Testing Tunneling quantum simulators

Ian Moss

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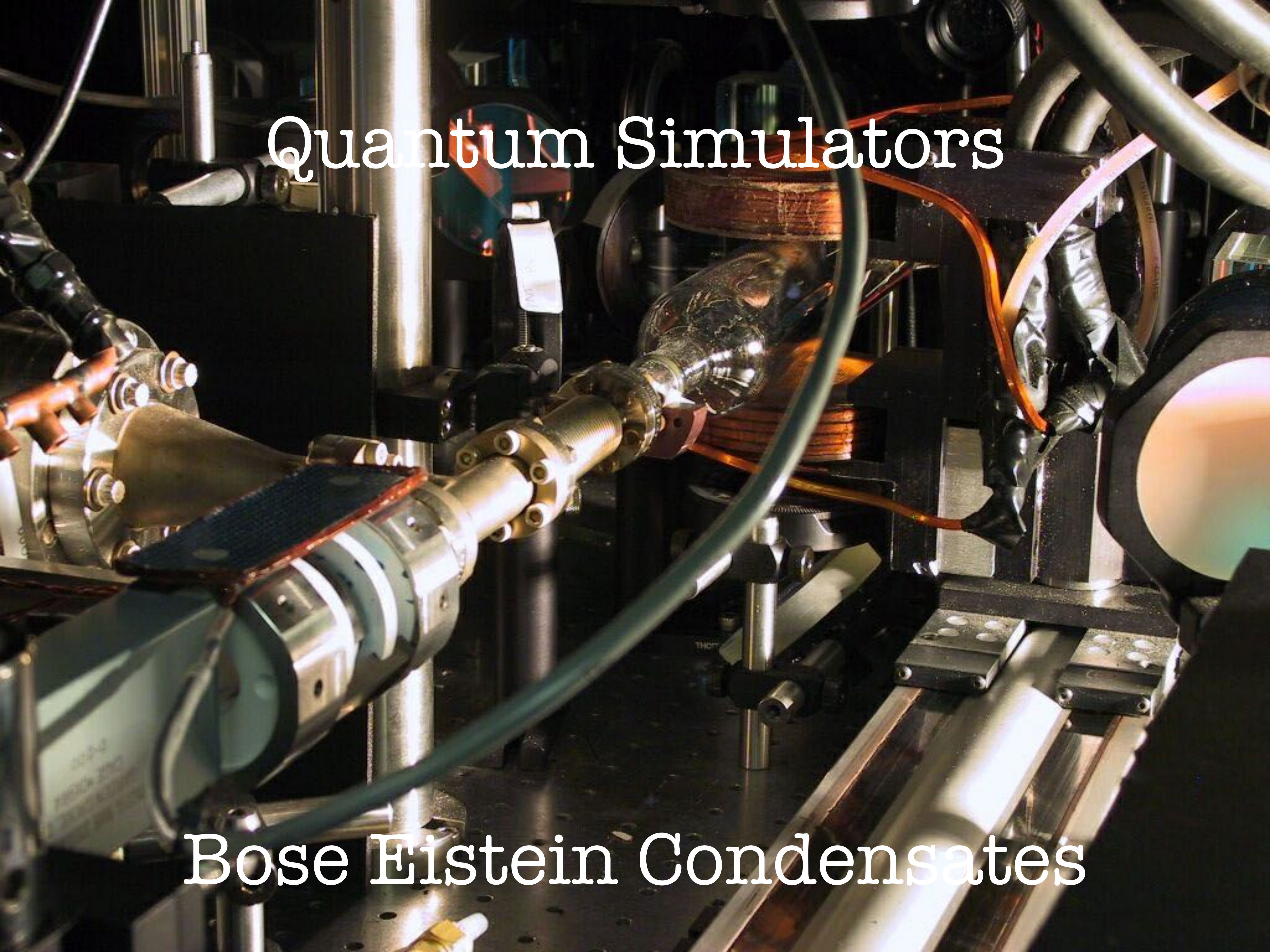
Tom Billam, Ruth Gregory, Florent Michel, IGM 1811.09169


Quantum Simulators
Bose Eistein Condensates

Quantum Simulators

- Homogeneous
- Metastable
- Relativistic

Homogeneous

trap width L

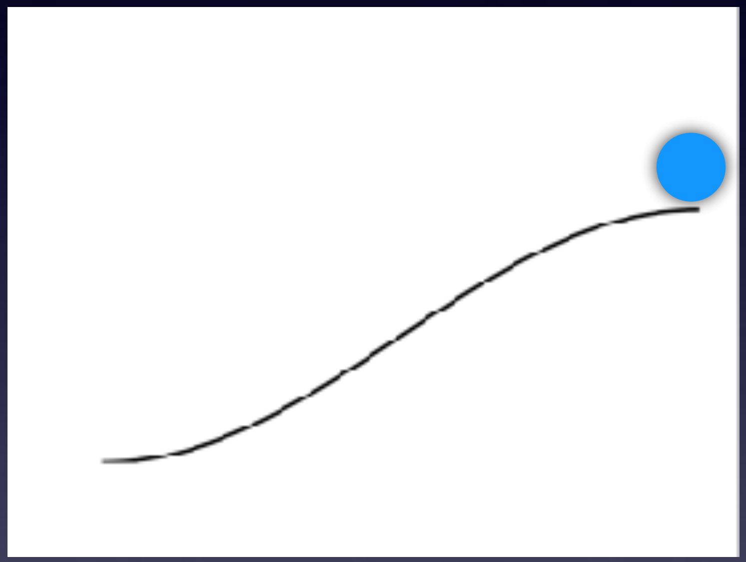
healing length ξ

scattering length a_s

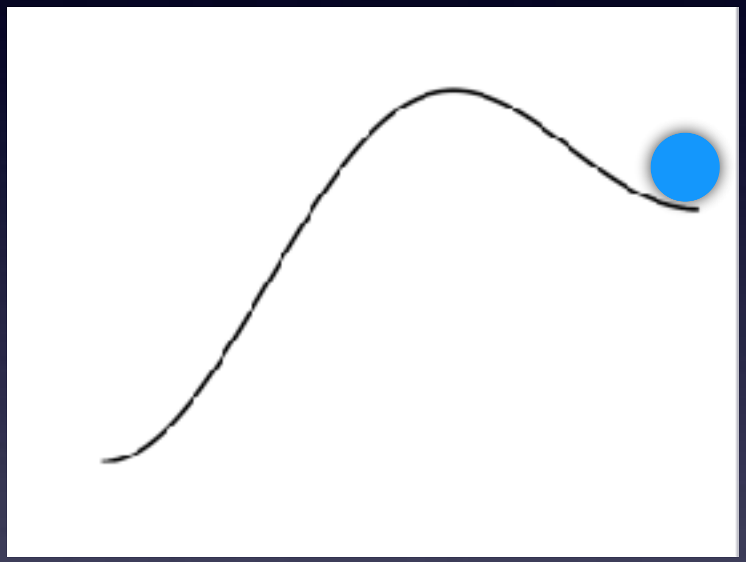
transverse dimensions $a_\perp$

$\xi << L$ $\quad$ $a_\perp << \xi$

Metastable



2nd order phase transition*



vacuum decay

*e.g. Comeron,Larcher,Dalfovo,Proukakis 1905.05263

Relativistic

sound speed $c_s \equiv c$ $c_s = 1/m\xi$

expansion $\xi \to \xi(t)$ $a_\perp \to a_\perp(t)$

Toy model*

Spinor gas model of Fialko et al:

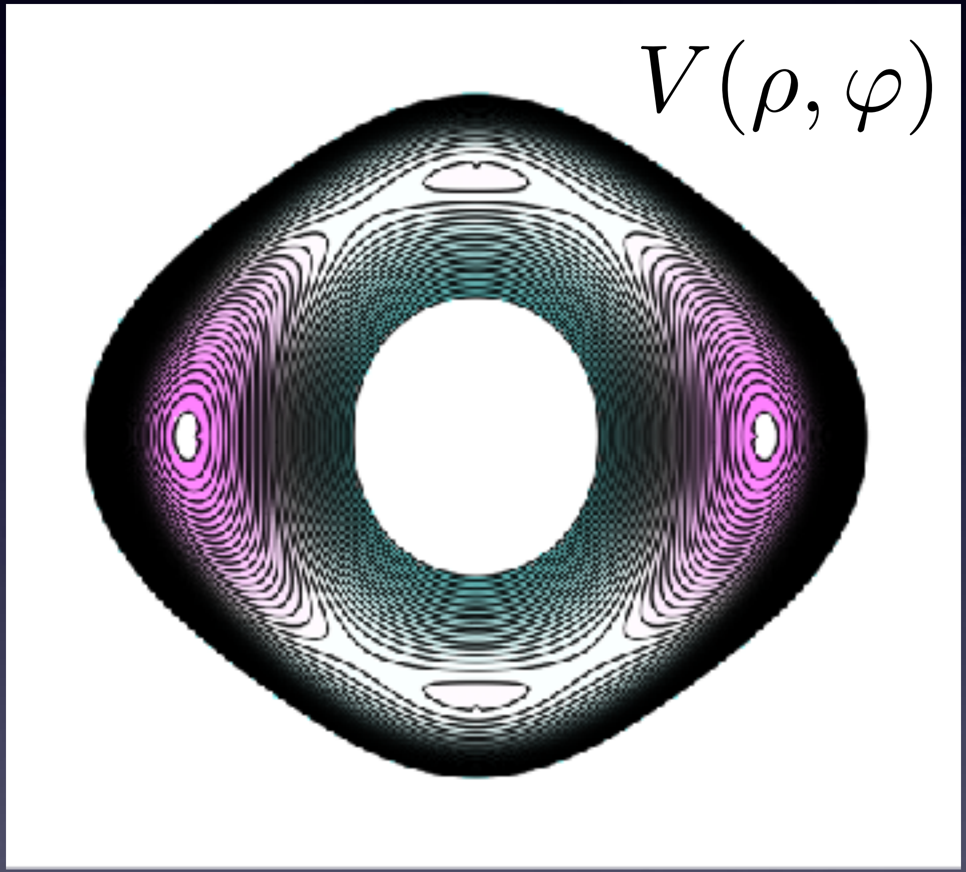
- Magnetic field-two Zeeman levels ψ_i
- RF field mixing ϵ
- Modulation of the RF field λ
- Collisions g

$$H = \int d^n x dt \left(-\frac{1}{2m} \psi^\dagger \nabla^2 \psi + V \right)$$

$$V = \frac{g}{2} (\psi_i^\dagger)^2 (\psi_i)^2 - \mu \psi^\dagger \psi - g\epsilon^2 \psi^\dagger J_x \psi + \frac{g\epsilon^2 \lambda^2}{4} (\psi^\dagger J_y \psi)^2$$

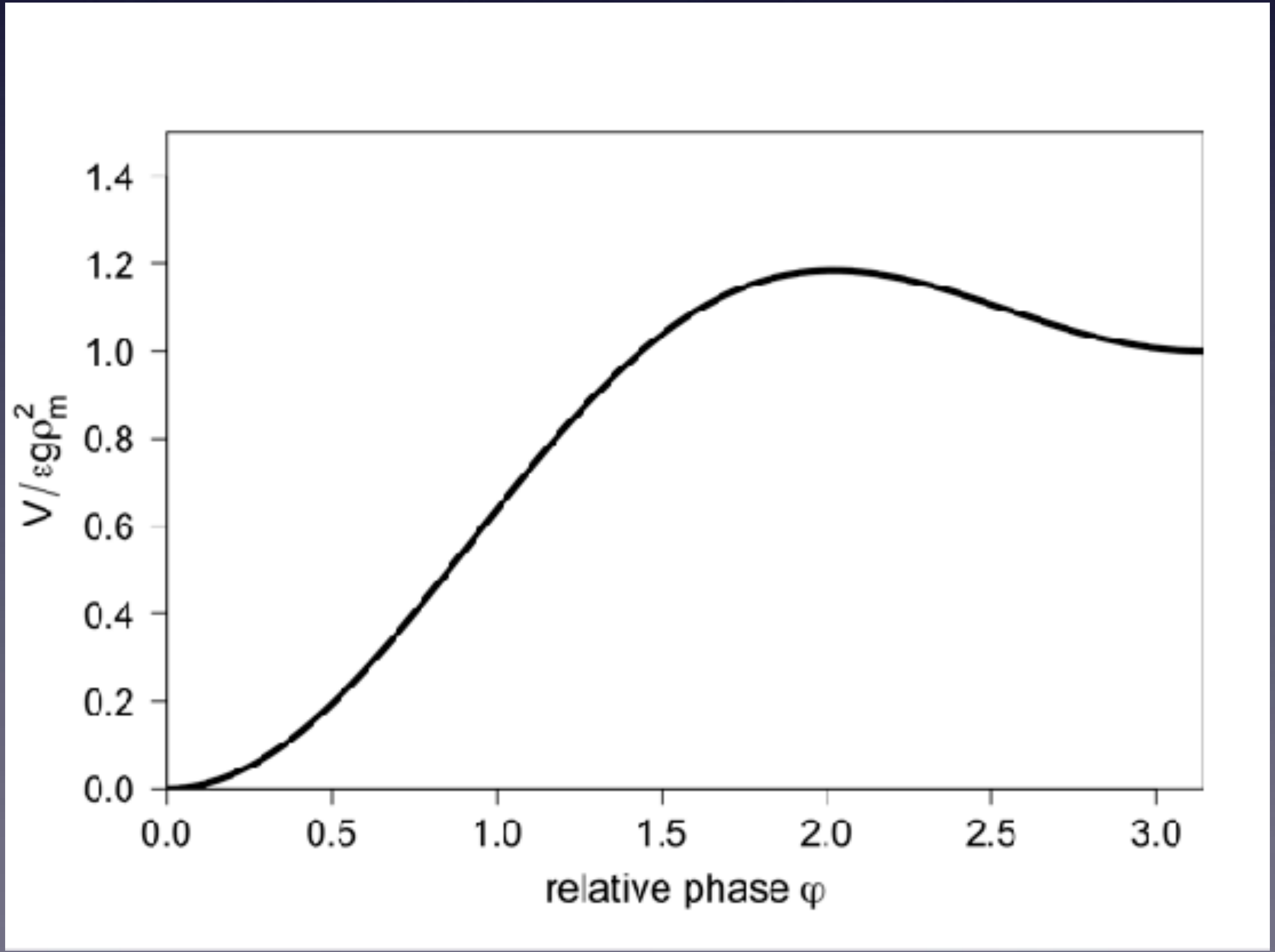
*Fialko, Opanchuk, Siderov, Drummond, Brand JPhys B (2017)

Metastability



V

$$\psi_i = \rho_m^{1/2}(1 \pm \epsilon\sigma)e^{\pm\varphi/2}$$



Relativistic

Action

$$S_L[\psi,\bar{\psi}] = \int d^n x dt \left(-i\bar{\psi}\dot{\psi} - \mathcal{H}\right)$$

$$\psi_i = \rho_m^{1/2}(1 \pm \epsilon\sigma)e^{\pm i\varphi/2}$$

At leading order in epsilon, after scaling $(c_s = 1)$

$$S_L = \epsilon^2 \int d^n x dt \left(-\frac{1}{2}\dot{\varphi}^2 + (\nabla\varphi)^2 + V(\varphi)\right)$$

Klein-Gordon system-but beyond this limit?

Vacuum decay: BEC's

Vacuum decay in a non-relativistic system

$$\langle \Psi_{\rm f} | e^{-i\hat{H}t - i\mu\hat{N}t} | \Psi_{\rm i} \rangle = \int D\psi D\bar{\psi} e^{-iS_L[\psi,\bar{\psi}]}$$

In imaginary time

$$Z(\mu) = \int D\psi D\bar{\psi} e^{-S(\psi,\bar{\psi})}$$

Euclidean action

$$S[\psi,\bar{\psi}] = \int d^n x d\tau \left(\bar{\psi}\dot{\psi} - \mathcal{H}(\psi,\bar{\psi}) \right)$$

Decay Rates

Vacuum decay rate in a volume $\mathcal{V}$

$$\Gamma = \mathcal{V} \left| \frac{\det' S''[\psi_b]}{\det S''[\psi_{\rm fv}]} \right|^{-1/2} \left(\frac{S[\psi_b]}{2\pi} \right)^{N/2} e^{-S[\psi_b]}$$

Where

$$S'[\psi_b] = \delta S/\delta\psi = 0$$

Instanton solutions

$$\dot{\psi}_i = \frac{1}{2m}\nabla^2\psi_i + \frac{\partial V}{\partial \overline{\psi}_i}$$

$$\dot{\overline{\psi}}_i = -\frac{1}{2m}\nabla^2\overline{\psi}_i - \frac{\partial V}{\partial \psi_i}$$

Where $\psi \to \psi_{\mathrm{fv}}$

For vacuum decay we must have $\psi_i^\dagger \neq \bar{\psi}_i$

'Rotate' the contour in $D\psi D\bar{\psi}$

Klein-Gordon limit

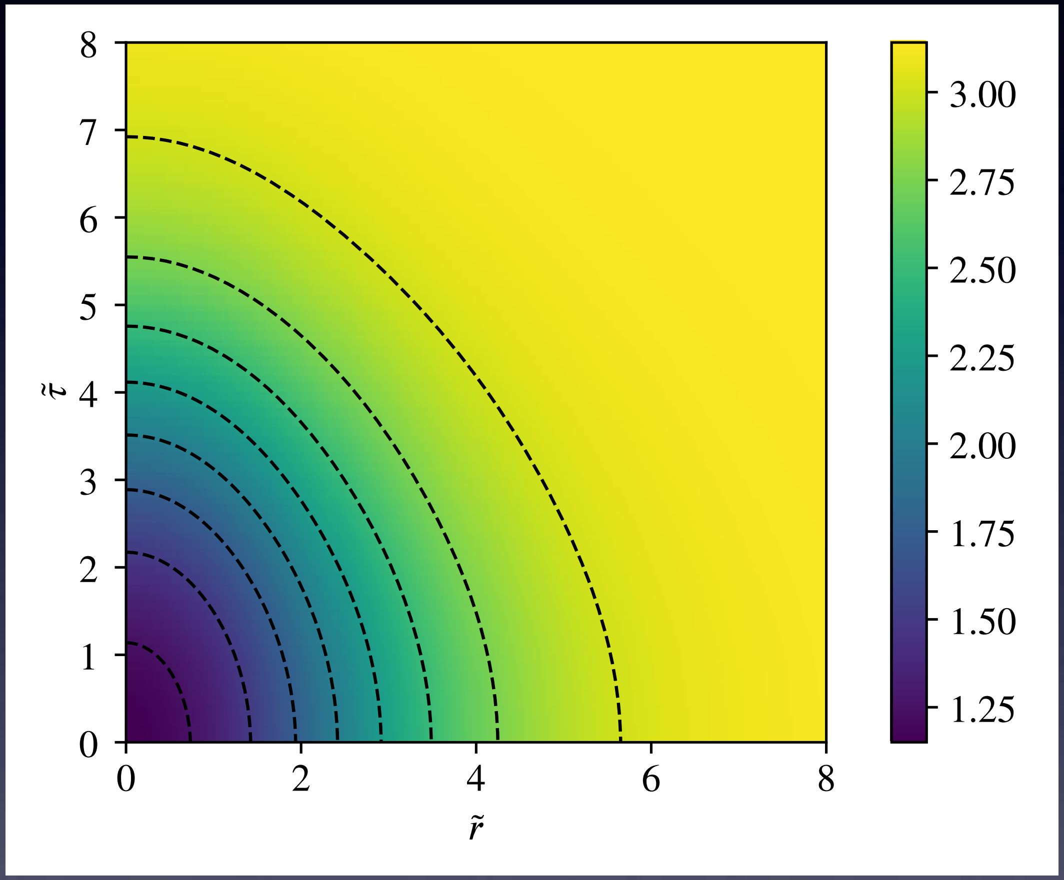
Take $\psi_i = \rho_m^{1/2}(1 \pm \epsilon\sigma)e^{\pm\varphi/2}$ $\sigma = -i\dot{\varphi}/\rho_m$

$$S = \epsilon^2 \int d^n x d\tau \left(\frac{1}{2}\dot{\varphi}^2 + \frac{1}{2}(\nabla\varphi)^2 + V(\varphi) \right)$$

False vacuum $\varphi \to \pi$

O(n+1) symmetry $\varphi = \varphi(\sqrt{\mathbf{x}^2 + \tau^2})$

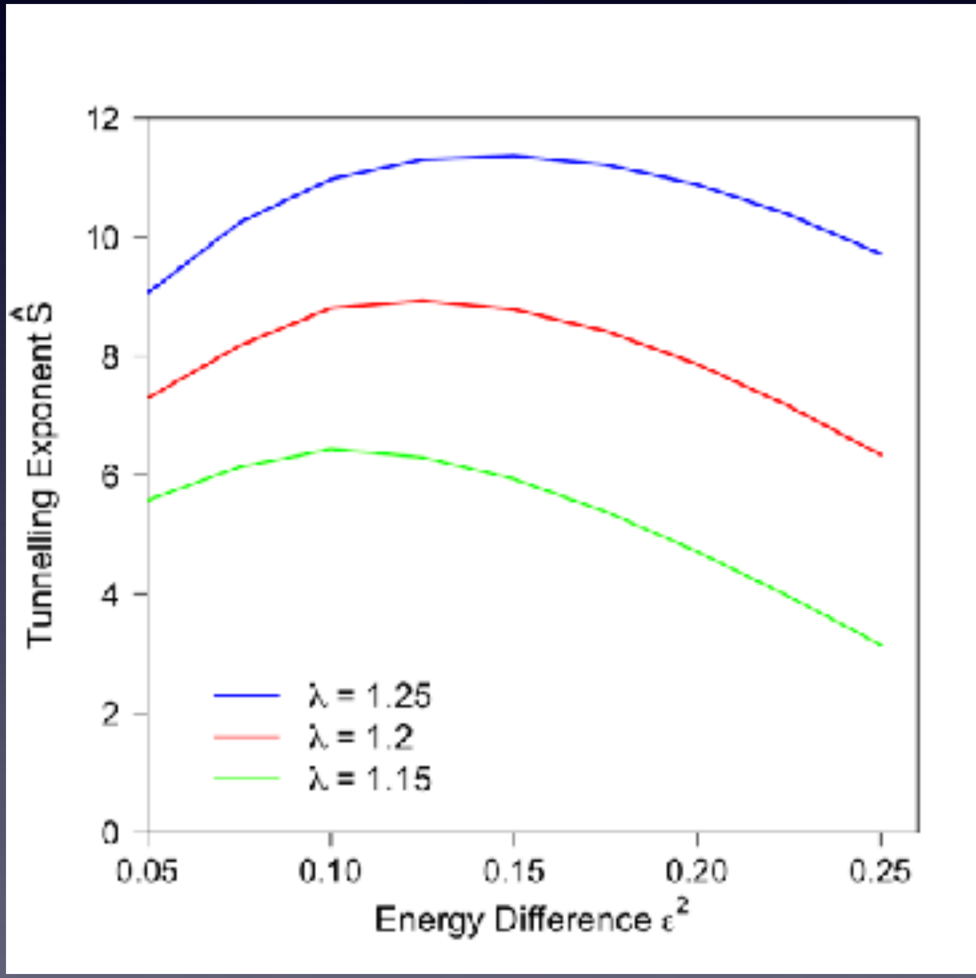
Vacuum decay rates (2D)



Instanton

Not quite O(3)

Decay exponent



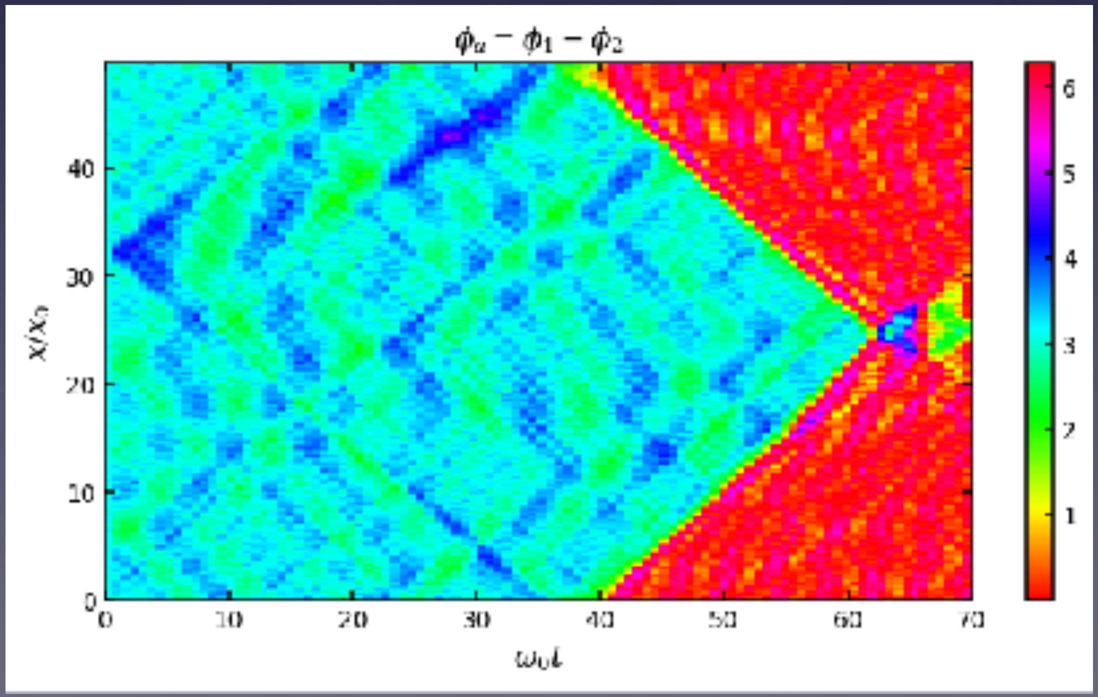
$$\Gamma = Ae^{-\rho_m R_0^2 \hat{S}}$$

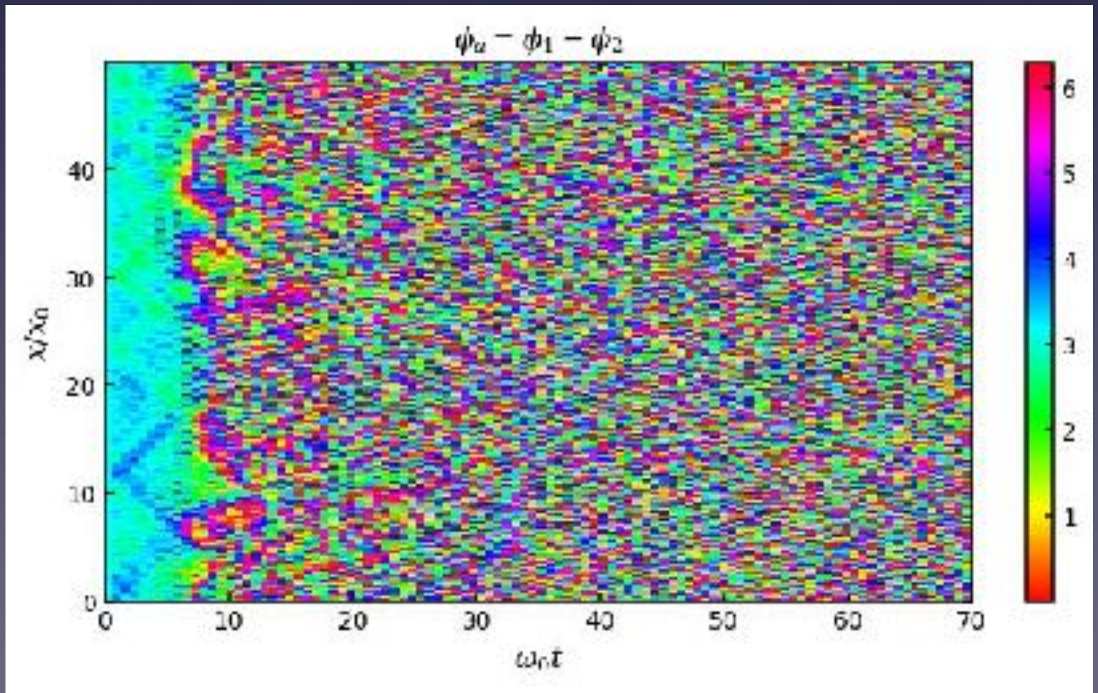
$$R_0 = \xi/2\epsilon$$

Parametric instability

$$V = \frac{1}{4}\left(\frac{\bar{\psi}_i\psi_i}{\rho_m} - 1\right)^2 - \frac{\epsilon^2}{2}\frac{\bar{\psi}J_x\psi}{\rho_m}(1 + \lambda\omega\cos\omega t)$$

False vacuum classically unstable (Braden, Johnson, Peiris, Weinfurtner 1712.02356)





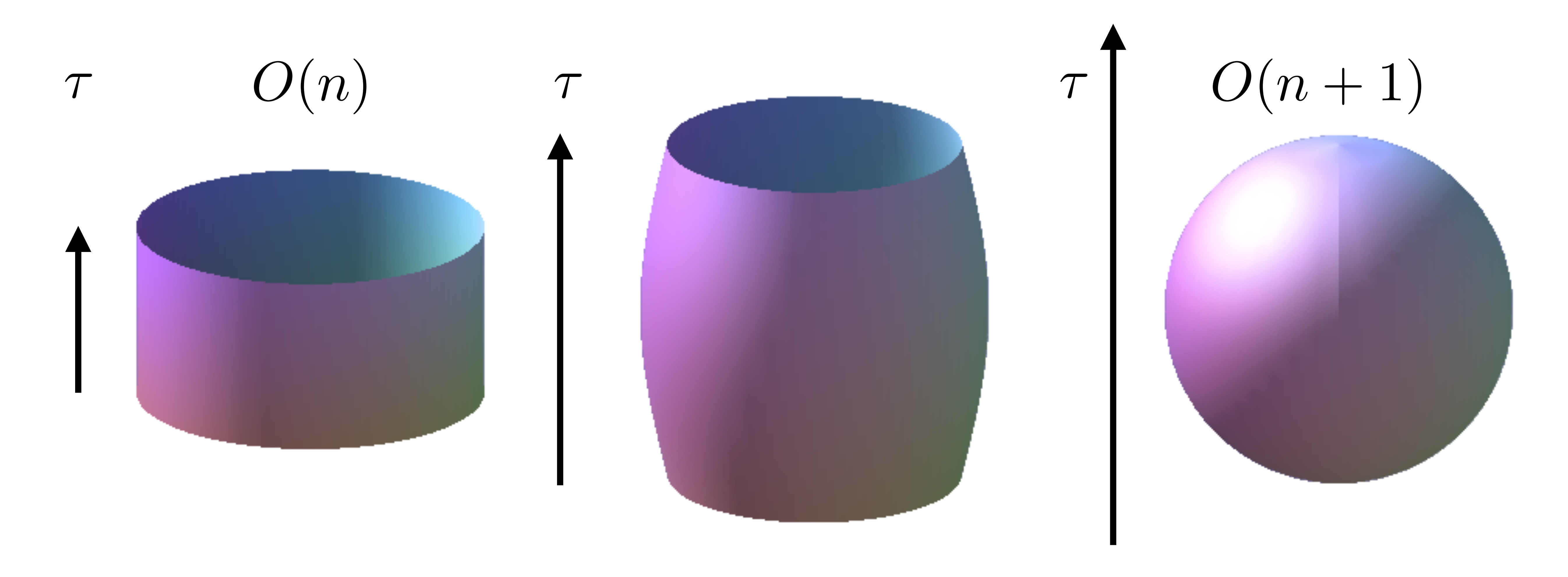
Andrew Grozek

$$\Delta x < (800\mathrm{Hz}/\omega)^{-1/2}$$

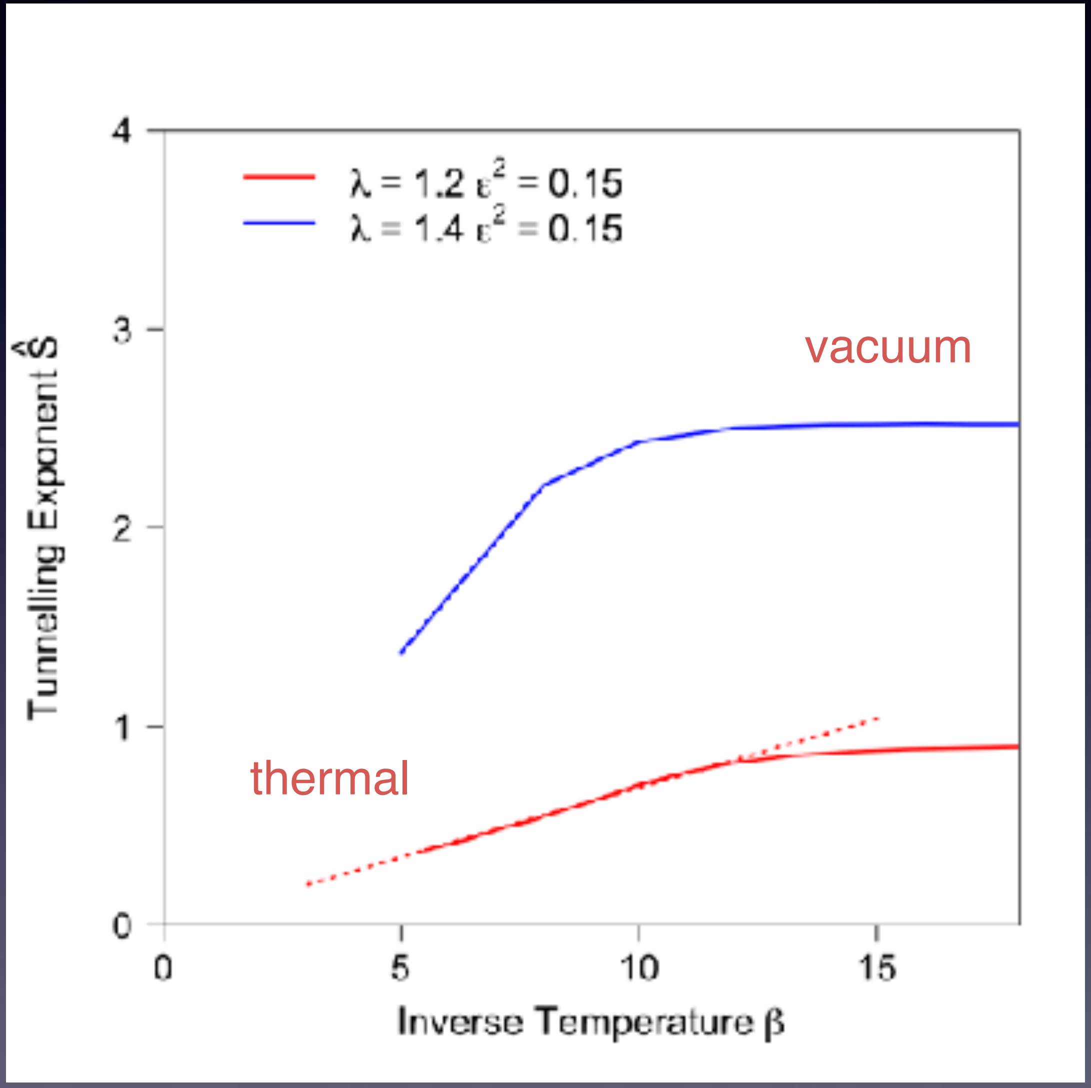
Thermal/Vacuum

Make the imaginary time variable periodic

$$Z(T) = \int D\varphi \, e^{-S[\varphi]} \qquad \varphi(\mathbf{x}, \tau + \beta) = \varphi(\mathbf{x}, \tau)$$



Thermal/Vacuum



$$\Gamma = Ae^{-\rho_m R_0^2 \hat{S}}$$


End
of part II