Experimental Quantum Cosmology: Probing Analogue TransPlanckian Physics in Dipolar Bose-Einstein Condensates

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Probing the Transplanckian World with Analogue Gravity

Analogue "Gravity" $\equiv$

Fundamental Kinematical Effects [Visser, PRL **80** (1998)]

For Quantum Fields

Propagating In Curved Spacetime

(Spacetime Fixed by Experimentalist!)

In Particular:

I. Extreme Spacetime Curvatures

II. Impact of Transplanckian Physics

Can be Investigated in Analogue Experiments

Examples for Kinematical ($\equiv$ non-Einstein Equation) Effects in BECs

- Hawking Radiation
  [Garay et al, PRL **85** (2000); Barceló et al, IJMPB **18** (2003)
  Carusotto et al, NJP **10** (2008); de Nova et al, Nature **569** (2019)]

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- Cosmological Particle Production, Sakharov Oscillations
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- Universe Inflation and Mode Freezing
  [URF & Schützhold, PRA **70** (2004); Eckel et al, PRX **8** (2018)]

I. Cosmological Particle Production (CPP)

Time Dependent Variation of Spacetime Metric
in Expanding Cosmos

OR Time Dependence of Boundary Conditions in Cavity
(Dynamical Casimir Effect)

CPP Concept: Schrödinger, Physica **6** (1939)
"*The proper vibrations of the expanding universe*"

QFT Elaboration of CPP: Parker, PRL **21** (1968)

Dynamical Casimir: Moore, J Math Phys **11** (1970)

CPP aka Dynamical Casimir

(aka Parametric Downconversion in Quantum Optics)

Dynamical Casimir or CPP in BECs

Simple Recipe: Do Something Explicitly Time-Dependent to System

For Example:

(a) Coupling Constant(s) $g_c(t)$ [Feshbach Resonance]

(b) Trapping $V_{\rm trap} = V_{\rm trap}(t)$

[$\omega_{\rm trap} = \omega_{\rm trap}(t)$ or Split into Double Well]

$\Longrightarrow$ CPP Generic Effect of Mode Mixing

(Conversion of Positive to Negative Frequency Modes)

II. Scale Invariance of Cosmological Power Spectrum

Cosmological Density Perturbations ($\equiv$ Inflaton) in

Inflationary Quantum Cosmology

Guth, PRD **23** (1981); Linde, PLB **108** (1982)

Inflaton Mode Freezing After Crossing of Cosmological Horizon

Standard Quantum Mechanism for Galaxy Formation:

Formation of Inhomogeneities in Initially Homogeneous Universe

Mukhanov, Physical Foundations of Cosmology (CUP 2005)

Chapter 1 of Transplanckian Analogue Simulation:

Inflation and Scale Invariance of Power Spectrum

Exponential De Sitter Evolution of Cosmological Scale Factor

$$a(\tau) = \exp[H\tau] \qquad H \propto \sqrt{\Lambda} \equiv \text{Hubble Constant}$$

Consider Power Spectrum of Phase-Phase (Inflaton) Correlator

$$\langle 0|\delta\hat{\phi}(\mathbf{x},\tau)\delta\hat{\phi}(\mathbf{y},\tau)|0\rangle := \frac{1}{V}\sum_{\mathbf{k}} P(k) e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}$$

Late Cosmological Times τ: $\quad P(k) \to \frac{\hbar H}{\pi m c_0^2} k^{-2}$

Definition of Scale Invariance:

$$\Delta^2(k) := k^2 P(k) \quad \text{Independent of } k$$

Hallmark Signature of Inflationary Quantum Cosmology

Note: Use D=2 Variant Here $[P(k) \propto k^{-D}]$

Trans-Planckian Problem of Inflationary Cosmology

Comoving Wavelengths $\lambda \sim \mathcal{O}(\lambda_{PL})$

Probe Planck Scale During Early Cosmological Stages

Scale Invariance Broken by Trans-Planckian Effects??

"Trans-Planckian Problem of Inflationary Cosmology"

Martin & Brandenberger, PRD **63** (2001)

Problem: Dispersion Relation Not Known for "Real" Gravity

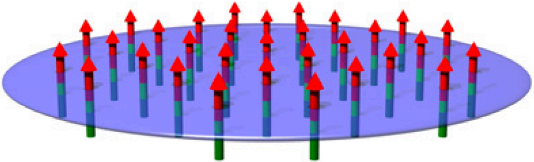
No Quantum Gravity Exists!

$\Longrightarrow$ Take Analog Model with Well-Known Microscopics:

Condensed Matter Physics Comes to the Rescue...

Volovik, "The Universe in a Helium Droplet", Oxford UP

Dipolar Bose-Einstein Condensate



$$V_{dd}(\boldsymbol{r}) = \frac{3g_d}{4\pi}\frac{1-3z^2/|\boldsymbol{r}|^2}{|\boldsymbol{r}|^3} \qquad g_d \propto \mu^2_{m,e}$$

Pancake With $\perp$-Polarized Dipoles For Stability

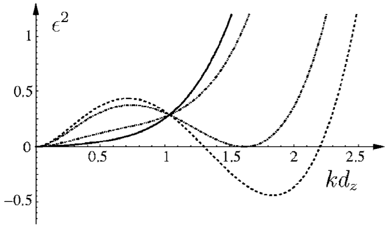
Large Magnetic Dipole Moment μ_m: Cr, Dy, Er

(Tune in Addition Contact Coupling with Feshbach...)

Pfau (Stuttgart), Lev (Stanford), Ferlaino (Innsbruck)
Modugno (Firenze, Pisa) [more to come...]

Electrically Dipolar BEC: Ongoing Work in Progress

(Quasi-2D) Bogoliubov Excitations



$\zeta = kd_z$

$$\frac{\varepsilon^2}{(\hbar\omega_{z,0})^2} = A\zeta^2\left(1 - \frac{3R}{2}\zeta \exp[\zeta^2/2]\,\mathrm{erfc}[\zeta/\sqrt{2}]\right) + \frac{\zeta^4}{4}$$

$A \propto \rho_0 d_z^2 g_d / R$ Chemical Potential

$R = \sqrt{\pi/2}(1 + g_c/2g_d)^{-1} \simeq \sqrt{\pi/2}$ Coupling Ratio

URF, PRA Rapids **73** (2006)

Definition of "Trans-Planckian" Physics

Beyond Linear (Sound) Part:

Physics not (Pseudo-)Lorentz Invariant

Large Dipolar Coupling ($R \to \sqrt{\pi/2}$): Curvature Negative

Dispersion Develops **"Roton" Minimum**

Impact of High-Momentum, Low-Energy Quasi-Particles

On (small-$\boldsymbol{k}$) Quantum Fields in Curved Spacetime?

Question Here: Scale Invariance Maintained?

Scaling Expansion of Dipolar BEC

Scaling Ansatz [Kagan et al., Castin & Dum (1996); Gritsev et al (2010)]

$$\mathbf{x} := \frac{\mathbf{r}}{b(t)} \qquad \tau := \int_0^t \frac{dt'}{b^2(t')} \qquad \Psi(\mathbf{r}, t) := \exp\left[i\frac{mr^2}{2\hbar}\frac{\partial_t b}{b}\right]\frac{\psi(\mathbf{x}, \tau)}{b}$$

provided

$$\boxed{\frac{b^3\partial_t^2 b + b^4\omega^2(t)}{\omega_0^2} = \frac{g_c(t)}{g_{c,0}b} = \frac{g_d(t)}{g_{d,0}b}}$$

Exact Solution for Constant g_c, g_d:

(Slow) Square Root Expansion

$$\boxed{b(t) = \sqrt{4Ht+1}, \quad \omega^2(t) = \omega_0^2/b^5 + 4H^2/b^4}$$

Robertson-Walker (a) and BEC (b) Universes Have

Scale Factor Relation $\quad a^2 = b \quad$ (Contact Interaction: $\quad a = b$)

Time-Dependent Bogoliubov Mode Functions

Analytical Solution when $b(t) = \sqrt{4Ht+1}$

$$\delta\tilde{\phi}_k(s) = s\sqrt{\frac{\pi\hbar VH}{4mc_0^2k^2}}\Big[J_1(s)+iY_1(s)\Big] := h_k(s)$$

Initial Free Oscillation $\Longrightarrow$ Overdamped for Large s

$$s(t) = \frac{c_0/H}{a(t)/k} \equiv \frac{\text{Hubble radius}}{\text{rescaled wavelength}}$$

Modes $h_k(s)$ Correspond to

Instantaneous **Bunch-Davies Vacuum**

[Chernikov & Tagirov; Schomblond & Spindel, Ann Inst H Poincaré **IX** & **XXV** (1968 & 1976); Bunch & Davies, Proc Roy Soc A **360** (1978)]

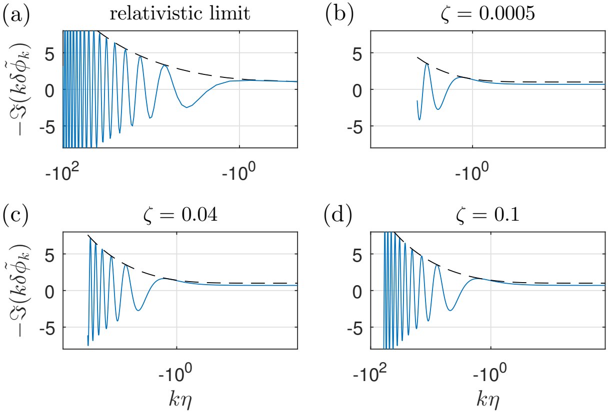
"BD Vacuum" $\equiv$ (Quasi-)Particle-Vacuum Free-Falling Observer

$\Rightarrow$ Yields in (Formally) Infinite Past

BEC Quasi-Particle Vacuum in Minkowski Space

Horizon Crossing and Mode Freezing

In Conformal Time: $ds^2 = a^2[d\eta^2 - d\mathbf{x}^2]$

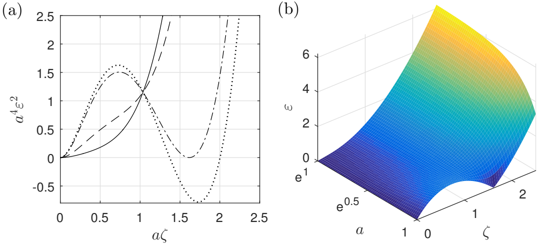


“Galaxy Formation” in Analogue Inflationary Quantum Cosmology

$\Longleftrightarrow$ Bogoliubov Modes Get “Frozen In”

Violation of Scale Invariance?

Microscopic Excitation Spectrum During Expansion



Expansion: Gas Becomes More Dilute

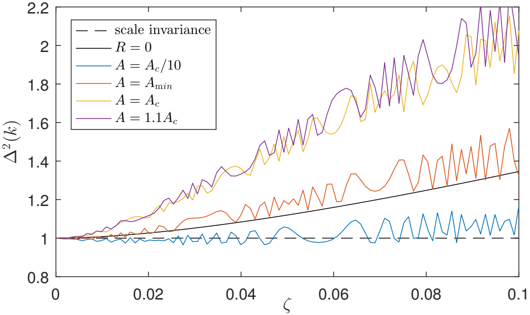
However: Initial "Roton" Minimum Leaves Imprint

On Power Spectrum of Phase-Phase Correlations

$\Longrightarrow$ Window on Transplanckian Physics

In Early (Analogue) Universe!

Phase-Phase Correlations in Fourier Space: Power Spectrum



Significant Violations of Scale Invariance

For Almost Critical (Initial) Spectrum

Roton Minimum Imprint!

Adiabaticity of Mode Propagation

Violated for Deep Minimum

Chä & URF, PRL **118,** 130404 (2017)

Chapter 2 of Transplanckian Analogue Simulation:

Roton Entanglement and Steering in Quenched Condensates

Dynamical Casimir Effect (aka Cosmological Particle Production)

Here: Created by Quench of Sound Speed

$\Longrightarrow$ Pairs of Quasiparticles Created with Momenta $\mathbf{k}$ and $-\mathbf{k}$

Enhanced Production Near Roton Minimum (Heisenberg Helps...)

Quasiparticle Pairs Entangled/Steered?

Dilute Gas: Tractable with Time-Dependent Bogoliubov Theory

Entanglement and Steering Primer

Entanglement $\equiv$ Nonseparability of Quantum States

Entanglement: Schrödinger, Naturwissenschaften **23** (1935)

Steerable States $\equiv$ Subset of Nonseparable States

Steering: Schrödinger, Math Proc Cambr Phil Soc **31** (1935)

Bipartite Steerable States Have "Half" of Bell-Nonlocality:

Alice Performs Uncharacterized Measurements
Not Accessible (Hidden) to **B**ob

$\Longrightarrow$ Inferred Standard Dev of Noncommuting $\mathbf{A}, \mathbf{B}$ Observables
Can Violate Standard Heisenberg Uncertainty Relation

Steerable Quantum States

(a) Subset of Entangled States

(b) Superset of Bell States

Entanglement and Steering Criteria for CV Systems

Density Fluctuation Operator

$$\frac{\delta\hat{\rho}_{\mathbf{k}}(\tau)}{\rho_0} = (u_{\mathbf{k}} + v_{\mathbf{k}})(\hat{\varphi}_{\mathbf{k}}(\tau) + \hat{\varphi}^{\dagger}_{-\mathbf{k}}(\tau))$$

$u_{\mathbf{k}}, v_{\mathbf{k}}$ Bogoliubov Transformation Amplitudes

$\hat{\varphi}_{\mathbf{k}}$ Quasiparticle Field Operators

Density-Density Correlation Function

$$G_{2,\mathbf{k}}(\tau) = \frac{\langle|\delta\hat{\rho}_{\mathbf{k}}(\tau)|^2\rangle}{\rho_0^2} = (u_{\mathbf{k}} + v_{\mathbf{k}})^2(2n_{\mathbf{k}} + 1 + 2\Re[c_{\mathbf{k}}e^{-2i\omega_{\mathbf{k}}\tau}])$$

$n_{\mathbf{k}} = \langle\hat{b}^{\dagger}_{\mathbf{k}}\hat{b}_{\mathbf{k}}\rangle$ Mean Quasiparticle Occupation Number

$c_{\mathbf{k}} = \langle\hat{b}_{\mathbf{k}}\hat{b}_{-\mathbf{k}}\rangle$ Quasiparticle Pair Amplitude

Assessing Entanglement and Steering

Density-Density Correlations:

Experimentally Accessible Measure of Quasiparticle Entanglement

Robertson, Michel, Busch & Parentani
PRA **89** (2014), PRD **95** & **96** (2017)

Finazzi & Carusotto, PRA **90** (2014)

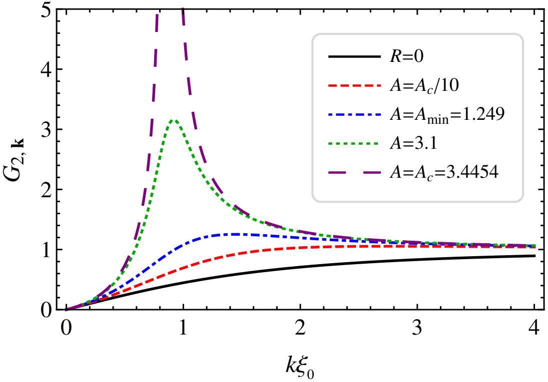
$$G_{2,\mathbf{k}}(\tau) < G_{2,\mathbf{k}}^{\mathrm{vac}} = (u_{\mathbf{k}} + v_{\mathbf{k}})^2 \quad \text{[Nonseparable]}$$

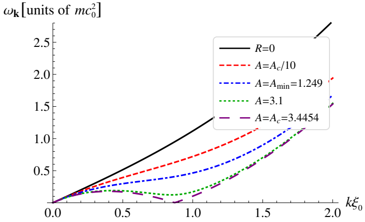
$$G_{2,\mathbf{k}}(\tau) < \frac{1}{2} G_{2,\mathbf{k}}^{\mathrm{vac}} \quad \text{[Steerable, More Strict Criterion!]}$$

$\Longrightarrow$ Vacuum Amplitude of Density-Density Correlations

Decides On Entanglement and Steering

Stationary Density-Density Correlations



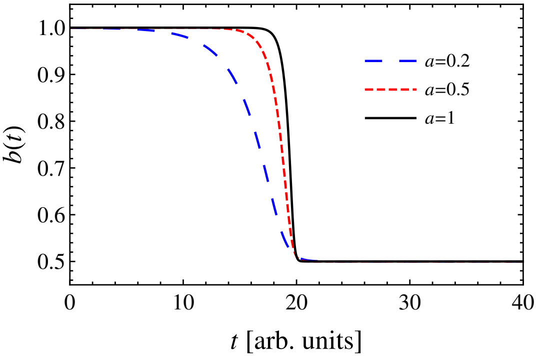


Correlation Peak at Roton Minimum

Contact Interactions: No Peak

Quench of Sound Speed

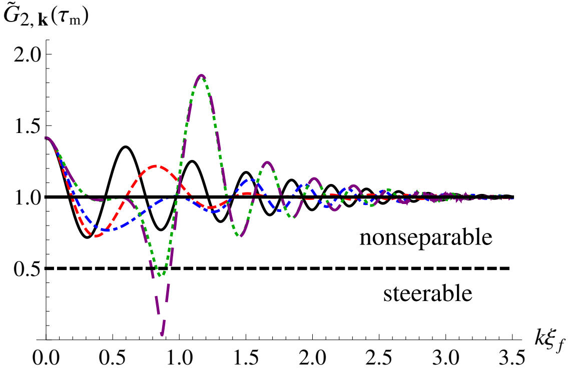
Scale Factor $b(t)$ of Gas in Lab Time

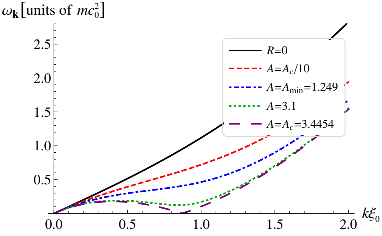


$\Longrightarrow$ Gas Contraction

Roton-Enhanced Entanglement and Steering

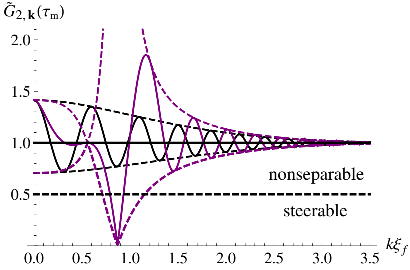
Normalized Correlations $\tilde{G}_{2,\mathbf{k}} := G_{2,\mathbf{k}}/G_{2,\mathbf{k}}^{\text{vac}}$





Increasing Roton Depth ⇒ Increased Entanglement Potential

Visualization with Envelopes and $T \neq 0$



Temperature $T = 0$ $\qquad$ $T = mc_0^2$

Black: Solely Contact Interactions

Purple: Critical Dipole-Dipole Interaction Dominated Gas

$\Longrightarrow$ Roton Minimum Generally Strongly Enhances Entanglement and Steering

Tian, Chä & URF, Phys. Rev. A **97,** 063611 (2018)

Summary

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  First Such Analogue Gravity System!
  More Typically: No Violations
  [cf Niemeyer and Parentani, PRD **64** (2001)]

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  Details of Transplanckian Dispersion Matter
  Chä & URF, PRL **118,** 130404 (2017) [in Supplement]

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- Strong Quantum Fluctuations of Electric-Dipole BEC:
  Possible Influence on Early (Pre-Metric?) Stage
  of Cosmological Evolution

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  Tian, Chä & URF, Phys. Rev. A **97,** 063611 (2018)

- Ad: Analogue Gravity Postdoc Positions Available!

  Reversing Rutherford: "When a young man in my lab uses the word *Universe*, I tell him it is time for him to leave"

Queen's Prophecy

Don't stop me now (1978):

Queen's Prophecy

Don't stop me now (1978):

"I'm traveling at the speed of light

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I wanna make a supersonic man out of you"

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So: Don't Stop Analogue Gravity Now!