Breakdown of Quasilocality in Long-Range Quantum Lattice Models

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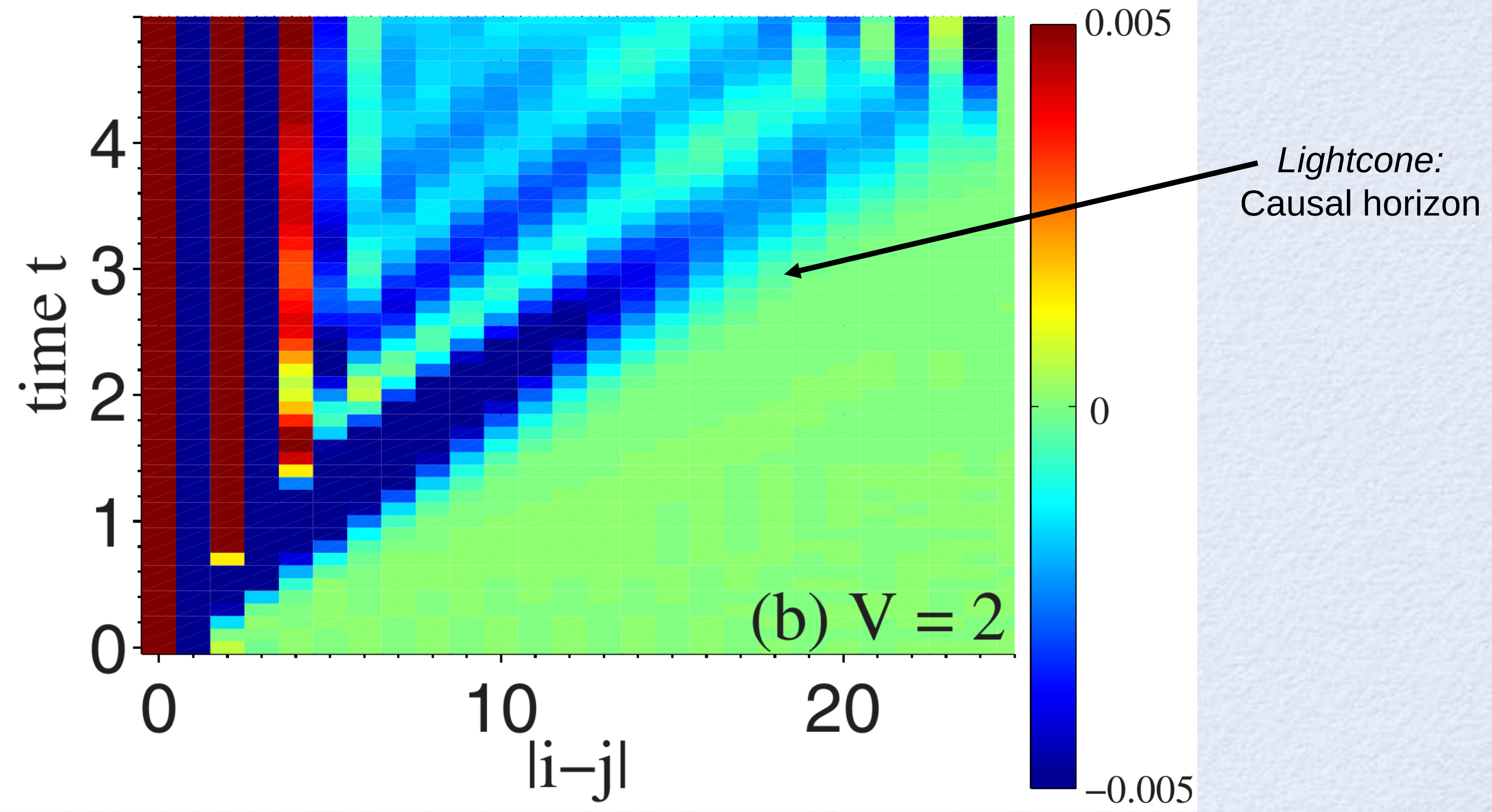
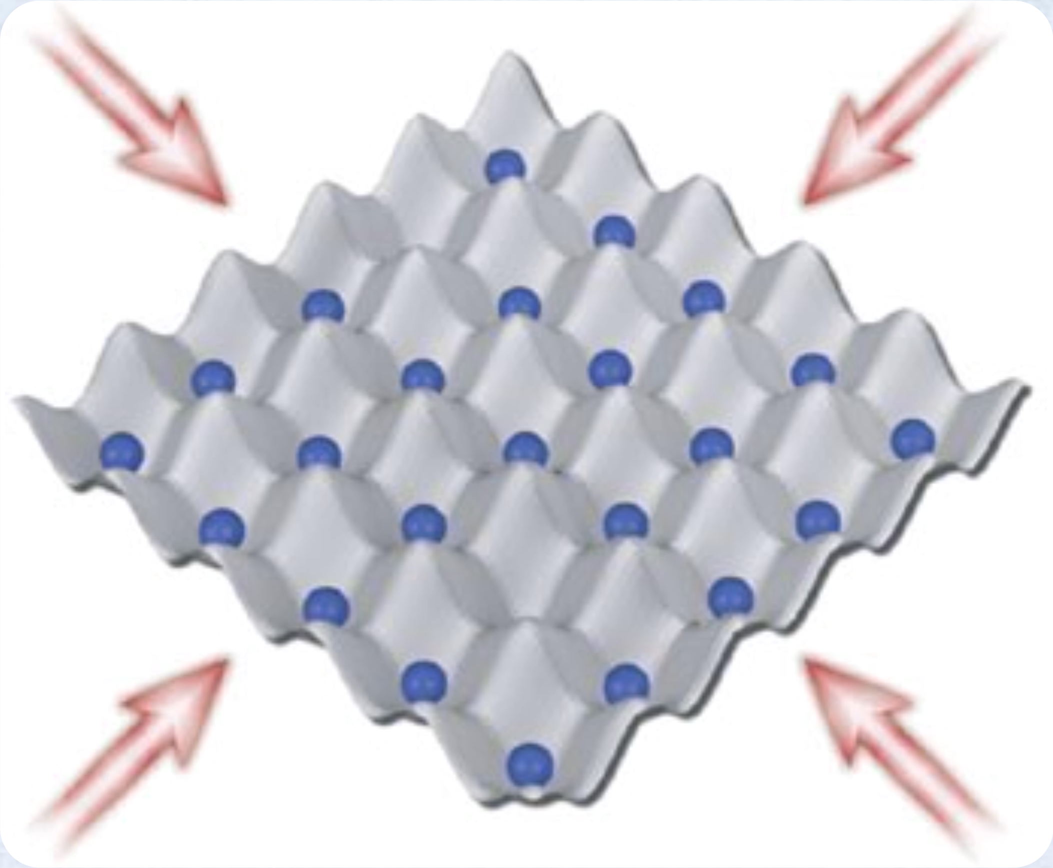
Topic of this talk:



How does information or correlations spread after pushing a system out-of-equilibrium?

Optical Lattices & Lattice Models



Standing waves of laser light: periodic structures

Mechanism: Stark-Effect

➠ Induced dipolemoment in neutral atoms leads to a trapping force in the periodic potential: “Crystals of Light”

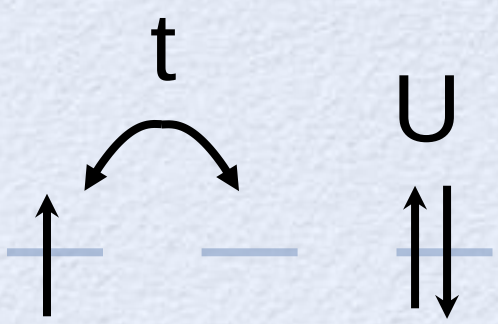
Optical Lattices & Lattice Models

Typical lattice models:

Hubbard model

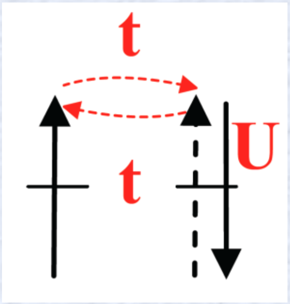
(e.g., cuprates):

$$\mathcal{H} = -t \sum_{\langle ij \rangle, \sigma} \left[c^\dagger_{i+1,\sigma} c_{i,\sigma} + h.c. \right] + U \sum_i n_{i,\uparrow} n_{i,\downarrow}$$



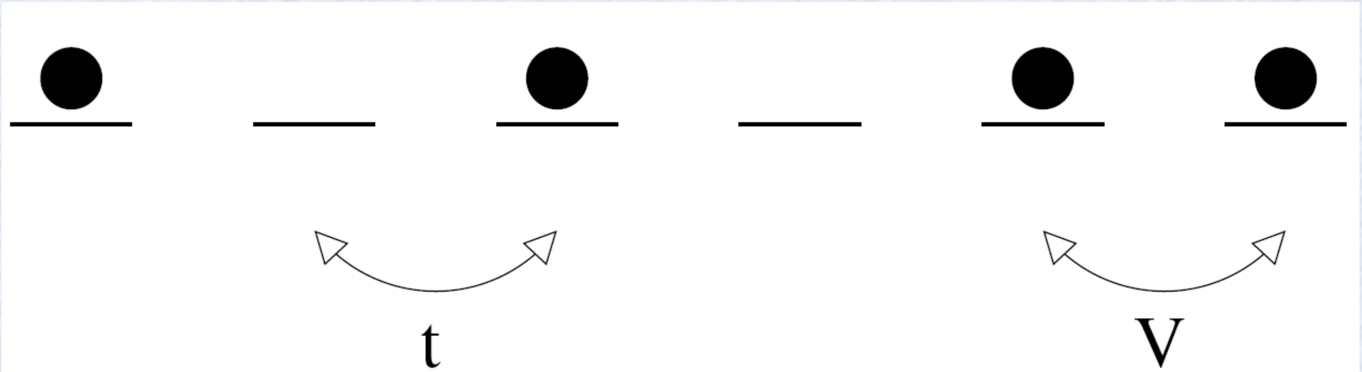
Heisenberg exchange: 2[nd] order perturbation theory for U >> t

$$J\, \vec{S}_1 \cdot \vec{S}_2 \qquad J = \frac{4t^2}{U}$$



(e.g., quantum magnets)

‚Spinless Fermions‘
(e.g., fully polarized extended Hubbard model):



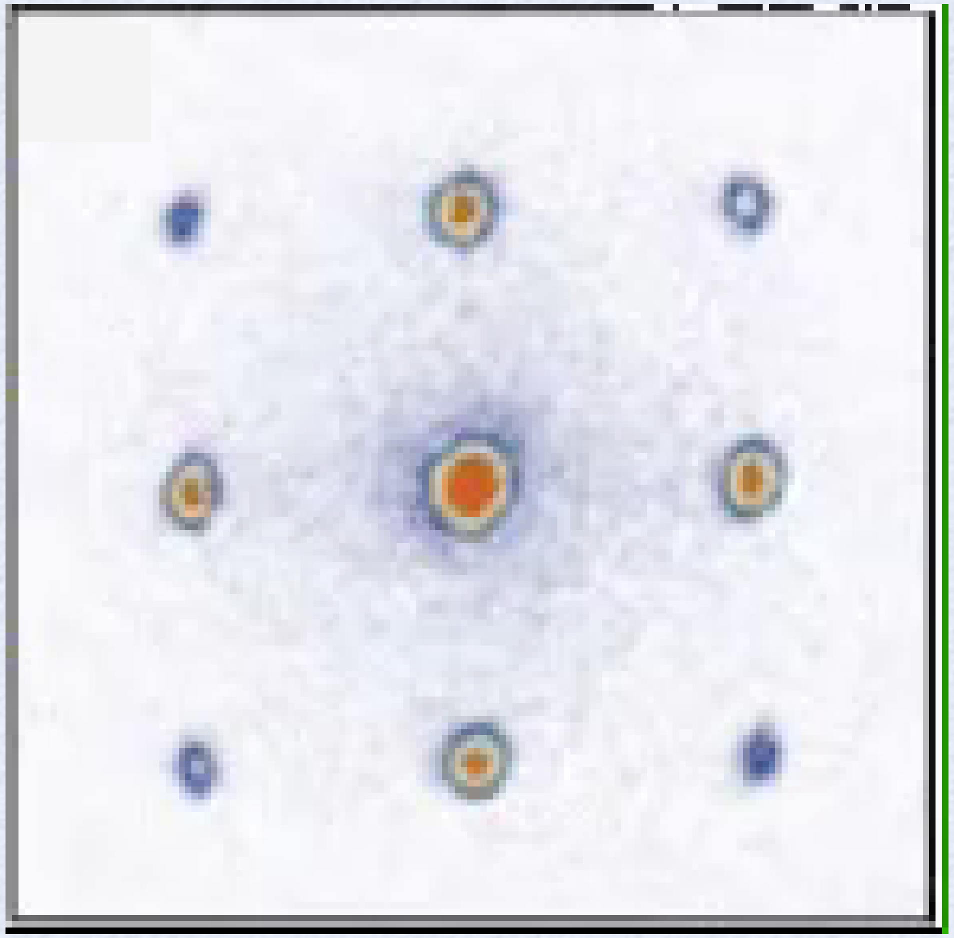
$$\hat{H} = -t \sum_j \left(c^\dagger_{j+1} c_j + h.c. \right) + V \sum_j \hat{n}_j \hat{n}_{j+1}$$

Out-of-Equilibrium Dynamics: Quantum Quenches

Example: “time-of-flight measurements” (Bragg scattering)

“Quantum Quench”:
➠ sudden change of interaction strength

$U_0 \rightarrow U$



«Collapse and Revival of a Bose-Einstein-Condensate»

[M. Greiner et al., Nature (2002)]

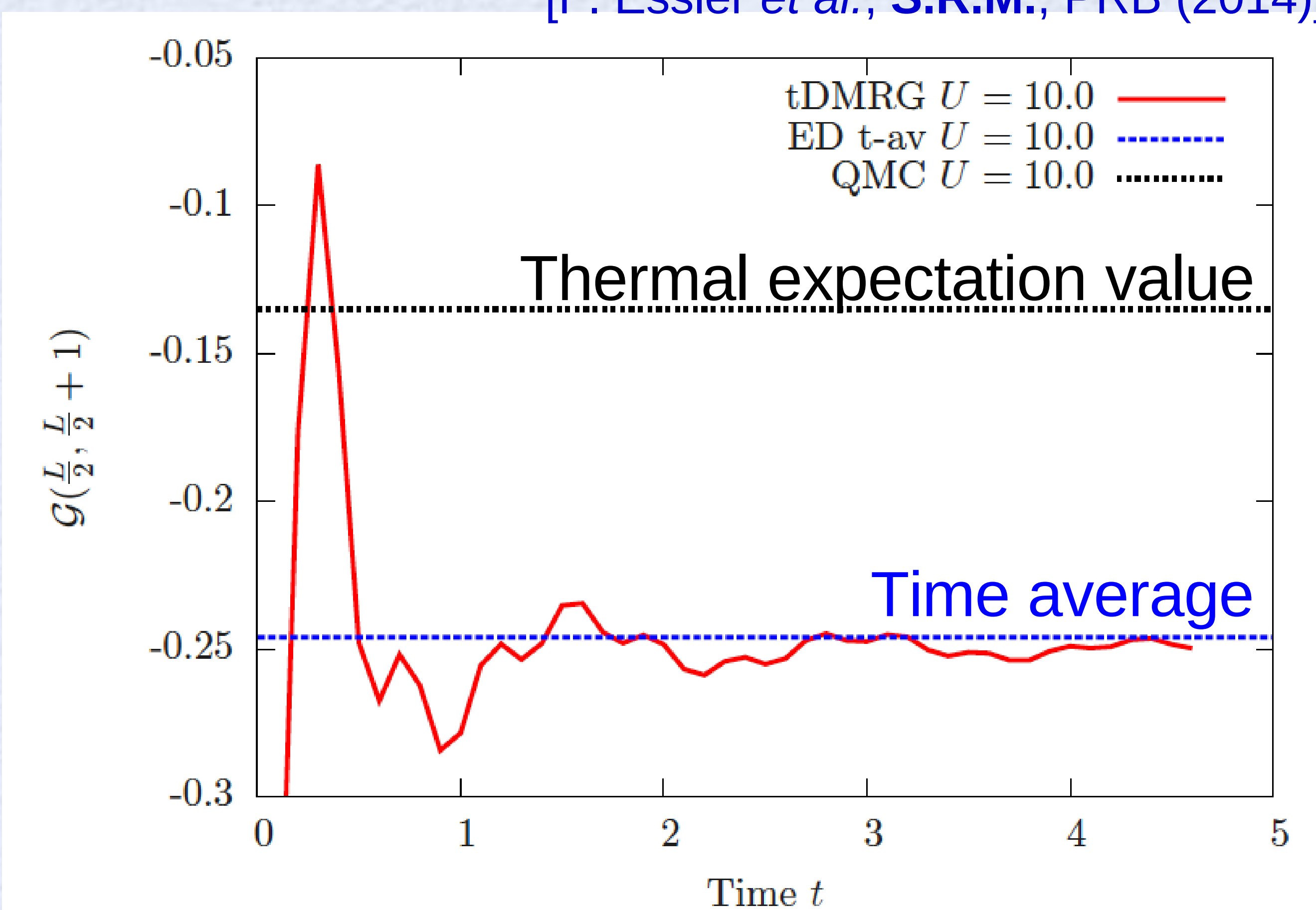
e.g.,
S.R.M. *et al.*, PRB (2009),
S.R.M. *et al.*, PRL (2007),

Out-of-Equilibrium Dynamics: Quantum Quenches

Typical behavior after a quantum quench:

Relaxation (dephasing)

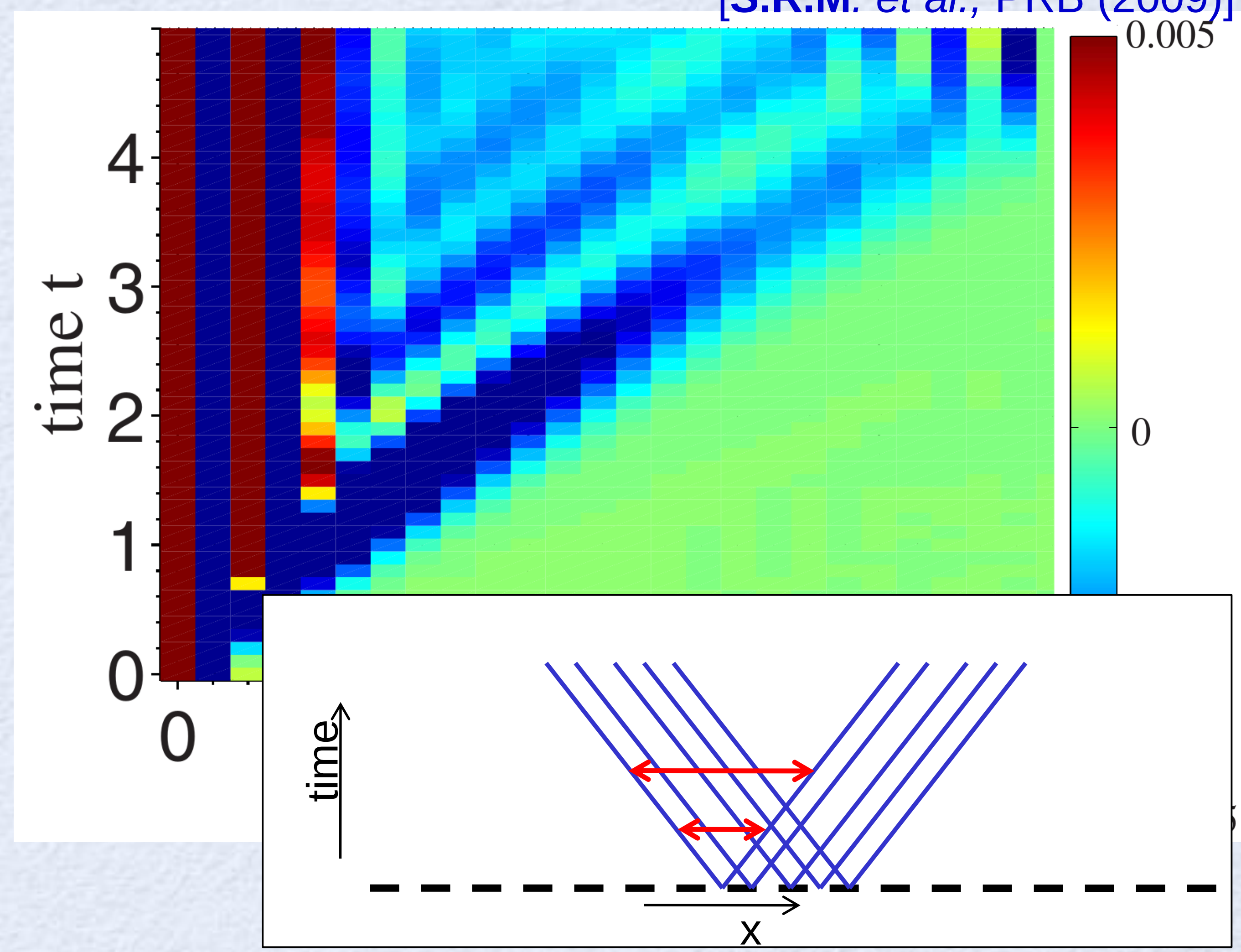
[F. Essler *et al.*, **S.R.M.**, PRB (2014)]



- Does the system reach a thermal state?
- On which time scale?

Lightcone effect

[**S.R.M**. *et al.*, PRB (2009)]



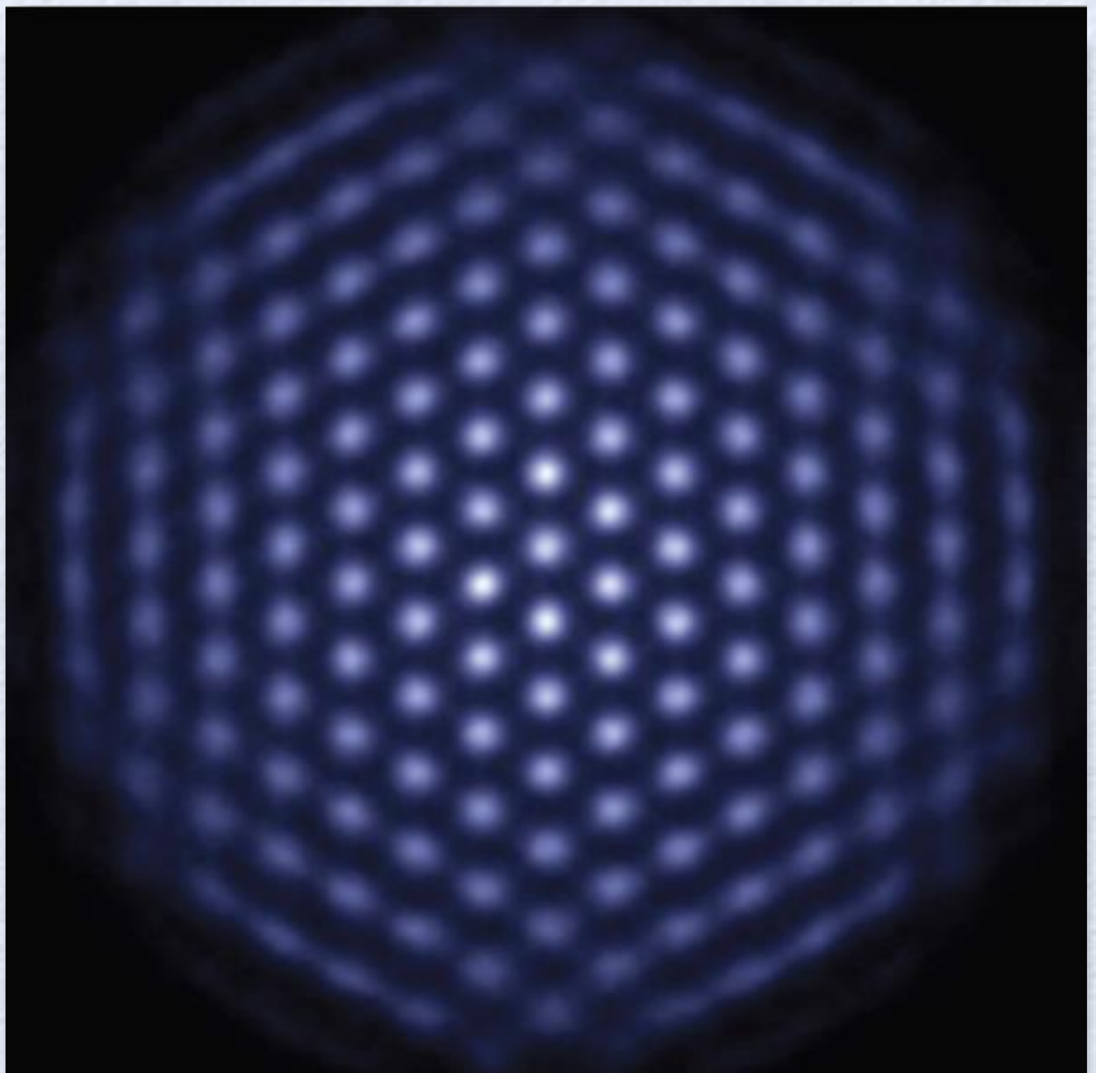
- Quasiparticles move ballistically through the system: lightcone

[P. Calabrese & J. Cardy, PRL (2006)]

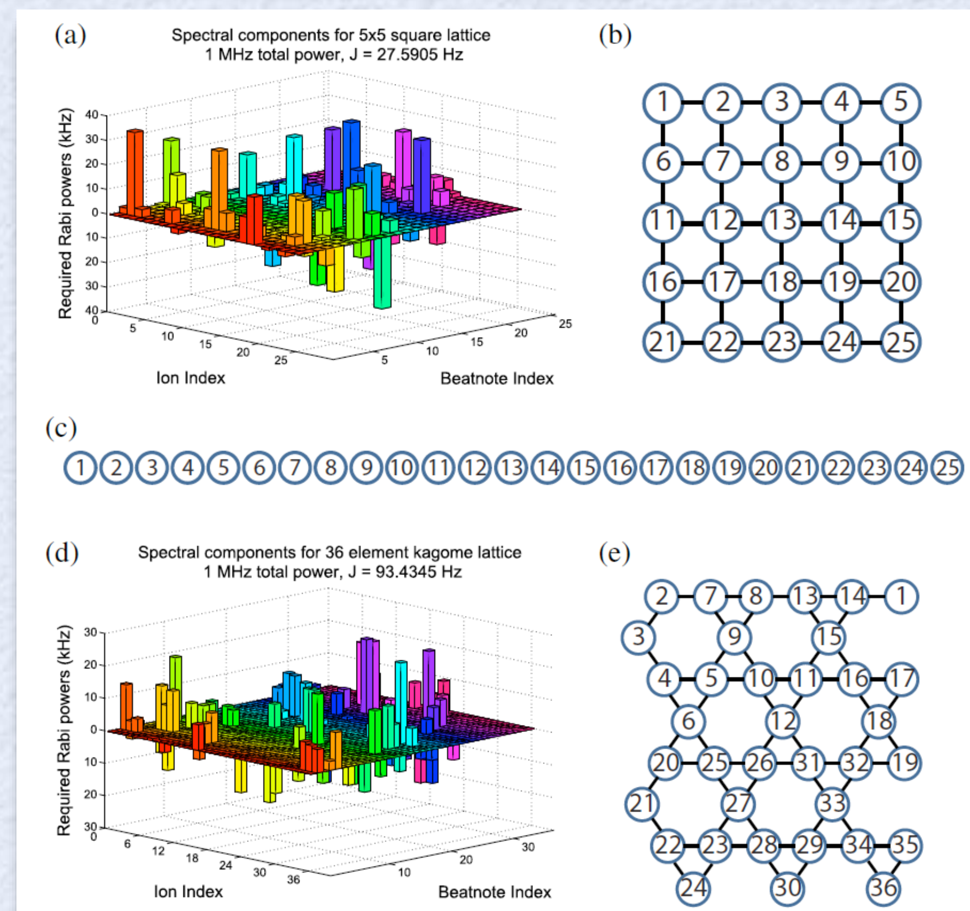
- Propagation of information

Variable interaction range: Ions in a Trap

$^{9}Be^{+}$ ions in a Penning trap (NIST Boulder)
[J.W. Britton et al., Nature **484**, 489 (2012)]



$^{171}Yb^{+}$ ions (JQI/NIST Maryland)
[K. Kim et al., Nature **465**, 590 (2010);
R. Islam et al., Nature Comm. **2**,377 (2011);
NJP and more...]



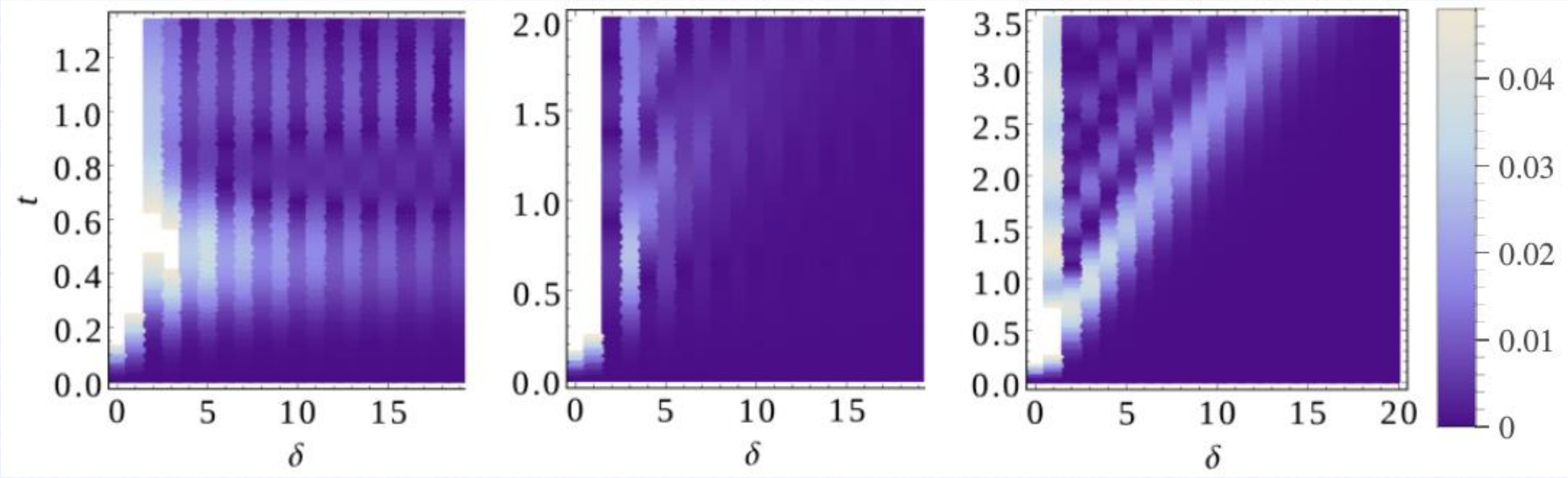
Realization of Ising models with transverse field on variety of lattices:

Interactions ~ $1/r^{\alpha}$

Two natural questions:

I. Time evolution of correlations?

II. Vary the exponent of the long-range interaction?



„Light-cone“and emergence of a causal region vs. Instantaneous propagation of information

Quasilocality: Lieb-Robinson bound

QM nonrelativistic:
local perturbations can have immediate effect everywhere

But: very small for short-range, finite-d systems:
light-cone, quasilocality & Lieb-Robinson-bound:

$$\|[O_A(t), O_B(0)]\| \leq C\|O_A\|\|O_B\| \min(|A|, |B|) \mathrm{e}^{[v|t| - d(A,B)]/\xi}$$

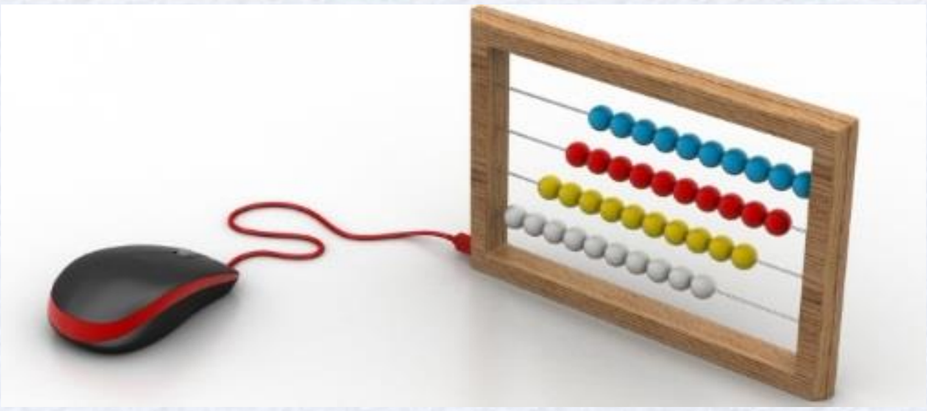
Long-range interactions ~ $r^{-\alpha}$?

$$\|[O_A(t), O_B(0)]\| \leq C\|O_A\|\|O_B\| \frac{\min(|A|, |B|)(e^{v|t|} - 1)}{(\mathrm{d}(A, B) + 1)^\alpha}$$

(Koma&Hastings 2006)

Logarithmic behaviour $v|t| > \ln\left[1 + \frac{\epsilon[1 + \mathrm{d}(A, B)]^\alpha}{\min(|A|, |B|)}\right]$

Numerical Methods



➠ How to treat correlated quantum systems on a computer?

[**Recent Review Article:**
S. Paeckel, T. Köhler, A. Swoboda, **S.R.M.**, U. Schollwöck, and C. Hubig,
arXiv:1901.05824, Ann. of Physics (in press)]

My Main Tool:
Matrix Product State Algorithms

Basic idea: data compression ("quantum version")

Original – 2.4 MB | Compressed 10x 257 KB | Compressed 20x 122 KB

→ Graphics (acoustics, signal transmission, etc.)

Key aspect:
Ignore modes that cannot be resolved (by the ear, the screen, ...) – excellent quality with much smaller amount of data.

➠ Control parameter here: entanglement.

MPS Algorithms: Key Aspects

S.R. White, PRL (1992); U. Schollwöck, RMP (2005)/Ann. Phys. (2011); **R.M. Noack & S.R.M., AIP (2005)**

Schmidt decomposition:

$$|\psi\rangle = \sum_{j=1}^{\dim\mathcal{H}} w_j\,|\alpha\rangle_j\,|\beta\rangle_j \approx \sum_{j=1}^{m} w_j\,|\alpha\rangle_j\,|\beta\rangle_j$$

A $\quad |\alpha\rangle_j$	B $\quad |\beta\rangle_j$

Approximation: $m \ll \dim\mathcal{H}$
(e.g., 1000 sites:
$\dim\mathcal{H} = 2^{1000} > (1\ \text{googol})^3$.
Typical choice: $m = 800$)

$|\alpha\rangle_j,\ |\beta\rangle_j$: eigenstates of the reduced density matrix of A or B

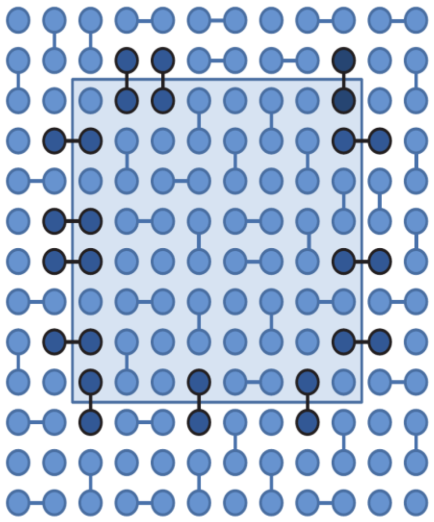
- *very powerful in 1D*
- nonequilibrium, finite-T linear-response dynamics

A. Daley *et al.*, J.Stat. (2004);
S.R. White & A.E. Feiguin, PRL (2004);
S.R.M. *et al.*, AIP (2005);
R.M. Noack, **S.R.M.** *et al.*, Springer Lect. Notes (2008);
A.C. Tiegel, **S.R.M.**, *et al.*, PRB(R) (2014)
Recent Review: S. Paeckel *et al.*, arXiv:1901.05824.

Key: entanglement entropy $\qquad S = -\sum_j w_j^2 \log w_j^2$

➥ the larger the entanglement in the system, the larger *m*

2D:



Problem in 2D:
“area law of entanglement” - *m* grows exponentially with system size

➥ Frontier of today’s efforts.

[See, e.g., E.M. Stoudenmire & S.R. White, Ann. Rev. Cond. Mat. Phys. (2012).]

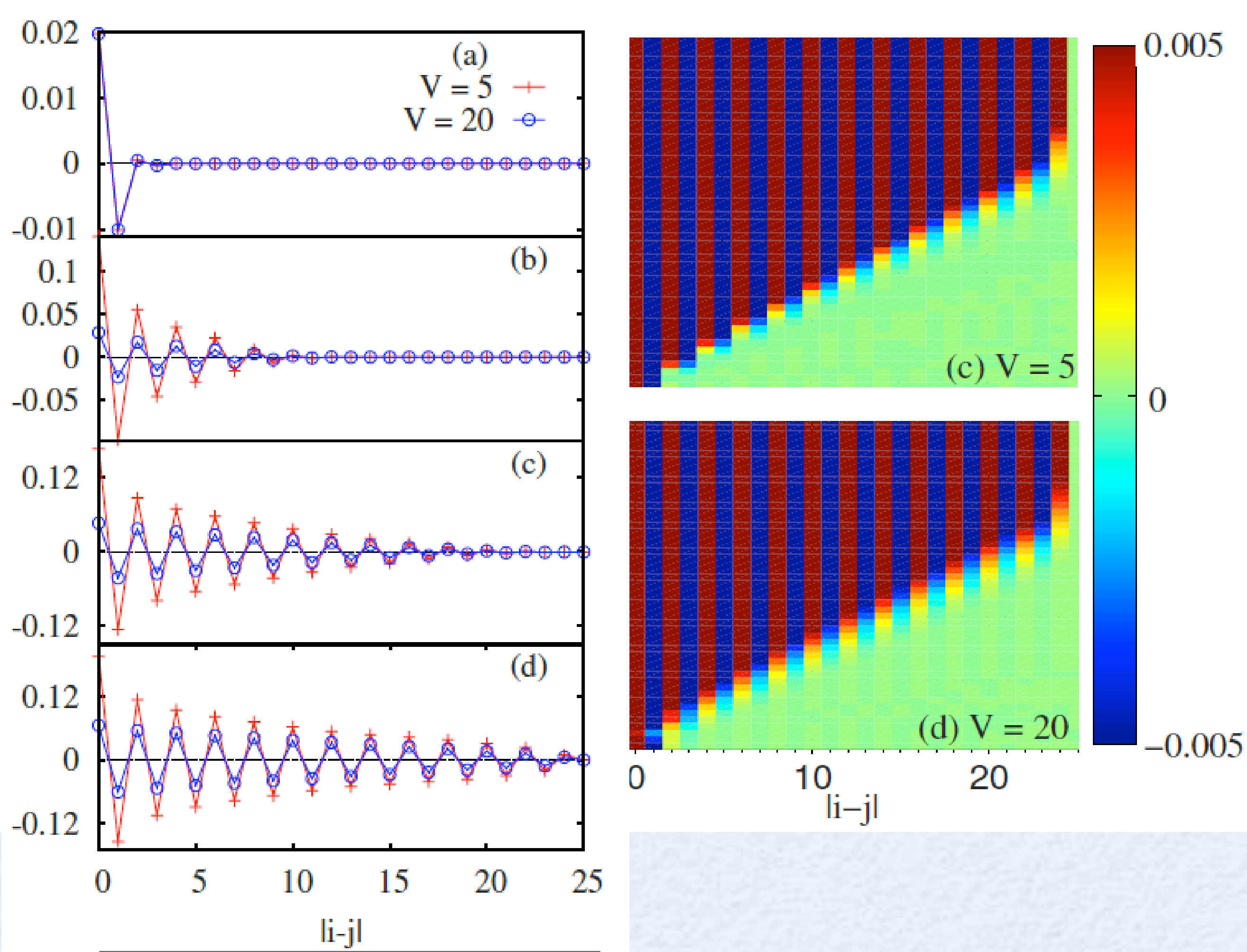
Horizons

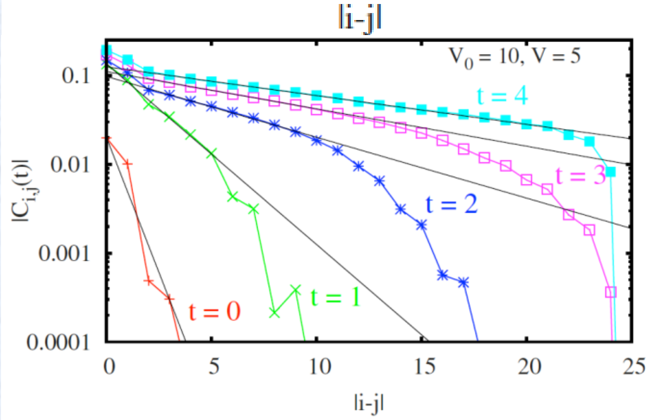
- ➠ Do we always see the lightcone?

Short-range systems: Nature of the light-cone

[S.R. Manmana, S. Wessel, R.M. Noack, and A. Muramatsu, PRB **79**, 155104 (2009)]

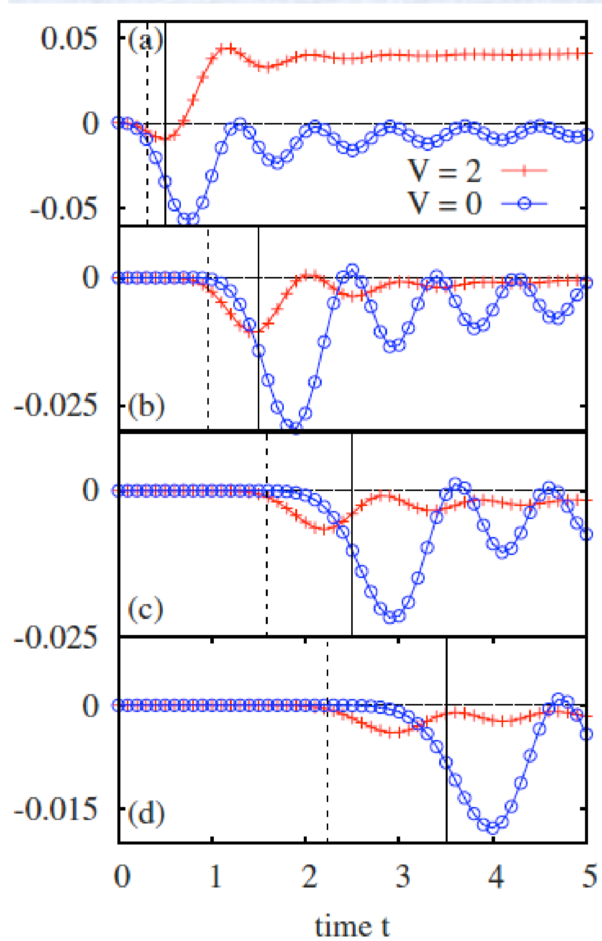
$$\langle n_i n_j \rangle - \langle n_i \rangle \langle n_j \rangle$$





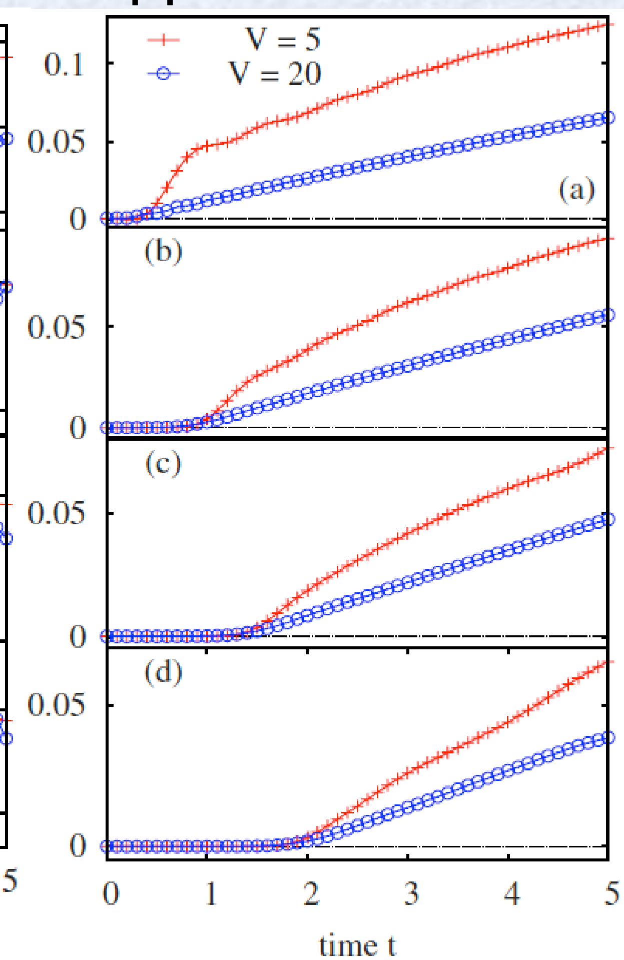
➠ Correlation length *growing* in time

Metallic Phase:



➠ Dip moving *ballistically* through the system

Gapped Phase:



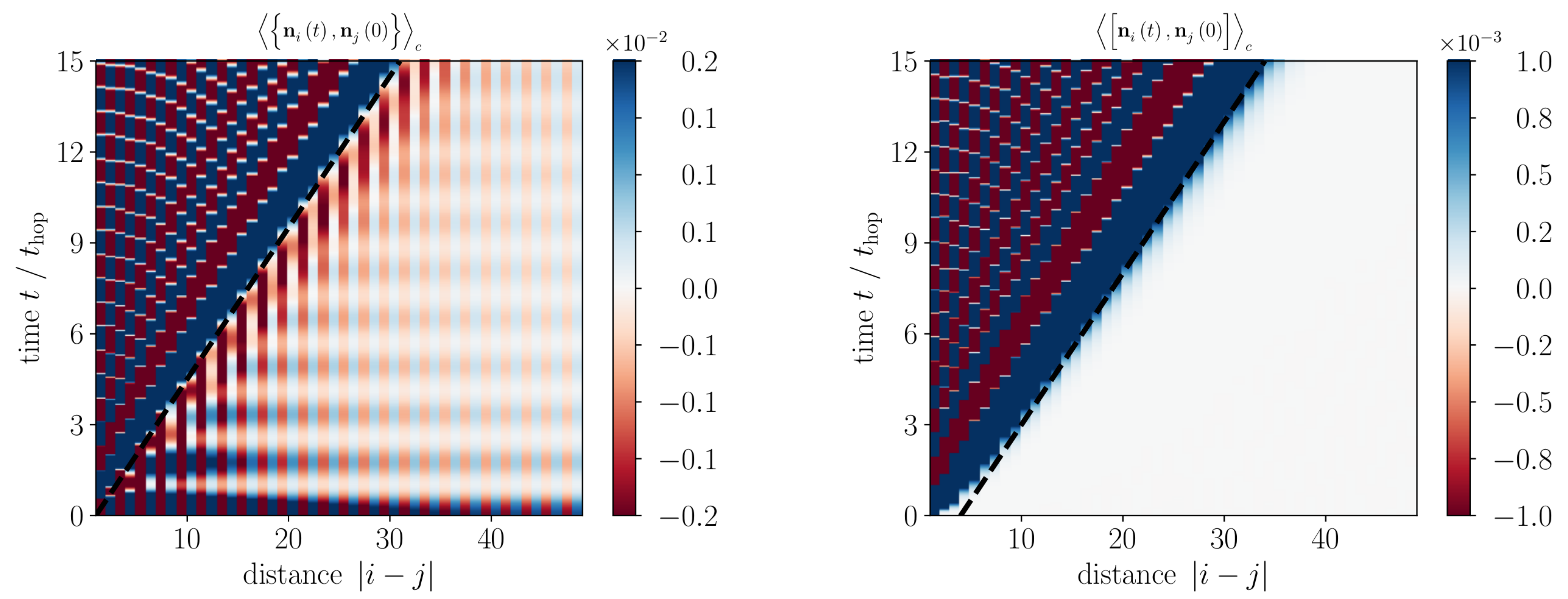
➠ *Onset* of correlations moving “*ballistically*”

Light cones: Propagation of Information?

[K. Harms, L. Cevolani, S. Kehrein, and S.R. Manmana, in preparation]

Anticommutator: *Correlation functions*

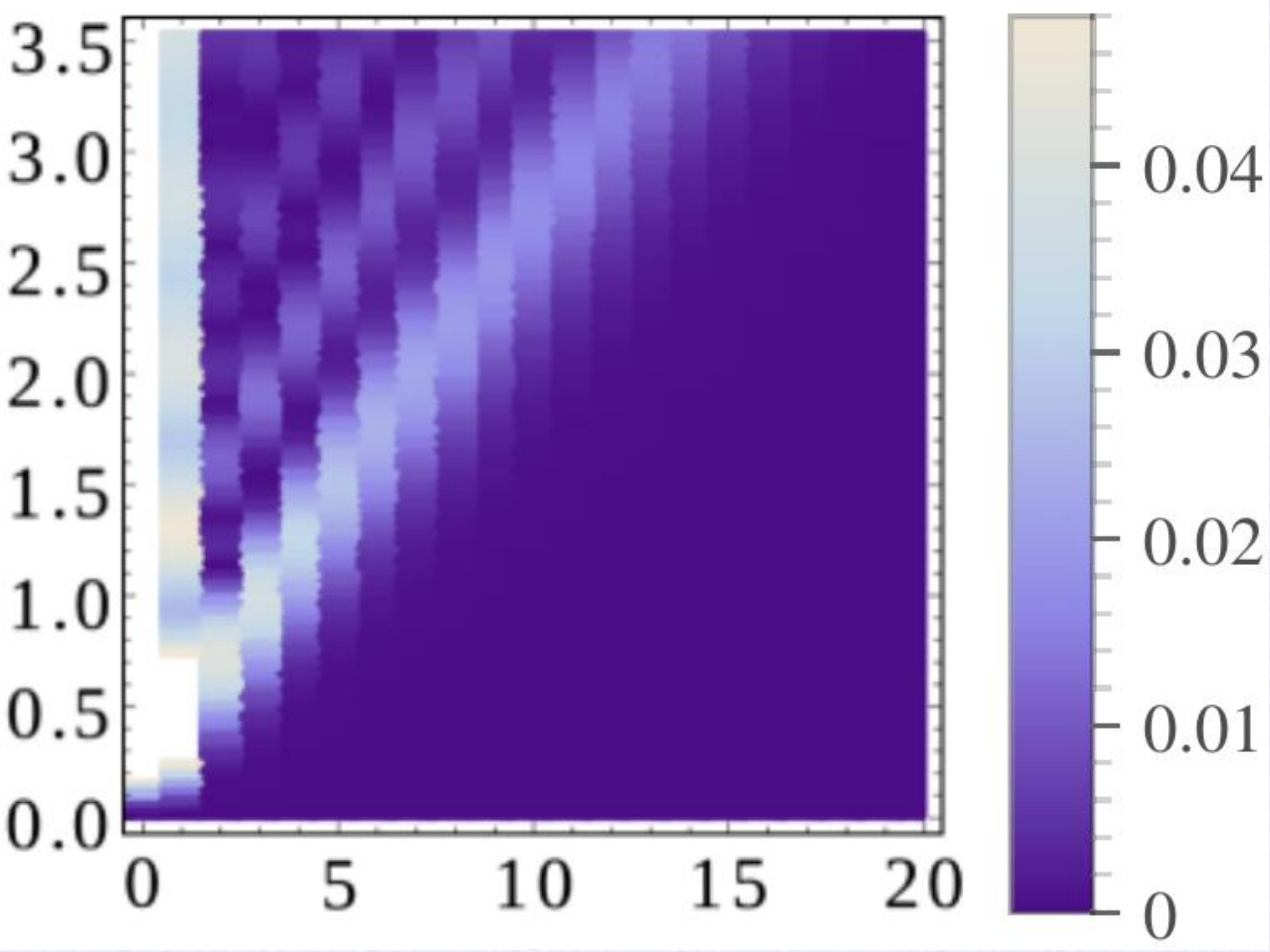
Commutator: *Susceptibilities*



- Lieb-Robinson theorem is – in general – only valid for *commutators, i.e., susceptibilities*
- Correlation functions can also have a signal *outside* the lightcone – suppressed, if exponentially decaying in initial state (e.g., product state)

Light cone with Dipolar interactions

α=3:

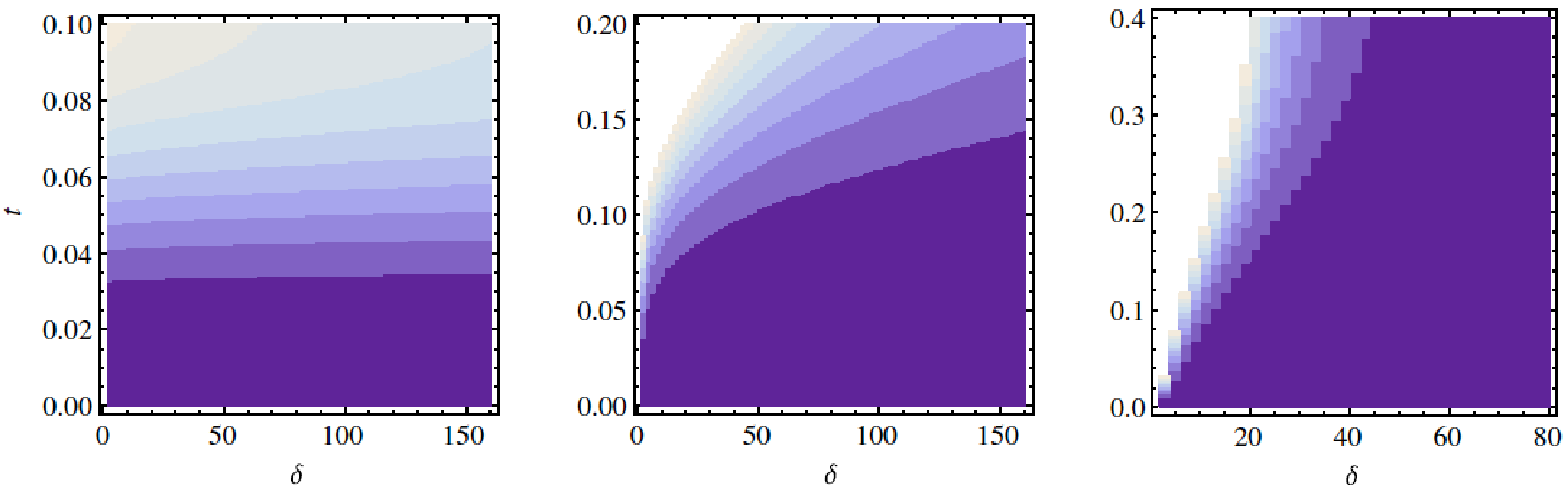


Looks quite linear!

Long-range Interactions: Causal Horizon vs. Immediate Spread

[J. Eisert, M. v.d. Worm, S.R. Manmana, and M. Kastner, PRL **111**, 260401 (2013)]

α = 1/4 α = 3/4 α = 3/2



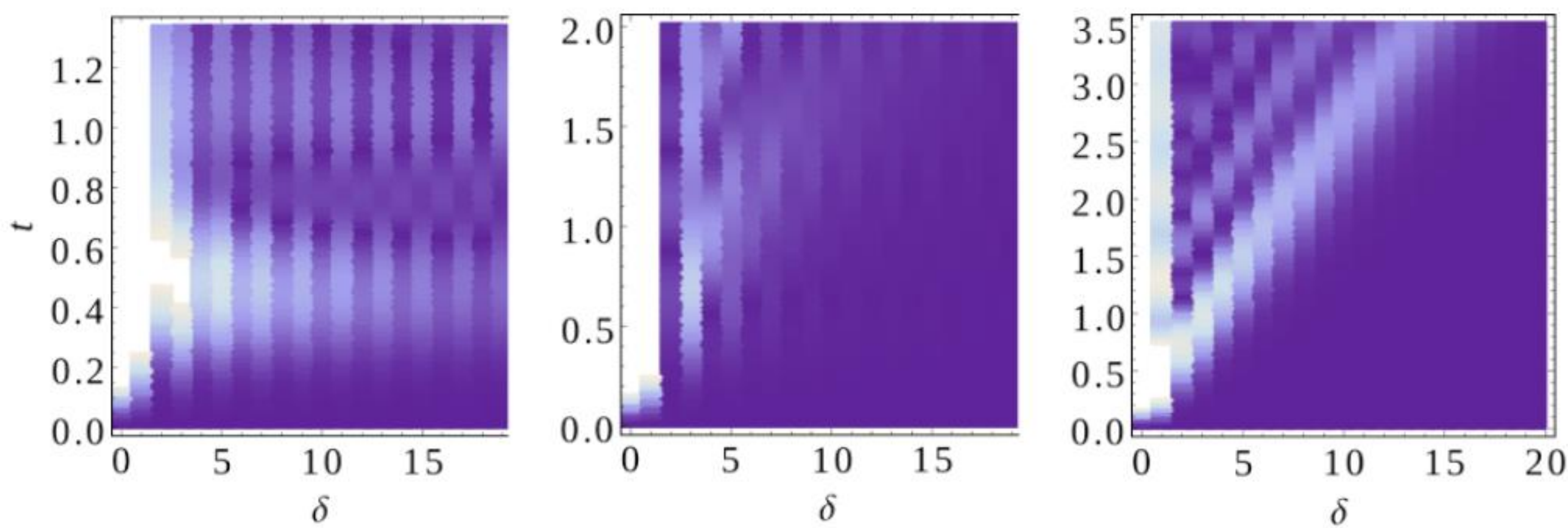
Ising,
L=1001

$v|t| > \epsilon\delta^q$

power law
shape

α = 3

XXZ,
L=40

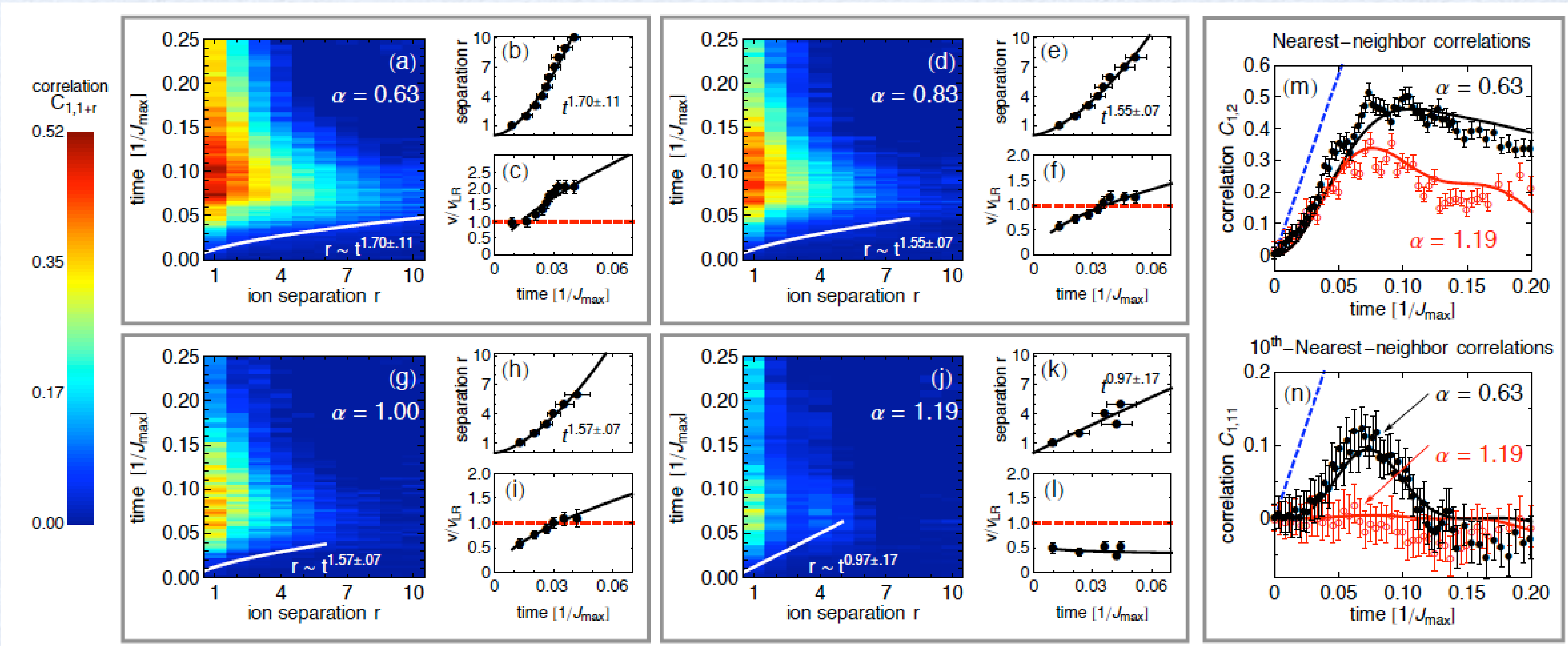


generic initial state: causal region appears for α > D
product initial state: causal region appears for α > D/2

Ion-Trap-Experiments

Interactions ~ $1/r^\alpha$

[P. Richerme et al., Nature **511**,198 (2014)]

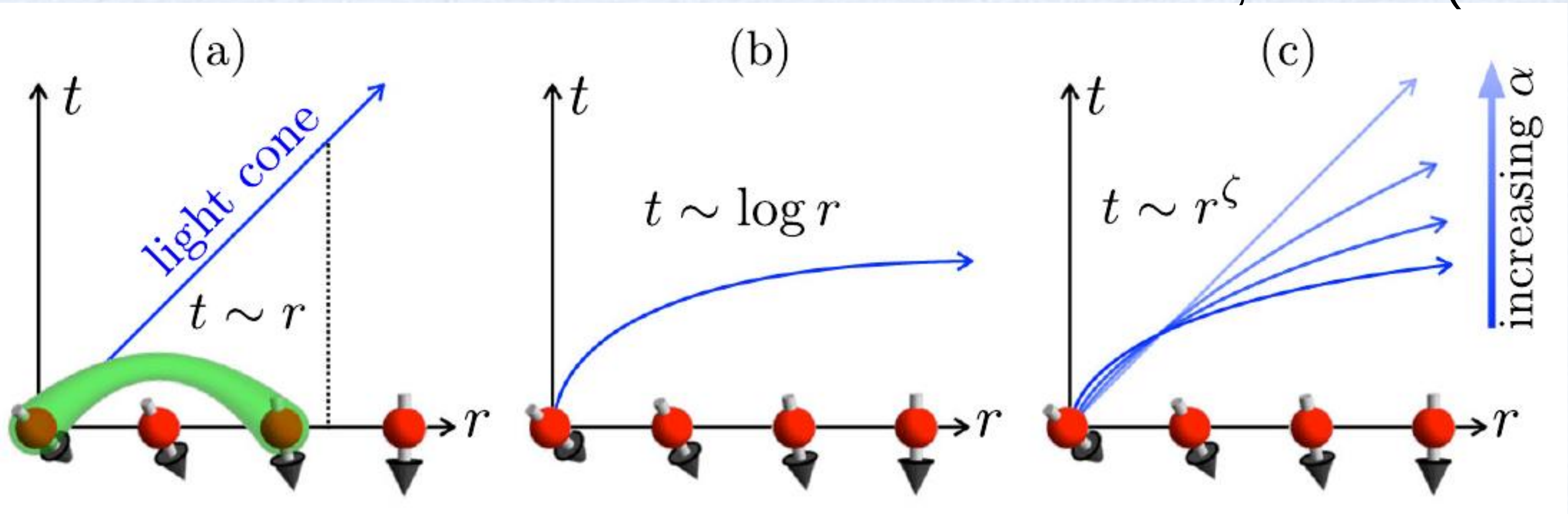


Not a linear ‘region of causality’, but curved!

Algebraic bounds for causality?

Proposed behaviours:

Lieb-Robinson Hastings-Koma Foss-Feig et al.
PRL **114**, 157201 (2015)



$$\mathscr{C}_r(t) \sim \exp[vt - r/t^{\gamma}] + \frac{t^{\alpha(1+\gamma)}}{r^{\alpha}}$$

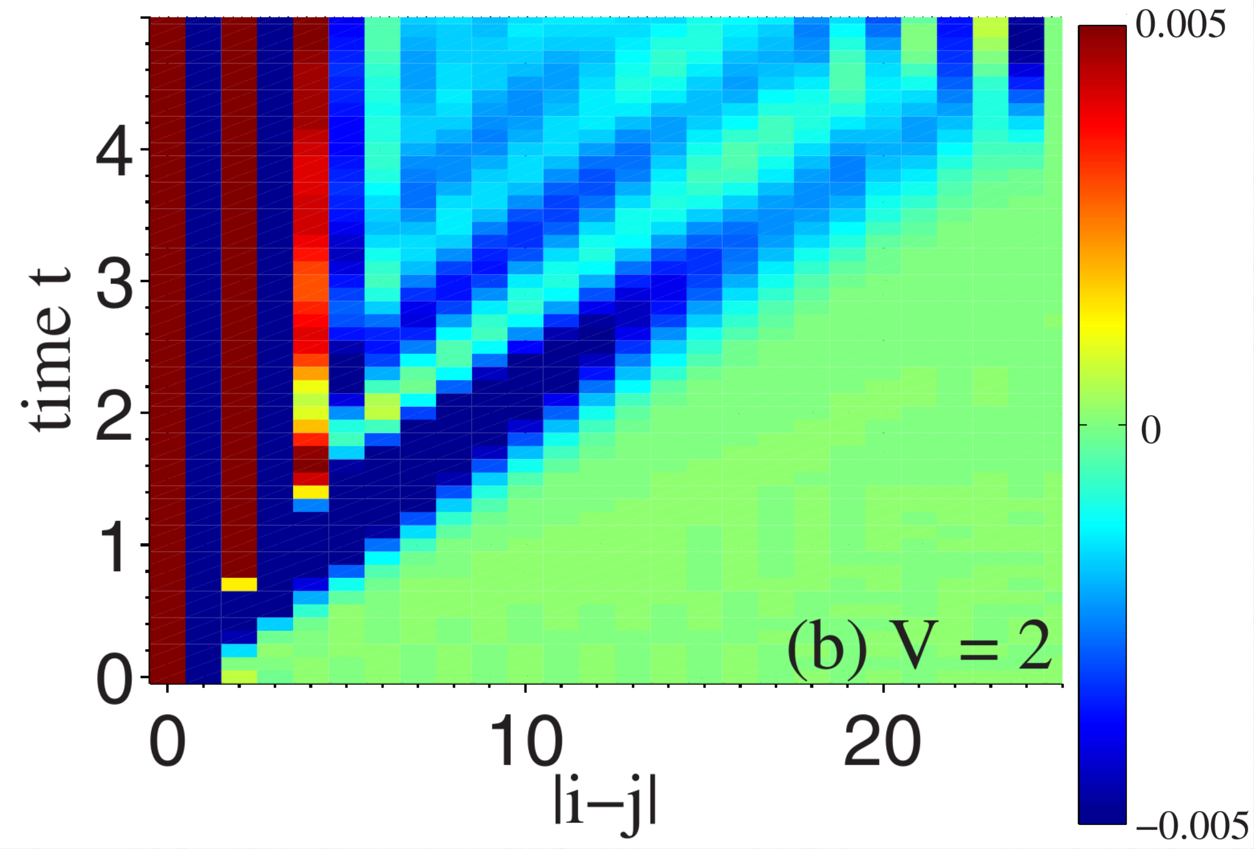
- α>2D: *algebraic* shape of the light-cone rather than logarithmic
- Becomes increasingly linear as α grows

Conclusions & Outlook

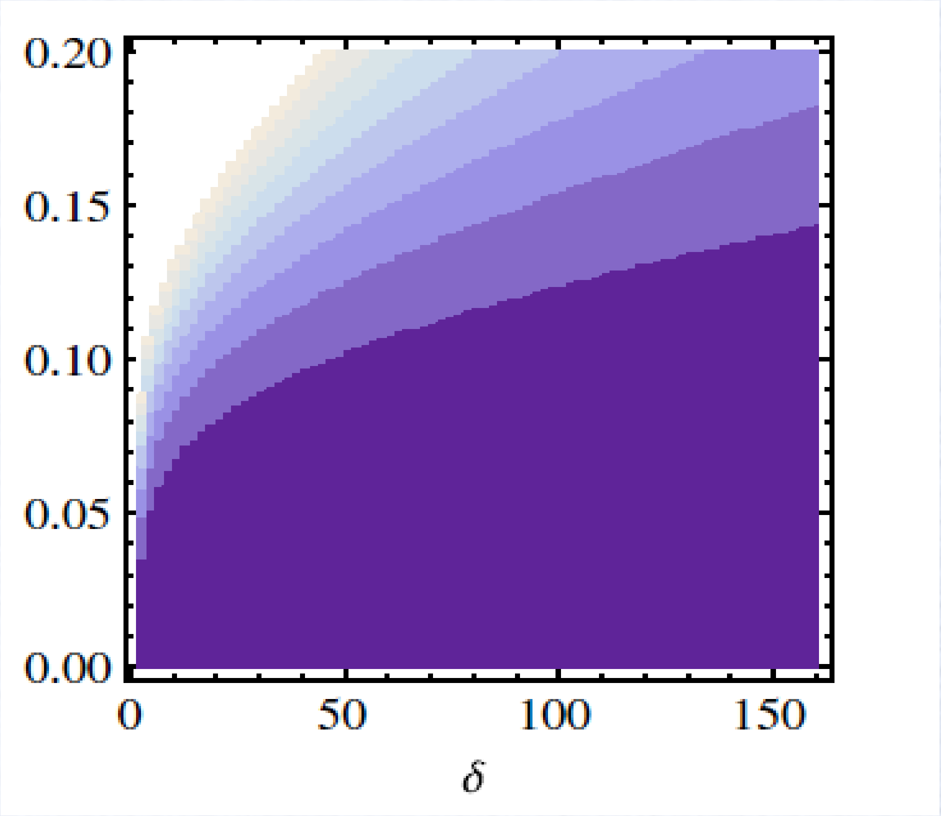
Light cone effect:

Short range interactions

long range interactions



α = 3/4



General form of the Lieb-Robinson bound for long-range interactions?