Thermodynamics of 2+1 flavor Polyakov-loop quark-meson model under external magnetic field

Xiang Li

Peking University & ECT

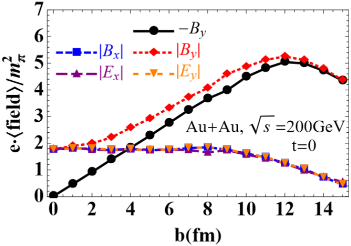
"*Continuum Functional Methods for QCD at New Generation Facilities*"

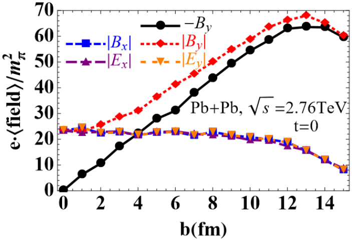
Xiang Li,Wei-jie Fu, and Yu-xin Liu, PRD 99, 074029 (2019).

Motivation

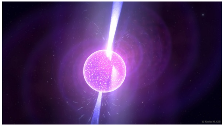
Phenomenological

- Large collider [1] ($m_\pi^2 \approx 0.02\ \mathrm{GeV}^2$)





- Neutron star
  magnetic field origins from a magnetohydrodynamic dynamo process. (wiki)

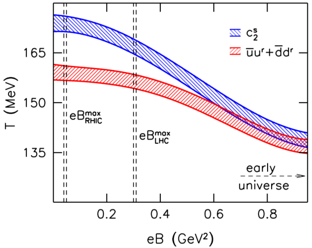


[1] Wei-Tian Deng et al, Phys. Rev. C 85 044907 (2012).

Motivation

Theoretical

Lattice QCD predicts inverse magnetic catalysis that chiral critical temperature *decreases* with the ascending of magnetic field.[2]



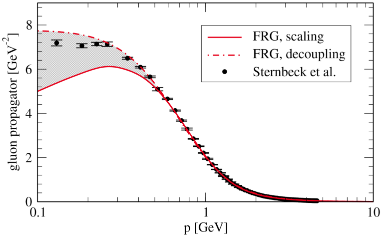
While phenomenological models (such as NJL, QM, and χPT) predict *opposite* effect[3]

[2] G. S. Bali et al, JHEP 1202 (2012) 044.

[3] K. Klimenko, Z. Phys. C 54 323 1992.

Approach

Polyakov-loop quark meson model (PQM)



[4]

$$\mathcal{L} = \bar{\psi}(\partial\!\!\!/ + ieA_\mu^{em}\gamma_\mu + iA_4\gamma_4 + g\Sigma_5)\psi + Tr[\mathcal{D}_\mu\Sigma\cdot(\mathcal{D}_\mu\Sigma)^\dagger] + U(\Sigma) -c[det(\Sigma) + det(\Sigma^\dagger)] - Tr[H(\Sigma+\Sigma^\dagger)] + V_{glue}(\Phi)^5 \quad (1)$$

[4] A. K. Cyrol et al, Phys. Rev. D 94 054005 (2016).

[5] B. J. Schaefer et al, Phys. Rev. D 76 074023 (2007).

Deconfinement aspect

$$\Phi = \frac{1}{N_c} \mathrm{Tr}\ \mathrm{P}\ \exp(i \int_0^\beta d\tau A_4) \qquad (2)$$

If quark is extremely heavy

$$(\frac{1}{i}\frac{\partial}{\partial t} - \vec{\tau} \cdot \vec{A^0})\psi(\vec{r}, t) = 0$$ [6]

Polyakov-loop is related to the free energy of a single-quark system

$$\Phi = e^{-F_q} \begin{cases} 0 & F_q \to \infty \quad \textit{confinement} \\ 1 & F_q \to 0 \quad \textit{deconfinement} \end{cases} \qquad (3)$$

[6] L. D. McLerran et al, Phys. Rev. D 24, 450 (1981).

$$\frac{V_{glue}(\Phi,\bar{\Phi})}{T^4} = -\frac{b_2(T)}{2}\Phi\bar{\Phi} - \frac{b_3}{6}(\Phi^3+\bar{\Phi}^3) + \frac{b_4}{4}(\Phi\bar{\Phi})^2$$

$$b_2(T) = a_1 + \frac{a_2}{1+t} + \frac{a_3}{(1+t)^2} + \frac{a_4}{(1+t)^3}\,^{7} \tag{4}$$

V_{poly} is added to imitate the effect from confinement.

$$\tilde{n}_f(E_f,\Phi) = \frac{1+2\Phi e^{\beta E_f}+\Phi e^{2\beta E_f}}{1+3\Phi e^{\beta E_f}+3\Phi e^{2\beta E_f}+e^{3\beta E_f}} \tag{5}$$

Statistical confinement is achieved at low temperature, i.e., $\Phi \sim 1$.

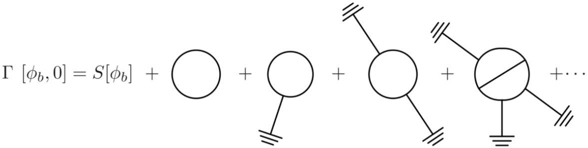
$$\tilde{n}_f = \frac{1}{e^{3\beta E_f}+1} \tag{6}$$

[7] C. Ratti et al, Phys. Rev. D 73, 014019 (2005).

Approach

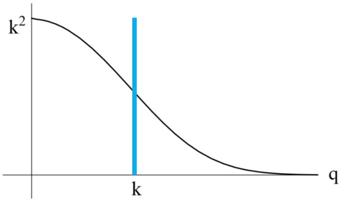
Functional renormalization group

- perturbative expansion
- mean-field approximation (neglects meson's fluctuation)
- functional renormalization group (FRG)[8]



A momentum-dependent mass term $R_k(p)$ is assigned to each propagator in the loop. Effective action gets modified $\Gamma \to \Gamma_k$.

[8] C. Wetterich, Phys. Lett. B 301, 90 (1993).

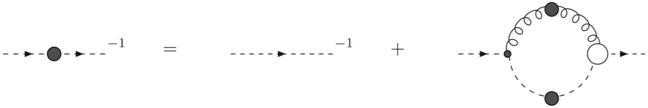


High energy mode is integrated gradually as the scale k go down.
An abstract functional differential equation can be obtained

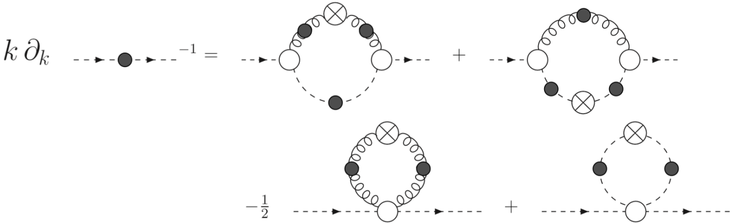
$$\partial_k \Gamma_k = \frac{1}{2}\mathrm{Tr}\left[\frac{\partial_k R_k^B}{\Gamma_k^{2B} + R_k^B}\right] - \mathrm{Tr}\left[\frac{\partial_k R_k^F}{\Gamma_k^{2F} + R_k^F}\right] \tag{7}$$

Vertex expansion

Dyson-Schwinger equation



Functional renormalization group


[9]

[9]C. S. Fischer et al, Annals Phys. 324 2408 (2009).

Local potential approximation(LPA)

A truncation for Γ_k is necessary

$$\begin{aligned}\Gamma_k = & \int d^4x\ \bar{\psi}(\not{D} - i\gamma_4 A_4 + g\Sigma_5)\psi \\ & + \mathrm{Tr}[\mathcal{D}_\mu \Sigma \cdot (\mathcal{D}_\mu \Sigma)^\dagger] + U_k(\rho_1, \rho_2) \\ & - h_x\sigma_x - h_y\sigma_y - c_a\xi + V_{glue}(\Phi) \end{aligned} \tag{8}$$

Employing the optimized regulator $R_k(p)$[10]

$$\begin{aligned}\partial_k U_k = & \frac{k^4}{12\pi^2}\Big\{ \sum_b \alpha_b(k)\frac{1}{E_b}[1 + 2n_b(E_b)] \\ & - \sum_{f=u,d,s} \alpha_f(k)\frac{1}{E_f}[1 - 2\tilde{n}_f(E_f, \Phi)]\Big\}^{11}\end{aligned} \tag{9}$$

[10] D. F. Litim, Phys. Rev. D 64, 105007 (2001).

[11] K. Kamikado, and T. Kanazawa, JHEP 1501 (2015) 129.

$$U_k = \sum_{i+2j=0}^{5} \frac{a_{i,j}(k)}{i!j!}[\rho_1 - \rho_1(k)]^i[\rho_2 - \rho_2(k)]^j \tag{10}$$

$$\rho_1 = \mathrm{Tr}\left[\Sigma \cdot \Sigma^\dagger\right],$$

$$\rho_2 = \mathrm{Tr}\left[(\Sigma \cdot \Sigma^\dagger - \frac{1}{3}\rho_1)^2\right] \tag{11}$$

Substituting into (9) and gain non-perturbative β functions.

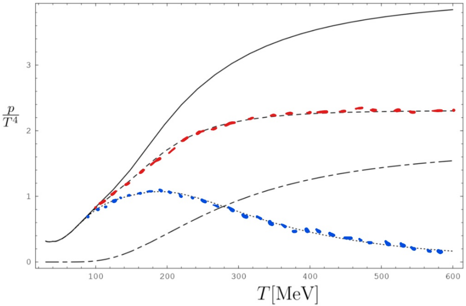
$$k\frac{da_{i,j}}{dk} = \beta_{i,j}(a_{m,n}, k, T, eB) \tag{12}$$

In summary

$$\Gamma_k \xrightarrow{Parameterized} a_{i,j}(k) \xrightarrow{FRG} \beta_{i,j} \xrightarrow{Mathematica} \Gamma$$

Compansate

The theory is only valid below $T \sim \frac{\Lambda}{2\pi} \sim 160MeV$



[12]

Extending Λ to ∞

$$\Gamma_{\Lambda}(0,0) \rightarrow \Gamma_{\Lambda}(0,0) + \Delta\Gamma$$
$$\Delta\Gamma = \Gamma_{\Lambda}(T, eB) - \Gamma_{\Lambda}(0,0) \tag{13}$$

[12] J. Braun et al, Phys. Rev. D 70 085016 (2004).

$$\begin{aligned}\Delta\Gamma_\Lambda[T,eB] \quad &= [\Gamma_\Lambda(T,eB)-\Gamma_\Lambda(0,0)] \\ &= [\Gamma_\Lambda(T,eB)-\Gamma_\infty(T,eB)]-[\Gamma_\Lambda(0,0)-\Gamma_\infty(0,0)] \\ &= \int_\infty^\Lambda dk\ [\partial_k\Gamma_k(T,eB)-\partial_k\Gamma_k(0,0)]\,. \end{aligned} \tag{14}$$

[13,14]

Meson decouples at high energy region so

$$\Delta U_\Lambda[T,eB] = \int_\infty^\Lambda dk\ \frac{k^4}{12\pi^2}\sum_{f=u,d,s}\alpha_f(k)\ \frac{2\tilde{n}_f(E_f,\Phi)}{E_f}\,, \tag{15}$$

[13] J. Braun, M. Leonhardt, and J. M. Pawlowski, arXiv:1806.04432.
[14] W. J. Fu, J. M. Pawlowski, and F. Rennecke, arXiv:1808.00410.

Result

The initial conditions.

g	h_x/Λ^3	h_y/Λ^3	c_a/Λ	a_{10}/Λ^2	a_{20}	a_{01}	Λ/GeV
6.5	0.121^3	0.336^3	4.808	0.56^2	26	50	1

Calculated hadronic observables (in MeV).

f_π	f_K	m_π	m_K	m_σ	m_{f_0}
92.0	113.4	138.3	495.1	601.8	1234.7
$m_{\eta'}$	m_η	m_{a_0}	m_κ	$m_{u,d}$	m_s
962.8	539.1	1013.7	1102.5	299.0	438.1

Order parameters

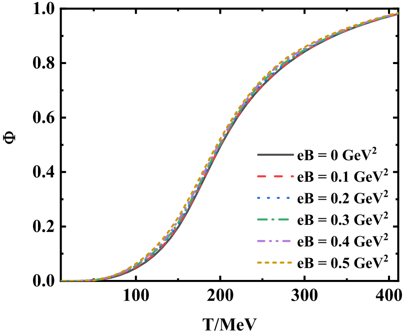


Figure: Calculated Polyakov-loops.

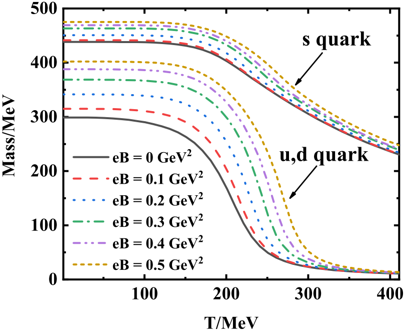


Figure: Calculated quark masses.

Deconfinement phase transition

$V_{glue}(T, eB?)$

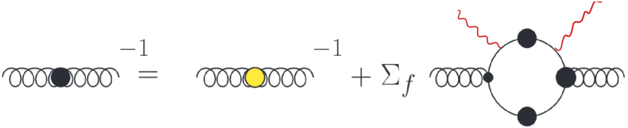
Chiral phase transition

Inverse case

Ordinary procedure

$$\partial_\mu \to \partial_\mu + ieA_\mu^{em}$$

Magnetic field couples with gluon indirectly



[15]

Indirect effect on strong interaction should be considered

$$G(\bar{\psi}\psi)^2 \to G(eB)\ (\bar{\psi}\psi)^2 \quad (16)$$

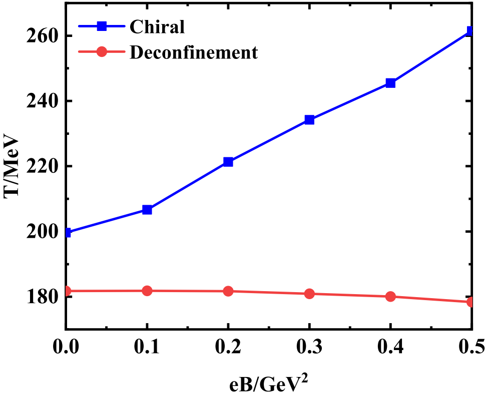
[16]

$$V_{glue}(T, eB?) \quad (17)$$

[15] N. Mueller et al, Phys.Rev. D 91 116010 (2015).

[16] R. L. S. Farias eta la, Phys. Rev. C 90 025203 (2014).

Phase diagram



Chiral and deconfinement phase transition lines will split with the increasing magnetic field.

Meson spectral

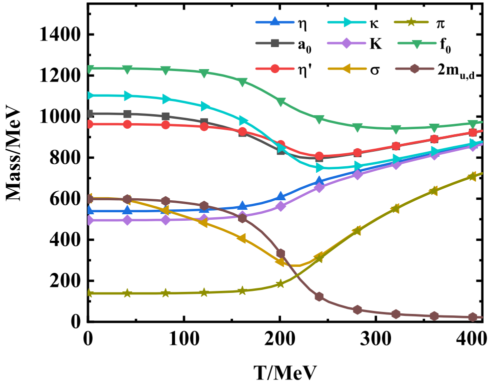


Figure: Calculated meson masses and twice light quark mass.

At low temperature (chiral symmetry broken), pion dominates.
At high temperature (chiral symmetry restored), quark dominates.

Pressure and entropy[17,18]

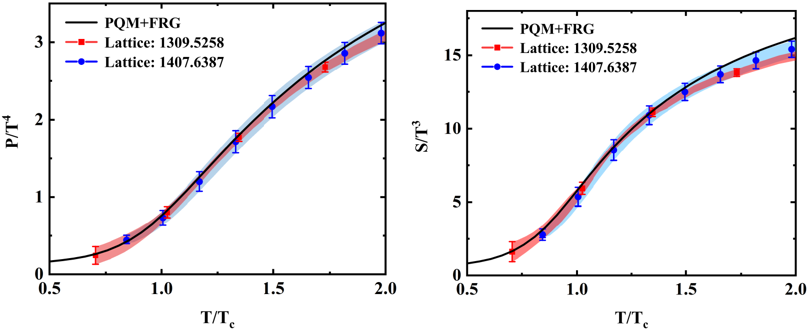


Figure: Calculated normalized pressure and normalized entropy density as functions of normalized temperature at $eB = 0$.

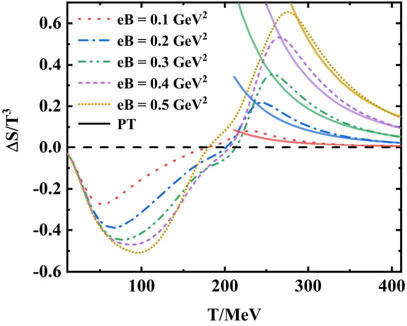
[17] A. Bazavov et al, Phys. Rev. D 90, 094503 (2014).
[18] S. Borsányi et al, Phys. Lett. B 730, 99 (2014).

Magnetic induced entropy

$$E^2 = m^2 + p_z^2 + (2l + 1 - s)eB$$

Mesons' energy get increased by eB at least (l=0, s=0)

Quarks encounter dimension reduction (3d→1d) and their energy get decreased (l=0, s=1)



$$S(T, eB) = \frac{\partial P(T, eB)}{\partial T}$$
$$\Delta S = S(T, eB) - S(T, 0)$$

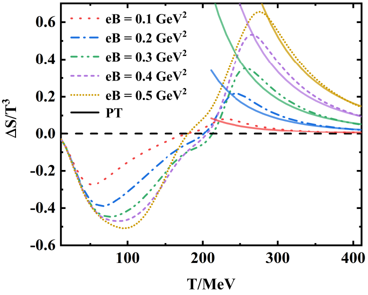


Figure: PQM+FRG

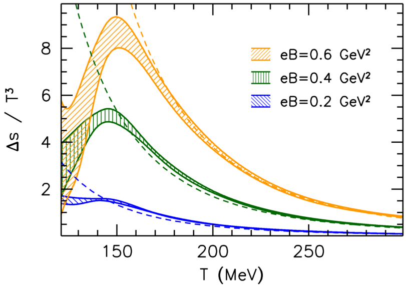


Figure: Lattice

Entropy varies more violently with magnetic field in Lattice.[19]

[19] G. S. Bali et al, JHEP 1408 (2014) 177

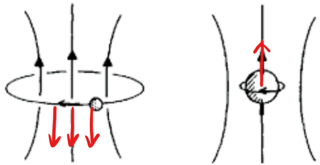
Magnetic susceptibility χ

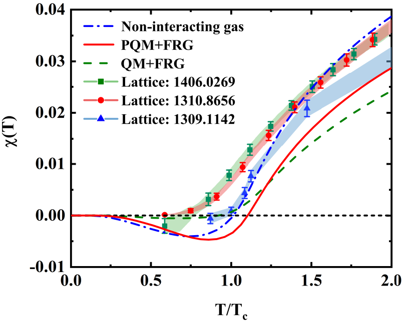
$$\tilde{\chi}(T) = \frac{\partial^2 P}{\partial (eB)^2}$$

$$\chi(T) = \tilde{\chi}(T) - \tilde{\chi}(0)$$

Diamagnetic at low temperature due to the orbital motion of mesons.

Paramagnetic at high temperature due to the spin orientation of quarks.





critical temperature in MeV

T_{QM}	T_{PQM}	T_{Gas}	$T_{Lattice}$
171	199	199	154

Noninteracting gas approximation

Approximate the system as noninteracting quark and meson gas,

$$\chi(T) = \chi_q(T) + \chi_m(T) \tag{18}$$

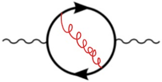
$$\begin{aligned}
\chi_q(T) = \frac{N_c}{6\pi^2}\sum_f q_f^2 \quad & \left\{ 2\int_0^\infty dx\, \frac{\tilde{n}_f(\sqrt{x^2+m_f^2(T)}, \Phi(T))}{\sqrt{x^2+m_f^2(T)}} \right. \\
& \left. -\ln\left(\frac{m_f(0)}{m_f(T)}\right) \right\}, \\
\chi_m(T) = -\frac{1}{48\pi^2}\sum_b \quad & \left\{ 2\int_0^\infty dx\, \frac{n_b(\sqrt{x^2+m_b^2(T)})}{\sqrt{x^2+m_b^2(T)}} \right. \\
& \left. +\ln\left(\frac{m_b(0)}{m_b(T)}\right) \right\}
\end{aligned} \tag{19}$$

[20]

[20] K. Kamikado, and T. Kanazawa, JHEP 1501 (2015) 129.

Adler function $D(Q^2)$[21,22,23]

$$D(Q^2) = -12\pi^2 Q^2 \frac{d}{dQ^2}\Pi(Q^2)$$



$$3Q^2\Pi(Q^2) = i\int d^4x e^{iq\cdot x}\langle 0|TJ^{\mu}(x)J_{\mu}(0)|0\rangle$$

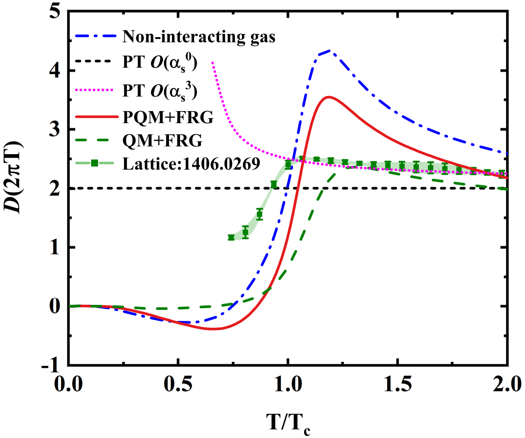
Virtual photon's momentum Q is replaced by Matsubara frequency.

$$Q \leftrightarrow 2\pi T$$

$$D(2\pi T) = 6\pi^2 T\frac{\partial\chi(T)}{\partial T} \qquad (20)$$

$$\begin{aligned}
&\int d^4x\,\langle 0|TJ_{\mu}(x)J_{\mu}(0)|0\rangle \\
&= \frac{1}{e^2}\int d^4x \frac{\delta^2 W[A]}{\delta A_{\mu}(x)\delta A_{\mu}(0)} \\
&= \frac{1}{e^2\beta V}\int d^4x d^4y \frac{\delta^2 W[A]}{\delta A_{\mu}(x)\delta A_{\mu}(y)} \\
&\stackrel{?}{=} \frac{\partial^2 P}{\partial(eB)^2} \quad (W[A] = \beta VP) \\
&= \tilde{\chi}(T) \qquad (21)
\end{aligned}$$

[21] S. L. Adler. Phys. Rev. D10, 3714 (1974).
[22] P. A. Baikov et al. Phys. Rev. Lett. 104, 132004 (2010).
[23] G. S. Bali et al, J. High Energy Phys. 1408 (2014) 177.

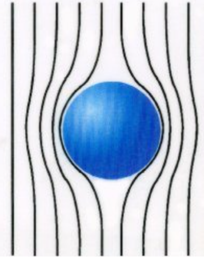


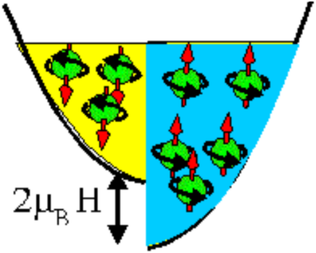
QM model has no gluon's effect — the lowest order $\mathcal{O}(\alpha_s^0)$

PQM model has gluon's effect — higher order $\mathcal{O}(\alpha_s^3)$

Intuitive picture

low temperature	**high temperature**
chiral symmetry breaking	chiral symmetry restored
quark pairs ($\pi \cdots$) dominate	free quarks dominate
Landau diamagnetic	Pauli paramagnetic





Summary

- PQM system displays magnetic catalysis effect.
- The response behaviors of PQM system experience a transition as the temperature rises.

Thank you for your listenning.