Four-quark state in continuum QCD

Gao Fei

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Continuum Functional Methods for QCD at New Generation Facilities
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Outline

1. **Charmonium like XYZ states**
   - background
   - Preliminary study on some states
2. **From quark to fourquark state**
3. **Numerical results**

Charmonium like XYZ states

Experimentally:

Recently, a family of charmonium-like states have been observed by several experimental collaborations such as CLEO-c, BaBar, Belle, BESIII, CDF, D∅, LHCb, CMS and so on.

It provides us a good chance to deeply understand the complicated non-perturbative behavior of QCD in the low energy regime.

Charmonium like XYZ states

Experimental status of XYZ states:

1.BaBar

- discovery of many charmonium like states, for example, Y(4260), Y(4360), Y(4660).

2.Belle

- discovery of many charmonium like states, for example, X(3872),Y(3940) Z^+(4430).

3.BESIII

- observation of Z_c(3900)

4.CDF and D∅

- confirming X(3872), observation of Y(4140).

Charmonium like XYZ states

5.CLEO-c

- confirming the observation of Y(4260)
- confirming the charged state $Z_c(3900)$

6. LHCb

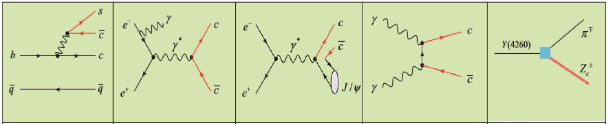
- measurement of spin-parity quantum number of X(3872)

7.CMS

- contribution of searching X(3872) and Y(4140)

The production mechanisms are:

- B meson decays
- e^+e^- annihilation
- double charmonium production
- $\gamma\gamma$ fusion (only for $I^G J^{PC} = 0^+0^{++}, 0^+2^{++}$)



$X(3872)$	$Y(4260)$	$X(3940)$	$X(3915)$	$Z_c(3900)$
$Y(3940)$	$Y(4008)$	$X(4160)$	$X(4350)$	$Z_c(4025)$
$Z^+(4430)$	$Y(4360)$		$Z(3930)$	$Z_c(4020)$
$Z^+(4051)$	$Y(4630)$			$Z_c(3885)$
$Z^+(4248)$	$Y(4660)$			
$Y(4140)$				
$Y(4274)$				
$Z_c^+(4200)$				
$Z^+(4240)$				
$X(3823)$				

Figure 2: (Color online) Five groups of the charmonium-like states corresponding to five production mechanisms.

Charmonium like XYZ states

It arouses great theoretical interests on exploring the components and the structure of these new states.

The possible structure of these new states can be:

- molecular state
- tetraquark state
- excited state
- hybrid state
- glueball
- kinematical effect (cusp effect)

Z_c

$Z_c(3900)$, $Z_c(4020)$ and $Z_c(4025)$

State	M (MeV)	Γ (MeV)	Process (decay mode)	Experiment
$Z_c(3900)$	$3899.0 \pm 3.6 \pm 4.9$	$46 \pm 10 \pm 20$	$e^+e^- \to Y(4260) \to \pi^- + (J/\psi\pi^+)$	BESIII [64]
	$3894.5 \pm 6.6 \pm 4.5$	$63 \pm 24 \pm 26$	$e^+e^- \to Y(4260) \to \pi^- + (J/\psi\pi^+)$	Belle [124]
	$3886 \pm 4 \pm 2$	$37 \pm 4 \pm 8$	$e^+e^- \to \psi(4160) \to \pi^- + (J/\psi\pi^+)$	Xiao *et al.* [61]
$Z_c(3885)$	$3882.2 \pm 1.1 \pm 1.5$	$26.5 \pm 1.7 \pm 2.1$	$e^+e^- \to Y(4260) \to \pi^- + (D\bar{D}^*)^+$	BESIII [159, 162]
$Z_c(4020)$	$4022.9 \pm 0.8 \pm 2.7$	$7.9 \pm 2.7 \pm 2.6$	$e^+e^- \to Y(4260) \to \pi^- + (h_c\pi^+)$	BESIII [160]
$Z_c(4025)$	$4026.3 \pm 2.6 \pm 3.7$	$24.8 \pm 5.6 \pm 7.7$	$e^+e^- \to Y(4260) \to \pi^- + (D^*\bar{D}^*)^+$	BESIII [161]

- $Z_c(3900)$ was argued to be $I^GJ^P = 1^+1^+$ assuming the orbital angular momentum between the J/Ψ and π is zero.

Z_c

Theoretically,

- no bound state of $D\bar{D}^*$ molecular states with such masses in QCD sum rules and lattice QCD simulation.
- diquark picture can be suitable, with cu/s diquark and $\bar{c}\bar{d}/\bar{s}$ antidiquark
- With the color interaction model, $Z_c(3900)$ and $Z_c(4025)$ are good molecular candidates,while $Z_c(4200)$ can be axial-vector hidden-charm tetraquark state.

X(3872)

Table 1: The resonance parameters of the $X(3872)$ and its observed productions and decay channels. Here the $X(3872)$ is abbreviated as X.

Experiment	Mass [MeV]	Width [MeV]	Productions and Decay Modes	J^{PC}
Belle [63]	$3872 \pm 0.6 \pm 0.5$	< 2.3	$B \to KX(\to \pi^+\pi^- J/\psi)$	
Belle [75]	–	–	$B \to KX(\to \gamma J/\psi\,,\, \omega J/\psi \to \pi^+\pi^-\pi^0 J/\psi)$	$C = +1$
Belle [76]	$3875.4 \pm 0.7^{+0.4}_{-1.7} \pm 0.9$	–	$B \to KX(\to D^0\bar{D}^0\pi^0)$	$1^{++}/2^{++}$
Belle [77]	$3871.46 \pm 0.37 \pm 0.07$	–	$B \to KX(\to \pi^+\pi^- J/\psi)$	
Belle [78]	$3872.9^{+0.6+0.4}_{-0.4-0.5}$	$3.9^{+2.8+0.2}_{-1.4-1.1}$	$B \to KX(\to D^{*0}\bar{D}^0)$	
Belle [79]	–	–	$B \to KX(\to \gamma J/\psi)$	
Belle [80]	$3871.84 \pm 0.27 \pm 0.19$	< 1.2	$B \to KX(\to \pi^+\pi^- J/\psi)$	
CDF [67]	$3871.3 \pm 0.7 \pm 0.4$	–	$p\bar{p} \to$ anything $+ X(\to \pi^+\pi^- J/\psi)$	
CDF [81]	–	–	$p\bar{p} \to$ anything $+ X(\to \pi^+\pi^- J/\psi)$	$C = +1$
CDF [82]	–	–	$p\bar{p} \to$ anything $+ X(\to \pi^+\pi^- J/\psi)$	$1^{++}/2^{-+}$
CDF [83]	$3871.61 \pm 0.16 \pm 0.19$	–	$p\bar{p} \to$ anything $+ X(\to \pi^+\pi^- J/\psi)$	
DØ [68]	$3871.8 \pm 3.1 \pm 3.0$	–	$p\bar{p} \to$ anything $+ X(\to \pi^+\pi^- J/\psi)$	
BaBar [84]	3873.4 ± 1.4	–	$B^- \to K^- X(\to \pi^+\pi^- J/\psi)$	
BaBar [85]	$3871.3 \pm 0.6 \pm 0.1$	< 4.1	$B^- \to K^- X(\to \pi^+\pi^- J/\psi)$	
	$3868.6 \pm 1.2 \pm 0.2$	–	$B^0 \to K^0 X(\to \pi^+\pi^- J/\psi)$	
BaBar [86]	–	–	$B \to KX(\to \gamma J/\psi)$	$C = +1$
BaBar [87]	$3875.1^{+0.7}_{-0.5} \pm 0.5$	$3.0^{+1.9}_{-1.4} \pm 0.9$	$B \to KX(\to \bar{D}^{*0}D^0)$	
BaBar [88]	$3871.4 \pm 0.6 \pm 0.1$	< 3.3	$B^+ \to K^+ X(\to \pi^+\pi^- J/\psi)$	
	$3868.7 \pm 1.5 \pm 0.4$	–	$B^0 \to K^0 X(\to \pi^+\pi^- J/\psi)$	
BaBar [89]	–	–	$B \to KX(\to \gamma J/\psi\,,\, \to \gamma\psi(3686))$	
BaBar [90]	$3873.0^{+1.8}_{-1.6} \pm 1.3$	–	$B \to KX(\to \omega J/\psi \to \pi^+\pi^-\pi^0 J/\psi)$	2^-
LHCb [91]	$3871.95 \pm 0.48 \pm 0.12$	–	$pp \to$ anything $+ X(\to \pi^+\pi^- J/\psi)$	
LHCb [70]	–	–	$pp \to$ anything $+ X(\to \pi^+\pi^- J/\psi)$	1^{++}
LHCb [92]	–	–	$pp \to$ anything $+ X(\to \gamma J/\psi\,,\, \to \gamma\psi(3686))$	
CMS [73]	–	–	$pp \to$ anything $+ X(\to \pi^+\pi^- J/\psi)$	
BESIII [93]	$3871.9 \pm 0.7 \pm 0.2$	< 2.4	$e^+e^-[\to Y(4260)] \to \gamma X(\to \pi^+\pi^- J/\psi)$	

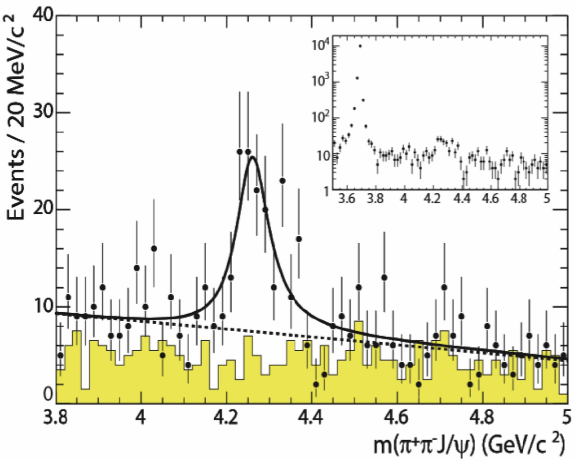
X(3872)

Theoretically,

- $D\bar{D}^*$ molecular state in pion exchange effective interaction model, chiral perturbation theory, QCD sum rules and lattice QCD simulations.
- axial vector diquark-antidiquark state in Constituent quark model

Y(4260)

Y(4260) from e^+e^- annihilation, and the quantum number is $J^{PC} = 1^{--}$.



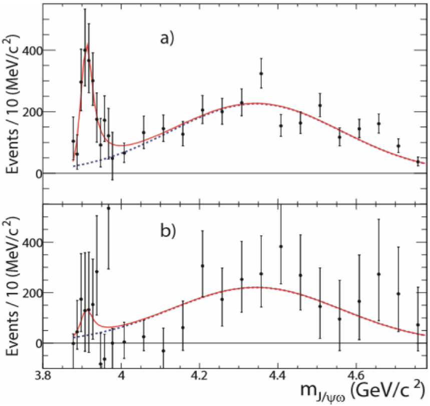
Y(4260)

Theoretically,

- the mass does not match with any 1^{--} excited states
- the decay modes disfavor all the possible molecular states and also glueball
- charmonium hybrid is strongly favored.

Y(3940)and Y(4140)

Y(3940)and Y(4140) from B meson decay



Experiment	$Y(4140)$
CDF [69]	$M = 4143.0 \pm 2.9 \pm 1.2, \Gamma = 11.7^{+8.3}_{-5.0} \pm 3.7$
CDF [100]	$M = 4143.4^{+2.9}_{-3.0} \pm 0.6, \Gamma = 15.3^{+10.4}_{-6.1} \pm 2.5$
DØ [102]	$M = 4159.0 \pm 4.3 \pm 6.6, \Gamma = 19.9 \pm 12.6^{+1.0}_{-8.0}$
CMS [74]	$M = 4148.0 \pm 2.4 \pm 6.3, \Gamma = 28^{+15}_{-11} \pm 19$

Y(3940)and Y(4140)

Theoretically

- Y(3940)and Y(4140) can be interpreted in QCD sum rules or pion exchange effective interaction model, as $D^*\bar{D}^*$ and $D_s^*\bar{D}_s^*$ molecular states, respectively. The quantum number can be either 0^{++} or 2^{++}
- Y(3940) is also argued to be $\chi_{c0}(2P)$ or 2^{++} diquark-antidiquark state.

Dyson-Schwinger equation

The Dyson-Schwinger equations (DSEs):

a collection of coupled integral equations that provide for a symmetry-preserving treatment of the continuum bound-state problem, have been widely employed to compute hadron spectra and interactions.

Here we would like to generalize this scheme into the four-quark stage.

quark propagator

The quark propagator DSE with contact model can be written as:

$$S_f^{-1}(p) = i\gamma \cdot p + m_f + \frac{16\pi}{3}\frac{\alpha_{\rm IR}}{m_G^2}\int \frac{d^4q}{(2\pi)^4}\gamma_\mu S_f(q)\gamma_\mu \,, \qquad (1)$$

where m_f is the quark's current-mass. The integral is quadratically divergent; but when it is regularised in a Poincaré-invariant manner, the solution is

$$S_f(p)^{-1} = i\gamma \cdot p + M_f \,, \qquad (2)$$

quark propagator

The regularisation is as following:

$$\frac{1}{s+M^2} = \int_0^\infty d\tau\, \mathrm{e}^{-\tau(s+M^2)}$$

$$\to \int_{\tau_{\mathrm{uv}}^2}^{\tau_{\mathrm{ir}}^2} d\tau\, \mathrm{e}^{-\tau(s+M^2)} \tag{3}$$

$$= \frac{\mathrm{e}^{-(s+M^2)\tau_{\mathrm{uv}}^2} - \mathrm{e}^{-(s+M^2)\tau_{\mathrm{ir}}^2}}{s+M^2}\,, \tag{4}$$

quark propagator

Consequently, the gap equation becomes

$$M_f = m_f + M_f \frac{4\alpha_{\rm IR}}{3\pi m_G^2}\, \mathcal{C}_0^{\rm iu}(M_f^2)\,, \tag{5}$$

where

$$\begin{aligned} \mathcal{C}_0^{\rm iu}(\sigma) &= \int_0^\infty ds\, s \int_{\tau_{\rm uv}^2}^{\tau_{\rm ir}^2} d\tau\, {\rm e}^{-\tau(s+\sigma)} \\ &= \sigma\left[\Gamma(-1,\sigma\tau_{\rm uv}^2) - \Gamma(-1,\sigma\tau_{\rm ir}^2)\right], \end{aligned} \tag{6}$$

with $\Gamma(\alpha, y)$ being the incomplete gamma-function.

Bethe-Salpeter equation

We then get the information of quark. The two body bound state can be studied via Bethe-Salpeter equation. The BS amplitude for a pseudoscalar or a vector meson constituted from a valence f-quark and valence g-antiquark has the following restricted form:

$$\Gamma_{0^-}(Q) = \gamma_5 \left[iE_{0^-} + \frac{1}{2M_R} \gamma \cdot Q F_{0^-} \right] , \tag{7}$$

$$\Gamma_\mu^{1^-}(Q) = \gamma_\mu^\perp E_{1^-}(Q) \, , \tag{8}$$

Here, Q is the bound-state's total momentum, $Q^2 = -m_{0^-}^2$, m_{0^-} is the meson's mass; and $M_R = M_f M_g / [M_f + M_g]$, with $M_{f,g}$ being the relevant dressed-quark masses obtained from the contact-interaction gap equations, described above.

Bethe-Salpeter equation

The amplitude is determined by the following equation:

$$\Gamma_{0^-,1^-}(Q) = -\frac{16\pi}{3}\frac{\alpha_{\rm IR}}{m_G^2}\int\frac{d^4t}{(2\pi)^4}\gamma_\mu S_f(t+Q)\Gamma_{0^-,1^-}(Q)S_g(t)\gamma_\mu\,. \tag{9}$$

From here, using the symmetry-preserving regularisation scheme, which requires, in the spirit of dimensional regularisation,

$$0 = \int_0^1 d\alpha\left[\mathcal{C}_0^{\rm iu}(\omega_{fg}(\alpha,Q^2)) + \mathcal{C}_1^{\rm iu}(\omega_{fg}(\alpha,Q^2))\right], \tag{10}$$

where $\mathcal{C}_1^{\rm iu} = -d\mathcal{C}_0^{\rm iu}(\sigma)/d\sigma$ and $\hat{\alpha} = 1-\alpha$, $\omega_{fg}(\alpha,Q^2) = M_f^2\hat{\alpha} + \alpha M_g^2 + \alpha\hat{\alpha}Q^2$. The BS amplitude for the respective diquark is quite similar. The amplitude of J^P diquark can be directly obtained from that for a J^{-P} meson by multiplying the meson kernel by a factor of $1/2$.

Bethe-Salpeter equation

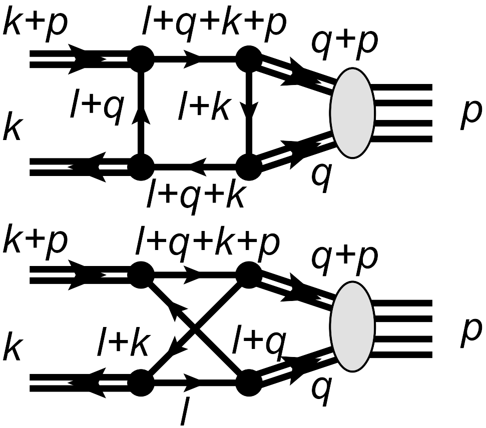
P. L. Yin, C. Chen, G. Krein, C. D. Roberts, J. Segovia, S. S. Xu,arXiv:1903.00160

meson, M	E_M	F_M	m_M^{CI}	f_M^{CI}	$m_M^{\mathrm{e/l}}$	$f_M^{\mathrm{e/l}}$
π	3.59	0.47	0.14	0.10	0.14	0.092
K	3.82	0.56	0.50	0.11	0.50	0.11
ρ	1.53		0.93	0.13	0.78	0.15
K^*	1.63		1.03	0.12	0.89	0.16
ϕ	1.74		1.13	0.12	1.02	0.17
D	3.11	0.36	1.92	0.16	1.87	0.15(1)
D_s	3.25	0.49	2.01	0.17	1.97	0.18
D^*	1.21		2.14	0.15	2.01	0.17(1)
D_s^*	1.23		2.23	0.16	2.11	0.19
η_c	3.28	0.73	2.98	0.24	2.98	0.24
J/ψ	1.21		3.19	0.20	3.10	0.29
B	1.67	0.095	5.41	0.17	5.30	0.14(2)
B^*	0.70		5.46	0.16	5.33	0.12
B_s	1.79	0.14	5.50	0.18	5.37	0.16
B_s^*	0.71		5.56	0.16	5.42	0.15(1)
B_c	3.38	0.61	6.28	0.27	6.28	0.35
B_c^*	1.37		6.38	0.23	6.33	0.30(1)
η_b	3.18	0.81	9.40	0.41	9.40	0.41(2)
Υ	1.50		9.49	0.38	9.46	0.46

diquark	mass	E	F
$[ud]$	0.77	2.74	0.31
$[us]$	0.93	2.88	0.39
$[uc]$	2.15	1.97	0.22
$[sc]$	2.26	1.99	0.29
$[ub]$	5.51	1.05	0.059
$[sb]$	5.60	1.05	0.083
$[cb]$	6.48	1.42	0.25
$\{uu\}$	1.06	1.31	
$\{us\}$	1.16	1.36	
$\{ss\}$	1.26	1.43	
$\{uc\}$	2.24	0.89	
$\{sc\}$	2.34	0.87	
$\{cc\}$	3.30	0.69	
$\{ub\}$	5.53	0.51	
$\{sb\}$	5.62	0.50	
$\{cb\}$	6.50	0.62	
$\{bb\}$	9.68	0.48	

four quark state

After having the information of two-body bound states, it can be then generalized for computing the four-quark state in the picture of meson-meson and diquark-antidiquark bound state.



The bound state equation can be expressed in the Feynman diagram, in the following computation we only take the diagonal contribution into account. The respective eigenstate equation is:

$$\Pi_{\mu\beta\gamma}(k,P) = \int d^4l d^4q Tr[\Gamma_\mu(P)S(l+q+k+P)\bar{\Gamma}_\nu(P) \times S(l+k)\Gamma_\alpha(P)S(l+q+k)\bar{\Gamma}_\beta(P)S(l+q)] \times D_{\nu\rho}(q+P)D_{\alpha\lambda}(q)\Pi_{\rho\lambda\gamma}(q,P) \tag{11}$$

four-quark state

For Scalar fourquark state amplitude, we simply employ:

$$\Pi_S = T_S(k, P) \tag{12}$$

and for vector we employ

$$\Pi_V = T_V(k, P)(k_\mu - k \cdot P P_\mu / P^2). \tag{13}$$

After then we can solve the eigenstate equation and obtain the bound state mass.

light sector

We here employ this scheme for the $\bar{u}\bar{d}ud$ four-quark system.
If considering σ meson is a $\bar{\pi} - \pi$ bound state, then the mass will be $m_\sigma = 0.515$ GeV.
The mass for scalar diquark-antidiquark bound state will be 1.110 GeV.

- The diquark-antidiquark bound state is much heavier than the respective meson-meson bound state because the mass of *ud* diquark is 0.77 GeV while the pion mass is only 0.14 GeV.

It reveals that σ meson is dominant by $\pi - \bar{\pi}$ bound state

This diquark-antidiquark bound state with 1.110 GeV is close to $f(1370)$ state.

light sector

The $f(980)$ state can be consisted of $\bar{K}K$ bound state which mass is 1.02 GeV.

- The mass of the respective diquark-antidiquark bound state is 2.02 GeV. It becomes heavier also because the diquark component mass is heavier than Kaon.

charmonium like XYZ states

Here we focus on the charmonium like XYZ states with possible components of $c\bar{c}q\bar{q}$.
For 0^{++} states, it can be composed of two pseudoscalar or two vector meson-antimeson, and the counterpart in diquark-antidiquark picture.

0^{++}	$\bar{D}D$	$[uc]\overline{[uc]}$	$\bar{D}_s D_s$	$[sc]\overline{[sc]}$
	3.36 GeV	/	3.44 GeV	/
0^{++}	$\bar{D}^* D^*$	$\{uc\}\overline{\{uc\}}$	$\bar{D}^*{}_s D^*_s$	$\{sc\}\overline{\{sc\}}$
	/	/	/	/

- for both pictures, vector components can not form a bound state
- pseudoscalar meson-antimeson components can be a bound state, but the mass is too light compared to any observed states.
- 0^{++} states are more likely excited states of χ_c.

charmonium like XYZ states

For 1^{++} states, we try the possible structure with one pseudoscalar meson and one vector antimeson, or the respective diquark-antidiquark.

1^{++}	$\bar{D}D^*$	$[uc]\overline{\{uc\}}$	$\bar{D}_sD_s^*$	$[sc]\overline{\{sc\}}$
	3.910 GeV	/	4.195 GeV	/

- no bound states in diquark-antidiquark picture.
- the mass of $\bar{D}D^*$ is 3910 MeV, which is very close to $X(3872)$,
- the state of $\bar{D}_sD_s^*$ with 4195 MeV might contribute to $X(4140)$ state.

charmonium like XYZ states

For 1^{+-} states, we employ two vector meson-antimeson or respective diquark-antidiquark.

1^{+-}	$\bar{D}^* D^*$	$\{uc\}\overline{\{uc\}}$	$\bar{D}^*{}_s D_s^*$	$\{sc\}\overline{\{sc\}}$
	3.946 GeV	4.235 GeV	4.244 GeV	4.144 GeV

- both pictures are approved of being bound states.
- the mass of bound state with $\bar{D}^* D^*$ is close to $Z_c(3900)$, and the counterpart with diquark-antidiquark components is about 300 MeV heavier which can be $Z_c(4200)$
- changing u/d quark to s quark, people can also obtain a bound state with similar mass. Noticing that in this case, the bound state with diquark-antidiquark components is lighter.

What we obtained:

- Sigma meson with only pi-pi component is 515 MeV;
- Neither meson nor diquark component are approved for 0^{++} state, 0^{++} states are more likely excited states of χ_c.
- The mass of $D\bar{D}^*$ 1^{++} state is 3910 MeV, close to X(3872).
- The mass of $D^*\bar{D}^*$ 1^{+-} state is 3940 MeV, close to Z_c(3910).

Thank you