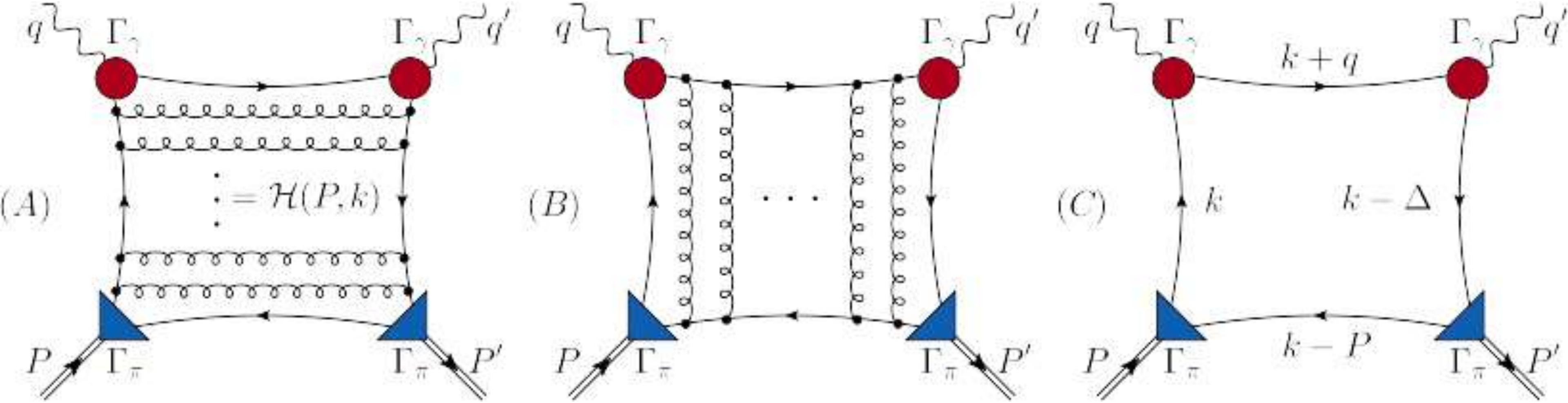
- Simpler case = pion form factor = "Three-point" function
  - line (*a*) = momentum *k*
  - line (*b*) = *k+q*
  - line (*c*) = *k-P*
- Suppose finally one were to add a gluon emitted from (*a*) and reabsorbed by (*b*)
  - over-counting because the contribution is already included in RL-truncation of Γ_γ
- Indeed, no matter which line or lines one chooses to emit and reabsorb a single gluon, the contribution generated is already included in either S, Γ_π, Γ_γ



No Corrections in RL truncation

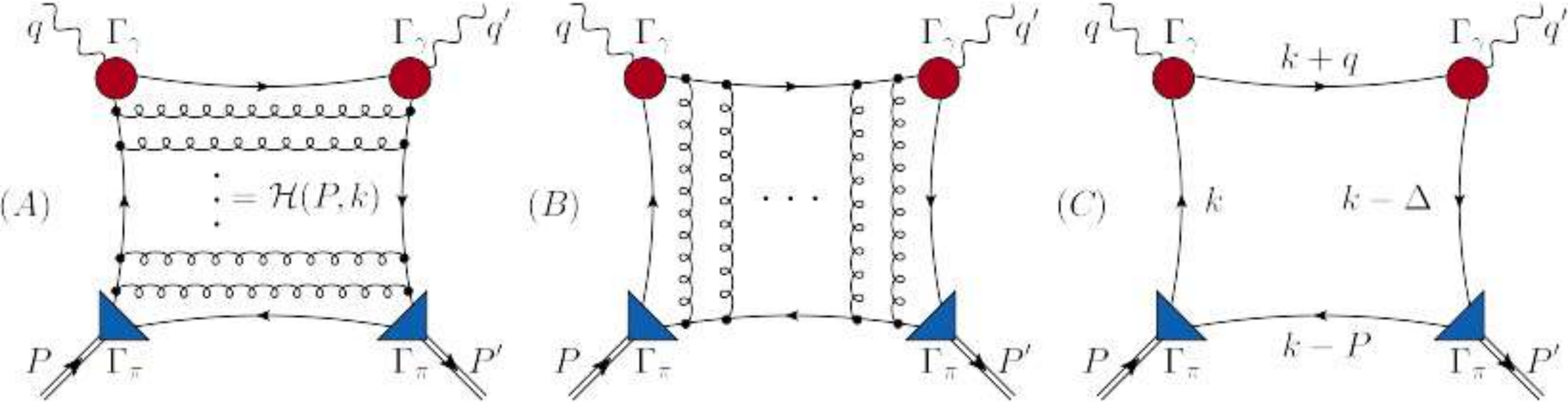
Symmetry, Symmetry Breaking, & Pion Parton Distributions

- ➢ Using Rainbow-Ladder Truncation, what collection of diagrams is necessary and sufficient to preserve all Ward-Green-Takahashi identities?



- ➢ $\gamma^*(q) + \pi(P) \rightarrow \gamma^*(q) + \pi(P) = (A) + (B) - (C)$

Symmetry, Symmetry Breaking, & Pion Parton Distributions

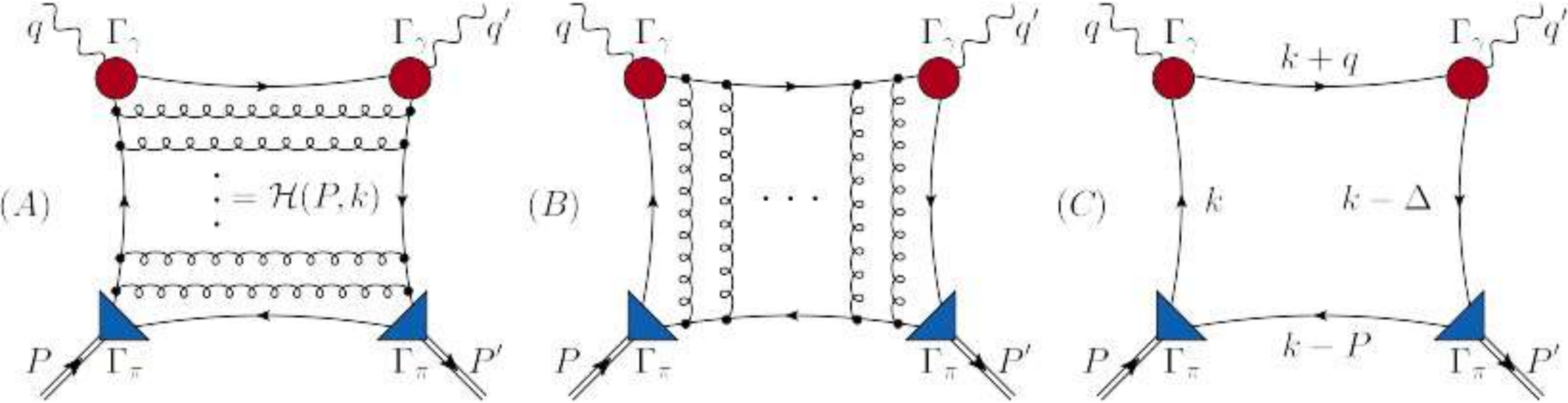


- In the forward and Bjorken limits, (*C*) is textbook *handbag* contribution to γ*(q) + π(P) → γ* (q) + π(P)
- Often been used alone to estimate $q^\pi(x;\zeta_H)$
- If Γ_π is assumed to be momentum-independent (consistent treatment of contact interaction) & Poincaré-invariant regularisation is employed, then (*C*) yields result that preserves both the baryon-number and momentum sum-rules

$$\int_0^1 dx\, q^\pi(x;\zeta_H) = 1\,,$$

$$\int_0^1 dx\, x\, q^\pi(x;\zeta_H) = \frac{1}{2}\,.$$

Symmetry, Symmetry Breaking, & Pion Parton Distributions



- However, if regularisation introduces a mass-scale and/or the quark-antiquark interaction is momentum-dependent, then the result obtained from (*C*) violates one or both of the sum rules
- Therefore, (*C*) alone is a poor approximation when realistic interactions are used.

$$\xcancel{\int_0^1 dx\, q^\pi(x;\zeta_H) = 1\,,}$$

$$\xcancel{\int_0^1 dx\, x\, q^\pi(x;\zeta_H) = \frac{1}{2}\,.}$$

Symmetry, Symmetry Breaking, & Pion Parton Distributions

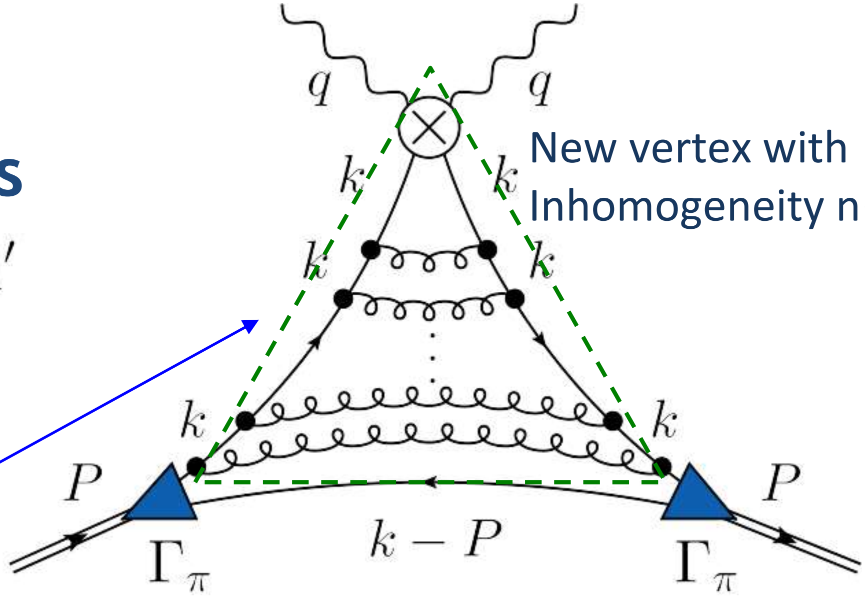
- γ*(q) + π(P) → γ* (q) + π(P)
  = (A) + (B) – (C)
- Bjorken limit for DIS
  (A) + (B) – (C) → (A’) + (B’)

$$q^\pi_A(x;\zeta_H) = N_c \mathrm{tr}\int_{dk} i\Gamma_\pi(k_\eta,-P) \times S(k_\eta)\, i\Gamma^n(k;x;\zeta_H)\, S(k_\eta)\, i\Gamma_\pi(k_{\bar\eta},P)\, S(k_{\bar\eta})$$

$$q^\pi_{BC}(x;\zeta_H) = N_c \mathrm{tr}\int_{dk} \Gamma^n_\pi(k_\eta,-P;\zeta_H) \times S(k_\eta)\Gamma_\pi(k_{\bar\eta},P)\, S(k_{\bar\eta})$$

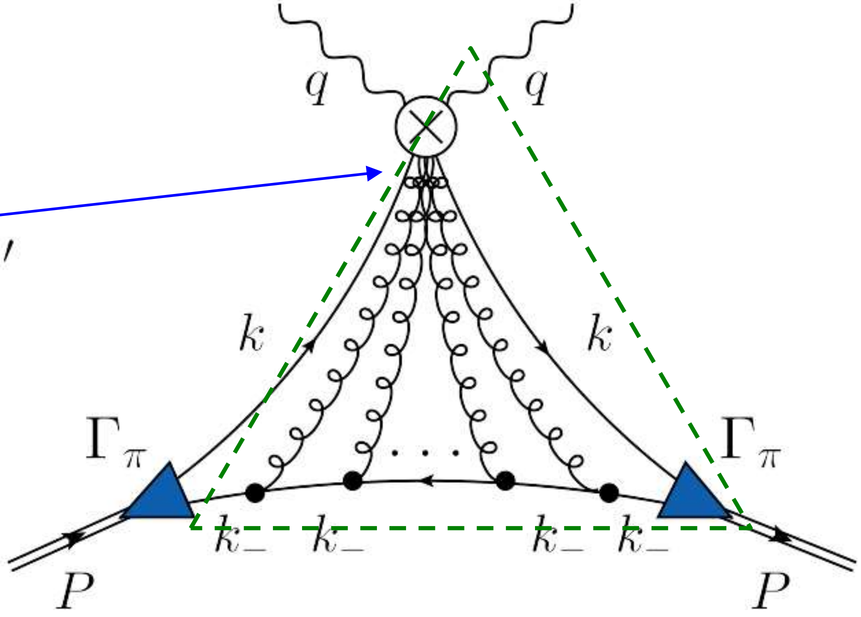
Pierced pion Bethe-Salpeter amplitude computed by summing infinitely many insertions of $[\delta_n^x(k_\eta)\, n\cdot\partial S(k_\eta)]$

A'



New vertex with Inhomogeneity $n\cdot\gamma\ \ \delta_n^x(k_\eta)$

B'



$$q^\pi(x;\zeta_H) = q^\pi_A(x;\zeta_H) + q^\pi_{BC}(x;\zeta_H)$$

Basic features of the pion valence-quark distribution function, Lei Chang, Cédric Mezrag, Hervé Moutarde, Craig D. Roberts, Jose Rodríguez-Quintero, arXiv:1406:5450 [nucl-th], Phys. Lett. B **737** (2014)pp. 23–29

Symmetry, Symmetry Breaking, & Pion Parton Distributions

- Ward identity approximation

$$i\Gamma^n(k;x;\zeta_H) = \delta_n^x(k_\eta) n\cdot\partial_{k_\eta} S^{-1}(k_\eta)\,,$$
$$\Gamma_\pi^n(k_\eta,-P;\zeta_H) = n\cdot\partial_{k_\eta}\Gamma_\pi(k_\eta,-P;\zeta_H)\,,$$

in which case

$$q^\pi(x;\zeta_H) = N_c \mathrm{tr}\int_{dk} \delta_n^x(k_\eta)$$
$$\times\, n\cdot\partial_{k_\eta}\left[\Gamma_\pi(k_\eta,-P)S(k_\eta)\right]\Gamma_\pi(k_{\bar\eta},P)\,S(k_{\bar\eta})$$

- $q^\pi(x\,;\zeta_H) = q^\pi(1-x\,;\zeta_H)$

☑

$$\int_0^1 dx\, q^\pi(x;\zeta_H) = 1\,,$$
$$\int_0^1 dx\, x q^\pi(x;\zeta_H) = \frac{1}{2}\,.$$

Symmetry, Symmetry Breaking, & Pion Parton Distributions

- Symmetry-preserving RL-truncation computation of $q^{\pi}(x\,;\zeta_H)$
- Question = What is ζ_H?
- In first applications of DSE approach to hadron observables [Frank:1995uk, Maris:1997tm] (and many that have followed):
  - renormalisation scale was chosen deep in the spacelike region: $\zeta = \zeta_{19} := 19$ GeV
  - primarily to ensure simplicity in nonperturbative renormalisation procedure
- This choice entails that the dressed quasiparticles obtained as solutions to the DSEs remain intact and thus serve as the dominant degrees-of-freedom for all observables.
  - adequate for infrared quantities, such as hadron masses
    - flexibility of model parameters and the bridge with QCD enable valid predictions to be made.

Symmetry, Symmetry Breaking, & Pion Parton Distributions

- However, renormalisation scale $\zeta = \zeta_{19}$ generates errors in form factors and parton distributions
  - Form factors … correct power-law behaviour is obtained, but the scaling violations deriving from anomalous operator dimensions are wrong (see, *e.g*. [Maris:1998hc])
  - Parton distributions .. natural connection between renormalisation scale and reference scale for evolution equations is lost
    - again because parton loops are suppressed when renormalising a RL truncation study at deep spacelike momenta so computed anomalous dimensions are wrong.

Symmetry, Symmetry Breaking, & Pion Parton Distributions

- Solution to these problems = renormalise DSEs at hadronic scale, where dressed quasiparticles are the correct degrees-of-freedom.
  - Recognises that a given meson's Poincaré covariant wave function and correlated vertices, too, must evolve with ζ [Lepage:1979zb, Efremov:1979qk, Lepage:1980fj]
- Such evolution enables dressed-quark and -antiquark degrees-of-freedom, in terms of which the wave function is expressed at a given scale $\zeta^2 = Q^2$ to *undress* …
  - split into less well-dressed partons via emission of gluons and sea quarks in the manner prescribed by QCD dynamics.
- These effects are automatically incorporated in bound-state problems when complete quark-antiquark scattering kernel is used
  - aspects are lost when kernel is truncated, *e.g*. RL truncation.
- Therefore renormalise DSEs at hadronic scale $\zeta = \zeta_H$

Hadronic Scale = ζ_H

- A natural value for the hadronic scale, ζ_H, must now be determined.
- Recall QCD's process-independent effective charge
- This running-coupling saturates in the infrared:

  $\alpha_{PI}(k^2=0) \approx \pi$

  owing to dynamical generation of gluon mass-scale
- These features and a smooth connection with are expressed in the following algebraic expression

$$\alpha_{\rm PI}(k^2) = \frac{4\pi}{\beta_0 \ln[(m_\alpha^2 + k^2)/\Lambda_{\rm QCD}^2]}$$

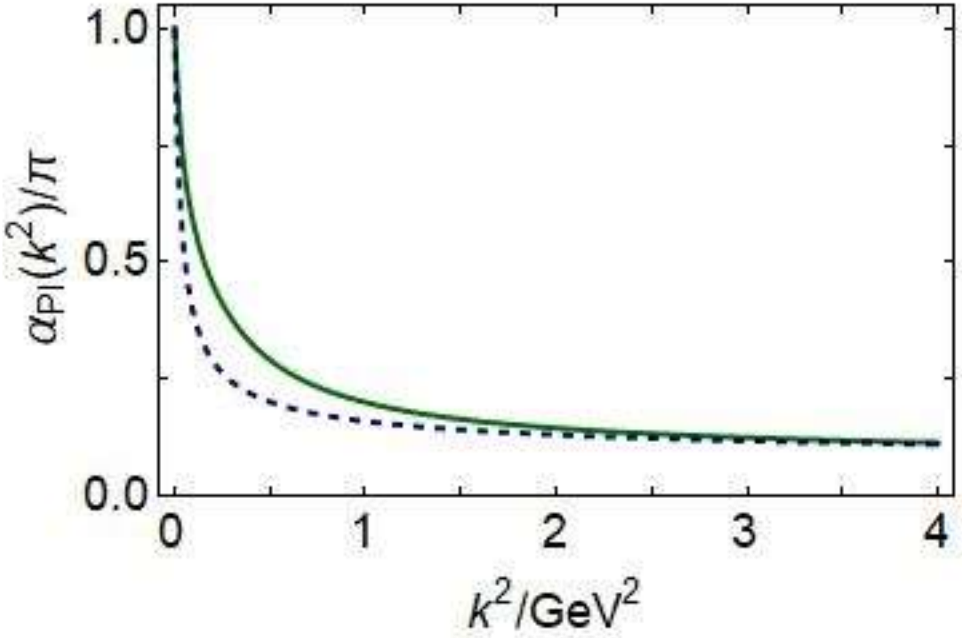
$\beta_0 = 11 - (2/3)\, n_f$

$m_\alpha = 0.3$ GeV $\sim \Lambda_{QCD}$

PI Effective Charge

$$\alpha_{\rm PI}(k^2) = \frac{4\pi}{\beta_0 \ln[(m_\alpha^2 + k^2)/\Lambda_{\rm QCD}^2]}$$

- ✓ Solid Green = original
- ✓ Dashed Blue = simplified expression
- ✓ No practical difference
- ✓ m_α = 0.3 GeV ~ Λ_{QCD}
  - ✓ Very natural value for IR mass-scale
- ✓ Evidently, m_α = essentially nonperturbative scale whose existence ensures that modes with $k^2 \leq m_\alpha^2$ are screened from interactions.
- ✓ m_α therefore serves to define the natural boundary between soft and hard physics



Identify $\zeta_H = m_\alpha$

Calculating $q^\pi(x,\zeta_H)$

- Reconstruct $q^\pi(x,\zeta_H)$ from Mellin moments

$$
\begin{aligned}
\langle x^m\rangle^\pi_{\zeta_H} &= \int_0^1 dx\, x^m q^\pi(x;\zeta_H) \\
&= \frac{N_c}{n\cdot P}\mathrm{tr}\int_{dk}\left[\frac{n\cdot k_\eta}{n\cdot P}\right]^m \Gamma_\pi(k_{\bar\eta},P)\,S(k_{\bar\eta}) \\
&\quad \times n\cdot\partial_{k_\eta}\left[\Gamma_\pi(k_\eta,-P)S(k_\eta)\right]
\end{aligned}
$$

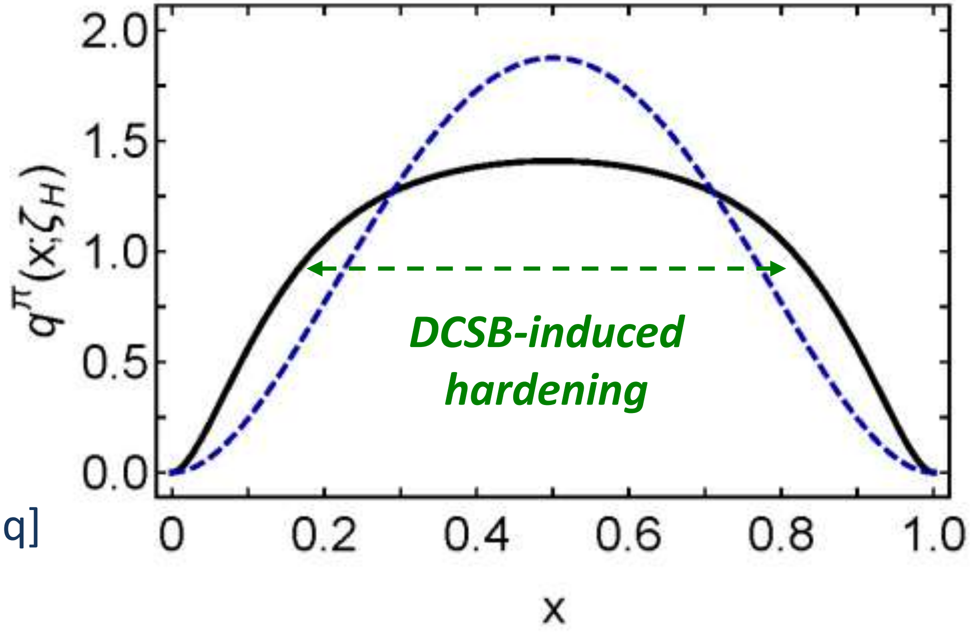
- Every moment is finite.
- However, direct calculation using numerically determined inputs for S & Γ_π is difficult in practice owing to an amplification of oscillations produced by the $[n\cdot k_\eta]^m$ factor
- Working hard, can compute m = 0, 1, 2, 3, 4, 5
- Subsequently employ methods from complex analysis to construct two analytic approximations (m=0…3 & m=0…5) that enable estimates of additional moments m = 6, 7, 8, 9, 10
- Reconstruct $q^\pi(x,\zeta_H)$ from this information on first 11 moments

$$
\begin{aligned}
q^\pi(x;\zeta_H) &= 213.32\, x^2(1-x)^2 \\
&\quad \times\left[1-2.9342\sqrt{x(1-x)}+2.2911\, x(1-x)\right]
\end{aligned}
$$

$q^\pi(x,\zeta_H)$

$$q^\pi(x;\zeta_H) = 213.32\, x^2 (1-x)^2 \times \left[1 - 2.9342\sqrt{x(1-x)} + 2.2911\, x(1-x)\right]$$

- Dashed Blue = scale-free parton-model-like result 30 x^2 $(1-x)^2$
- Solid Black = DSE prediction
- ✓ DSE distribution computed with realistic inputs is a broad concave function.
- ✓ Similar effect observed in the pion's leading-twist valence-quark distribution amplitude [Chang:2013pq] & those of other mesons



- ✓ Cause is identical: $q^\pi(x,\zeta_H)$ is hardened owing to DCSB = a realisation of the mechanism responsible for the emergence of mass in the Standard Model
- ✓ DCSB is expressed in momentum-dependence of all QCD Schwinger functions.
- ✓ Therefore manifest in pointwise behaviour of wave functions, elastic and transition form factors, *etc.*; and as now seen, also in parton distributions.
- ✓ Expected, given the connection between light-front wave functions and parton distributions

Evolution of $q^{\pi}(x,\zeta_H)$

- $q^{\pi}(x,\zeta_H)$ computed at $\zeta_H = m_\alpha$ … but …
  - existing lQCD calculations of low-order moments and phenomenological fits to pion parton distributions are typically quoted at $\zeta_2 = 2$ GeV
  - and the scale relevant to the E615 data is $\zeta_5 = 5.2$ GeV
- Therefore employ leading-order QCD evolution
  - $q^{\pi}(x,\zeta_H) \to q^{\pi}(x,\zeta_2) \to q^{\pi}(x,\zeta_5)$

  using the process-independent running coupling
- Notably, given that $\zeta_H = m_\alpha$ is fixed by our analysis, all results are predictions
- $\alpha_{PI}(\zeta_H)/(2\pi) = 0.20$ & $[\alpha_{PI}(\zeta_H)/(2\pi)]^2 = 0.04$

  … so LO evolution should serve as a good approximation
- Results reported with $\zeta_H \to (1 \pm 0.1)\, \zeta_H$

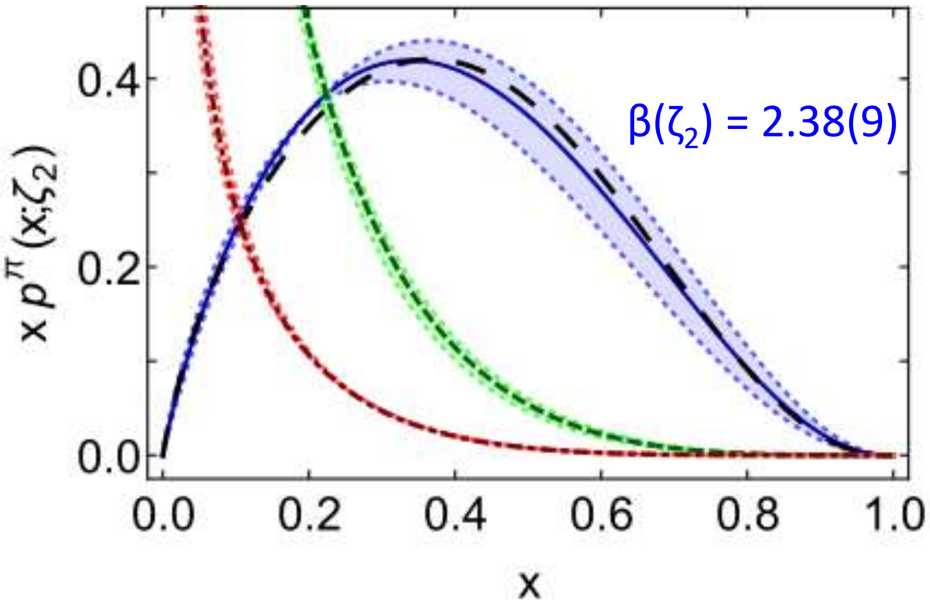
$q^{\pi}(x,\zeta_H) \to q^{\pi}(x,\zeta_2)$

- ✓ Nonsinglet evolution for valence-quark

$$\langle 2x \rangle_q^{\pi} = 0.48(3)$$

- ✓ Dashed black curve = [Hecht:2000xa]
- ✓ Valence-quarks carry only ½ pion's light-front momentum
- ✓ Pion is solely bound-state of dressed-quark and dressed-antiquark at ζ_H
- ✓ Glue and sea distributions are zero at ζ_H
- ✓ *g* & *S* distributions are generated by singlet evolution on $\zeta > \zeta_H$

$$\langle x \rangle_g^{\pi} = 0.41(2)\,, \quad \langle x \rangle_{\rm sea}^{\pi} = 0.11(2)$$



$\beta(\zeta_2) = 2.38(9)$

FIG. 6. Pion valence-quark momentum distribution function, $xq^{\pi}(x;\zeta)$, evolved $\zeta_H \to \zeta_2 = 2\,\text{GeV}$ – solid (blue) curve embedded in shaded band; and long-dashed (black) curve – ζ_2 result from Ref. [12]. Eqs. (39), (40): gluon momentum distribution in pion, $xg^{\pi}(x;\zeta_2)$ – dashed (green) curve within shaded band; and sea-quark momentum distribution, $xS^{\pi}(x;\zeta_2)$ – dot-dashed (red) curve within shaded band. In all cases, the shaded band indicates the effect of $\zeta_H \to \zeta_H(1 \pm 0.1)$.

$q^\pi(x,\zeta_H) \rightarrow q^\pi(x,\zeta_2)$

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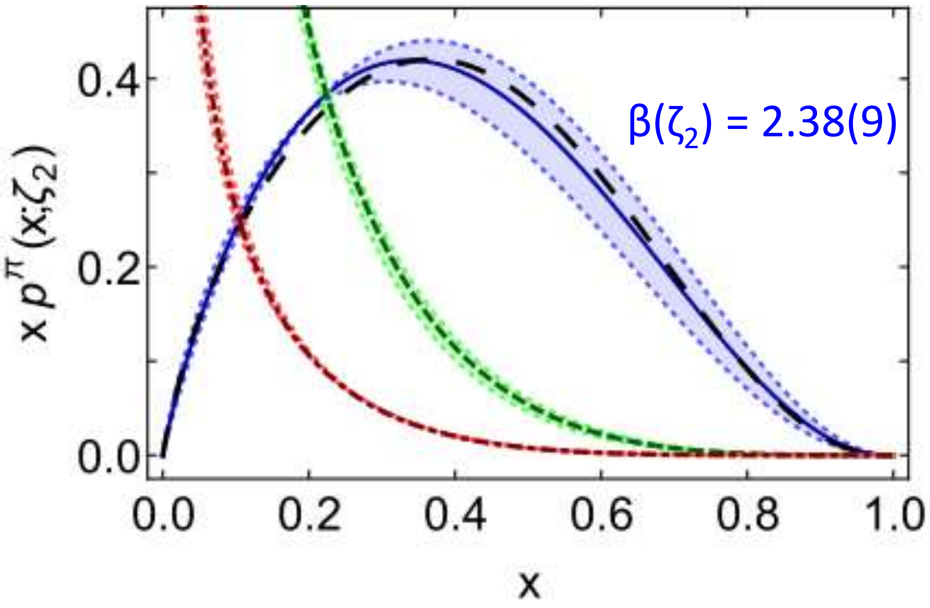
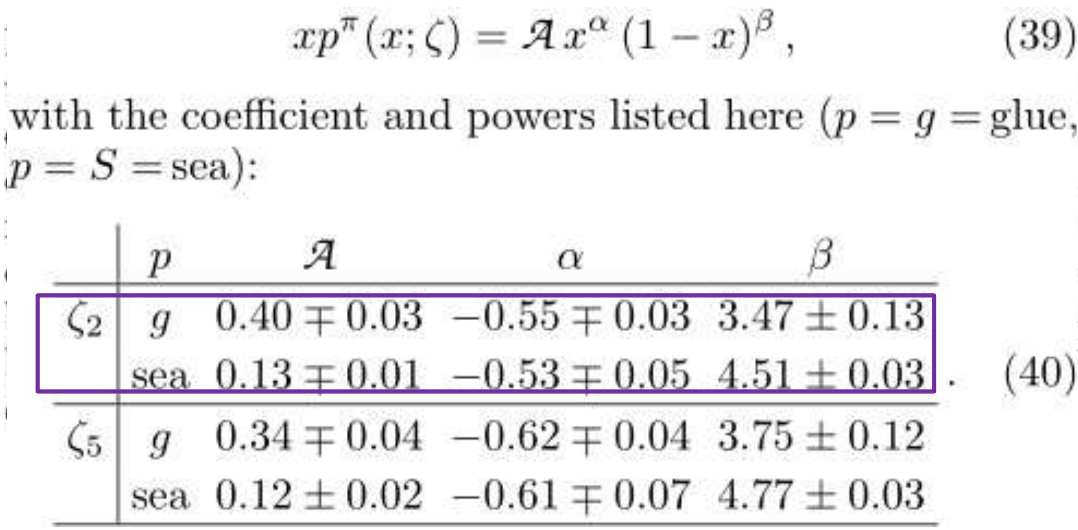




$$x p^\pi(x;\zeta) = \mathcal{A}\, x^\alpha\, (1-x)^\beta\,, \tag{39}$$

with the coefficient and powers listed here ($p = g =$ glue, $p = S =$ sea):

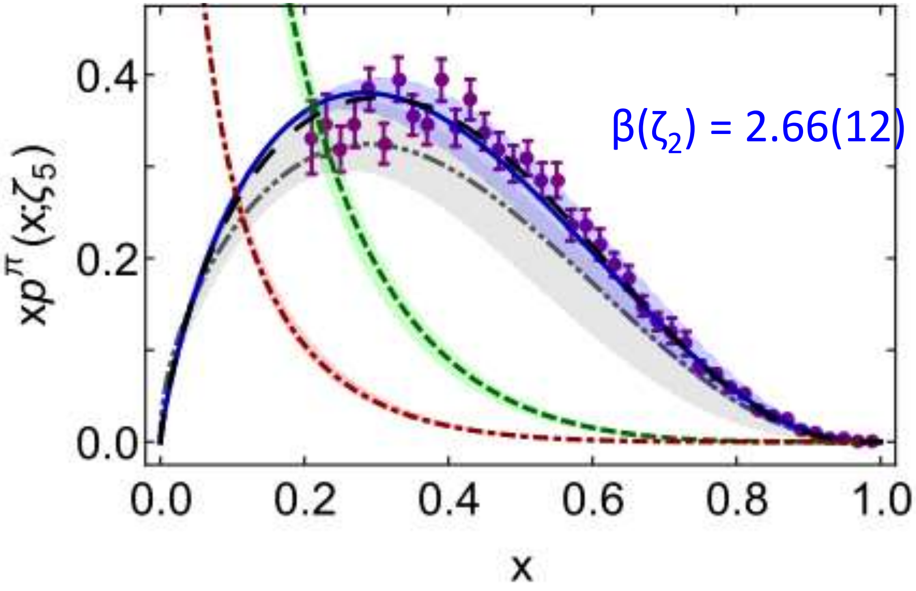
	p	$\mathcal{A}$	α	β
ζ_2	g	0.40 ∓ 0.03	-0.55 ∓ 0.03	3.47 ± 0.13
	sea	0.13 ∓ 0.01	-0.53 ∓ 0.05	4.51 ± 0.03
ζ_5	g	0.34 ∓ 0.04	-0.62 ∓ 0.04	3.75 ± 0.12
	sea	0.12 ± 0.02	-0.61 ∓ 0.07	4.77 ± 0.03

. (40)

$q^\pi(x,\zeta_H) \to q^\pi(x,\zeta_5)$

- ✓ Solid Blue = nonsinglet evolution for valence-quark
  $$\langle 2x\rangle^\pi_q = 0.42\ (3)$$
- ✓ Dashed black curve = [Hecht:2000xa]
- ✓ Valence-quarks carry less-than ½ pion’s light-front momentum
- ✓ *g* & *S* distributions are generated by singlet evolution on $\zeta > \zeta_H$

$$\langle x\rangle^\pi_g = 0.45(1)\,,\quad \langle x\rangle^\pi_{\rm sea} = 0.14(2)$$



$\beta(\zeta_2)$ = 2.66(12)

FIG. 7. Pion valence-quark momentum distribution function, $xq^\pi(x;\zeta)$, evolved $\zeta_H \to \zeta_5 = 5.2\,\text{GeV}$ – solid (blue) curve embedded in shaded band; and long-dashed (black) curve – ζ_5 result from Ref. [12]. Gluon momentum distribution in pion, $xg^\pi(x;\zeta_2)$ – dashed (green) curve within shaded band; and sea-quark momentum distribution, $xS^\pi(x;\zeta_2)$ – dot-dashed (red) curve within shaded band. See Eqs. (39), (40). In all the above cases, the shaded band indicates the effect of $\zeta_H \to \zeta_H(1\pm0.1)$. Dot-dot-dashed (grey) curve within shaded band – lQCD result [31]. Data (purple) from Ref. [9], rescaled according to the analysis in Ref. [14].

$q^\pi(x,\zeta_H) \to q^\pi(x,\zeta_5)$

- ✓ Solid Blue = nonsinglet evolution for valence-quark
  $$\langle 2x\rangle_q^\pi = 0.42\,(3)$$
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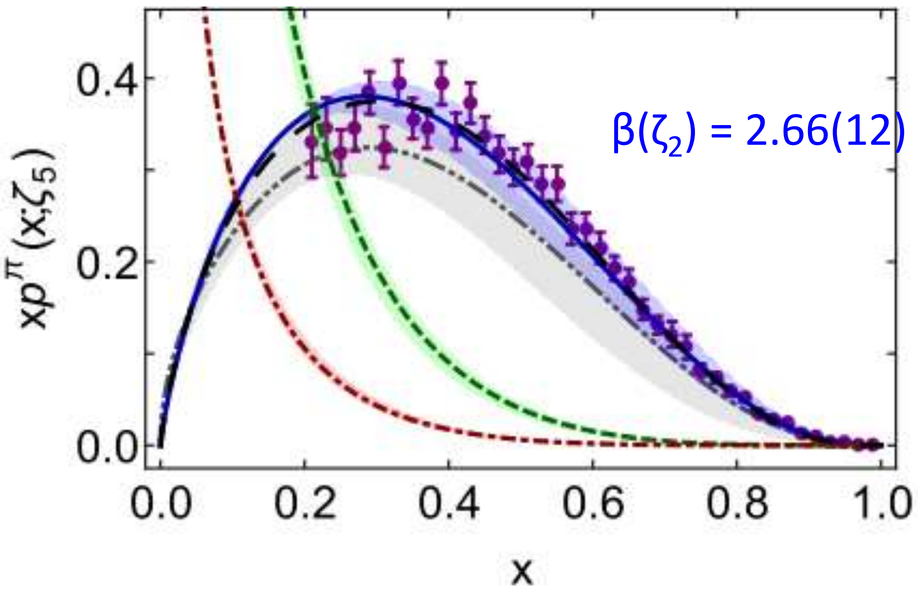

$$xp^\pi(x;\zeta) = \mathcal{A}\, x^\alpha\,(1-x)^\beta\,, \qquad (39)$$

with the coefficient and powers listed here ($p = g =$ glue, $p = S =$ sea):

	p	$\mathcal{A}$	α	β
ζ_2	g	0.40 ∓ 0.03	-0.55 ∓ 0.03	3.47 ± 0.13
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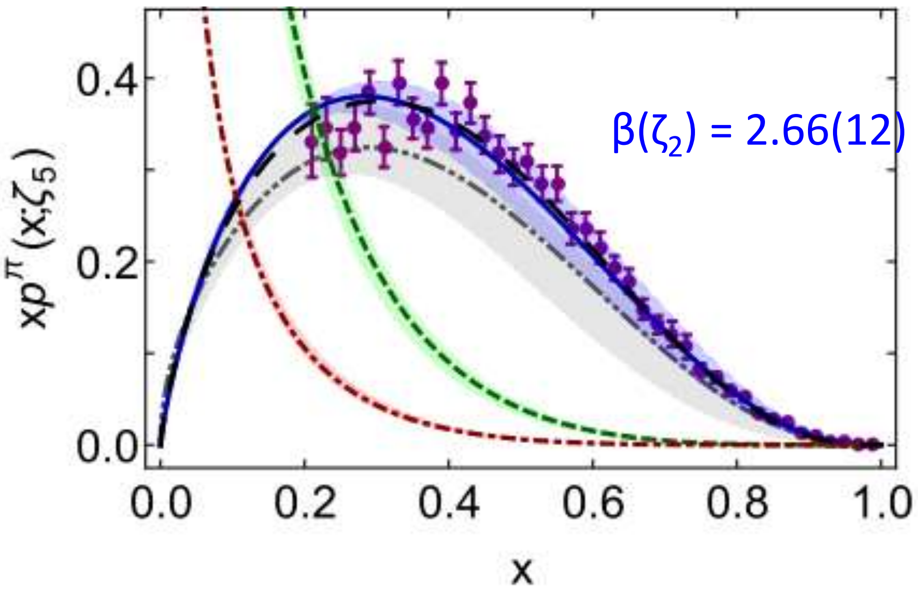
$q^{\pi}(x,\zeta_H) \to q^{\pi}(x,\zeta_5)$

- ✓ Solid Blue = nonsinglet evolution for valence-quark

$$\langle 2x \rangle^{\pi}_{q} = 0.42\ (3)$$

- ✓ Dashed black curve = [Hecht:2000xa]
- ✓ Valence-quarks carry less-than ½ pion's light-front momentum
- ✓ dot-dot-dashed (grey) = lQCD result for the pion valence-quark distribution function [Sufian:2019bol]
- ✓ Pointwise form of the lQCD prediction agrees with our result (within errors).
- ✓ ***Significant***: two disparate treatments of the pion have arrived at the same prediction for $q^{\pi}(x,\zeta_5)$



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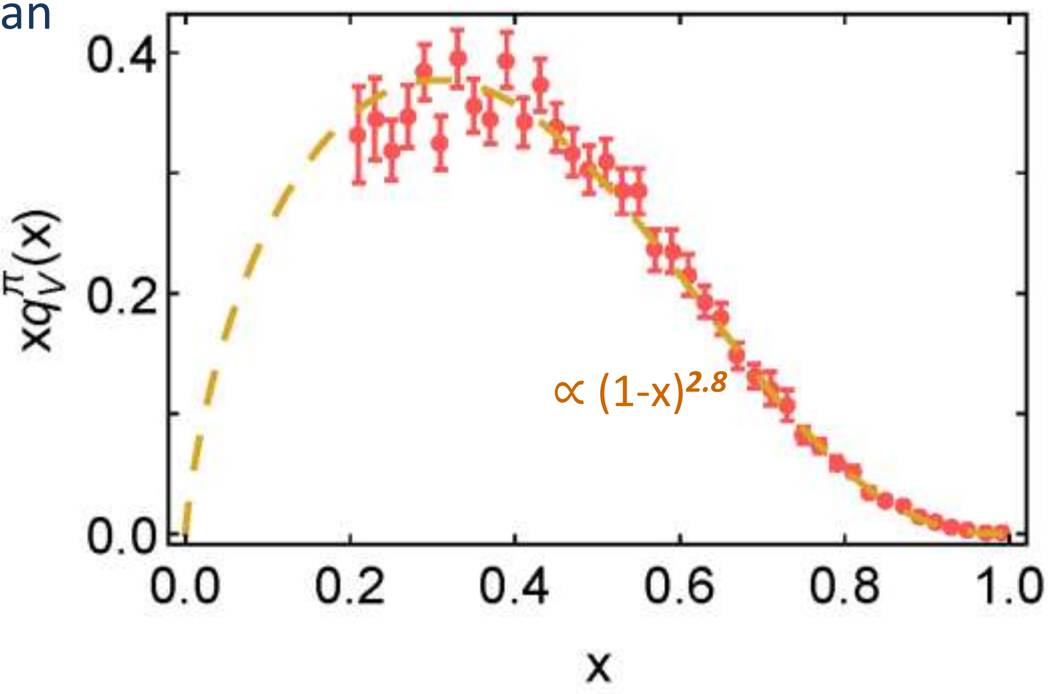
π valence-quark distributions 20 Years of Evolution

- 1989 … Conway *et al*. Phys. Rev. D **39**, 92 (1989)
  - Leading-order analysis of Drell-Yan data
- 2000 … Hecht *et al*. Phys.Rev. C **63** (2001) 025213
  - QCD-connected model prediction
- 2010/02 … Controversy highlighted: Holt & Roberts, Rev. Mod. Phys. **82** (2010) 2991-3044
- 2010/09 … Reconsideration of data: Aicher *et al*., Phys. Rev. Lett. **105** (2010) 252003
  - Consistent next-to-leading-order analysis
  - Large-*x* power-law → 2.6(1)



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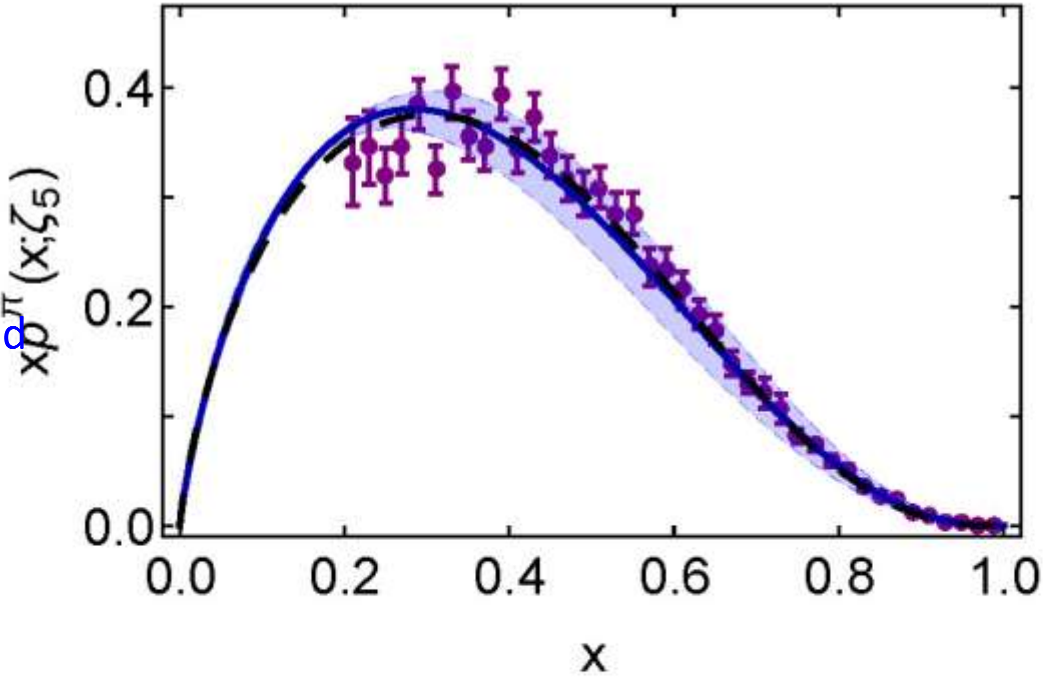

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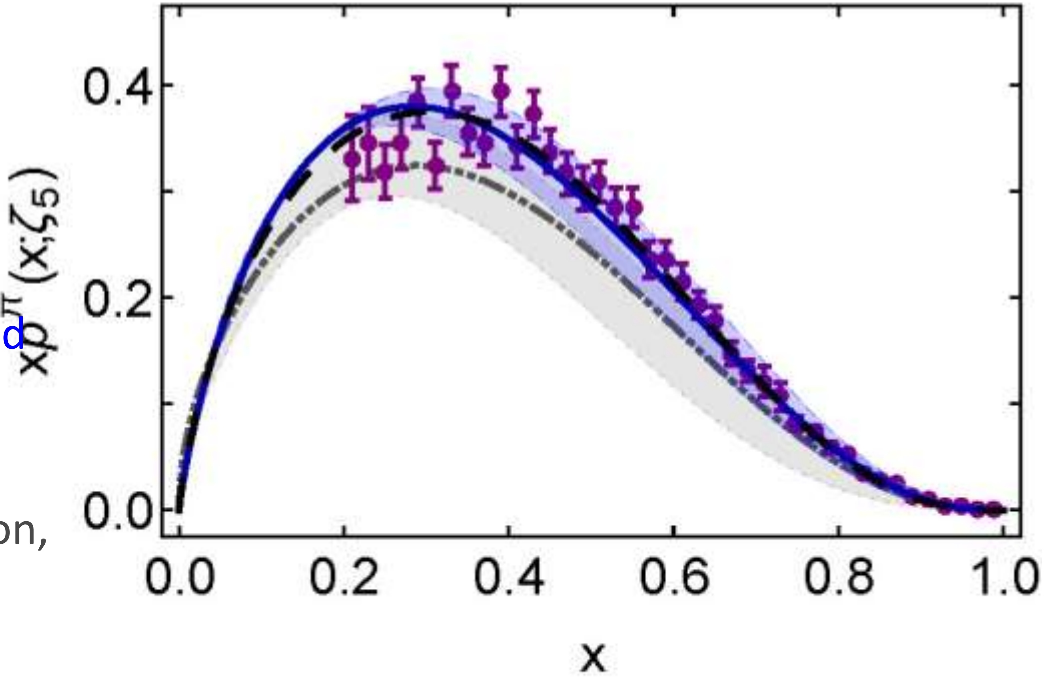
- 2019/01 … Sufian, *et al.*
  - 1^{st} exploratory lattice-QCD calculation,
  - using lattice-calculable matrix element obtained through spatially-separated current-current correlations in coordinate space
  - $m_\pi^2 = 9\ m_\pi^2$-physical

Large-*x* exponent and momentum$^{\zeta = 5\text{ GeV}}$

Continuum … 2.7(1) & <2*x*> = 0.42±0.04

Lattice … 2.5(6) & <2x> = 0.34±0.03



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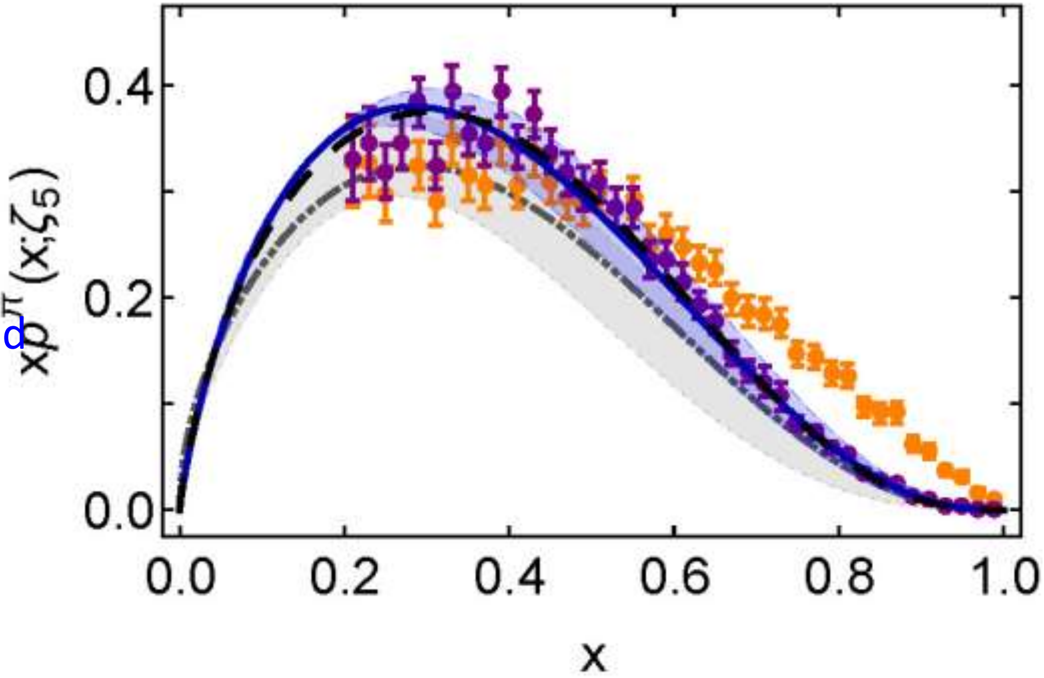


💣 Modellers still insist on ignoring QCD & its symmetries

💣 Phenomenologists question analysis of Aicher *et al.*

Symmetry, Symmetry Breaking, & Pion Parton Distributions

- Complete, symmetry-preserving analysis using precisely same approach as that which predicted the pion elastic and transition form factors
  - Unified treatment of all pion properties within one framework that possesses a traceable connection to QCD.
- Unambiguous definition of hadronic scale, ζ, through connection between one-loop running at high momenta and α_{PI} saturation at low momenta
  - eliminates uncertainty and phenomenological fitting procedures so that results are truly predictions
- Introducing SPM for moments
  - new numerical technique producing reliable estimate of twice as many moments as every before obtained
- Exposing DCSB effect on PDF
  - revealing impact of emergence of mass in the Standard Model
- Illustration of equivalent predictions from continuum and lattice QCD
- Realistic predictions for glue and sea content of pion
  - results match phenomenological analyses and our they are predictions for the pointwise behavior.

π & *K* PDFs

- Urgent need for Newer Data
  - Persistent controversy regarding the Bjorken-$x \simeq 1$ behaviour of the pion's valence-quark PDF
  - Single modest-quality measurement of $u^K(x)/u^\pi(x)$ *(1980)* cannot be considered definitive.
- Approved experiment, using tagged DIS at JLab 12, should contribute to a resolution of pion question
- Similar technique might also serve for the kaon … experiment approved
- Future:
  - New mesonic Drell-Yan measurements at modern facilities (COMPASS at LHC) could yield valuable information on π and *K* PDFs
    - Discussed extensively in "Letter of Intent: A New QCD facility at the M2 beam line of the CERN SPS (COMPASS++/AMBER)" [http://arxiv.org/abs/arXiv:1808.00848]
  - Two-jet experiments at the large hadron collider;
  - EIC would be capable of providing access to π and *K* PDFs through measurements of forward nucleon structure functions.

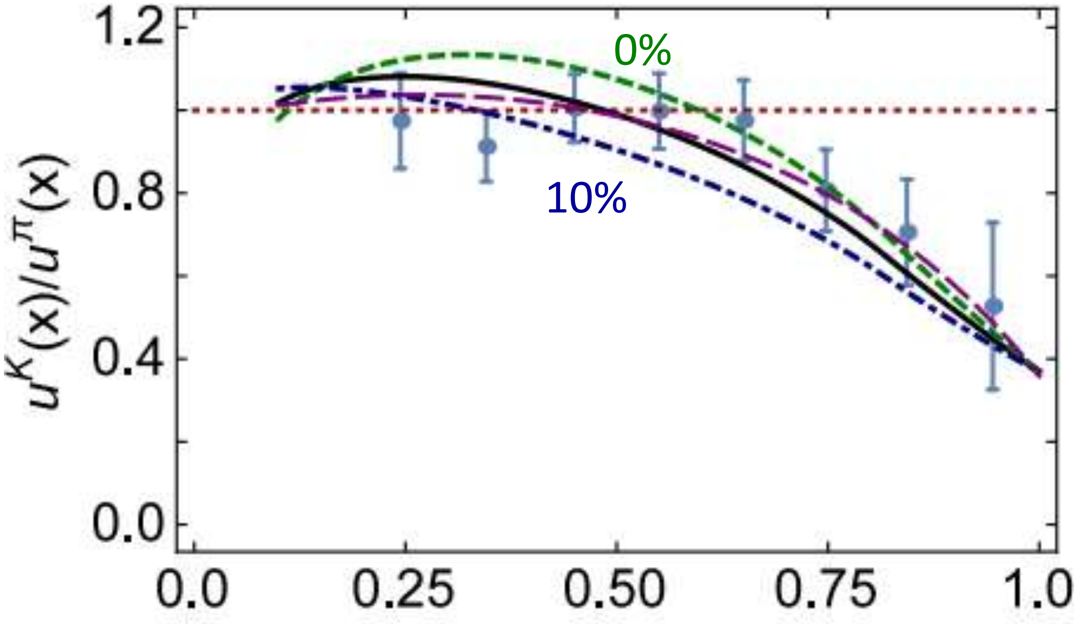
Kaon's gluon content

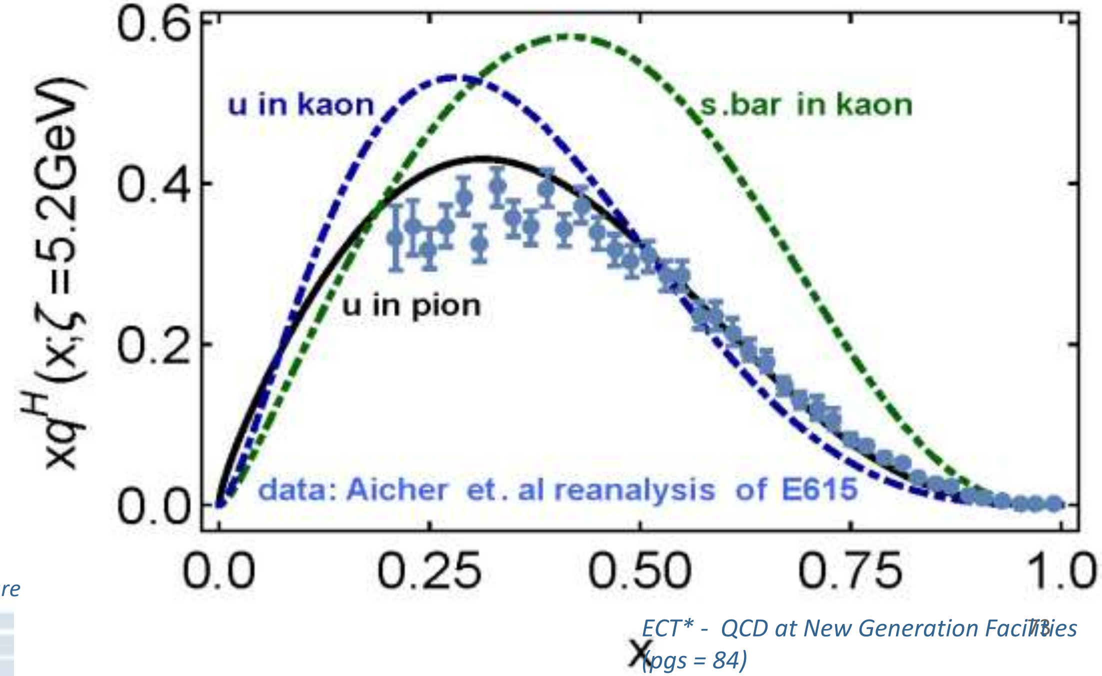
- $\langle x\rangle_g^K(\zeta_H) = 0.05 \pm 0.05$
  $\Rightarrow$ Valence quarks carry 95% of kaon's momentum at ζ_H
- DGLAP-evolved to ζ_2

q	$\langle x\rangle_q^K$	$\langle x^2\rangle_q^K$	$\langle x^3\rangle_q^K$
u	0.28	0.11	0.048
$\bar{s}$	0.36	0.17	0.092

Valence-quarks carry ⅔ of kaon's light-front momentum

Cf. Only ½ for the pion





Valence-quark distribution functions in the kaon and pion,
Chen Chen, Lei Chang *et al.*
arXiv:1602.01502 [nucl-th], Phys. Rev. D**93** (2016) 074021/1-11

π & *K* PDFs

- Marked differences between *π* & *K* gluon content
  - ζ_H:
    - Whilst $\frac{1}{3} \sim \frac{1}{5}$ of pion's light-front momentum carried by glue
    - $Only\ \frac{1}{20}$ of the kaon's light-front momentum lies with glue
  - $\zeta_2^2 = 4\ \text{GeV}^2$
    - Glue carries $\frac{1}{2}$ of pion's momentum and $\frac{1}{3}$ of kaon's momentum
  - Evident in differences between large-*x* behaviour of valence-quark distributions in these two mesons
- Signal of Nambu-Goldstone boson character of *π*
  - Nearly complete cancellation between one-particle dressing and binding attraction in this almost-massless pseudoscalar system

$$2\ \text{Mass}_Q + U_g \approx 0$$

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π & *K* PDFs

- Existing textbook description of Goldstone's theorem via pointlike modes is *outdated* and *simplistic*

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π & *K* PDFs

- The appearance of Nambu-Goldstone modes in the Standard Model is far more interesting
  - Nambu-Goldstone modes are nonpointlike!
  - Intimately connected with origin of mass!
  - Possibly/Probably(?) inseparable from expression of confinement!

- Difference between gluon content of *π* & *K* is measurable … using well-designed EIC
- Write a definitive new chapter in future textbooks on the Standard Model


Electron Ion Collider:
The Next QCD Frontier

Epilogue

Epilogue

- QCD = plausibly a well-defined quantum field theory,

  The only one we've ever produced

  ∴ crucial to understand all its consequences

- Reveal the content of strong-QCD

Continuum Functional Methods

- Demand a traceable connection to QCD
- Requires that there must be a defined notion of improvement
  - One must know what is omitted and know that a strategy exists for its incorporation
  - Does not entail that one must immediately pursue such improvement
  - Sufficient to know the phenomena for which such improvements are important so that any modelling is well-informed
  - Strive to provide indications of reliability of predictions

    = a confidence band

20 years of

Epilogue

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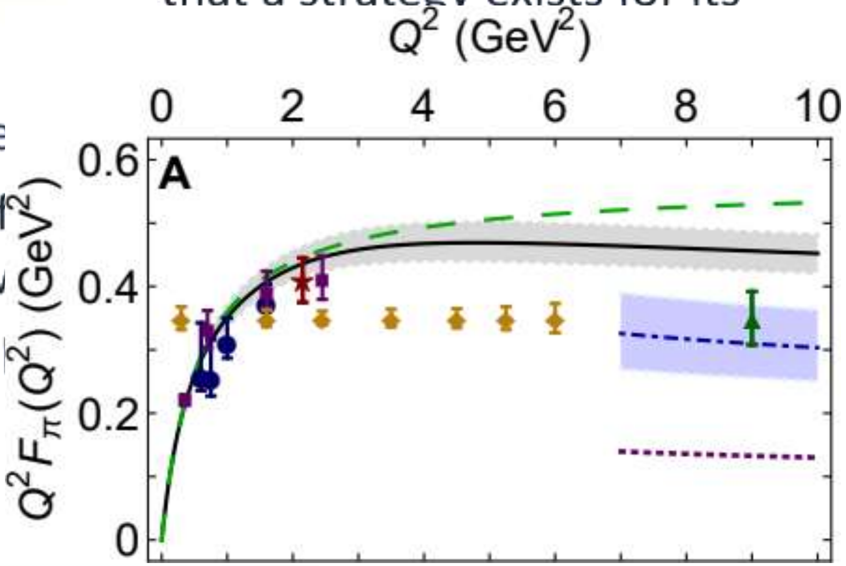
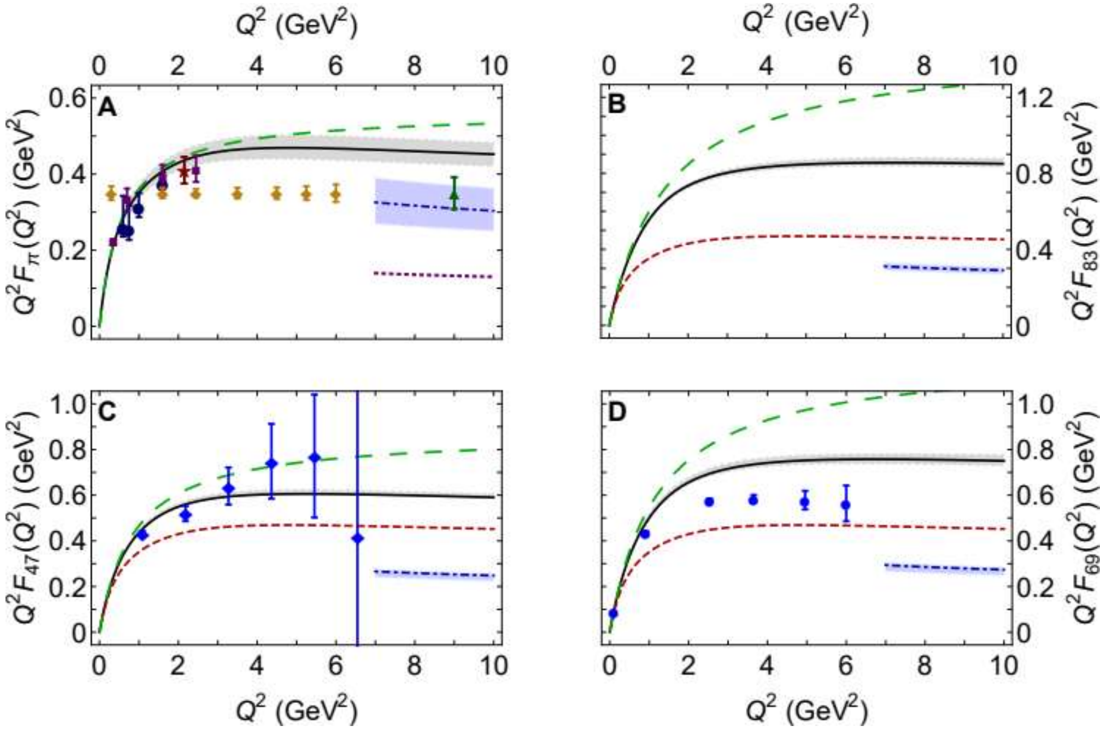


Mass-dependence of pseudoscalar meson elastic form factors,
Muyang Chen, Minghui Ding, Lei Chang and Craig D. Roberts,
arXiv:1808.09461 [nucl-th], Phys. Rev. D Rapid Comm. (*in press*)

Predictions

- ✓ DSE RL approach to quark-antiquark bound-states
- ✓ Used to determine electromagnetic form factors of pion-like mesons with masses m_{0^-}/GeV=0.14, 0.47, 0.69, 0.83
- ✓ Spacelike domain that extends to $Q^2 \lesssim 10$ GeV^2.
- ✓ Results enable direct comparisons with contemporary lattice-QCD calculations of heavy-pion form factors at large values of momentum transfer and aid in understanding them.
- ✓ Reveal, *inter alia*,
  - that form factor of the physical pion provides the best opportunity for verification of the factorised hard-scattering formula relevant to this class of exclusive processes
  - This capacity diminishes steadily as the meson mass increases.



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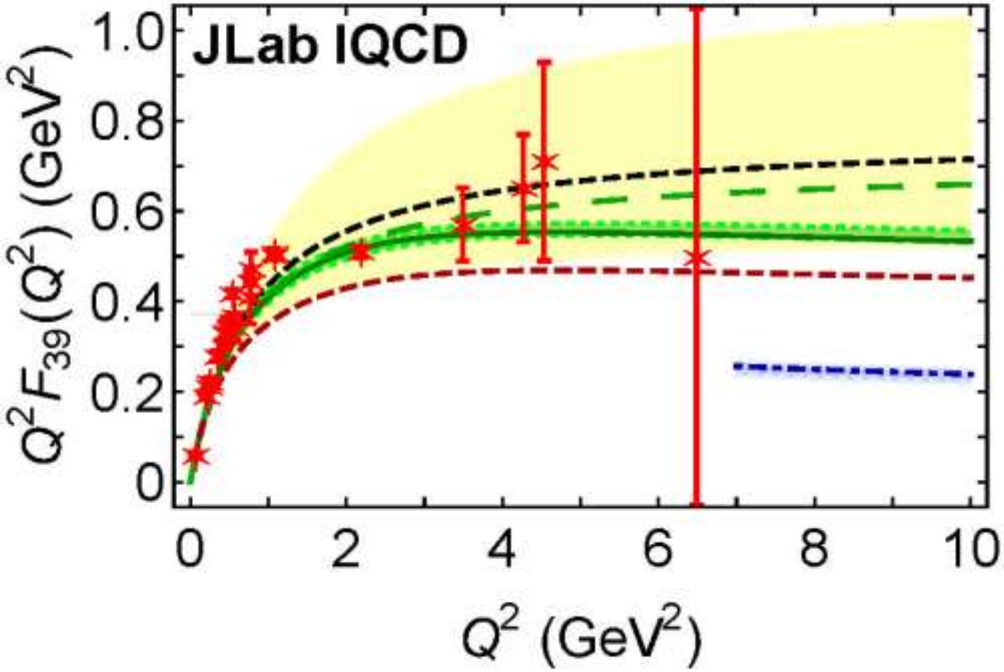


Thankyou