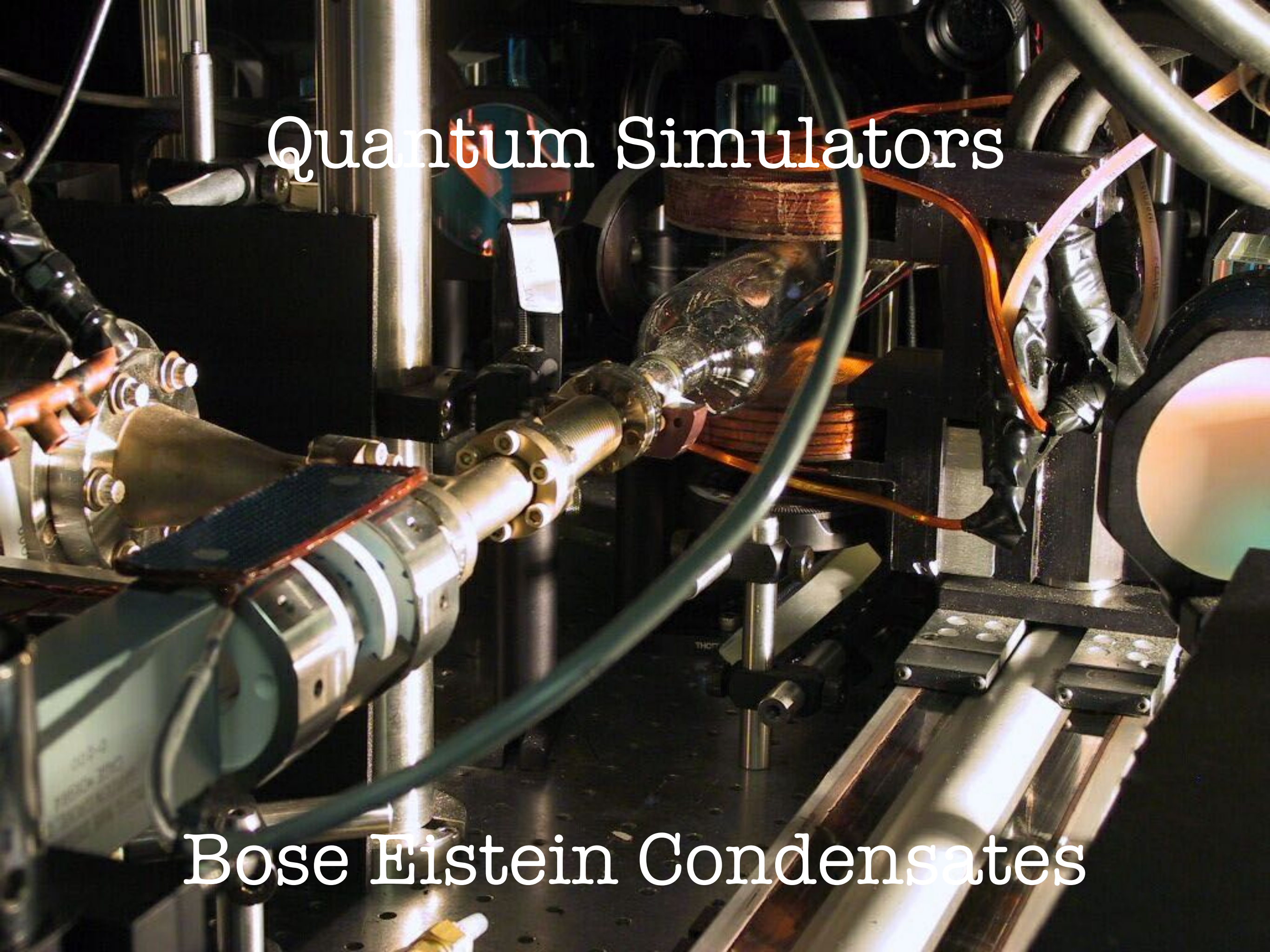


Testing Tunneling quantum simulators

Ian Moss
July 2019

Tom Billam, Ruth Gregory, Florent Michel, IGM 1811.09169

Quantum Simulators

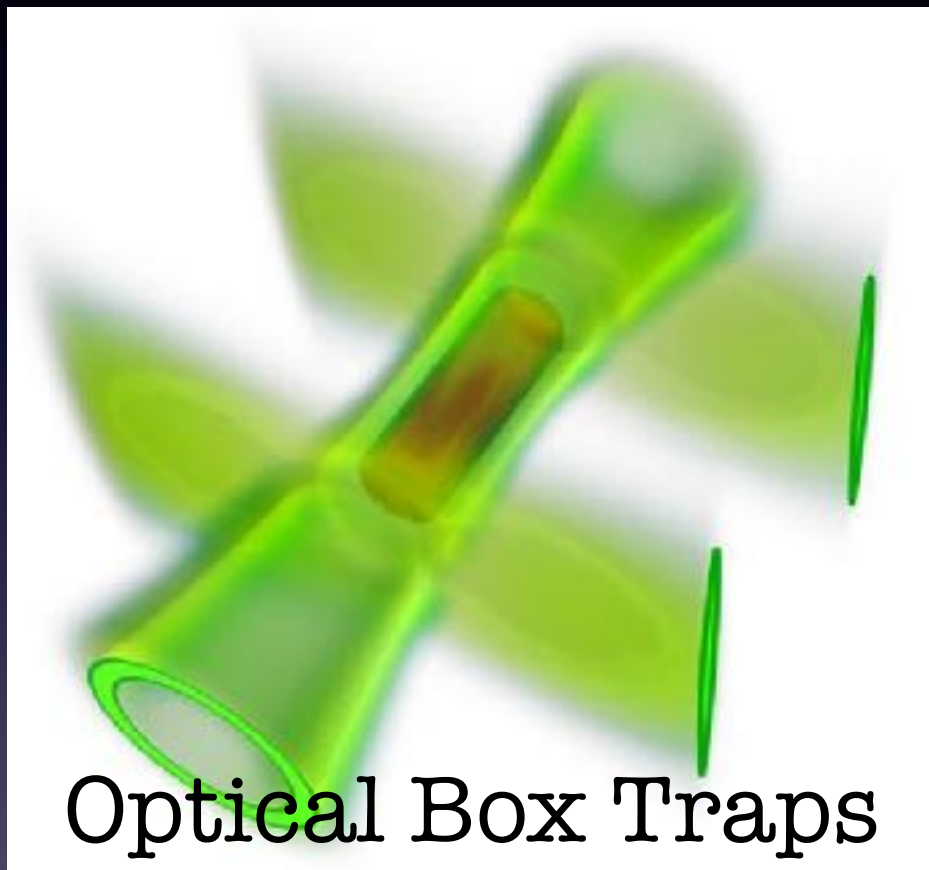
A detailed photograph of a quantum simulator experimental setup. The central component is a horizontal glass tube containing a glowing orange light, likely a laser or a heated gas. This tube is connected to various mechanical and electrical components. To the left, there's a complex assembly of metal parts, including a large cylindrical component with a blue mesh. To the right, a large circular lens or mirror is visible, reflecting some light. The entire setup is mounted on a metal frame with various adjustment screws and supports. The lighting is focused on the central tube, creating a warm, orange glow.

Bose Einstein Condensates

Quantum Simulators

- Homogeneous
- Metastable
- Relativistic

Homogeneous



trap width L

healing length ξ

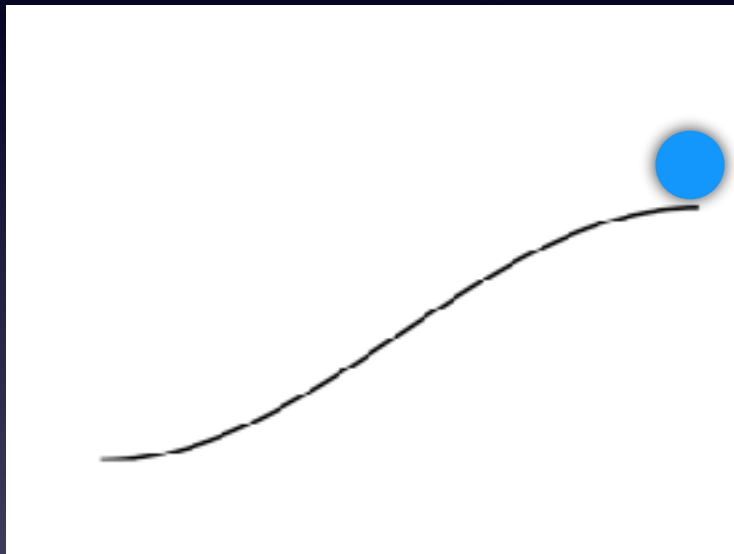
scattering length a_s

transverse dimensions $a_{\perp}$

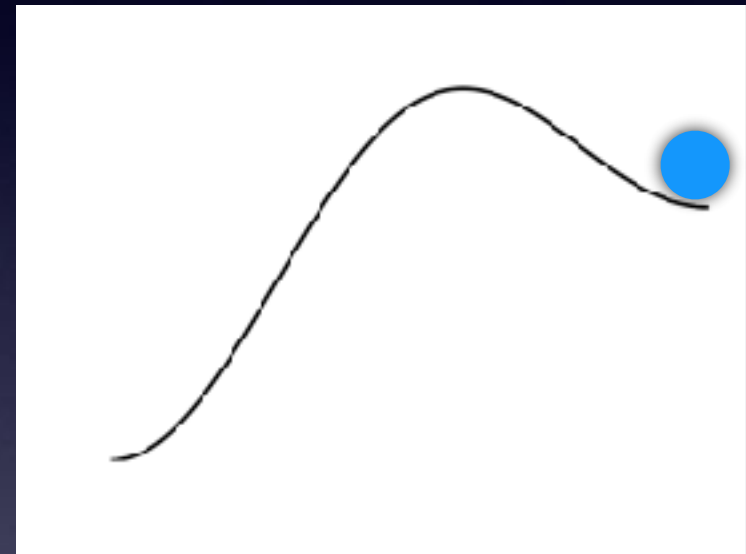
$$\xi \ll L$$

$$a_{\perp} \ll \xi$$

Metastable



2nd order phase
transition*



vacuum decay

*e.g. Comeron, Larcher, Dalfovo, Proukakis 1905.05263

Relativistic

sound speed $c_s \equiv c$ $c_s = 1/m\xi$

expansion $\xi \rightarrow \xi(t)$ $a_\perp \rightarrow a_\perp(t)$

Toy model*

Spinor gas model of Fialko et al:

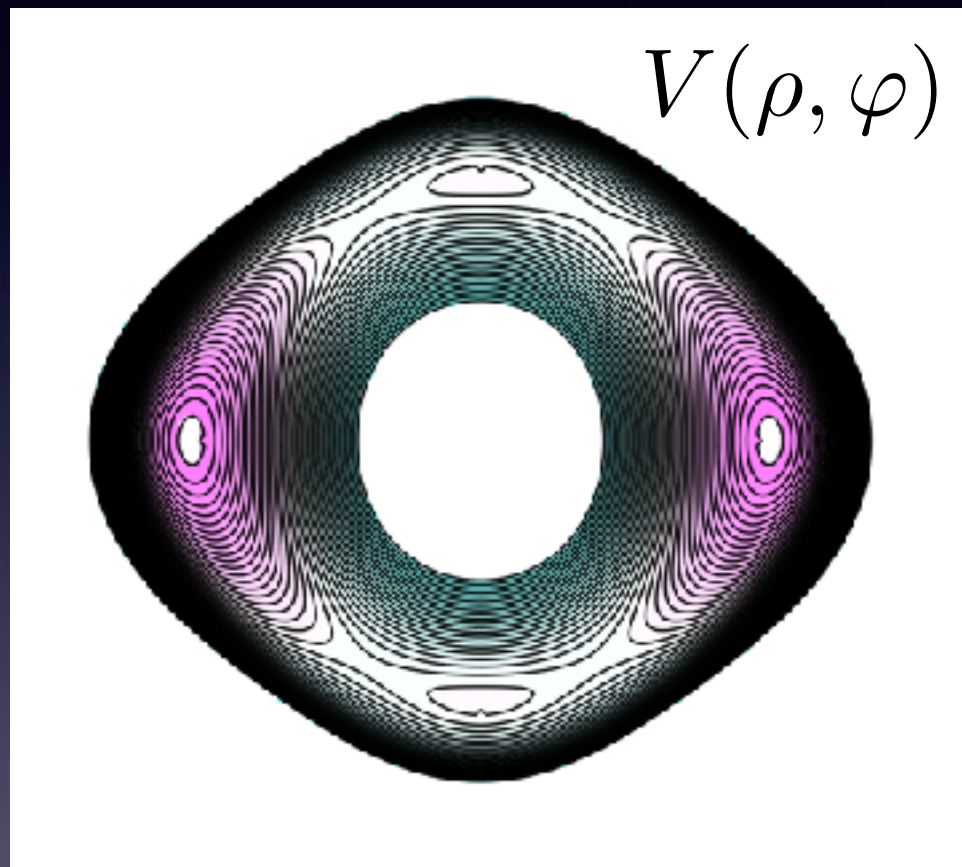
- Magnetic field-two Zeeman levels ψ_i
- RF field mixing ϵ
- Modulation of the RF field λ
- Collisions g

$$H = \int d^n x dt \left(-\frac{1}{2m} \psi^\dagger \nabla^2 \psi + V \right)$$

$$V = \frac{g}{2} (\psi_i^\dagger)^2 (\psi_i)^2 - \mu \psi^\dagger \psi - g \epsilon^2 \psi^\dagger J_x \psi + \frac{g \epsilon^2 \lambda^2}{4} (\psi^\dagger J_y \psi)^2$$

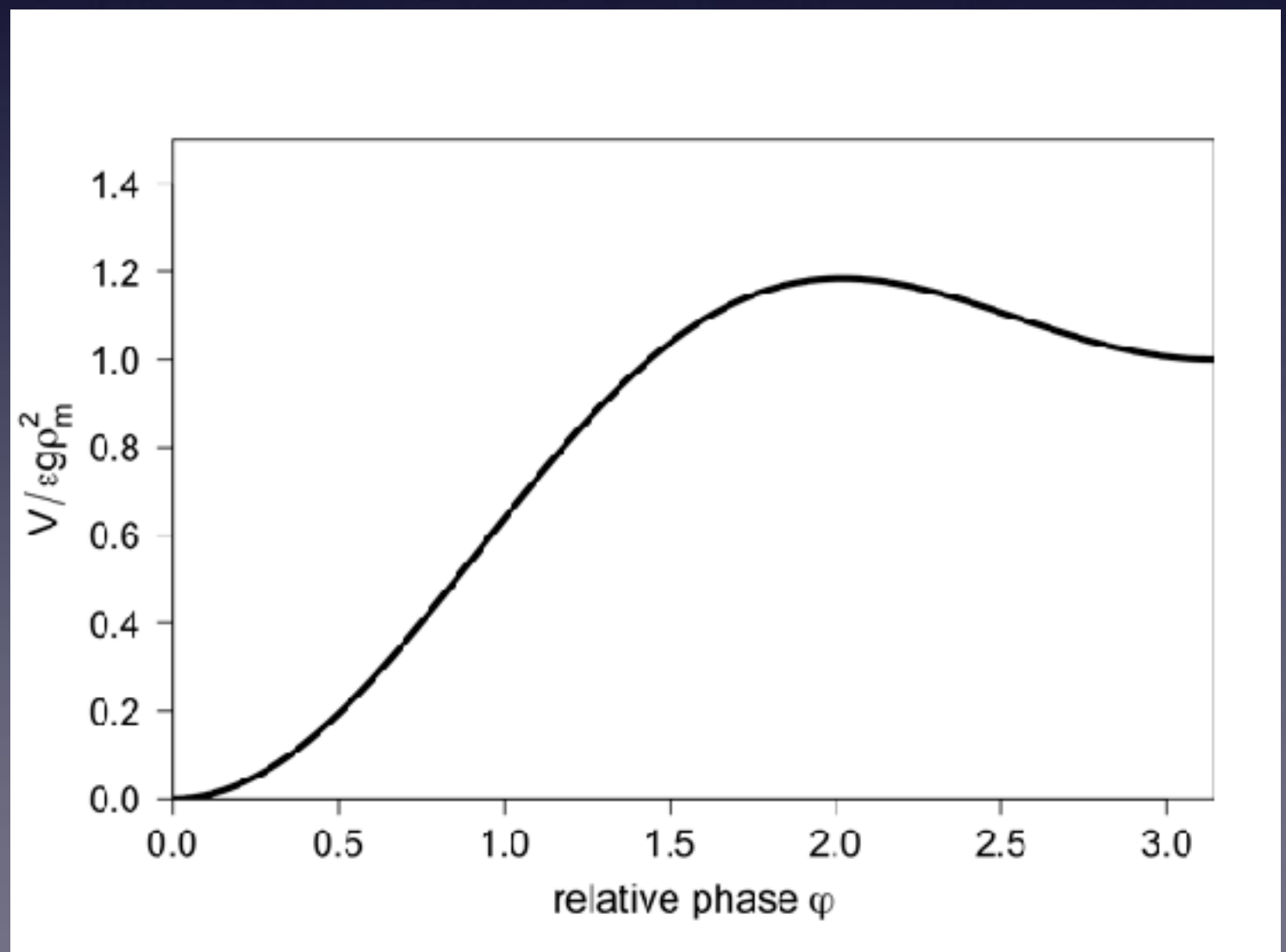
*Fialko, Opanchuk, Siderov, Drummond, Brand JPhys B (2017)

Metastability



V

$$\psi_i = \rho_m^{1/2} (1 \pm \epsilon \sigma) e^{\pm \varphi/2}$$



Relativistic

Action

$$S_L[\psi, \bar{\psi}] = \int d^n x dt \left(-i\bar{\psi}\dot{\psi} - \mathcal{H} \right)$$

$$\psi_i = \rho_m^{1/2} (1 \pm \epsilon \sigma) e^{\pm i\varphi/2}$$

At leading order in epsilon, after scaling ($c_s = 1$)

$$S_L = \epsilon^2 \int d^n x dt \left(-\frac{1}{2}\dot{\varphi}^2 + (\nabla\varphi)^2 + V(\varphi) \right)$$

Klein-Gordon system-but beyond this limit?

Vacuum decay: BEC's

Vacuum decay in a non-relativistic system

$$\langle \Psi_f | e^{-i\hat{H}t - i\mu\hat{N}t} | \Psi_i \rangle = \int D\psi D\bar{\psi} e^{-iS_L[\psi, \bar{\psi}]}$$

In imaginary time

$$Z(\mu) = \int D\psi D\bar{\psi} e^{-S(\psi, \bar{\psi})}$$

Euclidean action

$$S[\psi, \bar{\psi}] = \int d^n x d\tau \left(\bar{\psi} \dot{\psi} - \mathcal{H}(\psi, \bar{\psi}) \right)$$

Decay Rates

Vacuum decay rate in a volume $\mathcal{V}$

$$\Gamma = \mathcal{V} \left| \frac{\det' S''[\psi_b]}{\det S''[\psi_{fv}]} \right|^{-1/2} \left(\frac{S[\psi_b]}{2\pi} \right)^{N/2} e^{-S[\psi_b]}$$

Where

$$S'[\psi_b] = \delta S / \delta \psi = 0$$

Instanton solutions

$$\dot{\psi}_i = \frac{1}{2m} \nabla^2 \psi_i + \frac{\partial V}{\partial \bar{\psi}_i}$$

$$\dot{\bar{\psi}}_i = -\frac{1}{2m} \nabla^2 \bar{\psi}_i - \frac{\partial V}{\partial \psi_i}$$

Where $\psi \rightarrow \psi_{\text{fv}}$

For vacuum decay we must have $\psi_i^\dagger \neq \bar{\psi}_i$

‘Rotate’ the contour in $D\psi D\bar{\psi}$

Klein-Gordon limit

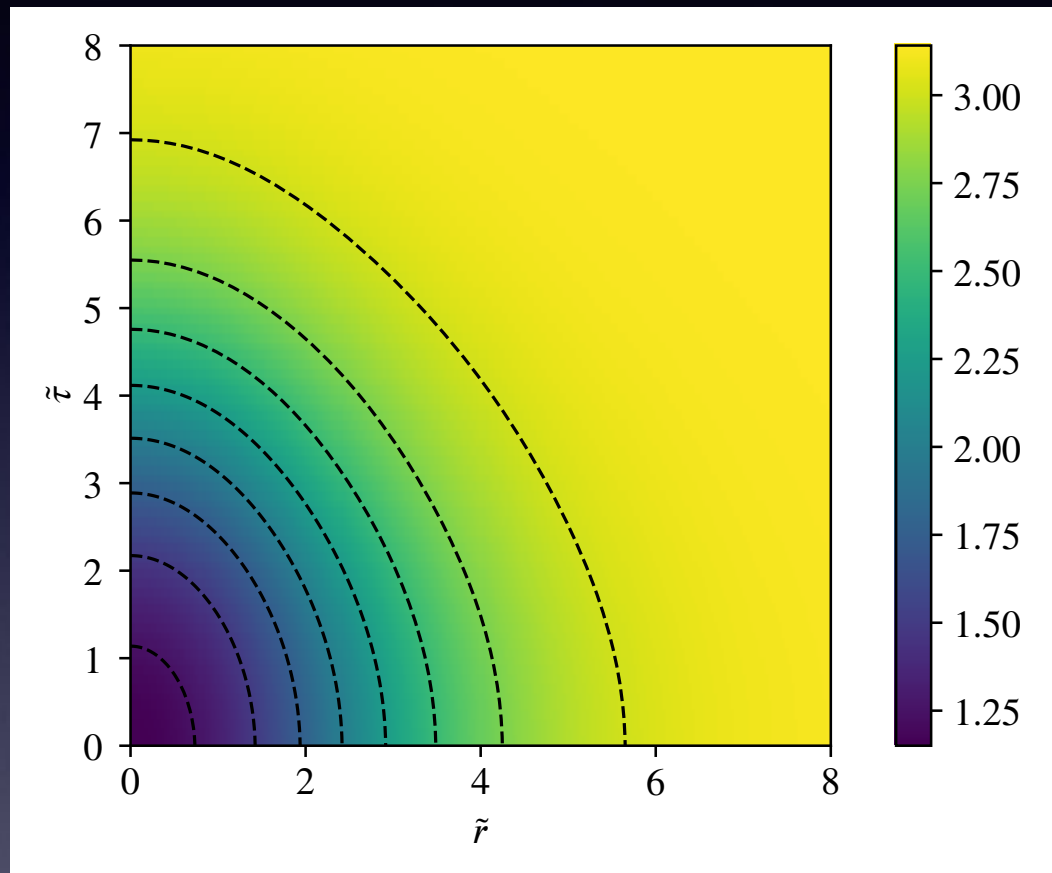
Take $\psi_i = \rho_m^{1/2}(1 \pm \epsilon\sigma)e^{\pm\varphi/2}$ $\sigma = -i\dot{\varphi}/\rho_m$

$$S = \epsilon^2 \int d^n x d\tau \left(\frac{1}{2} \dot{\varphi}^2 + \frac{1}{2} (\nabla \varphi)^2 + V(\varphi) \right)$$

False vacuum $\varphi \rightarrow \pi$

$O(n+1)$ symmetry $\varphi = \varphi(\sqrt{\mathbf{x}^2 + \tau^2})$

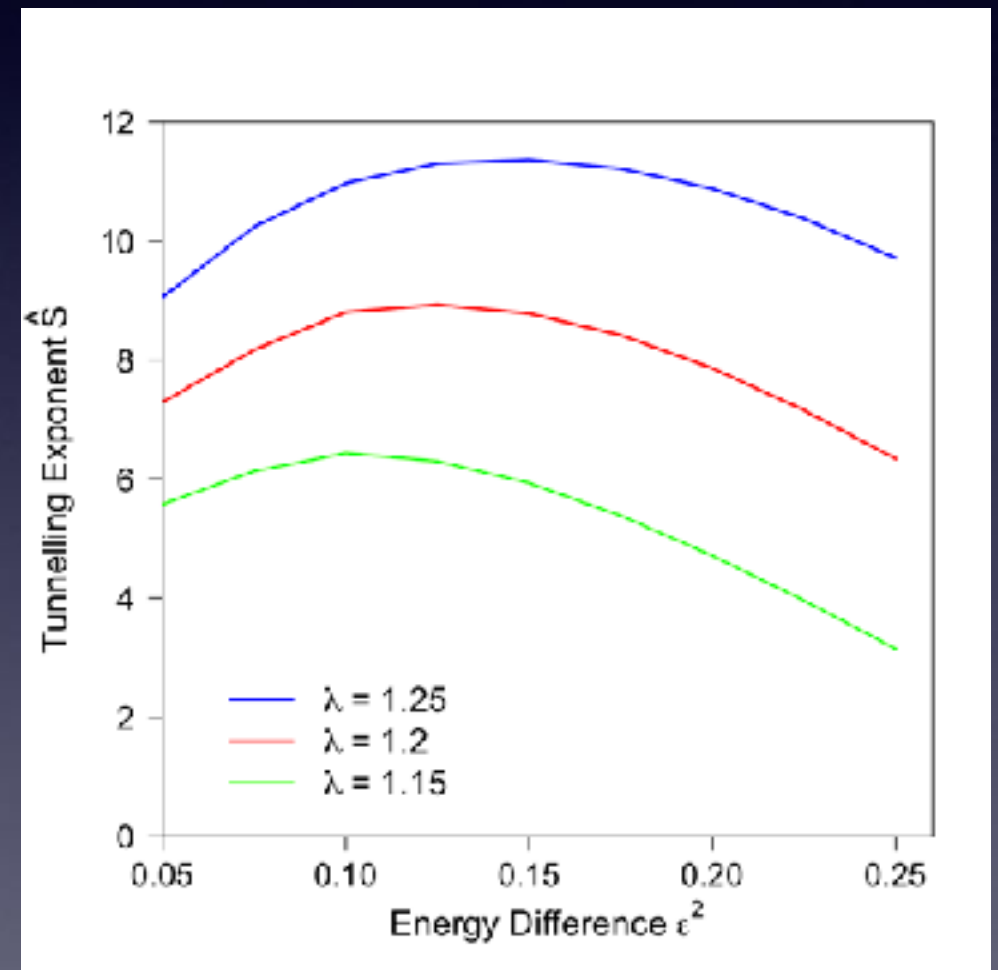
Vacuum decay rates (2D)



Instanton

Not quite $O(3)$

Decay exponent



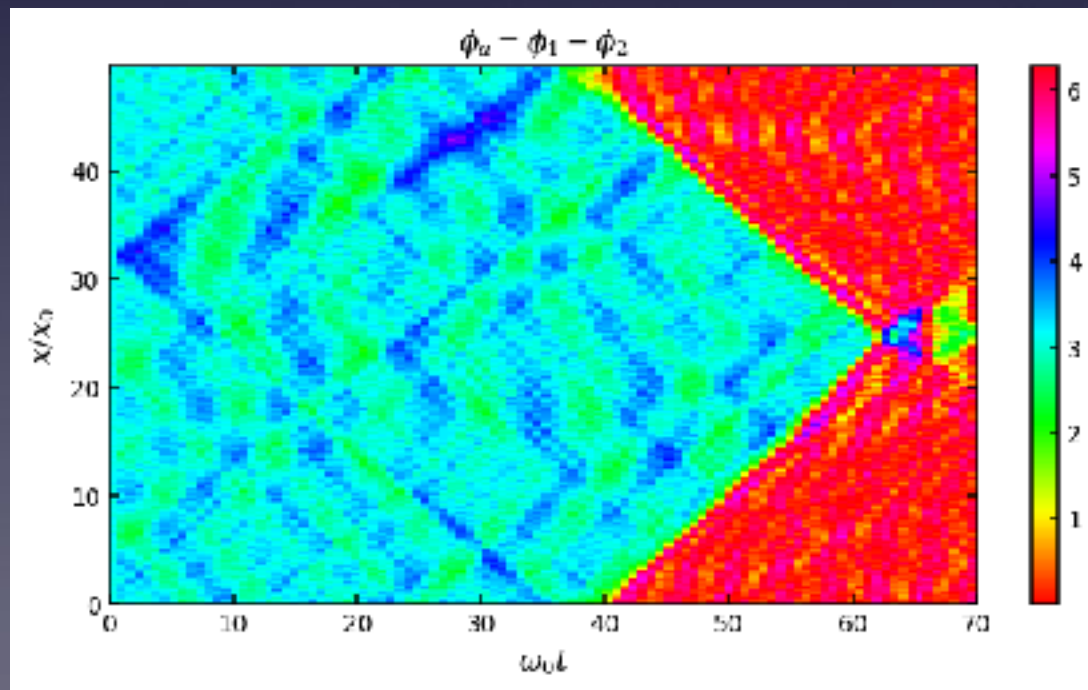
$$\Gamma = A e^{-\rho_m R_0^2 \hat{S}}$$

$$R_0 = \xi / 2\epsilon$$

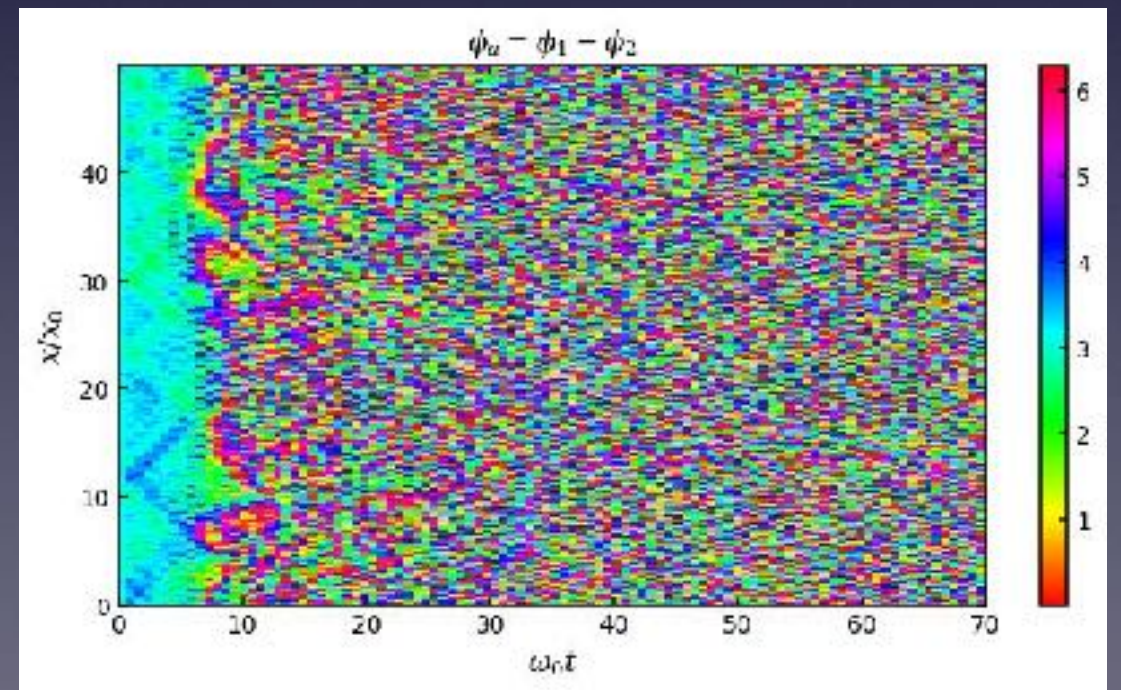
Parametric instability

$$V = \frac{1}{4} \left(\frac{\bar{\psi}_i \psi_i}{\rho_m} - 1 \right)^2 - \frac{\epsilon^2}{2} \frac{\bar{\psi} J_x \psi}{\rho_m} (1 + \lambda \omega \cos \omega t)$$

False vacuum classically unstable (Braden, Johnson, Peiris, Weinfurtner 1712.02356)



Andrew Grozek

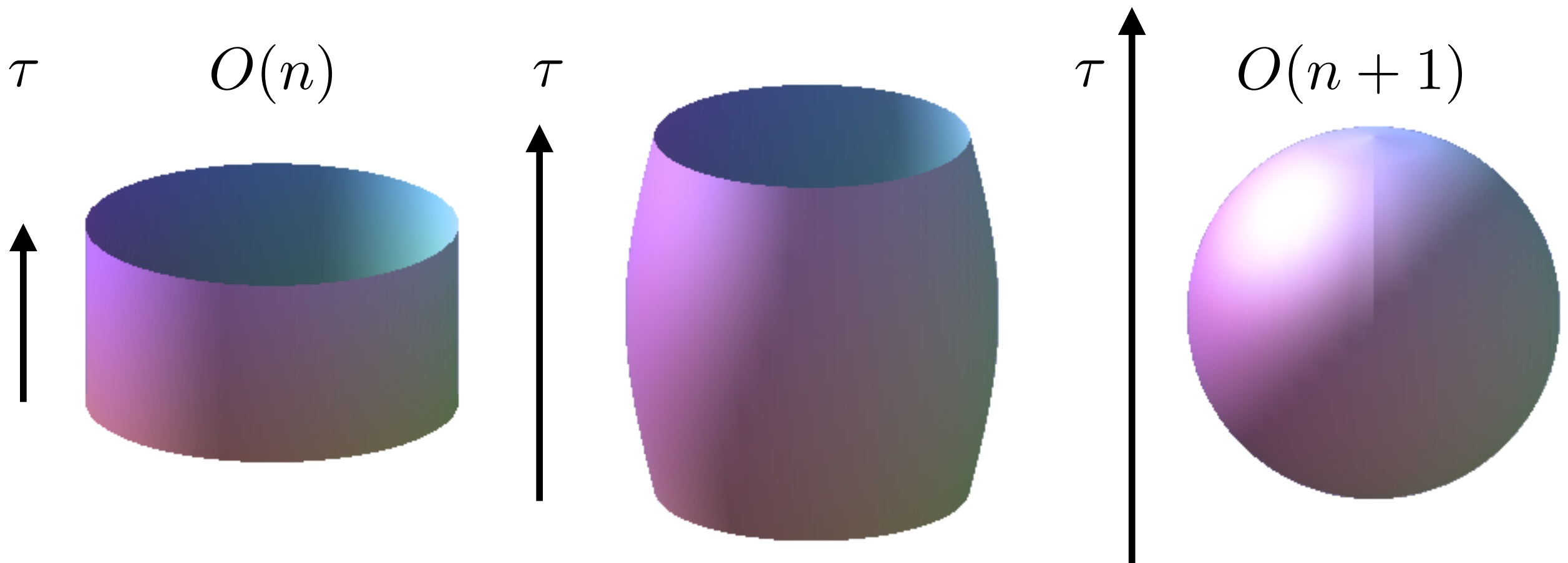


$$\Delta x < (800\text{Hz}/\omega)^{-1/2}$$

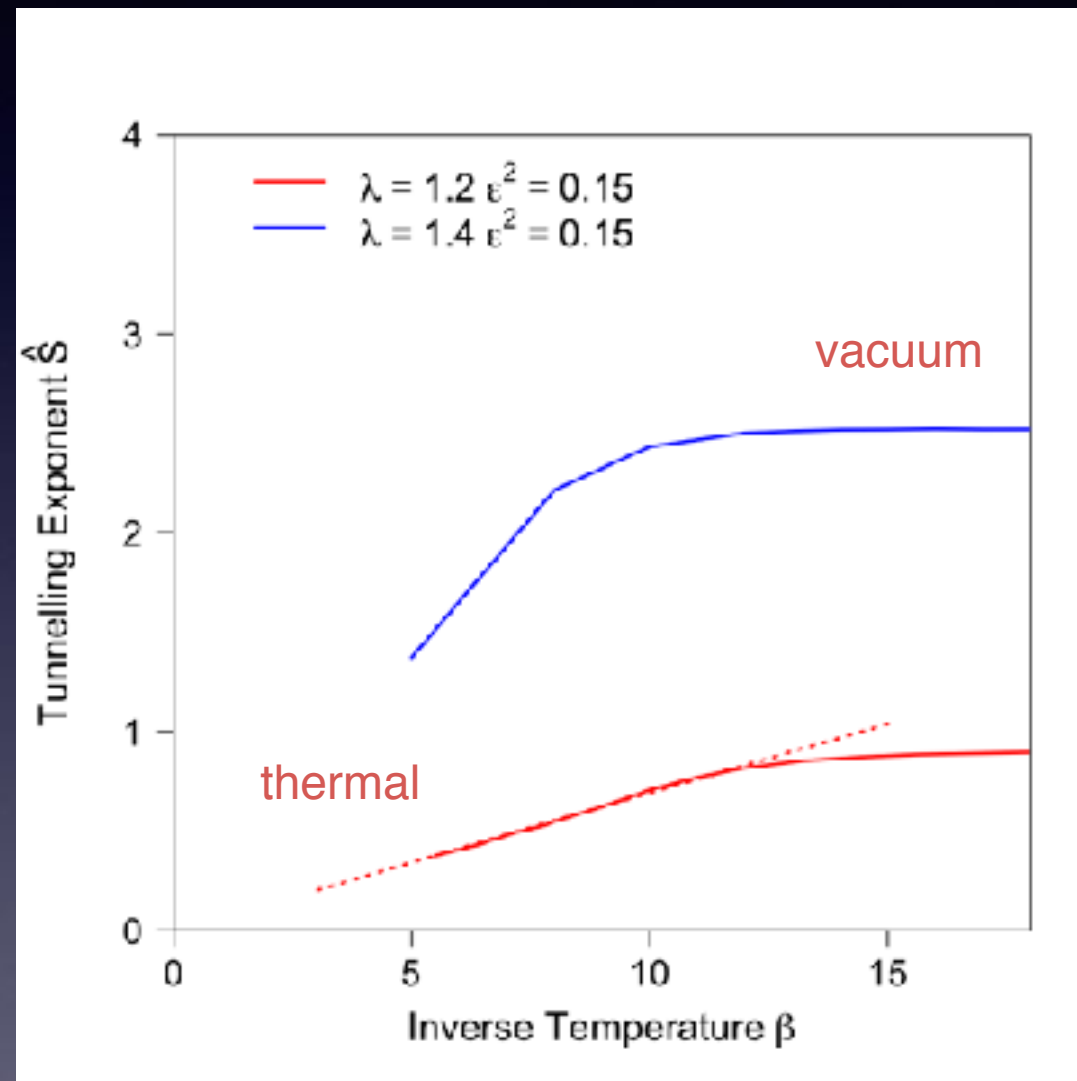
Thermal/Vacuum

Make the imaginary time variable periodic

$$Z(T) = \int D\varphi e^{-S[\varphi]} \quad \varphi(\mathbf{x}, \tau + \beta) = \varphi(\mathbf{x}, \tau)$$



Thermal/Vacuum



$$\Gamma = A e^{-\rho_m R_0^2 \hat{S}}$$



End
of part II