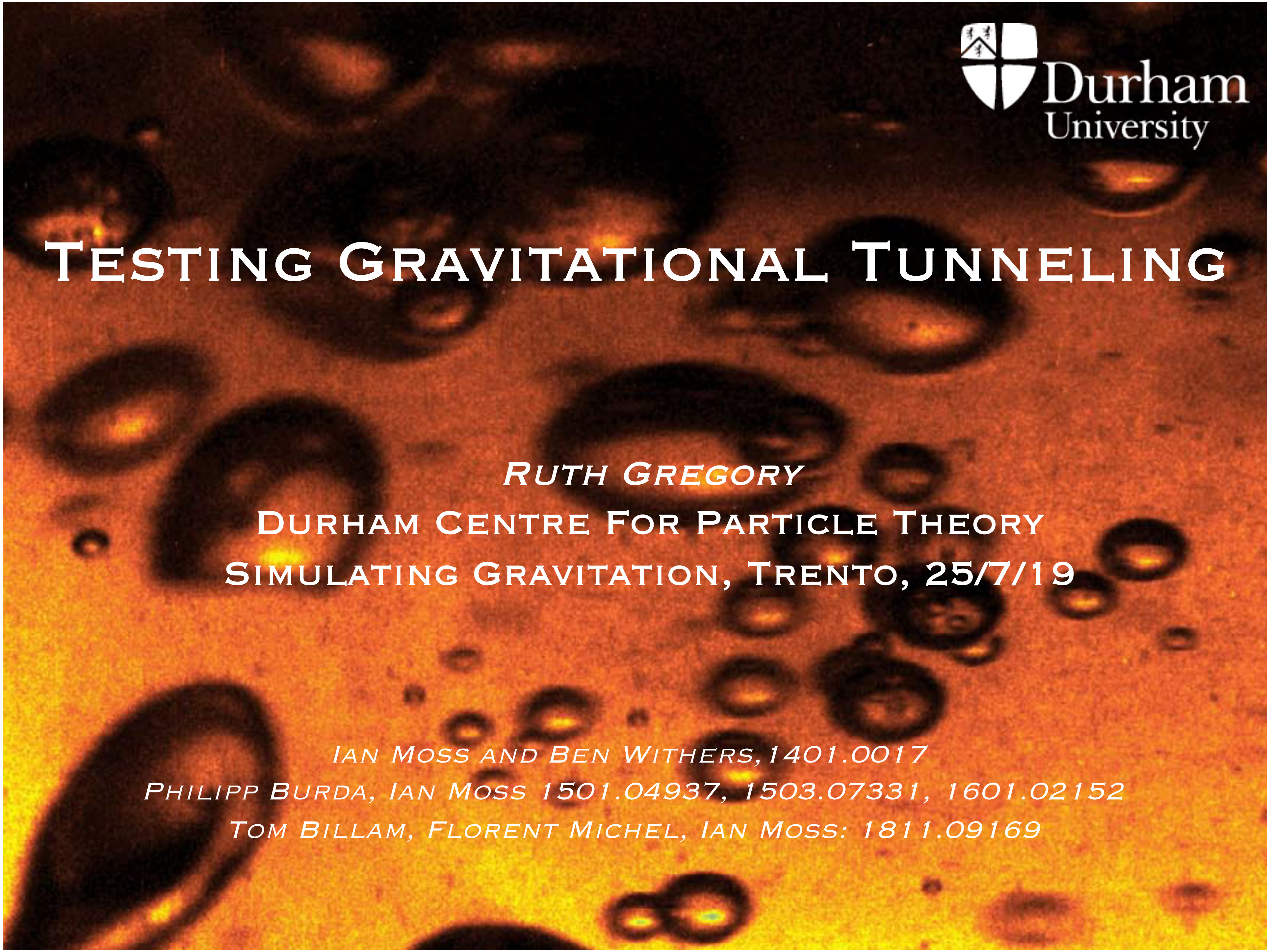


Durham University

# Testing Gravitational Tunneling


*Ruth Gregory*

Durham Centre For Particle Theory


Simulating Gravitation, Trento, 25/7/19

*Ian Moss and Ben Withers,1401.0017*

*Philipp Burda, Ian Moss 1501.04937, 1503.07331, 1601.02152*

*Tom Billam, Florent Michel, Ian Moss: 1811.09169*

# Much ado about Nothing!

How do we define the “vacuum”?

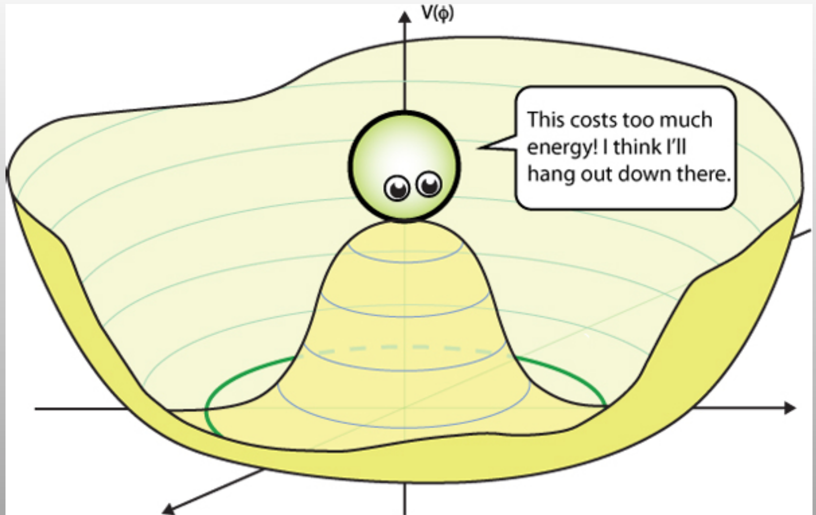
Is it simply “empty space”?

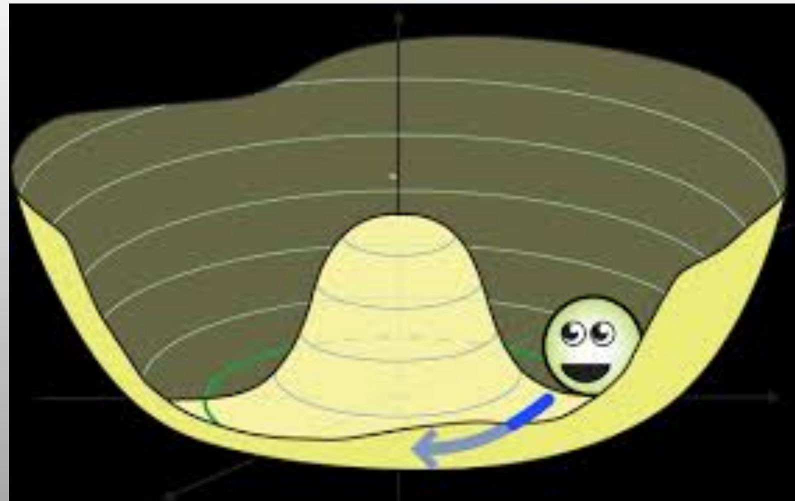
Can we tell?

Is it stable?

# The Higgs Mechanism

Spontaneous symmetry breaking is central to the Standard Model. The Higgs defines our vacuum, and determines the masses of the SM particles. Even without quantum fluctuations, the vacuum can be complex and structured. However, we still have the intuitive notion that it is the lowest energy state.

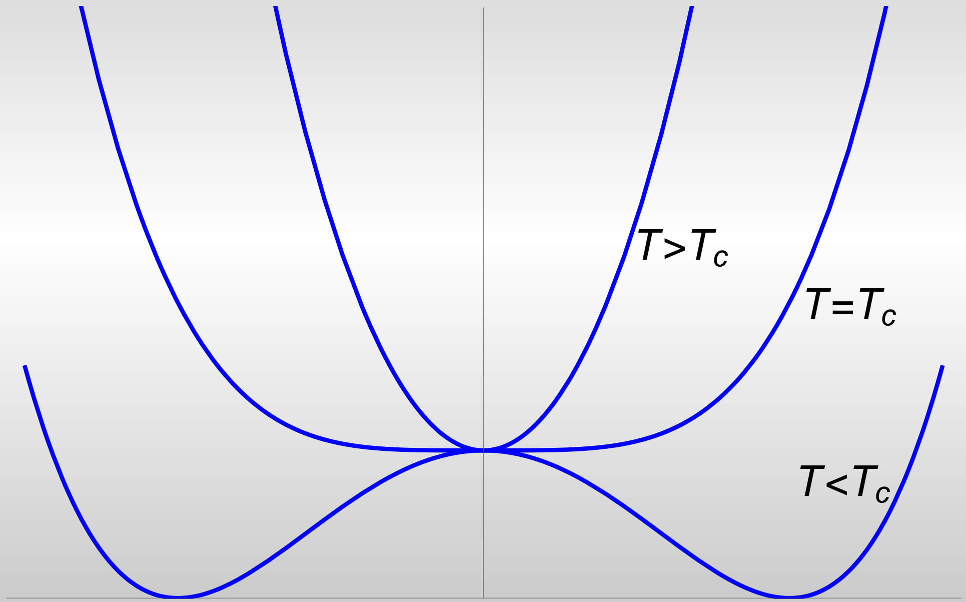


# Phase Transitions

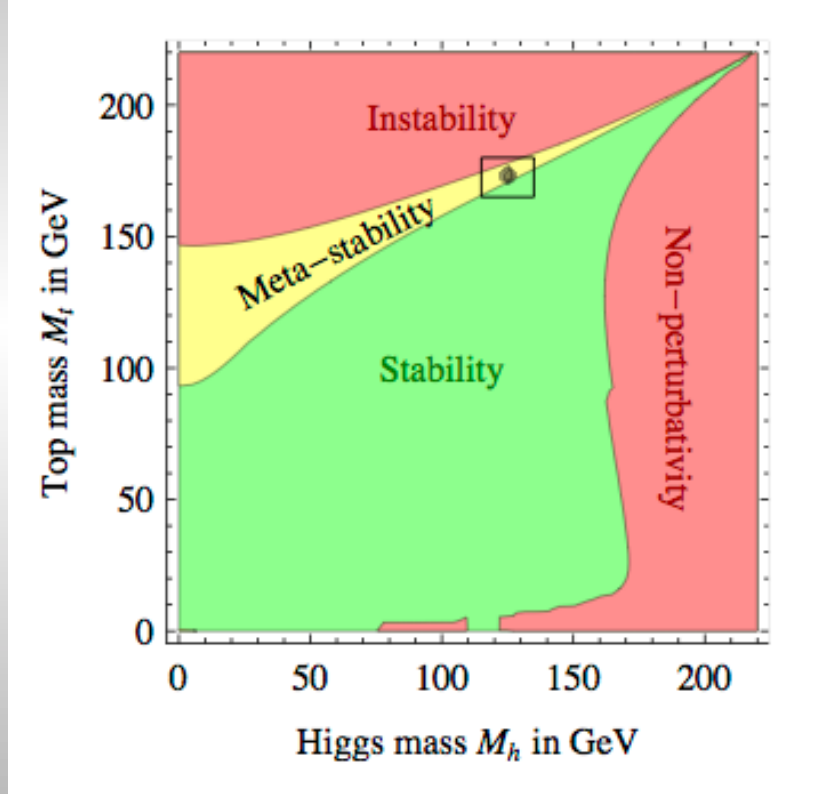
The potential depends on scale – in cosmology we expect phase transitions in the early universe as the universe cools.

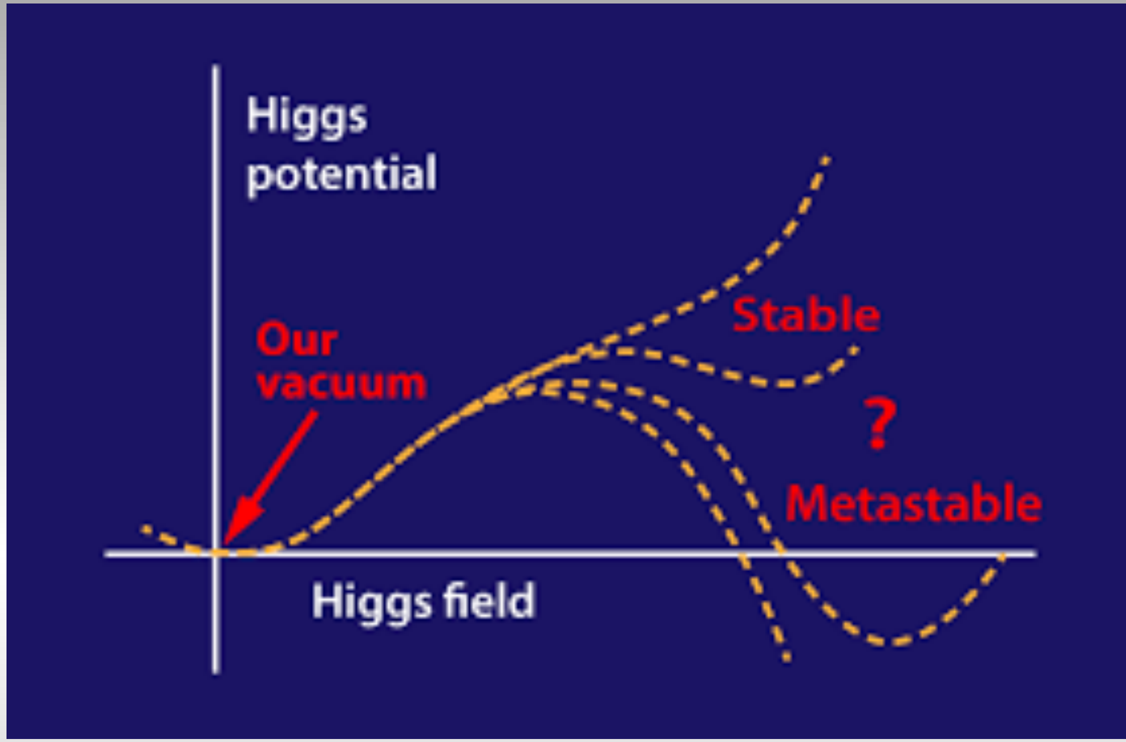
What is surprising is that the universe might have one more phase transition up its sleeve!



# Higgs Vacuum?

We can calculate the energy of the Higgs vacuum at different scales using masses of other fundamental particles (top quark). The LHC tells us that we seem to be in a sweet spot between stability and instability – metastability.

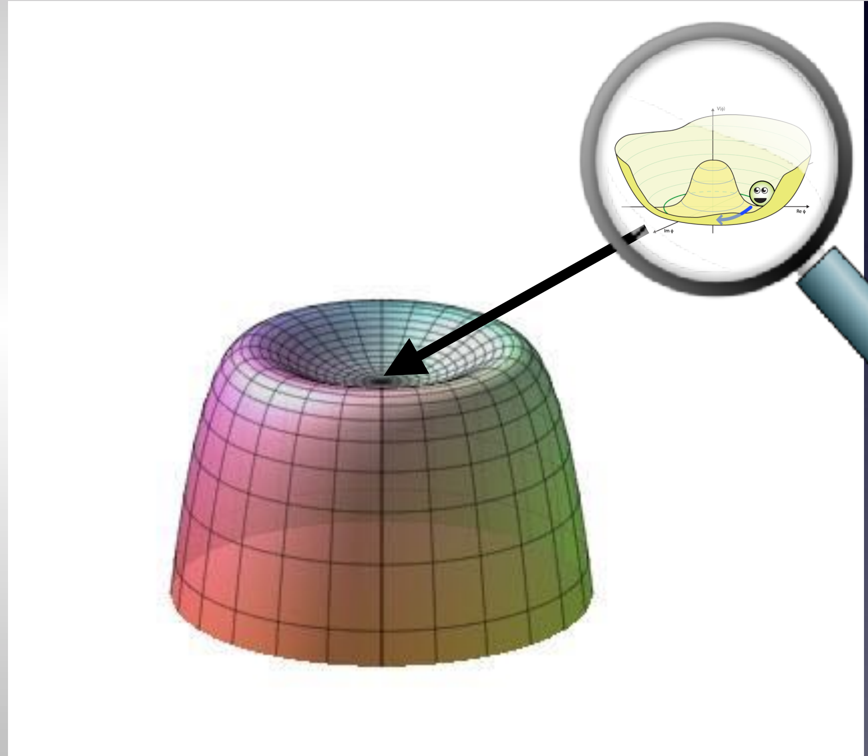





As we run to very high energies, the Higgs self-coupling can become negative.

# Higgs Vacuum?

The bigger picture from the standard model tells us our universe is…

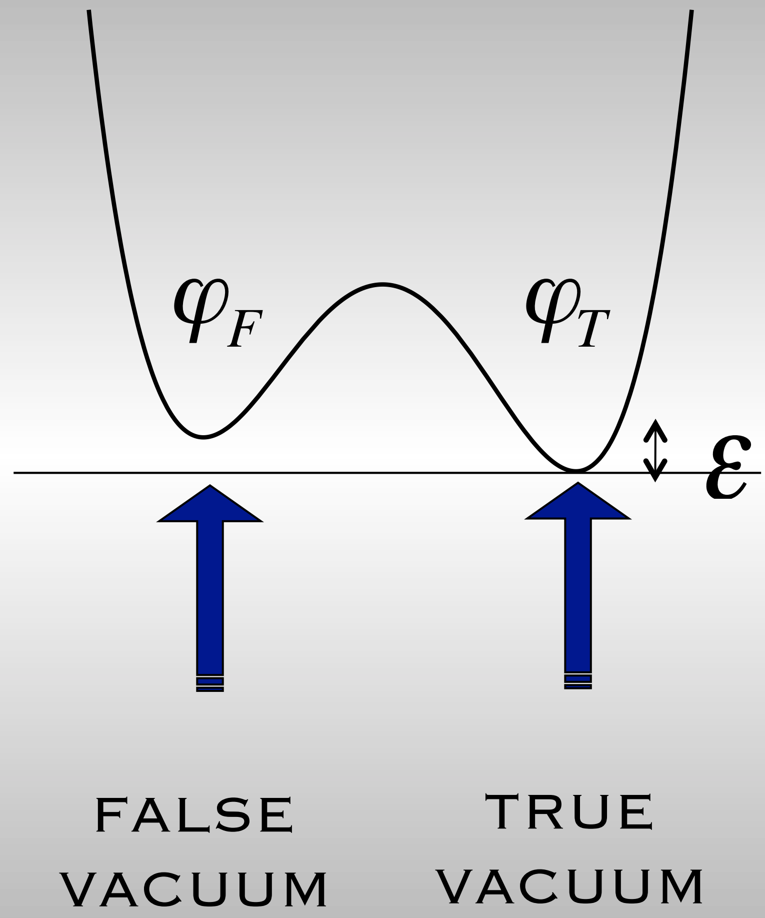


*….not entirely stable!*

# False and True Vacuum

We call this local – not global – minimum a false vacuum, and expect there is a tunneling process to the true minimum / true vacuum.

This will give a first order phase transition, where we tunnel from one local energy minimum to a region with lower overall energy.

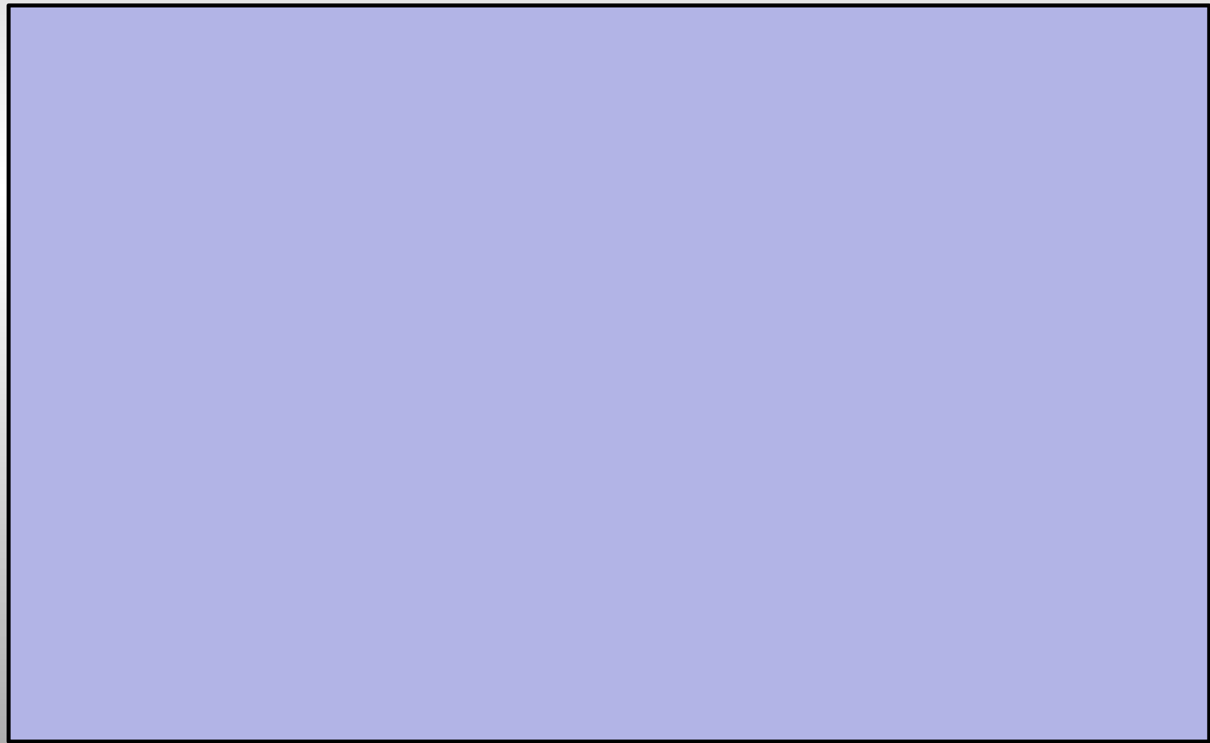


# First Order Phase Transition

A first order phase transition proceeds by bubble nucleation – in this case of true vacuum within false. This is described by quantum mechanical tunnelling, and was explored by Coleman and collaborators in the 70's and 80's.

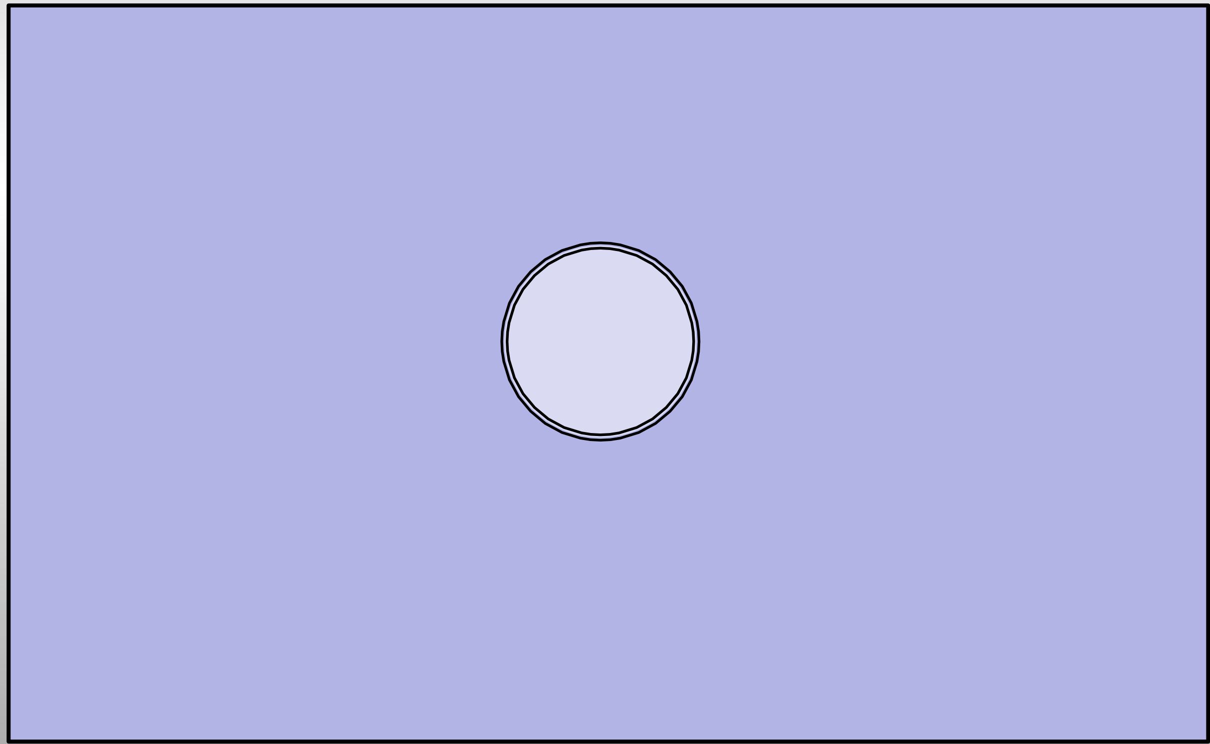
# Coleman, 1977

Via quantum uncertainty, a bubble of true vacuum suddenly appears in the false vacuum



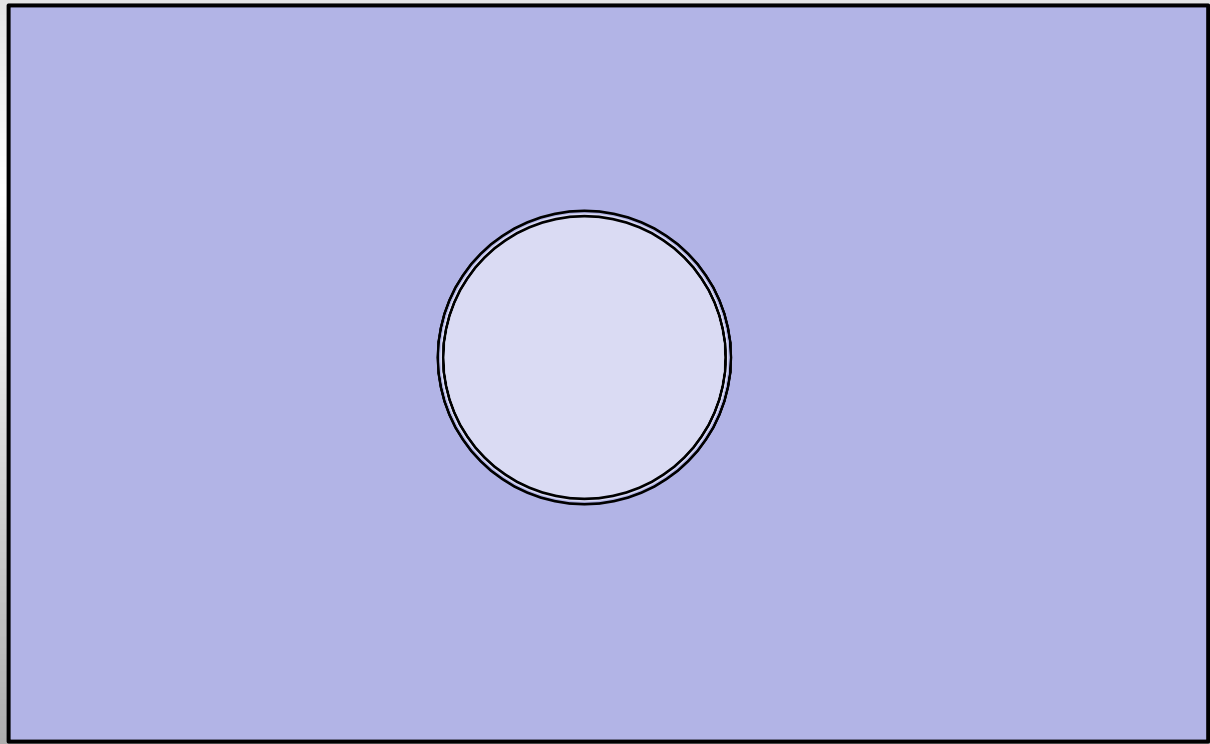
# Coleman, 1977

Via quantum uncertainty, a bubble of true vacuum suddenly appears in the false vacuum,



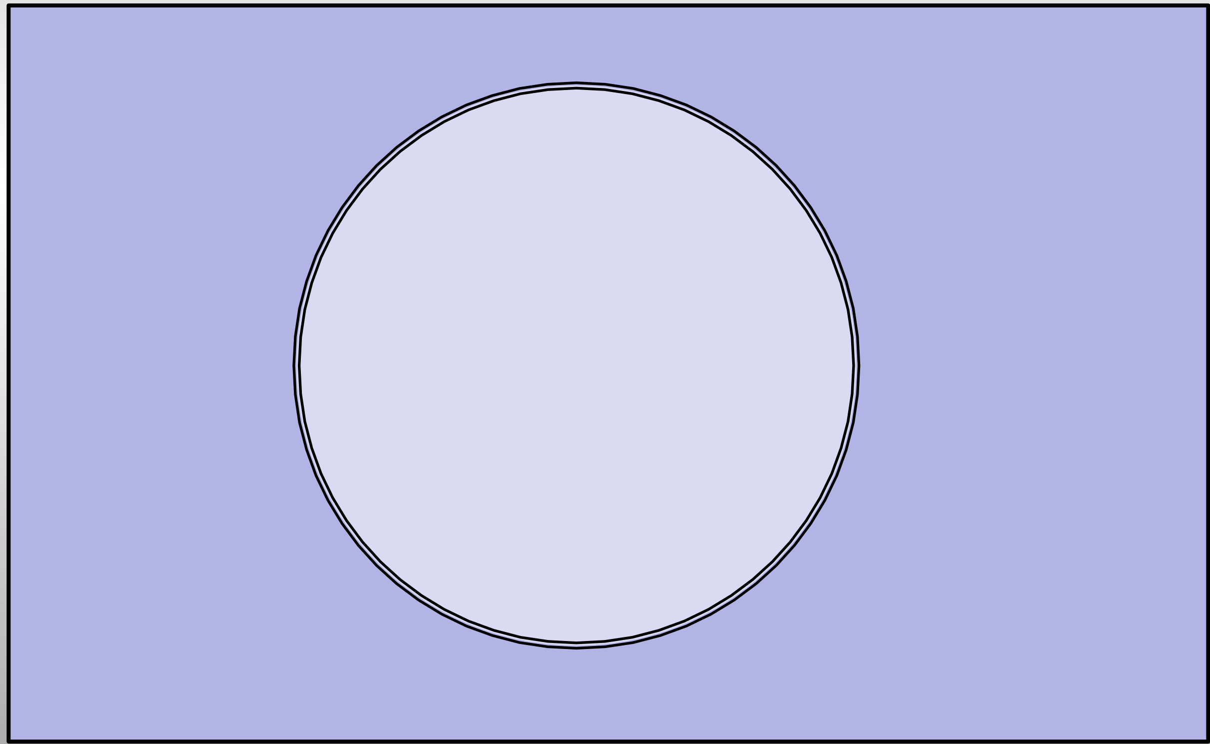
# Coleman, 1977

Via quantum uncertainty, a bubble of true vacuum suddenly appears in the false vacuum, then



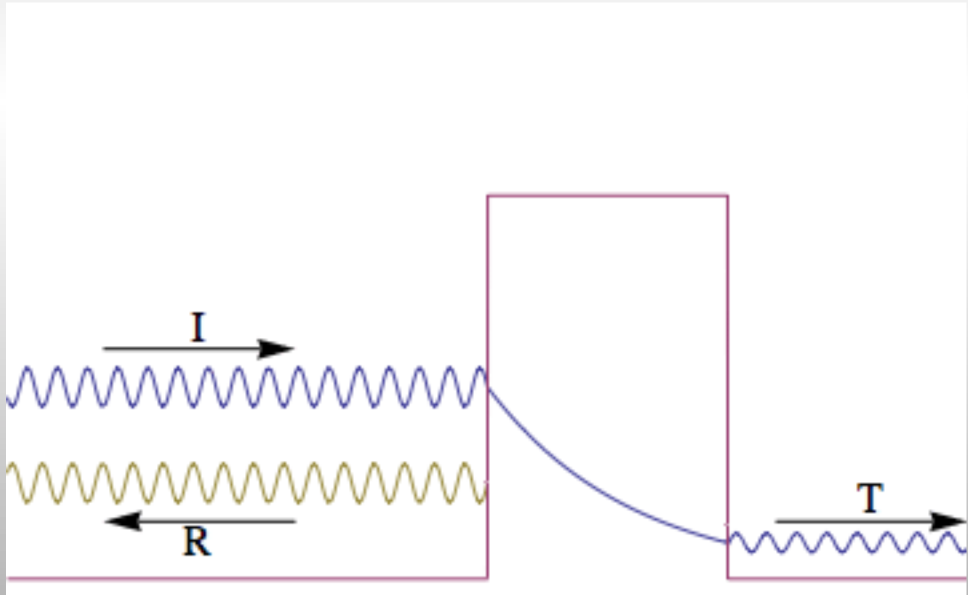
# Coleman, 1977

Via quantum uncertainty, a bubble of true vacuum suddenly appears in the false vacuum, then expands.



# How to Calculate?

First meet tunneling in the Schrodinger equation. Standard 1+1 Schrodinger tunneling exactly soluble. Recall tunnelling probabilities exponentially suppressed.



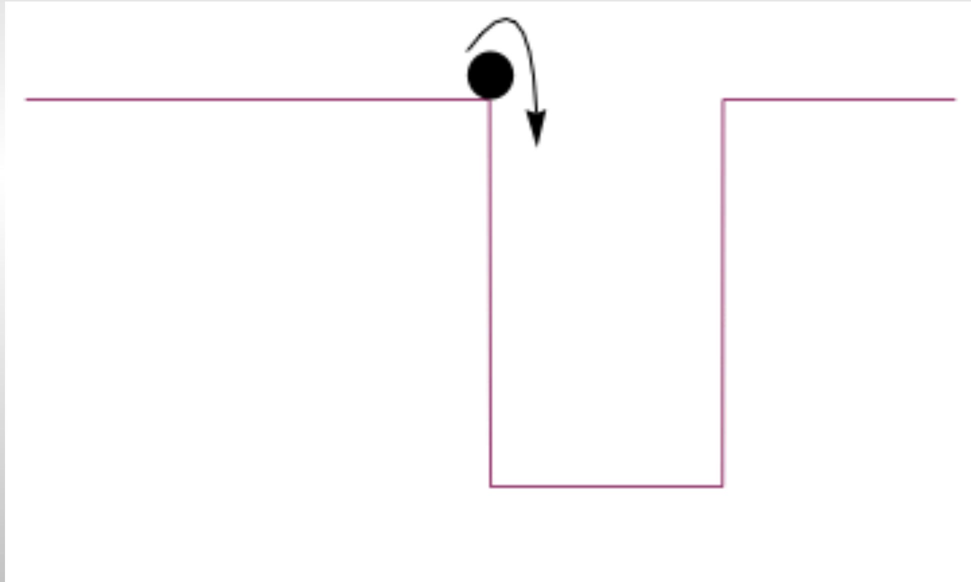

$$|T|^2 = \frac{1}{1 + \frac{V_0^2 \sinh^2 \Omega d}{4E(V_0 - E)}} \approx e^{-2\Omega d}$$

$$\Omega^2 = \frac{2m}{\hbar^2}(V_0 - E)$$

$$\Omega d = \frac{1}{\hbar}\int_0^d \sqrt{2m(V_0 - E)}dx$$

# Euclidean Perspective

A simple and intuitive way of extracting this leading order behaviour is to take “classical” motion in Euclidean time:



$$\boxed{t \to i\tau}$$

$$\frac{1}{2}\left(\frac{dx}{d\tau}\right)^2 = \Delta V$$

$$\int \sqrt{2\Delta V}\,dx = \int 2\Delta V\,d\tau = \int \left(\Delta V + \frac{1}{2}\left(\frac{dx}{d\tau}\right)^2\right) d\tau = \int L_E\,d\tau$$

# Euclidean 'Motion'

For more general potentials, this gives an intuitive visualisation of the tunneling amplitude calculation.
The particle rolls from the (now) unstable point to the "exit" and back again – a "bounce".

In QFT, we construct instantons as solutions to the Euclidean equations of motion. Can think of this by analogy to Schrodinger problem, or via functional Schrodinger approach.

# Goldilocks Bubble

Back to Coleman's bubble: If a bubble fluctuates into existence, we gain energy from moving to true vacuum, but the bubble wall costs energy.

Too small and the bubble has too much surface area – recollapses.

Too large and it is too expensive to form.

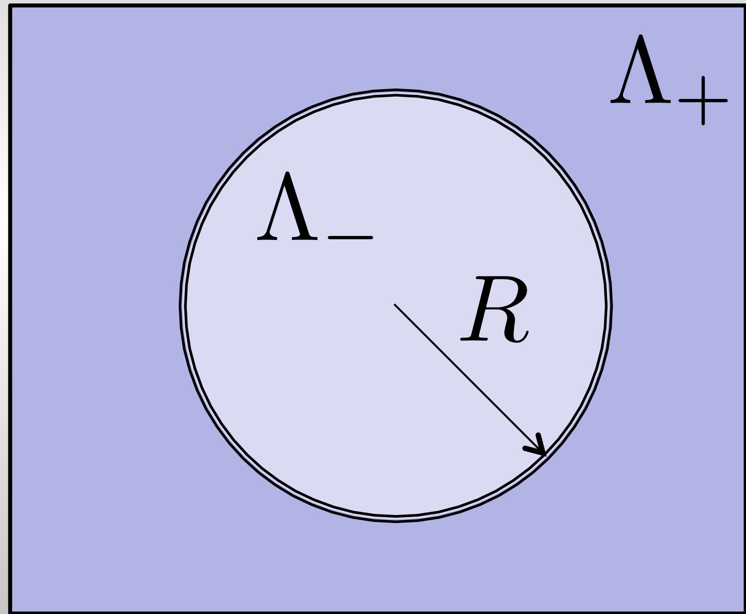
"Just Right" means the bubble will not recollapse, but is still "cheap enough" to form.

# Euclidean Action

This corresponds beautifully to the Euclidean calculation of the tunneling solution: "The Bounce"

ENERGY COST $\sigma \times 2\pi^2 R^3$

ENERGY GAIN $\varepsilon \times \pi^2 R^4/2$




Solution stationary wrt R,

$$\Rightarrow R = 3\sigma/\varepsilon$$

$$B = \frac{\pi^2 R^3}{2}(-\sigma + \varepsilon R) \sim \frac{27\pi^2}{2}\frac{\sigma^4}{\varepsilon^3}$$

# Coleman Bounce

This gives us the bubble radius, and the amplitude for the decay – backed up by full field theory calculations.

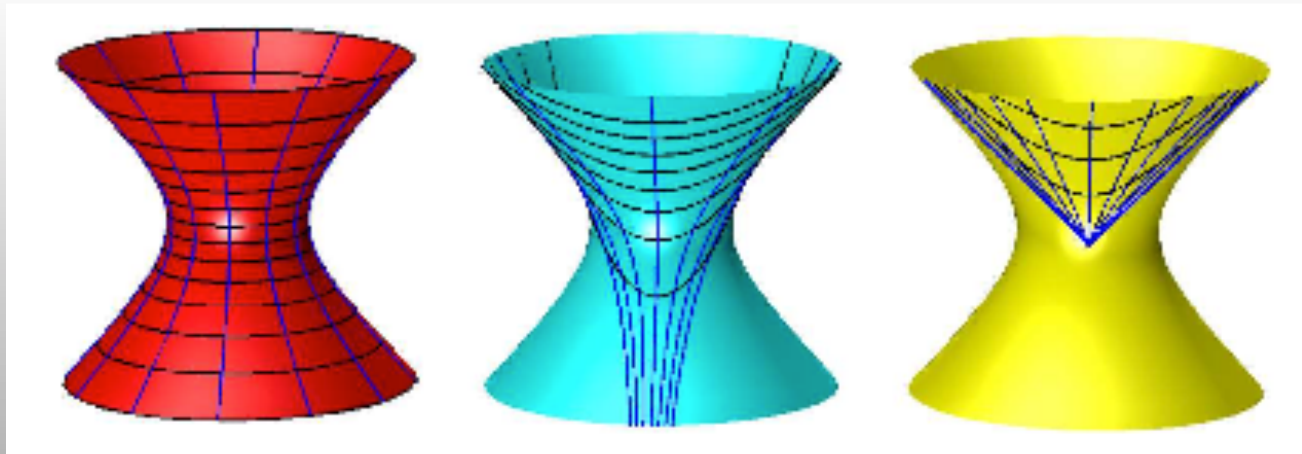
$$B = \frac{\pi^2 R^3}{2}(-\sigma + \varepsilon R) \sim \frac{27\pi^2}{2}\frac{\sigma^4}{\varepsilon^3}$$

Tunneling amplitude:

$$\mathcal{P} \sim e^{-\mathcal{B}/\hbar}$$

# Gravity and the Vacuum

But vacuum energy gravitates – e.g. a positive cosmological constant gives us de Sitter spacetime – so we must add gravity to this picture



# Quantum Gravity?

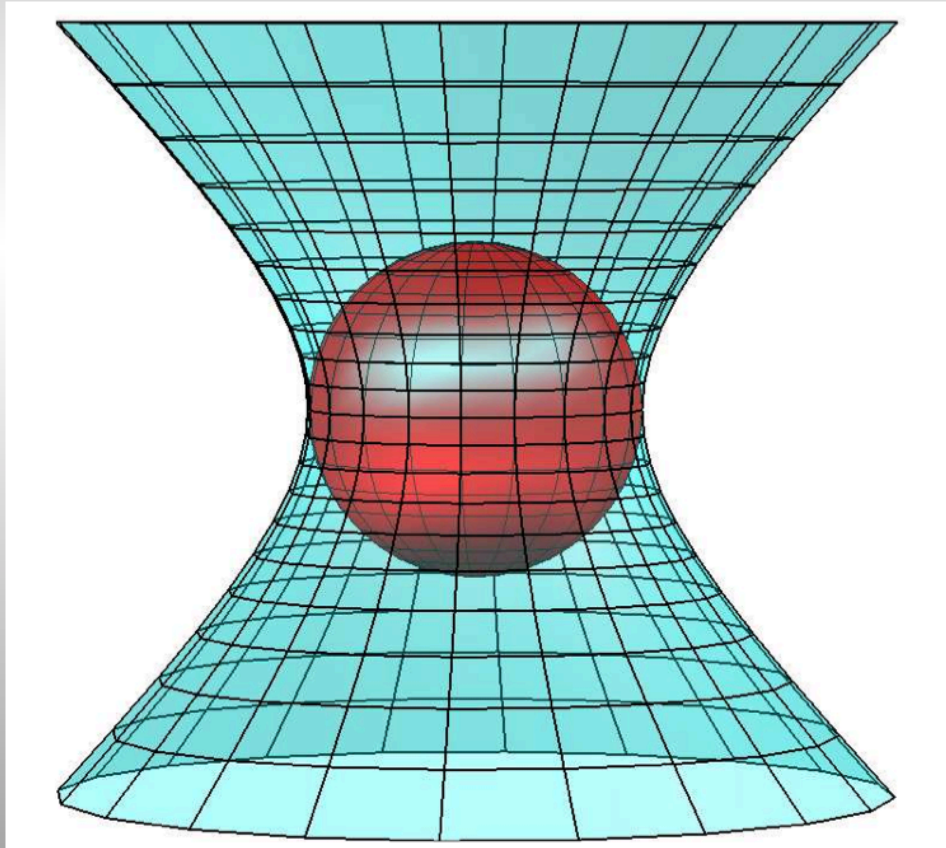
Although we do not have an uncontested theory of quantum gravity, we do have ideas on how quantum effects in gravity behave below the Planck scale.

# Quantum Effects in Gravity

Below the Planck scale, we expect that spacetime is essentially classical, but that gravity can contribute to quantum effects through the wave functions of fields, and through the back-reaction of quantum fields on the spacetime.

We use this in black hole thermodynamics, cosmological perturbation theory, and for non-perturbative solutions in field theory, this method is particularly unambiguous, but can we test these ideas in a broader sense?

De Sitter spacetime has a Lorentzian (real time) and Euclidean (imaginary time) spacetime. The real time expanding universe looks like a hyperboloid and the Euclidean a sphere:

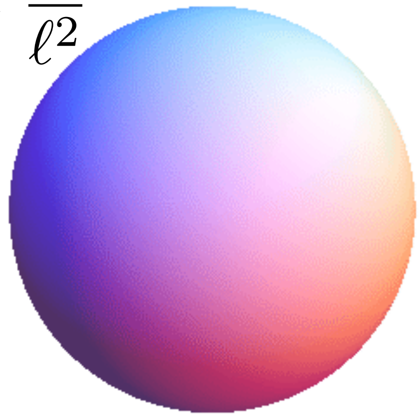


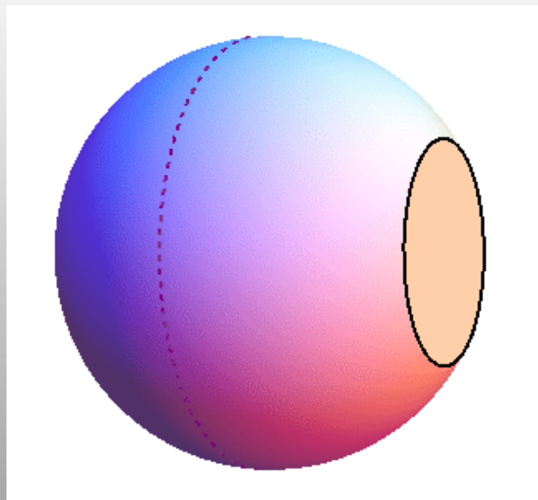
Our instanton must cut the sphere and replace it with flat space (true vacuum).

# Coleman de Luccia (CDL)

Coleman and de Luccia showed how to do this with a bubble wall: Euclidean de Sitter space is a sphere, of radius $\ell$ related to the cosmological constant. The true vacuum has zero cosmological constant, so must be flat.

$$\Lambda = \frac{3}{\ell^2}$$





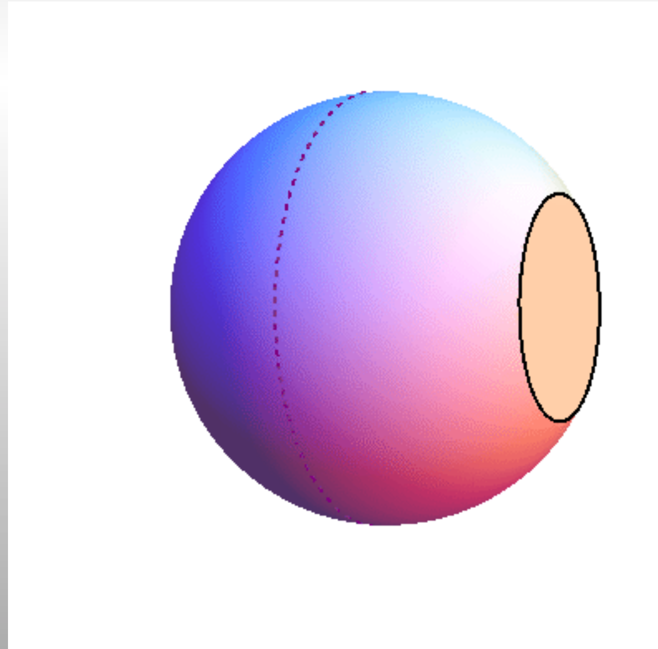
The bounce looks like a truncated sphere.

*Coleman and de Luccia, PRD21 3305 (1980)*

# Goldilocks with Gravity

We can play the same “Goldilocks bubble” game – finding the cost of making this truncated sphere, but adding in the effect of gravity.

$$\mathcal{B}(R) = \frac{4}{3}\pi^2\varepsilon\ell^4\left[1-\left(1-\frac{R^2}{\ell^2}\right)^{\frac{3}{2}}\right] - 2\pi^2\varepsilon\ell^2R^2 + 2\pi^2\sigma R^3$$



# CDL Action

Once again, too small a bubble will recollapse, and large bubbles are harder to make, so there is a “just right” bubble that corresponds to a solution of the Euclidean Einstein equations that we can find either numerically with the full field theory, or analytically if we take our bubble wall to be thin, and we can find our instanton action.

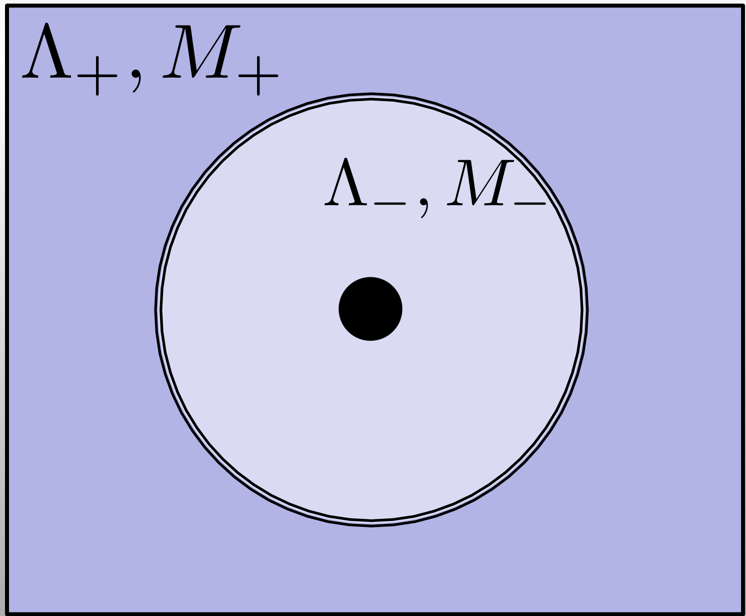
$$\begin{aligned}\mathcal{B} &= -\frac{\Lambda}{8\pi G}\int_{\mathrm{int}} d^4x\sqrt{g} - \frac{\sigma}{2}\int_{\mathcal{W}} d^3x\sqrt{h} \\ &= \frac{\pi\ell^2}{4G}(1-\cos\chi_0)^2 = \frac{\pi\ell^2}{G}\frac{16\bar{\sigma}^4\ell^4}{(1+4\bar{\sigma}^2\ell^2)^2}\end{aligned}$$

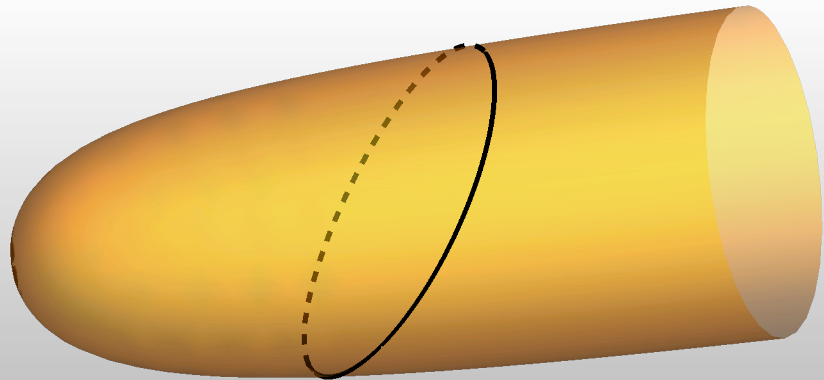
# Not Entirely Stable?

But these calculations are very ideal – the universe is empty and featureless – what if we throw in a little impurity?

# Tweaking CDL

A black hole is an inhomogeneity, and also exactly soluble:



*RG, Moss & Withers, 1401.0017*

# Goldilocks Black Hole Bubbles

- The bubble with a black hole inside, can have a different mass term outside (seed).
- The solution in general depends on time, but for each seed mass there is a unique bubble with lowest action.
- For small seed masses this is time, but the bubble has no black hole inside it – no remnant black hole.

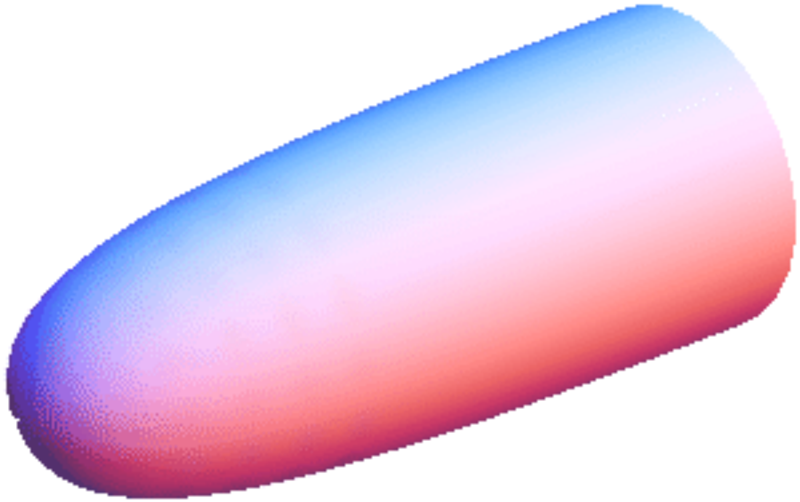
- For larger seed masses the bubble does not depend on Euclidean time, and has a remnant black hole.

This last case is the relevant one – the action is the difference in entropy (area) between the seed and remnant black holes!

# Technical Aside:

## Euclidean Black Holes

In Euclidean Schwarzschild, to make the black hole horizon regular, we must have $\tau$ periodic. This “explains” black hole temperature, but also sets a specific value, $8\pi GM$.



$$ds^2 = \left(1 - \frac{2GM}{r}\right) d\tau^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\Omega_{II}^2$$

$$\sim \rho^2 d\left(\frac{\tau}{4GM}\right)^2 + d\rho^2 + (2GM)^2 d\Omega_{II}^2$$

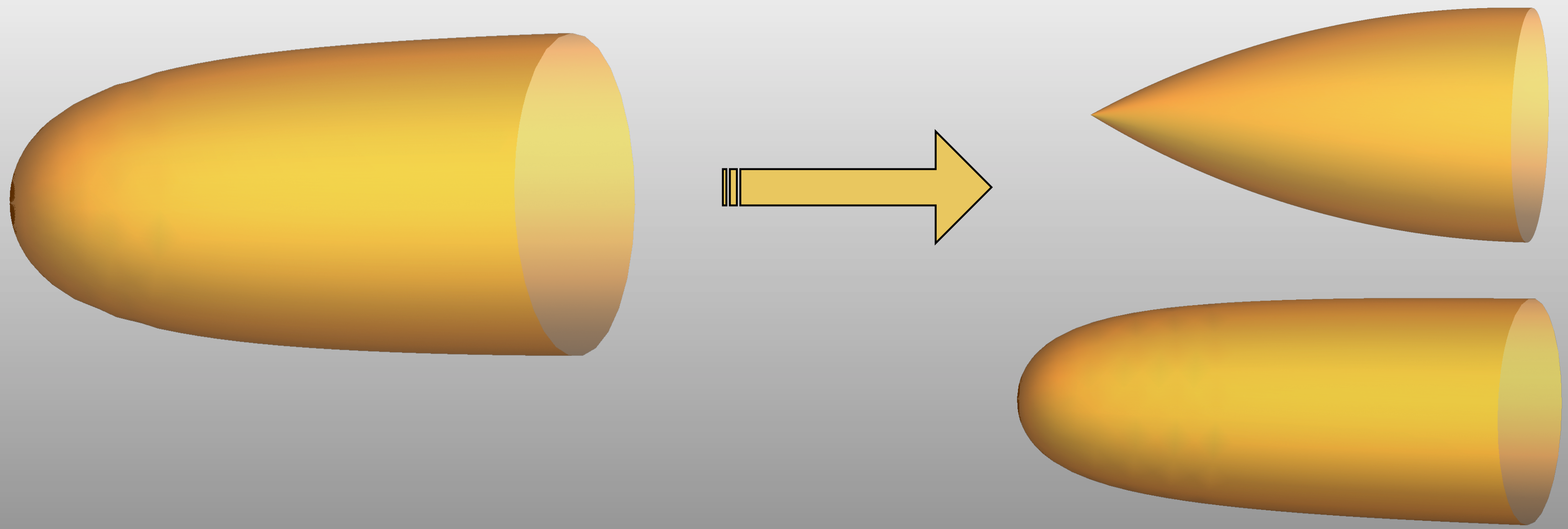
$$\rho^2 = 8GM(r - 2GM) \qquad \tau \sim \tau + 8\pi GM$$

# Technical Aside:

## Conical Deficits

For different seed and remnant masses the periodicity is different – we need to deal with conical deficits. This technicality is **crucial** to the calculation, and give a much lower instanton action.

To subtract off the false vacuum background, we must shrink the time circles to fit



# Black Hole Bounces

Balance of action changes because of periodic time:

$$B \sim \sigma \times 4\pi R^2 L - \varepsilon \times \frac{4}{3}\pi R^3 L$$

$$R \sim 2\sigma/\varepsilon$$

$$B \sim \frac{\sigma^3}{\varepsilon^2} L$$

The result is that the action is the difference in entropy of the seed and remnant black hole masses:

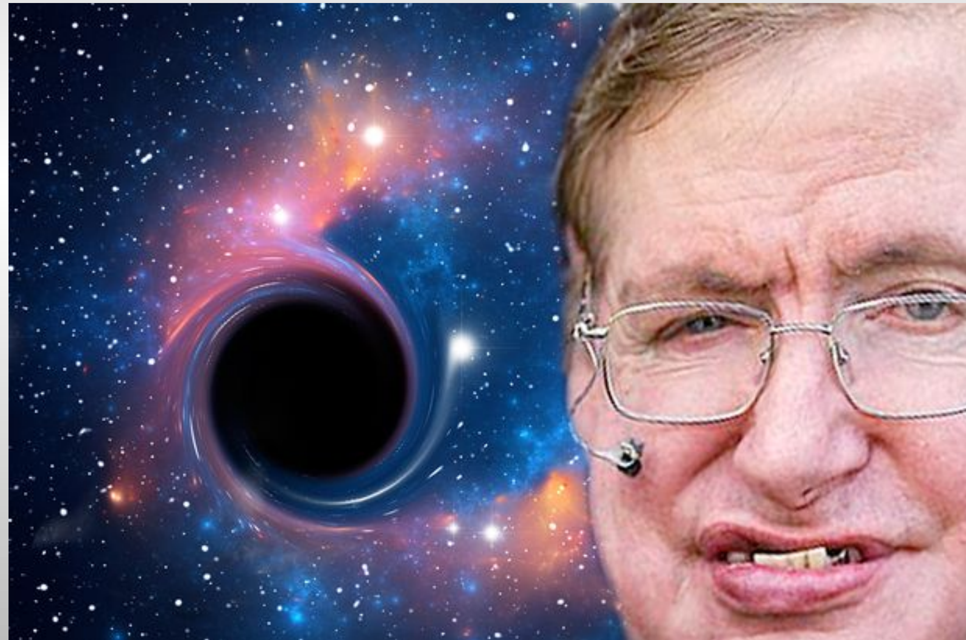
$$B \sim \mathcal{A}_+ - \mathcal{A}_-$$

Seeded tunneling is much more likely than CDL!

# The fate of the black hole?

Vacuum decay is not all that can happen! Hawking tells us that black holes are black bodies, and radiate:

$$T_H = \frac{\hbar c^3}{8\pi G M k_B}$$



So we must compare evaporation rate to tunneling half-life.

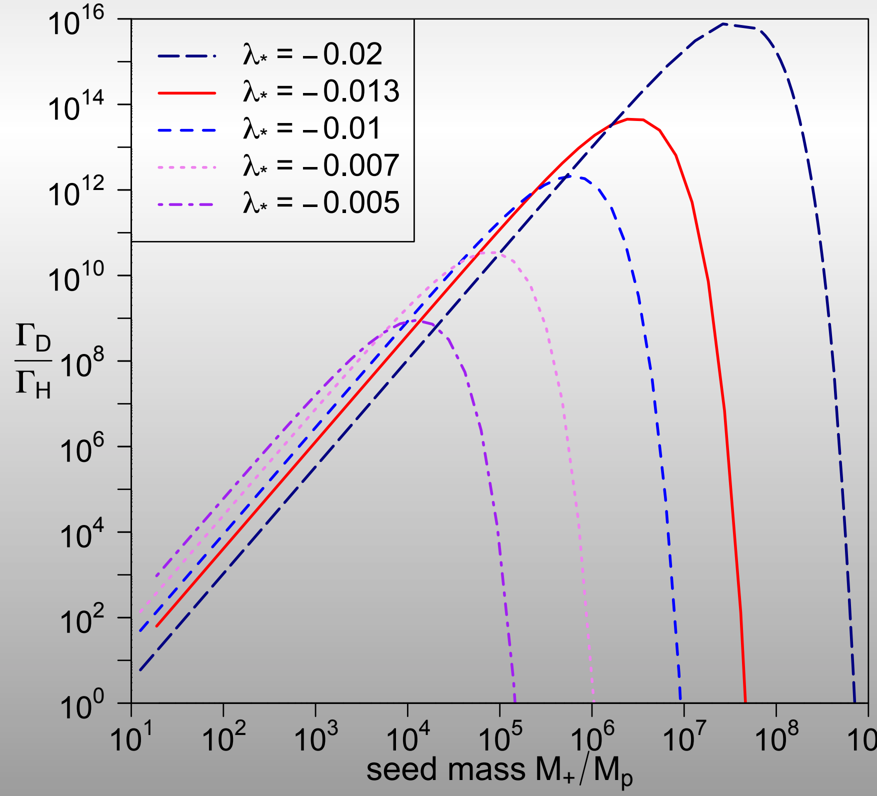
# Decay of the Universe

We compare the seeded tunneling rate to the rate of evaporation for primordial black holes:

*Page, PRD-76*

$$\Gamma_H \approx 3.6 \times 10^{-4}(G^2 M_*^3)^{-1}$$

And find that a PBH will result in Higgs vacuum decay!




*Burda, RG, Moss, 1501.04937, 1503.07331,1601.02152*

# Why Analog Counterpart?

- The Euclidean method is a tool – but how much does it capture of the real process? Should we be trying other techniques?
- QM tunnelling well tested, but QFT tunnelling is another matter.
- Can we construct analog system that mimics a relativistic (or not!) metastable vacuum?
- And can we test seeded tunnelling?