# Hawking radiation in an exactly solvable model of BEC

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A. Parola *et al*., Europhys. Lett. **119** 50002 (2017) - M.Tettamanti *et al*., Phys. Rev. D **99** 045014 (2019)

# Exactly solvable models

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**One dimensional model of Hard Core Bosons (Tonks-Girardeau gas) flowing against an obstacle**

1. Follow the formation dynamics of the sonic horizon
2. Determine the asymptotic stationary quantum state
3. Verify the presence of thermal phonons at the Hawking-Unruh temperature
4. Study correlations between phonons in the upstream/downstream region

*Generic behavior of 1D interacting Bose gases*

S.Giovanazzi PRL 2005

# Take away message

**The solution is obtained by use of many body methods only**

- No reference to analogue gravity/effective theory arguments
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**The thermal nature of the phonon emission is not always achieved**

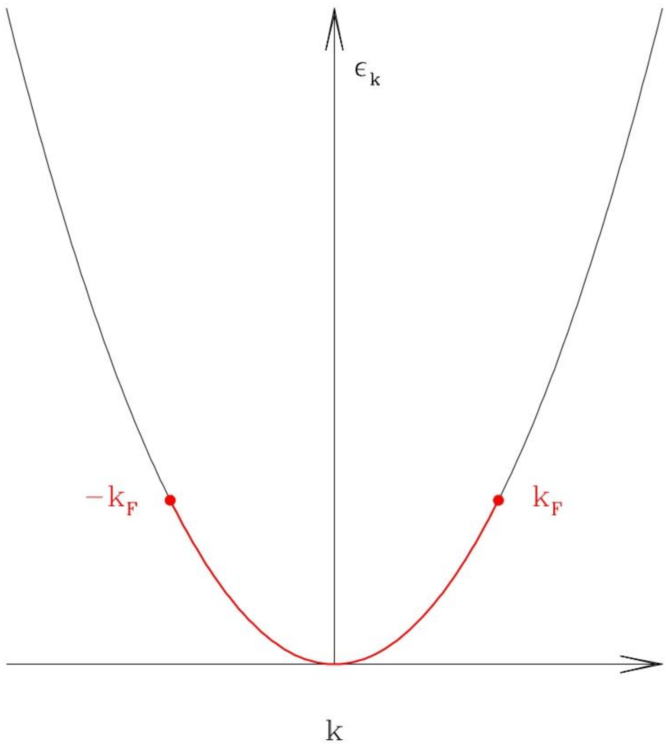
A strict requirement is the decoupling between phonon dynamics (the quantum field) and the matter flow (the analogue metric)

# The model

*A one dimensional gas of hard core bosons has the same energy spectrum and density correlations of a one dimensional Fermi gas (Girardeau 1960)*

1. The ground state of a free Fermi gas is obtained filling the energy levels with

$$|k| < k_F$$



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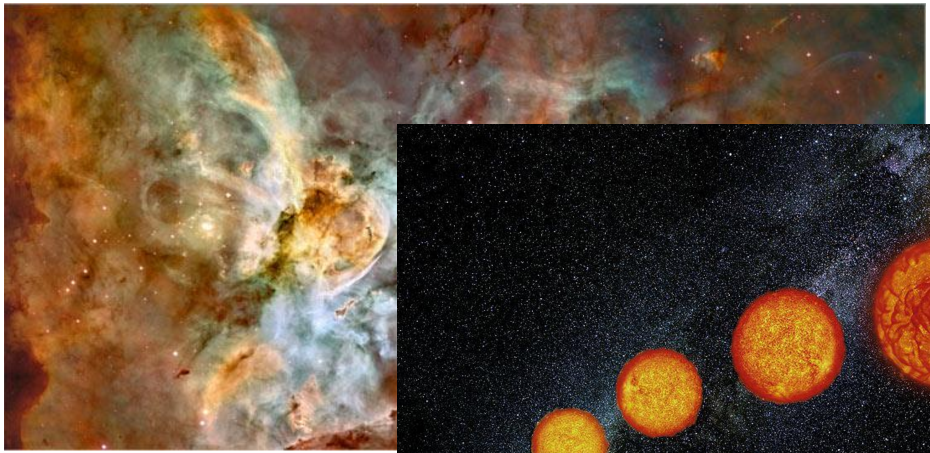

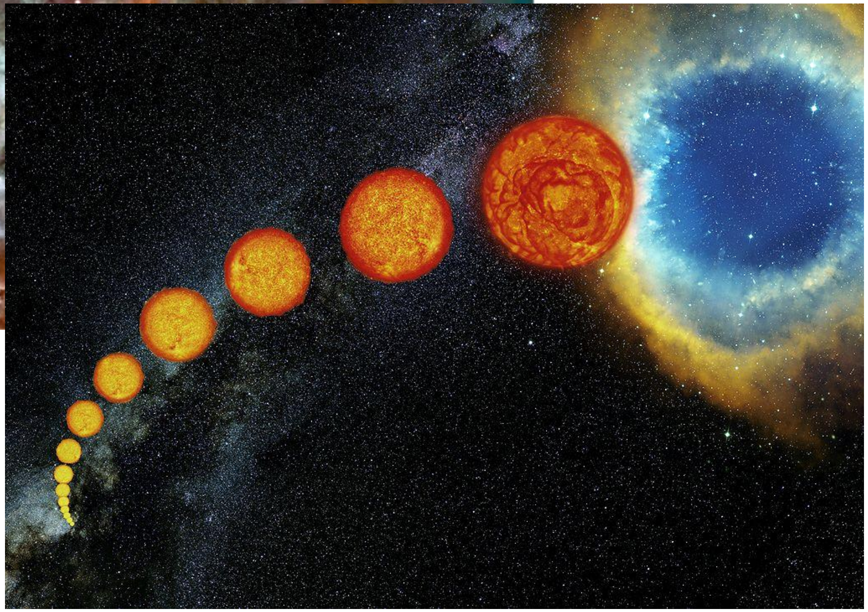

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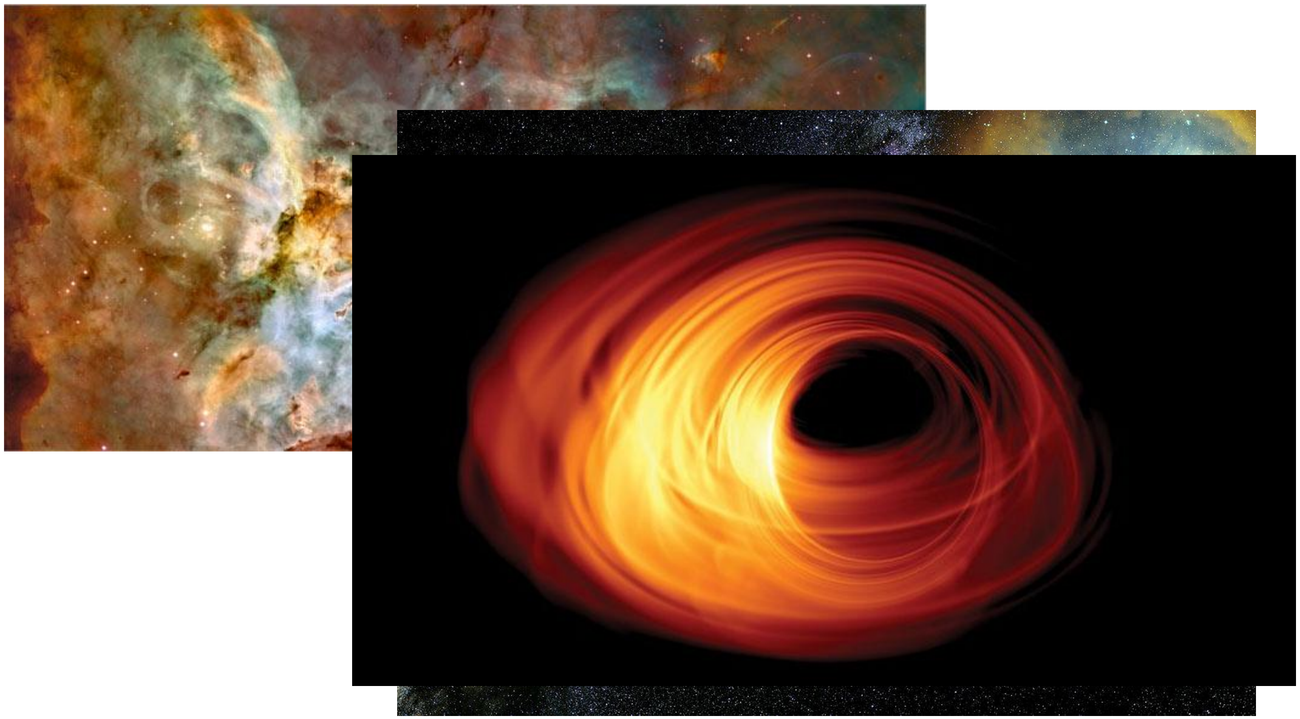

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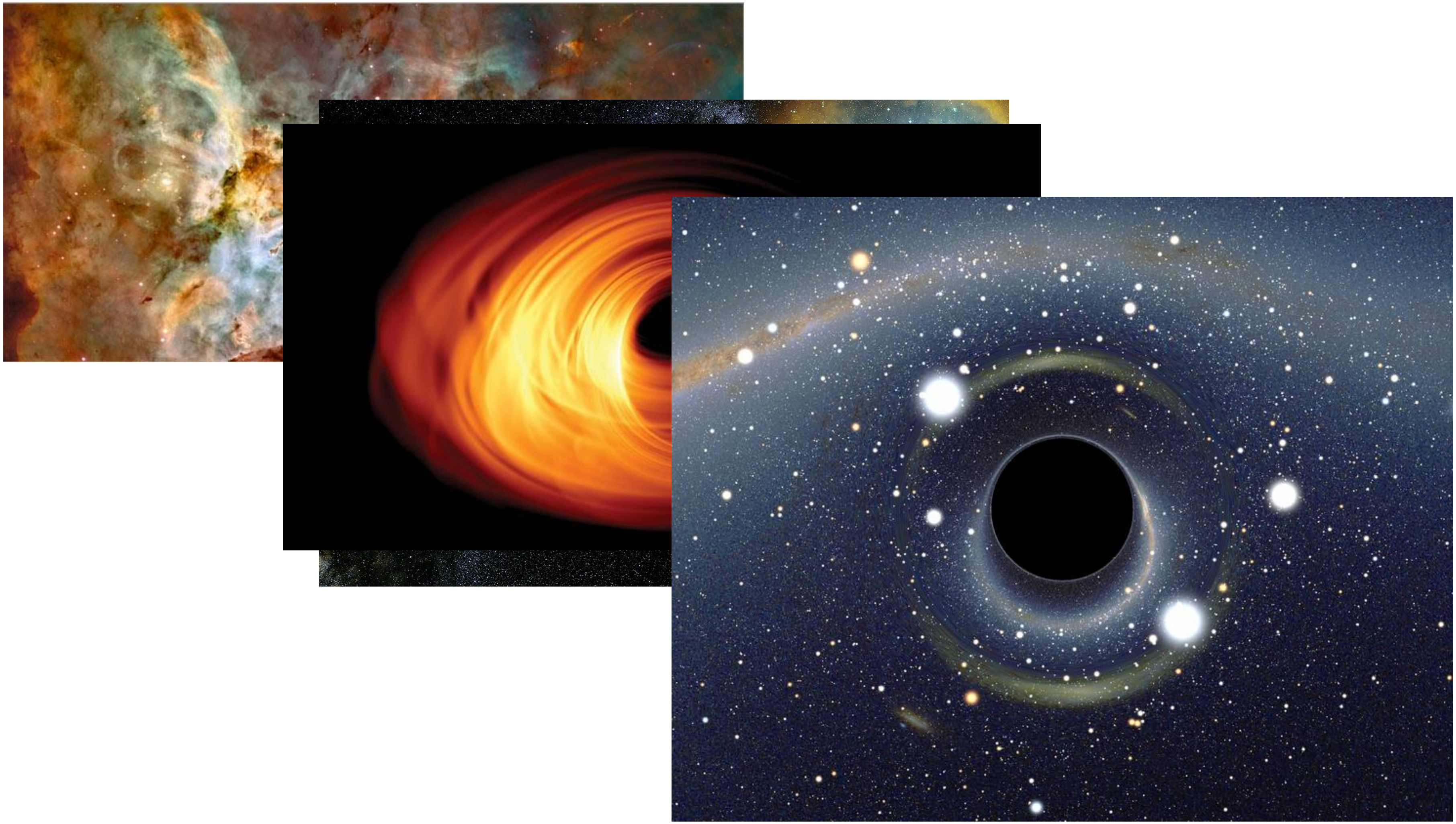

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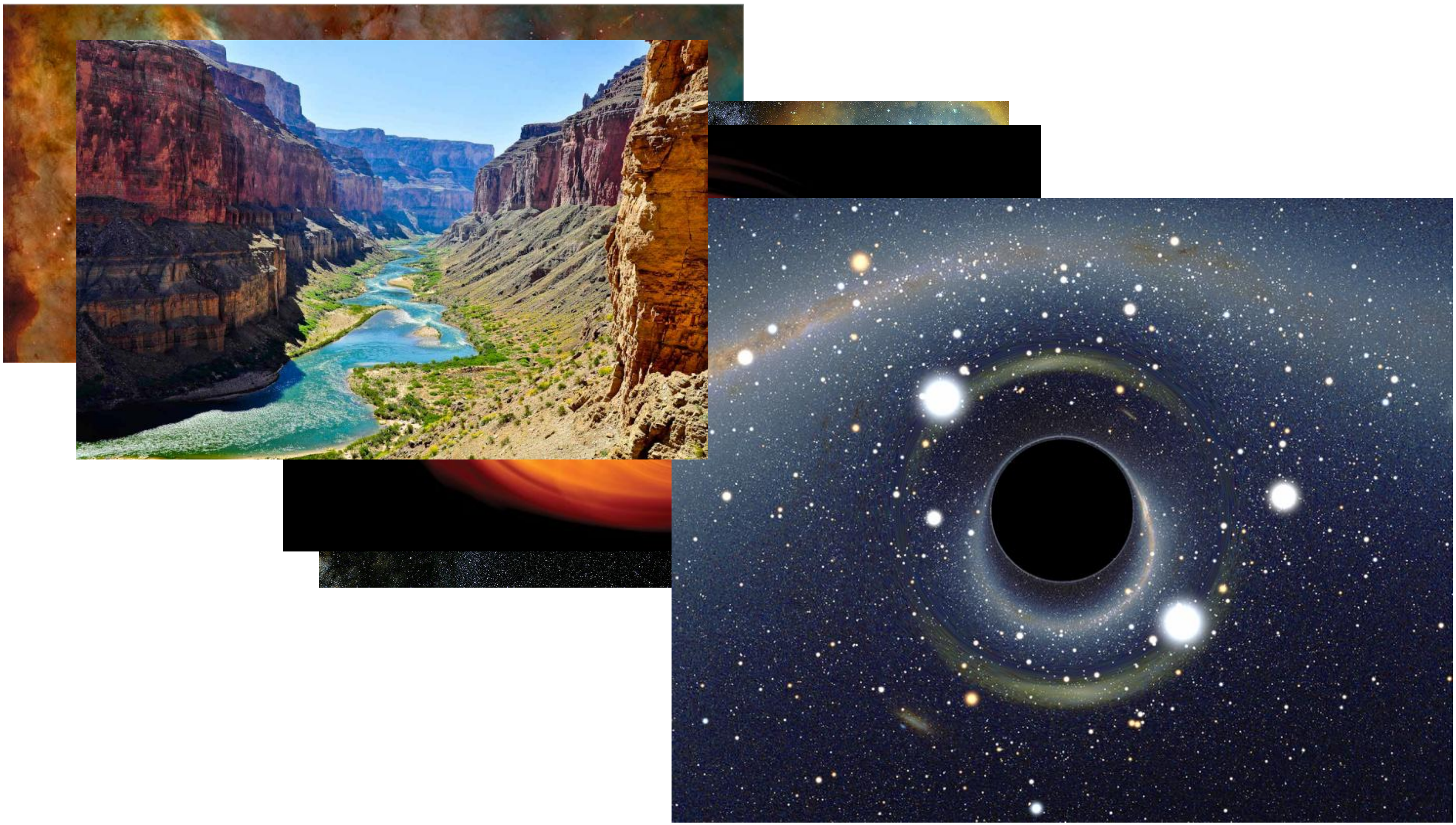

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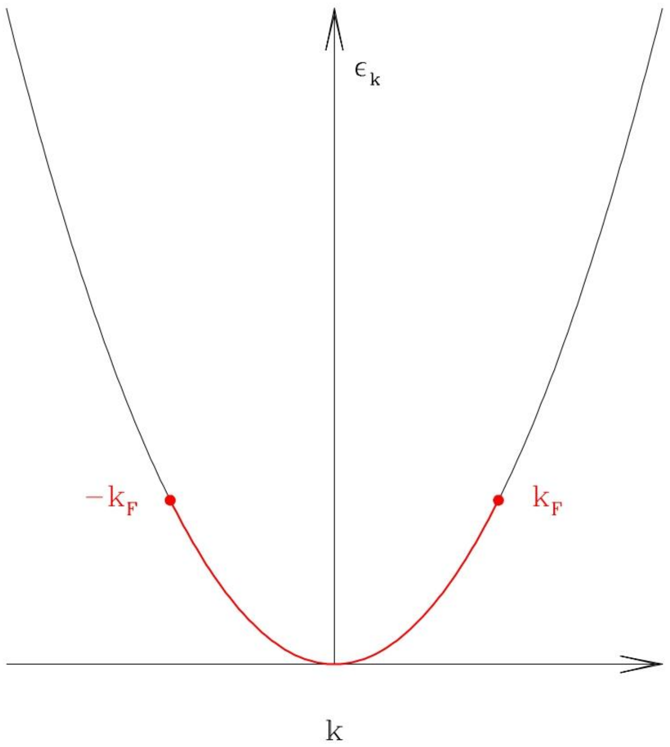

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1. The ground state of a free Fermi gas is obtained filling the energy levels with

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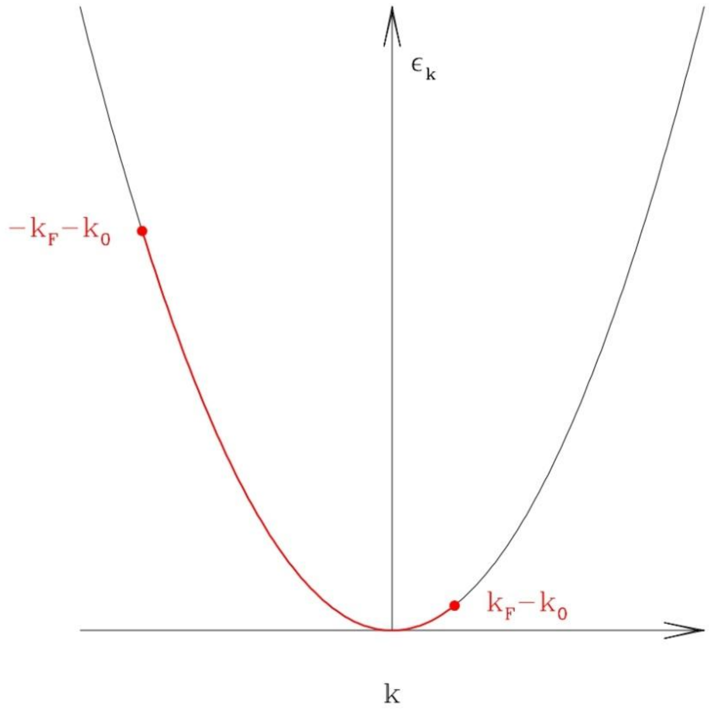
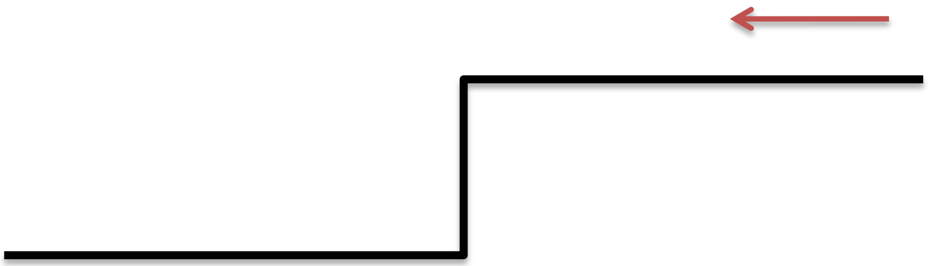
2. Setting the Fermi gas in motion shifts the Fermi points by $-k_0$
This state is clearly stationary



# Quantum Quench: Waterfall potential

Perform a *quantum quench* by switching on an external potential
eg. a sharp step (waterfall) potential

$$V(x) = \begin{cases} 0 & \text{for} \qquad x < 0 \\ \frac{\hbar^2 Q^2}{2m} & \text{for} \qquad x > 0 \end{cases}$$



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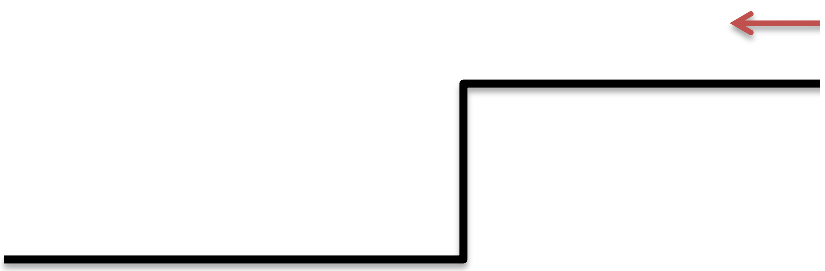

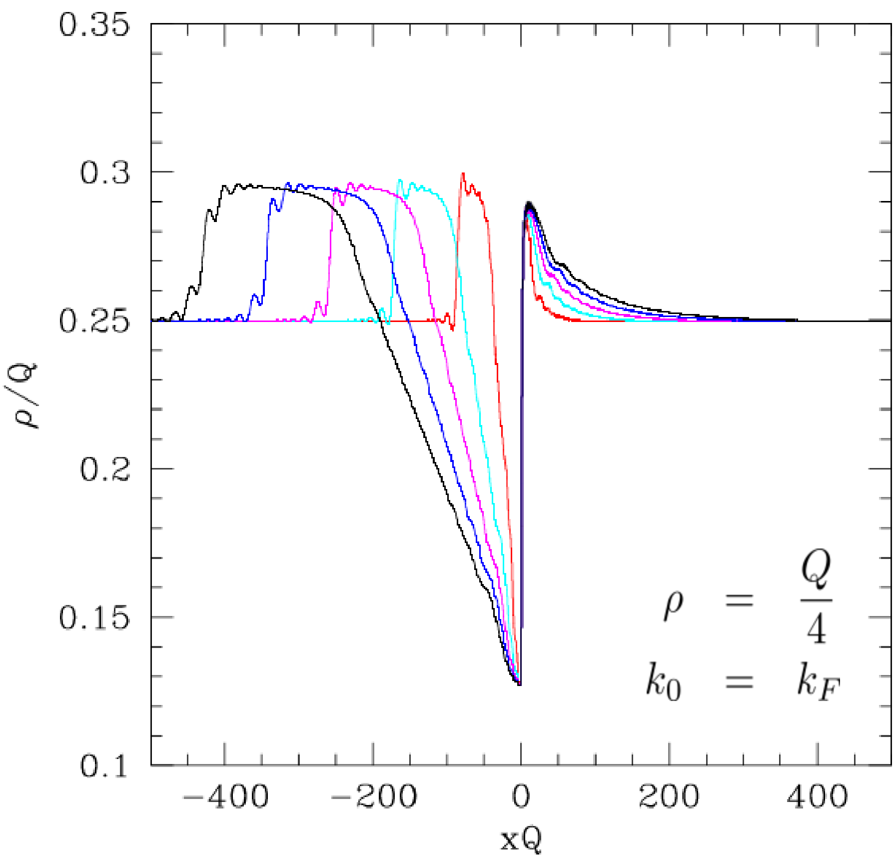


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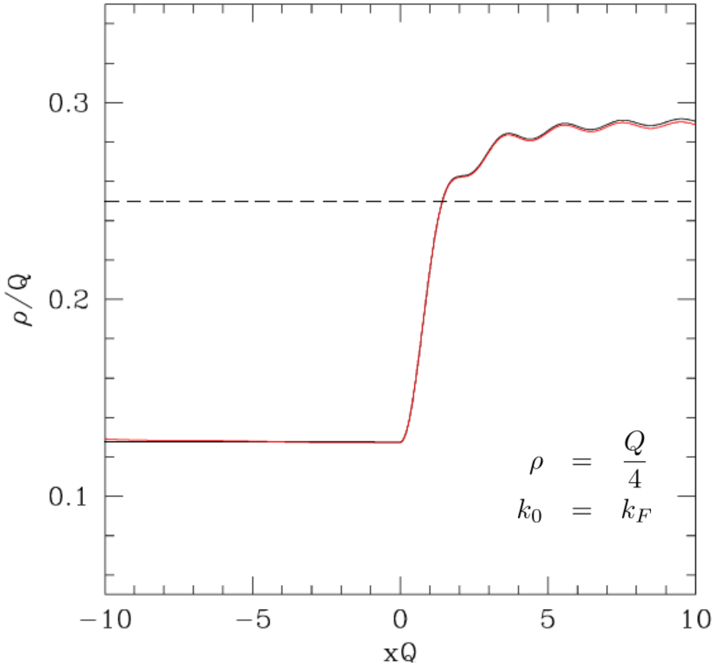
# Stationary state

After a transient, a stationary state is reached (starting from the region near the step).
*The stationary (pure) state is built out of the scattering states of the step potential in the interval* $-k_F - k_0 < k < k_F - k_0$



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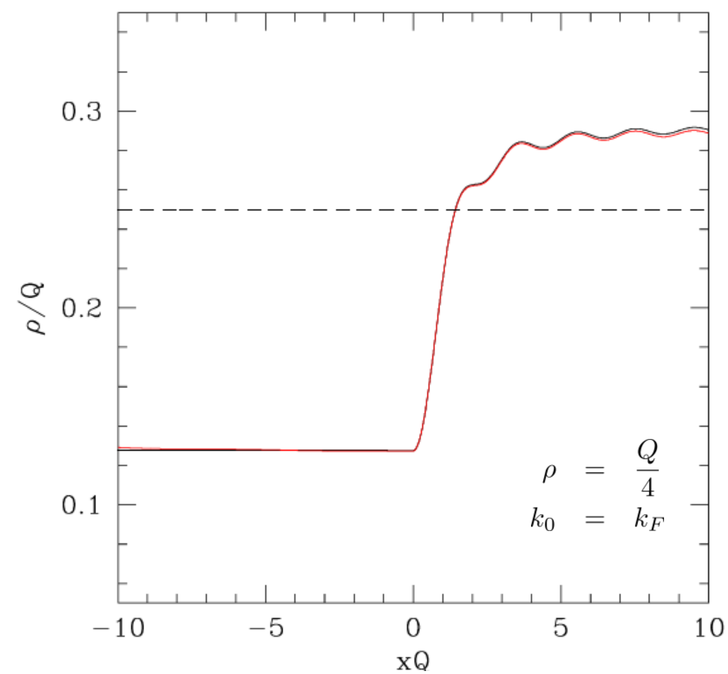
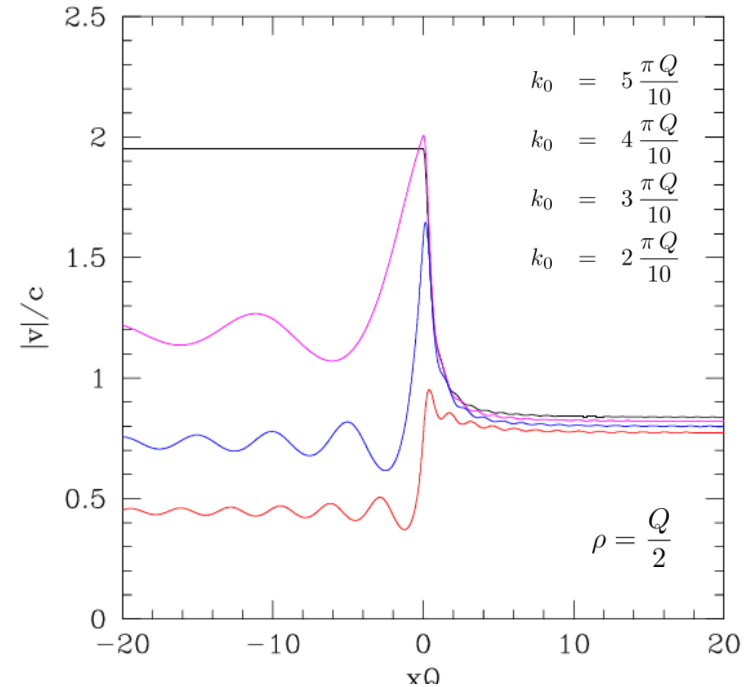



A sonic horizon is formed for sufficiently high initial velocity

# Absence of Thermalization

Let's take for simplicity $k_0 = k_F$ (only left moving fermions)
Far from the waterfall in the upstream region $x \to \infty$ each single particle wave-function has the form

$$\psi_k(x) = \frac{1}{\sqrt{2\pi}} \left[ e^{ikx} + R_k \, e^{-ikx} \right] \qquad \text{for} \qquad k < 0$$

The local density is then given by

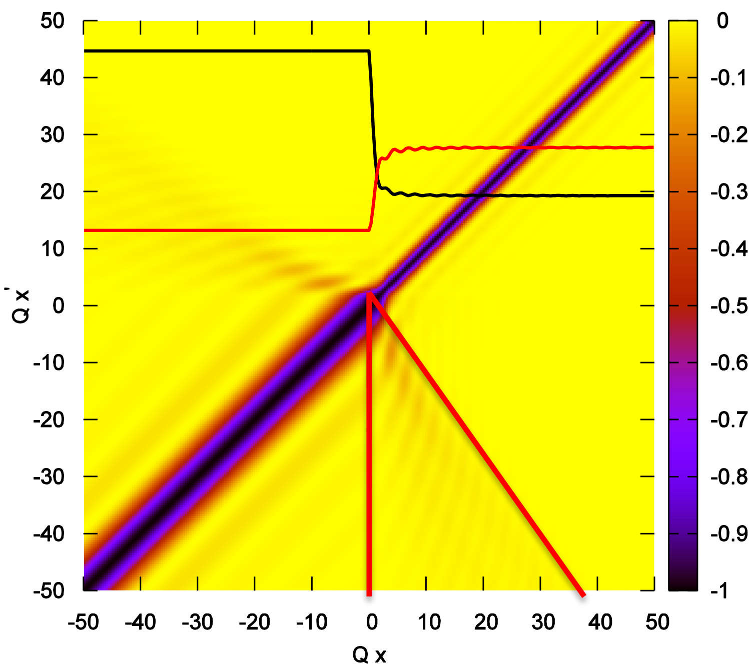
$$\rho(x) = \int_{-k_F - k_0}^{0} dk \, |\psi_k(x)|^2 = \int_{-k_F - k_0}^{0} \frac{dk}{2\pi} + \int_{0}^{k_F + k_0} \frac{dk}{2\pi} \, |R_k|^2$$

which corresponds to a *fictitious* momentum distribution

$$f(k) = \begin{cases} 1 & \text{for} \quad -k_F - k_0 < k < 0 \\ \left[ \sqrt{\frac{k^2}{Q^2} + 1} - \frac{k}{Q} \right]^4 & \text{for} \quad 0 < k < k_F + k_0 \end{cases}$$

Quasi-particles are excited but the tail is not thermal

# Density correlations




$$h(x,x') = \frac{< n(x)n(x') >}{< n(x) >< n(x') >} - 1$$

Correlations between the subsonic and supersonic regions are present but they appear as a *band* rather than a sharp line as expected

$$k_0 = k_F = \frac{\pi}{5} Q$$

# What is going wrong ?

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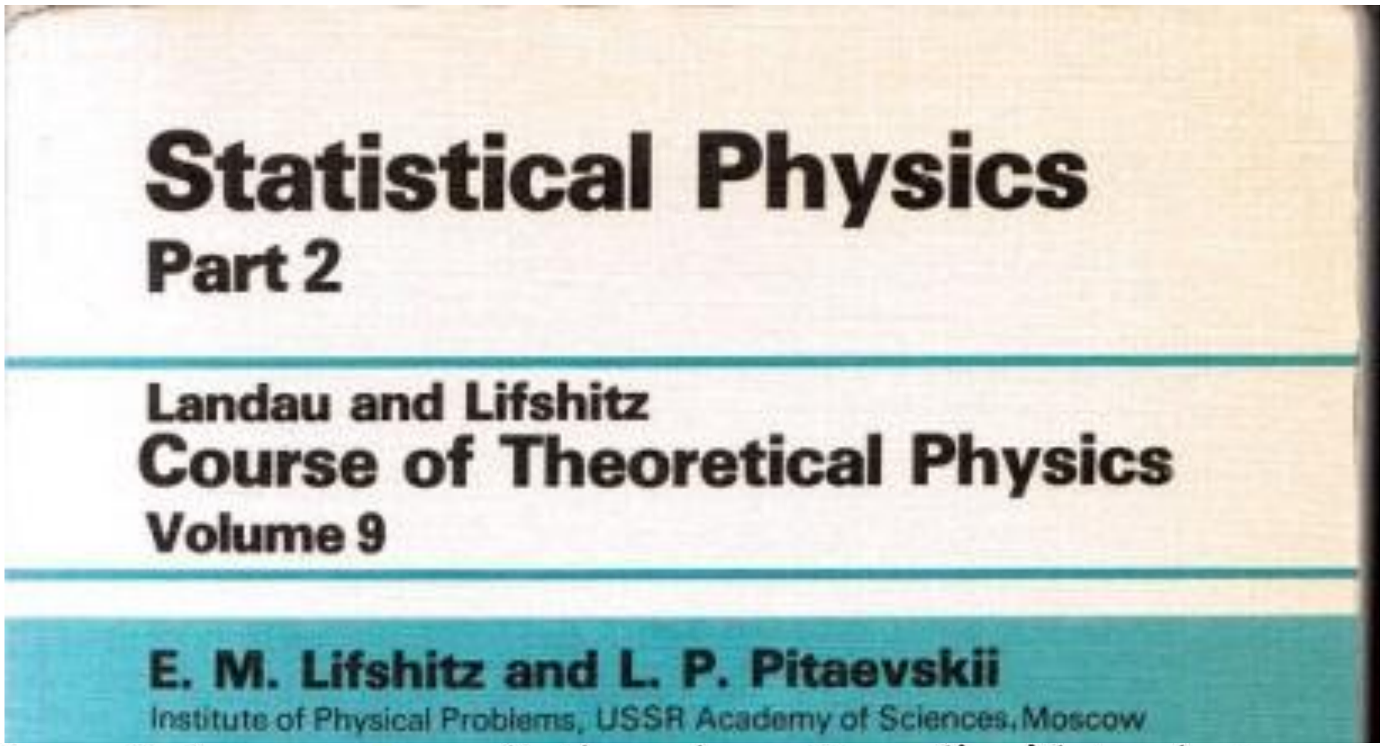



The number of elementary excitations in a Bose liquid tends to zero as $T \to 0$, and at low temperatures, when their density is sufficiently small, the quasi-particles may be regarded as not interacting with one another, i.e. as forming an ideal Bose gas.

# What is going wrong ?

Quasi-particles behave as a free quantum field only at low density/low energy.

For the waterfall potential the density of excited quasi-particles is not small

**The gravitational analogy breaks down**

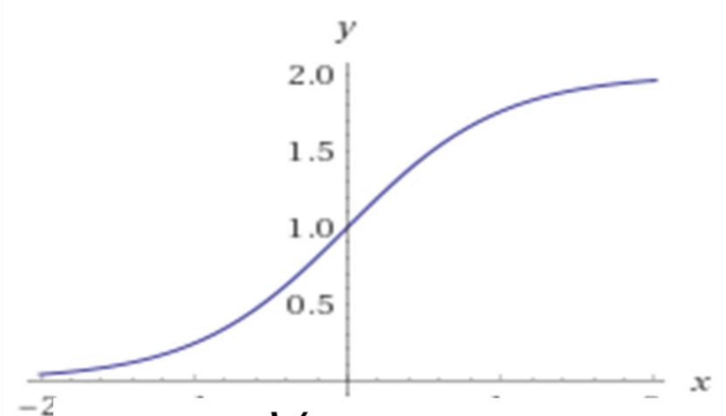
The

$T \to 0$

quasi-

formi

# Smooth Step




$$V(x) = \frac{V_0}{2}\left[1 + \tanh(\alpha x)\right]$$

We can fix this problem by taking a sufficiently smooth potential

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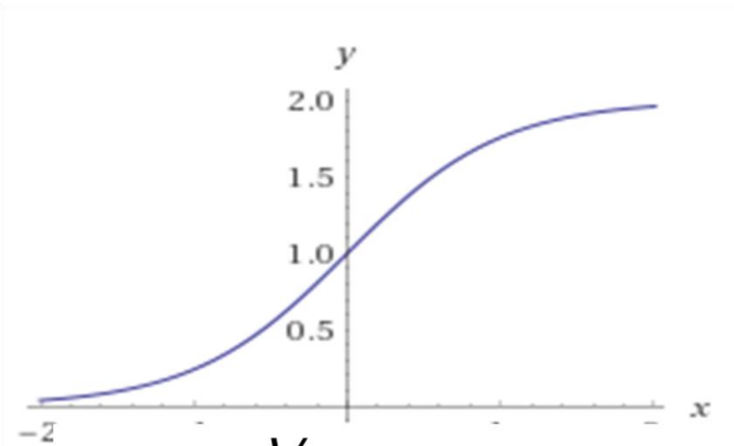


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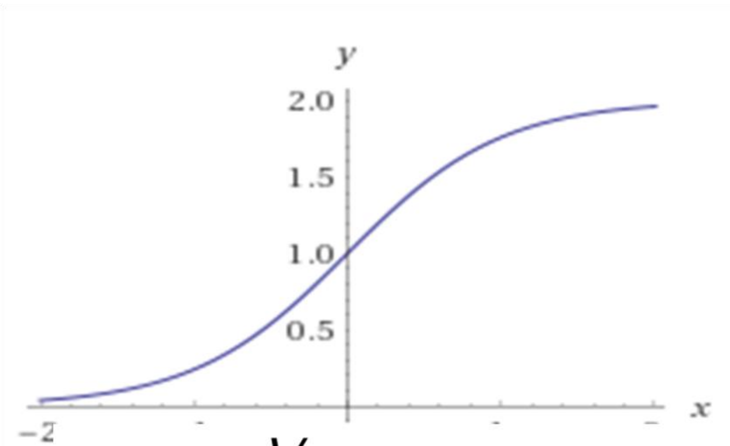


$$V(x) = \frac{V_0}{2}\left[1 + \tanh(\alpha x)\right] \longrightarrow \alpha \to 0$$

We can fix this problem by taking a sufficiently smooth potential ↓ $\alpha \to 0$

1. We recover a *fictitious* momentum distribution but the tail is still not thermal.

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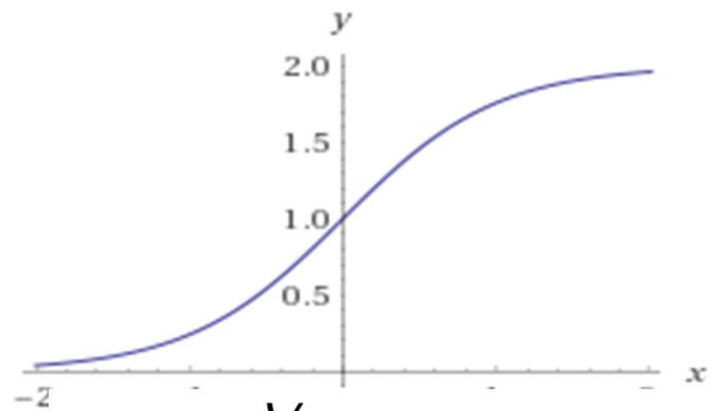


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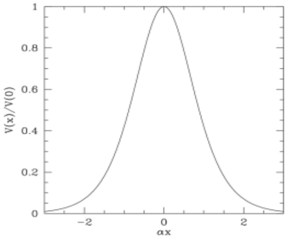
We can fix this problem by taking a sufficiently smooth potential

1. We recover a *fictitious* momentum distribution but the tail is still not thermal.
2. Furthermore, under a certain threshold the sonic horizon disappears

# Smooth Barrier

We can change the form of the potential
and study the smooth limit

$$V(x) = \frac{V_0}{\cosh^2(\alpha x)} \qquad \alpha \to 0$$



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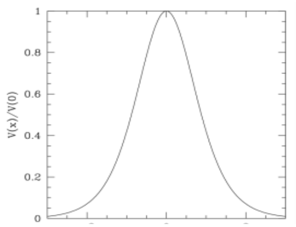


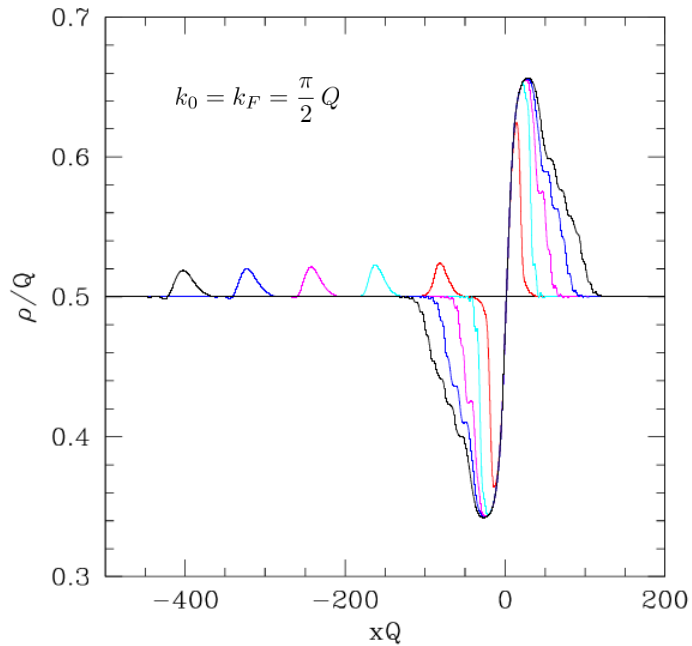


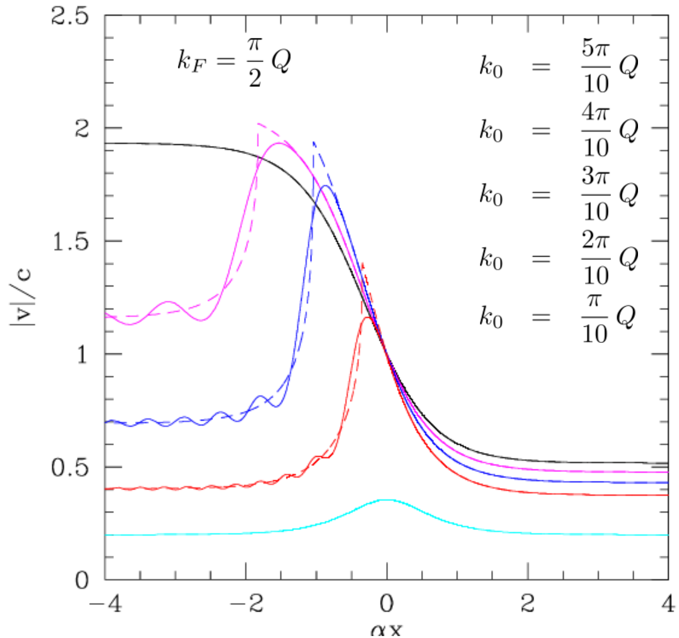

# Thermalization !

In the "Minkowski" (subsonic) region at $x \to \infty$
the Fermi gas is described by the effective distribution

$$f(k) = \begin{cases} 1 & \text{for} \quad -k_F - k_0 < k < 0 \\ |R_k|^2 & \text{for} \quad 0 < k < k_F + k_0 \end{cases} \qquad \text{with} \qquad |R_k|^2 = \frac{1}{e^{\frac{\alpha}{2\pi}(k-Q)} + 1}$$

**Thermal equilibrium distribution (only if a horizon is present!)**

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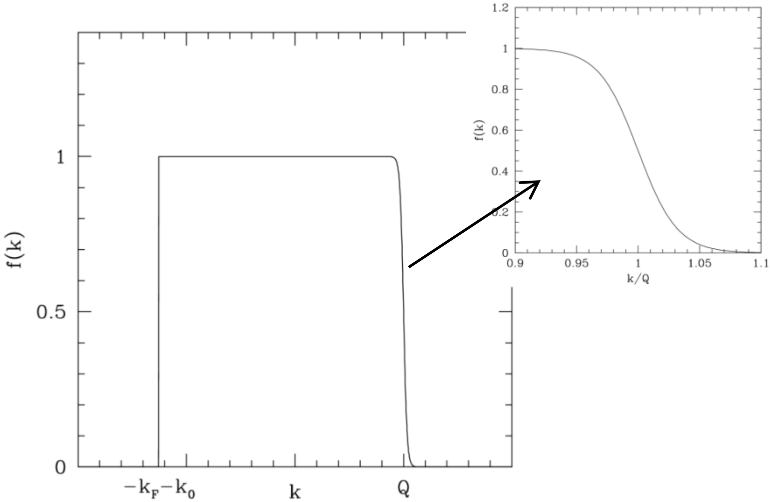
**Thermal equilibrium distribution (only if a horizon is present!)**

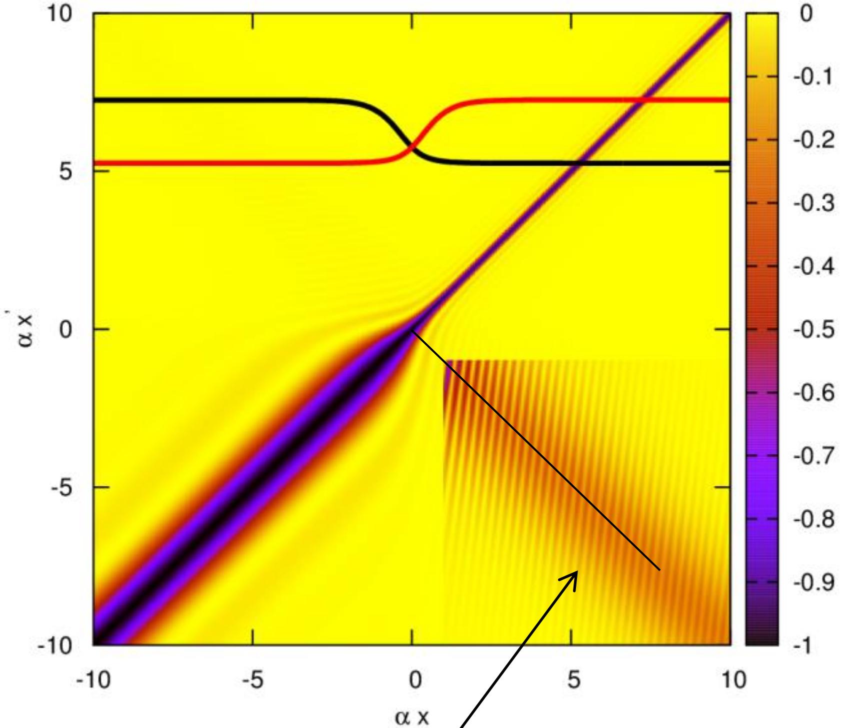
$$k_B\, T_H = \alpha \frac{\hbar^2\, Q}{2\pi m} = \frac{\hbar\, \kappa}{2\pi}$$

$$\kappa = \frac{\mathrm{d}}{\mathrm{d}x}\,[c(x) - |v(x)|]\Big|_{x=0}$$



# Correlations

$$k_0 = k_F = \frac{\pi}{5} Q$$




Blown up 50 times !

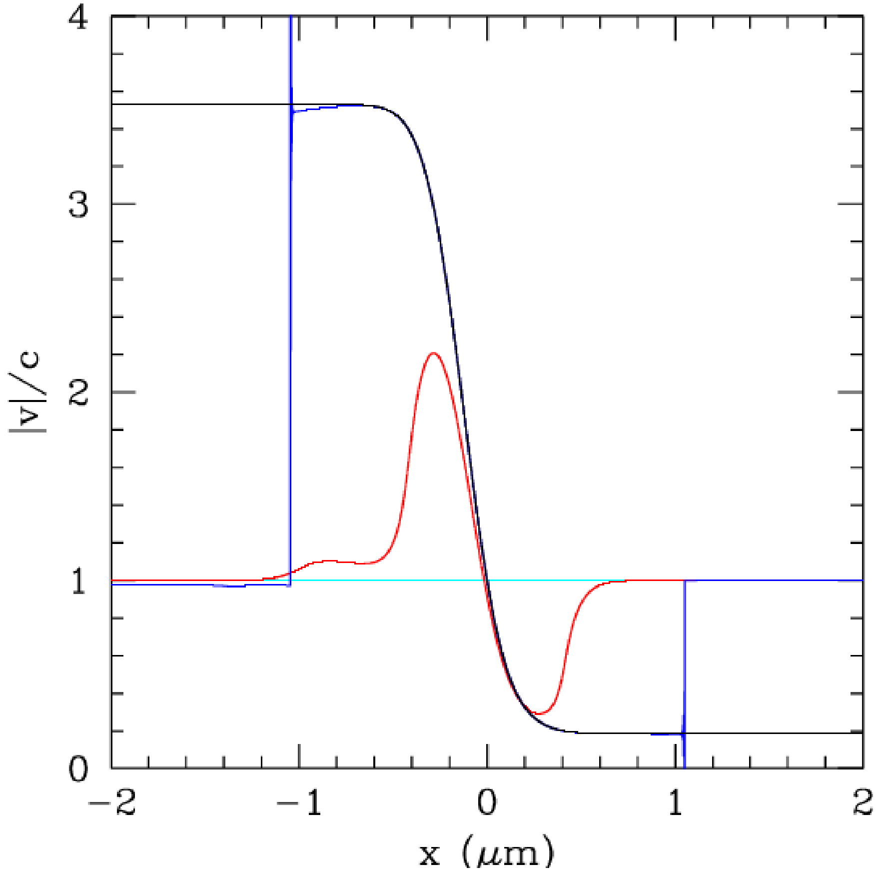
$$h(x,x') = \frac{< n(x)n(x') >}{< n(x) >< n(x') >} - 1$$

$$\propto \left[\frac{\alpha}{\cosh \frac{\alpha}{2}(x+x')}\right]^2$$

For $k_0 = k_F$ (only left moving particles)

$$\frac{|x'|}{x} = \frac{c_L + |v_L|}{c_R - |v_R|} = 1$$

# Experiments



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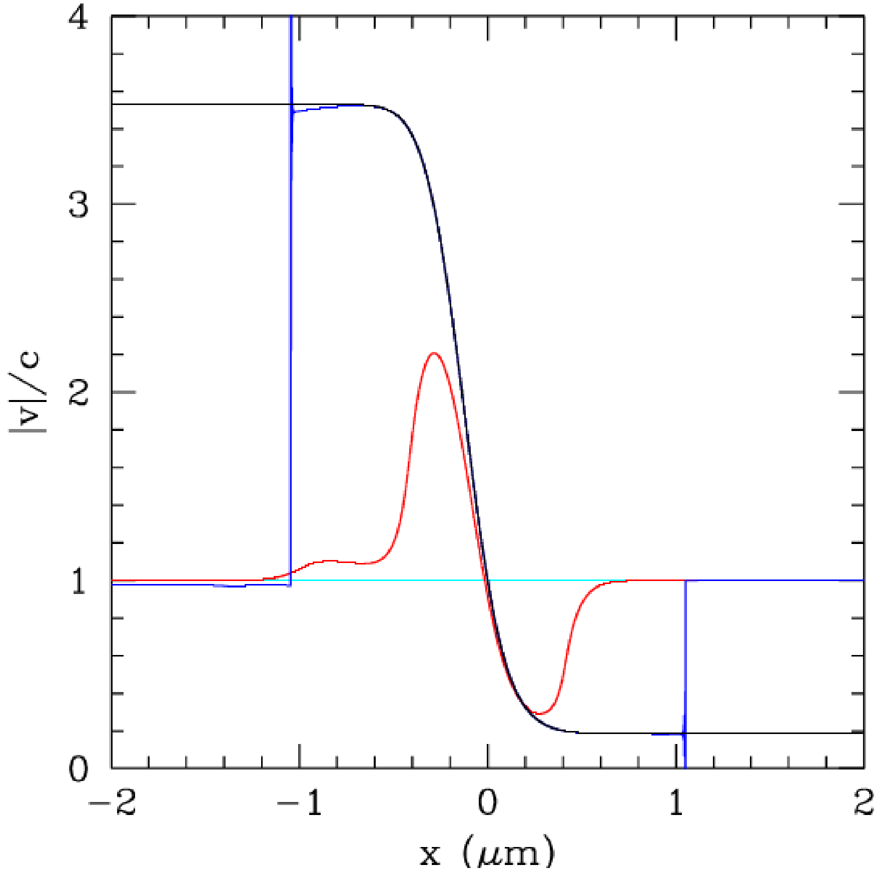


Rubiudium BEC with

1. Cylindrical transverse trap

$a_\perp = 0.25\,\mu\mathrm{m},$

2. “Flat” longitudinal trap

length $L \gtrsim 10\mu\mathrm{m}$

3. Initial density

$\rho_0 = 3.8\,10^3\,\mu\mathrm{m}^{-1}$

4. Initial velocity

$v_0 \sim 18\ \mathrm{mm/s}$

5. Barrier-like obstacle

$V(x) = V_0\, e^{-(\alpha x)^2} \quad V_0 \sim 3.6\,\mu\mathrm{K}$

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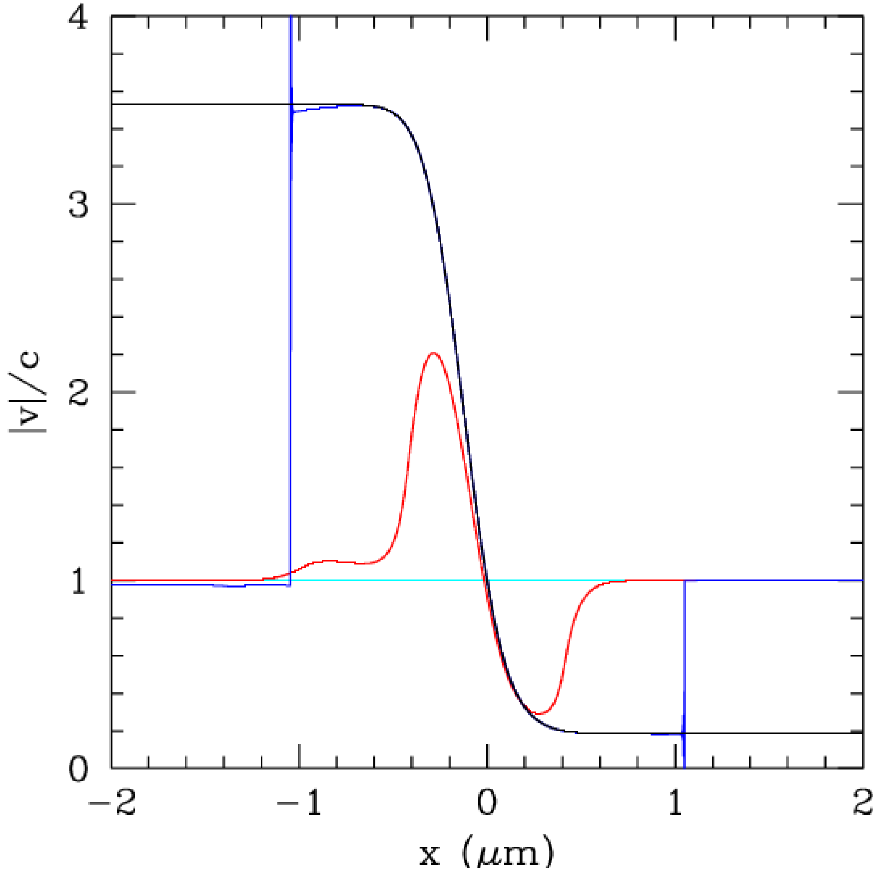


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$t = 0.2\ \text{ms}$ $T_H \sim 100\ \text{nK}$,

# Conclusions

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4. Analysis validated with a semiclassical approach + experimental insight

# Outlooks

Apply the same model to other phenomena

↓

WH dynamics, BH laser effect , Dynamical Casimir effect …

WE'RE OPEN TO SUGGESTIONS!