Analogue Kerr black hole and Penrose process

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Plan

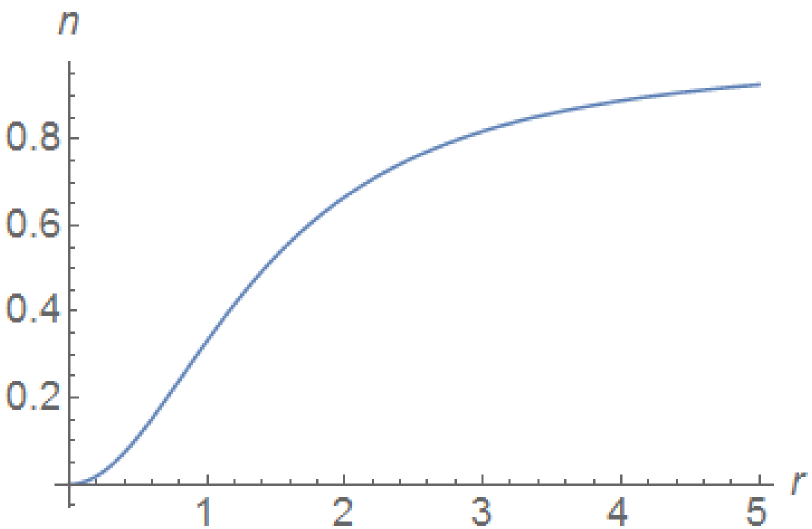
- Analogue physics with quantum vortices
- 2D acoustic Kerr black hole
- Penrose process
- Conclusion

Analogue physics with vortices

- Maxwell 1861:
  - Magnetic field by analogy with fluid dynamics
- Popov 1973:
  - 2D quantum vortices mapped to 3D magnetostatic problem
  - 2+1 relativistic electrodynamics:
    - vortices = charges
    - phonons = photons
- Other works:
  - 2+1 QED with vortices (Arovas, 1997), Maxwell's equations
  - Cosmic strings (Volovik, 1995)
  - Half-vortices as magnetic monopoles (Solnyshkov, 2012)

Quantum vortex

- Quantum fluid described by a wave function
- Phase gradient determines velocity $$\nabla \times \mathbf{v} = 0$$
- Velocity curl zero everywhere
- Except zero-density points – quantum vortices
- Quantized circulation (phase winding) $$\oint \nabla \varphi \cdot \mathbf{dl} = 2\pi n$$



Wave function of a single vortex:

$$\psi \approx \frac{r/\xi}{\sqrt{2+(r/\xi)^2}} e^{i\phi}$$

V.N. Popov, JETP 37, 341 (1973)

Quantum vortex as a relativistic charge

- Action for a vortex = action for a relativistic charge

$$S = -m_0 c \int ds + \int \frac{e}{c} \frac{dx_\alpha}{ds} A^\alpha ds$$

- Relativistic particle in an electromagnetic field F

$$\frac{d^2 x^\mu}{ds^2} = \frac{q}{m_V} F^{\mu\beta} \frac{dx^\alpha}{ds} \eta_{\alpha\beta}$$

- In curved spacetime:

$$\boxed{\frac{d^2 x^\mu}{ds^2} = -\Gamma^\mu_{\alpha\beta} \frac{dx^\alpha}{ds} \frac{dx^\beta}{ds}} + \frac{q}{m_V} F^{\mu\beta} \frac{dx^\alpha}{ds} g_{\alpha\beta}$$

Time-like geodesics

The motion of the vortex is described by the acoustic metric!

Christoffel symbols: $\Gamma^\mu_{\alpha\beta} = 0.5 g^{\mu\nu} (\partial g_{\nu\alpha} / \partial x^\beta + \partial g_{\nu\beta} / \partial x^\alpha - \partial g_{\alpha\beta} / \partial x^\nu)$

Kerr black hole features

- Rotating black hole
- Maximal angular momentum $a/M=1$
- Strong frame dragging
- Two particular surfaces
  - Horizon (light cannot get out)
  - Static limit (light cannot go against the rotation)
- Penrose process (extraction of the rotation energy)
- Ergosphere

PRD 70, 124006 (2004).

Kerr metric for a condensate in 2D

Equatorial plane only!

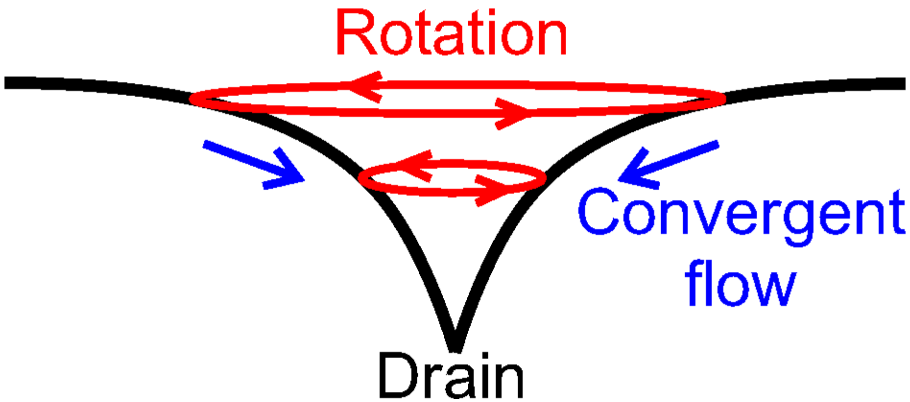
Kerr

$$g_{\mu\nu}^{Kerr} = \begin{pmatrix} -\left(1-\frac{2M}{r}\right) & 0 & -\frac{4aM}{r} \\ 0 & \frac{r^2}{r^2-2Mr+a^2} & 0 \\ -\frac{4aM}{r} & 0 & \left(r^2+a^2+\frac{2a^2M}{r}\right) \end{pmatrix}$$

Condensate

$$g_{\mu\nu} = \frac{mn}{c}\begin{pmatrix} -\left(c^2-v^2\right) & 0 & -2rv_\phi \\ 0 & \left(1-\frac{v_r^2}{c^2}\right)^{-1} & 0 \\ -2rv_\phi & 0 & r^2 \end{pmatrix}$$

Acoustic metric



- Quantized rotation = vortices in the BH
- Need to determine c, v, v_r

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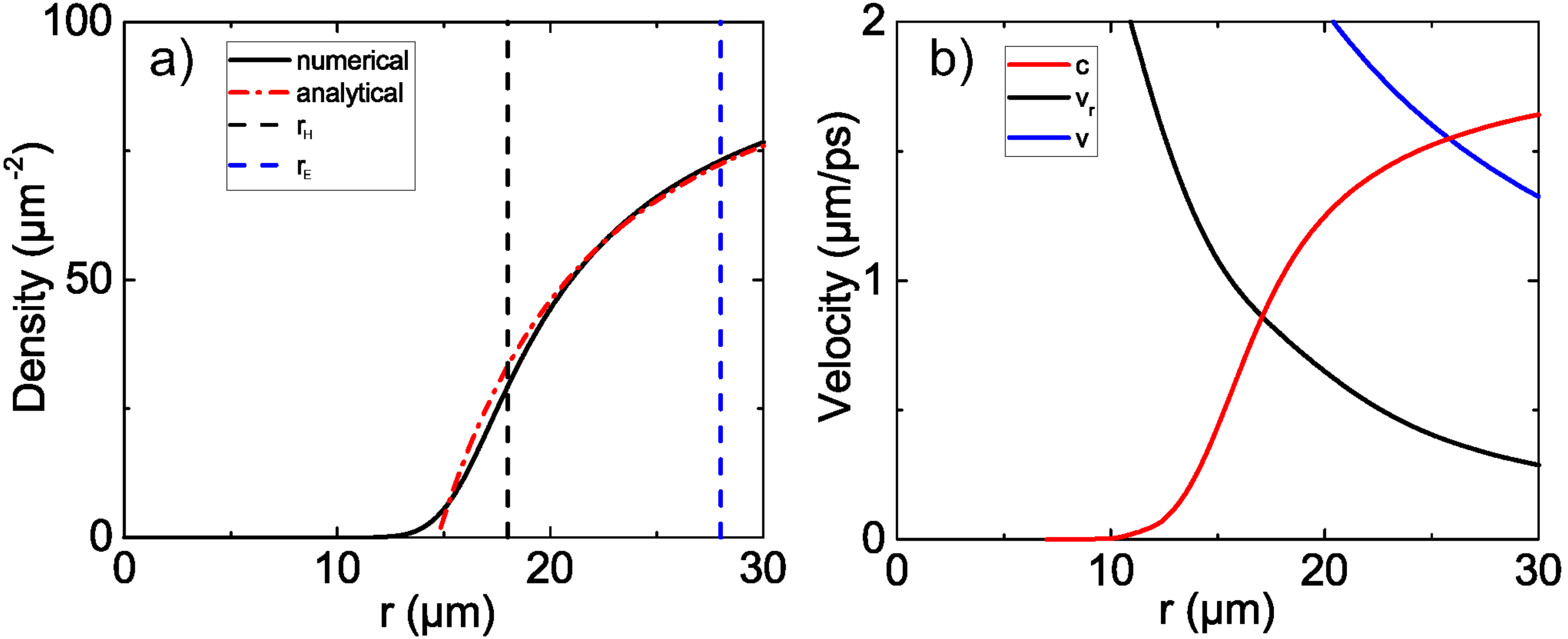
Condensate wavefunction

$$\nabla \times \mathbf{v} \sim \nu \delta(\mathbf{r})$$
$$\nabla \cdot \mathbf{v} \sim -\zeta \delta(\mathbf{r})$$

$$\psi(r,\phi) = \sqrt{n_\infty}\left(1 - \xi^2 \frac{\nu^2 + \zeta^2}{r^2}\right)\exp\left(i\left(\zeta \ln\frac{r}{\xi} + \nu\phi\right)\right)$$

Asymptotic series expansion at large r



Parameters typical for exciton-polariton condensates

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Analogue Kerr BH parameters

- Comparing the metric element g_{rr} we obtain

$$M_{cond} \sim r_H \sim \zeta\xi$$

Mass is controlled by the drain

- Maximal number of vortices in an analogue BH

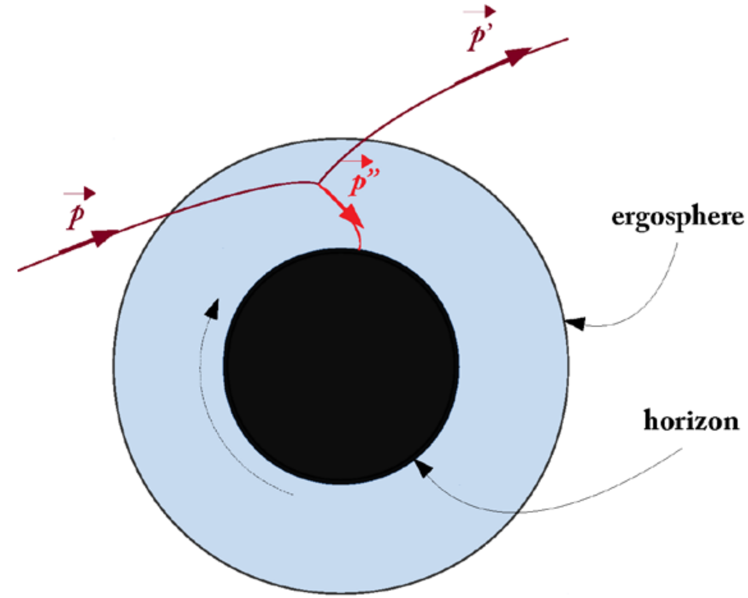
$$\nu_{\max} \sim \frac{r_H}{\xi}, a_{\max} \sim r_H$$

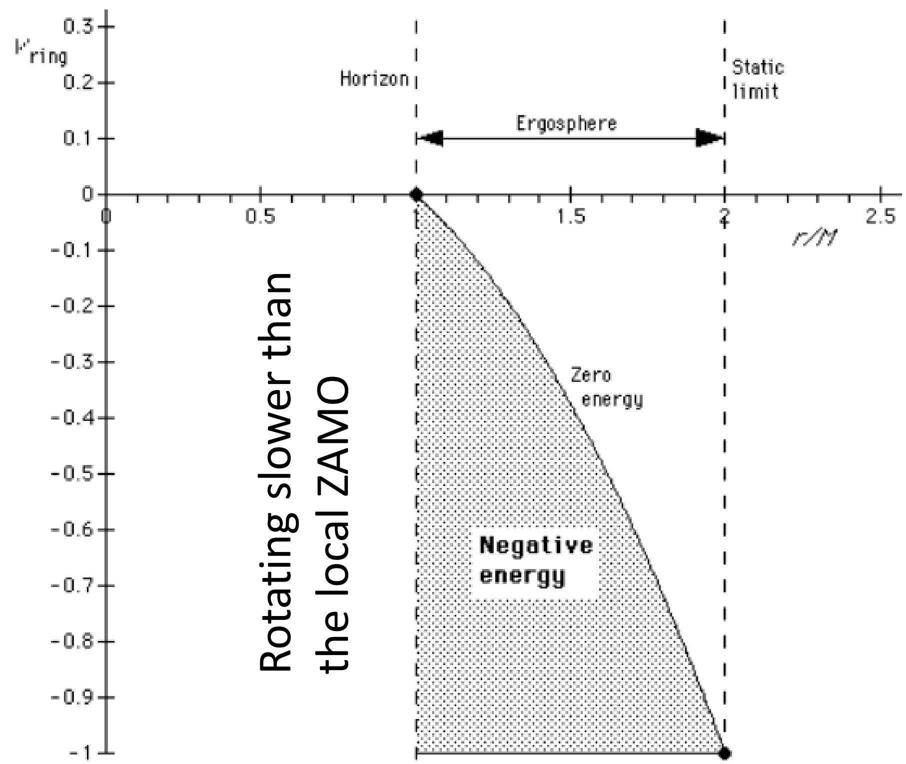
- Maximal angular momentum $\frac{a_{\max}}{M} \sim 1$

Vortices inside the BH are distributed along the horizon

Penrose process

- A particle p falls into the ergosphere
- It splits into two (p' and p'')
- p'' falls into the BH
  - p'' had negative energy!
- p' escapes





- Negative energy does not mean an antiparticle
- One needs more than mc^2 to get the particle away from BH

E.F. Taylor, Exploring Black Holes

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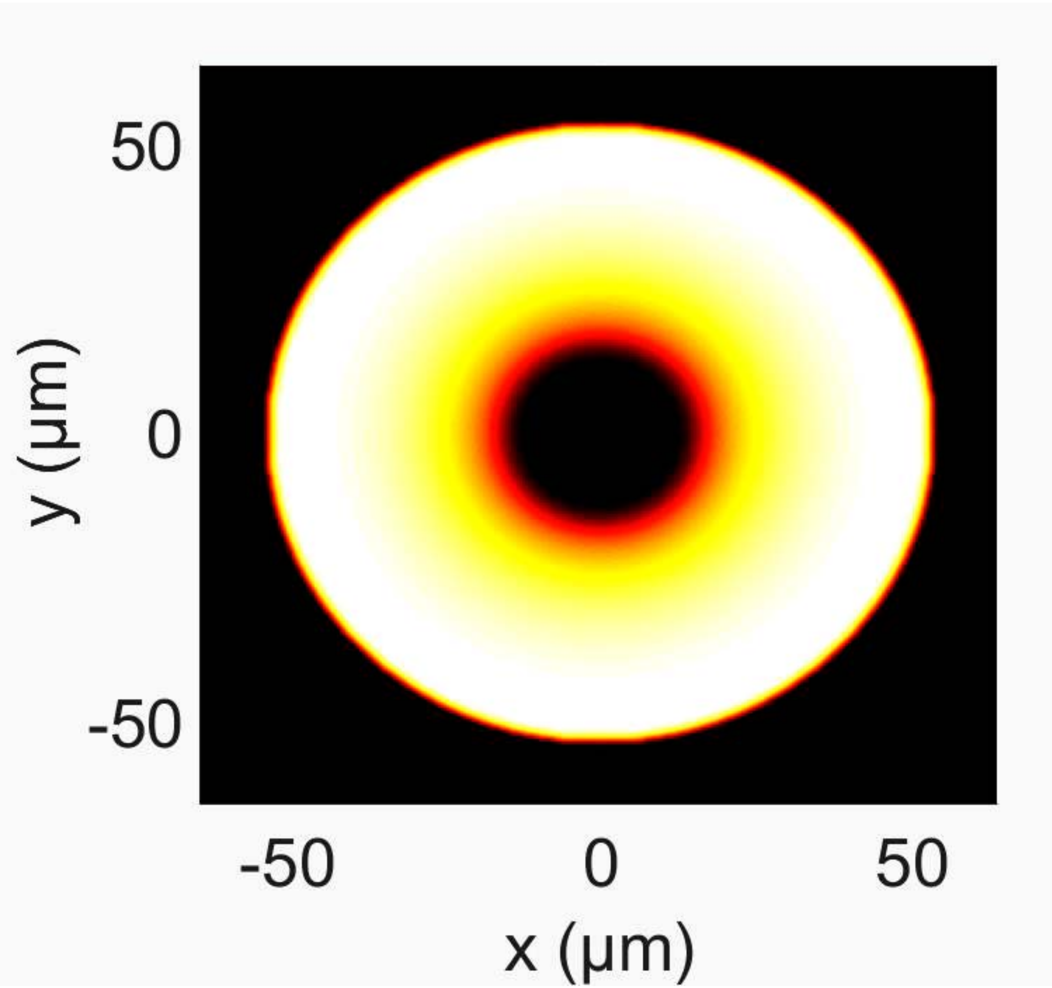
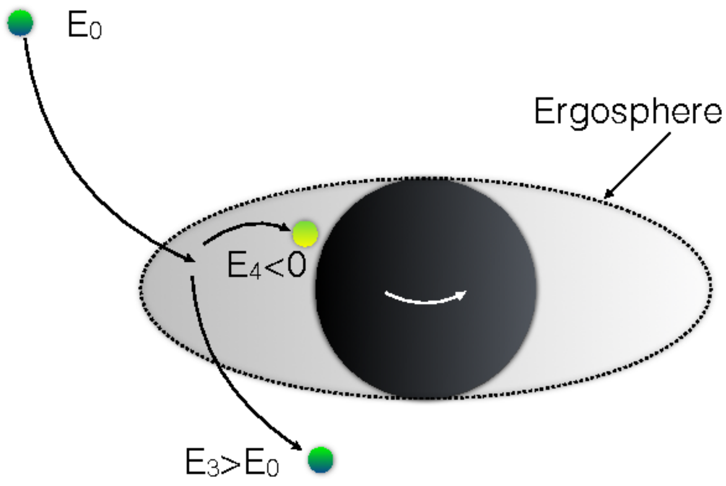
Penrose process





- A vortex-antivortex pair is formed from a density dip
- The anti-vortex falls into BH
- The vortex escapes to infinity
- BH rotation decreases (energy loss)

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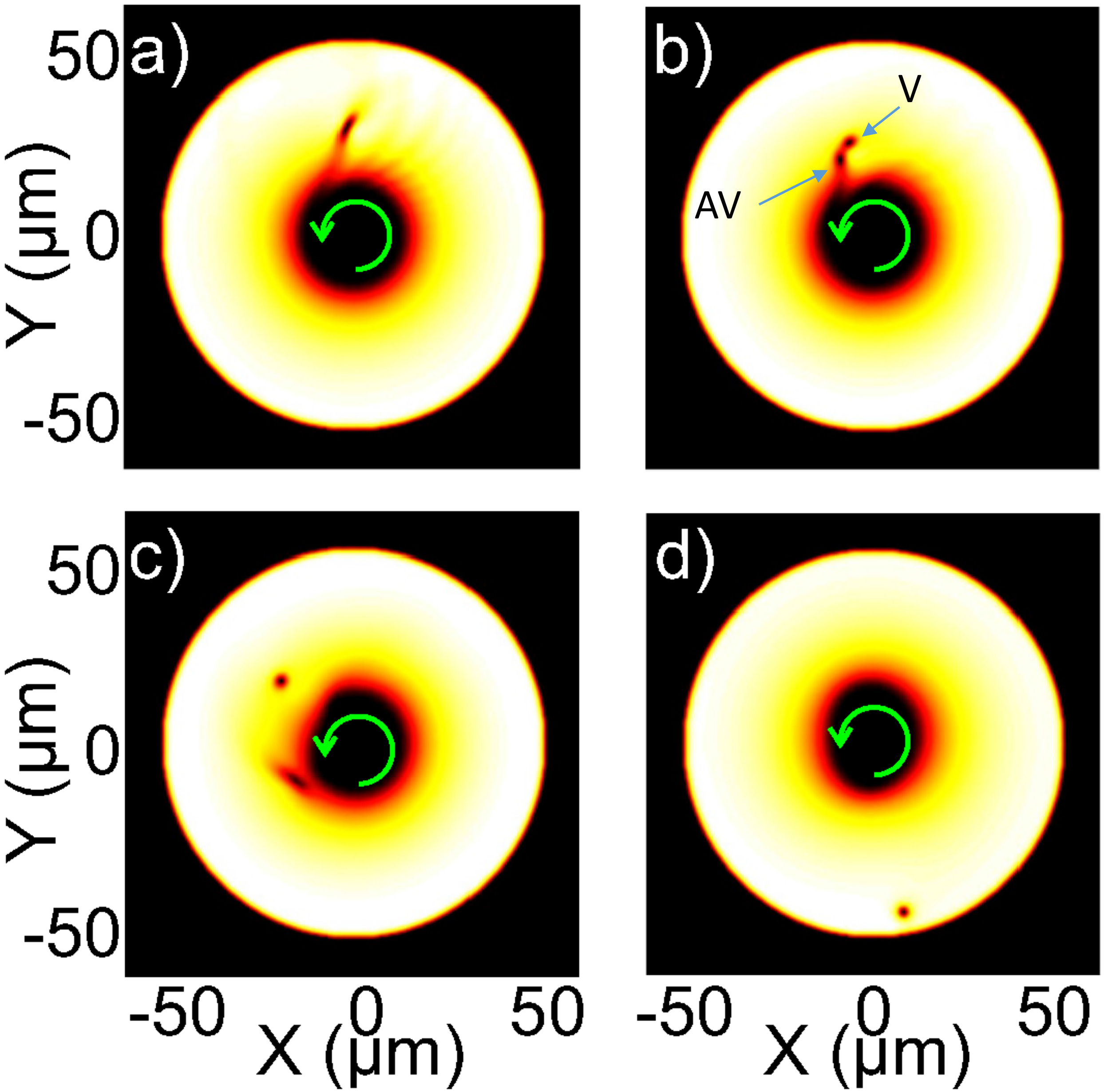
Penrose process: snapshots



- V/AV **interaction** slows the AV
- AV rotates ***slower*** than the condensate
- $E_{AV}<0$
- V gains energy from AV and escapes

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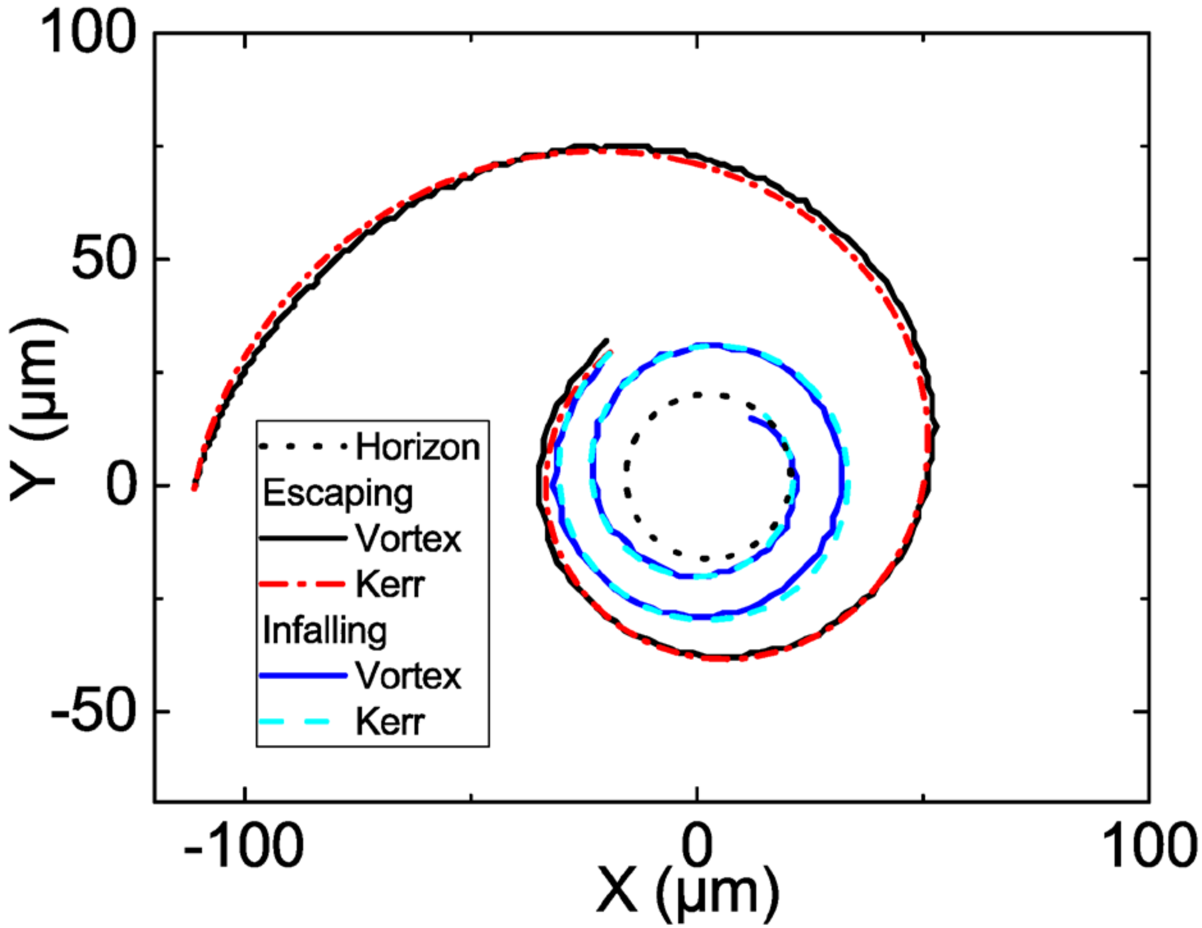
Vortex trajectories and time-like geodesics of the Kerr metric



Hamilton equations for time-like Kerr geodesics

$$\dot{r} = \frac{\Delta}{\Sigma} p_r$$

$$\dot{p}_r = -\left(\frac{\Delta}{2\Sigma}\right)' p_r^2 + \left(\frac{R}{2\Delta\Sigma}\right)'$$

$$\dot{\phi} = -\frac{1}{2\Delta\Sigma}\frac{\partial}{\partial L} R$$

where

$$\Sigma = r^2$$

$$\Delta = r^2 - 2Mr + a^2$$

$$R = P^2 - \Delta\left(r^2 + \left(L - aE\right)^2\right)$$

$$P = E\left(r^2 + a^2\right) - aL$$

Both metrics dominated by the divergent term $g_{rr} \sim (r - r_H)^{-1}$

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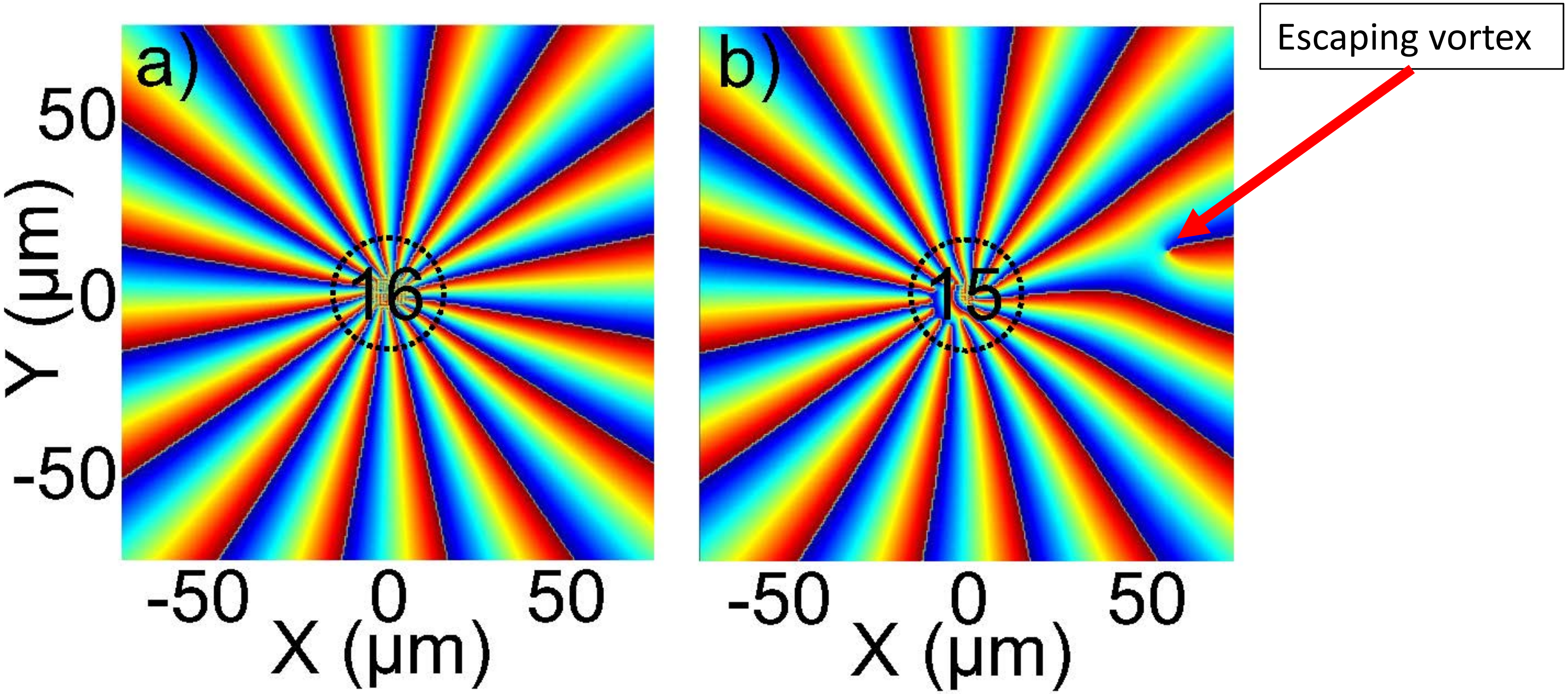
Phase of the condensate



Outlook

- Dynamical metric
  - The angular momentum is not fixed externally
- Natural presence of quantum fluctuations
  - Towards quantum gravity
  - Comparable scales of quantum and gravitational effects
- Control of quantum fluctuations
  - Interactions
  - Particle mass

$$\frac{n^{(1)}(0) - n^{(1)}(\infty)}{n^{(1)}(0)} \sim \frac{\alpha m}{\hbar^2}$$

- Thermal fluctuations negligible

$$n^{(1)}(s) \sim \left(\frac{s_T}{s}\right)^{\nu}, \ \nu = \frac{k_B T m}{2\pi\hbar^2 n_s}$$

Conclusions

- Quantum vortices – relativistic massive test particles
- Analogue electrodynamics in curved spacetimes

More info: PRB 99, 214511 (2019)